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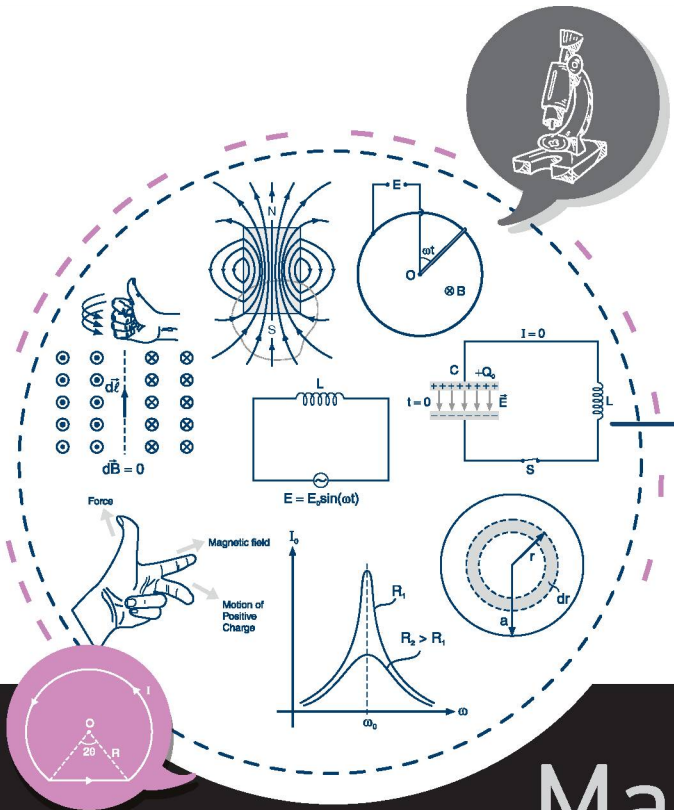

JEE ADVANCED PHYSICS

Magnetic Effects of Current & Electromagnetic Induction

Pearson

Rahul Sardana

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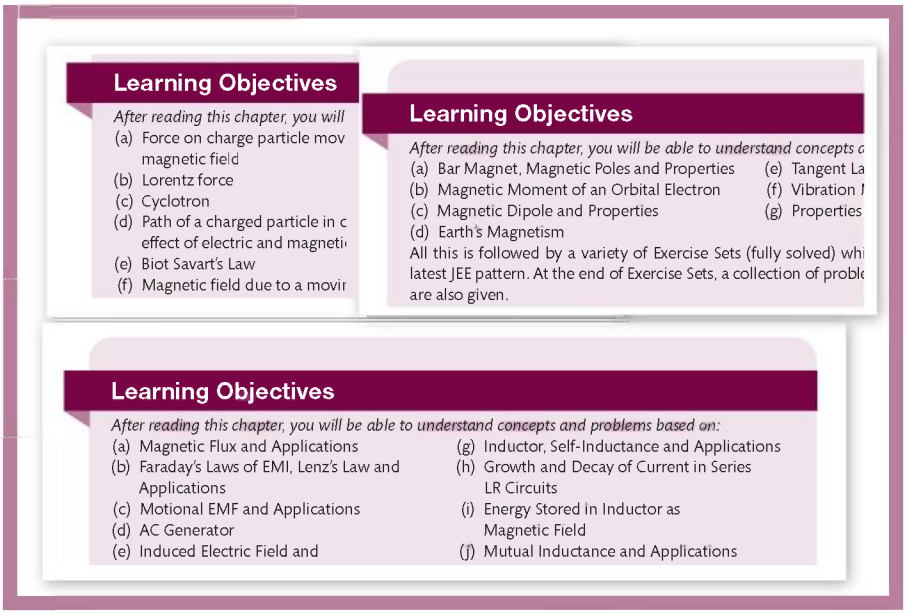
CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



Learning Objectives

After reading this chapter, you will

(a) Force on charge particle mov
magnetic field
(b) Lorentz force
(c) Cyclotron
(d) Path of a charged particle in c
effect of electric and magneti
(e) Biot Savart's Law
(f) Magnetic field due to a movi

Learning Objectives

After reading this chapter, you will be able to understand concepts c

(a) Bar Magnet, Magnetic Poles and Properties
(b) Magnetic Moment of an Orbital Electron
(c) Magnetic Dipole and Properties
(d) Earth's Magnetism
(e) Tangent La
(f) Vibration
(g) Properties

All this is followed by a variety of Exercise Sets (fully solved) whi
latest JEE pattern. At the end of Exercise Sets, a collection of probl
are also given.

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Magnetic Flux and Applications
(b) Faraday's Laws of EMI, Lenz's Law and Applications
(c) Motional EMF and Applications
(d) AC Generator
(e) Induced Electric Field and
(g) Inductor, Self-Inductance and Applications
(h) Growth and Decay of Current in Series LR Circuits
(i) Energy Stored in Inductor as Magnetic Field
(j) Mutual Inductance and Applications

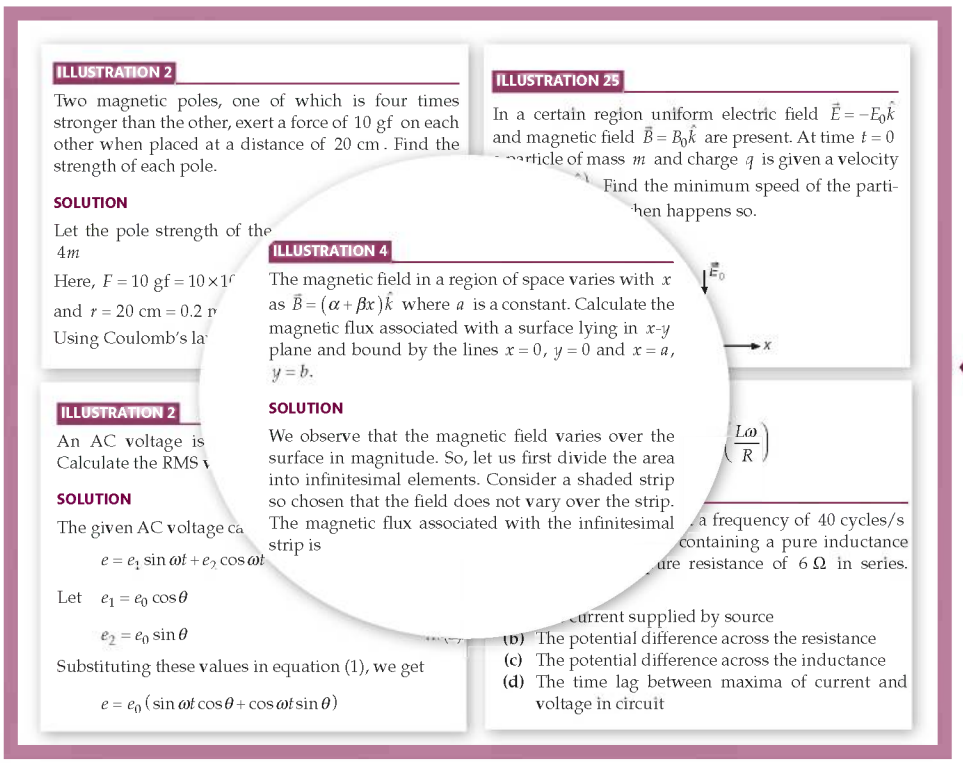


ILLUSTRATION 2

Two magnetic poles, one of which is four times stronger than the other, exert a force of 10 gf on each other when placed at a distance of 20 cm. Find the strength of each pole.

SOLUTION

Let the pole strength of the
$4m$

Here, $F = 10$ gf $= 10 \times 1$

and $r = 20$ cm $= 0.2$ r

Using Coulomb's la

ILLUSTRATION 2

An AC voltage is
Calculate the RMS v

SOLUTION

The given AC voltage ca

$$e = e_1 \sin \omega t + e_2 \cos \omega t$$

Let $e_1 = e_0 \cos\theta$

$e_2 = e_0 \sin\theta$

Substituting these values in equation (1), we get

$$e = e_0 (\sin \omega t \cos\theta + \cos \omega t \sin\theta)$$

ILLUSTRATION 4

The magnetic field in a region of space varies with x as $\vec{B} = (\alpha + \beta x)\hat{k}$ where a is a constant. Calculate the magnetic flux associated with a surface lying in x-y plane and bound by the lines $x = 0$, $y = 0$ and $x = a$, $y = b$.

SOLUTION

We observe that the magnetic field varies over the surface in magnitude. So, let us first divide the area into infinitesimal elements. Consider a shaded strip so chosen that the field does not vary over the strip. The magnetic flux associated with the infinitesimal strip is

ILLUSTRATION 25

In a certain region uniform electric field $\vec{E} = -E_0\hat{k}$ and magnetic field $\vec{B} = B_0\hat{k}$ are present. At time $t = 0$ particle of mass m and charge q is given a velocity
Find the minimum speed of the parti-
hen happens so.

$\vec{E}_0$

x

$\left(\frac{L\omega}{R}\right)$

a frequency of 40 cycles/s
containing a pure inductance
ure resistance of 6 Ω in series.

current supplied by source
(b) The potential difference across the resistance
(c) The potential difference across the inductance
(d) The time lag between maxima of current and voltage in circuit

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

Test Your Concepts-I

Based on Force and Fleming's Left Hand Rule

(Solutions on page H.3)

1. Can you find the dimensional formula for the ratio $\frac{E}{B}$, where E denotes the electric field and B denotes the magnetic field?
2. If $\vec{F}$ denotes the magnetic force acting on a charge q entering a field $\vec{B}$ with a velocity $\vec{v}$, then find
   (a) $\vec{v}\cdot(\vec{v}\times\vec{B})$
   the particle is found to be, $\vec{a}=\left(x\hat{i}+4\hat{j}+2\hat{k}\right)\ \text{ms}^{-2}$. Find the value of x.
4. A particle with mass 1.81×10^{-3} kg and a charge of 1.22×10^{-8} C has, at a given instant, a velocity $\vec{v}=\left(3\times10^{4}\ \text{ms}^{-1}\right)\hat{j}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\vec{B}=(1.63\ \text{T})\hat{i}+(0.98\ \text{T})\hat{j}$?

Test Your Concepts-I

Based on Bar Ma

1. The work done in turning a magnet of magnetic moment M by an angle 90° from the meridian is two times the corresponding work done to turn it through an angle of θ. Calculate θ.
2. Calculate the geometric length of a bar magnet that has a magnetic length of 10 cm.
3. The distance between the poles of a horse shoe magnet is 0.1 m and its pole strength is 0.01 Am. Calculate the induction of magnetic field at a point midway between the poles.

Test Your Concepts-I

Base

1. Show that average heat produced during a cycle of ac is same as produced by dc with $I=I_{\text{rms}}$.
2. Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by $V=200\sin(\omega t)+100\sin(3\omega t)+50\sin(5\omega t)$.
3. The current flowing through a resistor R varies as $i=kt$, where k is constant. Calculate the rms current for first 2 seconds.
4. Calculate the RMS value of the voltage (for a cycle)

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

(a) The above results can also be expressed in terms of the specific charge $\left(\frac{q}{m}\right)$, sometimes denoted by α. So, we get $R=\frac{v}{\alpha B}$, $T=\frac{2\pi}{\alpha B}$, $\omega=\alpha B$, $f=\frac{\alpha B}{2\pi}$

(b) As discussed, T, f and ω are independent of the velocity v, however R (radius of curvature) is directly proportional to v. So, for two identical charged particles (having sa... tering at right angles to the field $\vec{B}$ ... v_1 and v_2, where $v_2>v_1$, ...

Problem Solving Techr

(a) If the growth current ... decay current by I_d, then ... from the start, we have

$$I_g+I_d=I_0=\frac{E}{R}$$

(b) Theoretically the current grows from zero to I_0 (or decays from I_0 to zero) when $t\to\infty$, however practically, it is observed that the current grows from zero to I_0 (or decays from I_0 to zero) in five time constants i.e., as $t\to\frac{5L}{R}$.

Problem Solving Technique(s)

(a) Other formulae for magnetic moment M

(i) $M=ni\pi r^2$

(ii) $M=\frac{evr}{2}=\frac{er^2\omega}{2}=\frac{er^2 2\pi f}{2}=\frac{er^2\pi}{T}$

(iii) $M=\frac{eL}{2m}$, where L = Angular Momentum o... the Electron

(iv) $M=n\mu_B$

(b) About 90% of magnetic moment is due to spir... ...n of electrons whereas 10% part is due to ...tion.

...oment of an atom is equa...

Problem Solving Technique(s)

Following points are worth noting regarding the Biot Savart Law.

(a) Magnitude of $d\vec{B}$ is given by,

$$|d\vec{B}|=\frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

...nique(s)

...agram for a pure element (e.g. ...ce or an inductance) either of ...can be plotted along x-axis ...but ...r diagrams for a combination of elements is drawn then the quantity which remains constant for the combination must be plotted along x-axis So, we observe that

(a) in series circuits current has to be plotted along

Conceptual Note(s)

If the point is at a large distance from the strip i.e., $b \gg w$

$$\log_e\left[1+\frac{w}{b}\right]=\frac{w}{b}-\frac{w^2}{2b^2}+\ldots\ldots=\frac{w}{b}$$

$$\Rightarrow\ B=\frac{\mu_0}{4\pi}\frac{2I}{w}\times\frac{w}{b}=\frac{\mu_0 I}{2\pi a}$$

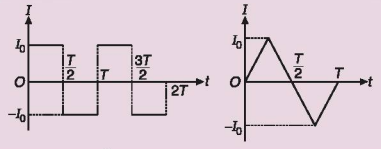
So, we observe that for a distant point, the strip just behaves like a current carrying wire which justifies the above result obtained by approximation.

Conceptual Note(s)

It was thought that, like two types of electric charges, there are two types of magnetic charges (or poles). However, every effort to find such magnetic charges, or to isolate the poles of a magnet have failed. If we break a magnet into two parts, the two pieces become two new magnets, each having both N-pole and S-pole. Even if we break up a magnet into the electrons and nuclei, it will be found that even these are magnetic dipoles. Thus, **a magnetic monopole does not exist.**

Conceptual Note(s)

An alternating current change its direction of flow periodically. For a half cycle, it flows in one direction and for next half cycle, it flows in opposite direction. The following graphs can also be a good representation of alternating currents



Conceptual Note(s)

(a) If $\vec{\ell}$ is a vector directed along the direction of induced current, then a general notation for induced emf ξ is

$$\xi=\vec{B}\cdot(\vec{\ell}\times\vec{v})=(\vec{v}\times\vec{B})\cdot\vec{\ell}$$

So, for emf or induced current to exist we must make sure that $\vec{B}$, $\vec{\ell}$ and $\vec{v}$ must never be coplanar.

(b) In general, the motional emf around a closed conducting loop can also be written as

$$\xi=\oint(\vec{v}\times\vec{B})\cdot d\vec{\ell}=\oint\vec{B}\cdot(d\vec{\ell}\times\vec{v})$$

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

SOLVED PROBLEMS

PROBLEM 1

A particle having charge $-q$ and mass $m = 2.58 \times 10^{-15}$ kg is travelling through a region containing a uniform magnetic field $\vec{B}=-(0.12\ \text{T})\hat{k}$. At a particular instant of time the velocity of the particle is $\vec{v}=(1.05\times10^6\ \text{ms}^{-1})(-3\hat{i}+4\hat{j}+12\hat{k})$ and the force $\vec{F}$ on the particle has a magnitude of 1.25 N.

(a) Calculate the charge q.
(b) Calculate the acceleration $\vec{a}$ of the particle.
(c) Explain why the path of the particle is a helix, and determine the radius of curvature R of the

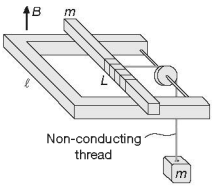
(b) Since, $\Sigma\vec{F}=m\vec{a}$, so $\vec{a}=\frac{\vec{F}}{m}$.

From equation (1), we have

$$\vec{F}=-q(1.26\times10^5)(4\hat{i}+3\hat{j})$$

PROBLEM 2

A pair of parallel horizontal conducting rails of negligible resistance is shorted at one end of it and is fixed on the table as shown in Figure.



A constant magnetic field B exists perpendicular to the table. The distance between the rails is l. An inductor of negligible resistance and mass m can slide on the rails smoothly. The inductor is connected to the mass m by a light inextensible string passing over a light frictionless pulley as shown. Calculate velocity of the rod as a function of displacement form

PROBLEM 3

A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given horizontal component of earth's field = 0.34 G.

SOLUTION

When magnet is placed with its north pole pointing south, then neutral point is obtained on its axial line.

$\Rightarrow\quad B_{\text{axial}} = B_H$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Corre
which ONLY ONE is correct.

1. Two observers moving with
that a point charge produces
the same point A. Their relati
allel to $\vec{r}$, where $\vec{r}$ is the pos

LINKED COMPREHENSION TYPE QUESTION

This section contains Linked Comprehension Type Question
followed by questions. Each question has four choices (A), (
competitiveness there may be a few questions that may hav

Comprehension I

MULTIPLE CORRECT CHOICE TYPE QUESTION

This section contains Multiple Correct Choice Type Question
which ONE OR MORE is/are correct.

1. A co-axial cable consists of a thin inner current carry-
the axis of a hollow current
B_2 be the magnetic
en the conductors and out-
tively
nductors carrying equal cur-

INTEGER/NUMERICAL ANSWER TYPE QUESTI

In this section, the answer to each question is a numerical valu
given in the question(s).

1. In a certain region surrounding the origin of the coor-
dinates, $\vec{B} = 5\times10^{-4}$ T and $\vec{E} = 5\hat{k}$ Vm^{-1}. A proton
enters the fields at the origin with an initial velocity
$\vec{v}_0 = 2.5\times10^{5}\hat{i}$ ms^{-1}. Describe the proton's motion

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having
correct. Each question contains STATEMENT 1 and STATEME

Bubble (A) If both statements are TRUE and STATEMENT
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 i
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 i

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ	ⓣ
C	Ⓟ	Ⓠ	ⓡ	Ⓢ	ⓣ
D	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advaned

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.

ARCHIVE: JEE MAIN

1.

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which

1. [II

Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D),

1. [

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is
STATEM
Bubble (
2 is the
Bubble (

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two colum
in COLUM
ments in C
given state
with ONE
appropriat

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. **[JEE (Advanced) 2018]**
In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uni-

CHAPTER 1: MAGNETIC EFFECTS OF CURRENT

Test Your Concepts-I (Based on Force and Fleming's Left Hand Rule)

1. Since $v = \frac{E}{B}$

   $\Rightarrow \left[\frac{E}{B}\right] = [\text{Velocity}] = LT^{-1}$

2. Since $\vec{F} = q(\vec{v} \times \vec{B})$

   By properties of cross product, we know that

   However

   $\vec{F} = \left(-3.4\times10^{-7}\hat{i} + 7.4\times10^{-7}\hat{j}\right)$ N ...(2)

   From (1) and (2), by equating the components, we get

   $\Rightarrow \quad -3.4\times10^{-7} = -1.25qv_y$

   $\Rightarrow \quad v_y = \frac{-3.4\times10^{-7}}{-(1.25)(-5.6\times10^{-9})} = -48.6 \text{ ms}^{-1}$

   Similarly, we get

Single Correct Choice Type Questions

1. $B = \frac{\mu_0 q}{4\pi}\left(\frac{\vec{v}\times\vec{r}}{r^3}\right)$

   $\vec{v}$ is the velocity of charge with respect to observer. Let the observers A and B have velocities $\vec{v}_A$ and $\vec{v}_B$ and since both see that a point charge produces same magnetic field at

   $\vec{v}_{CA}\times\vec{r} = \vec{v}_{CB}\times\vec{r}$

   where $\vec{v}_{CA}$ is vel

   velocity of charge

   $\Rightarrow \quad (\vec{v}_C - \vec{v}_A)\times\vec{r}$

   $\Rightarrow \quad (\vec{v}_A - \vec{v}_B)\times\vec{r}$

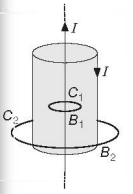
   $\Rightarrow \quad (\vec{v}_A - \vec{v}_B) \| \vec{r}$

   So, the relative ve

   Hence, the correc

Multiple Correct Choice Type Questions

1. We apply Ampere's Law to the co-axial circular loops C_1 and C_2 with corresponding magnetic fields B_1 and B_2 respectively. For C_1 and C_2 we have the OPTION (B) correct. On the other hand, when the currents are in same direction, $B_1 \neq 0$ and $B_2 \neq 0$. Hence, the OPTIONS (C) and (D) are also correct.



and (D) are correct.

Matrix Match/Column Match Type Questions

1. A → (q)
   B → (r)
   C → (s)
   D → (s)

   At (a, a), B due to both conductors is along $+z$ direction. At $(-a, -a)$, B due to both conductors is along $-z$ direction

   At $(a, -a)$ and $(-a, a)$ B due to both conductors is zero.

2. A → (p, q)
   B → (q)

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions at *sardanaeducation@gmail.com*.

I hope this book will nourish you with the concepts involved so that you get a great rank at JEE. PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

You can reach him through email at sardanaeducation@gmail.com.

CHAPTER 1

Magnetic Effects of Current

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

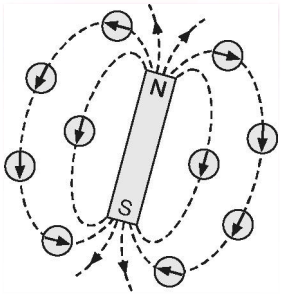
(a) Force on charge particle moving in magnetic field
(b) Lorentz force
(c) Cyclotron
(d) Path of a charged particle in combined effect of electric and magnetic field
(e) Biot Savart's Law
(f) Magnetic field due to a moving charge
(g) Application of Biot Savart's Law and Solenoid
(h) Ampere's Circuital Law (ACL) and Application
(i) Magnetic force on a current carrying conductor placed in magnetic field
(j) Magnetic pressure and magnetic field energy
(k) Magnetic dipole
(l) Gyromagnetic ratio (GMR)
(m) Torque on a current loop placed in uniform field
(n) Magnetic dipole moment
(o) Work done to change orientation of current carrying coil in magnetic field
(p) Moving coil galvanometer
(q) Magnetic force between two straight parallel current carrying wires

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION TO MAGNETIC FIELD AND MOTION OF CHARGED PARTICLES IN MAGNETIC FIELD

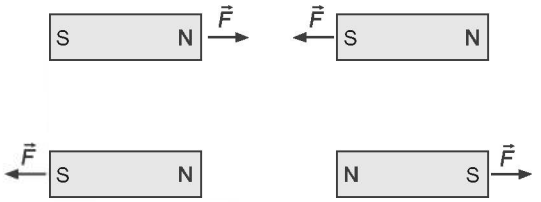
INTRODUCTION

We are aware of the fact that a charged object produces an electric filed $\vec{E}$ at all points in space. In a similar manner, a bar magnet is a source of a magnetic field $\vec{B}$. This can be readily demonstrated by moving a compass near the magnet. The compass needle always aligns itself along the direction of the magnetic field produced by the magnet, as shown in Figure.



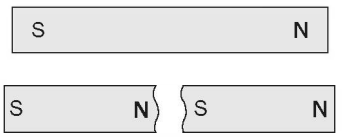
Magnetic field produced by a bar magnet

The bar magnet consists of two poles, which are designated as the north (N) and the south (S). Magnetic fields are strongest at the poles. The magnetic field lines leave from the north pole and enter the south pole. When holding two bar magnets close to each other, the like poles repel each other while the opposite or unlike poles attract.



Magnets attracting and repelling

Unlike electric charges which can be isolated, the two magnetic poles always exist in a pair. Whenever you break a bar magnet, two new bar magnets are obtained, each with a north pole and a south pole as shown.



Magnetic monopoles do not exist in isolation

In other words, "magnetic monopoles do not exist in isolation", although they are of theoretical interest.
How do we define the magnetic field $\vec{B}$?

In the case of an electric field $\vec{E}$, we have already seen that the field is defined as the force per unit charge, given by

$$\vec{E} = \frac{\vec{F}_e}{q}$$

However, due to the absence of magnetic monopoles, $\vec{B}$ must be defined in a different way.

FORCE ON A CHARGED PARTICLE IN MAGNETIC FIELD AND THE DEFINITION OF A MAGNETIC FIELD (B)

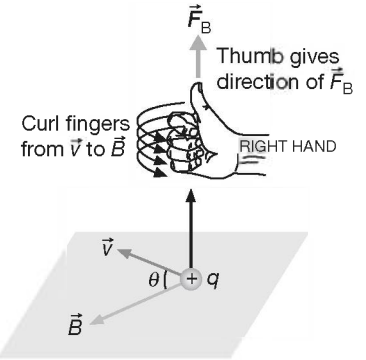
To define the magnetic field at a point, consider a particle of charge q moving with a velocity $\vec{v}$. Experimentally, we have the following observations.

(a) The magnitude of the magnetic force $\vec{F}_B$ or $\vec{F}_m$ exerted on the charged particle is proportional to both q and v.

(b) The magnitude and direction of $\vec{F}$ depends on $\vec{B}$ and $\vec{v}$.

(c) The magnetic force $\vec{F}$ vanishes when $\vec{v}$ is parallel to $\vec{B}$. However, when $\vec{v}$ makes an angle θ with $\vec{B}$, the direction of $\vec{F}$ is normal to the plane containing $\vec{v}$ as well as $\vec{B}$ and the magnitude of $\vec{F}_B$ is proportional to $\sin\theta$.

(d) When the sign of the charge of the particle is changed from positive to negative (or vice versa), the direction of the magnetic force also reverses.



From the above observations we conclude that this magnetic force must be expressed vectorially as

$$\vec{F} = q(v \times \vec{B}) \qquad \ldots(1)$$

The above expression is regarded as the working definition of the magnetic field at a point in space. The magnitude of $\vec{F}_B$ is given by

$$F = |q| vB \sin\theta \qquad \ldots(2)$$

$$\Rightarrow \quad B = \frac{F}{qv\sin\theta}$$

If $q = +1$ coulomb, $v = 1\ \text{ms}^{-1}$, $\theta = 90°$ then

$$B = F(\text{numerically})$$

Hence, magnetic field is numerically equal to the force experienced by a charged particle of +1 coulomb when it enters a magnetic field with a velocity of $1\ \text{ms}^{-1}$ at right angles to the field.
The SI unit of magnetic field is the tesla (T).

$$1\ \text{tesla} = 1\ \text{T} = 1\ \frac{\text{newton}}{\left(\text{coulomb}\,\frac{\text{meter}}{\text{second}}\right)}$$

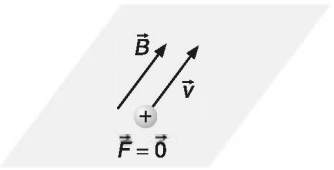
$$\Rightarrow \quad 1\ \text{tesla} = 1\ \frac{\text{N}}{\text{Cms}^{-1}} = 1\ \frac{\text{N}}{\text{Am}}$$

Another commonly used non-SI unit for $\vec{B}$ is the gauss (G), where $1\,T = 10^{-4}\,G$. Instruments used to measure magnetic field are sometimes called as Gaussmeters.

The magnetic force has several important properties:

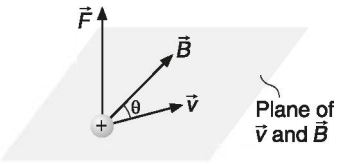
(a) Only a moving charge experiences a magnetic force. There is no magnetic force on a charge at rest $(v = 0)$ in a magnetic field.

(b) There is no force on a charge moving parallel $(\theta = 0°)$ or antiparallel $(\theta = 180°)$ to a magnetic field.



There is no force on the charge moving parallel to $\vec{B}$

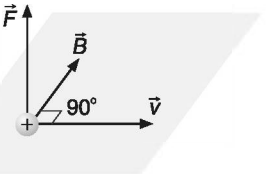
(c) When there is a force, the force is perpendicular to both $\vec{v}$ and $\vec{B}$.



The magnetic force is perpendicular to $\vec{v}$ and $\vec{B}$. Its magnitude is $qvB\sin\theta$

(d) The force on a negative charge is in the direction opposite to $\vec{v} \times \vec{B}$.

(e) For a charge moving perpendicular to $\vec{B}$ $(\theta = 90°)$, the magnitude of the magnetic force is maximum given by $F_{max} = |q|vB$.



The magnetic force is maximum when the charge moves perpendicular to $\vec{B}$.

DIRECTION OF $\vec{F}$ (THE MAGNETIC FORCE)

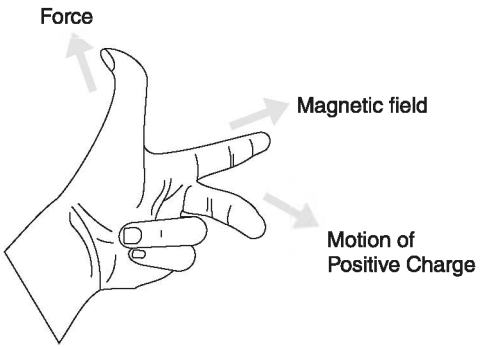
The direction of the magnetic force $\vec{F}$, can be found by using either of the two methods

(a) Fleming's Left Hand Rule

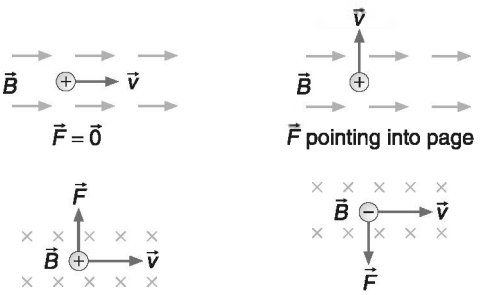
(b) Right Hand Thumb Rule (also used to find the direction of cross product of two vectors)

Fleming's Left Hand Rule

Stretch the first finger, the middle finger, and the thumb of left hand in such a way that they are mutually perpendicular to each other.



First finger points in the direction of Field, middle finger points in the direction of motion of positive charge (or the velocity of positive charge) then thumb gives the direction of force experienced by the positive charge.



Magnetic Forces on a Moving Charge

Right Hand Thumb Rule

To use this method for finding the direction of magnetic force $(\vec{F})$ simply follow the steps given.

STEP-1:

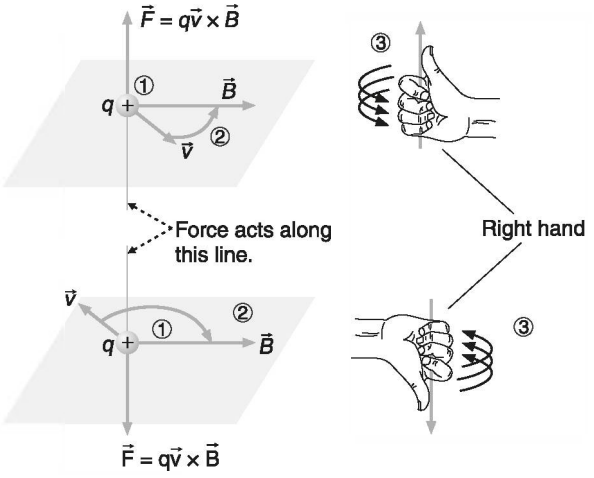
Place $\vec{v}$ (of the positive charge) and $\vec{B}$ tail to tail.

STEP-2:

Curl fingers of right hand from $\vec{v}$ to $\vec{B}$ (in the $\vec{v}$-$\vec{B}$ plane), as shown.

STEP-3:

The magnetic force $(\vec{F})$ acts along the line normal to the $\vec{v}$-$\vec{B}$ plane i.e., when we curl the fingers of the right hand from $\vec{v}$ to $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane, then direction of thumb gives the direction of the magnetic force as shown in Figure.



STEP-4:

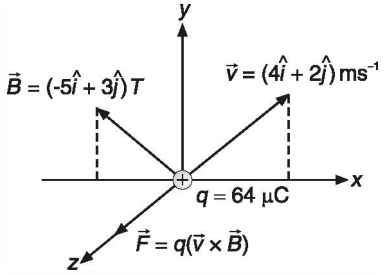
If the charge is negative, then the direction of the magnetic force is opposite to that given by the Right-Hand Rule. So, just find the direction of force as if charge were positive and then just reverse it.

ILLUSTRATION 1

A 64 μC charge travelling with velocity $\vec{v} = \left(4\hat{i} + 2\hat{j}\right)$ ms^{-1} enters a region where there is a magnetic field $\vec{B} = \left(-5\hat{i} + 3\hat{j}\right)$ T. Find the magnetic force on the charge.

SOLUTION

A charge q moving with velocity $\vec{v}$ entering a magnetic field $\vec{B}$ is shown in Figure.



Because both $\vec{v}$ and $\vec{B}$ lie in the xy-plane, we feel that F should be along the z-axis. We use the determinant method to compute the cross product indicated by equation.

$$\vec{F} = q(\vec{v} \times \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 0 \\ -5 & 3 & 0 \end{vmatrix}$$

$$\Rightarrow \quad \vec{F} = (64 \times 10^{-6})(12+10)\hat{k}$$

$$\Rightarrow \quad \vec{F} = 1.41 \times 10^{-3}\hat{k} \text{ N}$$

Problem Solving Technique(s)

(a) Since $\vec{F} = q(\vec{v} \times \vec{B})$, so from the property of cross product of two vectors we conclude that $\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$.

$\Rightarrow \quad \vec{F} \perp \vec{v}$ and $\vec{F} \perp \vec{B}$

$\Rightarrow \quad \vec{F} \cdot \vec{v} = 0$ and $\vec{F} \cdot \vec{B} = 0$

If $\vec{a}$ is the acceleration of the particle then $\vec{a} \cdot \vec{v} = 0$ and $\vec{a} \cdot \vec{B} = 0$.

(b) From above property we can also conclude that $\vec{F}$ is always normal to the plane containing $\vec{v}$ and $\vec{B}$.

(c) Since $\vec{F} = q(\vec{v} \times \vec{B})$, so we observe that apart from other factors $\vec{F}$ depends upon sign of q also. Mathematically

$$\vec{F} = \begin{cases} \text{parallel to } \vec{v} \times \vec{B}, \text{ if } q \text{ is positive,} \\ \text{antiparallel to } \vec{v} \times \vec{B} \text{ if } q \text{ is negative or} \\ \text{parallel to } \vec{B} \times \vec{v}, \text{ if } q \text{ is negative} \end{cases}$$

(d) Since we have noted that $\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$, so the displacement of a charged particle in a field will always be perpendicular to the force. Due to this, we have

$$dW = \vec{F} \cdot d\vec{\ell} = q(\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$\Rightarrow \quad dW = q(\vec{v} \times \vec{B}) \cdot (\vec{v}dt) = 0$$

This makes us conclude that the magnetic force does no work to displace a particle in a magnetic field (both varying and static) Further since, according to Work - Energy Theorem, we have

Work Done = Change in KE

$$\Rightarrow \quad 0 = \Delta K$$

$$\Rightarrow \quad K_f = K_i$$

$$\Rightarrow \quad |\vec{v}_{\text{final}}| = |\vec{v}_{\text{initial}}| \text{ (in magnitude only)}$$

$$\Rightarrow \quad v_x^2 + v_y^2 + v_z^2 = u_x^2 + u_y^2 + u_z^2$$

i.e., a magnetic force is just capable of changing the direction of velocity of the charged particle without changing its magnitude.

Hence, keep in mind that a **magnetic force is incapable of speeding up or slowing down a charge particle**. However, it can just change the direction of the velocity of the particle (without any change in its magnitude).

(e) $F = qvB\sin\theta$ has a maximum value qvB when $\theta = 90°$ i.e., whenever a charged particle enters at right angles to the field, then it will experience a maximum magnetic force. So, when, $\theta = \dfrac{\pi}{2}$

$$F_{max} = qvB$$

(f) By convention, the magnetic field perpendicular to and directed out of the page (i.e., a field that has its tips of arrows coming towards you) is represented by $\odot$ and the magnetic field perpendicular to and directed inside the page (i.e., a field that has its tips of arrows directed away from you) is represented by $\otimes$, as shown.

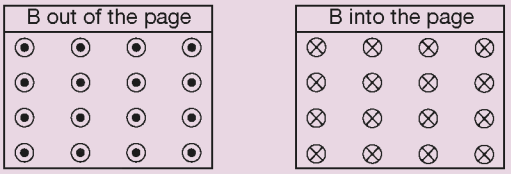


ILLUSTRATION 2

A charged particle moving in a magnetic field $\vec{B} = (\hat{i} - \hat{j})$ T has an acceleration of $(2\hat{i} + \alpha\hat{j})$ ms^{-2} at an instant. Calculate the value of α.

SOLUTION

Since, $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow \quad \vec{a} = \frac{q}{m}(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad \vec{a} \perp \vec{B}$$

$$\Rightarrow \quad \vec{a} \cdot \vec{B} = 0$$

$$\Rightarrow \quad (2\hat{i} + \alpha\hat{j})(\hat{i} - \hat{j}) = 0$$

$$\Rightarrow \quad 2 - \alpha = 0$$

$$\Rightarrow \quad \alpha = 2$$

ILLUSTRATION 3

A particle having charge $q = 10\ \mu C$ moves in uniform magnetic field with velocity $v_1 = 10^6$ ms^{-1} at angle $45°$ with x-axis in the x-y plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ ms^{-1} along the z-axis it experiences a force F_2 in y direction. Find the magnitude and direction of the magnetic field. Also find the magnitude of the force F_2.

SOLUTION

Since we know that $\vec{F} = q(\vec{v} \times \vec{B})$

For the first case, we get

$$-(5\sqrt{2} \times 10^{-3})\hat{k} = (10^{-5})\left(\frac{10^6}{\sqrt{2}}\right)(\hat{i} + \hat{j}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\Rightarrow \quad -(5\sqrt{2} \times 10^{-3})\hat{k} = \frac{10}{\sqrt{2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

$$\Rightarrow \quad -(5\sqrt{2} \times 10^{-3})\hat{k} = \frac{10}{\sqrt{2}}\left[B_z\hat{i} - B_z\hat{j} + (B_y - B_x)\hat{k}\right]$$

$$\Rightarrow \quad B_z = 0 \text{ and } B_y - B_x = -10^{-3} \text{ T} \qquad \ldots(1)$$

For the second case, we have

$$F_2\hat{j} = (10^{-5})(10^6)\hat{k} \times (B_x\hat{i} + B_y\hat{j})$$

$$\Rightarrow \quad F_2\hat{j} = 10\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ B_x & B_y & 0 \end{vmatrix} = 10B_x\hat{j} - 10B_y\hat{i} \qquad \ldots(2)$$

From (2), we get

$$B_y = 0$$

$$\Rightarrow \quad B_x = 10^{-3} \text{ T and } F_2 = 10B_x = 10^{-2} \text{ N}$$

$$\Rightarrow \quad \vec{B} = (10^{-3} \text{ T})\hat{i} \text{ and } F_2 = 10^{-2} \text{ N}$$

ILLUSTRATION 4

A particle of charge $q > 0$ is moving at speed v in the $+z$-direction through a region of uniform magnetic field $\vec{B}$. The magnetic force on the particle is $\vec{F} = F_0(3\hat{i} + 4\hat{j})$, where F_0 is a positive constant.

(a) Calculate the components B_x, B_y, and B_z, or at least as many as possible from the information given.

(b) If $|\vec{B}| = \frac{6F_0}{qv}$, calculate the left-out component(s) of $\vec{B}$, if any.

SOLUTION

(a) $\vec{v} = v\hat{k}$ and $\vec{F} = F_0\left(3\hat{i} + 4\hat{j}\right)$

Let $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Since, $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow \vec{F} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & v \\ B_x & B_y & B_z \end{vmatrix}$$

$$\Rightarrow \vec{F} = q\left[\hat{i}\left(-vB_y\right) - \hat{j}\left(-vB_x\right) + \hat{k}(0-0)\right]$$

Since, $\vec{F} = F_0\left(3\hat{i} + 4\hat{j}\right)$

$$\Rightarrow F_0\left(3\hat{i} + 4\hat{j}\right) = -\left(qvB_y\right)\hat{i} + \left(qvB_x\right)\hat{j}$$

$$\Rightarrow B_y = -\frac{3F_0}{qv} \text{ and } B_x = \frac{4F_0}{qv}$$

However, nothing can be said about B_z with the information provided

(b) $|\vec{B}| = \frac{6F_0}{qv}$

$$\Rightarrow B_x^2 + B_y^2 + B_z^2 = \frac{36F_0^2}{q^2v^2}$$

$$\Rightarrow \frac{16F_0^2}{qv^2} + \frac{9F_0^2}{q^2v^2} + B_z^2 = \frac{36F_0^2}{q^2v^2}$$

$$\Rightarrow B_z = \pm\sqrt{11}\left(\frac{F_0}{qv}\right)$$

ILLUSTRATION 5

A group of particles is travelling in a magnetic field of unknown magnitude and direction. It is observed that a proton moving at 1.5 kms^{-1} in the $+x$ direction experiences a force of 2.25×10^{-16} N in the $+y$ direction, and an electron moving at 4.75 kms^{-1} in the $-z$ direction experiences a force of 8.5×10^{-16} N. Calculate the magnitude and direction of the magnetic field.

SOLUTION

Let $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Since the proton moving along $+x$ direction experiences a force in the $+y$ direction and also we know that this force must be perpendicular to $\vec{v}$ as well as $\vec{B}$. Hence, for the proton the magnetic force given by

$$\vec{F} = +e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

$$\Rightarrow \vec{F} = +e\left[\hat{i}(0-0) - \hat{j}\left(v_xB_z - 0\right) + \hat{k}\left(v_xB_y - 0\right)\right]$$

$$\Rightarrow \vec{F} = +ev_x\left(-B_z\hat{j} + B_y\hat{k}\right) \quad \ldots(1)$$

Since $\vec{F} = 2.25\times10^{-16}$ N along $+y$ direction

$$\Rightarrow \vec{F} = \left(2.25\times10^{-16}\right)\hat{j} \text{ N} \quad \ldots(2)$$

On equating (1) and (2), we observe that

$$B_y = 0 \text{ and } -ev_xB_z = 2.25\times10^{-16}$$

$$\Rightarrow B_z = \frac{2.25\times10^{-16}}{1.6\times10^{-19}\times1.5\times10^3} = 0.938 \text{ T} = 938 \text{ mT}$$

Further, for the electron we have $\vec{v} = -v_z\hat{k}$ and $|\vec{F}| = 8.5\times10^{-16}$ N i.e., here we have just been provided with F without direction i.e., only with magnitude.

Since $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow \vec{F} = -e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -v_z \\ B_x & 0 & B_z \end{vmatrix}$$

$$\Rightarrow \vec{F} = -e\left[\hat{i}(0-0) - \hat{j}\left(0 + v_zB_x\right) + \hat{k}(0-0)\right]$$

$$\Rightarrow \vec{F} = +e\left(v_zB_x\right)\hat{j} \text{ where } v_z = 4.75\times10^3 \text{ kms}^{-1}$$

Since $|\vec{F}| = 8.5\times10^{-16}$ N

$$\Rightarrow ev_z|B_x| = 8.5\times10^{-16}$$

$$\Rightarrow |B_x| = \frac{8.5\times10^{-16}}{\left(1.6\times10^{-19}\right)\left(4.75\times10^3\right)} = 1.12 \text{ T}$$

$$\Rightarrow B_x = \pm1.12 \text{ T}$$

$$\Rightarrow \vec{B} = \left(\pm1.12\hat{i} + 0.938\hat{k}\right) \text{ T}$$

$$\Rightarrow |\vec{B}| = \sqrt{(\pm1.12)^2 + (0.938)^2} = 1.46 \text{ T}$$

Direction of $\vec{B}$

Let $\vec{B}$ make an angle θ with $+x$-axis, then

$$\tan\theta = \frac{B_z}{B_x} = \frac{-0.938}{\pm 1.12}$$

$$\Rightarrow \quad \theta = \pm 40^\circ$$

i.e., $\vec{B}$ is in the xz plane and makes an angle of 40° with $+x$-axis either in anticlockwise sense or in clockwise sense.

ILLUSTRATION 6

A particle of mass 0.195 g carries a charge of -2.5×10^{-8} C. The particle is given an initial horizontal velocity that is due north and has magnitude 4×10^4 ms^{-1}. Calculate the magnitude and direction of the minimum magnetic field that keeps the particle moving in the earth's gravitational field in the same horizontal, northward direction?

SOLUTION

For the net force to be zero, the magnetic force (F_m) must balance the weight (W) of the particle
Now, for equilibrium we have

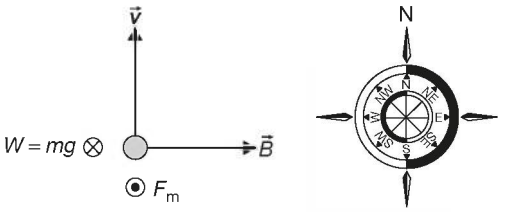
$$F_m = W$$

$$\Rightarrow \quad |q|vB\sin\theta = W$$

$$\Rightarrow \quad B = \frac{W}{|q|v\sin\theta} = \frac{mg}{|q|v\sin\theta}$$

Since, we are asked to calculate the minimum B, so we must have $\sin\theta = MAX = 1$, i.e., $\theta = 90^\circ$ i.e., $\vec{v} \perp \vec{B}$. Hence

$$B_{\min} = \frac{mg}{|q|v} = \frac{(0.195\times10^{-3})(9.8)}{(2.5\times10^{-8})(4\times10^4)} = 1.91 \text{ T}$$



Since the velocity of the particle is due North and the weight of the particle $W(=mg)$ is acting towards the centre of the earth, which happens to be a direction denoted by $\otimes$ shown in figure. Now for the negative charge that moves due North, the magnetic force that balances W must be acting in the outward direction $\odot$, just opposite to W so as to balance it. Please note that the direction of the force on negative charge will be opposite to that obtained by Fleming's Left Hand Rule (specified for positive charge). So, be careful while finding the direction of $\vec{F}_m$. Hence to conclude we get to have B acting towards the East.

Test Your Concepts-I

Based on Force and Fleming's Left Hand Rule

(Solutions on page H.3)

1. Can you find the dimensional formula for the ratio $\frac{E}{B}$, where E denotes the electric field and B denotes the magnetic field?
2. If $\vec{F}$ denotes the magnetic force acting on a charge q entering a field $\vec{B}$ with a velocity $\vec{v}$, then find
   (a) $\vec{v}\cdot(\vec{v}\times\vec{B})$
   (b) $\vec{B}\cdot(\vec{v}\times\vec{B})$
   (c) $\vec{v}\times(\vec{v}\times\vec{B})$
3. A charged particle is projected in a magnetic field $\vec{B} = (3\hat{i} + 2\hat{j} - 10\hat{k})\times10^{-2}$ T. The acceleration of the particle is found to be, $\vec{a} = (x\hat{i} + 4\hat{j} + 2\hat{k})$ ms^{-2}. Find the value of x.
4. A particle with mass 1.81×10^{-3} kg and a charge of 1.22×10^{-8} C has, at a given instant, a velocity $\vec{v} = (3\times10^4 \text{ ms}^{-1})\hat{j}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\vec{B} = (1.63 \text{ T})\hat{i} + (0.98 \text{ T})\hat{j}$?
5. A particle with charge -5.6 nC is moving in a uniform magnetic field $\vec{B} = -(1.25 \text{ T})\hat{k}$. The magnetic force on the particle is measured to be
   $$\vec{F} = -(3.4\times10^{-7} \text{ N})\hat{i} + (7.4\times10^{-7} \text{ N})\hat{j}$$

(a) Calculate all the components of the velocity of the particle that you can calculate from this information.

(b) Are there components of the velocity that are not determined by the measurement of the force? Explain.

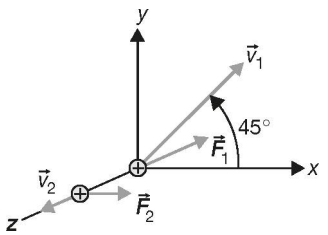
(c) Calculate the scalar product $\vec{v}\cdot\vec{F}$. What is the angle between $\vec{v}$ and $\vec{F}$?

6. To hit a target from several meters away with a charged coin having a mass of 5 g and a charge of $+2500\ \mu C$, a coin is given an initial velocity of $12.8\ ms^{-1}$. A downward, uniform electric field with field strength $27.5\ NC^{-1}$ exists throughout the region. Assuming that you aim directly at the target and fire the coin horizontally, what magnitude and direction of uniform magnetic field are needed in the region for the coin to hit the target?

7. A charged particle of charge 1 mC and mass 2 g is moving with a speed of $5\ ms^{-1}$ in a uniform magnetic field of 0.5 T. Calculate the maximum acceleration of the charged particle.

8. When a proton has a velocity $\vec{v}=\left(2\hat{i}+3\hat{j}\right)\times10^6\ ms^{-1}$ it experiences a force $\vec{F}=-\left(1.28\times10^{-13}\hat{k}\right)$ N. When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?

9. When a particle of charge $q>0$ moves with a velocity of $\vec{v}_1$ at $45°$ from the $+x$-axis in the xy-plane, a uniform magnetic field exerts a force $\vec{F}_1$ along the $-z$-axis. When the same particle moves with a velocity $\vec{v}_2$ with the same magnitude as $\vec{v}_1$ but along the $+z$-axis, a force $\vec{F}_2$ of magnitude F_2 is exerted on it along the $+x$-axis.



(a) What are the magnitude (in terms of q, v_1 and F_2) and direction of the magnetic field?

(b) What is the magnitude of $\vec{F}_1$ in terms of F_2?

10. A charged particle of specific charge (i.e. charge unit mass) $0.2\ Ckg^{-1}$ has velocity $\left(2\hat{i}-3\hat{j}\right)\ ms^{-1}$ at an instant in a uniform magnetic field $\left(5\hat{i}+2\hat{j}\right)$ T. Calculate the acceleration of the particle at this instant.

11. A proton moving in a uniform magnetic field with a velocity $\vec{v}_1=10^6\hat{i}\ ms^{-1}$ experiences a force $\vec{F}_1=1.2\times10^{-16}\hat{k}$ N. A second proton moving in the same field with a velocity $\vec{v}_2=2\times10^6\hat{j}\ ms^{-1}$ experiences a force $\vec{F}_2=-4.16\times10^{-16}\hat{k}$ N. Calculate the magnetic field $\vec{B}$? Give your answer as a magnitude and an angle measured CCW from the $+x$-axis.

12. A particle with charge 9.45×10^{-8} C is moving in a region where there is a uniform magnetic field of 0.45 T in the $+x$-direction. At a particular instant of time the velocity of the particle has components

$v_x=-1.68\times10^4\ ms^{-1}$,

$v_y=-3.11\times10^4\ ms^{-1}$ and

$v_z=5.85\times10^4\ ms^{-1}$

What are the components of the force on the particle at this time?

13. A small ball having mass m, charge q is suspended from a rigid support by means of a light inextensible string of length l. It is made to revolve on a horizontal circular path in a uniform magnetic field B directed vertically upwards. If the time period of revolution of the ball is T_0 and the thread is always tight, then calculate the radius of circular path on which the ball moves.

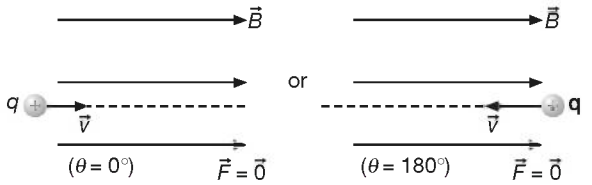
PATH OF A CHARGED PARTICLE IN UNIFORM MAGNETIC FIELD

When a charged particle moves in a magnetic field, it is acted upon by a magnetic force given by $|\vec{F}| = qvB\sin\theta$ and the motion is determined by Newton's Laws.

The path of a charged particle in uniform magnetic field depends on the angle θ (the angle between $\vec{v}$ and $\vec{B}$). Based on the different values of θ, following three cases are possible.

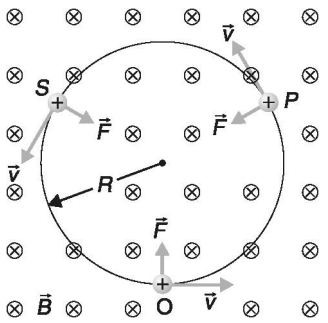
CASE-1: When θ is 0° or 180° (= π radian)

Since $|\vec{F}| = qvB\sin\theta$, so when θ is either 0° or 180° i.e. the charged particle enters the field with a velocity either parallel to it $(\theta = 0°)$ or anti parallel to the field $(\theta = 180°)$, then we have $|\vec{F}| = 0$. Hence, path of the charged particle is a straight line (undeviated) when it enters parallel or antiparallel to the magnetic field.



CASE-2: When the charged particle moves perpendicular to the field, i.e., $\theta = 90°$ or $\frac{\pi}{2}$ radian

Consider a positive point charge q of mass m at **point** O, moving with a velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ directed into the plane of the page as shown in Figure.



The vectors $\vec{v}$ and $\vec{B}$ are perpendicular to each other initially as well as at later instants of time. The magnetic force $\vec{F} = q(\vec{v}\times\vec{B})$ will have a magnitude $F = qvB\sin 90° = qvB$ and a direction as shown in the figure. We observe that the force is always perpendicular to $\vec{v}$ and hence cannot change the magnitude of the velocity, but can change its direction only.

To understand this concept differently, we know that the magnetic force has got no component parallel to the motion of the particle (it always possesses component perpendicular to the particle's motion) and hence cannot do any work on the particle. This is true even if the magnetic field is non-uniform. So, always keep in mind that the motion of a charged particle under the action of a magnetic field alone is always a motion with constant speed.

So, we see that in the situation shown, the magnitudes of both $\vec{F}$ and $\vec{v}$ are constant. At points P and S, the directions of force and velocity have changed as shown, but their magnitudes are still the same. The particle therefore moves under the influence of a constant-magnitude force which is always at right angles to the velocity of the particle and we see that the particle's path is a circle of radius R, traced out with constant speed v. The centripetal acceleration of the particle is $\frac{v^2}{R}$ and the only force acting on the particle is the magnetic force, so from Newton's Second Law, we get

$$F = |q|vB = \frac{mv^2}{R}$$

$$\Rightarrow \quad R = \frac{mv}{|q|B}$$ (radius of a circular orbit in a magnetic field)

Since $p = mv$ = momentum of the particle, so we can also write $R = \frac{mv}{qB} = \frac{p}{qB}$.

If the charge q is negative, the particle traces the circle in clockwise sense.

The angular speed ω of the particle can be found by using $v = R\omega$. So, we get

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$$

The number of revolutions per unit time is $f = \frac{\omega}{2\pi}$.

This frequency f is independent of the radius R of the path. It is called the **cyclotron frequency**.

If the charged particle has a kinetic energy K, then we have

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Rightarrow \quad p = \sqrt{2mK}$$

$$\Rightarrow \quad R = \frac{mv}{|q|B} = \frac{p}{|q|B} = \frac{\sqrt{2mK}}{|q|B}$$

Also, there is a possibility that a charged particle is first accelerated through a potential V to gain a velocity v.

Then, $\frac{1}{2}mv^2 = |q|V$

$$\Rightarrow \quad v = \sqrt{\frac{2|q|V}{m}}$$

$$\Rightarrow \quad R = \frac{mv}{|q|B} = \frac{m}{|q|B}\sqrt{\frac{2|q|V}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{|q|}} = \frac{\sqrt{2|q|mV}}{|q|B}$$

So, finally, we observe that the radius of the particle moving in a circle is given by

$$R = \frac{mv}{|q|B} = \frac{p}{|q|B} = \frac{\sqrt{2mK}}{|q|B} = \frac{\sqrt{2|q|mV}}{|q|B} = \frac{1}{B}\sqrt{\frac{2mV}{|q|}}$$

where

$p =$ momentum of the particle $= mv$

$K =$ kinetic energy of the particle $= \frac{1}{2}mv^2$

$V =$ electrostatic potential through which the charge is accelerated to gain a speed v.

If T be the period of revolution of the particle, then we have

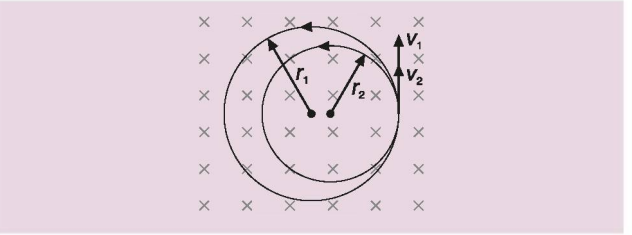
$$T = \frac{2\pi R}{v}$$

$$\Rightarrow \quad T = \frac{2\pi}{v}\left(\frac{mv}{|q|B}\right) = \frac{2\pi m}{|q|B} = \frac{1}{f}$$

$$\Rightarrow \quad T = \frac{2\pi m}{|q|B} = \frac{1}{f}$$

So, we observe that T (the period of revolution), f (the frequency of revolution) and ω (the angular frequency or the angular velocity) are the independent of v.

So, if two identical charge particles are given different speeds from a same point in the same direction, then they will return to the initial point simultaneously.



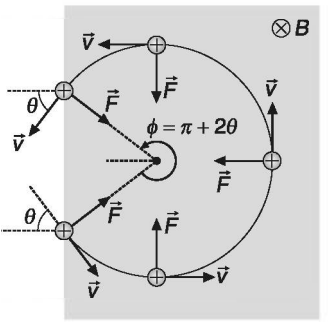
Also, the kinetic energy of the particle is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m}$$

CHARGED PARTICLE ENTERING INTO MAGNETIC FIELD REGION FROM OUTSIDE

When a charged particle enters a region of uniform magnetic field, such that the particle moves in a circular arc, but cannot complete a full circle.

CASE-1: For a positively charged particle



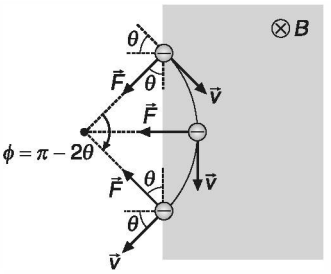
Time for which the charged particle stays inside the field is given by

$$\phi = \omega t \text{, where } \phi = \pi + 2\theta$$

$$\Rightarrow \quad t = \frac{\phi}{\omega} \text{, where } \omega = \frac{qB}{m}$$

$$\Rightarrow \quad t = \left(\frac{m}{qB}\right)\phi = \frac{m(\pi + 2\theta)}{qB}$$

CASE-2: For a negatively charged particle



Time for which the charged particle stays inside the field is given by

$$\phi = \omega t \text{, where } \phi = \pi - 2\theta$$

$$\Rightarrow \quad t = \frac{\phi}{\omega} \text{, where } \omega = \frac{qB}{m}$$

$$\Rightarrow \quad t = \left(\frac{m}{qB}\right)\phi = \frac{m(\pi - 2\theta)}{qB}$$

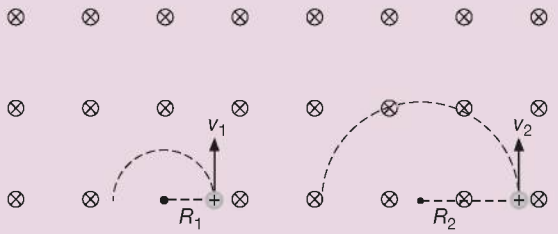
Problem Solving Technique(s)

(a) The above results can also be expressed in terms of the specific charge $\left(\frac{q}{m}\right)$, sometimes denoted by α. So, we get $R = \frac{v}{\alpha B}$, $T = \frac{2\pi}{\alpha B}$, $\omega = \alpha B$, $f = \frac{\alpha B}{2\pi}$

(b) As discussed, T, f and ω are independent of the velocity v, however R (radius of curvature) is directly proportional to v. So, for two identical charged particles (having same q and m) entering at right angles to the field $\vec{B}$ with different speeds v_1 and v_2, where $v_2 > v_1$, we get

$$T_1 = T_2, \; f_1 = f_2 \text{ and } \omega_1 = \omega_2$$

However, $R_2 > R_1$ (as shown)



(c) For a particle moving with velocity at right angles to the field, the plane of the circle is normal to the magnetic field. If the magnetic field is along z-direction, the circular path is in x-y plane. In other words, the plane of the circle is actually the plane that contains both $\vec{F}$ and $\vec{v}$ which of course happens to be normal to $\vec{B}$.

Also note that F acts towards the centre of the circle.

(d) The speed of the particle does not change in magnetic field (as discussed already).

Hence, if v_0 be the speed of the particle, then velocity of particle at any instant of time will be given by

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \text{ where } v_x^2 + v_y^2 = v_0^2$$

If $\vec{u}$ be the initial velocity of the particle, then

$$\vec{u} = u_x\hat{i} + u_y\hat{j}$$

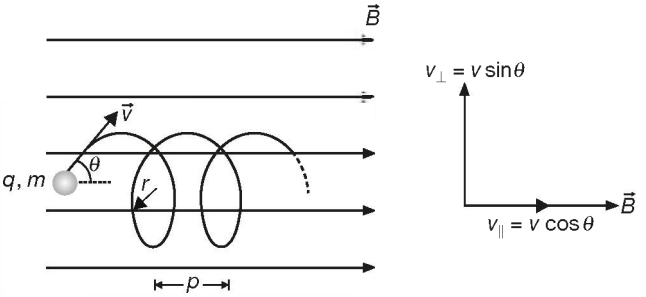
However, $|\vec{v}| = |\vec{u}| = v_0$

$$\Rightarrow \quad u_x^2 + u_y^2 = v_x^2 + v_y^2 = v_0^2$$

This strategy will help us solve problems easily.

CASE-3: When the charged particle moves with velocity not perpendicular to the field i.e. $\theta \neq 0°, 90°, 180°$

If the direction of the initial velocity of the charged particle is not perpendicular to the field, then the velocity can be resolved into two components, one along $\vec{B}$ and other perpendicular to $\vec{B}$. Now since the magnetic force has got no component parallel to the field (remember that $\vec{F} \perp \vec{B}$) so the component of velocity parallel to the field remains constant.



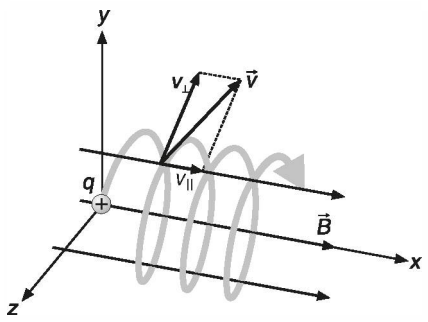
If $v_\parallel$ be the component of velocity parallel to the field and $v_\perp$ be the component of velocity perpendicular to the field. Since $\vec{B}$ cannot influence the component of velocity parallel to it, so we have

$$v_\parallel = v\cos\theta = \text{constant}$$

Also, we have

$$v_\perp = v\sin\theta$$

This component of velocity $(v_\perp)$ perpendicular to field gives a circular path and the component of velocity $(v_\parallel)$ parallel to the field $(v_\parallel)$ gives a straight line path. The resultant path is a helical path called **helix** as shown in Figure.



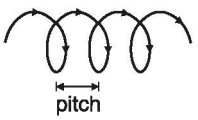
The radius of this helical path is,

$$r = \frac{mv_\perp}{|q|B} = \frac{mv\sin\theta}{|q|B}$$

Time period and frequency do not depend on velocity and so they are given by,

$$T = \frac{2\pi m}{|q|B} \text{ and } f = \frac{|q|B}{2\pi m}$$

Pitch (p) of the helical path is defined as the horizontal forward distance travelled by the particle in one complete cycle as shown in Figure.



So, mathematically pitch is given by

$$p = v_\parallel T$$

$$\Rightarrow \quad p = (v\cos\theta)\left(\frac{2\pi m}{|q|B}\right)$$

$$\Rightarrow \quad p = \frac{2\pi mv\cos\theta}{|q|B}$$

Problem Solving Technique(s)

(a) The plane of the circle of the helix is normal to the magnetic field.

(b) The axis of the helix is parallel to magnetic field.

(c) The particle while moving in helical path in magnetic field touches the line passing through the starting point parallel to the magnetic field after every pitch.

For example, a charged particle is projected from origin in a magnetic field (along x-direction) at angle θ from the x-axis as shown. As the velocity vector $\vec{v}$ makes an angle θ with $\vec{B}$ its path is a helix. The plane of the circle of the helix is y-z (perpendicular to magnetic field) and axis of the helix is parallel to x-axis. The particle while moving in helical path touches the x-axis after every pitch, i.e., it will touch the x-axis at a distance.

$$x = np \text{ where } n = 0, 1, 2 \ldots$$

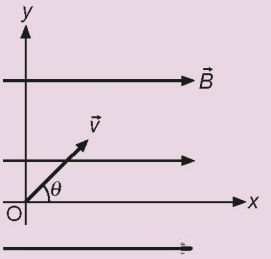
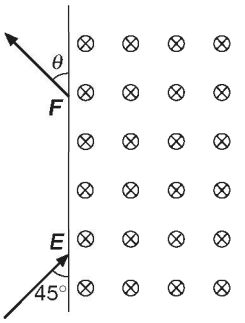


ILLUSTRATION 7

A particle of mass $m = 1.6\times10^{-27}$ kg and charge $q = 1.6\times10^{-19}$ C enters a region of uniform magnetic field of strength 1 T along the direction shown in figure. The speed of the particle is 10^7 ms^{-1}.



(a) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(b) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

SOLUTION

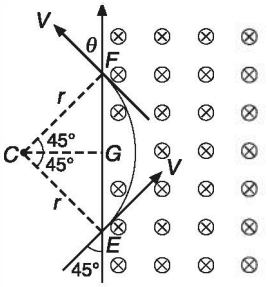
Inside a magnetic field speed of charged particle does not change. Further, velocity is perpendicular to magnetic field in both the cases hence path of the particle in the magnetic field will be circular.

Centre of circle can be obtained by drawing perpendicular to velocity (or tangent to the circular path) at

E and F. Radius and angular speed of circular path would be

$$r = \frac{mv}{qB} \quad \text{and} \quad \omega = \frac{qB}{m}$$

(a) From the figure we observe that



(a)

$$\angle CFG = 90° - \theta$$

and $\angle CEG = 90° - 45° = 45°$

Since, $CF = CE$

$\Rightarrow \angle CFG = \angle CEG$

$\Rightarrow 90° - \theta = 45°$

$\Rightarrow \theta = 45°$

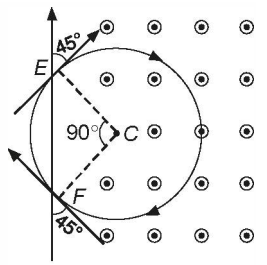
Further, $FG = GE = R\cos 45°$

$$\Rightarrow EF = 2FG = 2r\cos 45° = \frac{2mv\cos 45°}{qB}$$

$$EF = \frac{2\left(1.6\times10^{-27}\right)\left(10^{7}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(1.6\times10^{-19}\right)(1)} = 0.14 \text{ m}$$

In this case, the particle completes $\frac{1}{4}$th of circle in the magnetic field.

(b) In this case, the particle will complete $\frac{3}{4}$th of circle in the magnetic field.



(b)

Hence, the time spent in the magnetic field is

$$t = \frac{3}{4}(\text{time period of circular motion})$$

$$t = \frac{3}{4}\left(\frac{2\pi m}{qB}\right) = \frac{3\pi m}{2qB}$$

$$\Rightarrow t = \frac{(3\pi)\left(1.6\times10^{-27}\right)}{(2)\left(1.6\times10^{-19}\right)(1)}$$

$$\Rightarrow t = 4.712\times10^{-8} \text{ s} \cong 47 \text{ ns}$$

ILLUSTRATION 8

A beam of protons with a velocity 4×10^{5} ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix.

SOLUTION

$$r = \frac{mv\sin\theta}{qB} = \frac{\left(1.67\times10^{-27}\right)\left(4\times10^{5}\right)(\sin 60°)}{\left(1.6\times10^{-19}\right)(0.3)}$$

$$\Rightarrow \quad r = 1.2\times10^{-2} \text{ m} = 1.2 \text{ cm}$$

Since, $p = \left(\frac{2\pi m}{qB}\right)(v\cos\theta)$

$$\Rightarrow \quad p = \frac{(2\pi)\left(1.67\times10^{-27}\right)\left(4\times10^{5}\right)(\cos 60°)}{\left(1.6\times10^{-19}\right)(0.3)}$$

$$\Rightarrow \quad p = 4.37\times10^{-2} \text{ m} = 4.37 \text{ cm}$$

Problem Solving Technique(s)

MOTION OF CHARGED PARTICLE

STEP-1

In analysing the motion of a charged particle in electric and magnetic fields, you will apply Newton's Second Law of motion, with the net force given by

$$\Sigma\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Often other forces such as gravity can be neglected.

STEP-2

Read and analyse the problem carefully. The use of components is the most efficient approach. Select a coordinate system and then express all vector quantities (including $\vec{E}$, $\vec{B}$, $\vec{v}$, $\vec{F}$, and $\vec{a}$) in terms of their components in this system.

STEP-3

If the particle moves perpendicular to a uniform magnetic field, the trajectory is a circle whose radius and angular speed are given by

$$R=\frac{mv}{|q|B} \text{ and } \omega=\frac{|q|B}{m}$$

STEP-4

If our calculation involves a more complex trajectory then use $\Sigma\vec{F}=m\vec{a}$ in component form $\Sigma F_x=ma_x$, and so forth. This approach is particularly useful when both electric and magnetic fields are present.

STEP-5

If $\vec{u}=u_x\hat{i}+u_y\hat{j}$ be the initial velocity of the particle and $\vec{v}=v_x\hat{i}+v_y\hat{j}$ be the final velocity of the particle in a magnetic field, then

$$|\vec{v}|=|\vec{u}|$$

$$\Rightarrow\ v_x^2+v_y^2=u_x^2+u_y^2$$

ILLUSTRATION 9

A particle having mass m and charge q, enters a uniform magnetic field $\vec{B}=-B_0\hat{k}$ with velocity $\vec{v}=v_0\hat{i}$ from the origin. Find the time dependence of velocity and position of the particle.

SOLUTION

In such type of problems, we should follow the steps given.

STEP-1:

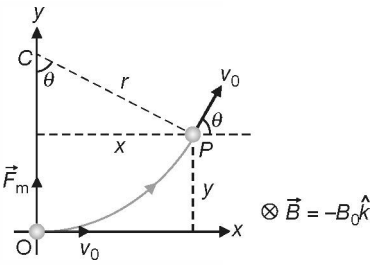
First of all see the angle between $\vec{v}$ and $\vec{B}$. Because it only decides the path of the particle. Here, the angle is $90°$. Therefore, the path is a circle.

STEP-2:

If it is a circle, see the plane of the circle (normal to the magnetic field). Here the plane is x-y.

STEP-3:

Apply Fleming's Left Hand Rule to find the sense of rotation of the particle, which happens to be counter clockwise as shown in Figure.



$OC = CP$ = radius of circle

STEP-4:

Now find the deviation and radius of the particle,

$$\theta=\omega t=\left(\frac{qB_0}{m}\right)t \text{ and } r=\frac{mv_0}{qB_0}$$

STEP-5:

Now from the figure, find $\vec{v}(t)$ and $\vec{r}(t)$.

Since velocity of the particle at any time t is,

$$\vec{v}(t)=v_x\hat{i}+v_y\hat{j}=v_0\cos\theta\hat{i}+v_0\sin\theta\hat{j}$$

$$\Rightarrow\ \vec{v}(t)=v_0\cos(\omega t)\hat{i}+v_0\sin(\omega t)\hat{j}, \text{ where}$$

$$\omega=\frac{qB_0}{m}$$

Position of particle at time t is,

$$\vec{r}(t)=x\hat{i}+y\hat{j}=r\sin\theta\hat{i}+(r-r\cos\theta)\hat{j}$$

$$\vec{r}(t)=r\left((\sin\theta)\hat{i}+(1-\cos\theta)\hat{j}\right)$$

Substituting the values of r and θ, we have

$$\vec{r}(t)=\frac{mv_0}{qB_0}\left(\sin\left(\frac{qB_0t}{m}\right)\hat{i}+\left(1-\cos\left(\frac{qB_0t}{m}\right)\right)\hat{j}\right)$$

ILLUSTRATION 10

A particle of charge q and mass m is projected from origin with velocity $\vec{v}=v_0(\hat{i}-\hat{k})$ in a uniform magnetic field $\vec{B}=-B_0\hat{k}$. Find the velocity and position of the particle as a function of time t.

SOLUTION

Let us first calculate the angle between $\vec{v}$ and $\vec{B}$. If θ is the angle between $\vec{v}$ and $\vec{B}$, then

$$\theta=\cos^{-1}\left(\frac{\vec{v}\cdot\vec{B}}{|\vec{v}||\vec{B}|}\right)=\cos^{-1}\left(\frac{B_0v_0}{\sqrt{2}v_0B_0}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Since this happens to be an angle other than 0°, 90° and 180°, hence, the path followed by the particle is a helix. The axis of the helix is along z-axis (parallel to $\vec{B}$) and plane of the circle of helix is the xy plane (normal to $\vec{B}$). So, in x-y plane, the velocity components and x and y co-ordinates are same as the ones calculated in **ILLUSTRATION 9**. The only change is in the values along z-axis. Velocity component in this direction will remain unchanged while the z-coordinate of particle at time t would be $v_z t$.

Velocity of particle at time t is,

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{v}(t) = v_0\cos\left(\frac{qB_0t}{m}\right)\hat{i} + v_0\sin\left(\frac{qB_0t}{m}\right)\hat{j} - v_0\hat{k}$$

v_x and v_y have been calculated in the similar manner as done in **ILLUSTRATION 9.**

The position of the particle at time t would be,

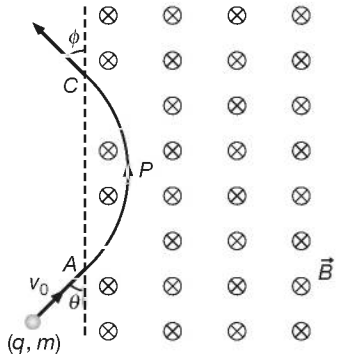
$$\vec{r}(t) = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where, $z = v_z t = -v_0 t$ and x and y are same as calculated in **ILLUSTRATION 9**. Hence,

$$\vec{r} = \frac{mv_0}{qB_0}\left[\sin\left(\frac{qB_0t}{m}\right)\hat{i} + \left(1 - \cos\left(\frac{qB_0t}{m}\right)\right)\hat{j}\right] - v_0t\hat{k}$$

ILLUSTRATION 11

A charged particle of mass m, charge q enters a uniform magnetic field $\vec{B}$ with speed v_0 at angle θ as shown in figure.

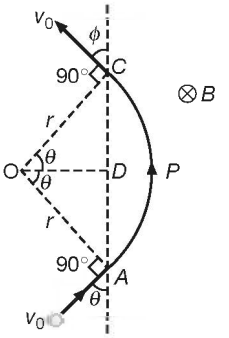


Calculate the

(a) angle ϕ at which it leaves the magnetic field.
(b) time spent by the particle in magnetic field and
(c) separation AC.

SOLUTION

(a) Here, velocity of the particle is in the plane of paper while the magnetic field is perpendicular to the paper inwards. i.e., angle between $\vec{v}$ and $\vec{B}$ is 90°.



So, the path is a circle, having radius $r = \frac{mv_0}{qB}$

If, O is the centre of the circle, then in ΔAOC,

$$\angle OCD = \angle OAD$$

$$\Rightarrow \quad 90^\circ - \phi = 90^\circ - \theta$$

$$\Rightarrow \quad \phi = \theta$$

(b) $\angle COD = \angle DOA = \theta$

$$\{\because \angle OCD = \angle OAD = 90^\circ - \theta\}$$

$$\Rightarrow \quad \angle AOC = 2\theta$$

$$\text{Length of arc } APC = r(2\theta) = \left(\frac{2mv_0}{qB}\right)\theta$$

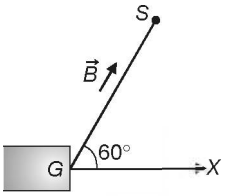
$$\Rightarrow \quad t_{APC} = \frac{APC}{v_0} = \frac{2m\theta}{qB}$$

(c) Separation $AC = 2(AD) = 2(r\sin\theta)$

$$AC = \left(\frac{2mv_0}{qB}\right)\sin\theta$$

ILLUSTRATION 12

An electron gun G emits electrons of energy 2 keV travelling in the positive X-direction. The electrons are required to hit the spot S where $GS = 0.1$ m, and the line GS makes an angle of 60° with the x-axis as shown in figure. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.



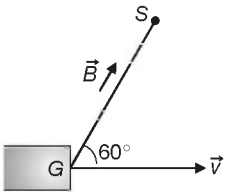
SOLUTION

Kinetic energy of electron, $K = \frac{1}{2}mv^2 = 2$ keV

So, the speed of electrons hitting the spot S is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times 2\times 1.6\times 10^{-16}}{9.1\times 10^{-31}}}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v = 2.65\times 10^{7}\ \text{ms}^{-1}$$



Since, the velocity $(\vec{v})$ of the electron makes an angle of $\theta = 60°$ with the magnetic field $\vec{B}$, the path will be a Helix of pitch p.

So, the particle will hit S if we have

$GS = np$, where $n = 1, 2, 3, 4, \ldots$

and p = pitch of helix $= \frac{2\pi mv\cos\theta}{qB}$

However, for B to be minimum we must have $n = 1$

$$\Rightarrow \quad GS = p = \frac{2\pi m}{qB}v\cos\theta$$

$$\Rightarrow \quad B = B_{\min} = \frac{2\pi mv\cos\theta}{q(GS)}$$

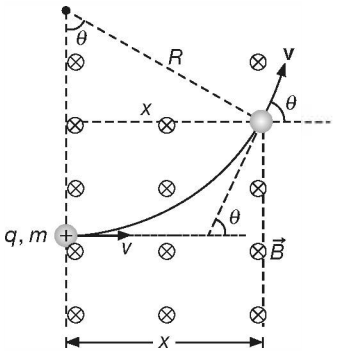
Substituting the values, we get

$$B_{\min} = \frac{(2\pi)(9.1\times 10^{-31})(2.65\times 10^{7})\left(\frac{1}{2}\right)}{(1.6\times 10^{-19})(0.1)}\ \text{T}$$

$$\Rightarrow \quad B_{\min} = 4.73\times 10^{-3}\ \text{T}$$

DEVIATION OF A CHARGED PARTICLE IN MAGNETIC FIELD

Consider a charged particle of mass m, charge q entering a uniform magnetic field $\vec{B}$ at right angles with speed v as shown in Figure.



Let the magnetic field extend in a region of space upto a length x. The path of the particle is a circle of radius R, where

$$R = \frac{mv}{|q|B}$$

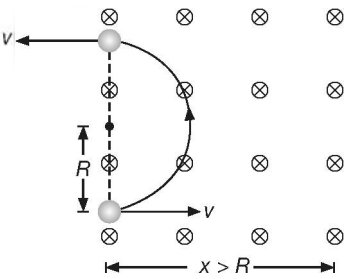
The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field. The angular deviation θ can be found and expressed as a function of time t or in terms of the length of the magnetic field space.

(a) After time t, deviation will be,

$$\theta = \omega t = \left(\frac{|q|B}{m}\right)t \ \text{ as } \ \omega = \frac{|q|B}{m}$$

(b) In terms of the length of the magnetic field (i.e., when the particle leaves the magnetic field) the deviation will be,

$$\theta = \begin{cases} \sin^{-1}\left(\frac{x}{R}\right) & \text{for} \quad x \le R \\ \pi\,(\text{or } 180°) & \text{for} \quad x > R \end{cases}$$



Please note that the deviation, when x is slightly less than R is $\frac{\pi}{2}$. Also note that the deviation suffered by a charged particle is given by

$$\theta = \sin^{-1}\left(\frac{x}{R}\right)$$

$$\Rightarrow \quad \sin\theta = \frac{x}{R} \ \text{ where}$$

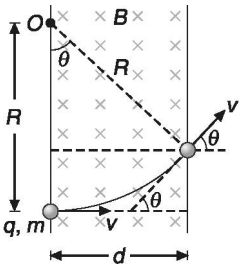
$$R = \frac{mv}{|q|B} = \frac{p}{|q|B} = \frac{\sqrt{2mE}}{|q|B} = \frac{\sqrt{2m|q|V}}{|q|B}$$

ILLUSTRATION 13

An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion, if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 tesla. Given: mass of α-particle 6.4×10^{-27} kg.

SOLUTION

The situation is shown in figure.



When a charged particle with charge q is accelerated through a potential difference V volt , then it gains a speed given by

$$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}} \qquad \ldots(1)$$

When α-particle enters the magnetic field, it moves in a circle of radius R as shown in figure.

Since $R = \frac{mv}{qB}$

$$\Rightarrow \quad R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

Given that $A = 0.1$ m , $B = 0.1$ T $V = 10^4$ volt

Also, $q_\alpha = 2e = 2\times1.6\times10^{-19} = 32\times10^{-19}$ C

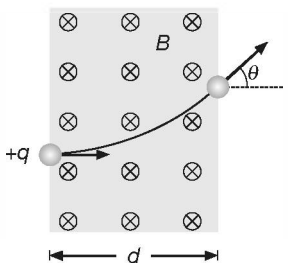
and $m_\alpha = 6.4\times10^{-27}$ kg

$$\Rightarrow \quad \sin\theta = (0.1)(0.1)\sqrt{\frac{3.2\times10^{-19}}{2\times6.4\times10^{-27}\times10^4}}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

ILLUSTRATION 14

A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B .

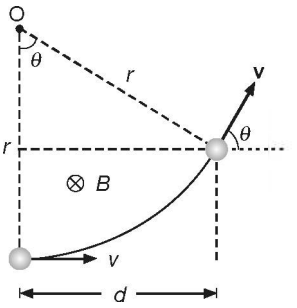


Find the angle of deviation θ of the particle as it comes out of the magnetic field if width d of the region is

(a) $\frac{mv}{2qB}$ (b) $\frac{mv}{qB}$ (c) $\frac{2mv}{qB}$

SOLUTION

(a) The radius of the circular orbit is given by $r = \frac{mv}{qB}$.

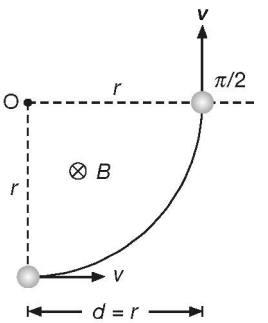


So, the angle of deviation for $d = \frac{mv}{2qB} = \frac{r}{2}$ is

$$\sin\theta = \frac{d}{r} = \frac{1}{2}$$

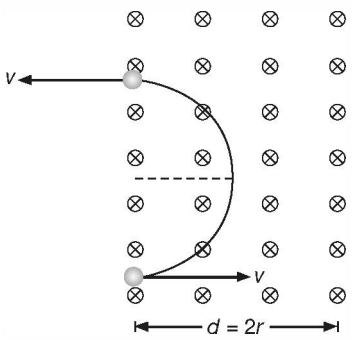
$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} = 30^\circ$$

(b) When $d = \frac{mv}{qB} = r$, the charged particle deviates through an angle of $\frac{\pi}{2}$ as shown in Figure.



So, we have $\theta = \frac{\pi}{2}$

(c) When $d = 2\left(\frac{mv}{qB}\right) = 2r$, the charged particle completes one semi-circle and deviates through π , as shown in Figure.



So, we get $\theta = \pi$

ILLUSTRATION 15

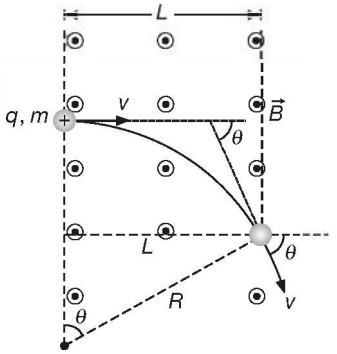
The region between $x = 0$ and $x = L$ is filled with uniform steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of the magnetic field. Neglecting gravity throughout the question, find the

(a) value of L if the particle emerges from the region of magnetic field with its final velocity at an angle $30°$ to its initial velocity.

(b) final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now expands upto 2.1 L.

SOLUTION

The situation is shown in Figure.



(a) For $\theta = 30°$

Since, we know that $\sin\theta = \dfrac{L}{R}$

where, $R = \dfrac{mv_0}{qB_0}$

$$\Rightarrow \sin(30°) = \frac{L}{\frac{mv_0}{qB_0}}$$

$$\Rightarrow \frac{1}{2} = \frac{qB_0L}{mv_0}$$

$$\Rightarrow L = \frac{mv_0}{2qB_0}$$

(b) Since, we know that $\sin\theta = \dfrac{L}{R}$

$$\Rightarrow \sin(30°) = \frac{L}{R}$$

$$\Rightarrow \frac{1}{2} = \frac{L}{R}$$

$$\Rightarrow L = \frac{R}{2}$$

Now when $L' = 2.1L = \dfrac{2.1R}{2} = 1.05R$

$$\Rightarrow L' > R$$

Therefore, deviation of the particle is $\theta = 180°$, so that

$$\vec{v}_f = -v_0\hat{i} = \vec{v}_B \text{ and } t_{AB} = \frac{T}{2} = \frac{\pi m}{qB_0}$$

ILLUSTRATION 16

A proton is at rest at the plane vertical boundary of a region containing a uniform vertical magnetic field B. An alpha particle moving horizontally makes a head-on elastic collision with the proton. Immediately after the collision, both particles enter the magnetic field, moving perpendicular to the direction of the field. The radius of the proton's trajectory is R. Find the radius of the alpha particle's trajectory.

SOLUTION

Let u represent the original speed of the alpha particle. Let v_α and v_p represent the particles speeds after the collision. By Law of Conservation of Momentum, we have $4m_pu = 4m_pv_\alpha + m_pv_p$ and the relative velocity equation gives

$$u - 0 = v_p - v_\alpha \qquad \{\because e = 1\}$$

Eliminating u,

$$4v_p - 4v_\alpha = 4v_\alpha + v_p$$

$$\Rightarrow 3v_p = 8v_\alpha$$

$$\Rightarrow v_\alpha = \frac{3}{8}v_p$$

For the proton's motion in the magnetic field,

$$\sum F = ma$$

$$\Rightarrow \quad ev_pB\sin(90°)=\frac{m_pv_p^2}{R}$$

$$\Rightarrow \quad \frac{eBR}{m_p}=v_p$$

For the alpha particle,

$$q_\alpha v_\alpha B\sin 90°=\frac{m_\alpha v_\alpha^2}{r_\alpha}$$

$$\Rightarrow \quad (2e)v_\alpha B=\frac{(4m_p)v_\alpha^2}{r_\alpha}$$

$$\Rightarrow \quad r_\alpha=\frac{2m_pv_\alpha}{eB}$$

$$\Rightarrow \quad r_\alpha=\frac{2m_p}{eB}\frac{3}{8}v_p=\frac{2m_p}{eB}\frac{3}{8}\frac{eBR}{m_p}=\frac{3}{4}R$$

ILLUSTRATION 17

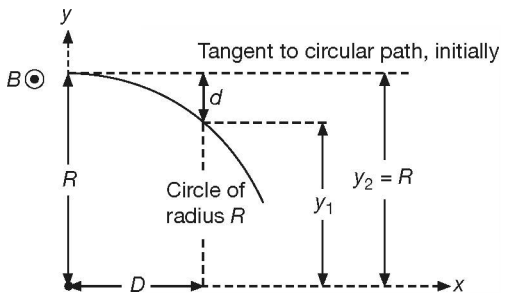
In the electron gun of a TV picture tube the electrons (charge $-e$, mass m) are accelerated by a voltage V. After leaving the electron gun, the electron beam travels a distance D to the screen through a region having transverse magnetic field of magnitude B and no electric field. Sketch the path of the electron beam in the tube. Show that the approximate deflection of the beam due to this magnetic field is

$$d=\frac{BD^2}{2}\sqrt{\frac{e}{2mV}}$$

Calculate the value of d if $V=750$ V, $D=50$ cm and $B=5\times10^{-5}$ T. Is this deflection significant?

SOLUTION

The path is sketched in figure. It is circular with radius $R=\frac{mv}{qB}$. Here we must note that $R\gg D$



Since the motion is circular, therefore we have $x^2+y^2=R^2$. So, at $x=D$ we get $y_1=\sqrt{R^2-D^2}$ (path of deflected particle).

$y_2=R$ (equation for tangent to the circle, path of undeflected particle).

$$d=y_2-y_1=R-\sqrt{R^2-D^2}$$

$$\Rightarrow \quad d=R-R\sqrt{1-\frac{D^2}{R^2}}$$

$$\Rightarrow \quad d=R\left[1-\sqrt{1-\frac{D^2}{R^2}}\right]$$

When $R\gg D$ we get,

$$d\approx R\left[1-\left(1-\frac{1}{2}\frac{D^2}{R^2}\right)\right]=\frac{D^2}{2R}$$

For a particle moving in a magnetic field,

$$R=\frac{mv}{qB}$$

But $\frac{1}{2}mv^2=qV$

$$\Rightarrow \quad R=\frac{1}{B}\sqrt{\frac{2mV}{q}}$$

Thus, the deflection d is given by

$$d\approx\frac{D^2B}{2}\sqrt{\frac{q}{2mV}}=\frac{D^2B}{2}\sqrt{\frac{e}{2mV}}$$

$$\Rightarrow \quad d=\frac{(0.5)^2(5\times10^{-5})}{2}\sqrt{\frac{1.6\times10^{-19}}{2(9.11\times10^{-31})(750)}}$$

$$\Rightarrow \quad d=0.067\text{ m}=6.7\text{ cm}$$

Since we see that $d\approx13\%$ of D, which is fairly significant.

Also, here we observe that

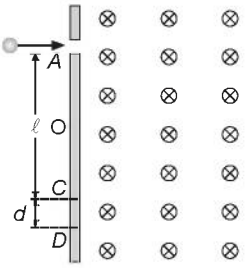
$$R=\frac{1}{B}\sqrt{\frac{2mV}{e}}=\frac{D^2}{2d}=\left(\frac{D}{2d}\right)D=3.7D$$

$$\Rightarrow \quad \left(\frac{R}{D}\right)^2=14$$

So, the approximation made i.e., $R\gg D$ is valid.

ILLUSTRATION 18

A beam of equally charged particles after being accelerated through a voltage V enters into a magnetic field B as shown in the figure. It is found that all the particles hit the plate between C and D. Find the ratio between the masses of the heaviest and lightest particles of the beam.



SOLUTION

When accelerated through a potential V, the particles gain a velocity v such that

$$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

When the particles enter the field, they follow a circular track $\left(\because \vec{v} \perp \vec{B}\right)$ of radius r, where

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\Rightarrow \quad r \propto \sqrt{m}$$

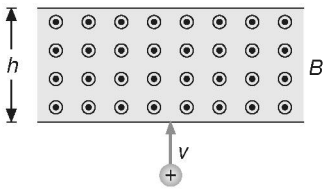
$$\Rightarrow \quad m \propto r^2$$

i.e., the heavier particles will hit the point D and have a radius $(d+l)$ whereas the light particles will hit the point C and have a radius l.

$$\Rightarrow \quad \frac{m_{\text{Heavy}}}{m_{\text{Light}}} = \left(\frac{d+l}{l}\right)^2 = \left(1+\frac{d}{l}\right)^2$$

ILLUSTRATION 19

As shown in figure, a particle of mass m having positive charge q is initially travelling with velocity $v\hat{j}$. At the origin of coordinates it enters a region between $y=0$ and $y=h$ containing a uniform magnetic field $B\hat{k}$ directed perpendicularly out of the page.



(a) What is the critical value of velocity, $v = v_c$, such that the particle just reaches $y = h$? Describe the path of the particle under this condition and predict its final velocity.

(b) Specify the path that the particle takes and its final velocity, if v is less than the critical value.

(c) Specify the path that the particle takes and its final velocity if v is greater than the critical value.

SOLUTION

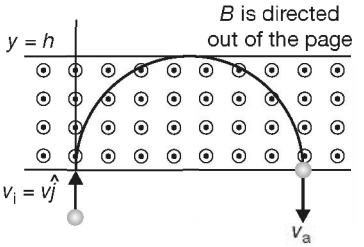
When the particle is in the field, then it follows a circular path of radius r.

Since $qvB = \frac{mv^2}{r}$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

(a) For the particle just to reach h, we have

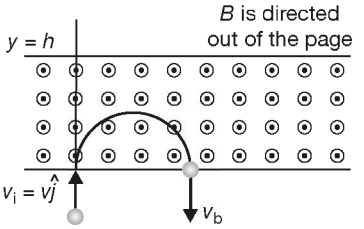
$$r = h = \frac{mv_c}{qB}$$



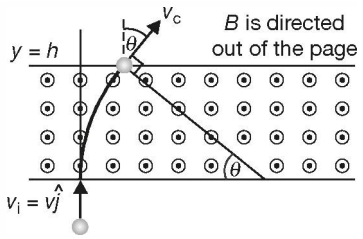
$$\Rightarrow \quad v_c = \frac{qBh}{m},$$

So, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h,\ 0,\ 0)$ with velocity $v_f = -v_c\hat{j}$

(b) When $v < v_c$ i.e., $v < \frac{qBh}{m}$, the particle will move in a smaller semicircle of radius $r = \frac{mv}{qB} < h$. It will leave the field at $(2r,\ 0, 0)$ with velocity $v_f = -v\hat{j}$



(c) When $v > \frac{qBh}{m}$, the particle moves in a circular arc of radius $r = \frac{mv}{qB} > h$, centred at $(r,\ 0,\ 0)$. The arc subtends an angle given by $\theta = \sin^{-1}\left(\frac{h}{r}\right)$.



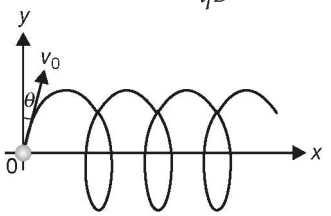
It will leave the field at the point with coordinates $[r(1-\cos\theta),\ h,\ 0]$ with velocity $v_f = v\sin\theta\hat{i} + v\cos\theta\hat{j}$

ILLUSTRATION 20

A particle of mass m and change q is lying at the origin in a uniform magnetic field B directed along x-axis. At time $t=0$, it is given a velocity v_0 at an angle θ with the y-axis in the xy-plane. Find the coordinates of the particle after one revolution.

SOLUTION

After one revolution the particle will have a forward motion equal to the pitch of the helix, so its x component equals pitch $p = \dfrac{2\pi m v_0 \cos(90-\theta)}{qB}$



$$\Rightarrow \quad x = \frac{2\pi m v_0 \sin\theta}{qB}$$

However, y and z coordinates still continue to be zero. Hence, after one revolution, the coordinates are

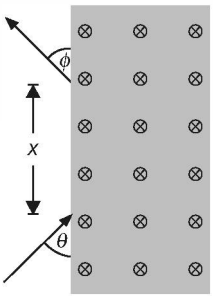
$$(x,\ y,\ z) \equiv \left(\frac{2\pi m v_0 \sin\theta}{qB},\ 0, 0\right)$$

Test Your Concepts-II

Based on Charged Particle in a Magnetic Field

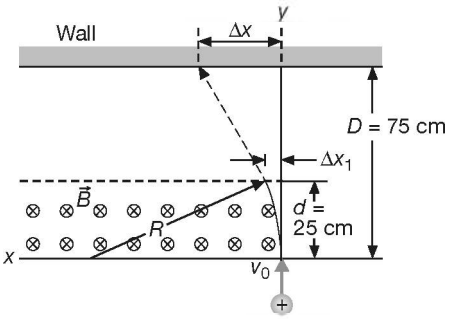
(Solutions on page H.5)

1. In a region of space, a beam of charged particles travels in a straight line. What can be the conclusion made by you about the magnetic field in the region. Assume that no electric field exists in the region.
2. In a region of space where only magnetic field is present, an electron beam projected along positive x-axis deflects along the positive y-axis. What is the direction of the field?
3. Can a charged particle be accelerated by a magnetic field? Can its speed be increased?
4. An electron and a proton are projected with same velocity perpendicular to a magnetic field.
   (a) Which particle will describe the smaller circle?
   (b) Which particle will have greater frequency?
5. A proton moving in the plane of the page has a kinetic energy of 6 MeV. A magnetic field of magnitude $B = 1$ T is directed into the page. The proton enters the magnetic field with its velocity vector at an angle $\theta = 45°$ to the linear boundary of the field as shown in figure.



   (a) Find x, the distance from the point of entry to where the proton will leave the field.
   (b) Determine ϕ, the angle between the boundary and the proton's velocity vector as it leaves the field.
6. A particle with charge 2.15 μC and mass 3.2×10^{-11} kg is initially travelling in the $+y$-direction with a speed $v_0 = 1.45\times10^5$ ms^{-1} and then enters a region containing a uniform magnetic field that is directed inwards as shown. The magnitude of the field is 0.42 T. The region extends

a distance of 25 cm along the initial direction of travel; 75 cm from the point of entry into the magnetic field region is a wall. The length of the field free region is thus 50 cm. When the charged particle enters the magnetic field, it follows a curved path whose radius of curvature is R. It then leaves the magnetic field after a time t_1, having been deflected a distance Δx_1. The particle then travels in the field free region and strikes the wall after undergoing a total deflection Δx.



(a) Determine the radius R of the curved part of the path.

(b) Determine t_1, the time the particle spends in the magnetic field.

(c) Determine Δx_1, the horizontal deflection at the point of exit from the field.

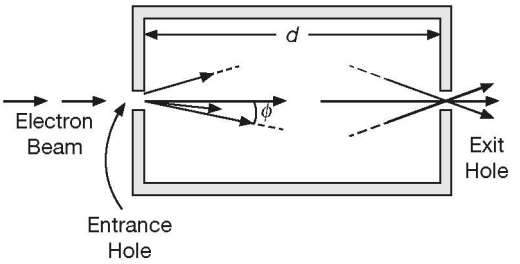
(d) Determine Δx, the total horizontal deflection.

7. A proton of charge e and mass m enters a uniform magnetic field $\vec{B} = B\hat{i}$ with an initial velocity $\vec{v} = v_x\hat{i} + v_y\hat{j}$. Find an expression in unit-vector notation for its velocity at time t.

8. A singly charged ion of mass m is accelerated from rest by a potential difference ΔV. It is then deflected by a uniform magnetic field (perpendicular to the ion's velocity) into a semicircle of radius R. Now a doubly charged ion of mass m' is accelerated through the same potential difference and deflected by the same magnetic field into a semicircle of radius $R' = 2R$. What is the ratio of the masses of the ions?

9. A negatively charged particle (of mass m and charge q) is thrown with speed v along x-axis in a region of uniform, magnetic field B directed along positive z-axis. If thickness of the region is $d\left(< \frac{mv}{qB}\right)$, calculate the angular deviation suffered by the particle as it comes out of the magnetic field.

10. Electrons in a beam are accelerated from rest through a potential difference ΔV. The beam enters an experimental chamber through a small hole, as shown in figure. The electron velocity vectors lie within a narrow cone of half angle ϕ oriented along the beam axis. Calculate the magnitude of a uniform magnetic field directed parallel to the axis to focus the beam, so that all of the electrons can pass through a small exit hole on the opposite side of the chamber after travelling a length d in the chamber.



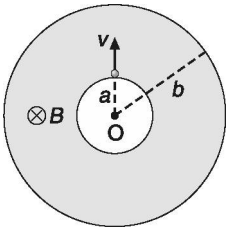
11. A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$) and an alpha particle (charge $+2e$, mass $4m_p$) are accelerated through a common potential difference ΔV. Each of the particles enters a uniform magnetic field $\vec{B}$, with its velocity in a direction perpendicular to $\vec{B}$. The proton moves in a circular path of radius r_p. Calculate the radii of the circular orbits for the deuteron, r_d and the alpha particle, r_α, in terms of r_p.

12. One electron collides elastically with a second electron initially at rest. After the collision, the radii of their trajectories are 1 cm and 2.4 cm. The trajectories are perpendicular to a uniform magnetic field of magnitude 0.044 T. Calculate the energy (in keV) of the incident electron.

13. A particle of mass m and charge q is accelerated by a potential difference V and made to enter a magnetic field region at an angle θ with the field. At the same moment another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field induction is B. What should be the speed of second particle so that both the particles meet again and again after regular minimum interval of time? Also find the time

interval after which they meet and the distance travelled by the second particle during that interval.

14. A uniform magnetic field B exists in the annular space enclosed between two cylindrical shells of the inner radius a and outer radius of b as shown in Figure.



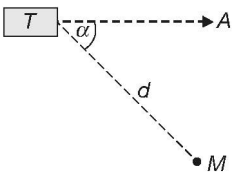
An electron is projected from the surface of inner cylindrical shell perpendicular to it with some initial velocity. The magnetic field is directed along the common axis of the cylindrical shells. Calculate the maximum initial velocity with which this electron should be projected so that it will not hit the inner surface of the outer shell.

15. An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1 mT. The angular momentum of the electron about the centre of the circle is 4×10^{-25} Js. Find the
(a) the radius of the circular path and
(b) the speed of the electron.

16. A charged particle having mass m and charge q is accelerated by a potential difference V. It flies through a region having uniform transverse magnetic field B. The field occupies a region of space d. Find the time interval for which it remains inside the magnetic field.

17. A beam of protons with a velocity of 2×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Calculate the radius of helical path followed by the proton beam and pitch of the helix.

18. An electron gun emits electrons accelerated by potential difference of 1000 V along direction TA as depicted in figure. We want the electrons leaving T to strike target M in the direction making an angle $\alpha = 60°$ with direction TA at a distance d from T. Given $d = 5$ cm.



(a) Calculate the uniform magnetic field, which is applied normal to the plane ATM so that the electrons leaving T strike the target M.
(b) Also calculate the minimum value of B applied parallel to TM so that the electrons leaving T strike the target M.

LORENTZ FORCE

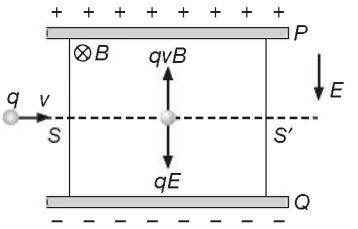
When a particle is subjected to both electric and magnetic fields in the same region, the total force on it is called the **Lorentz force**.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

We know that the path of particle in a uniform magnetic field is a helix and the path in a uniform electric field is *parabola*. When a particle is subject to both electric and magnetic fields, the motion is in general quite complex. However, the special cases in which the fields are either *parallel* to *perpendicular* to each other are simple to analyse.

CROSSED ELECTRIC AND MAGNETIC FIELDS: VELOCITY SELECTOR

Suppose both the uniform electric and the magnetic fields are applied in the region between two parallel plates P and Q as shown in Figure.



If $\vec{F}=\vec{0}$, then $\vec{E}+\vec{v}\times\vec{B}=\vec{0}$

$\Rightarrow \quad \vec{E}=-\vec{v}\times\vec{B}$

$\Rightarrow \quad \vec{E}=\vec{B}\times\vec{v}$

i.e. $\vec{E}$ must be perpendicular to both $\vec{B}$ and $\vec{v}$.

A positive charge q enters through the slit S perpendicular to both $\vec{E}$ and $\vec{B}$. It is acted upon by forces qE downwards and qvB upwards. If the two forces are equal in magnitude the particle will pass through the region undeflected. So, for equilibrium, we have

$$F_m = F_e$$

$$\Rightarrow \quad qvB = qE$$

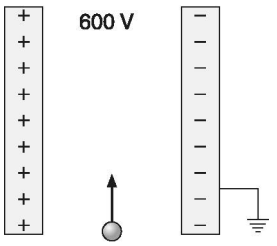
$$\Rightarrow \quad v=\frac{E}{B}$$

This situation is obtained when all three $(v,\ E \text{ and } B)$ are mutually perpendicular

Only those particles will pass out of the slit S' which satisfy the above relation. This device is used to select charged particles of a particular speed out of a beam having particles which move at different speeds and hence, it is called a **velocity selector**.

ILLUSTRATION 21

A potential difference of 600 V is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 ms^{-1} moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects). (Charge of the electron $=1.6\times10^{-19}$ C).



SOLUTION

Electron passes undeviated, so

$$|\vec{F}_e|=|\vec{F}_m|$$

$$\Rightarrow \quad eE=eBv$$

Since, $E=\frac{V}{d}$

$$\Rightarrow \quad B=\frac{E}{v}=\frac{V/d}{v}$$

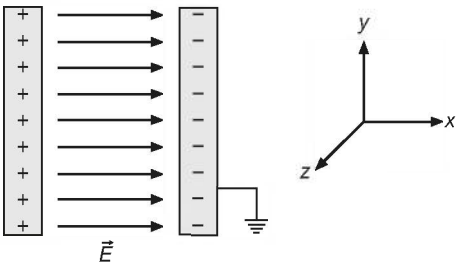
$$\Rightarrow \quad B=\frac{V}{vd}$$

Substituting the values, we have

$$\Rightarrow \quad B=\frac{600}{3\times10^{-3}\times2\times10^6}=0.1 \text{ T}$$

Further, for the electron to pass undeviated, direction of $\vec{F}_e$ should be opposite of $\vec{F}_m$ and both must have same magnitude.

Since, $\vec{E}$ is in positive x- direction as shown in Figure, so the electrostatic force on the electron will be opposite to $\vec{E}$ i.e. along negative x- direction and hence the magnetic force $\vec{F}_m=q(\vec{v}\times\vec{B})$ must be directed along positive x- direction.



Therefore $\vec{v}\times\vec{B}$ should be in positive x- direction and hence the magnetic field $\vec{B}$ should be in negative z- direction (or perpendicular) to paper inwards, because velocity of electron is in positive y- direction.

ILLUSTRATION 22

A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with a velocity 1.28×10^6 ms^{-1} in the $+X$ direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x=E_y=0$, $E_z=-102.4$ kVm^{-1} and $B_x=B_z=0$, $B_y=8\times10^{-2}$ Wbm^{-2}. The particle enters this region at the origin at time $t=0$. Determine the location (x, y and z co-ordinates) of the particle at $t=5\times10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t=7.45\times10^{-6}$ s ?

SOLUTION

$$\vec{F}_e = q\vec{E} = \left(1.6\times10^{-19}\right)\left(-102.4\times10^{3}\right)\hat{k}$$

$$\Rightarrow \quad \vec{F}_e = -\left(1.6384\times10^{-14}\right)\hat{k}\ \text{N}$$

$$\vec{F}_m = q\left(\vec{v}\times\vec{B}\right)$$

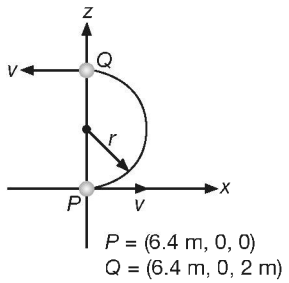
$$\Rightarrow \quad \vec{F}_m = \left(1.6\times10^{-19}\right)\left(1.28\times10^{6}\,\hat{i}\times 8\times10^{-2}\,\hat{j}\right)$$

$$\Rightarrow \quad \vec{F}_m = \left(1.6384\times10^{-14}\right)\hat{k}\ \text{N}$$

Since, $\vec{F}_e + \vec{F}_m = \vec{0}$, so net force on the charged particle is zero and hence the particle will move undeviated. In time $t = 5\times10^{-6}$ s, the x-coordinate of particle will become,

$$x = v_x t = \left(1.28\times10^{6}\right)\left(5\times10^{-6}\right) = 6.4\ \text{m}$$

The y and z-coordinates will be zero.



At $x = 5\times10^{-6}$ s, electric field is switched-off. Only magnetic field is left which is perpendicular to its velocity. Hence, path of the particle will now become circular. Plane of circle will be normal to magnetic field i.e., x-z plane. Radius and angular velocity of circular path is given by

$$r = \frac{mv}{qB} = \frac{\left(10^{-26}\right)\left(1.28\times10^{6}\right)}{\left(1.6\times10^{-19}\right)\left(8\times10^{-2}\right)} = 1\ \text{m}$$

$$\omega = \frac{qB}{m} = \frac{\left(1.6\times10^{-19}\right)\left(8\times10^{-2}\right)}{\left(10^{-26}\right)} = 1.28\times10^{6}\ \text{rads}^{-1}$$

In the remaining time, i.e.,

$$(7.45-5)\times10^{-6} = 2.45\times10^{-6}\ \text{s}$$

Angle rotated by particle, in this time is

$$\theta = \omega t = \left(1.28\times10^{6}\right)\left(2.45\times10^{-6}\right)$$

$$\Rightarrow \quad \theta = 3.14\ \text{rad} \approx 180^{\circ}$$

The z-coordinate of particle is then given by

$$z = 2r = 2\ \text{m}$$

and y-coordinate will be zero.

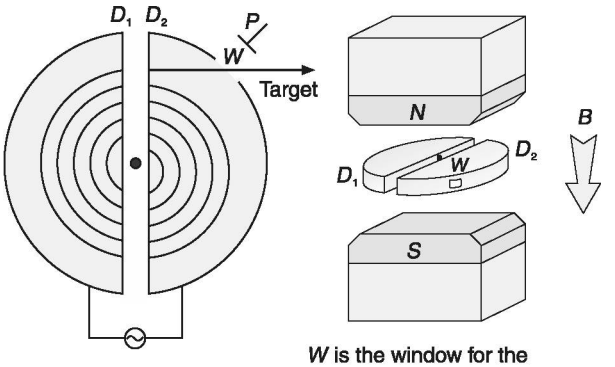
Position of particle at

$$t = 5\times10^{-6}\ \text{s is } P \equiv (6.4\ \text{m}, 0, 0) \text{ and at}$$

$$t = 7.45\times10^{-6}\ \text{s is } Q \equiv (6.4\ \text{m}, 0, 2\ \text{m})$$

THE CYCLOTRON

It is a device to accelerate charged particles to high speeds. The alternating potential difference source accelerates the particles in the gap between the Dees D_1 and D_2 and the magnetic field makes them move in circular orbits. The frequency of the alternating potential difference is made equal to the cyclotron frequency $\frac{qB}{2\pi m}$ so that the polarity of the dees is reversed exactly when the particle comes back in the gap. It then gets accelerated towards the other dee. This process continues till the particle is taken out through a gap with the help of a deflector.



If r is the radius of the chamber then the speed of the particle circulating at this radius is

$$v = \frac{qBr}{m}$$

Therefore, the kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m}$$

PATH OF A CHARGED PARTICLE IN BOTH ELECTRIC AND MAGNETIC FIELD

When the particle is exposed to simultaneous electric and magnetic field, then the force $\vec{F}$ acting on the particle is

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

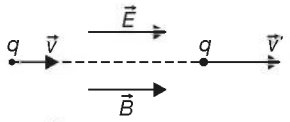
$$\Rightarrow \quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

CASE-1: When $\vec{v}$, $\vec{E}$ and $\vec{B}$ are Collinear

As the particle is moving parallel or antiparallel to the magnetic field, the magnetic force on it will be zero. The electric force $\vec{F}_e$ will produce an acceleration,

$$\vec{a} = \frac{\vec{F}_e}{m} = \frac{q\vec{E}}{m}$$

The particle follows a straight-line path with change in speed. So, in this situation speed, velocity, momentum and kinetic energy all will change, without change in direction, of motion as shown in figure.



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are collinear

At any instant, the velocity of the particle is $v = at = \left(\frac{qE}{m}\right)t$, displacement is $s = \frac{1}{2}at^2 = \left(\frac{qE}{2m}\right)t^2$.

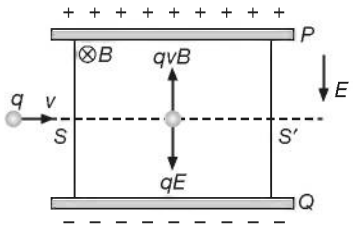
CASE-2: $\vec{v}$, $\vec{E}$ and $\vec{B}$ are Mutually Perpendicular (just as the case of velocity selector)

In this situation we consider a specific case when $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = \vec{0}$$

$$\Rightarrow \quad \vec{a} = \left(\frac{\vec{F}}{m}\right) = \vec{0}$$

The particle will pass through the field with same velocity as shown in Figure.



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular

So, in this situation, we have

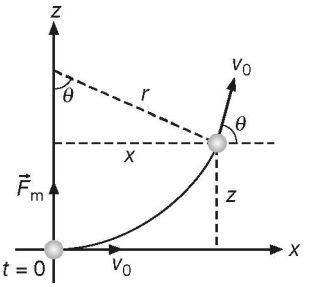
$$F_e = F_m$$

$$\Rightarrow \quad qE = qvB$$

$$\Rightarrow \quad v = \frac{E}{B}$$

CASE-3: When $\vec{E} \parallel \vec{B}$ and particle velocity is perpendicular to both of these fields.

Consider a particle of charge q and mass m released from the origin with velocity $\vec{v} = v_o\hat{i}$ into a region of uniform electric and magnetic fields parallel to y-axis. i.e., $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$ as shown in Figure.



The electric field accelerates the particle in y-direction, i.e., y component of velocity goes on increasing with acceleration,

$$a_y = \frac{F_y}{m} = \frac{F_e}{m} = \frac{qE_0}{m} \qquad \ldots(1)$$

The magnetic field rotates the particle in a circle in x-z plane (perpendicular to magnetic field). The resultant path of the particle is a helix with increasing pitch. The axis of the plane is parallel to y-axis. Velocity of the particle at time t would be

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

where, $v_y = a_y t = \left(\frac{qE_0}{m}\right)t$

Since, $v_x^2 + v_z^2 = \text{constant} = v_0^2$ and $\theta = \omega t = \left(\frac{qB}{m}\right)t$

$$\Rightarrow \quad v_x = v_0\cos\theta = v_0\cos\left(\frac{qBt}{m}\right) \text{ and}$$

$$v_z = v_0\sin\theta = v_0\sin\left(\frac{qBt}{m}\right)$$

$$\Rightarrow \quad \vec{v}(t) = v_0\cos\left(\frac{qBt}{m}\right)\hat{i} + \left(\frac{qE_0}{m}t\right)\hat{j} + v_0\sin\left(\frac{qBt}{m}\right)\hat{k}$$

Similarly, position vector of particle at time t can be given by

$$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

where $x = r\sin\theta = \left(\frac{mv_0}{qB}\right)\sin\left(\frac{qBt}{m}\right)$

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2$$

and $z = r(1-\cos\theta) = \left(\frac{mv_0}{qB}\right)\left[\left\{1-\cos\left(\frac{qBt}{m}\right)\right\}\right]$

$$\Rightarrow \quad \vec{r}(t) = \left(\frac{mv_0}{qB}\right)\sin\left(\frac{qBt}{m}\right)\hat{i} + \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\hat{j} + \left(\frac{mv_0}{qB}\right)\left[\left\{1-\cos\left(\frac{qBt}{m}\right)\right\}\right]\hat{k}$$

CASE-4: When $\vec{E} \perp \vec{B}$ and the particle is released at rest from origin

Consider a particle of charge q and mass m emitted at origin with zero initial velocity into a region of uniform electric and magnetic fields. The field $\vec{E}$ is acting along x-axis and field $\vec{B}$ along y-axis i.e.,

$$\vec{E} = E_0\hat{i} \quad \text{and} \quad \vec{B} = B_o\hat{j}$$

Electric field will provide the particle an acceleration (and therefore a velocity component) in x-direction and the magnetic field will rotate the particle in x-z plane (perpendicular to $\vec{B}$). Hence, at any instant of time its velocity (and hence, position) will have only x and z components. Let at time t its velocity be,

$$\vec{v} = v_x\hat{i} + v_z\hat{k}$$

Net force on it at this instant is

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v}\times\vec{B})$$

$$\Rightarrow \quad \vec{F} = q\left[E_0\hat{i} + \left(v_x\hat{i} + v_z\hat{k}\right)\times\left(B_0\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{F} = q\left(E_0 - v_z B_0\right)\hat{i} + qv_x B_0\hat{k}$$

From Newton's Second Law, we have $\vec{F} = m\vec{a}$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = a_x\hat{i} + a_z\hat{k}$$

where, $a_x = \frac{q}{m}\left(E_0 - v_z B_0\right)$ ...(1)

and $a_z = \frac{q}{m}v_x B_0$ ...(2)

Differentiating Equation (1), w.r.t. time, we have,

$$\frac{da_x}{dt} = \frac{d^2v_x}{dt^2} = -\frac{qB_0}{m}\left(\frac{dv_z}{dt}\right)$$

Since, $\frac{dv_z}{dt} = a_z = \frac{qE_0}{m}v_x$

$$\Rightarrow \quad \frac{d^2v_x}{dt^2} = -\left(\frac{qB_0}{m}\right)^2 v_x \qquad \ldots(3)$$

Comparing this equation with the differential equation of SHM $\left(\frac{d^2y}{dt^2} = -\omega^2 y\right)$, we get

$$\omega = \frac{qB_0}{m}$$

and the general solution of Equation (3) is,

$$v_x = A\sin(\omega t + \phi) \qquad \ldots(4)$$

At time $t = 0$, $v_x = 0$, hence, $\phi = 0$

Again,

$$\frac{dv_x}{dt} = A\omega\cos(\omega t) \qquad \{\text{as } \phi = 0\}$$

From equation (1),

$$a_x = \frac{qE_0}{m} \text{ at } t = 0, \text{ as } v_z = 0 \text{ at } t = 0$$

$$\Rightarrow \quad A\omega = \frac{qE_0}{m}$$

$$\Rightarrow \quad A = \frac{qE_0}{m\omega}$$

Substituting $\omega = \frac{qB_0}{m}$, we get $A = \frac{qE_0}{m\left(\frac{qB_0}{m}\right)} = \frac{E_0}{B_0}$

Therefore, equation (4) becomes,

$$v_x = \frac{E_0}{B_0}\sin(\omega t) \text{ where } \omega = \frac{qB_0}{m} \qquad \ldots(5)$$

Now substituting value of v_x in equation (2), we get

$$a_z = \frac{dv_z}{dt} = \frac{qE_0}{m}\sin(\omega t)$$

$$\Rightarrow \quad \int_0^{v_z} dv_z = \frac{qE_0}{m}\int_0^t \sin(\omega t)\,dt$$

$$\Rightarrow \quad v_z = \frac{qE_0}{m\omega}(1-\cos(\omega t))$$

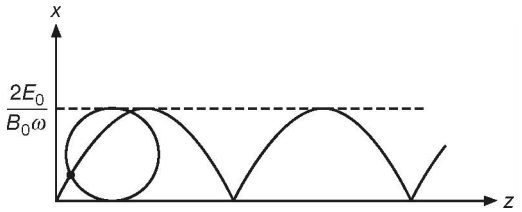
Again substituting $\omega = \frac{qB_0}{m}$, we get

$$v_z = \frac{E_0}{B_0}(1-\cos(\omega t)) \qquad \ldots(6)$$

On integrating equations for v_x and v_z, from (5) and (6), and knowing that at $t = 0$, $x = 0$ and $z = 0$, we get

$$x = \frac{E_0}{B_0\omega}(1-\cos\omega t) \text{ and } z = \frac{E_0}{B_0\omega}(\omega t - \sin\omega t)$$

These equations are the equations for a cycloid which is defined as the path generated by the point on the circumference of a wheel rolling on a ground.



In the present case, radius of the rolling wheel is $\frac{E_0}{B_0\omega}$.

The maximum displacement along x-direction is $\frac{2E_0}{B_0\omega}$.

The x-displacement becomes zero at $t = 0$, $\frac{2\pi}{\omega}$, $\frac{4\pi}{\omega}$, etc.

Problem Solving Technique(s)

MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELDS

STEP-1

In analysing the motion of a charged particle in electric and magnetic fields, you will apply Newton's Second Law of motion, with the net force given by

$$\Sigma\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

Often other forces such as gravity can be neglected.

STEP-2

Read and analyse the problem carefully. The use of components is the most efficient approach. Select a coordinate system and then express all vector quantities (including $\vec{E}$, $\vec{B}$, $\vec{v}$, $\vec{F}$, and $\vec{a}$) in terms of their components in this system.

STEP-3

If the particle moves perpendicular to a uniform magnetic field, the trajectory is a circle whose radius and angular speed are given by

$$R = \frac{mv}{|q|B} \text{ and } \omega = \frac{|q|B}{m}$$

STEP-4

If our calculation involves a more complex trajectory then use $\Sigma\vec{F} = m\vec{a}$ in component form $\Sigma F_x = ma_x$, and so forth. This approach is particularly useful when both electric and magnetic fields are present.

STEP-5

If $\vec{u} = u_x\hat{i} + u_y\hat{j}$ be the initial velocity of the particle and $\vec{v} = v_x\hat{i} + v_y\hat{j}$ be the final velocity of the particle in a magnetic field, then

$$|\vec{v}| = |\vec{u}|$$

$$\Rightarrow \quad v_x^2 + v_y^2 = u_x^2 + u_y^2$$

ILLUSTRATION 23

A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time $t = 0$, the velocity $\vec{v}_0$ of the particle is perpendicular to $\vec{E}$ (Assume that its speed is always $\ll c$, the speed of light in vacuum). Find the velocity $\vec{v}$ of the particle at time t. You must express your answer in terms of t, q, m the vectors $\vec{v}_0$, $\vec{E}$, $\vec{B}$ and their magnitudes v_0, E and B.

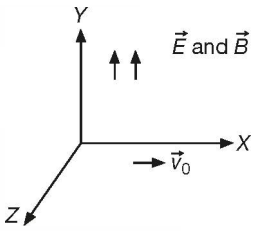
SOLUTION

Let us first find the unit vectors denoting directions of $\vec{E}$, $\vec{B}$, $\vec{v}_0$ and $\vec{F}$. So, we have

$$\hat{j} = \frac{\vec{E}}{E} \quad \text{or} \quad \hat{j} = \frac{\vec{B}}{B},\ \hat{i} = \frac{\vec{v}_0}{v_0}$$

$$\text{and} \quad \hat{k} = \frac{\vec{v}_0\times\vec{B}}{|\vec{v}_0\times\vec{B}|} = \frac{\vec{v}_0\times\vec{B}}{v_0B} \qquad \{\because \vec{v}_0 \perp \vec{B}\}$$

Force due to electric field will be along Y-axis. Magnetic force, acting along z-axis, will not affect the motion of charged particle in the direction of electric field (or Y-axis).

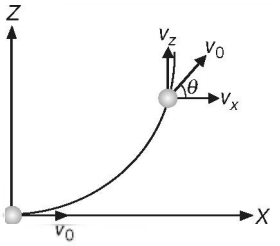


So, $a_y = \frac{F_e}{m} = \frac{qE}{m} = \text{constant}$

$$\Rightarrow \quad v_y = a_y t = \left(\frac{qE}{m}\right)t \qquad \ldots(1)$$

The charged particle under the action of magnetic field describes a circle in x-z plane (perpendicular to $\vec{B}$) with period and angular frequency given by

$$T = \frac{2\pi m}{qB} \quad \text{or} \quad \omega = \frac{2\pi}{T} = \frac{qB}{m}$$



Initially (i.e., at $t = 0$) velocity was along X-axis. Therefore, magnetic force $(\vec{F}_m)$ is given by $\vec{F}_m = q(\vec{v}_0 \times \vec{B})$ and is directed along positive Z-axis. Let v_0 make an angle θ with X-axis at time t, then $\theta = \omega t$. So, we get

$$v_x = v_0 \cos(\omega t) = v_0 \cos\left(\frac{qB}{m}t\right) \text{ and} \qquad \ldots(2)$$

$$v_z = v_0 \sin(\omega t) = v_0 \sin\left(\frac{qB}{m}t\right) \qquad \ldots(3)$$

Since, $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

So, from equations (1), (2) and (3), we get

$$\Rightarrow \quad v = v_0 \cos\left(\frac{qBt}{m}\right)\left(\frac{\vec{v}_0}{v_0}\right) + \frac{qEt}{m}\left(\frac{\vec{E}}{E}\right) + v_0 \sin\left(\frac{qBt}{m}\right)\left(\frac{\vec{v}_0 \times \vec{B}}{v_0 B}\right)$$

$$\Rightarrow \quad \vec{v} = \cos\left(\frac{qBt}{m}\right)(\vec{v}_0) + \left(\frac{qt}{m}\right)(\vec{E}) + \sin\left(\frac{qBt}{m}\right)\left(\frac{\vec{v}_0 \times \vec{B}}{B}\right)$$

The path of the particle will be a helix of increasing pitch. The axis of the helix is parallel to Y-axis.

ILLUSTRATION 24

A particle with specific charge s leaves the origin in the direction of x-axis with an initial velocity v_0. Uniform electric and magnetic fields with strength E and B are directed along the y-axis. Find the

(a) y-coordinate of the particle when it crosses the y-axis for nth time.

(b) angle α between the particle's velocity vector and the y-axis at that moment.

SOLUTION

(a) The path of the particle will be a helix of increasing pitch. The axis of the helix is parallel to y-axis (parallel to $\vec{E}$) and plane of circle of the helix is the xz plane (perpendicular to $\vec{B}$). The particle will cross the y-axis after time,

$$t = nT = n\left(\frac{2\pi m}{qB}\right) = \frac{2\pi n}{sB}$$

The y-coordinate of particle at this instant is,

$$y = \frac{1}{2}a_y t^2 \text{ where, } a_y = \frac{E_y}{m} = sE$$

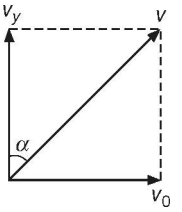
$$\Rightarrow \quad y = \frac{1}{2}(sE)\left(\frac{2\pi n}{sB}\right)^2$$

$$\Rightarrow \quad y = \frac{2\pi^2 n^2 E}{sB^2}$$

(b) At this moment, the y component of its velocity is,

$$v_y = a_y t = (sE)\left(\frac{2\pi n}{sB}\right) = 2\pi n\left(\frac{E}{B}\right)$$

The angle α between the particle's velocity vector and the y-axis at this moment as shown in Figure.



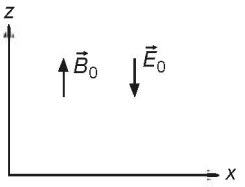
$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{v_0}{v_y}\right)$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi n E}\right)$$

Please note that v_0 is the resultant of the x and z components of the velocity of the particle and $\sqrt{v_x^2 + v_z^2} = v_0$

ILLUSTRATION 25

In a certain region uniform electric field $\vec{E} = -E_0\hat{k}$ and magnetic field $\vec{B} = B_0\hat{k}$ are present. At time $t = 0$ a particle of mass m and charge q is given a velocity $\vec{v} = v_0\left(\hat{j} + \hat{k}\right)$. Find the minimum speed of the particle and the time when happens so.



SOLUTION

$$\vec{E} = -E_0\hat{k} \text{ and } \vec{B} = B_0\hat{k}$$

At $t = 0$, velocity of the particle is $\vec{v} = v_0\hat{j} + v_0\hat{k}$

The electric field will not change the x component of velocity because it is acting along $-z$ direction.

The magnetic force, $\vec{F}_m$ is given by

$$\vec{F}_m = q(\vec{v} \times \vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v_0 & v_0 \\ 0 & 0 & B_0 \end{vmatrix}$$

$$\Rightarrow \quad \vec{F}_m = \left(qv_0B_0\right)\hat{i}$$

Since $\vec{F}_m$ is acting along $+x$ direction, so it will have no effect or change in the $\vec{v}$ (in the yz plane).

Further since $\vec{E} = -E_0\hat{k}$, so the electric force is going to change the velocity of the particle in yz plane.

$$\Rightarrow \quad \vec{v} = \vec{u} + \vec{a}t$$

$$\Rightarrow \quad \vec{v} = \left(v_0\hat{j} + v_o\hat{k}\right) - \left(\frac{qE_0}{m}\right)t\hat{k}$$

$$\Rightarrow \quad \vec{v} = v_0\hat{j} + \left(v_0 - \frac{qE_0t}{m}\right)\hat{k}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{v_0^2 + \left(v_0 - \frac{qE_0t}{m}\right)^2}$$

For $|\vec{v}|$ to be MINIMUM, we have

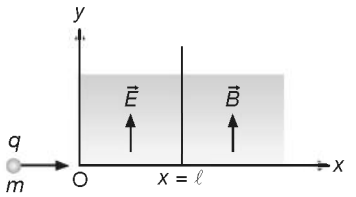
$$v_0 - \frac{qE_0t}{m} = 0$$

$$\Rightarrow \quad t = \frac{mv_0}{qE_0} \text{ and } v_{MIN} = |\vec{v}| = \sqrt{v_0^2 + 0^2}$$

$$\Rightarrow \quad v_{MIN} = v_0 \text{ at } t = \frac{mv_0}{qE_0}$$

ILLUSTRATION 26

A positively charged particle, having charge q, is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bounded by the lines $x = 0$ and $x = l$. Beyond the line $x = l$ (i.e., in the region $x \geq l$) there exists a magnetic field of strength B, directed along the positive y-axis. Find the



(a) distance of the point from x-axis where the particle meets the line $x = l$.

(b) pitch of the helix formed after the particle enters the region $x \geq l$.

Mass of the particle is m.

SOLUTION

Since the particle is accelerated through a potential V, so

$$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

(a) In region from $0 < x < \ell$, electric field E is present along y-axis. So, acceleration of particle is

$$a_y = \frac{qE}{m}$$

Under the influence of this acceleration a_y and initial velocity v (along x-axis) the particle will follow a parabolic path so that it hits the line $x = \ell$ at a distance y from x-axis.

IN REGION OF E

For motion along x-axis

$$v_x = u_x + a_xt$$

$$\Rightarrow \quad v_x = u_x = v \qquad \{\because a_x = 0\}$$

$$\text{and } l = u_xt = vt \qquad \ldots(1)$$

For motion along *y*-axis

$$y = u_y t + \frac{1}{2} a_y t^2$$

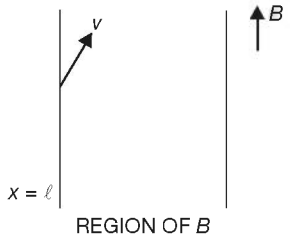
$$\Rightarrow \quad y = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{l}{v}\right)^2$$

$$\Rightarrow \quad y = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\frac{l^2}{\left(\frac{2qV}{m}\right)}$$

$$\Rightarrow \quad y = \frac{El^2}{4V}$$

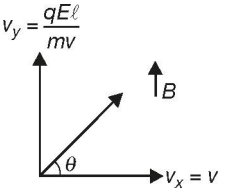
(b) Just when the particle is entering the "Region of B" beyond $x = l$, then it has a velocity $\vec{v}$ having two components v_x and v_y given by

$$v_x = v \text{ and } v_y = \left(\frac{qE}{m}\right)t = \frac{qEl}{mv}$$



Since $v_x \perp B$, so we shall get a helix whose pitch p is given by

$$p = v_y T$$



Please note that here we must understand that the pitch of the helix is due to the component of velocity parallel to the field and in this part of the question v_y happens to be parallel to B. So,

$$p = \frac{2\pi m(v_y)}{qB}$$

$$\Rightarrow \quad p = \left(\frac{2\pi m}{qB}\right)\left(\frac{qEl}{mv}\right)$$

$$\Rightarrow \quad p = \frac{2\pi El}{B\sqrt{2qVm}} = \frac{\pi El}{B}\sqrt{\frac{2m}{qV}}$$

Test Your Concepts-III

Based on Charged Particle in Magnetic and Electric Field

(Solutions on page H.9)

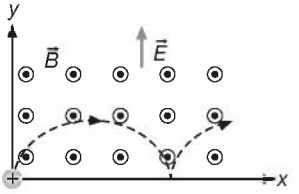
1. A positive charge $q = 3.2 \times 10^{-19}$ C moves with a velocity $\vec{v} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ ms^{-1} through a region where both a uniform magnetic field and a uniform electric field exist.

(a) Calculate the total force on the moving charge (in unit-vector notation), taking $\vec{B} = \left(2\hat{i} + 4\hat{j} + \hat{k}\right)$ T and $\vec{E} = \left(4\hat{i} - \hat{j} - 2\hat{k}\right)$ Vm^{-1}.

(b) What angle does the force vector make with the positive x-axis?

2. A velocity selector consists of electric and magnetic fields described by the expressions $\vec{E} = E\hat{k}$ and $\vec{B} = B\hat{j}$, with $B = 15$ mT. Find the value of E such that a 750 eV electron moving along the positive x-axis is undeflected.

3. A particle with mass m, charge q starts from rest at the origin shown in figure. There is a uniform electric field $\vec{E}$ in the +y-direction and a uniform magnetic field $\vec{B}$ directed out of the page. With our knowledge of Physics, we conclude that the path is a cycloid whose radius of curvature at the top points is twice the y-coordinate at that level.



(a) Explain why the path has this general shape and why it is repetitive.

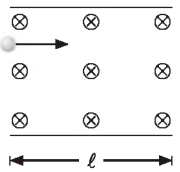
(b) Prove that the speed at any point is equal to $\sqrt{\frac{2qyE}{m}}$.

(c) Applying Newton's Second Law at the top point and assuming that the radius of curvature at the top point equals $2y$, prove that the speed at this point is $\frac{2E}{B}$.

4. A particle of mass m and charge q starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. If its velocity at $(x_0, 0, 0)$ is $(4\hat{i} + 3\hat{j})$. Then calculate x_0.

5. A particle of mass m and charge q starts moving from the origin under the action of an electric field $\vec{E} = E\hat{i}$ and magnetic field $\vec{B} = B\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. Calculate the time t after which the speed of the particle becomes $2v_0$.

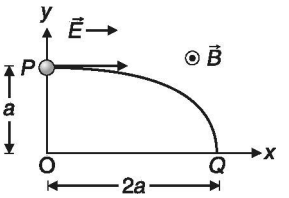
6. An electron moves straight inside a charged parallel plate capacitor of uniform surface charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Calculate the time for which the motion of electron in the capacitor is along a straight line.



7. A charge q of mass m moving with velocity $v_0\hat{i}$ enters a region of magnetic field $B_0\hat{i}$ and electric field $E_0\hat{i}$. If $\frac{E_0}{B_0} = v_0$, then discuss the motion of charged particle.

8. A charge q of mass m moving with velocity $v_0\hat{i} + v_0\hat{j}$ enters a region of magnetic field $B_0\hat{j}$ and electric field $E_0\hat{j}$. Calculate the pitch of the charged particle for this case.

9. A particle of mass m, charge q is moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$. It follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Calculate E, rate of work done by the electric field and magnetic field at the points P and Q.

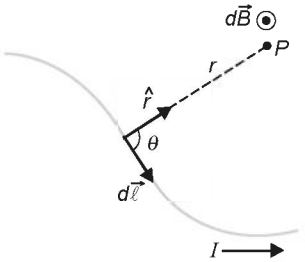


10. A charged particle moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively that co-exist in this region of space, then what conclusions can you make about the magnitudes of E and B in this region?

SOURCES OF MAGNETIC FIELD, BIOT SAVART'S LAW AND AMPERE'S CIRCUITAL LAW

BIOT SAVART'S LAW (BSL)

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire, they produce a current I. The magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire, $d\vec{l}$.



Magnetic field $d\vec{B}$ at point P due to a current carrying element $I\,d\vec{\ell}$

The current segment can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $Id\vec{l}$ and is also called as the **Current Element**.

Let r denote as the distance from the current source to the field point P, and $\hat{r}$ the corresponding unit vector (from the current element to the field point P). On the basis of experiments, Jean Biot and Felix Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{l}$ of a wire carrying a steady current I (or the current element $Id\vec{l}$).

(a) The vector $d\vec{B}$ is perpendicular to both $d\vec{l}$ (which points in the direction of the current) and the unit vector $\hat{r}$ directed from $d\vec{l}$ to P.

(b) The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{l}$ to P.

(c) The magnitude of $d\vec{B}$ is proportional to the current and to the magnitude (dl) of the length element $d\vec{l}$.

(d) The magnitude of $d\vec{B}$ is proportional to sine of the angle between the current element $(Id\vec{l})$ and $\hat{r}$.

The Biot Savart Law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{l}$.

So, mathematically, we write Biot Savart's Law as

$$d\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{Id\vec{l}\times\hat{r}}{r^2}\right) \qquad \ldots(1)$$

where μ_0 is a constant called the permeability of free space, having value

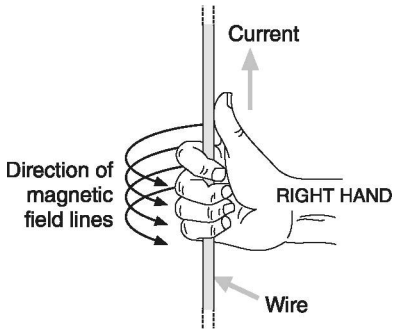
$$\mu_0 = 4\pi\times10^{-7}\ \text{TmA}^{-1} \qquad \ldots(2)$$

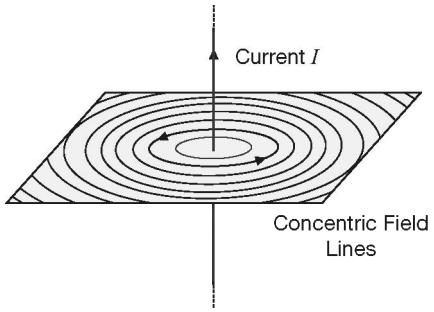
Magnitude of $d\vec{B}$ is given by

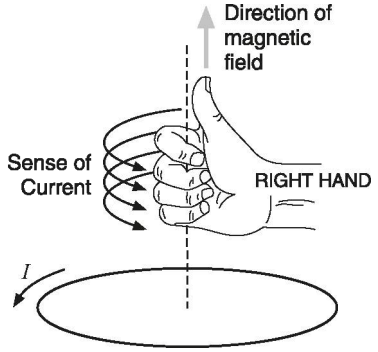
$$|d\vec{B}| = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)$$

The direction of $d\vec{B}$ at the field point P is found by using the RIGHT HAND THUMB RULE, according to which

'Curl the fingers of Right Hand in such a way that thumb points in the direction of current, then the curl of the fingers gives the direction of magnetic field and vice-versa i.e. if we curl the fingers in the sense of current (clock wise or counter clockwise), then direction of the thumb gives direction of the magnetic field.'







Notice that the expression is remarkably similar to the Coulomb's Law for the electric field due to a charge element dq, given by

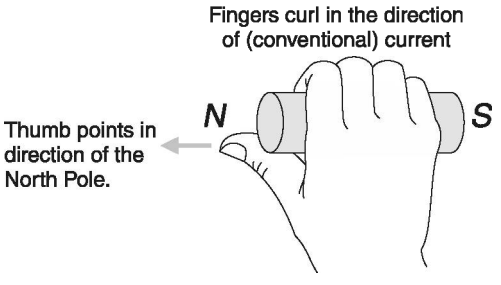
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Adding up these contributions to find the magnetic field at the field point P requires integrating over the entire current source. So, we get

$$\vec{B} = \int_{\text{wire}} d\vec{B} = \frac{\mu_0 I}{4\pi}\int_{\text{wire}} \frac{d\vec{l}\times\hat{r}}{r^2} \qquad \ldots(3)$$

The integral is a vector integral, which means that the expressions for $\vec{B}$ is actually obtained by calculating three integrals (one for each component) of $\vec{B}$. The vector nature of this integral appears in the cross product $Id\vec{l}\times\hat{r}$. Understanding how to evaluate this cross product and then perform the integral will be the key to use the Biot-Savart Law efficiently.

With the help of Right Hand Thumb Rule we can also find the polarity (North and South) of the current carrying conductor as shown.



Problem Solving Technique(s)

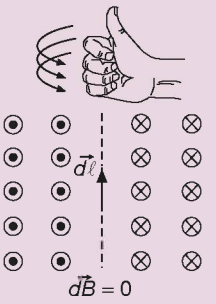
Following points are worth noting regarding the Biot Savart Law.

(a) Magnitude of $d\vec{B}$ is given by,

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

$|d\vec{B}|$ is zero at $\theta = 0°$ or $180°$ and maximum at $\theta = 90°$.

(b) For finding the direction of $d\vec{B}$, either of the following methods can be used.



(i) Since $d\vec{B} \| (d\vec{\ell}\times\hat{r})$. So, $d\vec{B}$ is along $d\vec{\ell}\times\vec{r}$ i.e., $d\vec{B} \perp d\vec{\ell}$ and $d\vec{B} \perp \hat{r}$.

(ii) If $d\vec{\ell}$ is in the plane of paper. $d\vec{B} = 0$ at all points lying on the straight line that passes through $d\vec{\ell}$. The magnetic field to the right of this line is in $\otimes$ direction and to the left of this line is in $\odot$ direction (simply use RIGHT HAND THUMB RULE)

MAGNETIC FIELD OF A MOVING POINT CHARGE

A point charge q, at rest, in the observer's inertial frame produces an electric field. If this charge moves relative to the observer's inertial system, then it also it produces a magnetic field. The magnitude of the magnetic field produced is proportional to the speed of the charge relative to the observer (provided speed $v < c$, the speed of light). An observer travelling with a moving charge (with the same v) detects no magnetic field.

The magnetic field vector $\vec{B}$ at point P at position vector $\vec{r}$ from the charge q moving with a velocity $\vec{v}$ is found by modifying the Biot Savart's Law.

MAGNETIC FIELD DUE TO UNIFORMLY MOVING CHARGE

Biot Savart's Law gives the magnetic field produced by a current or current element $Id\vec{l}$. Since $I = \frac{dq}{dt}$, we can write,

$$Id\vec{l} = \frac{dq}{dt}d\vec{l} = dq\frac{d\vec{l}}{dt} = \vec{v}dq$$

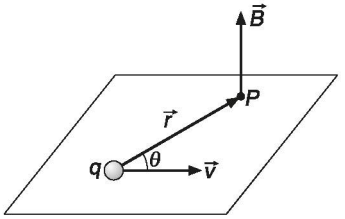
Since $d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{l}\times\vec{r}}{r^3}$

If a single charge q is moving with a velocity $\vec{v}$, it creates a magnetic field given by

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$$

If θ is the angle between $\vec{v}$ and $\vec{r}$ (or $\hat{r}$), then we get

$$|\vec{B}| = \frac{\mu_0}{4\pi}\left(\frac{qv\sin\theta}{r^2}\right)$$



For the expression $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ following points must be kept in mind.

(a) Direction of $\vec{B}$ is along $\vec{v}\times\vec{r}$ if q is positive and opposite to $\vec{v}\times\vec{r}$ if q is negative.

(b) $|\vec{B}|$ is zero at $\theta = 0°$ and $\theta = 180°$.

(c) $|\vec{B}|$ is and maximum at $\theta = 90°$.

(d) $|\vec{B}|$ is inversely proportional to r^2 and not r^3. i.e., $|\vec{B}| \propto \frac{1}{r^2}$.

(e) If a charge q_1 is moving with velocity $\vec{v}_1$ and another charge q_2 is moving with velocity $\vec{v}_2$ at position vector $\vec{r}$ relative to q_1, then force on q_2 is given by

$$\vec{F} = q_2(\vec{v}_2\times\vec{B})$$

Since we know that $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$, so we get

$$\vec{F} = q_2\left[(\vec{v}_2)\times\left(\frac{\mu_0}{4\pi}\frac{q_1}{r^3}\right)(\vec{v}_1\times\vec{r})\right]$$

$$\Rightarrow \quad \vec{F} = \frac{\mu_0}{4\pi}\frac{q_1q_2}{r^3}\left[(\vec{v}_2)\times(\vec{v}_1\times\vec{r})\right]$$

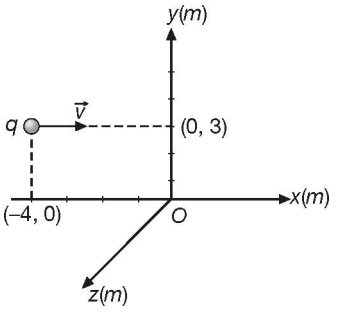
Since, $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$

$$\Rightarrow \quad \vec{F} = \frac{\mu_0}{4\pi}\frac{q_1q_2}{r^3}\left[(\vec{v}_2\cdot\vec{r})\vec{v}_1 - (\vec{v}_2\cdot\vec{v}_1)\vec{r}\right]$$

This corresponds to Coulomb's electrical force between the charges q_1 and q_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively relative to an observer at rest.

ILLUSTRATION 27

A point charge of magnitude $q = 4.5$ nC is moving with speed $v = 3.6\times10^7$ ms^{-1} parallel to the x-axis along the line $y = 3$ m. Calculate the magnetic field at the origin produced by this charge when the charge is at the point $P(-4, 3)$ m as shown in Figure.

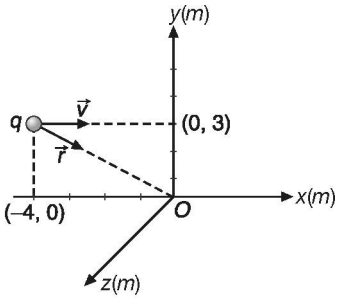


SOLUTION

The magnetic field is given by

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2},$$

where $\vec{v} = v\hat{i}$ and $\vec{r}$ is a vector drawn from the charge to the point at which field is to be calculated.



$$\Rightarrow \quad \vec{r} = \left(4\hat{i} - 3\hat{j}\right) \text{ m}$$

$$\Rightarrow \quad r = \sqrt{4^2 + 3^2} \text{ m} = 5 \text{ m}$$

Unit vector in the direction of $\vec{r}$ is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{4\hat{i} - 3\hat{j}}{5} = 0.8\hat{i} - 0.6\hat{j}$$

$$\Rightarrow \quad \vec{v} \times \hat{r} = \left(v\hat{i}\right) \times \left(0.8\hat{i} - 0.6\hat{j}\right) = -0.6v\hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\left(\frac{q\vec{v} \times \hat{r}}{r^2}\right) = \frac{\mu_0}{4\pi}\frac{q\left(-0.6v\hat{k}\right)}{r^2}$$

$$\Rightarrow \quad \vec{B} = -\left(10^{-7}\right)\frac{\left(4.5 \times 10^{-9}\right)(0.6)\left(3.6 \times 10^{7}\right)}{(5)^2}\hat{k}$$

$$\Rightarrow \quad \vec{B} = \left(-3.89 \times 10^{-10} \text{ T}\right)\hat{k}$$

ILLUSTRATION 28

A negative point charge $q = -7.2$ mC is moving in a reference frame. When the point charge is at the origin, the magnetic field it produces at the point $(25, 0, 0)$ cm is $\vec{B} = (6\ \mu\text{T})\hat{j}$, and its speed is 800 kms^{-1}.

(a) Find the x, y and z components of the velocity $\vec{v}_0$ of the charge.

(b) At this same instant, what is the magnitude of the magnetic field that the charge produces at the point $(0, 25, 0)$ cm?

SOLUTION

(a) Since $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\left(\vec{v}_0 \times \hat{r}\right)}{r^2}$, where

$$\hat{r} = \hat{i}, \ r = 0.25 \text{ m and } \vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} + v_{0z}\hat{k}$$

$$\Rightarrow \quad \vec{v}_0 \times \hat{r} = v_{0z}\hat{j} - v_{0y}\hat{k}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0z}\hat{j} - v_{0y}\hat{k}\right) = \left(6 \times 10^{-6} \text{ T}\right)\hat{j}$$

$$\Rightarrow \quad v_{0y} = 0$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{q}{r^2}v_{0z} = 6 \times 10^{-6} \text{ T}$$

$$\Rightarrow \quad v_{0z} = \frac{4\pi\left(6 \times 10^{-6} \text{ T}\right)(0.25 \text{ m})^2}{\mu_0\left(-7.2 \times 10^{-3} \text{ C}\right)} = -521 \text{ ms}^{-1}$$

Since $v_0^2 = v_{0x}^2 + v_{0y}^2 + v_{0z}^2$

$$v_{0x} = \pm\sqrt{v_0^2 - v_{0y}^2 - v_{0z}^2}$$

$$\Rightarrow \quad v_{0x} = \pm\sqrt{\left(800 \text{ ms}^{-1}\right)^2 - \left(-521 \text{ ms}^{-1}\right)^2}$$

$$\Rightarrow \quad v_{0x} = \pm 607 \text{ ms}^{-1}$$

The sign of v_{0x} isn't determined

(b) Now $\vec{r} = \hat{j}$ and

$$r = 0.25 \text{ m}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\left(\vec{v}_0 \times \hat{r}\right)}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0x}\hat{k} - v_{0z}\hat{i}\right)$$

$$\Rightarrow \quad |\vec{B}| = B = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}\sqrt{v_{0x}^2 + v_{0z}^2} = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_0$$

$$\Rightarrow \quad |\vec{B}| = \frac{\mu_0}{4\pi}\frac{\left(7.2 \times 10^{-3} \text{ C}\right)}{(0.25 \text{ m})^2}\left(800 \text{ ms}^{-1}\right)$$

$$\Rightarrow \quad |\vec{B}| = 9.2 \times 10^{-6} \text{ T} = 9.2\ \mu\text{T}$$

The magnetic field in part (b) doesn't depend on the sign of v_{0x}.

ILLUSTRATION 29

Point charges Q_1 and Q_2 are constrained to move along the x and y-axes, respectively, with the same uniform speed v. At time $t = 0$, both the charges are at the origin. Calculate the Lorentz force $\vec{F}$ acting on Q_2 due to the magnetic field of Q_1 at time t.

SOLUTION

The expression for the magnetic field $\vec{B}$ at a point P at position vector $\vec{r}$ from the charge Q moving with a velocity $\vec{v}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Q\left(\vec{v} \times \vec{r}\right)}{r^3}$$

Since both charges start from the origin at $t = 0$, so the position vector of the charge Q_1 is $\vec{r}_1 = \vec{v}_1 t$ and the position vector of the charge Q_2 is $\vec{r}_2 = \vec{v}_2 t$. The vector displacement $\vec{r}$ of Q_2 w.r.t. Q_1 is

$$\vec{r}_{12} = (\vec{v}_2 - \vec{v}_1)t$$

The magnetic field at the position of Q_2 due to the charge Q_1 is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Q_1(\vec{v}_1 \times \vec{r}_{12})}{r^3} = \frac{\mu_0}{4\pi}\frac{Q_1\vec{v}_1 \times (\vec{v}_2 - \vec{v}_1)t}{t^3|\vec{v}_2 - \vec{v}_1|^3}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{Q_1\vec{v}_1 \times (\vec{v}_2 - \vec{v}_1)}{t^2|\vec{v}_2 - \vec{v}_1|^3}$$

The force on a charge Q_2 moving with at velocity $\vec{v}_2$ in a magnetic field $\vec{B}$ is

$$\vec{F}_{21} = Q_2(\vec{v}_2 \times \vec{B})$$

$$\Rightarrow \quad \vec{F}_{21} = \left(\frac{\mu_0 Q_1 Q_2}{4\pi t^2|\vec{v}_2 - \vec{v}_1|^3}\right)\left[\vec{v}_2 \times \{\vec{v}_1 \times (\vec{v}_2 - \vec{v}_1)\}\right]$$

Since $\vec{v}_1 \times \vec{v}_1 = 0$, so we get

$$\vec{F}_{21} = \left(\frac{\mu_0 Q_1 Q_2}{4\pi t^2|\vec{v}_2 - \vec{v}_1|^3}\right)\left[\vec{v}_2 \times (\vec{v}_1 \times \vec{v}_2)\right] \qquad \ldots(1)$$

Taking $\vec{v}_1 = v\hat{i}$ and $\vec{v}_2 = v\hat{j}$, we get

$$\vec{v}_1 \times \vec{v}_2 = v^2\hat{k}$$

$$\vec{v}_2 \times (\vec{v}_1 \times \vec{v}_2) = v^3\hat{i} \text{ and}$$

$$|\vec{v}_1 - \vec{v}_2| = \sqrt{2}v$$

$$\Rightarrow \quad |\vec{v}_1 - \vec{v}_2|^3 = (2^{3/2})v^3$$

From equation (1), the magnetic force is given by

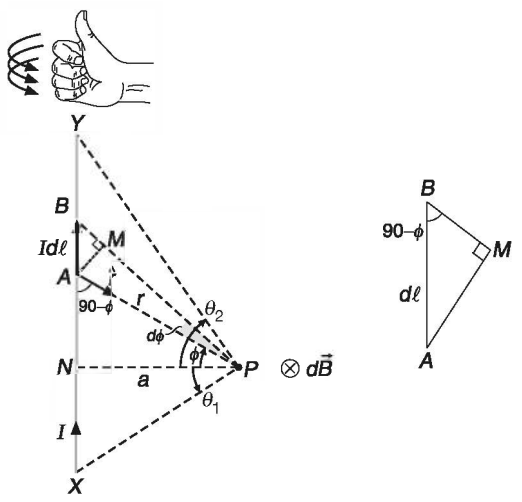
$$\vec{F}_{21} = \frac{\mu_0}{4\pi}\left(\frac{Q_1 Q_2}{2\sqrt{2}t^2}\right)\hat{i}$$

$$\Rightarrow \quad F_{21} = \frac{\mu_0 Q_1 Q_2}{8\pi t^2\sqrt{2}}, \text{ parallel to the } x\text{-axis.}$$

MAGNETIC FIELD AROUND A THIN, STRAIGHT CURRENT CARRYING CONDUCTOR

METHOD I

Consider a current carrying conductor XY carrying current I from X to Y. Let us find the magnetic field at the point P (at perpendicular distance a) due to the conductor. Let the angles subtended by the conductor at the point P be θ_1 and θ_2 as shown in Figure.



According to Biot Savart's Law, the field at P due to an infinitesimal current element $Id\vec{l}$ is given by

$$dB = |d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Id\ell\sin(90-\phi)}{r^2}$$

$$\{\because \angle PAN = 90 - \phi\}$$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\cos\phi}{r^2}\right), \text{ into the page} \qquad \ldots(1)$$

Since Idl is very small, so

$$\angle PAN \cong \angle PBN \cong (90-\phi)$$

Now, in triangle BAM, we have

$$\sin(90-\phi) = \frac{AM}{AB} = \frac{AM}{dl}$$

$$\Rightarrow \quad AM = dl\cos\phi$$

Substituting in (1), we get

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I(AM)}{r^2}$$

But $AM = rd\phi$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I(rd\phi)}{r^2} = \frac{\mu_0}{4\pi}\left(\frac{I}{r}\right)d\phi \qquad \ldots(2)$$

However, we observe that in triangle APN, $\cos\phi = \frac{a}{r}$

$$\Rightarrow \quad r = \frac{a}{\cos\phi} \qquad \ldots(3)$$

Substituting, (3) in (2), we get

$$dB = \frac{\mu_0}{4\pi}\left(\frac{I}{a}\right)\cos\phi d\phi \qquad \ldots(4)$$

Total field is obtained by integrating (4), over the complete wire. Hence

$$B=\int dB=\frac{\mu_0}{4\pi}\left(\frac{I}{a}\right)\int_{-\theta_1}^{\theta_2}\cos\phi\,d\phi=\frac{\mu_0 I}{4\pi a}\left(\sin\phi\Big|_{-\theta_1}^{\theta_2}\right)$$

$$\Rightarrow\quad B=\frac{\mu_0 I}{4\pi a}\left[\sin\theta_2-\sin(-\theta_1)\right]=\frac{\mu_0 I}{4\pi a}\left(\sin\theta_2+\sin\theta_1\right)$$

So finally, we get

$$B=\frac{\mu_0 I}{4\pi a}\left(\sin\theta_1+\sin\theta_2\right)\qquad\text{(inwards)}$$

WORD OF ADVICE

Please note that, $AB\neq rd\phi$ (or $d\ell\neq rd\phi$). However, $AM=rd\phi$.

MISCONCEPTION REMOVAL

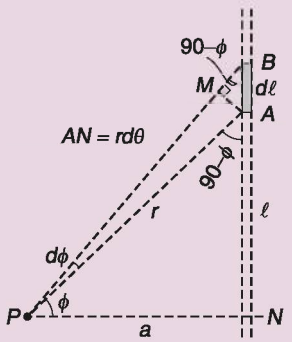
While attempting this problem, we may think that $d\ell=rd\phi$, but that would be a wrong step. From the magnified diagram of the element shown $PA=r$ and $PM=r$. So, its AM that equals $rd\theta$ and not AB. Since AB is extremely small, so $\angle NAP=\angle NBP=90-\theta$. Now, we observe the triangle ABM, then

$$\sin(90-\phi)=\frac{AM}{AB}=\frac{rd\phi}{d\ell}$$

$$\Rightarrow\quad d\ell=\frac{rd\phi}{\cos\phi}$$

$$\Rightarrow\quad d\ell=\frac{ad\phi}{\cos^2\phi}$$

$$\Rightarrow\quad d\ell=a\sec^2\phi\,d\phi$$

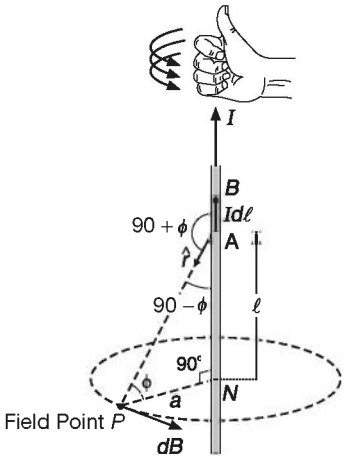


Otherwise we would have got $d\ell=a\sec\phi d\phi$ which definitely would not fetch us correct result for the magnetic field B.

METHOD II

According to Biot Savart's Law, $d\vec{B}$ due to infinitesimal element Idy is given by

$$dB=\frac{\mu_0}{4\pi}\frac{Idl\sin(90+\phi)}{r^2}$$



$$\Rightarrow\quad dB=\frac{\mu_0}{4\pi}\frac{I(dl\cos\phi)}{r^2}\qquad\ldots(1)$$

In triangle APN, we have

$$\tan\phi=\frac{l}{a}$$

$$\Rightarrow\quad l=a\tan\phi$$

$$\Rightarrow\quad dl=a\sec^2\phi d\phi\qquad\ldots(2)$$

Substitute (2) in (1), we get

$$dB=\frac{\mu_0}{4\pi}\frac{I\left(a\sec^2\phi d\phi\right)\cos\phi}{r^2}$$

$$\Rightarrow\quad dB=\frac{\mu_0}{4\pi}\frac{Ia\sec\phi d\phi}{r^2}$$

But $a\sec\phi=r$ (see from triangle APN)

$$\Rightarrow\quad dB=\frac{\mu_0}{4\pi}\left(\frac{Id\phi}{r}\right)=\frac{\mu_0}{4\pi}\left(\frac{I\cos\phi d\phi}{a}\right)\quad\left\{\because\ \frac{a}{r}=\cos\phi\right\}$$

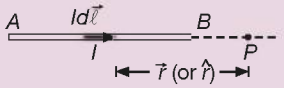
$$\Rightarrow\quad B=\int dB=\frac{\mu_0 I}{4\pi a}\int_{-\theta_1}^{\theta_2}\cos\phi d\phi=\frac{\mu_0 I}{4\pi a}\left(\sin\phi\Big|_{-\theta_1}^{\theta_2}\right)$$

$$\Rightarrow\quad B=\frac{\mu_0 I}{4\pi a}\left[\sin\theta_2-\sin(-\theta_1)\right]$$

$$\Rightarrow\quad B=\frac{\mu_0 I}{4\pi a}\left(\sin\theta_1+\sin\theta_2\right)\qquad\text{(inwards)}$$

Problem Solving Technique(s)

(a) If the point P is along the length of the wire, as shown, then $d\vec{l}$ and $\vec{r}$ or $\hat{r}$ will either be parallel or antiparallel, thus giving $\theta = 0°$ or $\theta = 180°$.



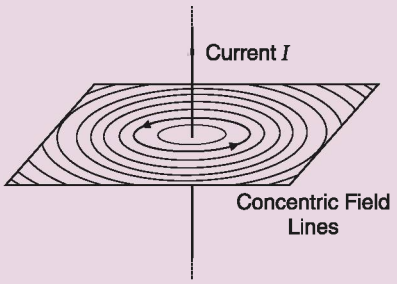
Hence, we get $d\vec{l} \times \hat{r} = \vec{0}$ or $d\vec{l} \times \vec{r} = \vec{0}$

So, at a point P along the length of wire, we have

$$\vec{B} = \vec{0}$$

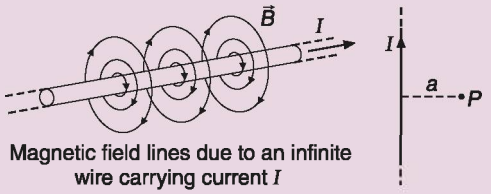
(b) Also note that for the points along the length of the wire (but not on it) the field is always zero.

(c) The field is always perpendicular to the plane containing the wire and the point. So, in a plane perpendicular to the wire the lines of force are concentric circles.



(d) For wire of infinite length, we have $\theta_1 \to \frac{\pi}{2}$ and $\theta_2 \to \frac{\pi}{2}$

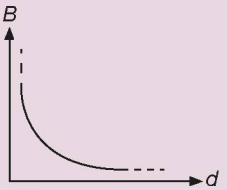
$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\mu_0 I}{2\pi a}$$



Magnetic field lines due to an infinite wire carrying current I

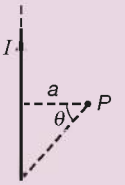
(e) For points at a perpendicular distance d from the wire, field B varies inversely with distance, so

$$B \propto \frac{1}{d} \qquad \left\{\text{and not } \frac{1}{d^2}\right\}$$



(f) For a semi-infinite wire, we have

$$B = \frac{\mu_0 I}{4\pi a}(1 + \sin\theta)$$

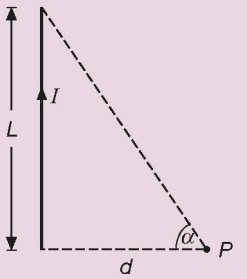


(g) For semi-infinite wire, the field at point P that lies on the perpendicular passing through the finite end of the wire is

$$B = \frac{\mu_0 I}{4\pi a}(1 + \sin\theta) = \frac{\mu_0 I}{4\pi a}$$

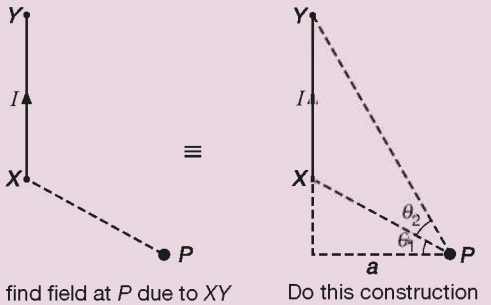
$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}$$

(h) If the wire is of finite length L and the point is near its one end, then $\theta_1 = \alpha$ and $\theta_2 = 0°$. Hence,



$$B = \frac{\mu_0 I}{4\pi d}\sin\alpha \text{ with } \sin\alpha = \frac{L}{\sqrt{(L^2 + d^2)}}$$

(i) Consider the arrangement shown in Figure.

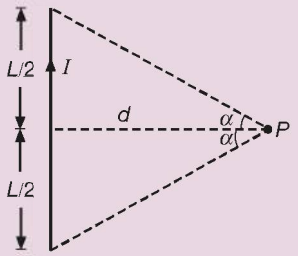


To find field at P due to XY Do this construction

The field due to current carrying wire *XY* at point *P* is given by

$$B=\frac{\mu_0 I}{4\pi a}(\sin\theta_2-\sin\theta_1)$$

(j) If the wire is of finite length *L* and the point lies at its perpendicular bisector, then $\theta_1=\theta_2=\alpha$ and hence we get



$$B=\frac{\mu_0 I}{2\pi d}\sin\alpha$$

where $\sin\alpha=\dfrac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+d^2}}=\dfrac{L}{\sqrt{(L^2+4d^2)}}$

where *L* is length of the wire.

ILLUSTRATION 30

An infinite wire passing through origin along the direction $\hat{i}+\hat{j}+\hat{k}$ carries a current I. Calculate the magnetic field due to the wire at point $(1,0,0)$ m.

SOLUTION

From the concept of three-dimensional geometry, we have

$$\vec{r}_\perp=\frac{\left|\vec{P_0P_1}\times\text{Directing }\vec{\text{Vector}}\right|}{\left|\text{Directing }\vec{\text{Vector}}\right|}$$

The directing vector $\left(\vec{DV}\right)$ of current is $\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{P}_0=(0,0,0)$, $\vec{P}_1=(1,0,0)$

$$\Rightarrow\quad \vec{P_0P_1}=\hat{i}$$

$$\Rightarrow\quad \vec{P_0P_1}\times\vec{DV}=\begin{vmatrix}\overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}}\\ 1 & 0 & 0\\ 1 & 1 & 1\end{vmatrix}$$

$$\Rightarrow\quad \vec{P_0P_1}\times\vec{DV}=\hat{i}(0-0)-\hat{j}(1-0)+\hat{k}(1-0)$$

$$\Rightarrow\quad \vec{P_0P_1}\times\vec{DV}=-\hat{j}+\hat{k}$$

$$\Rightarrow\quad \left|\vec{P_0P_1}\times\vec{DV}\right|=\sqrt{2}$$

Also, $\left|\vec{DV}\right|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$$\Rightarrow\quad \vec{r}_\perp=\frac{\left|\vec{P_0P_1}\times\vec{DV}\right|}{\left|\vec{DV}\right|}=\frac{\sqrt{2}}{\sqrt{3}}\text{ m}$$

Since $B=\dfrac{\mu_0 I}{2\pi r_\perp}$

$$\Rightarrow\quad B=\frac{\mu_0 I}{2\pi\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}\text{ T}$$

$$\Rightarrow\quad B=\frac{\sqrt{3}\mu_0 I}{2\sqrt{2}\pi}\text{ T}$$

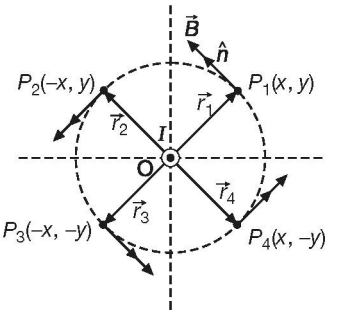
ILLUSTRATION 31

A long straight wire along the *z*-axis carries a current I in the positive *z* direction. On the $z=0$ plane, the magnetic field vector is $\vec{B}_1$ at the point $P_1(x,y)$, is $\vec{B}_2$ at the point $P_2(-x,y)$, is $\vec{B}_3$ at the point $P_3(-x,-y)$ and is $\vec{B}_4$ at the point $P_4(x,-y)$. Calculate $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$ and $\vec{B}_4$.

SOLUTION.

In magnitude, the field at point $P(x,y)$ is

$$B=\frac{\mu_0 I}{2\pi r}$$



However, vectorially the field is given by

$$\vec{B}=\left(\frac{\mu_0 I}{2\pi r}\right)\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}$.

For the point $P_1(x,y)$, we have

$$\vec{r}_1 = x\hat{i} + y\hat{j}$$

A unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}_1$ is given by

$$\hat{n}_1 = \frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$$

Because only then, we have $\vec{r}_1 \cdot \hat{n}_1 = 0$

$$\Rightarrow \quad \vec{B}_1 = \left(\frac{\mu_0 I}{2\pi\sqrt{x^2+y^2}}\right)\left(\frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}\right)$$

$$\Rightarrow \quad \vec{B}_1 = \frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i} - x\hat{j}}{x^2+y^2}\right)$$

Similarly, at the point $P_2(-x,y)$, we have

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi}\left(\frac{-y\hat{i} - x\hat{j}}{x^2+y^2}\right)$$

Similarly, at the point $P_3(-x,-y)$, we have

$$\vec{B}_3 = \frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i} - x\hat{j}}{x^2+y^2}\right)$$

Similarly, at the point $P_4(x,-y)$, we have

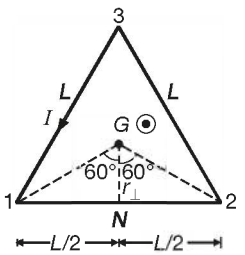
$$\vec{B}_4 = \frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i} + x\hat{j}}{x^2+y^2}\right)$$

ILLUSTRATION 32

Calculate the magnetic field at the centroid of an equilateral current carrying wire frame of length $3L$, carrying a current I.

SOLUTION

Let us find the field due to a single wire '12' first.



$$B_{12} = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\theta + \sin\theta)$$

where $\theta = 60° = \frac{\pi}{3}$ radian and in triangle $G1N$, we have

$$\tan 60° = \frac{N1}{GN} = \frac{\frac{L}{2}}{r_\perp}$$

$$\Rightarrow \quad r_\perp = \frac{L}{2\sqrt{3}}$$

$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi\left(\frac{L}{2\sqrt{3}}\right)}(\sin 60 + \sin 60) = \frac{3\mu_0 I}{2\pi L}, \odot$$

Each side will contribute the field of the same strength in the same direction. Hence, the total field is given by

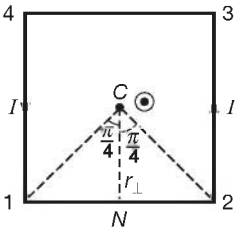
$$B_{\text{total}} = B = 3B_{12} = \frac{9\mu_0 I}{2\pi L}, \odot$$

ILLUSTRATION 33

Calculate the magnetic field at the centre of a current carrying squared wire frame of length $4L$, carrying a current I.

SOLUTION

Let us find the field due to a single wire '12' first.



$$B_{12} = \frac{\mu_0 I}{4\pi(r_\perp)}\left[\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

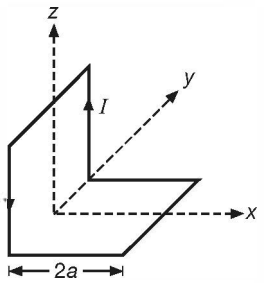
$$\Rightarrow \quad B_{12} = \frac{2\sqrt{2}\mu_0 I}{4\pi L}, \odot$$

Since each side will contribute the field of the same strength in the same direction. Hence, the total field is given by

$$B_{\text{total}} = 4B_{12} = \frac{8\sqrt{2}\mu_0 I}{4\pi L}, \odot$$

ILLUSTRATION 34

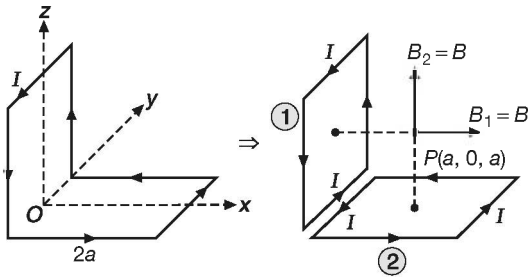
A non-planar loop of conducting wire carrying a current I is placed as shown in Figure.



Each of the straight sections of the loop is of length $2a$. Find the direction of the magnetic field due to this loop at the point $P(a, 0, a)$.

SOLUTION

Consider the bigger loop to be made up of two loops 1 and 2 as shown in Figure.



Magnetic field due to loop 1 and 2 at point P has same value say B. So,

$$\vec{B}_P = B\hat{i} + B\hat{k} = B(\hat{i} + \hat{k})$$

So, magnetic field points along the direction $\hat{n}$ given by

$$\hat{n} = \frac{\vec{B}_P}{|\vec{B}_P|} = \frac{B(\hat{i} + \hat{k})}{\sqrt{2}B} = \frac{\hat{i} + \hat{k}}{\sqrt{2}}$$

ILLUSTRATION 35

Calculate the magnetic field at the centre of a regular n sided current carrying regular polygon wire frame of length nL. Assume that the current in the wire frame is I. Also interpret the obtained result for $n \to \infty$.

SOLUTION

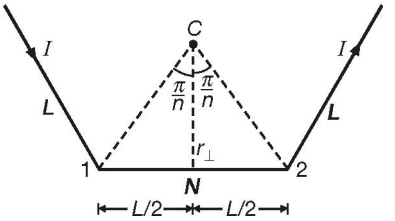
Let us find the field due to a single wire '12' first.

$$B_{12} = \frac{\mu_0 I}{4\pi(r_\perp)}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\right] \qquad \ldots(1)$$

In triangle $CN1$, we have

$$\tan\left(\frac{\pi}{n}\right) = \frac{N1}{CN} = \frac{\frac{L}{2}}{r_\perp}$$

$$\Rightarrow \quad r_\perp = \frac{L}{2\tan\left(\frac{\pi}{n}\right)} \qquad \ldots(2)$$



Substituting, the value of $r_\perp$ from (2) in (1), we get

$$B_{12} = \frac{\mu_0 I\left[2\tan\left(\frac{\pi}{n}\right)\right]}{4\pi L}\left[2\sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi L}\left[4\tan\left(\frac{\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)\right], \odot$$

Each side will contribute the field of the same strength in the same direction. Hence, the total magnetic field at the centre C of the polygon is given by

$$B_{\text{total}} = B = n(B_{12}), \odot$$

$$\Rightarrow \quad B_{\text{total}} = n\left(\frac{\mu_0 I}{4\pi L}\right)\left\{4\tan\left(\frac{\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)\right\}, \odot$$

Now, when $n \to \infty$ then the polygon approaches the circular shape. In that case, we have

$$2\pi r = nL$$

$$\Rightarrow \quad r = \frac{nL}{2\pi}$$

$$\Rightarrow \quad B = \lim_{n\to\infty} B_{\text{total}} = \frac{4\mu_0 I}{n(4\pi L)}\left(\lim_{n\to\infty} n^2 \tan\left(\frac{\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I \pi^2}{\pi(nL)}\lim_{n\to\infty}\left[\left(\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}\right)\left(\frac{\sin\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}\right)\right]$$

Since, $\lim_{n\to\infty}\dfrac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 1$ and $\lim_{n\to\infty}\dfrac{\sin\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 1$

$$\Rightarrow \quad B=\frac{\mu_0 I\pi}{nL}(1)(1)=\frac{\mu_0 I}{2\left(\frac{nL}{2\pi}\right)}=\frac{\mu_0 I}{2r}$$

$$\Rightarrow \quad B=\lim_{n\to\infty} B_{\text{total}}=\frac{\mu_0 I}{2r}=B_{\text{circle}}$$

So, in this problem, by discussing the limiting case condition, we have calculated the magnetic field at the centre of a circular loop (having infinite sides), which is given by

$$B_{\text{circle}}=\frac{\mu_0 I}{2r}$$

ILLUSTRATION 36

Find the magnetic induction at the centre of a rectangular wire frame whose diagonal is equal to $d=16$ cm and the angle between the diagonals is equal to $\varphi=30°$, carrying a current of $I=5$ A.

Take $\tan(15°)=0.27$ and $\sqrt{3}=1.73$.

SOLUTION

In such type of questions where we have been provided with the current value but not the direction, we can assume the direction of current ourselves and then express our results in accordance with the direction taken by us. Let the current flow in the frame from A to C to D to E to A (or in the counter clockwise sense). Then let us find the field due to AC at O. The field due to the wire AC subtending angles $60°$ and $60°$ at O is given by

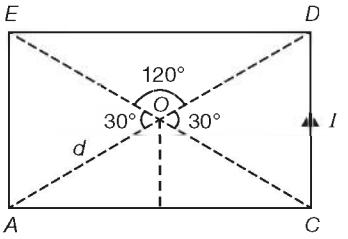
$$B_{AC}=\frac{\mu_0 I}{2\pi(r_\perp)}(\sin 60+\sin 60)$$

where $r_\perp=\frac{d}{2}\cos 60°=\frac{d}{4}$

$$\Rightarrow \quad B_{AC}=\frac{\mu_0 I}{2\pi\left(\frac{d}{4}\right)}(2\sin 60)=\frac{\mu_0 I}{2\pi\left(\frac{d}{4}\right)}\sqrt{3},\ \odot$$

$$\Rightarrow \quad B_{AC}=\frac{2\sqrt{3}\mu_0 I}{\pi d},\ \odot$$

Similarly, $B_{DE}=\frac{2\sqrt{3}\mu_0 I}{\pi d},\ \odot$



$AD = d = 16$ cm
$AO = OD = EO = OC = \frac{d}{2}$

Let us calculate field due to CD at O. Since CD subtends angles $15°$ and $15°$ at O and lies at $r_\perp=\frac{d}{2}\cos(15)$.

So, we get

$$B_{CD}=\frac{\mu_0 I}{2\pi\left(\frac{d}{2}\cos 15\right)}(\sin 15+\sin 15)$$

$$\Rightarrow \quad B_{CD}=\frac{2\mu_0 I}{\pi d}\tan(15),\ \odot$$

Similarly, $B_{EA}=\frac{2\mu_0 I}{\pi d}\tan(15),\ \odot$

So, $B_{\text{total}}=B_{AC}+B_{CD}+B_{DE}+B_{EA},\ \odot$

$$\Rightarrow \quad B_{\text{total}}=\frac{4\sqrt{3}\mu_0 I}{\pi d}+\frac{4\mu_0 I}{\pi d}\tan(15),\ \odot$$

$$\Rightarrow \quad B_{\text{total}}=\frac{4\mu_0 I}{\pi d}\left(\sqrt{3}+\tan 15\right),\ \odot$$

Since $I=5$ A, $d=16$ cm $=0.16$ m and $\tan(15)=0.27$

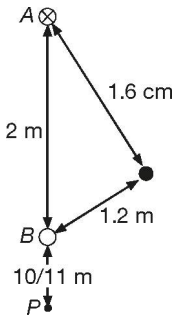
$$\Rightarrow \quad B_{\text{total}}=\frac{4\left(4\pi\times 10^{-7}\right)(5)}{(\pi)(0.16)}(1.73+0.27)$$

$$\Rightarrow \quad B_{\text{total}}=\frac{\left(16\times 10^{-7}\right)(5)(100)}{16}(2)$$

$$\Rightarrow \quad B_{\text{total}}=10^{-4}\ \text{T}=0.1\ \text{mT}$$

ILLUSTRATION 37

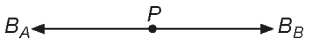
Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 A, directed into the plane of the paper as shown in Figure.



The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $\frac{10}{11}$ m from the wire B, is zero. Calculate the magnitude and direction of the current in B. Also calculate the magnitude of the magnetic field induction at the point S.

SOLUTION

At the point P, the field due to the wires A and B is shown in Figure.



For the net magnetic field at P to be zero, the direction of current at B should be perpendicular to paper outwards. Let current in the wire B be I_B. Then,

$$\frac{\mu_0}{2\pi}\frac{I_A}{\left(2+\frac{10}{11}\right)}=\frac{\mu_0}{2\pi}\frac{I_B}{\left(\frac{10}{11}\right)}$$

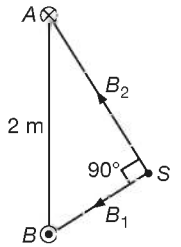
$$\Rightarrow \quad \frac{I_B}{I_A}=\frac{10}{32}$$

$$\Rightarrow \quad I_B=\frac{10}{32}\times I_A=\frac{10}{32}\times 9.6=3 \text{ A}$$

From the data provided in the question, we observe that

$$(AS)^2+(BS)^2=(AB)^2$$

$$\Rightarrow \quad \angle ASB=90^\circ$$



At S magnetic field due to I_A is B_1 given by

$$B_1=\frac{\mu_0}{2\pi}\frac{I_A}{(1.6)}=\frac{\left(2\times10^{-7}\right)(9.6)}{1.6}=12\times10^{-7} \text{ T}$$

and magnetic field due to I_B is B_2 given by

$$B_2=\frac{\mu_0}{2\pi}\frac{I_B}{(1.2)}=\frac{\left(2\times10^{-7}\right)(3)}{1.2}=5\times10^{-7} \text{ T}$$

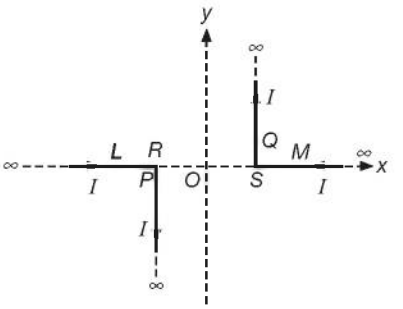
Since, B_1 and B_2 are mutually perpendicular. Net magnetic field at S is

$$B=\sqrt{B_1^2+B_2^2}=\sqrt{\left(12\times10^{-7}\right)^2+\left(5\times10^{-7}\right)^2}$$

$$\Rightarrow \quad B=13\times10^{-7} \text{ T}$$

ILLUSTRATION 38

A pair of stationary and infinitely long bent wires are placed in the xy plane as shown in figure. The wires carry currents of $I=10$ A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that $OS=OR=0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O.



SOLUTION

Magnetic field at O due to L and M is zero. Due to P magnetic field at O is

$$B_1=\frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{I}{OR}\right)=\frac{\left(10^{-7}\right)(10)}{0.02} \text{ T}=5\times10^{-5} \text{ T}$$

(perpendicular to paper outwards)

Similarly, field at O due to Q would be,

$$B_2=\frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{I}{OS}\right)=\frac{\left(10^{-7}\right)(10)}{0.02} \text{ T}=5\times10^{-5} \text{ T}$$

(perpendicular to paper outwards)

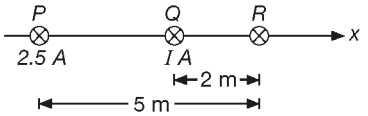
Since, both the fields are in same direction, net field will be sum of these two. Hence

$$B_{net} = B_1 + B_2 = 10^{-4}\ \text{T}$$

Direction of field is perpendicular to the paper outwards.

ILLUSTRATION 39

Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R (shown in figure).

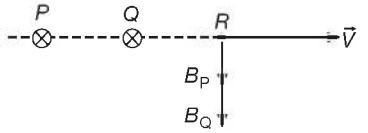


An electron moving with a velocity of 4×10^5 ms^{-1} along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

SOLUTION

Magnetic field at R due to both the wires P and Q will be downwards as shown in Figure.



Therefore, net field at R will be sum of these two.

$$B = B_P + B_Q = \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2} = \frac{\mu_o}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right)$$

$$B = \frac{\mu_0}{4\pi}(I+1) = 10^{-7}(I+1)$$

Net force on the electron will be

$$F_m = qvB\sin 90^\circ = qvB$$

$$\Rightarrow \quad 3.2\times10^{-20} = 1.6\times10^{-19}\left(4\times10^5\right)\left(10^{-7}\right)(I+1)$$

$$\Rightarrow \quad I+1 = 5$$

$$\Rightarrow \quad I = 4\ \text{A}$$

Net field at R due to wires P and Q is

$$B = 10^{-7}(I+1)\ \text{T} = 5\times10^{-7}\ \text{T} \qquad \{\because I = 4\ \text{A}\}$$

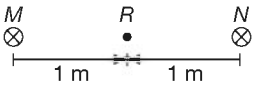
Magnetic field due to third wire carrying a current of 2.5 A should also be 5×10^{-7} T but in upward direction so, that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5\times10^{-7}$$

$$\Rightarrow \quad \frac{\left(2\times10^{-7}\right)(2.5)}{r} = 5\times10^{-7}\ \text{m}$$

$$\Rightarrow \quad r = 1\ \text{m}$$

So, the third wire can be placed at M or N as shown in figure.



If it is placed at M, then current in it should be outwards and if placed at N, then current be inwards.

ILLUSTRATION 40

A current I flows along a lengthy thin-walled tube of radius R with longitudinal slit of width w. Find the induction of the magnetic field inside the tube under the condition $w \ll R$.

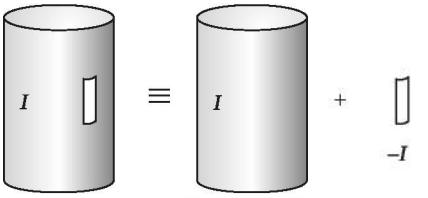
SOLUTION

The magnetic field due to a lengthy thin walled tube at any internal point is zero. When a longitudinal slit of width w is removed, then a net field will exist inside. The tube with a slit can be visualised as equivalent to a full tube carrying a current I and a slit carrying current $-I$ (i.e. current equal in magnitude and opposite in direction). So, the net field inside the tube is due to the slit carrying current I in opposite direction.

Since

$$B_{tube} = 0$$

$$\Rightarrow \quad B_{actual} = B_{slit} = \frac{\mu_0 I_{slit}}{2\pi R}$$



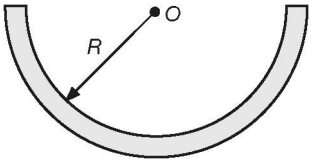
(Since width of slit is very small, so field due to slit is equal to that of a thin wire).
Further

$$I_{slit} = \left(\frac{I}{2\pi R}\right)w$$

$$\Rightarrow \quad B_{actual} = \frac{\mu_0\left(\frac{I}{2\pi R}\right)w}{2\pi R} = \frac{\mu_0 Iw}{4\pi^2 R^2}$$

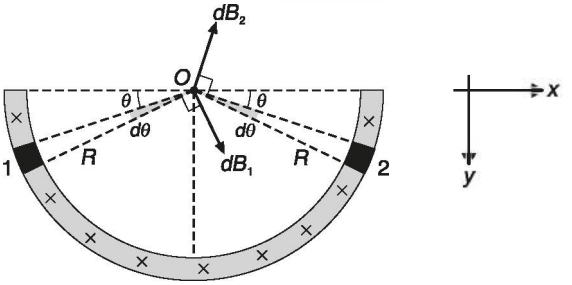
ILLUSTRATION 41

A uniform current I flows in a long straight semi-cylindrical shaped wire with cross-section having the form of a thin half-ring of radius R. Find the induction of the magnetic field at the point O.



SOLUTION

Please note that the diagram drawn shows the cross-sectional view of a straight wire. Consider a long wire (cross-sectional view in black), which is a part of the semi-cylindrical wire of radius R. Let this current carrying infinitesimal wire carry a current dI and be inclined to the x-axis at angle θ and subtends angle $d\theta$ at the centre. For the sake of convenient evaluation, we consider another wire element 2 (mirror image of 1). Due to symmetry of location of point O, both will give equal fields at O. (Please note that 1 and 2 have been attached to the fields just to identify which field is due to which element). Also note that while drawing dB never forget that $d\vec{B} \perp \vec{r}$ (or here $\vec{R}$).



Field due to an infinite wire carrying a current dI at distance R is

$$dB_1 = dB_2 = dB = \frac{\mu_0(dI)}{2\pi R}$$

where $dI = \frac{I}{(\pi R)}(Rd\theta)$

$$\Rightarrow \quad dI = \left(\frac{I}{\pi}\right)d\theta$$

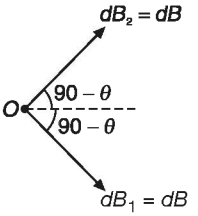
Now, on resolution of dB we observe the components $dB\cos\theta$ cancel. So,

$$B = B_{net} = \int\left(\begin{array}{c}\text{contribution due to}\\ \text{a single element}\end{array}\right)$$

$$\Rightarrow \quad B = \int dB\sin\theta$$

$$\Rightarrow \quad B = \int \frac{\mu_0 dI}{2\pi R}\sin\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi}\sin\theta d\theta$$



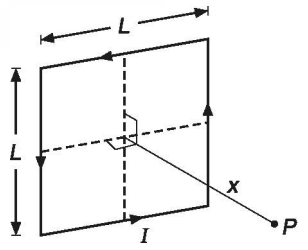
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi^2 R}\left(-\cos\theta|_0^{\pi}\right)$$

$$\Rightarrow \quad B = -\frac{\mu_0 I}{2\pi^2 R}(\cos\pi - \cos 0)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\pi^2 R}$$

ILLUSTRATION 42

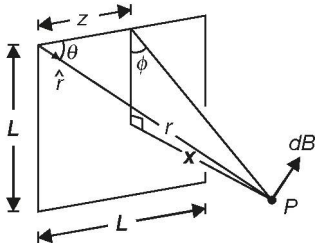
A wire is formed into the shape of a square loop of edge length L carrying a current I. Calculate the magnetic field at point P on the axis of the loop at a distance x from the centre of the loop.



SOLUTION

By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart Law and consider the plane of the square to be the yz plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by

$$\vec{B}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d\ell \times \hat{r}}{r^2}$$



From the figure we see that

$$r = \sqrt{x^2 + \left(\frac{L^2}{4}\right) + z^2}$$

and $|d\ell \times \hat{r}| = dz\sin\theta = dz\sqrt{\dfrac{\left(\frac{L^2}{4}\right)+x^2}{\left(\frac{L^2}{4}\right)+x^2+z^2}}$

By symmetry all components of the field $\vec{B}$ at P cancel except the components along x-axis (perpendicular to the plane of the square) and hence we have

$$B_{0x} = B_0\cos\phi \text{ where } \cos\phi = \frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+x^2}}$$

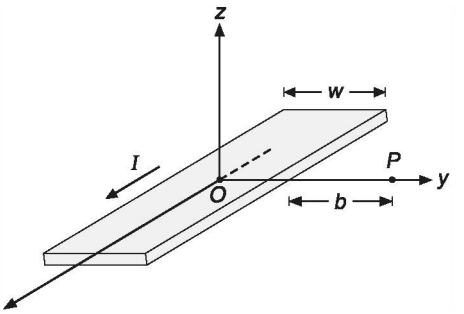
Therefore, $B_{0x} = \dfrac{\mu_0 I}{4\pi}\int_0^{\frac{L}{2}} \dfrac{\sin\theta\cos\phi\, dz}{r^2}$ and $B = 8B_{0x}$

Using the expressions given above for $\sin\theta$, $\cos\phi$ and r, we get

$$B = 8B_{0x} = \frac{\mu_0 I L^2}{2\pi\left(x^2+\frac{L^2}{4}\right)\left(\sqrt{x^2+\frac{L^2}{2}}\right)}$$

ILLUSTRATION 43

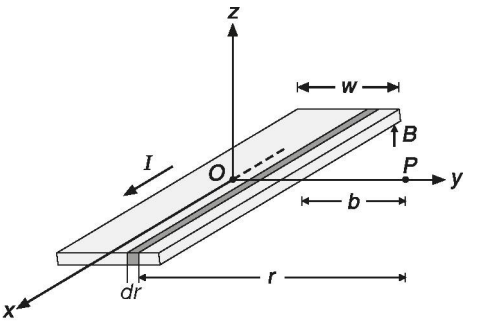
A very long, thin strip of metal of width w carries a current I along its length as shown. Find the magnetic field at the point P in the diagram. The point P is in the plane of the strip at distance b away from it.



SOLUTION

Consider a longitudinal infinitesimal element of the strip of width dr at a distance r from P is shown. The contribution to the field at point P due to the current di in the element dr is

$$dB = \frac{\mu_0 di}{2\pi r}, \text{ upwards towards } z\text{-axis}$$



where $di = I\left(\dfrac{dr}{w}\right)$

$$\Rightarrow \quad \vec{B} = \int d\vec{B} = \int_{b}^{b+w} \left(\frac{\mu_0 I dr}{2\pi w r} \right) \hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi w} \log_e \left(1 + \frac{w}{b} \right) \hat{k}$$

Conceptual Note(s)

If the point is at a large distance from the strip i.e., $b \gg w$

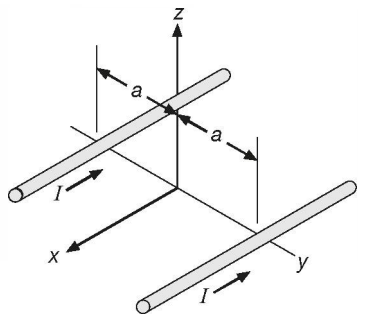
$$\log_e \left[1 + \frac{w}{b} \right] = \frac{w}{b} - \frac{w^2}{2b^2} + \ldots\ldots = \frac{w}{b}$$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi} \frac{2I}{w} \times \frac{w}{b} = \frac{\mu_0 I}{2\pi a}$$

So, we observe that for a distant point, the strip just behaves like a current carrying wire which justifies the above result obtained by approximation.

ILLUSTRATION 44

In figure, both currents in the infinitely long wires are in the negative x-direction.

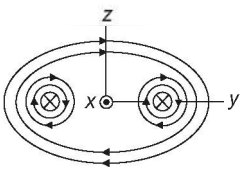


(a) Draw the magnetic field pattern in the yz plane.

(b) Calculate the distance d along the z-axis where the magnetic field is maximum.

SOLUTION

(a)

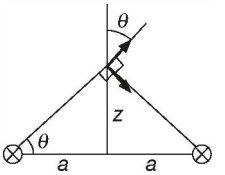


Currents are into the paper

(b) At a point on the z-axis, the contribution from each wire has magnitude $B = \frac{\mu_0 I}{2\pi\sqrt{a^2 + z^2}}$ and is perpendicular to the line from this point to the wire as shown in figure. Combining fields, the vertical components cancel while the horizontal components add, thus giving

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2 + z^2}} \sin\theta \right)$$

$$\Rightarrow \quad B_y = \frac{\mu_0 I}{\pi\sqrt{a^2 + z^2}} \left(\frac{z}{\sqrt{a^2 + z^2}} \right) = \frac{\mu_0 I z}{\pi\left(a^2 + z^2\right)}$$



At a distance z above the plane of the conductors

The condition for a maximum is

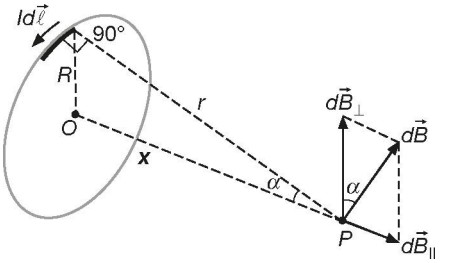
$$\frac{dB_y}{dz} = \frac{-\mu_0 I z (2z)}{\pi\left(a^2 + z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2 + z^2\right)} = 0$$

$$\Rightarrow \quad \frac{\mu_0 I}{\pi}\left(\frac{a^2 - z^2}{a^2 + z^2} \right) = 0$$

Thus, along the z-axis, the field is a maximum at $d = a$

FIELD AT AN AXIAL POINT OF A CURRENT CARRYING CIRCULAR LOOP

Consider a current carrying loop of radius R carrying a current I. Let us find the magnetic field at a point P on the axis of the loop at distance x from its centre as shown in Figure.



For this let us consider a current element $Id\vec{l}$ on circumference of the loop at distance r from P. Please note that r is same for all the current elements taken on the circumference of the loop.

Here, angle θ between the element $d\vec{l}$ and $\vec{r}$ is $\frac{\pi}{2}$ everywhere and r is same for all elements, so we have from Biot Savart's Law

$$dB = \frac{\mu_0}{4\pi}\frac{I(d\vec{l}\times\hat{r})}{r^2} \text{ where } d\vec{l}\times\hat{r} = dl\sin\theta = dl$$

$$\Rightarrow \quad dB = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl\sin(90°)}{r^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl}{r^2}$$

{Biot Savart's Law}

The field $d\vec{B}$ has components both along and perpendicular to the axis of the loop. Also, we observe $d\vec{B}$ to be perpendicular to $Id\vec{l}$ and $\vec{r}$ both. However, if we consider the contributions of the current elements that are diametrically opposite, we see that their components normal to the axis $d\vec{B}_{\perp}$ will cancel.

But, the component of $d\vec{B}$ along the axis is

$$dB_{\parallel} = dB\sin\alpha = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl}{r^2}\left(\frac{R}{r}\right) = \frac{\mu_0 IRdl}{4\pi r^3}$$

The total field is given by the integral of this expression over all elements, so

$$B = B_{\parallel} = B_{\text{axis}} = \int dB_{\parallel}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{r^3}\int_0^{2\pi R} dl = \left(\frac{\mu_0}{4\pi}\right)\frac{I2\pi R^2}{r^3}$$

Since, $r^2 = R^2 + x^2$, and if the coil has N turns, we get

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 IN}{(R^2+x^2)^{3/2}} = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}}$$

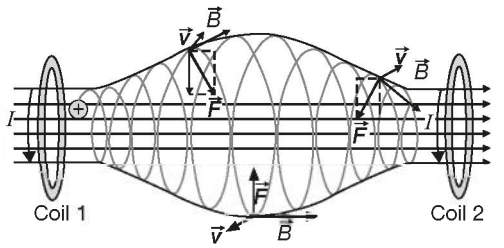
$$\Rightarrow \quad B = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}}, \text{ along the axis of the loop}$$

At the centre of the coil, we have $x = 0$, so

$$B_{\text{centre}} = \frac{\mu_0 NI}{2R}$$

CONCEPT OF MAGNETIC BOTTLE

Motion of a charged particle in a non-uniform magnetic field is more complex. A field produced by two circular coils separated by some distance is shown in Figure.

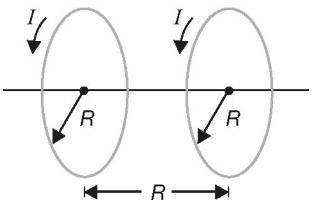


A magnetic bottle. Particles near either end of the region ecperience a magnetic force toward the center of the region.

Particles near either coil experience a magnetic force toward the centre of the region. Particles with appropriate speeds spiral repeatedly from one end of the region to the other and back. Since charged particles can be trapped in such a magnetic field so, it is called a **magnetic bottle**. This technique is used to confine very hot plasmas with temperatures of the order of 10^6 K. In a similar way the earth's non-uniform magnetic field traps charged particles coming from the sun in doughnut-shaped regions around the earth. These regions are called the **Van Allen Radiation Belts**.

ILLUSTRATION 45

Two circular coils of radius R, each with N turns, are perpendicular to a common axis. The centres of the coils are a distance R apart. Each coil carries a steady current I in the same direction, as shown in figure.



Show that the magnetic field on the axis at a distance x from the centre of one coil is

$$B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(2R^2+x^2-2Rx)^{3/2}}\right]$$

Also show that $\frac{dB}{dx}$ and $\frac{d^2B}{dx^2}$ are both zero at the point midway between the coils.

SOLUTION

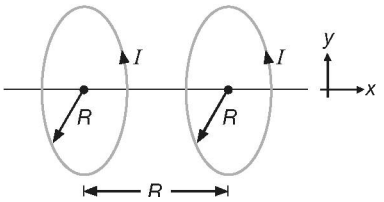
Let us use $B=\frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$ twice, for both the coils.

So, we get

$$B = B_{x_1} + B_{x_2}$$

$$B=\frac{N\mu_0 IR^2}{2}\left[\frac{1}{(x^2+R^2)^{3/2}}+\frac{1}{[(R-x)^2+R^2]^{3/2}}\right]$$

$$\Rightarrow \quad B=\frac{N\mu_0 IR^2}{2}\left[\frac{1}{(x^2+R^2)^{3/2}}+\frac{1}{(2R^2+x^2-2xR)^{3/2}}\right]$$



$$\frac{dB}{dx}=\frac{N\mu_0 IR^2}{2}\left[-\frac{3}{2}(2x)(x^2+R^2)^{-5/2}-\frac{3}{2}(2R^2+x^2-2xR)^{-5/2}(2x-2R)\right]$$

Substituting $x=\frac{R}{2}$ in this equation, we get

$$\frac{dB}{dx}=0$$

Further $\frac{d^2B}{dx^2}=\frac{d}{dx}\left(\frac{dB}{dx}\right)$, so we have

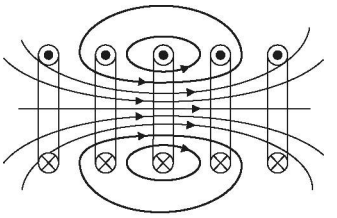
$$\Rightarrow \quad \frac{d^2B}{dx^2}=-\frac{3N\mu_0 IR^2}{2}\left[(x^2+R^2)^{-\frac{5}{2}}-5x^2(x^2+R^2)^{-\frac{7}{2}}+(2R^2+x^2-2xR)^{-\frac{5}{2}}\left(-5(x-R)^2(2R^2+x^2-2xR)^{-\frac{7}{2}}\right)\right]$$

Substituting $x=\frac{R}{2}$, we get $\frac{d^2B}{dx^2}=0$

This means the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called Helmholtz coils.

SOLENOID (AN INTRODUCTION)

Let us consider the field lines due to a current carrying coil having five turns as shown in Figure.



The field lines for five loops

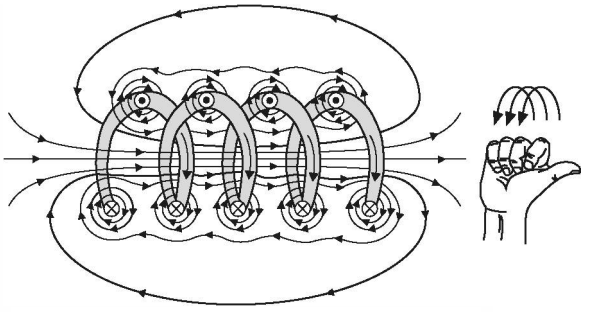
From the figure we observe that

(a) the field lines are always closed loops.
(b) very close to each wire the lines are circular (not shown).
(c) inside the coil the contributions from the different turns reinforce each other, so the field is strong.
(d) near the axis it is fairly uniform.
(e) outside the coil, the contributions from the various current elements tend to cancel, so the field is much weaker and the field outside the coil resembles that of a bar magnet so that one end of the coil acts as a north pole, whereas the other end acts as a south pole.

When the turns are tightly packed and their number becomes very large, the device is called a **solenoid**. The field within a long solenoid is quite uniform and strong, whereas it is essentially zero outside, as shown in figure. Let us now calculate the field strength along the axis of a long solenoid.

Field Along the Axis of a Solenoid

The name solenoid was first given by Ampere to a wire carrying a current I wound in a closely spaced spiral over a hollow cylindrical non-conducting core as shown in Figure.



If n is the number of turns per unit length of the coil, where each turn carries a current I uniformly wound round a cylinder (not shown) of radius R, then the number of turns in length dx are $n(dx)$. Thus, the magnetic field at the axial point O due to this element dx is,

$$dB = \frac{\mu_0}{2}\frac{I(ndx)R^2}{(R^2+x^2)^{3/2}}, \text{ along axis of solenoid}$$

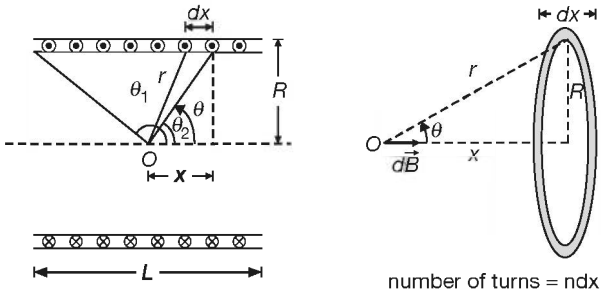
Also, we observe that

$$x = R\cot\theta$$

$$\Rightarrow \quad dx = -R(\text{cosec}^2\,\theta)d\theta$$

$$\Rightarrow \quad dB = -\frac{1}{2}\mu_0(nI\sin\theta)d\theta$$

Total field B due to the entire solenoid is, given by integrating the above expression within appropriate limits of θ as shown in Figure.



$$\Rightarrow \quad B = \frac{1}{2}\mu_0 nI\int_{\theta_1}^{\theta_2}(-\sin\theta)d\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)$$

CASE-1:

If the solenoid is very long $(L \gg R)$ and the point O is chosen at the middle, i.e., if $\theta_1 = 180°$ and $\theta_2 = 0°$, then, we get

$$B(\text{centre}) = \mu_0 nI \qquad \{\text{For } L \gg R\}$$

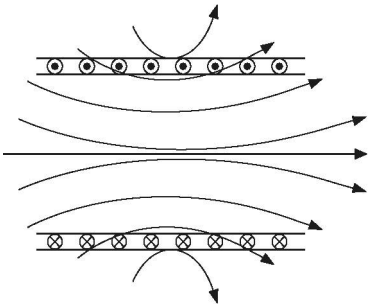
CASE-2:

If the solenoid is very long $(L \gg R)$ and the point O is taken at the ends of the solenoid, then

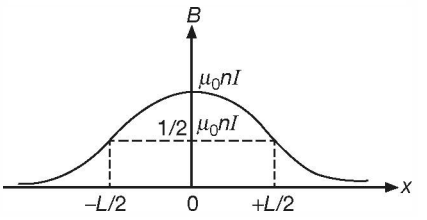
$$\theta_2 = 0°,\ \theta_1 = 90°$$

$$\Rightarrow \quad B(\text{end}) = \frac{1}{2}\mu_0 nI \qquad \{\text{For } L \gg R\}$$

Thus, the field at the end of a solenoid is just one half at the centre. The field lines are as shown in Figure.

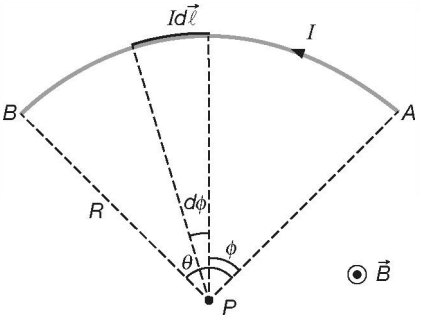


The field variation with distance along the axis of a solenoid is as shown in Figure.



FIELD DUE TO A CIRCULAR CURRENT CARRYING SEGMENT AT ITS CENTRE

Consider a circular segment AB of radius R as shown in Figure.



Let us find the magnetic field at the point P at its centre. We observe that

(a) each element is at the same distance from the centre, i.e., $r = R$ = constant,

(b) the angle between element $d\vec{l}$ and $\vec{r}$ is always $\frac{\pi}{2}$

The contribution of each element to $\vec{B}$ is in the same direction (i.e., out of the page if the current is anti-clockwise and into the page if clockwise). From Biot Savart's Law, we have

$$|\vec{B}| = \frac{\mu_0}{4\pi}\int \frac{I(d\vec{l}\times\hat{r})}{r^2} = \frac{\mu_0}{4\pi}\int_A^B \frac{Idl}{R^2}$$

But $dl = Rd\phi$,

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\int_0^\theta \frac{IRd\phi}{R^2}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 I}{4\pi R}\right)\theta$$

(a) Please note that, when we write $B = \left(\frac{\mu_0 I}{4\pi R}\right)\theta$, then the angle θ is in radian.

(b) If the loop is a quarter circle, $\left(\text{i.e., } \theta = \frac{\pi}{2}\right)$, then

$$B = \frac{\mu_0 I}{8R}$$

(c) If the loop is semi-circular (i.e., $\theta = \pi$), $B = \frac{\mu_0 I}{4R}$

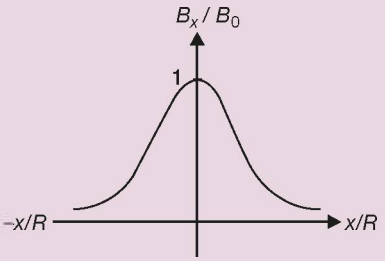
(d) If the loop is a three quarter circle $\left(\text{i.e., } \theta = \frac{3\pi}{2}\right)$, then

$$B = \frac{3\mu_0 I}{8R}$$

(e) If the loop is a full circle with N turns $(\text{i.e., } \theta = 2\pi N)$, then

$$B = \frac{\mu_0 NI}{2R}$$

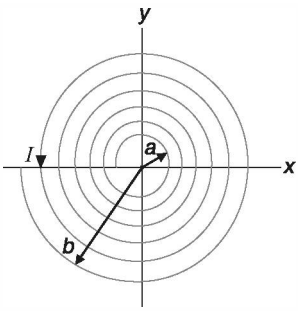
(f) The behavior of $\frac{B_x}{B_0}$ where $B_0 = B_{\text{centre}} = \frac{\mu_0 I}{2R}$ is the magnetic field strength at $x = 0$, as a function of $\frac{x}{R}$ is shown in Figure.



The ratio of the magnetic field, B_x / B_0, as a function of x/R

ILLUSTRATION 46

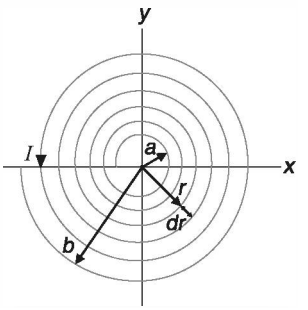
A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the x-y plane and a steady current I flows through the wire. Calculate the z component of the magnetic field at the centre of the spiral.



SOLUTION

We observe that the radial width $(b-a)$ of the coil is having N turns. So, number of turns per unit radial width of the coil is $n = \frac{N}{b-a}$.

Consider a circular coil of radius r, radial thickness dr as shown in Figure.



If dN be the number of turns in this radial thickness dr of the coil, then we have

$$dN = \frac{Ndr}{b-a}$$

If dB be the magnetic field (at the centre) due to this circular elemental coil of radius r having number of turns dN, then we have

$$dB = \frac{\mu_0 (dN) I}{2r}$$

$$\Rightarrow \quad dB = \frac{\mu_0 NI\, dr}{2(b-a)r}$$

To get the total magnetic field at the centre due to the coil integrating from $r = a$ to $r = b$, we get

$$B = \int_a^b dB = \int_a^b \frac{\mu_0 NI\, dr}{2(b-a)r} = \frac{\mu_0 NI}{2(b-a)} \int_a^b \frac{dr}{r}$$

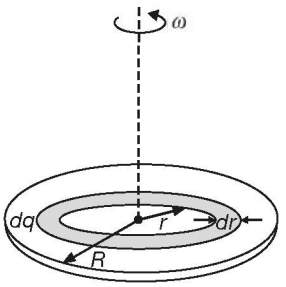
$$\Rightarrow \quad B = \frac{\mu_0 NI}{2(b-a)} \log_e r \Big|_a^b = \frac{\mu_0 NI}{2(b-a)} \log_e \left(\frac{b}{a}\right)$$

ILLUSTRATION 47

Calculate the magnetic field at the centre of a charged disc of radius R having a uniform charge Q, surface charge density σ rotating with angular velocity ω. Also calculate B in terms of σ.

SOLUTION

Consider a concentric infinitesimal ring of radius r, thickness dr, charge dq rotating with angular velocity ω as shown in Figure.



This rotating infinitesimal charge dq gives a current dI given by

$$dI = \frac{dq}{\pi} = \frac{\left(\frac{Q}{\pi R^2}\right)(2\pi r dr)}{\left(\frac{2\pi}{\omega}\right)}$$

$$\Rightarrow \quad dI = \left(\frac{Q\omega}{\pi R^2}\right) r\, dr$$

Since, we know that $dB = \frac{\mu_0 (dI)}{2r}$

$$\Rightarrow \quad dB = \frac{\mu_0}{2r}\left(\frac{Q\omega}{\pi R^2}\right) r\, dr$$

$$\Rightarrow \quad dB = \frac{\mu_0 Q\omega}{2\pi R^2} dr$$

$$\Rightarrow \quad B = \frac{\mu_0 Q\omega}{2\pi R^2} \int_0^R dr$$

$$\Rightarrow \quad B = \frac{\mu_0 Q\omega}{2\pi R}$$

Also $Q = \sigma(\pi R^2)$

$$\Rightarrow \quad B = \frac{\mu_0 \sigma(\pi R^2)\omega}{2\pi R}$$

$$\Rightarrow \quad B = \frac{\mu_0 \sigma R\omega}{2}$$

So, finally, we get

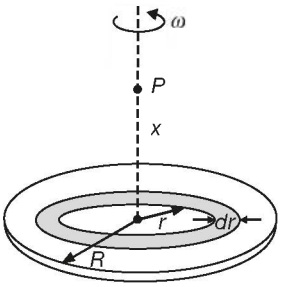
$$B = \frac{\mu_0 Q\omega}{2\pi R} = \frac{\mu_0 \sigma R\omega}{2}$$

ILLUSTRATION 48

A disc of radius R rotates with angular velocity ω about an axis perpendicular to its surface passing through centre. Assuming the surface charge density σ varies with r as $\sigma = \alpha r^2$, where r is the distance from its centre, find the magnetic induction on the axis of rotation at a point at distance x from the centre.

SOLUTION

Consider an infinitesimal concentric ring of radius r, thickness dr, having a charge dq on it as shown in Figure.



Then, $dq = (2\pi r dr)(\sigma)$

$$\Rightarrow \quad dq = (2\pi r dr)(\alpha r^2)$$

$$\Rightarrow \quad dq = 2\pi\alpha r^2 dr$$

If dI be the infinitesimal current due to rotation of the element of thickness dr, then

$$dI = \frac{dQ}{T} = \frac{2\pi\alpha r^3 dr}{2\pi/\omega}$$

$$\Rightarrow \quad dI = \alpha\omega r^3 dr$$

Magnetic induction at a distance x is given by

$$dB = \frac{\mu_0 (dI) r^2}{2\left(r^2 + x^2\right)^{3/2}}$$

$$\Rightarrow \quad dB = \frac{\mu_0 \alpha\omega r^5 dr}{2\left(r^2 + x^2\right)^{3/2}}$$

Total magnetic induction is obtained by integrating dB from zero to R. So, we get

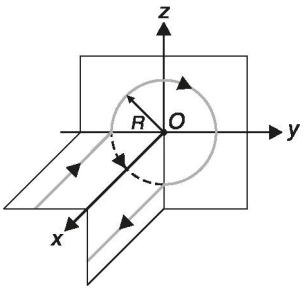
$$B = \int dB = \frac{\mu_0 \alpha\omega}{2} \int_0^R \frac{r^5 dr}{\left(r^2 + x^2\right)^{3/2}}$$

Integrating the above expression, we get

$$B = \frac{\mu_0 \alpha\omega}{2}\left[\frac{\left(R^2 + x^2\right)^{3/2}}{3} - 2x^2\left(R^2 + x^2\right)^{1/2} - \frac{x^4}{\left(R^2 + x^2\right)^{1/2}} + \frac{8}{3}x^3\right]$$

ILLUSTRATION 49

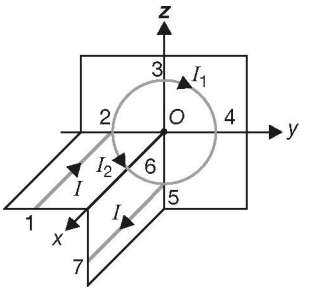
If the wire carrying a current I has the shape shown in figure, then calculate the magnetic field at the point O (both in magnitude and direction). The radius of the curved part of the wire is R, the linear parts of the wire are very long. Also assume the wires to be having uniform resistance.



SOLUTION

If we assume resistance of the quarter part '265' to be $R_2 = R$, then the resistance of three quarters '2345' will be $R_1 = 3R$. So, current passing through the two branches will have the ratio given by

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{R}{3R}$$



$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{3} \qquad \ldots(1)$$

Also $I_1 + I_2 = I$ $\qquad \ldots(2)$

So, from (1) and (2), we get

$$I_1 = \frac{I}{4} \text{ (from 2 to 3 to 4 to 5)}$$

and $\quad I_2 = \frac{3I}{4}$ (from 2 to 6 to 5)

So, $\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}\left(-\hat{k}\right)$

$$\vec{B}_{\text{arc }2345} = \frac{3\mu_0 I_1}{8R}\left(-\hat{i}\right) = \frac{3\mu_0 I}{32R}\left(-\hat{i}\right)$$

$$\vec{B}_{57} = \frac{\mu_0 I}{4\pi R}\left(-\hat{j}\right) \text{ and}$$

$$\vec{B}_{\text{arc }265} = \frac{\mu_0 I_2}{8R}\left(\hat{i}\right) = \frac{3\mu_0 I}{32R}\left(\hat{i}\right)$$

$$\Rightarrow \quad \vec{B} = \vec{B}_{12} + \vec{B}_{\text{arc }2345} + \vec{B}_{\text{arc }265} + \vec{B}_{57}$$

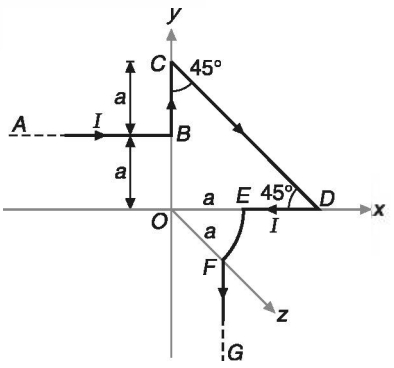
$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)\left[\hat{k} + \left(\frac{3\pi}{8}\right)\hat{i} - \left(\frac{3\pi}{8}\right)\hat{i} + \hat{j}\right]$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)\left(\hat{j} + \hat{k}\right)$$

$$B = |\vec{B}| = \frac{\sqrt{2}\mu_0 I}{4\pi R}$$

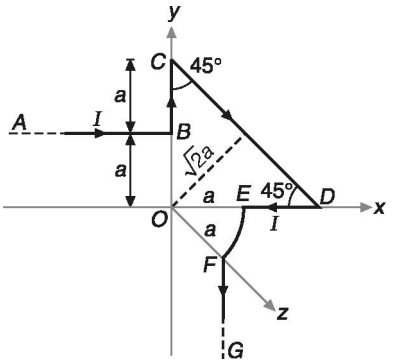
ILLUSTRATION 50

Calculate the magnetic field vector at the origin O due to the current carrying wire configuration shown in figure.



SOLUTION

Due to the wire segments BC and DE no magnetic field will be produced at O because O lies on the extended portions of BC and DE.



For the remaining wire segments the magnetic fields are given by

$$\vec{B}_{AB} = \frac{\mu_0 I}{4\pi a}(-\hat{k})$$

$$\vec{B}_{CD} = \frac{\mu_0 I}{4\pi(\sqrt{2}a)}\sqrt{2}(-\hat{k})$$

$$\vec{B}_{EF} = \frac{\mu_0 I}{8a}(-\hat{j})$$

$$\vec{B}_{FG} = \left(\frac{\mu_0 I}{4\pi a}\right)\hat{i}$$

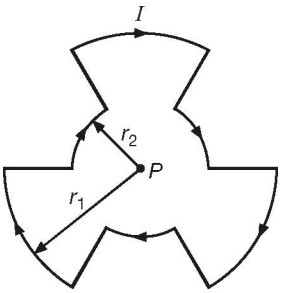
The resultant magnetic field at O due to all the wire segments is

$$\vec{B}_O = \vec{B}_{AB} + \vec{B}_{CD} + \vec{B}_{EF} + \vec{B}_{FG}$$

$$\Rightarrow \quad \vec{B}_0 = \frac{\mu_0 I}{4\pi a}\left(\hat{i} - \frac{\pi}{2}\hat{j} - 2\hat{k}\right)$$

ILLUSTRATION 51

A current I flows around a closed path in the horizontal plane of the circuit shown in figure. The path consists of six arcs with alternating radii r_1 and r_2 connected by radial segments. Each segment of arc subtends an angle of $60°$ at the common centre P, with $\frac{r_2}{r_1} = \frac{2}{3}$. This current path produces a magnetic field $\vec{B}$ at P. If the path is modified so that the ratio $\frac{r_2}{r_1} = \frac{1}{3}$, by what factor must I be multiplied in order that the field at P remain the same?



SOLUTION

Since $B_{\text{arc}} = \left(\frac{\mu_0 I}{4\pi r}\right)\theta$, where θ is in radian

When $\frac{r_2}{r_1} = \frac{2}{3}$, then the field at P is B given by

$$B = 3\left(\frac{\mu_0 I}{4\pi r_1}\right)\left(\frac{\pi}{3}\right) + 3\left(\frac{\mu_0 I}{4\pi r_2}\right)\left(\frac{\pi}{3}\right), \otimes$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4}\left(\frac{1}{r_1} + \frac{1}{r_2}\right), \otimes$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4r_1}\left(1 + \frac{r_1}{r_2}\right) = \frac{\mu_0 I}{4r_1}\left(1 + \frac{3}{2}\right)$$

$$\Rightarrow \quad B = \frac{5\mu_0 I}{8r_1}, \otimes \qquad \ldots(1)$$

Now, when $\frac{r_2}{r_1} = \frac{1}{3}$, let the current be kI so that B remains the same.

$$B = 3\left[\frac{\mu_0 (kI)}{4\pi r_1}\right]\left(\frac{\pi}{3}\right) + 3\left[\frac{\mu_0 (kI)}{4\pi r_2}\right]\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0(kI)}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0(kI)}{4r_1}\left(1+\frac{r_1}{r_2}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0(kI)}{4r_1}(1+3)$$

$$\Rightarrow \quad B=\frac{\mu_0(kI)}{r_1},\ \otimes \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{5\mu_0 I}{8r_1}=\frac{\mu_0(kI)}{r_1}$$

$$\Rightarrow \quad k=\frac{5}{8}$$

Please note that in both the cases the ratio $\frac{r_2}{r_1}=\frac{2}{3}$ and then the same ratio becomes $\frac{1}{3}$. So, the problem just wants to convey that the radius r_2 is now halved keeping r_1 constant.

Test Your Concepts-IV

Based on Biot Savart's Law and Applications

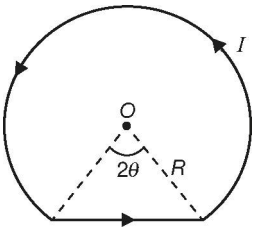
(Solutions on page H.11)

1. A current $I=1$ A circulates in a round thin-wire loop of radius $R=100$ mm. Find the magnetic induction
- **(a)** at the centre of the loop.
- **(b)** at the point lying on the axis of the loop at a distance $x=100$ mm from its centre.

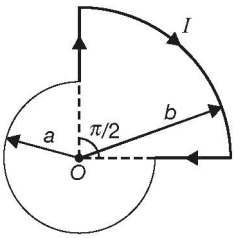
2. A current I flows along a thin wire shaped as a regular polygon with n sides which can be inscribed into a circle of radius R. Find the magnetic induction at the centre of the polygon. Analyse the obtained expression at $n\to\infty$.

3. Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 A each. Calculate the magnetic field at a point lying at a distance 0.1 m from both wires when the currents in the wires are in the same direction and when the currents in the wires are in the opposite direction.

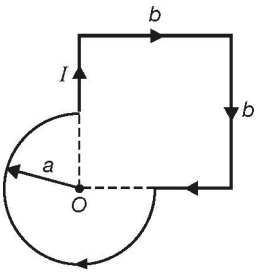
4. A current I flows along a thin wire shaped as shown in figure. The radius of a curved part of the wire is equal to R, the angle 2θ. Find the magnetic induction of the field at the point O.



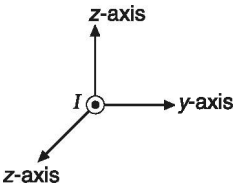
5. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is shown in figure.



6. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is shown in figure.

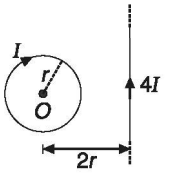


7. A long current carrying conductor carrying a current I lies along x-axis such that its one end is at origin, and the other end lies on positive x-axis at a large distance from origin. The current through the conductor is along positive x-direction as shown in Figure.

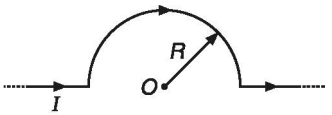


Calculate the magnetic field vector at points whose co-ordinates are $(0, a, 0)$, $(0, a, a)$ and $(-a, a, 0)$.

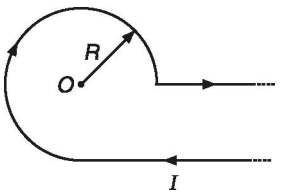
8. For the arrangement shown, calculate the magnetic field at point O.



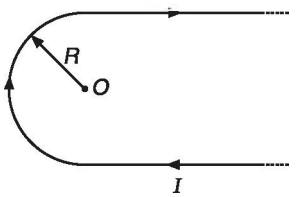
9. Calculate the magnetic field at the point O shown in figure. The radius of the curved part of the wire is R, the linear parts are assumed to be very long.



10. Find the magnetic field at the point O due to the current carrying wire having the shape shown in figure.

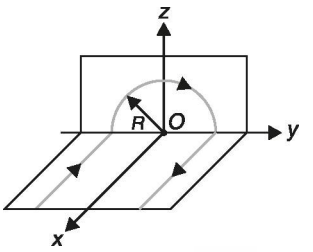


11. In the arrangement shown, calculate the magnetic flux density i.e., magnetic field at the point O.

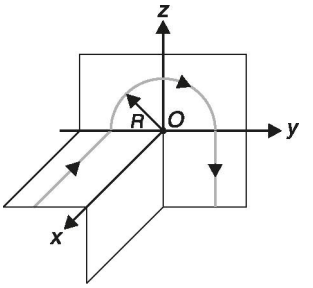


12. A very long wire carrying a current I is bent at right angles. Find the magnetic induction at a point lying on a perpendicular to the wire, drawn through the point of bending, at a distance ℓ from it.

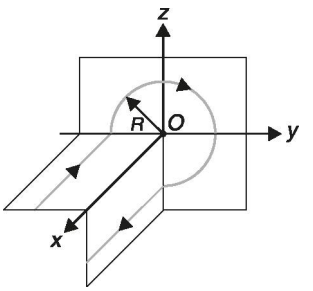
13. Calculate the magnetic field vector at the point O if the wire carrying a current I has the shape shown in figure. Also calculate its magnitude.



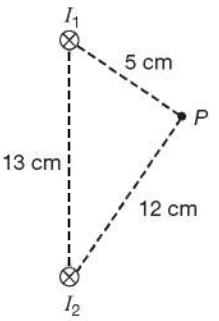
14. Calculate the magnetic field $\vec{B}$ (both in magnitude and direction) at the point O for the current carrying wire arrangement shown in figure.



15. In the current carrying wire arrangement shown, calculate the magnetic flux density i.e., magnetic field at the point O (both in magnitude and direction).

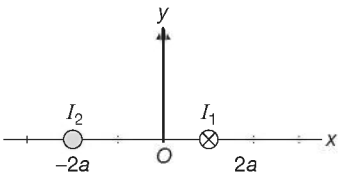


16. Two long, parallel conductors carry currents $I_1 = 3$ A and $I_2 = 3$ A, both directed into the page in figure. Determine the magnitude and direction of the resultant magnetic field at P.



17. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^{6} ms^{-1}. Compute the magnitude of the magnetic field that this motion produces at the location of the proton.

18. Two very long, straight, parallel wires carry currents that are directed perpendicular to the page, as in figure. Wire 1 carries a current I_1 into the page (in the $-x$ direction) and passes through the x-axis at $x=+a$. Wire 2 passes through the x axis at $x=-2a$ and carries an unknown current I_2. The total magnetic field at the origin due to the current carrying wires has the magnitude $\frac{2\mu_0 I_1}{2\pi a}$. The current I_2 can have either of two possible values.



(a) Find the value of I_2 with the smaller magnitude, stating it in terms of I_1 and giving its direction.

(b) Find the other possible value of I_2.

19. A very long wire 1 carries a current of 30 A to the left along the x axis. A second very long wire 2 carries current 50 A to the right along the line $y=\frac{2}{3}$ m, $z=0$

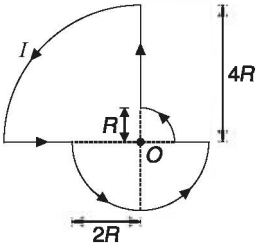
(a) Where in the plane of the two wires is the total magnetic field equal to zero?

(b) A particle with a charge of $-2\,\mu C$ is moving with a velocity of $150\hat{i}$ Mms^{-1} along the line $y=\frac{1}{3}$ m, $z=0$. Calculate the vector magnetic force acting on the particle.

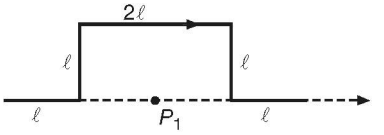
20. A non-conducting ring of radius R is uniformly charged with a total positive charge Q. The ring rotates at a constant angular speed w about an axis through its centre and perpendicular to its plane. Find the magnitude of the magnetic field on the axis of the ring at a point P, a distance $\frac{R}{2}$ from its centre.

21. A long straight wire lies on a horizontal table and carries a current I. In a vacuum, a proton moves parallel to the wire (opposite the current) with a constant speed of v at a distance d above the wire. Determine the value of d. You may ignore the magnetic field due to the Earth.

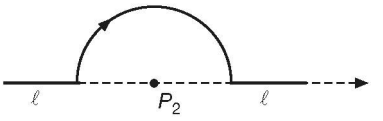
22. Calculate the magnetic field at the centre O of the current carrying loop shown in Figure.



23. A wire is bent into the shape shown in figure (a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in figure (b), and the magnetic field measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of $\frac{B_1}{B_2}$?

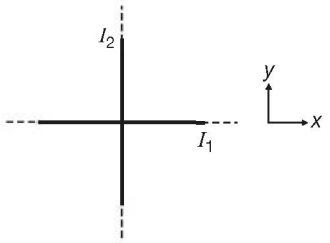


(a)

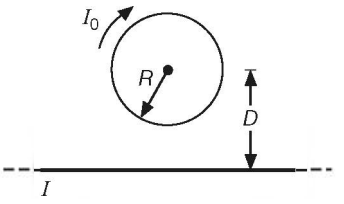


(b)

24. Two mutually perpendicular long conductors carrying currents I_1 and I_2 lie in one plane. Find the locus of points at which the magnetic induction is zero.



25. A circular loop has radius R and carries current I_0 in a clockwise direction. The centre of the loop is a distance D above a long, straight wire. What are the magnitude and direction of the current I in the wire if the magnetic field at the centre of the loop is zero?



AMPERE'S CIRCUITAL LAW (ACL)

Just as we use Gauss' Law to find the electric field at a point due to a charge distribution, similarly we shall be taking help of Ampere's Circuital Law (ACL) to find the magnetic field at a point due to a current carrying conductor.

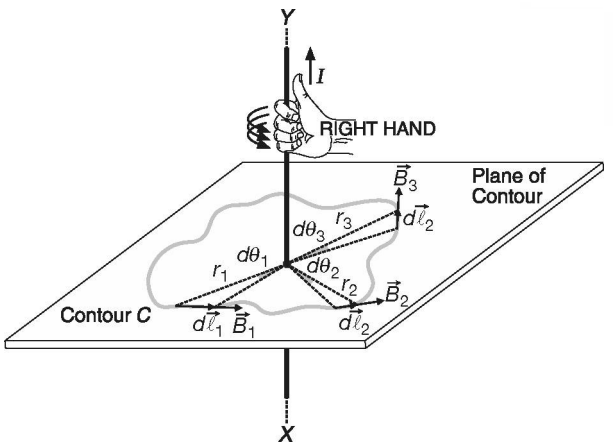
According to this Law, *"the line integral of the resultant magnetic field* $\left(\text{i.e., } \oint \vec{B}\cdot d\vec{l}\right)$ *along a closed contour* ***(also called Amperian Loop)*** *is* μ_0 *times the total current threading through the contour".*

Mathematically, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(\Sigma I) = \mu_0 I_{\text{net}}$$

Proof of Ampere's Circuital Law (ACL)

To prove the result obtained, let us consider a current carrying long wire XY as shown in Figure.



Let us draw a random contour C around the wire XY. Also, we have shown the plane which contains the contour C. Let us now randomly consider three infinitesimal elements $d\vec{l}_1$, $d\vec{l}_2$ and $d\vec{l}_3$ along the length of the contour (as shown). Let these elements be at distances r_1, r_2 and r_3 from the wire and they subtend angles $d\theta_1$, $d\theta_2$ and $d\theta_3$ at the wire. If $\vec{B}_1$, $\vec{B}_2$ and $\vec{B}_3$ be the magnetic fields due to the wire at these elements, then we observe that angle between $d\vec{l}_1$ and $\vec{B}_2$, between $d\vec{l}_2$ and $\vec{B}_2$, between $d\vec{l}_3$ and $\vec{B}_3$ is zero. Let us now calculate the integral $\oint \vec{B}\cdot d\vec{l}$.

From concepts of integral calculus, for a closed loop, we have

$$\oint \vec{B}\cdot d\vec{l} = \vec{B}_1\cdot d\vec{l}_1 + \vec{B}_2\cdot d\vec{l}_2 + \vec{B}_3\cdot d\vec{l}_3 + \ldots$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = B_1 dl_1\cos(0^\circ) + B_2 dl_2\cos(0^\circ) + \ldots$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = B_1 dl_1 + B_2 dl_2 + B_3 dl_3 + \ldots$$

Since $B_1 = \dfrac{\mu_0 I}{2\pi r_1}$, $B_2 = \dfrac{\mu_0 I}{2\pi r_2}$, $B_3 = \dfrac{\mu_0 I}{2\pi r_3}$ and so on

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\left(\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + \ldots\right)_{\text{in the closed loop}}$$

However, $\dfrac{dl_1}{r_1} = d\theta_1$, $\dfrac{dl_2}{r_2} = d\theta_2$, $\dfrac{dl_3}{r_3} = d\theta_3$ and so on

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\left(d\theta_1 + d\theta_2 + d\theta_3 + \ldots\right)_{\text{in the closed loop}}$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\oint d\theta$$

Since, for a closed loop, we have $\oint d\theta = 2\pi$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}(2\pi)$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

Conceptual Note(s)

(a) Generally, the **Amperian Loop** or the **Contour** is selected in a way such that at each point taken on the loop, either the magnetic field

(i) $\vec{B}$ is normal to the loop (because then $\oint \vec{B}\cdot d\vec{\ell} = 0$, since $\theta = 90°$)

(ii) $\vec{B}$ vanishes everywhere on the loop (because then too $\oint \vec{B}\cdot d\vec{\ell} = 0$)

(iii) $\vec{B}$ is tangential to the loop and has a non-zero constant magnitude all over the loop (because then

$$\oint \vec{B}\cdot d\vec{\ell} = \oint B(d\ell)\cos(0°) = B\oint d\ell,$$

which becomes really easy to evaluate just by knowing the shape of the contour.

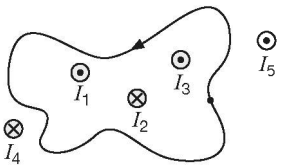
(b) The line integral is independent of the shape of the path and the placement of the current carrying wire in it.

(c) The statement $\oint \vec{B}\cdot d\vec{\ell} = 0$ does not necessarily imply that $B = 0$ everywhere along the path. However, we can conclude that no net current is threaded through the contour or the loop.

Sign Convention to be Followed for Currents while Applying ACL

Curl the fingers of the Right Hand in the sense of contour (Clockwise or Counter-Clockwise), then ***the currents along the direction of the thumb are taken as positive*** *and* ***the currents opposite to the direction of thumb are taken as negative.***

In the arrangement shown,



we observe that the currents I_1 and I_3 are along the direction of thumb when the fingers of the right hand are curled in the sense of contour i.e. counter clockwise, whereas I_2 points opposite to the direction of thumb. Also, the currents I_4 and I_5 are outside the contour, so according to Ampere's Circuital Law, we have

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 \sum I = \mu_0 (I_1 + I_3 - I_2)$$

AMPERE'S OBJECTION(S) TO BIOT SAVART'S LAW

Ampere had several objections to the work of Jean Biot and Felix Savart.

(a) He felt that their experiments were not precise enough for them to claim certainty in the $\sin\theta$ factor.

(b) He was uncomfortable with their use of "current elements", since isolated current elements do not exist as they are always part of a complete circuit.

Ampere's Line of Action

Consequently, he pursued his own line of experimental and theoretical research and obtained a different relation, now called Ampere's Law, between a current and the magnetic field it produces.

Although Ampere's Law could have been derived from the Biot-Savart's expression for $d\vec{B}$, but Ampere preferred not to do so. Instead, he made it possible by considering the field due to an infinite straight wire. He knew that the field lines due to an infinite straight wire are concentric circles and the magnitude of the field at a distance r from the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

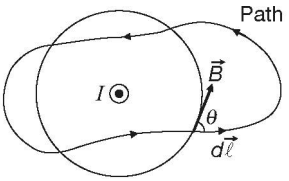
So, he had rewritten the above expression as

$$B(2\pi r) = \mu_0 I$$

and interpreted it as follows.

"$2\pi r$ is the length of a circular path around the wire, B is the component of the magnetic field tangential to the path, and I is the current through the area bounded by the path. Ampere generalized this result to paths and wires of any shape".

Figure shows a current coming out of the page and an arbitrary closed path around it.



A current is coming out of the page. According to Ampere's Law, around any closed path, enclosing the current I,

$$\oint \vec{B}.\, d\vec{\ell} = \mu_0 I$$

For an infinitesimal displacement $d\vec{l}$ along the path, the product of dl and the component of $\vec{B}$ along $d\vec{l}$ is

$$dl(B\cos\theta) = \vec{B}\cdot d\vec{l}$$

According to Ampere's Law, the sum (integral) of this product around a closed path is given by

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

where I is the net current flowing through the surface enclosed by the path.

The sense (clockwise or counter clockwise) in which the integral is to be evaluated is given by a right-hand rule, according to which when the thumb of the right hand points along the current, the curled fingers indicate the positive sense along the path.

Conceptual Note(s)

(a) The equation, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ is valid only for steady currents and nonmagnetic materials, such as Cu.

(b) The enclosed current may not have to flow in a wire. It may be a beam of charged particles which also constitutes a current.

(c) In order to use Ampere's Law to determine the magnetic field, it is necessary for the geometry of the current flow to possess sufficient symmetry so that the integral can be evaluated easily. One needs to know the field pattern and then to make a suitable choice for the path of integration.

(d) Ampere's Law in magnetism is analogous to Gauss's Law in electrostatics. In order to apply them, the system must posses certain symmetry. In the case of an infinite wire, the system possesses cylindrical symmetry and Ampere's Law can be readily applied. However, when the length of the wire is finite, Biot-Savart's Law must be used instead.

Biot-Savart's Law	$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2}$	General current source e.g. finite wire
Ampere's Law	$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$	Current source has certain symmetry e.g. infinite wire (cylindrical)

(e) Ampere's Law is applicable to the following current configurations:

(i) Infinitely long straight current carrying wire.

(ii) Infinitely large sheet of thickness b with a current density J.

(iii) Infinite solenoid.

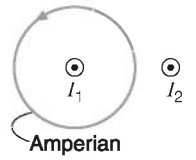
(iv) Toroid.

We shall examine all four configurations in detail.

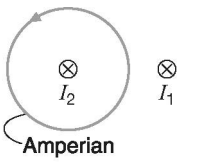
ILLUSTRATION 52

In the following arrangements shown, calculate $\oint \vec{B}\cdot d\vec{l}$ corresponding to the respective Amperian loops for which the direction in which the closed integral has to be performed is indicated by the arrows.

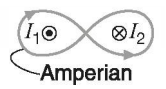
(a)



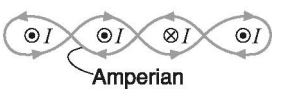
(b)



(c)



(d)



SOLUTION

To solve this problem, we must understand that when we curl the fingers of the Right Hand in the sense of contour (Clockwise or Counter-Clockwise), then the currents along the direction of the thumb are taken as positive and the currents opposite to the direction of thumb are taken as negative.

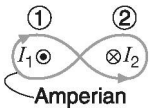
(a) The Amperian loop is traversed counter clockwise and hence the thumb will point outwards along I_1, so

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_1$$

(b) The Amperian loop is traversed counter clockwise and hence the thumb will point outwards opposite to I_2, so

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(-I_2) = -\mu_0 I_2$$

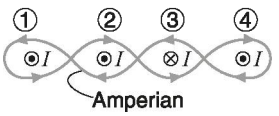
(c) The left side portion 1 of the Amperian loop is traversed clockwise, whereas the right-side portion 2 is traversed counter clockwise as shown in figure.



So, by applying the sign convention, we get that both I_1 and I_2 are negative, so we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(-I_1 - I_2) = -\mu_0(I_1 + I_2)$$

(d) The portion 1 of the Amperian loop is traversed counter clockwise, portion 2 clockwise, portion 3 counter clockwise and portion 4 clockwise as shown in figure.

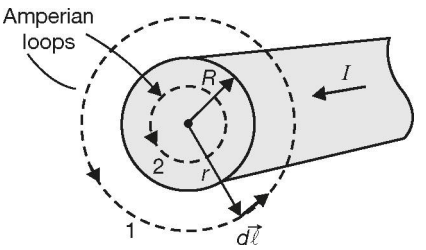


So, by applying the sign convention, we see that current for the loop 1 is to be taken as positive, for loop 2 to be taken as negative, for loop 3 to be taken as negative and for loop 4 to be taken as negative. So, we get

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(I - I - I - I) = -3\mu_0 I$$

MAGNETIC FIELD DUE TO A THICK CURRENT CARRYING WIRE

Consider a long straight wire of radius R carrying a current I of uniform current density, as shown in Figure.



Let us find the magnetic field everywhere i.e., outside, at the surface and inside the wire.

CASE-1: Outside

Outside the wire where $r > R$, the Amperian loop or the contour (circle 1) completely encircles the current, and hence $I_{enc} = I$.

Applying Ampere's Law, we get

$$\oint \vec{B}\cdot d\vec{l} = B\oint dl = B(2\pi r) = \mu_0 I$$

$$\Rightarrow \quad B_{outside} = \frac{\mu_0 I}{2\pi r}$$

CASE-2: At the Surface

Substituting $r = R$ in the above expression, we get

$$B_{surface} = \frac{\mu_0 I}{2\pi R}$$

CASE-3: Inside

Inside the wire where $r < R$, the amount of current encircled by the Amperian loop or the contour (circle 2) is proportional to the area enclosed, i.e.,

$$I_{encl} = \left(\frac{\pi r^2}{\pi R^2}\right) I = \left(\frac{r^2}{R^2}\right) I$$

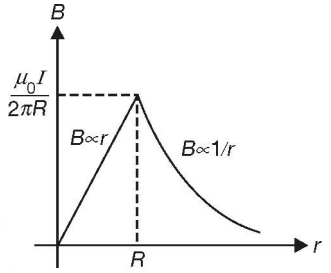
According to Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$\Rightarrow \quad B_{inside} = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

We see that the magnetic field is zero at the centre of the wire and increases linearly with r until $r = R$.

Outside the wire, the field falls off as $\frac{1}{r}$. The qualitative behaviour of the field is depicted as shown in Figure.



$$\Rightarrow \quad B = \begin{cases} \dfrac{\mu_0 I}{2\pi r} & r \geq R \text{ (outside and at the surface)} \\ \left(\dfrac{\mu_0 I}{2\pi R^2}\right) r & r < R \text{ (inside)} \end{cases}$$

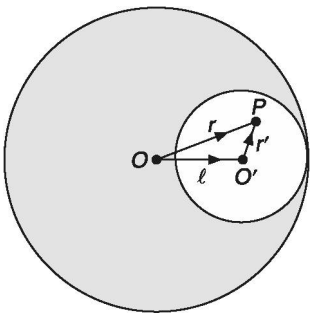
and $B_{\text{centre}} = 0$

ILLUSTRATION 53

Inside a homogeneous long straight current carrying wire of circular cross-section, there is a cylindrical cavity whose axis is parallel to the conductor axis and is displaced relative to it by a distance ℓ. A direct current of density J flows along the wire. Calculate the magnetic field inside the cavity.

SOLUTION

The arrangement for which the magnetic field $(\vec{B})$ has to be calculated at the point P inside the cavity is shown in Figure.



The magnetic field at a point P is

$$\vec{B}_P = \vec{B}_{\substack{\text{Full}\\\text{Cylinder}}} - \vec{B}_{\substack{\text{Cavity}\\\text{Cylinder}}} \qquad \ldots(1)$$

where $\vec{B}_0$ is the magnetic field at P due to the full conductor (without cavity) and $\vec{B}'$ is the magnetic field at P due to current flowing through the part of the conductor which has been removed in order to create the cavity.

According to Ampere's Circuital Law, the magnetic field at a distance r from the axis of solid cylinder (inside the solid cylinder) is given by

$$B(2\pi r) = \mu_0 \pi r^2 J$$

$$\Rightarrow \quad B = \frac{1}{2}\mu_0 J r \text{ (in magnitude)}$$

Vectorially, we have

$$\vec{B} = \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}\right)$$

The magnetic field due to the full cylinder is given by

$$\vec{B}_{\substack{\text{Full}\\\text{Cylinder}}} = \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}\right)$$

The magnetic field due to the cylindrical cavity is given by

$$\vec{B}_{\substack{\text{Cavity}\\\text{Cylinder}}} = \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}'\right)$$

Since $\vec{B}_P = \vec{B}_{\substack{\text{Full}\\\text{Cylinder}}} - \vec{B}_{\substack{\text{Cavity}\\\text{Cylinder}}}$

$$\Rightarrow \quad \vec{B}_P = \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}\right) - \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}'\right)$$

$$\Rightarrow \quad \vec{B}_P = \frac{\mu_0}{2}\vec{J} \times (\vec{r} - \vec{r}') \qquad \ldots(2)$$

In triangle $OO'P$, we have

$$\vec{l} + \vec{r}' = \vec{r}$$

$$\Rightarrow \quad \vec{r} - \vec{r}' = \vec{l}$$

So, equation (2) becomes

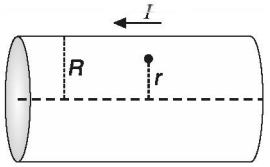
$$\vec{B}_P = \frac{\mu_0}{2}\left(\vec{J} \times \vec{l}\right)$$

In magnitude, we have

$$B_P = \frac{\mu_0 J l}{2}$$

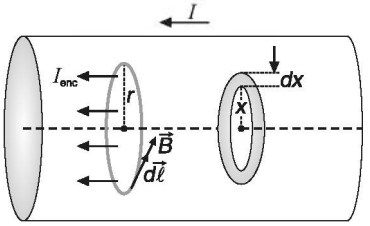
ILLUSTRATION 54

A long cylindrical conductor of radius R carries a current I as shown in figure. The current density J varies across the cross-section with the radial distance r as $J = kr^2$, where k is a constant. Find an expression for the magnetic field B at a distance $r(< R)$ from the axis of the cylindrical conductor.



SOLUTION

Consider an Amperian loop of radius r as shown in figure.



The distribution of current shows that the field will be same everywhere on the Amperian loop of radius r. If I_{enc} be the current threading through the Amperian loop, then by Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I_{enc}$$

$$\Rightarrow \quad B = \frac{\mu_0 I_{enc}}{2\pi r} \qquad \ldots(1)$$

Now, to find the current I_{enc} threading through the Amperian loop, consider a ring of radius x and thickness dx having area $dA = 2\pi x dx$. Since the current density J is defined as

$$J = \frac{dI}{dA}$$

$$\Rightarrow \quad dI = JdA \text{ , where } J = kx^2$$

$$\Rightarrow \quad I_{enc} = \int dI = \int JdA = \int_0^r kx^2 (2\pi x dx)$$

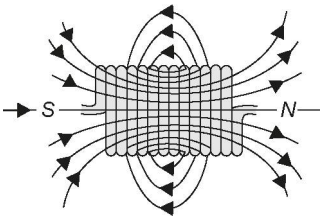
$$\Rightarrow \quad I_{enc} = 2\pi k \int_0^r x^3 dx = 2\pi k \left(\frac{x^4}{4}\Big|_0^r \right) = \frac{2\pi k r^4}{4}$$

Substituting this value of I_{enc} in equation (1), we get

$$B = \frac{\mu_0 k r^3}{4}$$

SOLENOID (REVISITED)

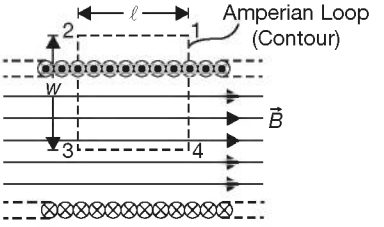
As studied and discussed earlier, a solenoid is a long coil of wire tightly wound in the helical form. The magnetic field lines of a solenoid carrying a steady current I is shown in Figure.



We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter.

For an ideal solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.

We shall be using Ampere's Law to calculate the magnetic field strength inside an ideal solenoid. The cross-sectional view of an ideal solenoid is shown in Figure.



Amperian loop for calculating the magnetic field of an ideal solenoid.

To compute $\vec{B}$, we consider a rectangular path of length l and width w and traverse the path in a counterclockwise manner. The line integral of $\vec{B}$ along this loop, 12341, is

$$\oint \vec{B}\cdot d\vec{l} = \int_1^2 \vec{B}\cdot d\vec{l} + \int_2^3 \vec{B}\cdot d\vec{l} + \int_3^4 \vec{B}\cdot d\vec{l} + \int_4^1 \vec{B}\cdot d\vec{l}$$

In the above, the contributions along sides 23 and 41 are zero because $\vec{B}$ is perpendicular to $d\vec{l}$. In addition, $\vec{B} = \vec{0}$ alongside 12 because the magnetic field is zero outside and non-zero only inside the solenoid. On the other hand, the total current enclosed by the contour (Amperian loop) is $I_{enc} = NI$, where N is the total number of turns. Applying Ampere's Law, we get

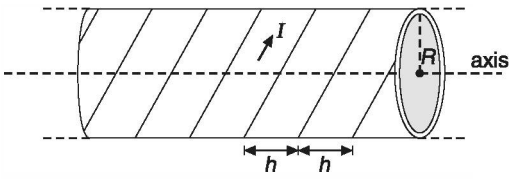
$$\oint \vec{B}\cdot d\vec{l} = Bl = \mu_0 (NI)$$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{l} = \mu_0 nI$$

where $n = \frac{N}{l}$ represents the number of turns per unit length.

ILLUSTRATION 55

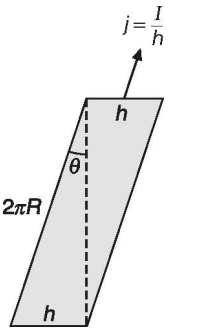
A thin conducting strip of width h is tightly wound in the shape of a very long cylindrical coil with cross-sectional radius R to make a single layer straight solenoid as shown in figure.



A direct current I flows through the strip. Find the magnetic induction inside and outside the solenoid as a function of the distance r from its axis.

SOLUTION

Let us consider one loop, then open it and draw it as shown in Figure.



(one loop completely opened)

Figure (a)

Since the current is flowing in a strip of width h so, the current per unit strip width is given by

$$j = \frac{I}{h}$$

This current per unit length (also called linear current density) can be resolved in two components j_x and j_y as shown in Figure.

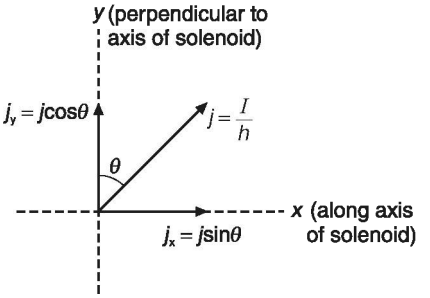


Figure (b)

Outside this type of solenoid, the magnetic field will be due to the component of current along the axis of solenoid as shown in Figure (c).

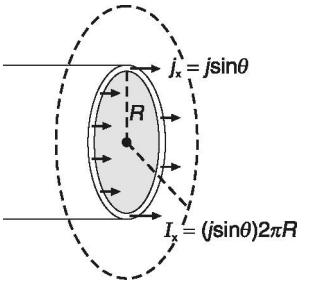


Figure (c)

For current density $j_x = j\sin\theta$ along the length of solenoid, by applying Amperes Circuital Law, we get

$$B_{\text{out}}(2\pi r) = \mu_0 I_x$$

where $I_x = j_x(2\pi R)$

$$\Rightarrow \quad I_x = j\sin\theta(2\pi R)$$

From Figure (a), we have

$$\sin\theta = \frac{h}{2\pi R}$$

$$\Rightarrow \quad I_x = j\left(\frac{h}{2\pi R}\right)(2\pi R) = jh$$

$$\Rightarrow \quad B_{\text{out}} = \frac{\mu_0 I_x}{2\pi r} = \frac{\mu_0 jh}{2\pi r} = \frac{\mu_0 I}{2\pi r}$$

Inside this type of solenoid, the magnetic field will be due to the component of current perpendicular to the axis of the solenoid. Due to component of current density perpendicular to length of solenoid, we have

$B = \mu_0 n I_y$, where

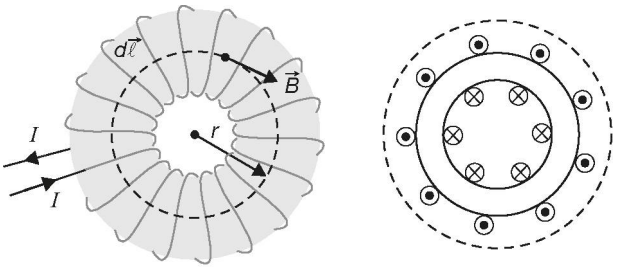
$$n = \frac{N}{Nh} = \frac{1}{h} \text{ and } I_y = (j\cos\theta)h = I\cos\theta$$

$$\Rightarrow \quad B_{\text{inside}} = \mu_0\left(\frac{1}{h}\right)(I\cos\theta)$$

$$\Rightarrow \quad B_{\text{inside}} = \frac{\mu_0 I}{h}\sqrt{1-\left(\frac{h}{2\pi R}\right)^2}$$

MAGNETIC FIELD DUE TO A TOROID

Consider a toroid which consists of N turns, as shown in Figure.



A toroid with N turns

Let us find the magnetic field due to the toroid. To calculate field, due to a toroid, we must evaluate $\oint \vec{B}\cdot d\vec{l}$ over the circle of radius r. Due to symmetry we see that the magnitude of the field is constant on this circle and tangent to it. So,

$$\oint \vec{B}\cdot d\vec{l} = Bl = B(2\pi r)$$

According to Ampere's Circuital Law, we have

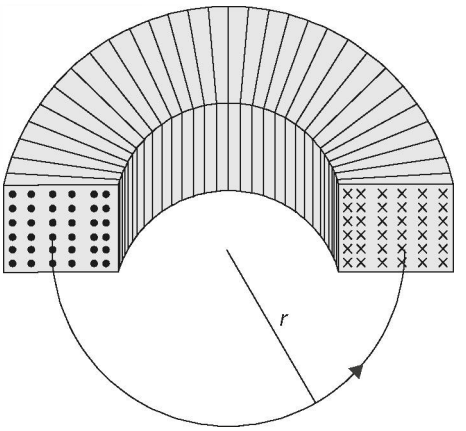
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{net}}$$

Since, the circular closed path surrounds N loops of the wire, each of which carries a current I, so we have $I_{\text{net}} = NI$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \mu_0(I_{\text{net}}) = \mu_0 NI$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 NI$$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi r}$$



Within a toroid, the field is constant along a given circle of radius r

This result shows that $B \propto \frac{1}{r}$ and hence is non-uniform in the region occupied by toroid.

However, if r is very large compared with the cross-sectional radius of the toroid, then the field is approximately uniform inside the toroid and is given by

$$B = \mu_0 nI$$

In this case, we observe that the number of turns per unit length of toroid is

$$n = \frac{N}{2\pi r}$$

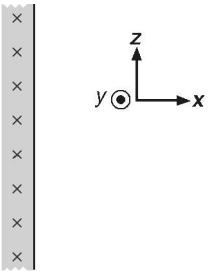
However, If the toroid has inner radius r_1 and an outer radius r_2, then number of turns per unit length of toroid is

$$n = \frac{N}{2\pi r_{av}} = \frac{N}{2\pi\left(\frac{r_1 + r_2}{2}\right)}$$

For an ideal toroid, in which turns are closely spaced, the external magnetic field is zero. This is because the net current passing through any circular path lying outside the toroid is zero. Therefore, from Ampere's Law we find that $B = 0$, in the regions exterior to the toroid.

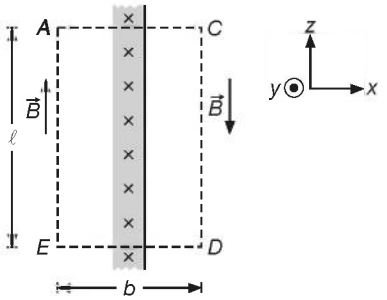
ILLUSTRATION 56

A thin infinitely large sheet lying in the yz plane carries a current of linear current density λ. The current is in the negative y-direction and λ represents the current per unit length measured along the z-axis. Find the magnetic field near the sheet.



SOLUTION

To evaluate the line integral in Ampere's Law, let us consider a rectangular path of length l, width b through the sheet as shown in Figure.



From symmetry we can argue that the magnetic field is constant over the sides of length l because sheet is infinitely long so it should not vary from point to point. The two sides of length b do not contribute to the line integral because the component of $\vec{B}$ along the direction of these paths is zero. The net current passing through the plane of the rectangle is λl. Applying Ampere's Law, we get

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_{\text{net}})$$

$$\Rightarrow \int_A^C \vec{B}\cdot d\vec{l} + \int_C^D \vec{B}\cdot d\vec{l} + \int_D^E \vec{B}\cdot d\vec{l} + \int_E^A \vec{B}\cdot d\vec{l} = \mu_0 (\lambda l)$$

$$\Rightarrow 0 + Bl + 0 + Bl = \mu_0 \lambda l$$

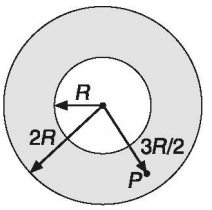
$$\Rightarrow B = \frac{\mu_0 \lambda}{2} = \text{constant}$$

The result shows that the magnetic field is independent of distance from the current sheet.

This is exactly similar to the case where the electric field due to an infinite sheet of charge does not depend on the distance from the sheet.

ILLUSTRATION 57

Figure shows the cross sectional view of a hollow cylindrical conductor with inner radius R and outer radius $2R$ carrying a uniformly distributed current I along it's axis.



Calculate the magnetic induction at point P at a distance $\frac{3R}{2}$ from the axis of the cylinder.

SOLUTION

Using Ampere's Circuital Law, the current inside the Amperian Loop passing through the point P is

$$I' = I\left(\frac{(3R/2)^2 - R^2}{(2R)^2 - R^2}\right) = \frac{5}{12}I$$

Since $\oint \vec{B}\cdot d\vec{l} = \mu_0 I'$

$$\Rightarrow B(2\pi r) = \mu_0 I', \text{ where } r = \frac{3R}{2}$$

$$\Rightarrow \quad 2\pi B\left(\frac{3R}{2}\right)=\mu_0\left(\frac{5}{12}I\right)$$

$$\Rightarrow \quad B=\frac{5\mu_0 I}{36\pi R}$$

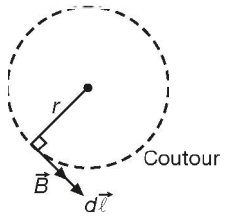
ILLUSTRATION 58

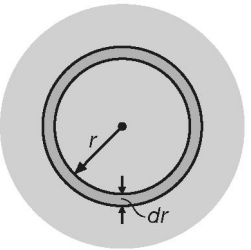
Find the current density as a function of distance r from the axis of a radially symmetrical parallel stream of electrons if the magnetic induction inside the stream varies as $B=br^{\alpha}$, where b and α are positive constants.

SOLUTION

According to Ampere's Circuital Law, we have

$$\int \vec{B}\cdot d\vec{l}=\mu_0\left(I_{\text{total}}\right)$$





Stream of Electrons

Since, $I_{\text{total}}=\int \vec{j}\cdot d\vec{A}=\int jdA$, where $dA=2\pi r dr$

$$\Rightarrow \quad B(2\pi r)=\mu_0\int jdA$$

$$\Rightarrow \quad B(2\pi r)=\mu_0\int j(2\pi r\,dr)$$

But $B=br^{\alpha}$

$$\Rightarrow \quad 2\pi b r^{\alpha+1}=\mu_0\int j(2\pi r dr)$$

Differentiating both sides, we get

$$\Rightarrow \quad 2\pi b(\alpha+1)r^{\alpha}dr=\mu_0\left(j2\pi r dr\right)$$

$$\Rightarrow \quad j=\frac{b(\alpha+1)}{\mu_0}r^{\alpha-1}$$

ILLUSTRATION 59

An infinitely long hollow conducting cylinder with inner radius $\frac{R}{2}$ and outer radius R carries a uniform current density along its length. Calculate the magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis of the cylinder. Also plot $|\vec{B}|$ vs r graph.

SOLUTION

Let r be the distance of a point from centre, then

For $r\le\frac{R}{2}$, using Ampere's circuital law,

$$\oint \vec{B}\cdot dl$$

$$\Rightarrow \quad Bl=\mu_0\left(I_{\text{in}}\right)$$

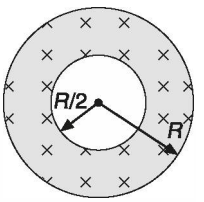
$$\Rightarrow \quad B(2\pi r)=\mu_0\left(I_{\text{in}}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0}{2\pi}\frac{I_{\text{in}}}{r} \qquad \ldots(1)$$

Since, $I_{\text{in}}=0$

$$\Rightarrow \quad B=0$$

If J be the current per unit area, then for $\frac{R}{2}\le r\le R$, we have



$$I_{\text{in}}=\left[\pi r^2-\pi\left(\frac{R}{2}\right)^2\right]J$$

Substituting in equation (1), we have

$$B=\frac{\mu_0}{2\pi}\frac{\left(\pi r^2-\pi\frac{R^2}{4}\right)J}{r}$$

$$\Rightarrow \quad B=\frac{\mu_0 J}{2r}\left(r^2-\frac{R^2}{4}\right)$$

At $r=\frac{R}{2}$, $B=0$

At $r=R$, $B=\frac{3\mu_0 JR}{8}$

For $r\ge R$, we have

$$I_{\text{in}}=I_{\text{Total}}=I(\text{say})$$

Therefore, substituting in equation (1), we have

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \quad B \propto \frac{1}{r}$$

The plot of $|\vec{B}|$ vs r graph is shown in Figure.

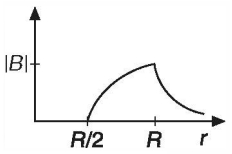


ILLUSTRATION 60

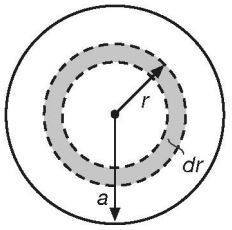
A long, straight, solid cylinder of radius a, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetrical about the cylinder axis, is not constant but varies with the radial distance r from the axis of the cylinder according to the relation given by

$$\vec{J} = \begin{cases} \dfrac{2I_0}{\pi a^2}\left(1-\left(\dfrac{r}{a}\right)^2\right)\hat{k} & \text{, for } r \le a \\ 0 & \text{, for } r \ge a \end{cases}$$

where I_0 is a constant having SI unit of current.
Show that the total current passing through the entire cross-section of the wire is I_0. Also, calculate the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis.
Using Ampere's Law, calculate the magnitude of the magnetic field $\vec{B}$ in the region $r \ge a$ and $r \le a$.

SOLUTION

Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross-section. Then use Ampere's Law to find $\vec{B}$ at the specified distance from the centre of the wire.
Let us divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure.



The current through each ring is $dI = JdA = J(2\pi r dr)$

$$dI = JdA = \frac{2I_0}{\pi a^2}\left[1-\left(\frac{r}{a}\right)^2\right]2\pi r dr$$

$$\Rightarrow \quad dI = \frac{4I_0}{a^2}\left[1-\left(\frac{r}{a}\right)^2\right]r dr$$

$$\Rightarrow \quad dI = \frac{4I_0}{a^2}\left[1-\left(\frac{r}{a}\right)^2\right]r dr \qquad \ldots(1)$$

The total current I is obtained by integrating dI over the entire cross-section of the conductor i.e. from $r = 0$ to $r = a$. So, we get

$$I = \int_0^a dI = \left(\frac{4I_0}{a^2}\right)\int_0^a\left(1-\frac{r^2}{a^2}\right)r dr$$

$$\Rightarrow \quad I = \left(\frac{4I_0}{a^2}\right)\left(\frac{r^2}{2}-\frac{1}{4}\frac{r^4}{a^2}\right)\Bigg|_0^a = \left(\frac{4I_0}{a^2}\right)\left(\frac{a^2}{2}-\frac{1}{4}\frac{a^4}{a^2}\right)$$

$$\Rightarrow \quad I = \left(\frac{4I_0}{a^2}\right)\left(\frac{a^2}{4}\right) = I_0$$

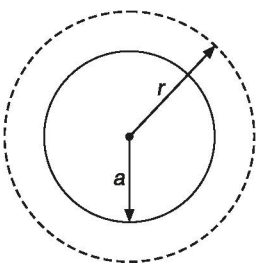
The current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis can simply be calculated by integrating equation (1), from $r = 0$ to $r = r$.

$$\Rightarrow \quad I = \int dI = \frac{4I_0}{a^2}\int_0^r\left[1-\left(\frac{r}{a}\right)^2\right]r dr$$

$$\Rightarrow \quad I = \frac{4I_0}{a^2}\left(\frac{r^2}{2}-\frac{1}{4}\frac{r^4}{a^2}\right)\Bigg|_0^r = \frac{4I_0}{a^2}\left(\frac{r^2}{2}-\frac{r^4}{4a^2}\right)$$

$$\Rightarrow \quad I = \frac{I_0 r^2}{a^2}\left(2-\frac{r^2}{a^2}\right) \qquad \ldots(2)$$

To find the magnitude of the magnetic field outside the wire i.e. for $r > a$, let us consider a circular Amperian loop of radius $r > a$, as shown in Figure.



By Ampere's Law, we have

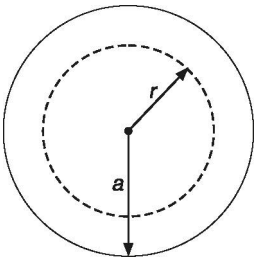
$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I_{enc}$$

Since, $I_{encl} = I_0$, because the path encloses the entire cylinder. So, we get

$$B(2\pi r) = \mu_0 I_0$$

$$\Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$

To find the magnitude of the magnetic field outside the wire i.e. for $r < a$, let us consider a circular Amperian loop of radius $r < a$, as shown in Figure.



By Ampere's Law, we have

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I_{enc}$$

From equation (2), for $r < a$, we get

$$I_{encl} = I = \frac{I_0 r^2}{a^2}\left(2 - \frac{r^2}{a^2}\right)$$

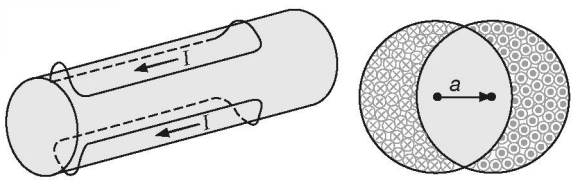
$$\Rightarrow \quad B(2\pi r) = \mu_0 I$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2}\left(2 - \frac{r^2}{a^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I_0 r}{2\pi a^2}\left(2 - \frac{r^2}{a^2}\right)$$

ILLUSTRATION 61

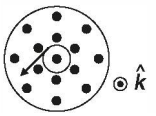
A long solenoid produces a uniform magnetic field directed along the axis of a cylindrical region. However, to produce a uniform magnetic field directed parallel to a diameter of a cylindrical region we can use the saddle coils illustrated in figure (a). The loops are wrapped over a somewhat flattened tube. Assume the straight sections of wire are very long. The end view of the tube, in figure (b) shows how the windings are applied. The overall current distribution is the superposition of two overlapping circular cylinders of uniformly distributed current, one towards the reader and one away from the reader. The current density J is the same for each cylinder. The position of the axis of one cylinder is described by a position vector a relative to the other cylinder. Prove that the magnetic field inside the hollow tube is $\frac{\mu_0 Ja}{2}$ downward.

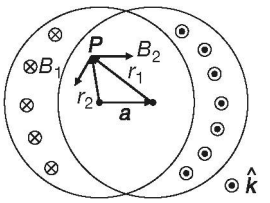


(a) (b)

SOLUTION

Here the use of vector methods will further simplify the calculations done. Let us consider first a solid cylindrical rod of radius R carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance r from its centre we consider a circular loop of radius r.





$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{inside}$$

$$B(2\pi r) = \left(\mu_0 \pi r^2\right) J$$

$$\Rightarrow \quad B = \frac{\mu_0 Jr}{2}$$

$$\Rightarrow \quad \vec{B} = \left(\frac{\mu_0 J}{2}\right)(\hat{k}\times\vec{r})$$

Now the total field at P inside the saddle coils is the field due to a solid rod carrying current toward you, centred at the head of vector $\vec{a}$, plus the field of a solid rod centred at the tail of vector $\vec{a}$ carrying current away from you.

$$\vec{B}_1 + \vec{B}_2 = \frac{\mu_0 J}{2}\hat{k}\times\vec{r}_1 + \frac{\mu_0 J}{2}(-\hat{k})\times\vec{r}_2$$

Now note $\vec{a}+r_1=\vec{r}_2$

$$\Rightarrow \quad \vec{B}=\vec{B}_1+\vec{B}_2=\frac{\mu_0 J}{2}\hat{k}\times\vec{r}_1-\frac{\mu_0 J}{2}\hat{k}\times(\vec{a}+\vec{r}_1)$$

$$\Rightarrow \quad \vec{B}=\frac{\mu_0 J}{2}(\vec{a}\times\hat{k})=\frac{\mu_0 Ja}{2} \text{ down in the diagram.}$$

ILLUSTRATION 62

A current I flows around a closed loop. It produces a field B_x directed along the axis of the loop. Integrate B_x from $-\infty$ to $+\infty$, i.e., calculate $\int_{-\infty}^{+\infty} B_x dx$. Explain the significance of your result.

SOLUTION

Since, $B_x=\dfrac{\mu_0 Ia^2}{2(x^2+a^2)^{3/2}}$

$$\Rightarrow \quad \int_{-\infty}^{+\infty} B_x dx=\int_{-\infty}^{\infty}\frac{\mu_0 Ia^2}{2(x^2+a^2)^{3/2}}dx=\frac{\mu_0 Ia^2}{2}\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^{3/2}}$$

Substitute $x=a\tan\theta$, we get

$$\int_{-\infty}^{\infty} B_x dx=\frac{\mu_0 Ia^2}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{a\sec^2\theta d\theta}{a^3\sec^3\theta}=\frac{\mu_0 Ia^2}{2a^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta d\theta$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} B_x dx=\frac{\mu_0 I}{2}\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} B_x dx=\frac{\mu_0 I}{2}\left[\sin\left(\frac{\pi}{2}\right)-\sin\left(-\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} B_x dx=\frac{\mu_0 I}{2}(2)$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} B_x dx=\mu_0 I$$

This is just what Ampere's Law tells us to expect if we imagine the loop runs along the x-axis closing itself at infinity. So, we get

$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0 I$$

Test Your Concepts-V

Based on Ampere's Circuital Law and Applications

(Solutions on page H.18)

1. A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{j}$. The current density, although symmetrical about the cylinder axis, is not constant and varies according to the relationship

$$\vec{j}=\begin{cases}\left(\dfrac{b}{r}\right)e^{\frac{(r-a)}{\delta}}\hat{k} & \text{for } r\le a\\ 0 & \text{for } r\ge a\end{cases}$$

where the radius of the cylinder is a, r is the radial distance from the cylinder axis, b is a constant and δ is a constant.

(a) Find the total current I_0, passing through the entire cross section of the wire in terms of b, δ and a.

(b) Use Ampere's Law to derive an expression for the magnetic field $\vec{B}$, in terms of I_0, in the region $r\ge a$.

(c) Obtain an expression for the current I, in terms of I_0, contained in a circular cross section of radius $r\le a$ and centred at the cylinder axis.

(d) Use Ampere's Law to derive an expression for the magnetic field $\vec{B}$ in the region $r\le a$.

2. A conductor is made in the form of a hollow cylinder with inner and outer radii a and b, respectively. It carries a current I uniformly distributed over its cross section. Find the magnitude of the magnetic field in the regions

(a) $r<a$

(b) $a<r<b$

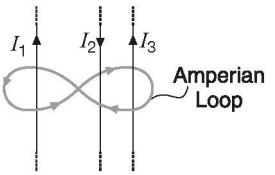
(c) $r>b$

3. A cylinder of radius $R = 0.5$ cm is actually made from a packed bundle of 100 long, straight, insulated wires.

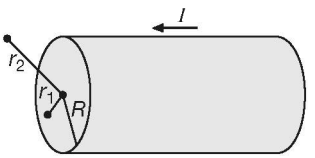
(a) If each wire carries 2 A, calculate the magnitude and direction of the magnetic force per unit length acting on a wire located 0.2 cm from the centre of the bundle.

(b) Consider a wire on the outer edge of the bundle. Will it experience a force greater or smaller than the value calculated in part (a)?

4. Write the equation for Ampere's circuital law for the Amperian loop shown in figure. The arrow marks show the sense in which the Amperian loop is taken.



5. A long cylindrical conductor of radius R carries a current I as shown in figure. The current density $\vec{J}$, however, is not uniform over the cross section of the conductor but is a function of the radius according to $J = br$, where b is a constant. Find an expression for the magnetic field B



(a) at a distance $r_1 < R$ and

(b) at a distance $r_2 > R$, measured from the axis.

6. A very large parallel-plate capacitor caries charge with uniform charge per unit area $+\sigma$ on the upper plate and $-\sigma$ on the lower plate. The plates are horizontal and both move horizontally with speed v to the right.

(a) Calculate the magnetic field between the plates.

(b) Calculate the magnetic field close to the plates but outside of the capacitor.

(c) Calculate the magnitude and direction of the magnetic force per unit area on the upper plate.

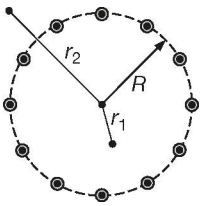
(d) At some extrapolated speed v, the magnetic force on a plate balances the electric force on the plate. Calculate v.

7. Consider the cross section of a non-conducting cylinder that has N wires parallel to the axis of the cylinder and uniformly spaced around the curved surface as shown in figure. The cylinder has a radius R and the current in each conductor is I and directed out of the plane of the figure. Assuming N to be a very large number and the radius of each wire to be small compared with the radius of the cylinder, find an expression for the magnetic field $\vec{B}$.

(a) at $r_1 < R$ and

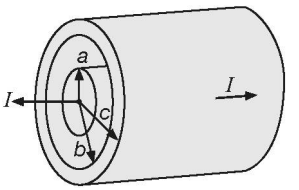
(b) at $r_2 > R$

(c) Calculate B when $N = 100$, $R = 5$ cm, $I = 10$ A and $r_2 = 15$ cm.



8. **(a)** For the coaxial cable shown, derive an expression for the magnitude of the magnetic field at points inside the central solid conductor $(r < a)$.

(b) For this coaxial cable derive an expression for the field within the tube $(b < r < c)$.
Interpret the results for $r = a$, $r = b$ and $r = c$.



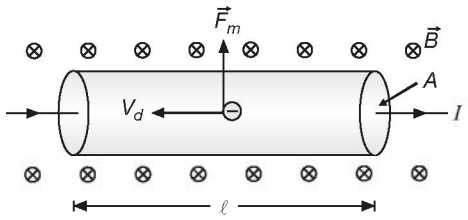
9. A column of electric current when passing through a plasma (ionized gas) makes filaments of current within the column which are magnetically attracted to one another. They can crowd together to yield a very great current density and a very strong magnetic field in a small region. Sometimes the current can be cut off momentarily by this pinch effect. The pinch effect can be demonstrated by making an empty aluminium can carry a large current parallel to its axis. Let R represent the radius of the can and I the upward current, uniformly distributed over its curved wall. Determine the magnetic field just inside the wall and just outside the wall. Also calculate the pressure on the wall.

10. Find the magnetic pressure which the lateral surface of a long straight solenoid with n turns per unit length experiences when a current I flows through it.

MAGNETIC FORCE ON CURRENT CARRYING CONDUCTORS, MAGNETIC MOMENT AND TORQUE

MAGNETIC FORCE ON A CURRENT CARRYING CONDUCTOR

Since a charged particle in motion experiences a magnetic force in a magnetic field. Hence a current carrying wire also experiences a force when placed in a magnetic field because of the fact that the current is due to the motion of collection of many charged particles. Hence, the resultant force exerted by the field on a current carrying wire is the vector sum of the individual forces exerted on all the charged particles constituting the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire. Consider a conducting wire of length l, area of cross-section A carrying a current I to be placed in a magnetic field $\vec{B}$ as shown in Figure.



Since the free electrons in the wire drift with a speed v_d opposite to the direction of current, so the magnetic force exerted on an electron is given by $\vec{f}$, where

$$\vec{f} = -e\left(\vec{v}_d \times \vec{B}\right)$$

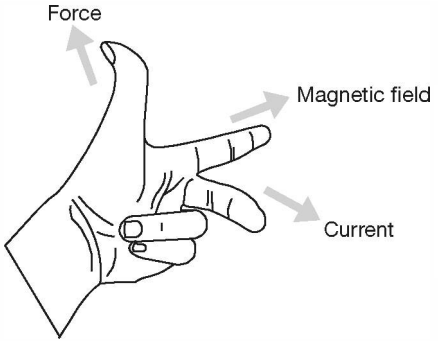
If n be the number of free electrons per unit volume of the wire, then total number of electrons in the wire having volume Al are, $N = n(Al)$. If $\vec{F}$ is the total force on the wire, then

$$\vec{F} = N\vec{f} = -e(nAl)\left(\vec{v}_d \times \vec{B}\right)$$

Since we know that $neAv_d = I$ and if we denote the length l along the direction of the current by $\vec{l}$, then the above equation becomes

$$\vec{F} = I(\vec{l} \times \vec{B}) \qquad \ldots(1)$$

The direction of $\vec{F}$ is given by Fleming's Left Hand Rule as discussed already. According to this rule, stretch the first finger, the middle finger and the thumb of the left hand in such a way that they are mutually perpendicular to each other such that the first finger points in the direction of field $(\vec{B})$, the middle finger points in the direction of conventional current (I), then the thumb gives the direction of magnetic force $(\vec{F})$.



For the above expression, following points are worth noting

(a) Here $\vec{l}$ is a vector that points in the direction of the current I and has a magnitude equal to the length.

(b) Magnitude of $\vec{F}$ is $F = BIl\sin\theta$, where θ is the angle between $\vec{l}$ and $\vec{B}$. F_m is zero for $\theta = 0°$ or $180°$ and maximum for $\theta = 90°$.

(c) The expression $\vec{F} = I(\vec{l} \times \vec{B})$ applies only to a straight segment of wire in a uniform magnetic field.

(d) For the magnetic force on an arbitrarily shaped wire segment, we consider the magnetic force exerted on a small segment of vector length $d\vec{l}$, then

$$d\vec{F} = I(d\vec{l} \times \vec{B}) \qquad \ldots(2)$$

The total force $\vec{F}_m$ acting on the wire, as shown earlier is calculated by integrating equation (2) over the length of the wire. So, we get

$$\vec{F} = I\int_P^R (d\vec{l} \times \vec{B})$$

Now let us consider two special cases involving $\vec{F} = I\int d\vec{l} \times \vec{B}$. *In both the cases we have taken the magnetic field to be constant in magnitude and direction.*

CASE-1:

A curved wire PQR (as shown in FIGURE 1) carries a current I and is located in a uniform magnetic field $\vec{B}$.

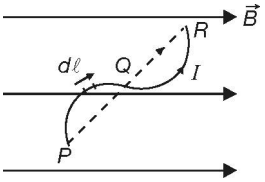


Figure 1

Since the field is uniform, so we can take $\vec{B}$ outside the integral to get

$$\vec{F} = I\left(\int_P^R d\vec{l}\right) \times \vec{B}$$

But the quantity $\int_P^R d\vec{l}$ represents the vector sum of all length elements from P to R. From the polygon law of vector addition this sum equals the vector $\vec{l}_{\text{effective}}$ directed from P to R. Hence, we can rewrite the expression as

$$\vec{F} = I\left(\vec{l}_{\text{effective}} \times \vec{B}\right)$$

$$\Rightarrow \quad \vec{F}_{PQR} = \vec{F}_{PR} = I\left(P\vec{R} \times \vec{B}\right), \text{ in a uniform field.}$$

CASE-2:

For an arbitrarily shaped closed loop carrying a current I placed in a uniform magnetic field (as shown in FIGURE 2).

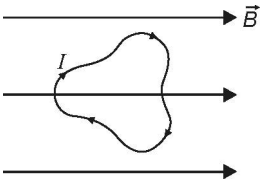


Figure 2

Here too, we again get the force acting on the loop as

$$\vec{F} = I\left(\int d\vec{l} \times \vec{B}\right)$$

However, this time we must take the vector sum of the length elements $d\vec{l}$ over the entire loop, so

$$\vec{F} = I\left(\oint d\vec{l}\right) \times \vec{B}$$

Now since, the set of length elements forms a closed polygon, hence the vector sum $\oint d\vec{l}$ must be zero. Thus, we get

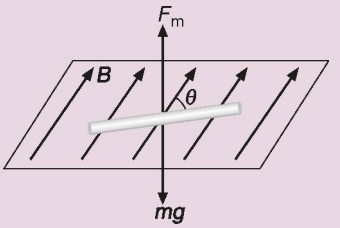
$$\vec{F} = 0$$

Hence the net magnetic force acting on any closed current loop in a uniform magnetic field is always zero.

Conceptual Note(s)

(a) Conductor to Appear Weightless (or Floating In Air)

A current carrying conductor is lying in a horizontal plane such that it is making an angle θ with the direction of magnetic field B, which also lies in the horizontal plane as shown in Figure.



Magnetic field B is inwards making an angle θ with the length of rod

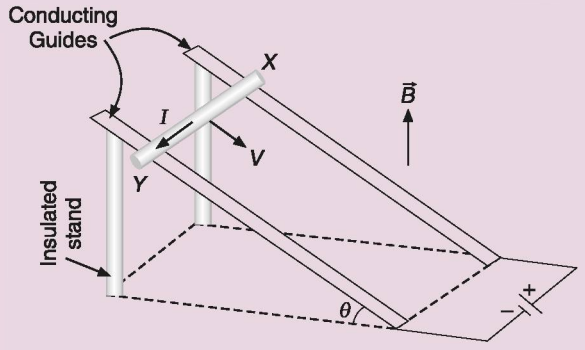
For the conductor to appear weightless, we have

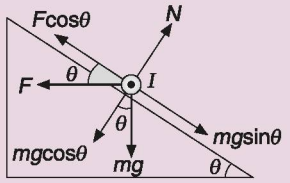
$$mg = BIL\sin\theta$$

$$\Rightarrow \quad I = \frac{mg}{BL\sin\theta}$$

(b) Sliding of conducting rod with constant velocity on inclined rails

When a conducting rod slides on conducting rails. In the following situation conducting rod XY slides with constant velocity (or is in equilibrium), when





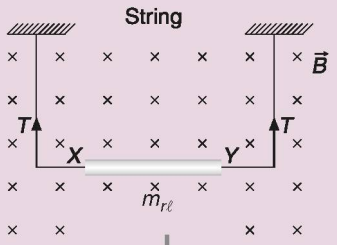
$$F\cos\theta = mg\sin\theta$$

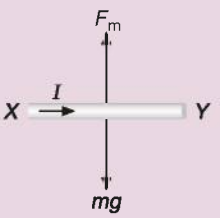
$$\Rightarrow \quad BIL\cos\theta = mg\sin\theta$$

$$\Rightarrow \quad B = \frac{mg}{IL}\tan\theta$$

(c) Tension less strings

In the following figure the value and direction of current through the conductor XY so that strings become tensionless i.e. when weight of the conductor XY is balanced by the magnetic force (F_m).





Hence direction of current is from $X \to Y$ and in balanced condition

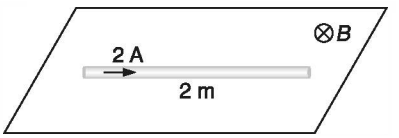
$$F_m = mg$$

$$\Rightarrow \quad BIL = mg$$

$$\Rightarrow \quad I = \frac{mg}{BL}$$

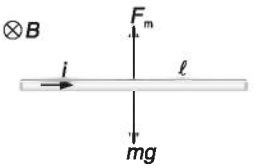
ILLUSTRATION 63

A straight wire having mass 400 g, length 2 m carrying a current of 2 A is suspended in mid air with the help of a uniform horizontal field B as shown in Figure. Calculate the value of the field for this to happen.



SOLUTION

Since the weight mg of the wire always acts vertically downwards, so for the weight to be balanced the magnetic force F_m on the wire must act upwards as shown in Figure.



So, we have

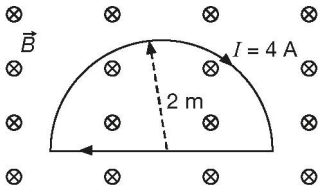
$$BiL\sin\theta = mg\text{, where } \theta = 90^\circ$$

$$\Rightarrow \quad B = \frac{0.4\times 9.8}{2\times 2\times \sin 90^\circ}$$

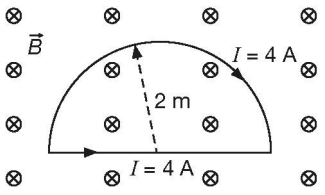
$$\Rightarrow \quad B = 0.98 \text{ T}$$

ILLUSTRATION 64

In the arrangement shown, a semi-circular wire loop is placed in a uniform magnetic field $B = 2$ T. The plane of the loop is perpendicular to the magnetic field. A current $I = 4$ A flows in the loop in the directions shown.



(a)

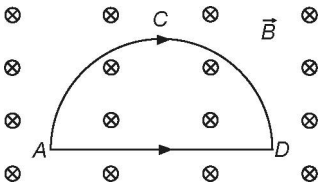


(b)

Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is 2 m.

SOLUTION

Since in FIGURE (a), the arrangement forms a closed loop and the current completes the loop. Hence the, net force on the loop in uniform field is zero.
In FIGURE (b) although the arrangement forms a closed loop, but the current does not complete the loop. Hence, net force is not zero.



Since $\vec{F}_{ACD} = \vec{F}_{AD}$ and

$$\vec{F}_{\text{loop}} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD} \qquad \{\because \vec{F}_{ACD} = \vec{F}_{AD}\}$$

$$\Rightarrow \quad \left|\vec{F}_{\text{loop}}\right| = F = 2\left|\vec{F}_{AD}\right|$$

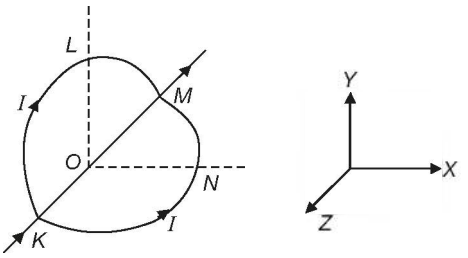
$$\Rightarrow \quad F = 2BIl_{\text{eff}}\sin\theta \qquad \{\text{where } l_{\text{eff}} = 2r = 4 \text{ m}\}$$

$$\Rightarrow \quad F = 4BIr\sin(90^\circ)$$

$$\Rightarrow \quad F = (4)(2)(4)(2) = 64 \text{ N}$$

ILLUSTRATION 65

A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semi-circles (KNM) lies in the X-Z plane and the other one (KLM) in the Y-Z plane with their centres at origin. Current I is flowing through each of the semicircles as shown in figure.



(a) A particle of charge q is released at the origin with a velocity $\vec{v} = -v_0\hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_0\hat{j}$ is applied, determine the force $\vec{F}_1$ and $\vec{F}_2$ on the semicircles KLM and KNM due to the field and the net force $\vec{F}$ on the loop.

SOLUTION

(a) Magnetic field $(\vec{B})$ at the origin is

$$\vec{B} = \begin{pmatrix}\text{Magnetic}\\ \text{field due to}\\ \text{semicircle } KLM\end{pmatrix} + \begin{pmatrix}\text{Magnetic field}\\ \text{due to other}\\ \text{semicircle } KNM\end{pmatrix}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4R}\left(-\hat{i}\right) + \frac{\mu_0 I}{4R}\left(\hat{j}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4R}\left(-\hat{i} + \hat{j}\right)$$

Since, magnetic force acting on the particle is given by $\vec{F} = q(\vec{v}\times\vec{B})$

$$\Rightarrow \quad \vec{F} = q\left[\left(-v_0\hat{i}\right)\times\left(-\hat{i} + \hat{j}\right)\right]\frac{\mu_0 I}{4R}$$

$$\Rightarrow \quad \vec{F} = -\left(\frac{\mu_0 q v_0 I}{4R}\right)\hat{k}$$

(b) $\vec{F}_{KLM} = \vec{F}_{KNM} = \vec{F}_{KM}$ and $\vec{F}_{KM} = BI(2R)\hat{i} = 2BIR\hat{i}$

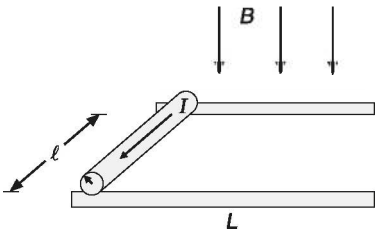
$$\Rightarrow \quad \vec{F}_1 = \vec{F}_2 = (2BIR)\hat{i}$$

Hence total force on the loop is $\vec{F} = \vec{F}_1 + \vec{F}_2$

$$\Rightarrow \quad \vec{F} = (4BIR)\hat{i}$$

ILLUSTRATION 66

A rod of mass m and radius R rests on two parallel rails that are a distance l apart and have a length L. The rod carries a current I in the direction shown in Figure.

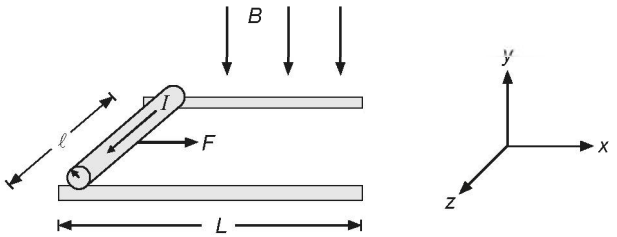


The rod rolls along the rails without slipping. A uniform magnetic field B is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails?

SOLUTION

The magnetic force acting on the rod is given by

$$\vec{F} = I(\vec{l} \times \vec{B}) = Il(\hat{k}) \times B(-\hat{j}) = (BIl)\hat{i}$$



From Work Energy Theorem we have

$$(K_{\text{trans}} + K_{\text{rot}})_{\text{initial}} + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_{\text{final}}$$

$$\Rightarrow \quad 0 + 0 + Fs\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, \text{ where } I = \frac{1}{2}mR^2$$

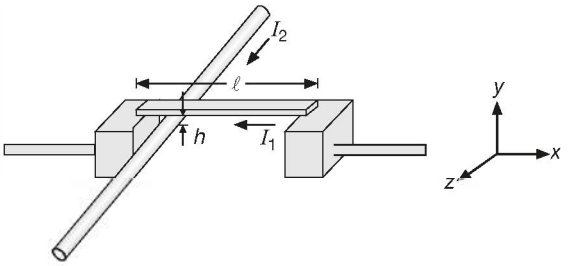
$$\Rightarrow \quad (BIl)L\cos 0^\circ = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow \quad BIlL = \frac{3}{4}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{4BIlL}{3m}}$$

ILLUSTRATION 67

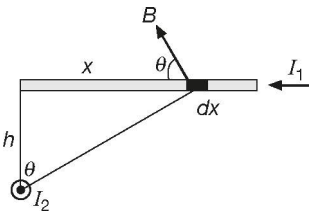
A thin copper bar of length l is supported horizontally by two (non-magnetic) contacts. The bar carries current I_1 in the $-x$ direction, as shown in Figure.



At a distance h below one end of the bar, a long straight wire carries a current I_2 in the z-direction. Determine the magnetic force exerted on the bar.

SOLUTION

At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{B} = \dfrac{\mu_0 I_2}{2\pi\sqrt{h^2 + x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \dfrac{x}{h}$ as shown in Figure.



This field exerts force on an element of the rod of length dx

$$d\vec{F} = I_1(d\vec{l} \times \vec{B}) = I_1 \frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2 + x^2}}\sin\theta$$

$$\Rightarrow \quad d\vec{F} = \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \frac{x}{\sqrt{h^2 + x^2}}, \text{ into the page}$$

$$\Rightarrow \quad d\vec{F} = \frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2 + x^2)}(-\hat{k})$$

The net force is the sum of the forces on all of the elements of the bar. So

$$\vec{F} = \int_{x=0}^{l} \frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2 + x^2)}(-\hat{k})$$

$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi}\int_0^l \frac{2x\,dx}{h^2 + x^2}$$

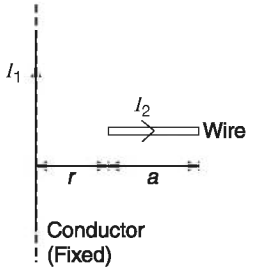
$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi}\log_e(h^2 + x^2)\Big|_0^l$$

$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi}\left[\log_e(h^2 + l^2) - \log_e h^2\right]$$

$$\Rightarrow \quad \vec{F} = \left(\frac{\mu_0 I_1 I_2}{4\pi}\right)\log_e\left(\frac{h^2 + l^2}{h^2}\right)(-\hat{k})$$

ILLUSTRATION 68

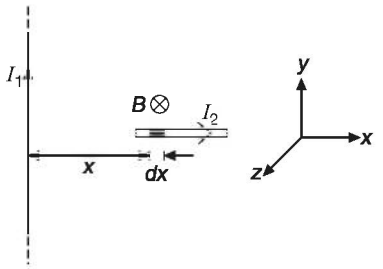
A straight current carrying wire of length a carrying a current I_2 is placed near a very long straight fixed conductor carrying a current I_1 as shown in Figure. Calculate the magnetic force acting on the wire due to the conductor.



SOLUTION

Dear Student, here we cannot directly use the formula $\vec{F} = I(\vec{l} \times \vec{B})$ to calculate force acting on the wire due to the conductor, because the magnetic field generated by the very long conductor is non-uniform and varies from one end of the wire to the other end.

So, to calculate the force, we have to consider an infinitesimal element of length dx at a distance x from the conductor as shown in Figure.



If $d\vec{F}$ be the force on this element due to the field of the conductor, then we have

$$d\vec{F} = I(d\vec{l} \times \vec{B})$$

where, $d\vec{l} = (dx)\hat{i}$ and $\vec{B} = \dfrac{\mu_0 I_1}{2\pi x}(-\hat{k})$

$$\Rightarrow \quad d\vec{F} = I(d\vec{l} \times \vec{B}) = \left(\frac{\mu_0 I_1 I_2 dx}{2\pi x}\right)(\hat{i} \times (-\hat{k}))$$

$$\Rightarrow \quad d\vec{F} = \left(\frac{\mu_0 I_1 I_2 dx}{2\pi x}\right)\hat{j}$$

Integrating, we get the net force on the wire to be

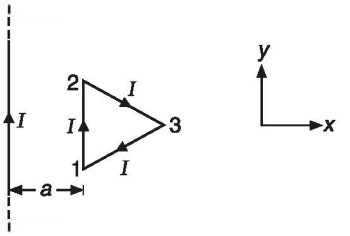
$$\vec{F} = \int d\vec{F} = \left(\frac{\mu_0 I_1 I_2}{2\pi}\right)\hat{j}\int_r^{r+a}\frac{dx}{x}$$

$$\vec{F} = \left(\frac{\mu_0 I_1 I_2}{2\pi}\right)\hat{j}\int_r^{r+a}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{2\pi}(\ln x)\Big|_r^{r+a} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{r+a}{r}\right)$$

ILLUSTRATION 69

An equilateral triangular frame with side a carrying a current I is placed at a distance a from an infinitely long straight wire carrying a current I as shown in the figure. One side of the frame is parallel to the wire. The whole system lies in the x-y plane. Find the magnetic force $\vec{F}$ acting on the frame.



SOLUTION

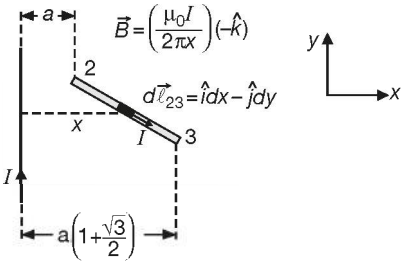
The net force on the frame is

$$\vec{F} = \vec{F}_{12} + \vec{F}_{23} + \vec{F}_{31}$$

Since, we known that $\dfrac{F}{l} = \dfrac{\mu_0 I I_2}{2\pi r}$

$$\Rightarrow \quad \vec{F}_{12} = \left(\frac{\mu_0 I^2 a}{2\pi a}\right)(-\hat{i}) = -\left(\frac{\mu_0 I^2}{2\pi}\right)\hat{i} \qquad \ldots(1)$$

To calculate $\vec{F}_{23}$ and $\vec{F}_{31}$, let us draw a figure.



Consider an infinitesimal length element $d\vec{l}_{23}$ on 23, then $d\vec{l}_{23} = \hat{i}dx - \hat{j}dy$. If $d\vec{F}_{23}$ is the force due to the field on this wire, then we have

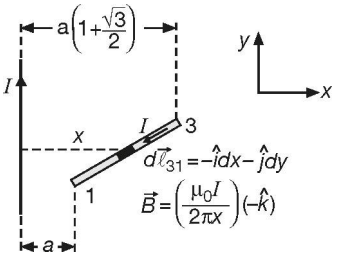
$$d\vec{F}_{23} = I(d\vec{l}_{23} \times \vec{B}), \text{ where } \vec{B} = \left(\frac{\mu_0 I}{2\pi x}\right)(-\hat{k})$$

$$\Rightarrow \quad d\vec{F}_{23} = I\left(\hat{i}dx - \hat{j}dy\right)\times\left(\frac{\mu_0 I}{2\pi x}\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad d\vec{F}_{23} = \frac{\mu_0 I^2}{2\pi}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & -dy & 0 \\ 0 & 0 & -\frac{1}{x} \end{vmatrix}$$

$$\Rightarrow \quad d\vec{F}_{23} = \frac{\mu_0 I^2}{2\pi}\left[\hat{i}\left(\frac{dy}{x}\right) + \hat{j}\left(\frac{dx}{x}\right)\right] \qquad \ldots(2)$$

Similarly, we shall now find $\vec{F}_{31}$. So, again consider an element $d\vec{l}_{31}$ on 31, then $d\vec{l}_{31} = -\left(\hat{i}dx + \hat{j}dy\right)$ as shown in figure.



If $d\vec{F}_{31}$ is the force due to the field on this wire, then we have

$$d\vec{F}_{31} = I\left(d\vec{l}_{31}\times\vec{B}\right), \text{ where } \vec{B} = \left(\frac{\mu_0 I}{2\pi x}\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad d\vec{F}_{31} = I\left(-\hat{i}dx - \hat{j}dy\right)\times\left(\frac{\mu_0 I}{2\pi x}\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad d\vec{F}_{31} = \frac{\mu_0 I^2}{2\pi}\left(\hat{i}dx + \hat{j}dy\right)\times\left(\frac{1}{x}\hat{k}\right)$$

$$\Rightarrow \quad d\vec{F}_{31} = \frac{\mu_0 I^2}{2\pi}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & 0 \\ 0 & 0 & \frac{1}{x} \end{vmatrix}$$

$$\Rightarrow \quad d\vec{F}_{31} = \frac{\mu_0 I^2}{2\pi}\left[\hat{i}\left(\frac{dy}{x}\right) - \hat{j}\left(\frac{dx}{x}\right)\right] \qquad \ldots(3)$$

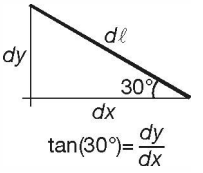
The resultant infinitesimal force on conductor 23 and 31 is given by

$$d\vec{F}_{321} = d\vec{F}_{23} + d\vec{F}_{31}$$

From (2) and (3), we get

$$d\vec{F}_{321} = \frac{\mu_0 I^2}{2\pi}\left[\hat{i}\left(\frac{2dy}{x}\right)\right]$$

To find the relation between dx and dy, we make use of the following figure.



Since, $dy = \frac{dx}{\sqrt{3}}$

$$\Rightarrow \quad d\vec{F}_{321} = \frac{\mu_0 I^2}{2\pi}\left[\hat{i}\left(\frac{2dx}{\sqrt{3}x}\right)\right]$$

Integrating, we get

$$\vec{F}_{321} = \frac{\mu_0 I^2}{\sqrt{3}\pi}\left[\int_a^{a+\frac{\sqrt{3}}{2}a}\frac{dx}{x}\right]\hat{i}$$

$$\Rightarrow \quad \vec{F}_{321} = \frac{\mu_0 I^2}{\sqrt{3}\pi}\left(\log_e x\Big|_a^{a+\frac{\sqrt{3}}{2}a}\right)\hat{i}$$

$$\Rightarrow \quad \vec{F}_{321} = \frac{\mu_0 I^2}{\sqrt{3}\pi}\left[\log_e\left(a+\frac{\sqrt{3}}{2}a\right) - \log_e a\right]\hat{i}$$

$$\Rightarrow \quad \vec{F}_{321} = \frac{\mu_0 I^2}{\sqrt{3}\pi}\left[\log_e\left(1+\frac{\sqrt{3}}{2}\right)\right]\hat{i}$$

So, from equations (1), (2) and (3), we get

$$\vec{F} = \vec{F}_{12} + \vec{F}_{23} + \vec{F}_{31} = \vec{F}_{12} + \vec{F}_{321}$$

$$\Rightarrow \quad \vec{F} = \left(\frac{\mu_0 I^2}{2\pi}\right)\hat{i} + \frac{\mu_0 I^2}{\sqrt{3}\pi}\left[\log_e\left(1+\frac{\sqrt{3}}{2}\right)\right]\hat{i}$$

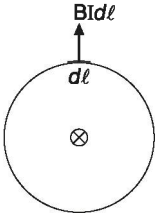
$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I^2}{\pi}\left[-\frac{1}{2} + \frac{1}{\sqrt{3}}\log_e\left(1+\frac{\sqrt{3}}{2}\right)\right]\hat{i}$$

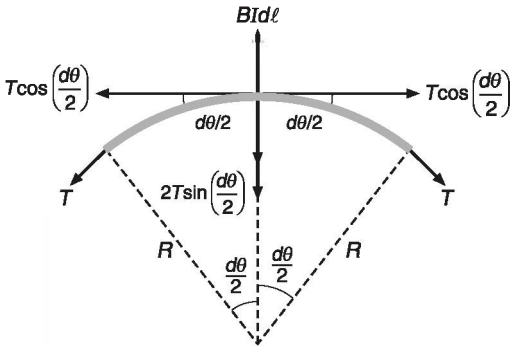
ILLUSTRATION 70

A coil carrying a current I is placed in a uniform magnetic field so that its axis coincides with the field direction. The single-layer winding of the coil is made of wire with diameter d, radius of turns is equal to R. At what value of the induction of the external magnetic field can the coil winding be ruptured? Assume the breaking stress of the wire to be σ_0.

SOLUTION

Each element of length dl experiences a force $BIdl$. This causes a tension T in the wire. Consider an infinitesimal arc element of length dl, subtending an angle $d\theta$ at the centre as shown in Figure.





For equilibrium, we have

$$2T\sin\left(\frac{d\theta}{2}\right)=BIdl$$

Since $d\theta$ is very small, so we get

$$\sin\left(\frac{d\theta}{2}\right)\simeq\frac{d\theta}{2}$$

$$\Rightarrow \quad Td\theta=BIdl$$

$$\Rightarrow \quad Td\theta=BI(Rd\theta)$$

$$\Rightarrow \quad T=BIR$$

The wire experiences a stress σ, which is

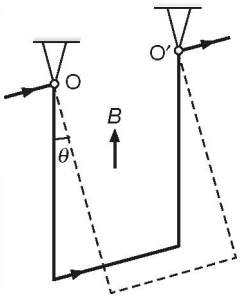
$$\sigma=\frac{\text{Force}}{\text{Area}}=\frac{BIR}{\frac{\pi d^2}{4}}$$

This must equal the breaking stress σ_0 for the winding to rupture. Thus,

$$B_{\max}=\frac{\pi d^2\sigma_0}{4IR}$$

ILLUSTRATION 71

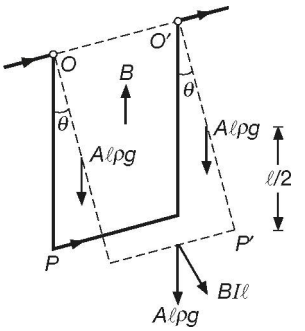
A wire with cross-sectional area A, density ρ bent to make three sides of a square can turn about a horizontal axis OO' as shown in Figure.



The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current I through the wire the latter deflects by an angle θ.

SOLUTION

The magnetic forces on the sides OP and $O'P'$ are directed along the same line, in opposite directions and have equal values, hence the net force as well as the net torque of these forces about the axis OO' is zero. The magnetic force on the segment PP' and the corresponding moment of this force about the axis OO' is effective and is deflecting in nature.



In equilibrium, shown by the dotted position, the deflecting torque must be equal to the restoring torque, developed due to the weight of the shape.
Let, the length of each side be l and ρ be the density of the material then,

$$BIl(l\cos\theta)=(Al\rho)g\left(\frac{l}{2}\sin\theta\right)+(Al\rho)g\left(\frac{l}{2}\sin\theta\right)+(Al\rho)g(\sin\theta)$$

$$\Rightarrow \quad Il^2B\cos\theta=2A\rho gl^2\sin\theta$$

$$\Rightarrow \quad B=\frac{2A\rho g}{I}\tan\theta$$

ILLUSTRATION 72

A conducting wire of length l is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current I is established in the rod are observed to be F_1 and $F_2\,(<F_1)$ for the two possible directions of the current through the rod respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.

SOLUTION

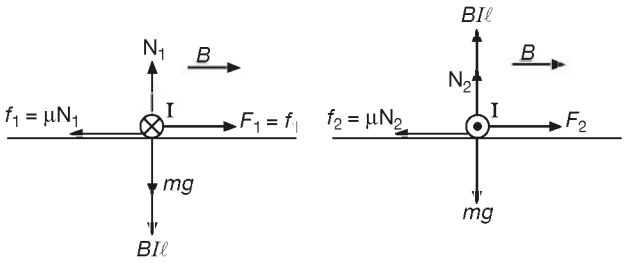
Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface. In one case magnetic force on the wire will be in upward direction while in the other case it will be in the downward direction. Hence normal reaction is given by

$$N = mg \pm BIl$$

$$\Rightarrow \quad f_{(\text{friction limiting})} = \mu(mg \pm BIl) \text{ as } F_1 > F_2$$

$$\Rightarrow \quad F_1 = f_1 = \mu(mg + BIl) \qquad \ldots(1)$$

and $F_2 = f_2 = \mu(mg - BIl)$ $\qquad \ldots(2)$



From equations (1) and (2),

$$\frac{F_1}{F_2} = \frac{mg + BIl}{mg - BIl}$$

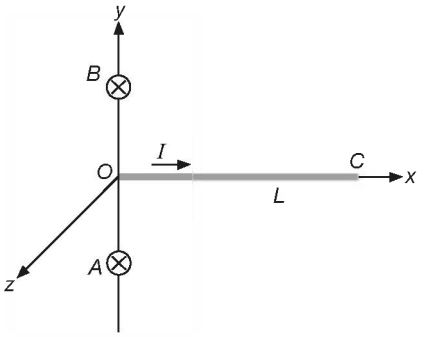
$$\Rightarrow \quad mg = BIl\left(\frac{F_1 + F_2}{F_1 - F_2}\right) \qquad \ldots(3)$$

From equations (1) and (3), we get

$$\mu = \frac{F_1 - F_2}{2BIl}$$

ILLUSTRATION 73

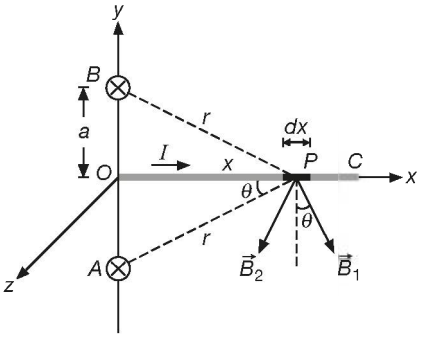
A straight segment OC (of length L) of a circuit carrying a current I is placed along the x-axis. Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed at $y = -a$ and $y = +a$ respectively, as shown in the Figure.



If the wires A and B each carry a current I into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed?

SOLUTION

The current I at A into the plane of the paper produces magnetic field B_1 at $P(x, 0, 0)$ in a direction perpendicular to AP, as shown in Figure.



The magnetic field B_2 which acts perpendicular to BP is due to the current I at B into the plane of paper. The total magnetic field at P due to the currents at A and B is obtained by the vector addition of the fields B_1 and B_2. On resolution, we observe that the x-components of B_1 and B_2 cancel each other whereas y-components add up and thus the net field of B_1 and B_2 would point towards $-y$ direction. Since

$$B = B_1 + B_2, \text{ where } B_1 = \frac{\mu_0 I}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \quad \vec{B} = \frac{2\mu_0 I}{2\pi r}\cos\theta\left(-\hat{j}\right) \qquad \ldots(1)$$

where θ is the angle which AP or BP makes with OP, (i.e., x-axis) and r is the length of AP or BP. If x is the length OP, then

$$r = \sqrt{x^2 + a^2} \text{ and } \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + a^2}}$$

Substituting these in equation (1), we get

$$\vec{B} = \left(\frac{\mu_0 I}{\pi}\right)\left(\frac{x}{x^2 + a^2}\right)\left(-\hat{j}\right)$$

The force dF acting on the current element $Idx\hat{i}$ due to the field B is given as

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$\Rightarrow \quad d\vec{F} = Idx\hat{i}\times(-\hat{j})\left(\frac{\mu_0 I}{\pi}\right)\left(\frac{x}{x^2+a^2}\right)$$

$$\Rightarrow \quad d\vec{F} = \frac{\mu_0 I^2}{\pi}(-\hat{k})\frac{xdx}{x^2+a^2}$$

Total force acting on the wire OC is obtaining by integrating the above expression within the limits from $x=0$ to $x=L$. So, we get

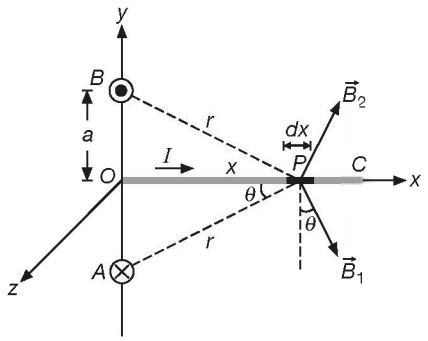
$$\vec{F} = \frac{\mu_0 I^2}{\pi}(-\hat{k})\int_0^L \frac{x\,dx}{x^2+a^2}$$

$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I^2}{2\pi}(-\hat{k})\log_e\left(\frac{x^2+a^2}{a^2}\right)$$

So, we get

$$F = \frac{\mu_0 I^2}{2\pi}\log_e\left(\frac{x^2+a^2}{a^2}\right), \text{along } -z \text{ axis}$$

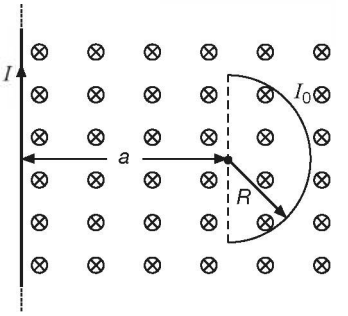
If the current in wire B is reversed, then the magnetic field B_2 at P due to the current I at B would act in the direction as shown in Figure.



On resolution, the resultant field of $\vec{B}_1$ and $\vec{B}_2$ will be along $+x$-axis i.e., along $Id\vec{l}$. So, force due to this arrangement of A and B on the conductor OC is zero.

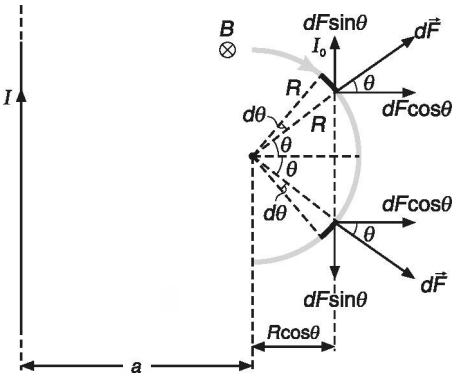
ILLUSTRATION 74

A semi-circular conductor of radius R carrying a current I_0 is coplanar with a very long straight conductor carrying a current I. The distance of the centre of the straight conductor is a from the straight conductor. Find the magnetic interaction force between the conductors for the position shown in Figure.



SOLUTION

According to Biot-Savart's Law the magnetic induction due to very long straight current I is directed normally into the plane of the figure. The magnitude of the magnetic induction due to straight current depends upon the perpendicular distance from it. Here the elements of semi-circular current are at different perpendicular distances from the straight current. Let us take an element of the circular current at an angle θ from horizontal as shown in the Figure.



Taken elemental wire experiences the Ampere force in accordance with the formula.

$d\vec{F} = I(d\vec{l}\times\vec{B})$, which gives

$d\vec{F} = I_0 Rd\theta\frac{\mu_0}{2\pi}\frac{I}{(a+R\cos\theta)}$, directed away from the centre

From the symmetry of the problem, all sine components cancel, hence the net Ampere force $\vec{F}$ is directed towards right and is given by

$$F_{net} = \int dF\cos\theta$$

$$\Rightarrow \quad F_{net} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mu_0}{2\pi} I I_0 R \frac{\cos\theta d\theta}{(a + R\cos\theta)}$$

$$\Rightarrow \quad F_{net} == \frac{\mu_0}{2\pi} I I_0 R \left[2\int_{0}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(a + R\cos\theta)} \right]$$

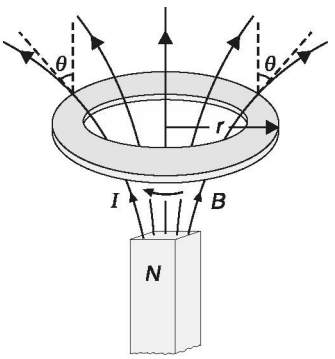
$$\Rightarrow \quad F_{net} = \frac{\mu_0}{\pi} I I_0 R \int_{0}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(a + R\cos\theta)}$$

$$\Rightarrow \quad F_{net} = \frac{\mu_0 I I_0}{\pi} \left(\frac{\pi}{2} - \frac{2a}{\sqrt{a^2 - R^2}} \tan^{-1} \sqrt{\frac{a-R}{a+R}} \right)$$

Please note that if the current I_0 were in a circular loop having its centre at O, then the two halves experience forces in the opposite directions. The left half experiences the Ampere force $F_{\text{Left Half}}$ (say) towards left and the right half experiences a net force $F_{\text{Right Half}}$ towards right and so the net Ampere force on the circular current loop becomes $\left(F_{\text{Left Half}} - F_{\text{Right Half}}\right)$, directed towards left.

ILLUSTRATION 75

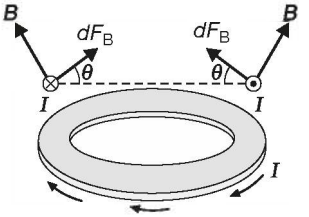
A non-uniform magnetic field exerts a net force on a magnetic dipole. A strong magnet is placed under a horizontal conducting ring of radius r that carries current I as shown in Figure.



If the magnetic field $\vec{B}$ makes an angle θ with the vertical at the ring's location, calculate the magnitude and direction of the resultant force on the ring?

SOLUTION

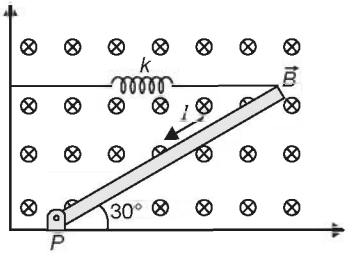
The magnetic force on each bit of ring is $I(d\vec{\ell} \times \vec{B}) = BI(d\ell)$ radially inward and upward, at angle θ above the radial line as shown in Figure.



The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $Id\ell B\sin\theta$ all add to give $F = I(2\pi r)B\sin\theta$, upwards.

ILLUSTRATION 76

A thin, uniform rod with negligible mass and length ℓ is attached to the floor by a frictionless hinge at point P. A horizontal spring with force constant k connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field B directed into the plane of the figure. There is current I in the rod, in the direction shown in Figure.



(a) Calculate the torque due to the magnetic force on the rod, for an axis at P. Is it correct to take the total magnetic force to act at the centre of gravity of the rod when calculating the torque? Explain.

(b) When the rod is in equilibrium and makes an angle of 30° with the floor, is the spring stretched or compressed?

(c) How much energy is stored in the spring when the rod is in equilibrium?

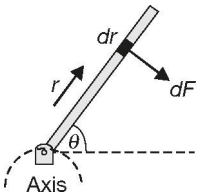
SOLUTION

Let us calculate the force and then the torque on each small section of the rod and then integrating to find the total magnetic torque. At equilibrium the torques

from the spring force and from the magnetic force cancel. If the spring stretches by x, then we have

$$U = \frac{1}{2}kx^2$$

which gives the energy stored in the spring.



Let us now divide the rod into infinitesimal sections of length dr, as shown in figure.

(a) The magnetic force on this section of the rod is $dF = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF = IBrdr$. The total torque is given by

$$\tau = \int d\tau = IB\int_0^{\ell} rdr = \frac{1}{2}I\ell^2B, \text{ clockwise.}$$

(b) F produces a clockwise torque so the spring force must produce a counter clockwise torque. The spring force must be to the left so as to keep the spring stretched.

(c) Since $\sum \tau = 0$, axis at hinge, counter clockwise torques positive

$$\Rightarrow (kx)\ell\sin(30°) - \frac{1}{2}I\ell^2B = 0$$

$$\Rightarrow x = \frac{BI\ell}{k}$$

$$\Rightarrow U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{BI\ell}{k}\right)^2 = \frac{B^2I^2\ell^2}{2k}$$

The total magnetic force $F = BI\ell$ acts at the centre of the rod. Also note that, we didn't include the torque due to gravity since the rod had negligible mass.

MAGNETIC PRESSURE AND MAGNETIC FIELD ENERGY

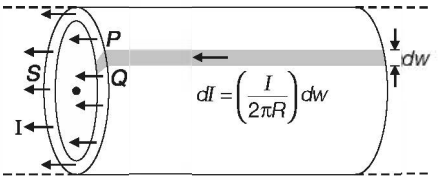
Since we know that a charged metal surface always experiences an electrostatic pressure on it in the outward direction due to its surface charge density (as already studied in Electrostatics).

Now let us consider the case of a hollow current carrying cylindrical conductor. For the case of a current carrying hollow conductor, the current flows at its surface. This current flowing on the surface can be thought to be flowing in parallel strips each carrying current in the same direction, due to which these strips must attract each other and hence the conductor surface experiences an inward magnetic pressure.

To calculate this magnetic pressure on the surface, let us consider a long straight hollow cylindrical shell carrying a current I. Since we know that at every point inside this hollow conductor, the magnetic field is zero, whereas just outside the conductor, the magnetic field is

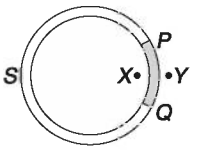
$$B = \frac{\mu_0 I}{2\pi R} \qquad \ldots(1)$$

Figure shows the front view and the cross sectional view of the hollow cylindrical conductor which shows an infinitesimal wire PQ of width dw and the remainder conductor shell PSQ.



Front View

Let us draw the cross-sectional view of the circular section and then consider two points X and Y just inside and outside the cylindrical shell as shown in the Figure.



Cross-sectional View

At both these points magnetic field must be the vector sum of the fields due to the infinitesimal wire and the remainder cylindrical shell. At point X, the magnetic field is zero and at point Y, magnetic field is given by

$$B = \frac{\mu_0 I}{2\pi R}.$$

If B_1 and B_2 be the magnetic fields at points X and Y due to the infinitesimal wire PQ (of width dw) and the remainder conductor shell PSQ, then we have

$$B_X = B_1 - B_2 = 0 \qquad \ldots(2)$$

$$B_Y = B_1 + B_2 = B = \frac{\mu_0 I}{2\pi R} \qquad \ldots(3)$$

From equations (2) and (3), we have

$$B_1 = B_2 = \frac{B}{2} \qquad \ldots(4)$$

This is exactly similar to the case studied in electrostatics. So, we see that, the elemental part PQ of the shell contributes exactly half of the magnetic field just outside the shell due to the entire current flowing in it.

If we consider a length l of the elemental wire PQ, then force experienced by it due to the remaining section PSQ is given by

$$dF = B_2(dI)l \text{, where } dI = \left(\frac{I}{2\pi R}\right)dw$$

$$\Rightarrow \quad dF = B_2\left(\frac{Idw}{2\pi R}\right)l$$

$$\Rightarrow \quad dF = \left(\frac{\mu_0 I}{4\pi R}\right)\left(\frac{Idw}{2\pi R}\right)l$$

The area of the elemental wire strip is $dA = ldw$, so, the inward magnetic pressure P_m on the elemental strip is given by

$$P_m = \frac{dF}{dA} = \frac{\mu_0 I^2}{8\pi^2 R^2} = \frac{1}{2\mu_0}\left(\frac{\mu_0{}^2 I^2}{4\pi^2 R^2}\right)$$

$$\Rightarrow \quad P_m = \frac{B^2}{2\mu_0} \qquad \ldots(5)$$

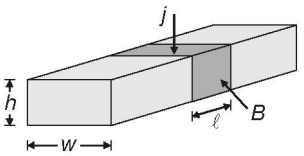
This expression in equation (5) represents the magnetic pressure due to a surface current and this is also equal to the magnetic energy density i.e. energy stored per unit volume by the magnetic field of the current carrying conductor.

Test Your Concepts-VI

Based on Force on Current Carrying Conductor

(Solutions on page H.21)

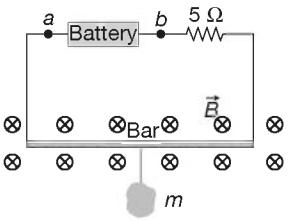
1. Liquid sodium, an excellent thermal conductor which melts at 99°C is used in some nuclear reactors to cool the reactor core. The liquid sodium is moved through the pipes of the reactor through Electromagnetic pumps. These pumps work on the concept that a moving charge in a magnetic field experiences a force. Consider a liquid metal to be in an electrically insulated pipe that has a rectangular cross-section of width w and height h. A uniform magnetic field $\vec{B}$ acts perpendicular to the section of pipe having a length ℓ as shown. Also, an electric field directed perpendicular to the pipe and to the magnetic field produces a current density j in the liquid.



(a) Calculate force that acts on the liquid in the arrangement shown.

(b) Calculate pressure increase, that the section of the liquid of length ℓ experiences in the magnetic field.

2. The circuit shown in figure is used to make a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.5 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60 cm long and is made of extremely light weight material. It is connected to the battery by thin vertical wires that can support no appreciable tension. All the weight of the suspended mass m is supported by the magnetic force on the bar. A resistor with $R = 5\ \Omega$ is in series with the bar and the resistance of the rest of the circuit is much less than this.

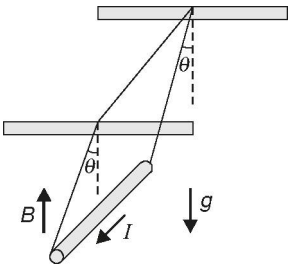


(a) Which point, a or b, should be the positive terminal of the battery?

(b) If the maximum terminal voltage of the battery is 175 V, calculate the greatest mass m that this instrument can measure.

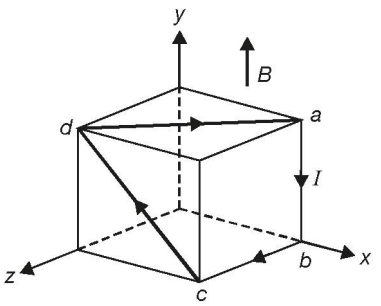
3. A 0.2 kg metal rod carrying a current of 10 A glides on two horizontal rails 0.5 m apart. What vertical magnetic field is required to keep the rod moving at a constant speed if the coefficient of kinetic friction between the rod and rails is 0.1?

4. A metal rod having a mass per unit length λ carries a current I. The rod hangs from two vertical wires in a uniform vertical magnetic field as shown in figure. The wires make an angle θ with the vertical when in equilibrium. Determine the magnitude of the magnetic field.

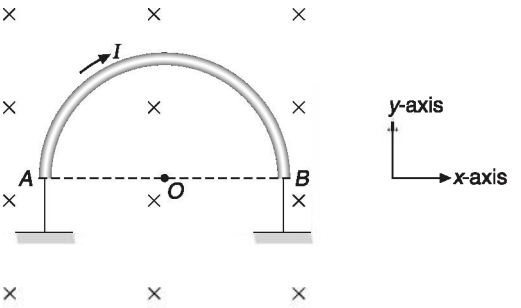


5. A wire having a mass per unit length of 0.5 gcm^{-1} carries a 2 A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

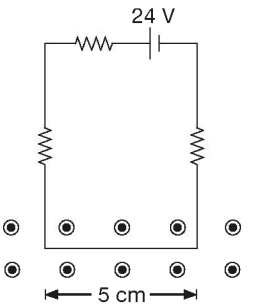
6. In the figure shown, the cube has each side of length ℓ. Four straight segments of wires ab, bc, cd and da forming a closed loop carrying a current I, in the direction shown are taken. A uniform magnetic field of magnitude B is existing in the positive y-direction. Determine the magnitude and direction of the magnetic force on each segment.



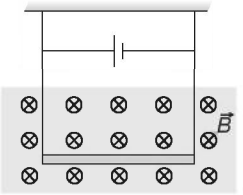
7. The top view of a conducting rod AB, bent to form a semicircle of radius R, carrying current I, placed on a smooth horizontal table is shown in Figure. It is subjected to a uniform magnetic field B perpendicular to its plane. The rod is held at rest in its position by two light strings attached to its ends. Calculate the magnetic force acting on the conductor AB and the tension in the strings.



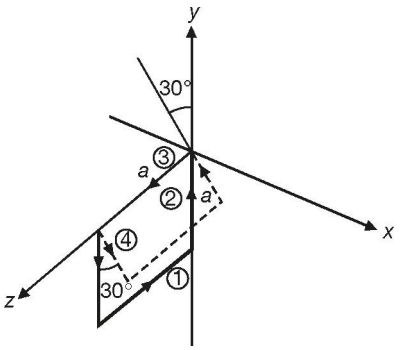
8. The circuit in figure consists of wires at the top and bottom and identical metal springs in the left and right sides. The upper portion of the circuit is fixed. The wire at the bottom has a mass of 10 g and is 5 cm long. The springs stretch 0.5 cm under the weight of the wire and the circuit has a total resistance of 12 Ω. When a magnetic field is turned on, directed out of the page, the springs stretch an additional 0.3 cm. What is the magnitude of the magnetic field?



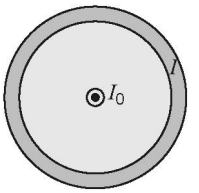
9. A conductor suspended by two flexible wires as shown in figure has a mass per unit length of 0.04 kgm^{-1}. What current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.6 T into the page? Also find the required direction for the current?



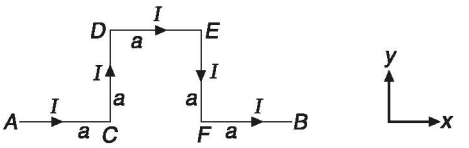
10. Consider a uniform square frame as shown in figure. Each side of the frame is of length a. The longitudinal mass density of the frame is λ. The frame carries an electric current I and is free to move around its upper side (side 3 in figure). Compute the magnetic field B in the $+y$ direction, required to hold the frame titled 30° away from the vertical axis.



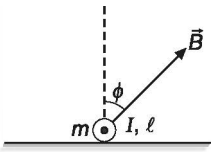
11. A long thin walled hollow cylinder of radius r is carrying a current I. A very long current carrying straight conductor is passing through the axis of the hollow cylinder, carrying a current I_0. Find the tension per unit length developed in the hollow cylinder due to the interaction of the straight current carrying conductor.



12. A uniform magnetic field $\vec{B} = B_0\hat{k}$ exists in a region. A current carrying wire $ACDEFB$ is placed in x-y plane as shown in Figure. Calculate the force acting on the wire AB, if each section of the wire is having length a.



13. A conducting rod of mass m, length l carrying a current I is subjected to a magnetic field of induction B as shown in Figure.



If the coefficients of friction between the conducting rod and rail is μ, find the value of I for which the rod starts sliding.

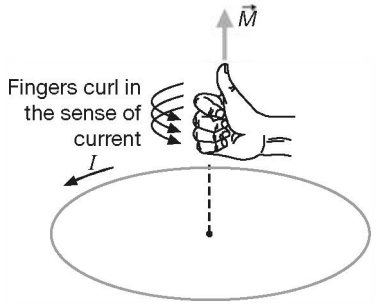
MAGNETIC DIPOLE

Just like a bar magnet , every current carrying loop behaves like a magnetic dipole and has two poles, South (S) and North (N) in which the magnetic field lines emanate from the north pole and after forming a closed path terminate at the south pole. Each magnetic dipole has some magnetic moment $(\vec{M})$. The magnitude of the dipole moment $\vec{M}$ is

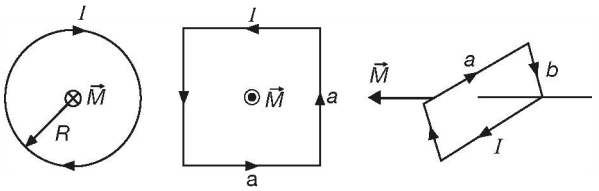
$$\vec{M} = N(I\vec{A})$$

where, N is the number of turns in the loop, I is current in the loop and $\vec{A}$ is area of cross-section of the loop. To find the direction of $\vec{M}$, we can make use of any of the following methods.

The vector $\vec{M}$ is along the normal to the plane of the loop and its orientation (up or down along the normal) is given by the right-hand rule.



For getting the direction of $\vec{M}$, just curl the fingers of your right hand around the perimeter of the loop in the sense of current as shown and extend your thumb so that it is perpendicular to the plane of the loop. The thumb direction gives the direction of $\vec{M}$.



(a) $M = I(\pi R^2)$ (b) $M = I(a^2)$ (c) $M = I(ab)$

Remark(s)

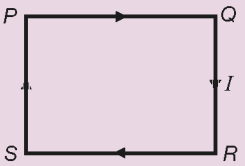
For calculating $\vec{M}$, let us discuss two more methods.

METHOD 1

This method is useful for calculating $\vec{M}$ for a rectangular or square loop.

The magnetic moment $(\vec{M})$ of the rectangular loop shown in figure is,

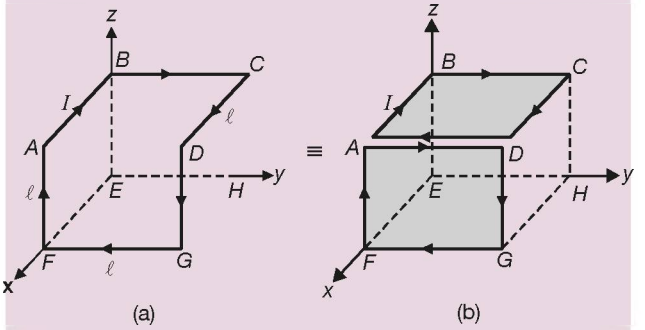
$$\vec{M} = I\left(\overrightarrow{PQ}\times\overrightarrow{QR}\right) = I\left(\overrightarrow{QR}\times\overrightarrow{RS}\right) = I\left(\overrightarrow{RS}\times\overrightarrow{SP}\right) = I\left(\overrightarrow{SP}\times\overrightarrow{PQ}\right)$$



Here the cross product of any two consecutive sides (taken in order) gives the area as well as the correct direction of $\vec{M}$ also.

METHOD 2

Sometimes a current carrying loop may not lie in a single plane. However, by assuming two equal and opposite currents in one branch (which obviously makes net current in that branch to be zero) two (or more) closed loops can be completed in different planes. Now the net magnetic moment of the given loop is the vector sum of individual loops.



For example, in figure (a), A loop carries a current I in the directions shown. To find the magnetic moment we assume two equal and opposite currents in wire AD and get two complete loops in two different planes (xy and yz). So,

$$\vec{M}_{ABCDA} = -I\ell^2\hat{k} \text{ and } \vec{M}_{ADGFA} = -I\ell^2\hat{i}$$

$$\Rightarrow \vec{M}_{net} = \vec{M}_{ABCDA} + \vec{M}_{ADGFA} = -I\ell^2\left(\hat{i}+\hat{k}\right)$$

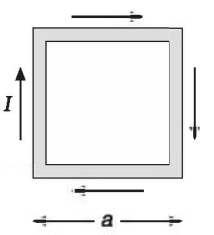
ILLUSTRATION 77

A given length of a constant current carrying straight wire is moulded into a square, equilateral triangle and a circular loop, each of one turn. Which loop has the maximum magnetic moment?

SOLUTION

Let L be the length of the wire to be bent in loops of various shapes and let I be the current in each loop and A_{sqaure}, $A_{triangle}$ and A_{circle} be the area of the square loop, triangular loop and circular loop. Then

For Square



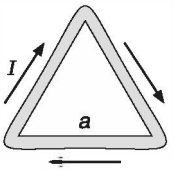
$$L = 4a$$

$$\Rightarrow a = \frac{L}{4}$$

$$\Rightarrow A_{square} = a^2 = \left(\frac{L}{4}\right)^2 = \frac{L^2}{16}$$

$$\Rightarrow M_{square} = IA_{square} = Ia^2 = \frac{IL^2}{16}$$

For Triangle



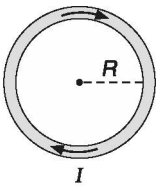
$$L = 3a$$

$$\Rightarrow \quad a = \frac{L}{3}$$

$$\Rightarrow \quad A_{\text{triangle}} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{L}{3}\right)^2 = \frac{\sqrt{3}L^2}{36}$$

$$\Rightarrow \quad M_{\text{triangle}} = IA_{\text{triangle}} = \frac{\sqrt{3}IL^2}{36}$$

For Circle



$$L = 2\pi R$$

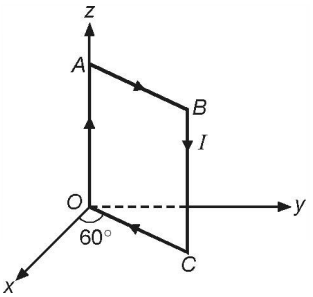
$$\Rightarrow \quad R = \frac{L}{2\pi}$$

$$\Rightarrow \quad A_{\text{circle}} = \pi R^2 = \pi\left(\frac{L^2}{4\pi^2}\right) = \frac{L^2}{4\pi}$$

$$\Rightarrow \quad M_{\text{circle}} = IA_{\text{circle}} = \frac{IL^2}{4\pi} = \text{MAXIMUM}$$

ILLUSTRATION 78

A square loop $OABCO$ of side l carries a current I and is placed as shown in Figure. Calculate the magnetic moment of the loop.



SOLUTION

As discussed, the magnetic moment of the loop is given by

$$\vec{M} = I(\vec{BC} \times \vec{CO})$$

where, $\vec{BC} = -l\hat{k}$

and $\vec{CO} = -l\cos 60°\hat{i} - l\sin 60°\hat{j} = -\frac{l}{2}\hat{i} - \frac{\sqrt{3}l}{2}\hat{j}$

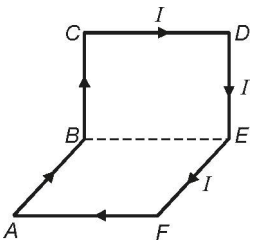
$$\Rightarrow \quad \vec{M} = I\left[(-l\hat{k}) \times \left(-\frac{l}{2}\hat{i} - \frac{\sqrt{3}l}{2}\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{M} = \frac{Il^2}{2}\left(-\sqrt{3}\hat{i} + \hat{j}\right)$$

$$\Rightarrow \quad |\vec{M}| = Il^2$$

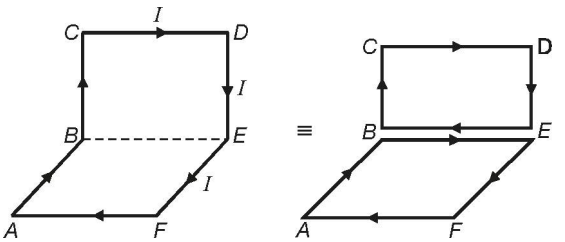
ILLUSTRATION 79

Find the magnitude of magnetic moment of the current carrying loop $ABCDEFA$. Each side of the loop is 10 cm long and current in the loop is $I = 2$ A.



SOLUTION

By assuming two equal and opposite currents in BE, two current carrying loops ($ABEFA$ and $BCDEB$) are formed as shown in Figure.



Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

where $M = IA = (2)(0.1)(0.1) = 0.02 \text{ Am}^2$

$$\Rightarrow \quad M_{\text{net}} = (\sqrt{2})(0.02) \text{ Am}^2$$

$$\Rightarrow \quad M_{\text{net}} = 0.028 \text{ Am}^2$$

GYROMAGNETIC RATIO (GMR)

Whenever a non-conducting uniformly charged body is rotated with some angular speed then the ratio of magnetic moment and angular momentum is constant which is equal to $\frac{q}{2m}$, where q is the charge and m the mass of the body. This ratio is called as the GYROMAGNETIC RATIO.

EXAMPLE

In case of a ring, of mass m, radius R and charge q distributed on its circumference, the angular momentum L of the ring about the said axis is given by

$$L = I\omega = (mR^2)\omega \qquad \ldots(1)$$

where, I is the moment of Inertia of the ring given by

$$I = mR^2$$

Magnetic moment is given by

$$M = iA = (qf)(\pi R^2)$$

where i is the current due to the circulating charge

$$\Rightarrow \quad i = \frac{q}{T} = qf$$

where f is the frequency given by $f = \dfrac{\omega}{2\pi}$

The magnetic moment of the loop is given by

$$M = iA = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{M}{L} = \frac{q}{2m}$$

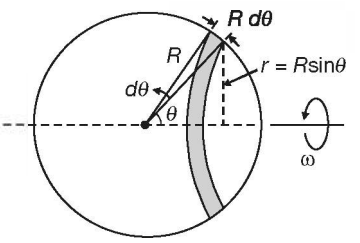
Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere.

ILLUSTRATION 80

A spherical shell of radius R, having charge Q rotates with angular velocity ω about an axis passing through its centre. Calculate the magnetic induction at the centre of the shell. Find the magnetic moment of the shell in terms of Q and express both the results in terms of σ. Also calculate the ratio of the magnetic moment $|\vec{M}|$ to the angular momentum of the rotating shell $|\vec{L}|$, called as GYROMAGNETIC RATIO. Can you conclude something significant from this ratio? Explain.

SOLUTION

The shell can be assumed to be made up of a number of co-axial rings. Consider one such ring of radius r, azimuth angle θ, thickness dr subtending an angle $d\theta$ at the centre, as shown in Figure.



If dq be the charge on the infinitesimal element, then

$$dq = \sigma dA \text{ where } dA = (2\pi r)dr,\ r = R\sin\theta$$

and $dr = Rd\theta$

$$\Rightarrow \quad dq = \frac{Q}{4\pi R^2}2\pi(R\sin\theta)(Rd\theta)$$

$$\Rightarrow \quad dq = \left(\frac{Q}{2}\right)\sin\theta d\theta$$

Now, this charge dq circulates about the axis shown, so the equivalent current produced by the element is

$$dI = \frac{dq}{\left(\frac{2\pi}{\omega}\right)} = \left(\frac{Q\omega}{4\pi}\right)(\sin\theta d\theta)$$

Since the magnetic field at the axis of the current carrying loop is given by

$$B_{\text{axis}} = \frac{\mu_0 I(\text{radius of ring})^2}{2(a^2+x^2)^{3/2}}, \text{ where}$$

$$(a^2+x^2)^{3/2} = R^3$$

So, the magnetic field produced due to an infinitesimal current carrying loop at the axis is

$$dB = \frac{\mu_0(dI)r^2}{2R^3}$$

$$\Rightarrow \quad dB = \frac{\mu_0 Q\omega(R\sin\theta)^2(\sin\theta d\theta)}{8\pi R^3}$$

$$\Rightarrow \quad dB = \frac{\mu_0 Q\omega}{8\pi R}\int_0^{\pi}\sin^3\theta d\theta$$

Since, $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$$\Rightarrow \quad \sin^3\theta = \frac{1}{4}[3\sin\theta - \sin(3\theta)]$$

$$\Rightarrow \quad B = \int dB = \frac{\mu_0 Q\omega}{8\pi R}\left[\frac{1}{4}\left(\int_0^{\pi}3\sin\theta d\theta - \int_0^{\pi}\sin(3\theta)d\theta\right)\right]$$

$$\Rightarrow \quad B=\int dB=\frac{\mu_0 Q\omega}{32\pi R}\left[3\left(-\cos\theta\Big|_0^{\pi}\right)+\frac{\cos(3\theta)}{3}\Big|_0^{\pi}\right]$$

$$\Rightarrow \quad B=\frac{\mu_0 Q\omega}{32\pi R}\left[3(2)+\frac{1}{3}(-2)\right]=\frac{\mu_0 Q\omega}{16\pi R}\left(3-\frac{1}{3}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0 Q\omega}{16\pi R}\left(\frac{8}{3}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0 Q\omega}{6\pi R} \qquad \{\text{in terms of } Q,\ \omega \text{ and } R\}$$

Now, since $Q=(4\pi R^2)\sigma$

$$\Rightarrow \quad B=\frac{\mu_0(4\pi R^2)\sigma\omega}{6\pi R}$$

$$\Rightarrow \quad B=\frac{2}{3}\mu_0\sigma R\omega \qquad \{\text{in terms of } \sigma,\ \omega \text{ and } R\}$$

Now to calculate the magnetic moment, we know that

$$dM=\begin{pmatrix}\text{Current in}\\ \text{the element}\end{pmatrix}\begin{pmatrix}\text{Area of the loop}\\ \text{formed by element}\end{pmatrix}$$

$$\Rightarrow \quad dM=(dI)(\pi r^2)$$

$$\Rightarrow \quad dM=\left(\frac{Q\omega}{4\pi}\right)(\sin\theta d\theta)(\pi R^2\sin^2\theta)$$

$$\Rightarrow \quad M=\int dM=\frac{Q\omega R^2}{4}\int_0^{\pi}\sin^3\theta d\theta$$

$$\Rightarrow \quad M=\frac{Q\omega R^2}{4}\left[\frac{1}{4}\int_0^{\pi}3\sin\theta d\theta-\int_0^{\pi}\sin(3\theta)d\theta\right]$$

$$\Rightarrow \quad M=\frac{Q\omega R^2}{16}\left[3(2)+\frac{1}{3}(-2)\right] \qquad \{\text{as solved earlier}\}$$

$$\Rightarrow \quad M=\frac{Q\omega R^2}{8}\left\{\frac{8}{3}\right\}=\frac{1}{3}Q\omega R^2$$

$\{\text{in terms of } Q,\ \omega \text{ and } R\}$

$$\Rightarrow \quad M=\frac{1}{3}(Q\omega R^2)$$

Since, $Q=(4\pi R^2)\sigma$

$$\Rightarrow \quad M=\frac{4}{3}(\pi\sigma R^4\omega) \qquad \{\text{in terms of } \sigma,\ \omega \text{ and } R\}$$

Let us also calculate the GYROMAGNETIC RATIO, denoted by γ, i.e., the ratio of magnetic moment to the angular momentum. So, we get

$$\text{Gyromagnetic Ratio, } \gamma=\frac{|\vec{M}|}{|\vec{L}|}=\frac{\frac{1}{3}Q\omega R^2}{\frac{2}{3}mR^2}=\frac{Q}{2m}$$

where m = mass of shell, Q = charge on shell.

Please note that, for a charged body of charge Q, mass m rotating about axis of symmetry will possess both angular momentum $(\vec{L})$ and the magnetic moment $(\vec{M})$, on account of its rotation. It can be proved (or also has been shown), that as long as the charge and mass are distributed uniformly on the body, its GYROMAGNETIC RATIO (γ) is always given by

$$\gamma=\frac{|\vec{M}|}{|\vec{L}|}=\frac{Q}{2m}$$

So, we could have calculated the magnetic moment from the expression of Gyromagnetic Ratio, according to which

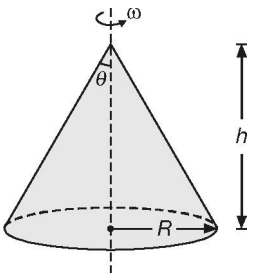
$$\gamma=\frac{|\vec{M}|}{|\vec{L}|}=\frac{Q}{2m}$$

$$\Rightarrow \quad \frac{|\vec{M}|}{\left(\frac{2}{3}mR^2\right)\omega}=\frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}|=\frac{1}{3}QR^2\omega$$

ILLUSTRATION 81

A charge Q is uniformly distributed over the slant surface of a thin walled right circular cone of semi-vertex angle θ and height h as shown in Figure.

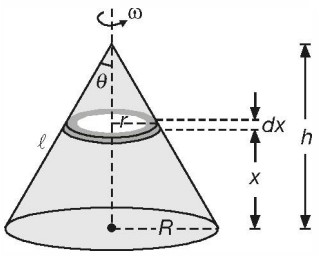


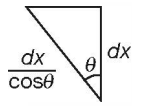
The cone is uniformly rotated about its axis at angular velocity ω. Calculate the magnetic dipole moment associated with the cone.

SOLUTION

$$M = IA = \left(\frac{\text{Charge Circulating}}{\text{Period of Revolution}}\right)(\text{Area of loop})$$

Let us consider a small, current carrying loop element, having thickness dx at a distance x from the base of the cone as shown in Figure.





Magnified view of element

If dM is the magnetic moment of this small loop having charge dq, radius r and rotating with angular velocity ω then we have

$$dM = iA = \left[\frac{dq}{\left(\frac{2\pi}{\omega}\right)}\right]A = \frac{\omega}{2\pi}(dq)A$$

where A is area of the loop or the area circulated by the charge in the loop. Also, we know that

$$dq = \sigma dA \text{, where } \sigma = \frac{Q}{\pi Rl},\ dA = (2\pi r)\left(\frac{dx}{\cos\theta}\right)$$

and l is the slant height of the cone given by $l = \frac{h}{\cos\theta}$

$$\Rightarrow \quad dM = \left(\frac{\omega}{2\pi}\right)\left(\frac{Q}{\pi Rl}\right)(2\pi r)\left(\frac{dx}{\cos\theta}\right)(\pi r^2) \qquad \ldots(1)$$

Also, from figure we observe that

$$\tan\theta = \frac{R}{h} = \frac{r}{h-x}$$

$$\Rightarrow \quad r = \left(\frac{h-x}{h}\right)R$$

So, from (1), we get

$$dM = \left(\frac{\omega}{2\pi}\right)\left(\frac{Q\cos\theta}{\pi Rh}\right)\left(\frac{2\pi^2 R^3}{\cos\theta}\right)\left(\frac{h-x}{h}\right)^3 dx$$

$$\Rightarrow \quad M = \int dM = \frac{\omega QR^2}{h^4}\int_0^h (h-x)^3\,dx$$

$$\Rightarrow \quad M = \frac{\omega QR^2}{h^4}\left(\frac{(h-x)^4}{-4}\right)\Bigg|_0^h$$

$$\Rightarrow \quad M = \frac{\omega QR^2}{h^4}\left(0 - \frac{h^4}{4}\right)$$

$$\Rightarrow \quad M = \frac{1}{4}Q\omega R^2 = \frac{1}{4}Q\omega h^2\tan^2\theta$$

Since we know that the Gyromagnetic ratio for such an arrangement is given by

$$\gamma = \frac{M}{L} = \frac{Q}{2m},$$

where M is to be calculated and $L = I\omega$. For a thin walled cone, $I = \frac{1}{2}mR^2$. So, we get

$$M = \frac{I\omega Q}{2m} = \frac{\left(\frac{1}{2}mR^2\right)\omega Q}{2m} = \frac{1}{4}Q\omega R^2$$

$$\Rightarrow \quad M = \frac{1}{4}Q\omega h^2\tan^2\theta$$

{we could have by passed the process of integration}

However, for the solid cone, $I = \frac{3}{10}MR^2$. So, our answer would be (if asked)

$$M = \frac{I\omega Q}{2m} = \frac{\left(\frac{3}{10}mR^2\right)\omega Q}{2m}$$

$$\Rightarrow \quad M = \frac{3}{20}Q\omega R^2 = \frac{3}{20}Q\omega h^2\tan^2\theta$$

ILLUSTRATION 82

A solid cylinder has length L and inner and outer radii R_1 and R_2 respectively. The cylinder carries a uniform charge density ρ and is rotating angular velocity ω about its axis. Calculate the magnetic moment of the cylinder. Assume the above cylinder to be solid of radius R, length L and let it carry a uniform charge density $+\rho$ from $r = 0$ to $r(< R)$ and an equal charge density of opposite sign $-\rho$, from $r(< R)$ to R. If the cylinder is rotating uniformly about its axis such that its magnetic moment is zero, then calculate r. Assume that the mass is distributed uniformly on each cylinder in both the cases.

SOLUTION

Since we know that the Gyro-magnetic Ratio

$$\frac{|\vec{M}|}{|\vec{L}|}=\frac{q}{2m} \qquad \ldots(1)$$

If m be the mass of the cylinder, then

$$L=\frac{1}{2}m\left(R_1^2+R_2^2\right)\omega \text{ and}$$

$$q=\rho V=\rho\pi\left(R_2^2-R_1^2\right)L$$

From equation (1), we get

$$|\vec{M}|=\frac{q}{2m}|\vec{L}|$$

$$\Rightarrow \quad |\vec{M}|=\frac{1}{2m}\left[\frac{1}{2}m\left(R_1^2+R_2^2\right)\omega\right]\left[\rho\pi\left(R_2^2-R_1^2\right)L\right]$$

$$\Rightarrow \quad |\vec{M}|=\frac{1}{4}\pi\rho L\omega\left(R_2^4-R_1^4\right)$$

Proceeding in a similar way, the magnetic moment of positively charged section of cylinder i.e. from $r=0$ to $r(<R)$ is

$$M_+=\frac{1}{4}\pi\rho\omega Lr^4$$

Similarly, magnetic moment of negatively charged section i.e. from $r(<R)$ to R is

$$M_-=-\frac{1}{4}\pi\rho\omega L\left(R^4-r^4\right)$$

The net magnetic moment should be zero, hence we have

$$M_++M_-=0$$

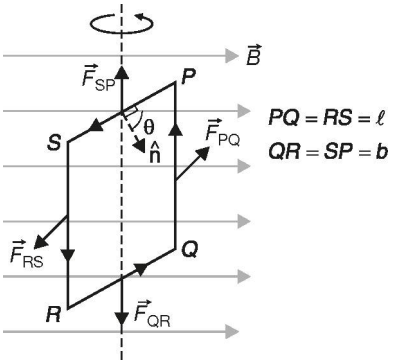
$$\Rightarrow \quad \frac{1}{4}\pi\rho\omega Lr^4=\frac{1}{4}\pi\rho\omega L\left(R^4-r^4\right)$$

$$\Rightarrow \quad 2r^4=R^4$$

$$\Rightarrow \quad r=\left(2^{-1/4}\right)R$$

TORQUE ON A CURRENT LOOP IN UNIFORM MAGNETIC FIELD: REVISITED

Consider a rectangular current carrying coil $PQRS$ having N turns and area A, placed in a uniform field B, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of B as shown in Figure.



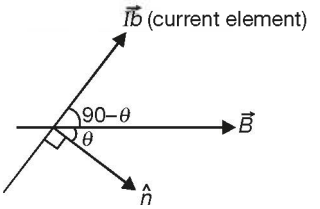
The coil experiences no net force and a torque given by

$$\tau=NBIA\sin\theta$$

This can be derived by simply calculating and drawing the forces on the current carrying conductors PQ, QR, RS and SP. The direction of the forces acting on the wires has been shown in the figure and direction has been found by using the Fleming's Left Hand Rule. Let $\vec{F}_{PQ}$, $\vec{F}_{QR}$, $\vec{F}_{RS}$ and $\vec{F}_{SP}$ the forces acting on the respective wires PQ, QR, RS and SP. Then

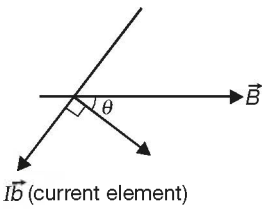
$$\left|\vec{F}_{PQ}\right|=BIl\sin 90^\circ=BIl\text{, inwards}$$

$$\left|\vec{F}_{QR}\right|=BIb\sin(90-\theta)=BIb\cos\theta\text{, downwards}$$



$$\left|\vec{F}_{RS}\right|=BIl\sin 90^\circ=BIl\text{, outwards}$$

$$\left|\vec{F}_{SP}\right|=BIb\sin(90+\theta)=BIb\cos\theta\text{, upwards}$$

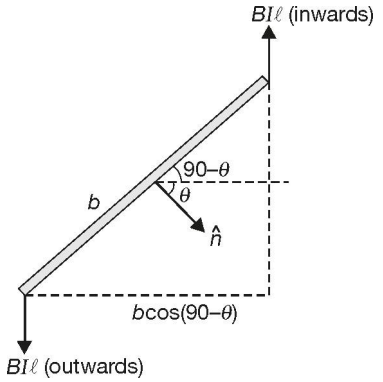


We observe that, these forces must be acting through the centres of the respective wires. Also, from the diagram we observe that $\vec{F}_{QR}$ and $\vec{F}_{SP}$ cancel (due

to the same magnitude and having the same line of action). Further, we see that $F_{PQ}(=BIl$, inwards$)$ and $F_{RS}(=BIl$, outwards$)$ have different lines of action and hence will make the loop experience a torque so that the loop rotates about the dotted axis. To calculate this torque let us take the overhead view of the loop and draw it. Since we know that

$$\tau = Fr_{\perp}$$

$$\Rightarrow \quad \tau = (\text{Either force})\begin{pmatrix}\text{Perpendicular distance}\\ \text{between the points of}\\ \text{application of forces}\end{pmatrix}$$



$$\Rightarrow \quad \tau = (BIl)(b\cos(90-\theta))$$

$$\Rightarrow \quad \tau = BI(lb)\sin\theta$$

$$\Rightarrow \quad \tau = B(IA)\sin\theta$$

Since we know that $IA = M$, where M is the Magnetic Dipole Moment, so we have

$$\tau = BM\sin\theta$$

If the loop has N turns, then we get

$$\tau = NBIA\sin\theta$$

So, we observe that

(a) τ is zero when $\theta = 0$, i.e., when the plane of the coil is perpendicular to the field.

(b) τ is maximum when $\theta = 90°$, i.e., the plane of the coil is parallel to the field.

$$\Rightarrow \quad \tau_{\max} = NBIA$$

The above expression is valid for coils of all shapes.

The quantity $I\vec{A}$ is a vector and is called the **Magnetic Moment of the loop**. So,

$$\vec{M} = I\vec{A}$$

Vectorially $\vec{\tau} = \vec{M}\times\vec{B}$

In magnitude, $\tau = MB\sin\theta$

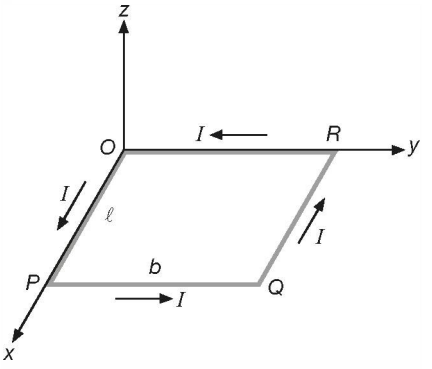
where θ is the angle between magnetic moment and magnetic field OR θ is the angle between normal to the coil and the magnetic field.

Problem Solving Technique(s)

Direction of $\vec{M}$ is found by using Right Hand Thumb Rule according to which "curl the fingers of right hand in the direction of circulation of conventional current, then the thumb gives the direction of $\vec{M}$".

MAGNETIC DIPOLE IN UNIFORM MAGNETIC FIELD: REVISITED USING VECTOR METHOD

Let us consider a rectangular current carrying loop $OPQRO$ of length l, width b placed in xy plane in a uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.



Let us find the net force and torque on the loop.

FORCE ON THE LOOP:

Net force on the loop is,

$$\vec{F} = \vec{F}_{OP} + \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RO}$$

Since $\vec{F} = I(\vec{l}\times\vec{B})$

$$\Rightarrow \quad \vec{F} = I\left[(\vec{OP}\times\vec{B}) + (\vec{PQ}\times\vec{B}) + (\vec{QR}\times\vec{B}) + (\vec{RO}\times\vec{B})\right]$$

$$\Rightarrow \quad \vec{F} = I\left[(\vec{OP} + \vec{PQ} + \vec{QR} + \vec{RO})\times\vec{B}\right] = \vec{0} \text{ (null vector)}$$

So, net force on the loop is $\vec{F} = \vec{0}$

$$\left\{\because \ \vec{OP} + \vec{PQ} + \vec{QR} + \vec{RO} \text{ forms a null vector}\right\}$$

TORQUE ON THE LOOP:

The current carrying loop $OPQR$ can be considered to be made up of four current carrying wires OP, PQ, QR and RO. Using $\vec{F} = I(\vec{l} \times \vec{B})$, we get

$$\vec{F}_{OP} = I(\overrightarrow{OP} \times \vec{B}) = I\left[(l\hat{i}) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_{OP} = Il\left(B_y\hat{k} - B_z\hat{j}\right)$$

$$\vec{F}_{PQ} = I(\overrightarrow{PQ} \times \vec{B}) = I\left[(b\hat{j}) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_{PQ} = Ib\left(-B_x\hat{k} + B_z\hat{i}\right)$$

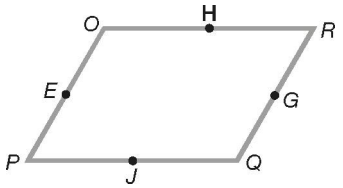
$$\vec{F}_{QR} = I(\overrightarrow{QR} \times \vec{B}) = I\left[(-l\hat{i}) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_{QR} = Il\left[-B_y\hat{k} + B_z\hat{j}\right]$$

$$\vec{F}_{RO} = I(\overrightarrow{RO} \times \vec{B}) = I\left[(-b\hat{j}) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_{RO} = Ib\left[B_x\hat{k} - B_z\hat{i}\right]$$

All these forces are acting at the centre of the wires. For example, $\vec{F}_{OP}$ will act at the centre of OP. When the forces are in equilibrium then the net torque about any point remains the same. Let us calculate the torque due to the forces about O. Let E, J, G and H are the mid-point of OP, PQ, QR and RO respectively.



Using $\vec{\tau} = \vec{r} \times \vec{F}$, we get

$$\vec{\tau}_O = \left(\overrightarrow{OE} \times \vec{F}_{OP}\right) + \left(\overrightarrow{OJ} \times \vec{F}_{PQ}\right) + \left(\overrightarrow{OG} \times \vec{F}_{QR}\right) + \left(\overrightarrow{OH} \times \vec{F}_{RO}\right)$$

$$\Rightarrow \quad \vec{\tau}_O = \left[\left(\frac{l}{2}\hat{i}\right) \times \left(Il\left(B_y\hat{k} - B_z\hat{j}\right)\right)\right] + \left[\left(l\hat{i} + \frac{b}{2}\hat{j}\right) \times \left(Ib\left(-B_x\hat{k} + B_z\hat{i}\right)\right)\right] + \left[\left(\frac{l}{2}\hat{i} + b\hat{j}\right) \times \left(Il\left(-B_y\hat{k} + B_z\hat{j}\right)\right)\right] + \left[\left(\frac{b}{2}\hat{j}\right) \times \left(Ib\left(B_x\hat{k} - B_z\hat{i}\right)\right)\right]$$

$$\Rightarrow \quad \vec{\tau}_O = (Ilb)B_x\hat{j} - (Ilb)B_y\hat{i}$$

$$\Rightarrow \quad \vec{\tau}_O = IA\left(B_x\hat{j} - B_y\hat{i}\right) \text{ where area of the loop is } A = lb \text{ and } \vec{A} = (lb)\hat{k}$$

$$\Rightarrow \quad \vec{\tau}_O = M\left(B_x\hat{j} - B_y\hat{i}\right) \text{ where } M = IA \text{ and } \vec{M} = I\vec{A}$$

This expression obtained can be rewritten as

$$\vec{\tau}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & M \\ B_x & B_y & B_z \end{vmatrix} = (M\hat{k}) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\Rightarrow \quad \vec{\tau}_0 = \vec{M} \times \vec{B}$$

The above equation for the torque is very similar to that of an electric dipole in an electric field. The similarity between electric and magnetic dipole extends even further as illustrated in the table shown.

S. No.	Similarity	Electric Dipole	Magnetic Dipole
1.	Magnitude	$\lvert\vec{p}\rvert = q(2d)$	$\lvert\vec{M}\rvert = NIA$
2.	Direction	from $-q$ to $+q$	from south (S) to north (N)
3.	Net force in uniform field	zero	zero
4.	Torque	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{M} \times \vec{B}$
5.	Potential energy	$U = -\vec{p} \cdot \vec{E}$	$U = -\vec{M} \cdot \vec{B}$
6.	Work done in rotating the dipole from θ_1 to θ_2	$W = pE(\cos\theta_1 - \cos\theta_2)$	$W = MB(\cos\theta_1 - \cos\theta_2)$
7.	Field along axis at farther points (Axial Line)	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\vec{p}}{r^3}\right)$	$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{r^3}\right)$
8.	Field perpendicular to axis at farther points (Equitorial Line)	$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}}{r^3}\right)$	$\vec{B} = -\frac{\mu_0}{4\pi}\left(\frac{\vec{M}}{r^3}\right)$
9.	At far points	$\lvert\vec{E}_{\text{axial}}\rvert = 2\lvert\vec{E}_{\text{equitorial}}\rvert$	$\lvert\vec{E}_{\text{axial}}\rvert = 2\lvert\vec{E}_{\text{equitorial}}\rvert$

Remark(s)

(a) The expressions for the magnetic dipole can be obtained from the expressions for the electric dipole by replacing $\vec{p}$ by $\vec{M}$ and ε_0 by $\frac{1}{\mu_0}$. Here μ_0 is called the permeability of free space. It is related with ε_0 and speed of light c as, $c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and it has the value, $\mu_0=4\pi\times10^{-7}\ \text{TmA}^{-1}$

(b) Dimensions of $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ are that of speed or $[LT^{-1}]$

Hence, $\left[\frac{1}{\sqrt{\varepsilon_0\mu_0}}\right]=[LT^{-1}]$

Problem Solving Technique(s)

(a) Note that although $\vec{\tau}=\vec{M}\times\vec{B}$ has been derived for a rectangular loop, it comes out to be true for any shape of loop.

(b) Magnitude of $\vec{\tau}$ is $MB\sin\theta$ or $NB(IA)\sin\theta$. Here θ is the angle between $\vec{M}$ and $\vec{B}$. Torque is zero when $\theta=0°$ or $180°$ and it is maximum at $\theta=90°$.
Also, we can say that θ is the angle between the normal to the coil and the magnetic field.

(c) If the loop is free to rotate in a magnetic field the axis of rotation becomes an axis parallel to $\vec{\tau}$ passing through the centre of mass of the loop.

ILLUSTRATION 83

A current loop with magnetic dipole moment $\vec{M}$ is placed in a uniform magnetic field $\vec{B}$, with its moment making angle θ with the field. With the arbitrary choice of $U=0$ for $\theta=90°$, prove that the potential energy of the dipole field system is $U=-\vec{M}\cdot\vec{B}$. Also calculate the maximum and minimum values of U, giving its significance.

SOLUTION

Since we have been given to take the arbitrary choice of $U=0$ at $\theta=90°$, so we shall proceed as follows. The torque due to the field on the current loop is

$$\vec{\tau}=\vec{M}\times\vec{B}$$

This torque of magnitude $MB\sin\theta$, tends to rotate the dipole of decreasing θ. Let this torque turn the dipole through an infinitesimal angle $d\theta$, then

$$dW=dU=\tau d\theta$$

$$\Rightarrow \int_0^U dU=\int_{\frac{\pi}{2}}^{\theta} MB\sin\theta d\theta$$

$$\Rightarrow U=MB\left(-\cos\theta\Big|_{\frac{\pi}{2}}^{\theta}\right)$$

$$\Rightarrow U=-MB\left(\cos\theta-\cos\frac{\pi}{2}\right)$$

$$\Rightarrow U=-MB\cos\theta$$

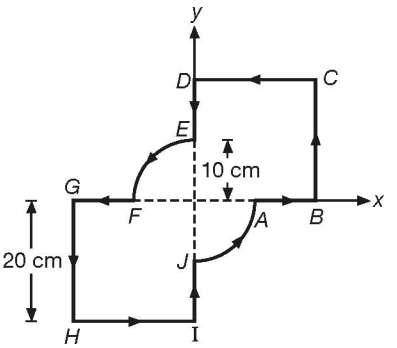
$$\Rightarrow U=-\vec{M}\cdot\vec{B}$$

Now, U is minimum when $\theta=0°$ i.e., $\vec{M}\parallel\vec{B}$ i.e., for a stable system we must have $U=U_{\min}=-MB$.

Further, U is maximum when $\theta=\pi$ i.e., $\vec{M}$ and $\vec{B}$ are antiparallel i.e., for an unstable system we have $U=U_{\max}=MB$.

ILLUSTRATION 84

A current $I=5$ A flows through a thin wire as shown in Figure.



(a) Find the magnetic field produced by the current at point O in the figure,

(b) If there exists an external magnetic field $\vec{B}=\left(14\hat{i}+14\hat{j}\right)$ T, calculate the torque acting on the wire.

Take $\pi=\frac{22}{7}$

SOLUTION

(a) Let 10 cm = a

$$B_0 = 2(B_{ABCDE} + B_{EF})$$

$$\Rightarrow \quad B_0 = 2\left[2\frac{\mu_0 I}{4\pi 2a}\left(\cos\frac{\pi}{4}+\cos\frac{\pi}{2}\right)+\frac{\mu_0 I}{4\pi a}\frac{\pi}{2}\right]$$

$$\Rightarrow \quad B_0 = \frac{\mu_0 I}{4\pi a}\left[\frac{2}{\sqrt{2}}+\pi\right] = \frac{10^{-7}\times 5}{0.1}[\sqrt{2}+3.14]$$

$$\Rightarrow \quad B_0 = 22.78\ \mu\text{T} \text{ in the direction } \hat{k}$$

(b) $\vec{M} = I\vec{A} = 5\left(8a^2+\frac{\pi a^2}{2}\right)\hat{k} = 5a^2\left(8+\frac{\pi}{2}\right)\hat{k}$

$$\vec{\tau} = \vec{M}\times\vec{B} = 5a^2\left(8+\frac{\pi}{2}\right)\hat{k}\times 14(\hat{i}+\hat{j})\ \text{Nm}$$

$$\Rightarrow \quad \vec{\tau} = (5)(14)\left(8+\frac{\pi}{2}\right)\hat{k}\times(\hat{i}+\hat{j})\ \text{Nm}$$

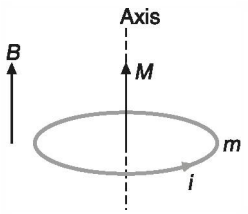
$$\Rightarrow \quad \vec{\tau} = 5a^2(112+22)(\hat{j}-\hat{i})\ \text{Nm}$$

$$\Rightarrow \quad \vec{\tau} = 5\times 0.01\times 134(\hat{j}-\hat{i})\ \text{Nm}$$

$$\Rightarrow \quad \vec{\tau} = 6.7(\hat{j}-\hat{i})\ \text{Nm}$$

ILLUSTRATION 85

A small circular coil of mass m consists of N turns of fine wire and carries a current i. The coil is located in a uniform magnetic field B with the coil axis originally parallel to the direction of field as shown in Figure.



Calculate the period of small angle oscillations of the coil axis around its equilibrium position, assuming friction force mechanical force to be absent. Now, if the coil is rotated through an angle of θ from its equilibrium position and then released, calculate the angular speed of the coil when it passes through the equilibrium position.

SOLUTION

Magnetic moment of coil is given by

$$M = N(iA) = N(\pi R^2)$$

Let the coil be displaced by a small angle θ, then torque on the coil is

$$\tau = -MB\sin\theta$$

The negative sign indicates that this torque is restoring in nature. Since θ is small, so $\sin\theta \approx \theta$, hence

$$\Rightarrow \quad I\alpha = -MB\theta, \text{ where } I = \frac{1}{2}mR^2$$

$$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\alpha = -Ni\pi R^2 B\theta$$

$$\Rightarrow \quad \alpha = -\left(\frac{2Ni\pi B}{m}\right)\theta$$

Comparing with $\alpha = -\omega^2\theta$ we get

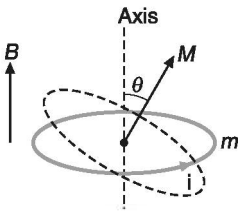
$$\omega = \sqrt{\frac{2Nl\pi B}{m}}$$

Time period T is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2Ni\pi B}}$$

Applying conservation of energy

$$(\text{KE}+\text{PE})_i = (\text{KE}+\text{PE})_f$$



$$\Rightarrow \quad 0 - MB\cos\theta = \frac{1}{2}I\omega^2 - MB\cos 0°$$

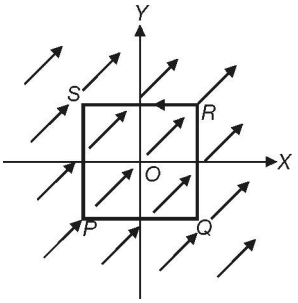
$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = MB(1-\cos\theta)$$

$$\Rightarrow \quad \frac{1}{4}mR^2\omega^2 = Ni\pi R^2 B(1-\cos\theta)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4Ni\pi B(1-\cos\theta)}{m}}$$

ILLUSTRATION 86

A uniform constant magnetic field $\vec{B}$ is directed at an angle of 45° to the X-axis in X-Y plane. $PQRS$ is a rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure with its sides parallel to X and Y axes. Each side of the frame is of mass M and length L.



(a) What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs (Δt is so short that any variation in the torque during this interval may be neglected).

Given: the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2$.

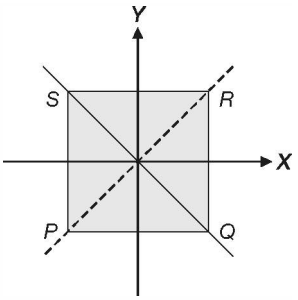
SOLUTION

Magnetic moment of the loop is given by

$$\vec{M} = (IA)\hat{k} = \left(I_0L^2\right)\hat{k}$$

Magnetic field is given by

$$\vec{B} = (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j} = \frac{B}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$$



(a) Torque acting on the loop,

$$\vec{\tau} = \vec{M}\times\vec{B} = \left(I_0L^2\hat{k}\right)\times\left[\frac{B}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)\right]$$

$$\Rightarrow \tau = \frac{I_0L^2B}{\sqrt{2}}\left(\hat{j}-\hat{i}\right)$$

$$\Rightarrow |\vec{\tau}| = I_0L^2B$$

(b) Since, torque is in $\left(\hat{j}-\hat{i}\right)$ direction or parallel to QS. Therefore, the loop will rotate about an axis passing through Q and S with an angular acceleration α, given by

$$\alpha = \frac{|\vec{\tau}|}{I_{QS}}$$

where I_{QS} is moment of inertia of loop about the axis QS. ***Please do not confuse moment of inertia with the current.***

From theorem of perpendicular axis, we get

$$I_{QS} + I_{PR} = I_{ZZ}$$

However, since $I_{QS} = I_{PR}$

$$\Rightarrow 2I_{QS} = I_{ZZ} = \frac{4}{3}ML^2$$

$$\Rightarrow I_{QS} = \frac{2}{3}ML^2$$

$$\Rightarrow \alpha = \frac{|\vec{\tau}|}{I_{QS}} = \frac{I_0L^2B}{\frac{2}{3}ML^2} = \frac{3}{2}\frac{I_0B}{M}$$

Since Δt is given very small, so we can assume τ and α almost constant because θ will not change appreciably. Hence, we can use,

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$$

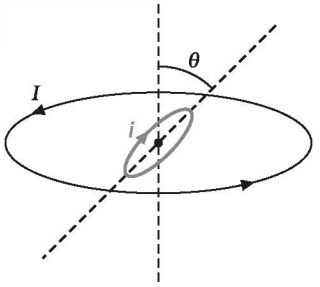
So, the angle by which the frame rotates in time Δt is

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$$

$$\Rightarrow \Delta\theta = \frac{3}{4}\frac{I_0B}{M}(\Delta t)^2$$

ILLUSTRATION 87

A large horizontal coil of radius R is carrying a current I. Another small coil of radius $r(\ll R)$ carrying a current i and N turns is placed at the centre of the large coil with its plane inclined at an angle θ with the axis of the large coil as shown in Figure. Calculate the torque experienced by the smaller coil.



SOLUTION

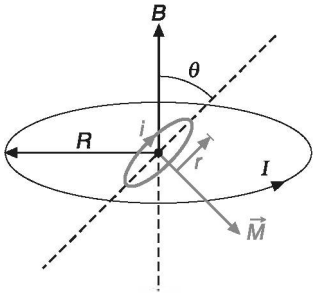
Magnetic moment of smaller coil is given by

$$M = Ni(\pi r^2) = Ni\pi r^2$$

Magnetic induction due to large coil at its centre is

$$B = \frac{\mu_0 I}{2R}$$

Angle between magnetic induction at centre of large coil and magnetic moment of smaller coil is $\frac{\pi}{2}+\theta$ as shown in Figure.



Torque on smaller coil due to the magnetic induction of larger coil at its centre is given by

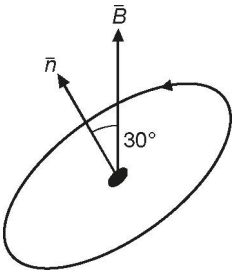
$$\tau = MB\sin\left(\frac{\pi}{2}+\theta\right)$$

$$\Rightarrow \quad \tau = Ni\pi r^2\left(\frac{\mu_0 I}{2R}\right)\cos\theta$$

$$\Rightarrow \quad \tau = \frac{\mu_0 ilN\pi r^2}{2R}\cos\theta$$

ILLUSTRATION 88

An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius R (shown in figure).



(a) Obtain an expression for the orbital magnetic moment of the electron.

(b) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane normal of the electron orbit makes an angle of $30°$ with the magnetic induction. Find the torque experienced by the orbiting electron.

SOLUTION

(a) In ground state $(n=1)$ according to Bohr's Theory

$$mvR = \frac{h}{2\pi}$$

$$\Rightarrow \quad v = \frac{h}{2\pi mR}$$

Now, time period,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{h/2\pi mR} = \frac{4\pi^2 mR^2}{h}$$

Magnetic moment $M = IA$, where

$$I = \frac{\text{charge}}{\text{time period}} = \frac{e}{\frac{4\pi^2 mR^2}{h}} = \frac{eh}{4\pi^2 mR^2}$$

and $A = \pi R^2$

$$\Rightarrow \quad M = \left(\pi R^2\right)\left(\frac{eh}{4\pi^2 mR^2}\right)$$

$$\Rightarrow \quad m = \frac{eh}{4\pi m}$$

Direction of magnetic moment $\vec{M}$ is perpendicular to the plane of orbit.

(b) Since, $\vec{\tau} = \vec{M}\times\vec{B}$

$$\Rightarrow \quad \vec{\tau} = MB\sin\theta$$

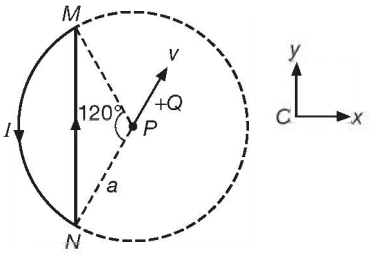
where $\theta = 30°$ is the angle between $\vec{M}$ and $\vec{B}$.

$$\Rightarrow \tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30^\circ = \frac{ehB}{8\pi m}$$

The direction of $\vec{\tau}$ is perpendicular to both $\vec{M}$ and $\vec{B}$.

ILLUSTRATION 89

A wire loop carrying a current I is placed in the x-y plane as shown in Figure.

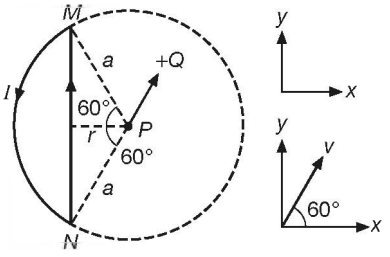


(a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\vec{V}$ along NP (shown in figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

SOLUTION

(a) Magnetic field at P due to arc of circle (shown in Figure) subtending an angle of 120° at centre would be



$$B_1 = \frac{1}{3}(\text{Field Due to a Circle})$$

$$\Rightarrow B_1 = \frac{1}{3}\left(\frac{\mu_0 I}{2a}\right) = \frac{\mu_0 I}{6a} \text{ (outwards)}$$

$$\Rightarrow B_1 = \frac{0.16\mu_0 I}{a} \text{ (outwards)}$$

$$\Rightarrow \vec{B}_1 = \left(\frac{0.16\mu_0 I}{a}\right)\hat{k}$$

Magnetic field due to straight wire NM at P is

$$B_2 = \frac{\mu_0 I}{4\pi r_\perp}(\sin 60^\circ + \sin 60^\circ)$$

Here, $r_\perp = r = a\cos 60^\circ$

$$\Rightarrow B_2 = \frac{\mu_0 I}{4\pi(a\cos 60^\circ)}(2\sin 60^\circ)$$

$$\Rightarrow B_2 = \frac{\mu_0 I}{2\pi a}\tan 60^\circ = \frac{0.27\mu_0 I}{a}(\text{inwards})$$

$$\Rightarrow \vec{B}_2 = -\frac{0.27\mu_0 I}{a}\hat{k}$$

So, $\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 = -\left(\frac{0.11\mu_0 I}{a}\right)\hat{k}$

The velocity of particle can be written as,

$$\vec{v} = (v\cos 60^\circ)\hat{i} + (v\sin 60^\circ)\hat{j} = \frac{v}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

Magnetic force on the moving charge is given by

$$\vec{F}_m = Q\left(\vec{v}\times\vec{B}\right) = \frac{0.11\mu_0 IQv}{2a}\hat{j} - \frac{0.11\sqrt{3}\mu_0 IQv}{2a}\hat{i}$$

Instantaneous acceleration $\vec{a}$ is given by

$$\vec{a} = \frac{\vec{F}_m}{m} = \frac{0.11\mu_0 IQv}{2am}\left(\hat{j} - \sqrt{3}\hat{i}\right)$$

(b) In uniform magnetic field, force on a current loop is zero. Further, magnetic dipole moment of the loop will be,

$$\vec{M} = (IA)\hat{k}$$

where A is the area of the loop given by

$$A = \frac{1}{3}\left(\pi a^2\right) - \frac{1}{2}(2a\sin 60^\circ)(a\cos 60^\circ)$$

$$\Rightarrow A = \frac{\pi a^2}{3} - \frac{a^2}{2}\sin 120^\circ = 0.61a^2$$

$$\Rightarrow \vec{M} = \left(0.61\ Ia^2\right)\hat{k}$$

Given, $\vec{B} = B\hat{i}$

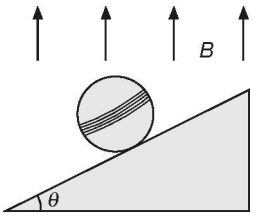
$$\Rightarrow \vec{\tau} = \vec{M}\times\vec{B} = \left(0.61\ Ia^2B\right)\hat{j}$$

ILLUSTRATION 90

A non-conducting sphere has mass 220 g and radius 20 cm. A flat compact coil of wire with 10 turns is wrapped tightly around it, with each turn concentric with the sphere. The sphere is placed on an inclined plane that slopes downward to the left, making an

angle θ with the horizontal, so that the coil is parallel to the inclined plane. A uniform magnetic field of 0.35 T vertically upward exists in the region of the sphere. What current in the coil will enable the sphere to rest in equilibrium on the inclined plane? Show that the result does not depend on the value of θ.

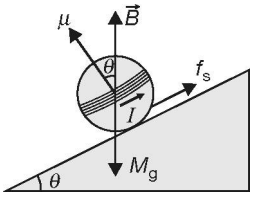
$\left(\text{Take } g = 10 \text{ ms}^{-2} \text{ and } \pi = \frac{22}{7}\right)$



SOLUTION

For Translational Equilibrium, we have

$$f_s - Mg\sin\theta = 0 \qquad \ldots(1)$$



For Rotational Equilibrium, if torques are taken about the centre of the sphere, the magnetic field produces a clockwise torque of magnitude $\mu B\sin\theta$, and the frictional force a counter clockwise torque of magnitude $f_s R$, where R is the radius of the sphere. Thus

$$f_s R - \mu B\sin\theta = 0 \qquad \ldots(2)$$

For avoiding confusion, we have taken the magnetic moment here to be $\vec{\mu}$ instead of M.
From (1), we have $f_s = Mg\sin\theta$. Substituting this in (2) and cancelling out $\sin\theta$, we get

$$\mu B = MgR \qquad \ldots(3)$$

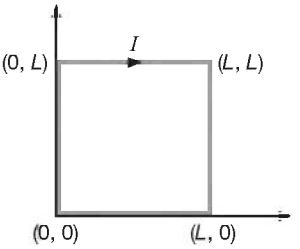
where $\mu = NI(\pi R^2)$. So, we get

$$I = \frac{Mg}{\pi NBR} = \frac{(0.22 \text{ kg})(10 \text{ ms}^{-2})}{\pi(10)(0.35 \text{ T})(0.2 \text{ m})} = 1 \text{ A}$$

The current must be counter clockwise as seen from above.

ILLUSTRATION 91

Figure shows a square loop of wire that lies in the xy-plane. The loop has corners at $(0, 0)$, $(0, L)$, $(L, 0)$ and (L, L) and carries a constant current I in the clockwise direction. The magnetic field is given by $\vec{B} = \left(\frac{B_0 y}{L}\right)\hat{i} + \left(\frac{B_0 x}{L}\right)\hat{j}$, where B_0 is a positive constant.

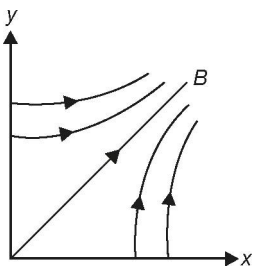


(a) Draw the magnetic field lines in the xy- plane.
(b) Find the magnitude and direction of the magnetic force exerted on each side of the loop.
(c) If the loop is free to rotate about the x-axis, find the magnitude and direction of the magnetic torque on the loop.
(d) If the loop is free to rotate about the y-axis, find the magnitude and direction of the magnetic torque on the loop.
(e) According to you, is the expression $\vec{\tau} = \vec{\mu}\times\vec{B}$, an appropriate description of the torque on this loop? If yes, then why and if no, then why not?

SOLUTION

Apply $d\vec{F} = I(d\vec{\ell}\times\vec{B})$ to each side of the loop $\vec{\tau} = \vec{r}\times\vec{F}$. For each side of the loop, $d\vec{\ell}$ is parallel to that side of the loop and is in the direction of I

(a) The magnetic field lines in the xy-plane are sketched in figure



(b) Side 1, that runs from $(0,\ 0)$ to $(0,\ L)$

$$\vec{F}=\int_0^L I(d\vec{\ell}\times\vec{B})=I\int_0^L\frac{B_0 y\,dy}{L}(-\hat{k})=-\frac{1}{2}B_0LI\hat{k}$$

Side 2, that runs from $(0,\ L)$ to $(L,\ L)$

$$\vec{F}=\int_0^L I(d\vec{\ell}\times\vec{B})=I\int_0^L\frac{B_0 x dx}{L}\hat{k}=\frac{1}{2}IB_0L\hat{k}$$

Side 3, that runs from $(L,\ L)$ to $(L,\ 0)$

$$\vec{F}=\int_0^L I(d\vec{\ell}\times\vec{B})=I\int_0^L\frac{B_0 y dy}{L}\hat{k}=+\frac{1}{2}IB_0L\hat{k}$$

Side 4, that runs from $(L,\ 0)$ to $(0,\ 0)$

$$\vec{F}=\int_0^L I(d\vec{\ell}\times\vec{B})=I\int_0^L\frac{B_0 x dx}{L}(-\hat{k})=-\frac{1}{2}IB_0L\hat{k}$$

(c) When free to rotate about the x-axis, the torques due to the forces on sides 1 and 3 cancel and the torque due to the forces on side 4 is zero. For side 2, we have $\vec{r}=L\hat{j}$. Therefore,

$$\vec{\tau}=\vec{r}\times\vec{F}=\frac{IB_0L^2}{2}\hat{i}=\frac{1}{2}IAB_0\hat{i}$$

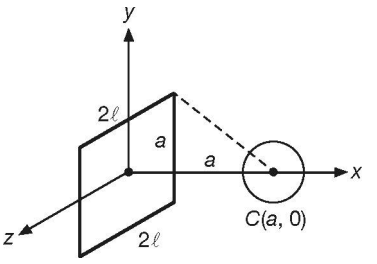
(d) When free to rotate about the y-axis, the torques due to the forces on sides 2 and 4 cancel and the torque due to the forces on side 1 is zero. For side 3, we have $\vec{r}=L\hat{i}$. Therefore,

$$\vec{\tau}=\vec{r}\times\vec{F}=\frac{IB_0L^2}{2}\hat{j}=-\frac{1}{2}IAB_0\hat{j}$$

(e) The equation for the torque $\vec{\tau}=\vec{\mu}\times\vec{B}$ is not appropriate, since the magnetic field is not constant.

ILLUSTRATION 92

A very small circular loop of radius r and carrying a current I_1 is placed in the x-y plane with its centre on x-axis at the point $C(a,\ 0)$. A square loop of side length 2ℓ carrying a current I_2 is fixed in the y-z plane with the centre of the loop at the origin. Calculate the torque exerted by the square loop on the circular loop.



SOLUTION

As derived earlier, B at the axis of a square loop of side L is given by

$$B_{\text{axis of square loop}}=\frac{\mu_0IL^2}{2\pi\left(x^2+\frac{L^2}{4}\right)\left(x^2+\frac{L^2}{2}\right)^{\frac{1}{2}}}$$

where x is the distance of the point at the axis from the centre of the loop.

However in this problem, we have the loop of side 2ℓ and the point at a distance $x=a$ from its centre, where the field has to be found. So,

$$B_{\substack{\text{square loop}\\ \text{at the axis}}}=\frac{\mu_0I_2(2\ell)^2}{2\pi\left(x^2+\frac{(2\ell)^2}{4}\right)\left(x^2+\frac{(2\ell)^2}{2}\right)^{\frac{1}{2}}}$$

$$\Rightarrow\quad B_{\substack{\text{Square loop}\\ \text{at the axis}}}=\frac{4\mu_0I_2\ell^2}{2\pi(x^2+\ell^2)\sqrt{x^2+2\ell^2}}$$

So, torque due to field of square loop on the circular loop is

$$\tau=MB\sin 90 \qquad \{\because \vec{M}\perp\vec{B}\}$$

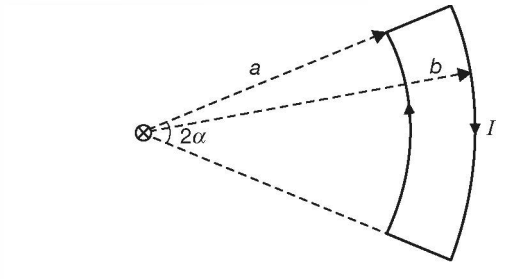
Since, $M=I_1(\pi r^2)$

$$\Rightarrow\quad \tau=I_1(\pi r^2)\frac{4\mu_0I_2\ell^2}{2\pi(x^2+\ell^2)\sqrt{x^2+2\ell^2}}$$

$$\Rightarrow\quad \tau=\frac{2\mu_0I_1I_2\ell^2r^2}{(x^2+\ell^2)\sqrt{x^2+2\ell^2}}$$

ILLUSTRATION 93

A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find the moment of Ampere's force acting on this loop.



SOLUTION

Consider a rectangular element of width dr and length $rd\theta$ as shown. If $d\vec{A}$ is the area of the infinitesimal element then

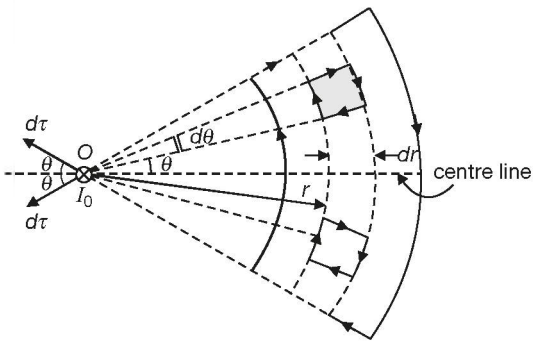
$$d\vec{A} = (dr)(rd\theta) = rdrd\theta$$

$$d\vec{A} = (rd\theta dr) \quad \{\text{inwards}\}$$

$$d\vec{M} = Id\vec{A} = I(rd\theta dr) \quad \{\text{inwards}\}$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I_0}{r} \quad \{\text{tangential clockwise}\}$$

$$\Rightarrow \quad d\tau = |d\vec{M} \times \vec{B}| = \frac{\mu_0 I I_0 d\theta dr}{2\pi} \quad \{\text{towards centre}\}$$



$$\Rightarrow \quad \tau = \int_{-\alpha}^{\alpha}\int_{a}^{b} d\tau \cos\theta$$

$$\Rightarrow \quad \tau = \frac{\mu_0 I I_0}{2\pi}\int_{-\alpha}^{\alpha}\int_{a}^{b} \cos\theta d\theta dr$$

$$\Rightarrow \quad \tau = \frac{\mu_0 I I_0 (b-a)\sin\alpha}{\pi}$$

INTERACTION ENERGY FOR A CURRENT CARRYING LOOP IN MAGNETIC FIELD

As already discussed in the chapter of electrostatics (while studying an electric dipole), the interaction potential energy of an electrical dipole with dipole moment p placed in an external uniform electric field E is

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

Similarly, for a magnetic dipole with dipole moment M placed in a magnetic field with magnetic induction B with θ as the angle between magnetic moment and magnetic induction, its interaction potential energy is given by

$$U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$$

Since $M = IA$, so the above expression for potential energy can also be written as

$$U = -(IA)B\cos\theta$$

Also, we must know that just like electric flux ϕ_E is defined as $\phi_E = EA\cos\theta$, the magnetic flux ϕ_B associated with the coil is

$$\phi_B = BA\cos\theta$$

$$\Rightarrow \quad U = -I\phi_B$$

Dear Student, we shall be discussing the Magnetic Flux in detail in Electromagnetic Induction.

WORK DONE TO CHANGE ORIENTATION OF A CURRENT CARRYING COIL IN MAGNETIC FIELD

Consider a current carrying coil to be rotated in a uniform magnetic field, such that angle between magnetic induction and magnetic moment of coil changes from θ_1 to θ_2. The potential energy of the coil in magnetic field for its initial and final state are given by

$$U_i = -MB\cos\theta_1 \text{ and}$$

$$U_f = -MB\cos\theta_2$$

Work done to change the orientation of the coil in this process is

$$W = U_f - U_i$$

$$\Rightarrow \quad W = (-MB\cos\theta_2) - (-MB\cos\theta_1)$$

$$\Rightarrow \quad W = -MB(\cos\theta_2 - \cos\theta_1) \qquad \ldots(1)$$

Also, we can think that to change the orientation of coil externally, we need to apply an external torque against the magnetic torque such that this external torque has magnitude equal to the magnetic torque. So, work done in changing the orientation in this case is given by

$$W = \int dW = \int \tau d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta$$

$$\Rightarrow \quad W = MB(-\cos\theta)\Big|_{\theta_1}^{\theta_2}$$

$$\Rightarrow \quad W = -MB(\cos\theta_2 - \cos\theta_1) \qquad \ldots(2)$$

So, we see that equations (1) and (2) are identical.
Since $M = IA$, so the above expression for work done can also be written as

$$W = -(IA)B(\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow \quad W = -I(BA\cos\theta_2 - BA\cos\theta_1)$$

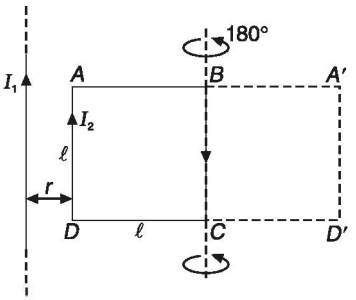
Since $BA\cos\theta_1 = \phi_1 = \phi_{\text{initial}}$ and $BA\cos\theta_2 = \phi_2 = \phi_{\text{final}}$

$$\Rightarrow \quad W = -I(\phi_{\text{final}} - \phi_{\text{initial}})$$

$$\Rightarrow \quad W = -I\Delta\phi$$

ILLUSTRATION 94

A square coil of edge l carrying a current I_2 is placed near to a long straight wire carrying current I_1. Calculate the work required to rotate the coil $ABCD$ about the axis along the edge BC through 180° to the dotted position $A'BCD'$ as shown in Figure.

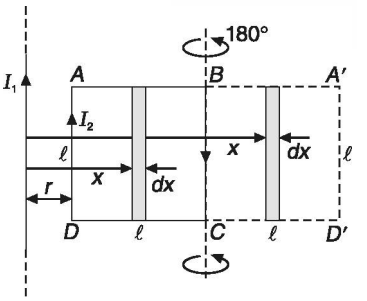


SOLUTION

Work done in the process of changing the orientation of a coil in the magnetic field of the infinite current carrying wire is given by

$$W = -I_2(\phi_f - \phi_i) \qquad \ldots(1)$$

where ϕ_f and ϕ_i are the flux associated with the square coil in the final and the initial position.
To calculate the magnetic flux associated with the square loop let us consider an infinitesimal strip element of length l, width dx at a distance x in the coil as shown in Figure.



The magnetic field B at the strip due to the infinite wire is

$$B = \frac{\mu_0 I_1}{2\pi x}$$

The magnetic flux associated with this infinitesimal strip is

$$d\phi = BdA$$

where $dA = ldx$ is the area of the strip.

$$\Rightarrow \quad d\phi = \left(\frac{\mu_0 I_1}{2\pi x}\right)ldx$$

Total magnetic flux associated with the square loop for the initial position of the loop is obtained by integrating $d\phi$. So, we get

$$\phi_i = \int BdA = \int_r^{r+l}\left(\frac{\mu_0 I_1}{2\pi x}\right)ldx = \left(\frac{\mu_0 I_1 l}{2\pi}\right)\int_r^{r+l}\frac{dx}{x}$$

$$\Rightarrow \quad \phi_i = \frac{\mu_0 I_1 l}{2\pi}(\ln x)\Big|_r^{r+l} = \frac{\mu_0 I_1 l}{2\pi}[\ln(r+l) - \ln r]$$

$$\Rightarrow \quad \phi_i = \frac{\mu_0 I_1 l}{2\pi}\ln\left(\frac{r+l}{r}\right)$$

Similarly, the final flux associated with the coil is

$$\phi_f = \int_{r+l}^{r+2l}\frac{\mu_0 I_1}{2\pi x}ldx = \frac{\mu_0 I_1 l}{2\pi}(\ln x)\Big|_{r+l}^{r+2l}$$

$$\Rightarrow \quad \phi_f = \frac{\mu_0 I_1 l}{2\pi}\ln\left(\frac{r+2l}{r+l}\right)$$

So, work done in this process is calculated by using equation (1), according to which

$$W = -I_2(\phi_f - \phi_i)$$

$$\Rightarrow \quad W = -I_2\left(\frac{\mu_0 I_1 l}{2\pi}\right)\left[\ln\left(\frac{r+2l}{r+l}\right) - \ln\left(\frac{r+l}{r}\right)\right]$$

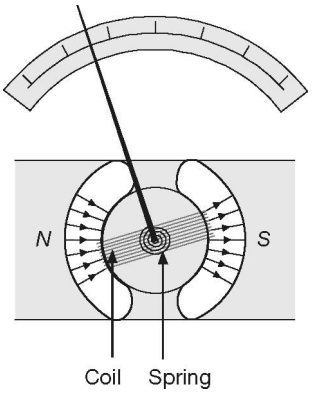
$$\Rightarrow \quad W = I_2\left(\frac{\mu_0 I_1 l}{2\pi}\right)\left[\ln\left(\frac{r+l}{r}\right) - \ln\left(\frac{r+2l}{r+l}\right)\right]$$

$$\Rightarrow \quad W = I_2\left(\frac{\mu_0 I_1 l}{2\pi}\right)\left[\ln\left(\frac{r+l}{r}\right) + \ln\left(\frac{r+l}{r+2l}\right)\right]$$

$$\Rightarrow \quad W = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left(\frac{(r+l)^2}{r(r+2l)}\right)$$

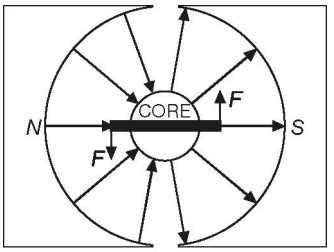
MOVING COIL GALVANOMETER (D'ARSOVNAL GALVANOMETER): RADIAL FIELD

In a moving coil galvanometer, the coil is suspended between the pole pieces of a strong horse-shoe magnet as shown in Figure.



Structure of a moving-coil galvanometer

The pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. This makes the field radial. In a radial field, the plane of the coil always remains parallel to the field.



$F = nBIl$

Therefore $\theta = 90°$ and the deflecting torque always has the value

$$\tau_{def} = NBIA$$

As the coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$$\tau_{rest} = C\alpha$$

where C is the torsional constant of the fibre.
When the coil is in equilibrium.

$$NBIA = C\alpha$$

$$\Rightarrow \quad I = \frac{C}{NBA}\alpha$$

$$\Rightarrow \quad I = K\alpha\,,$$

where $K = \dfrac{C}{NBA}$ is the galvanometer constant. This linear relationship between I and α makes the moving coil galvanometer useful for current measurement and detection.

Current Sensitivity

The current sensitivity of a moving coil galvanometer is defined as

$$S_I = \frac{\alpha}{I} = \frac{NBA}{C}$$

Thus in order to increase the sensitivity of a moving coil galvanometer, N, B and A should be increased and C should be decreased.

Voltage Sensitivity (S_V)

The twist angle per unit voltage is called Voltage Sensitivity. So,

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{S_I}{R} = \frac{NBA}{RC}$$

ILLUSTRATION 95

A moving coil galvanometer experiences torque ki where i is current. If N coils of area A each and moment of inertia I is kept in magnetic field B.

(a) Find k in terms of given parameters,

(b) If for current i deflection is $\dfrac{\pi}{2}$, find out torsional constant of spring.

(c) If a charge Q is passed suddenly through the galvanometer. Find out maximum angle of deflection.

SOLUTION

(a) The torque acting on the coil of moving coil galvanometer is

$$\tau = MB = ki \quad (\theta = 90^\circ)$$

$$\Rightarrow \quad k = \frac{MB}{i} = \frac{(NiA)B}{i} = NBA$$

(b) If K is torsional constant of the spring of galvanometer, then we know that

$$\tau = K\theta = NBiA$$

$$\Rightarrow \quad k = \frac{2NBiA}{\pi}$$

(c) $\tau = NBiA$

$$\Rightarrow \quad \int_0^t \tau dt = NBA \int_0^t i\, dt$$

$$\Rightarrow \quad I\omega = (NBA)Q$$

$$\Rightarrow \quad \omega = \frac{NBAQ}{I} \qquad \ldots(1)$$

At maximum deflection whole, kinetic energy (rotational) will be converted into potential energy of spring. If θ_{max} is the maximum deflection, then by Law of Conservation of Energy, we have

$$\frac{1}{2} I\omega^2 = \frac{1}{2} k\theta_{max}^2$$

Substituting the values, we get

$$\theta_{max} = Q\sqrt{\frac{\pi NBA}{2I}}$$

Test Your Concepts-VII

Based on Magnetic Moment and Torque

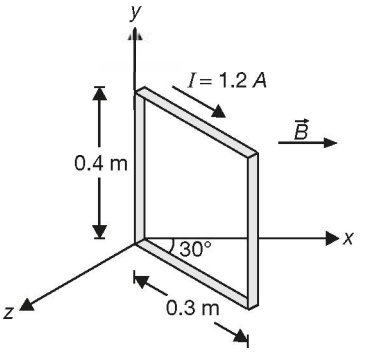
(Solutions on page H.24)

1. Consider an electron orbiting a proton and maintained in a fixed circular path of radius $R = 0.53$ Å by the Coulomb force. Treating the orbiting charge as a current loop, calculate the resulting torque when the system is in a magnetic field of 0.4 T directed perpendicular to the magnetic moment of the electron. Take $m_e = 9\times10^{-31}$ kg and $e = 1.6\times10^{-19}$ C.

2. If θ represents the angle between magnetic moment $\vec{M}$ of a current carrying loop and magnetic field $\vec{B}$ in the region in which the loop is placed. Assuming the reference of potential energy to be at $\theta = 0^\circ$, calculate the work done in rotating the loop from $\theta_1 = 60^\circ$ to $\theta_2 = 120^\circ$ and the potential energy of the loop at $\theta = 45^\circ$.

3. A current of 17 mA is maintained in a single circular loop of 2 m circumference. A magnetic field of 0.8 T is directed parallel to the plane of the loop.
(a) Calculate the magnetic moment of the loop.
(b) What is the magnitude of the torque exerted by the magnetic field on the loop?

4. A rectangular coil carrying a current of 1.2 A consists of $N = 100$ closely wrapped turns and has dimensions $a = 0.4$ m and $b = 0.3$ m. The coil is hinged along the y-axis, and its plane makes an angle $\theta = 30^\circ$ with the x-axis. The coil is placed in a field of 0.8 T directed along x-axis. Calculate the torque on the coil and the sense of rotation.

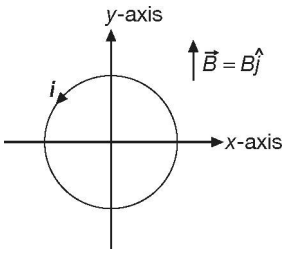


5. A long piece of wire with a mass of 0.1 kg and a total length of 4 m is used to make a square coil with a side of 0.1 m. The coil is hinged along a horizontal side, carries a 3.4 A current, and is placed in a vertical magnetic field with a magnitude of 0.01 T.
(a) Determine the angle that the plane of the coil makes with the vertical when the coil is in equilibrium.
(b) Find the torque acting on the coil due to the magnetic force at equilibrium.

6. A 40 cm length of wire carries a current of 20 A. It is bent into a loop and placed with its normal perpendicular to a magnetic field with a magnitude of 0.52 T. What is the torque on the loop if it is bent into
(a) an equilateral triangle?
(b) a square
(c) a circle?
(d) Which torque is greatest?

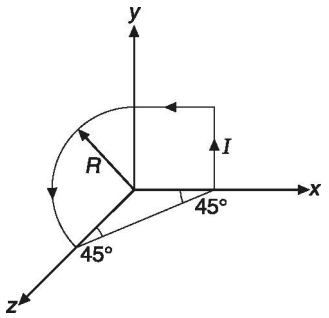
7. A wire is formed into a circle having a diameter of 10 cm and placed in a uniform magnetic field of 3 mT. The wire carries a current of 5 A. Find
(a) the maximum torque on the wire and
(b) the range of potential energies of the wire-field system for different orientations of the circle.

8. A wire ring of mass m has a radius R carries a current i. It is placed in x-y plane and is free to rotate about a diameter parallel to x-axis as shown in Figure.



A uniform magnetic field B is switched on along the y-axis. Calculate the angular speed acquired by the ring as it rotates through 90°.

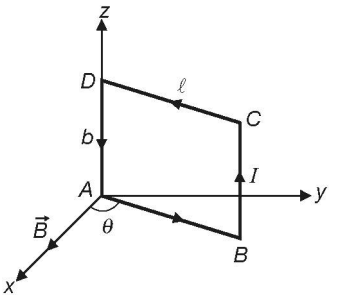
9. Calculate the magnetic moment of the current carrying loop shown in figure.



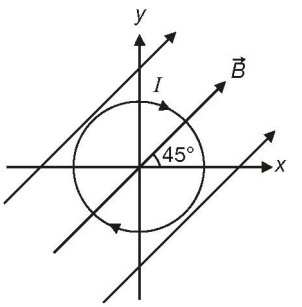
10. A rectangular galvanometer coil of area 5.0×10^{-4} m^2 having 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal magnetic field having a magnitude of 9×10^{-3} T. Calculate the torsional constant of the string in Nmrad^{-1} connected to the coil if a current of 0.20 mA produces an angular deflection of 18°.

11. Find an expression for the magnetic dipole moment and magnetic field induction at the center of a Bohr's hypothetical hydrogen atom in the nth orbit of the electron in terms of universal constants.

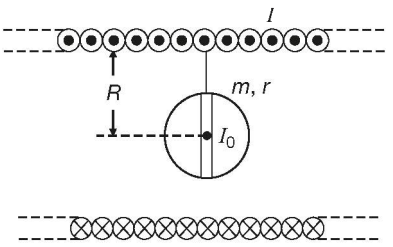
12. Consider a rectangular frame $ABCD$ whose sides are ℓ and b meters long. The frame holds N coils, each of which carries a current I. The frame is embedded in a uniform magnetic field $\vec{B}$ which forms an angle θ with the frame plane.
(a) Find the forces which act on the sides of the frame.
(b) Compute the torque about the axis DA.



13. A circular loop of radius $R = 20$ cm is placed in a uniform magnetic field $\vec{B} = 2$ T in x-y plane as shown in figure. The loop carries a current $I = 1$ A in the direction shown in figure. Find the magnitude and direction of torque acting on the loop.

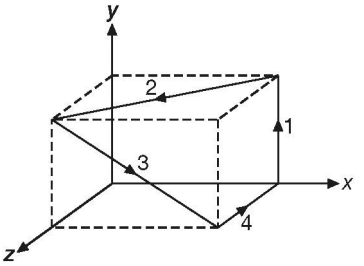


14. A small sphere of mass m and radius r is suspended by a silk thread inside a current carrying solenoid. The solenoid carries a current I and has n numbers of turns per unit length. The sphere is wrapped by a single turn of coil carrying a current I_0 as shown in the diagram. Find the



(a) maximum torque experienced by the sphere
(b) time period of small oscillation.

15. A wire carrying a current of 10 A is bent to pass through various sides of a cube of side 10 cm as shown in Figure.

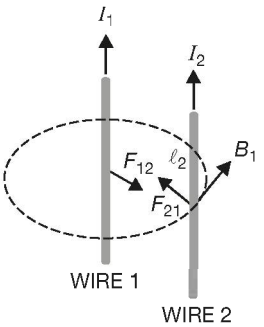


A magnetic field $\vec{B}=\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ T is present in the region. Calculate the net force on the loop, the magnetic moment of the loop and torque on the loop formed by the wires.

MAGNETIC FORCE BETWEEN TWO PARALLEL CURRENT CARRYING WIRES

CASE-1: Wires Carrying Currents In The Same Direction

Consider two long straight wires 1 and 2 kept parallel to each other at perpendicular separation r, carrying currents I_1 and I_2 respectively, in the same direction as shown in Figure.



The equal and opposite forces exerted by two parallel current carrying wires. The force is attractive in nature.

The wire 2 (carrying current I_2) lies in the magnetic field of wire 1 $\left(B_1=\dfrac{\mu_0 I_1}{2\pi r},\ \otimes\right)$ as a result of which the wire 2 experiences a magnetic force. The magnetic force on a length l_2 of wire 2 exerted by the field B_1 of the wire 1 (at wire 2) is given by

$$\vec{F}_{21}=I_2\left(\vec{l}_2\times\vec{B}_1\right) \qquad \{\text{towards wire 1}\}$$

where $\vec{l}_2$ is in the direction of I_2.

Since $\vec{l}_2\perp\vec{B}_1$ (inwards), so we get

$$\left|\vec{F}_{21}\right|=F_{21}=I_2 l_2 B_1\sin 90=I_2 l_2 B_1$$

Since, we have $B_1=\dfrac{\mu_0 I_1}{2\pi r}$, $\otimes$, so we get

$$\Rightarrow \quad F_{21}=I_2 l_2\left(\frac{\mu_0 I_1}{2\pi r}\right) \qquad \ldots(1)$$

The direction of $\vec{F}_{21}$ (towards wire 1) is found by using Fleming's Left Hand Rule.

Similarly, $\vec{F}_{12}$ on a length l_1 of the wire is

$$\vec{F}_{12}=I_1\left(\vec{l}_1\times\vec{B}_2\right) \qquad \{\text{Towards Wire 2}\}$$

Since, we have $B_2=\dfrac{\mu_0 I_2}{2\pi r}$, $\odot$, so we get

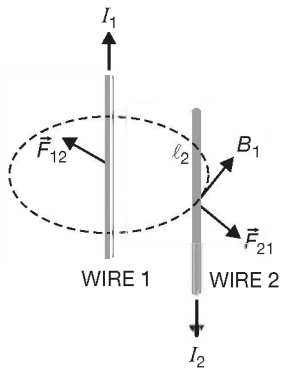
$$\Rightarrow \quad \left|\vec{F}_{12}\right|=F_{12}=I_1 l_1\left(\frac{\mu_0 I_2}{2\pi r}\right) \qquad \ldots(2)$$

From equation (1) and (2), we conclude that the force per unit length on either of the wire is equal

$$\frac{F_{21}}{l_2}=\frac{F_{12}}{l_1}=\frac{\mu_0 I_1 I_2}{2\pi r}$$

CASE-2: Wires Carrying Current In The Opposite Direction

Consider two long straight wires 1 and 2 kept parallel to each other at perpendicular separation r, carrying currents I_1 and I_2 respectively, in the opposite direction as shown in Figure.



The equal and opposite forces exerted by two parallel current carrying wires. Here the force is repulsive in nature.

The wire 2 (carrying current I_2) lies in the magnetic field of wire 1 $\left(B_1 = \frac{\mu_0 I_1}{2\pi r}, \otimes\right)$ as a result of which the wire 2 experiences a magnetic force. The magnetic force on a length l_2 of wire 2 exerted by the field B_1 of the wire 1 (at wire 2) is given by

$$\vec{F}_{21} = I_2\left(\vec{l}_2 \times \vec{B}_1\right) \qquad \{\text{away from wire 1}\}$$

where $\vec{l}_2$ is in the direction of I_2.

Since $\vec{l}_2 \perp \vec{B}_1$ (inwards), so we get

$$\left|\vec{F}_{21}\right| = F_{21} = I_2 B_1 l_2 \sin 90° = I_2 B_1 l_2$$

Also, $B_1 = \frac{\mu_0 I_1}{2\pi r}, \otimes$, so we get

$$\Rightarrow \quad F_{21} = I_2\left(\frac{\mu_0 I_1}{2\pi r}\right) l_2 \qquad \ldots(1)$$

The direction of $\vec{F}_{21}$ (towards wire 1) is found by using Fleming's Left Hand Rule.

Similarly, $\vec{F}_{12}$ on a length l_1 of the wire is

$$\vec{F}_{12} = I_1\left(\vec{l}_1 \times \vec{B}_2\right) \qquad \{\text{away from wire 2}\}$$

Since, we have $B_2 = \frac{\mu_0 I_2}{2\pi r}, \otimes$, so we get

$$\Rightarrow \quad \left|\vec{F}_{12}\right| = F_{12} = I_1\left(\frac{\mu_0 I_2}{2\pi r}\right) l_1 \qquad \ldots(2)$$

From equation (1) and (2), we conclude that the force per unit length on either of the wire is equal

$$\frac{F_{21}}{l_2} = \frac{F_{12}}{l_1} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Also, we see that the two current carrying wires carrying current in the opposite direction repel each other with a force per unit length given by

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Definition of One Ampere

Since we know that

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

If $I_1 = I_2 = I$ (say), $r = 1$ metre and $\frac{F}{l} = 2\times10^{-7}\ \text{Nm}^{-1}$, then we get

$$2\times10^{-7} = 2\times10^{-7} I^2 \quad \left\{\because\ \mu_0 = 4\pi\times10^{-7}\ \text{TmA}^{-1}\right\}$$

$$\Rightarrow \quad I^2 = 1$$

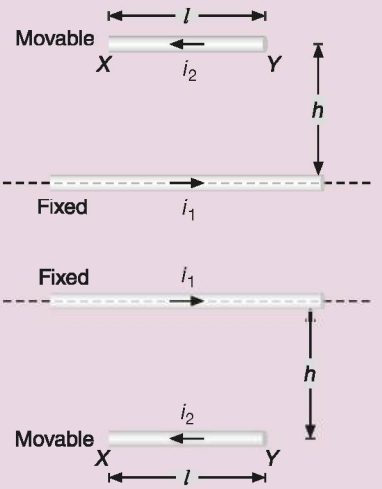
$$\Rightarrow \quad I = 1\ \text{A}$$

So, the current in the wire is one ampere when two identical current carrying wires placed at a separation of one metre, attract (or repel) each other with a force of 2×10^{-7} newton per metre of their length.

Problem Solving Technique(s)

(a) Equilibrium of a current carrying conductor

When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown in Figure



In both the situations for equilibrium of XY, we have

$$\begin{pmatrix}\text{Weight of}\\ \text{Conductor}\end{pmatrix}=\begin{pmatrix}\text{Upward Magnetic}\\ \text{Force}\end{pmatrix}$$

$$\Rightarrow \quad mg=\left(\frac{\mu_0 I_1 I_2}{2\pi h}\right)\ell$$

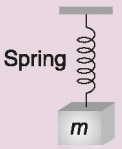
(b) In the first case if wire XY is slightly displaced from its equilibrium position, then it executes SHM and its time period is given by

$$T=2\pi\sqrt{\frac{h}{g}}$$

(c) If direction of current in movable wire is reversed then its instantaneous acceleration produced is $2g$ downwards.

(d) Current carrying spring

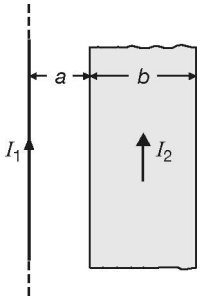
When current is passed through a spring, then it will contract, because the current flow in all the segments of the spring is in the same direction.



So, due to the current flow in the spring the spring contracts and the weight is lifted up.

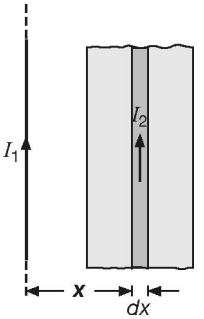
ILLUSTRATION 96

Two long thin parallel conductors of the shape shown in figure carry direct currents I_1 and I_2. The separation between the conductors is a, the width of the right hand conductor is equal to b as shown in Figure. With both conductors lying in one plane, find the magnetic interaction force between them reduced to a unit of their length.



SOLUTION

Consider an infinitesimal thin strip of "unit length" carrying a current dI as shown in Figure.



Then infinitesimal current dI flowing through this element is

$$dI=\left(\frac{I_2}{b}\right)dx$$

The field due to wire at the strip is

$$B=\frac{\mu_0 I_1}{2\pi x}$$

The force per unit length between the wire and the infinitesimal elemental wire is

$$dF=\frac{\mu_0 I_1 (dI)}{2\pi x}$$

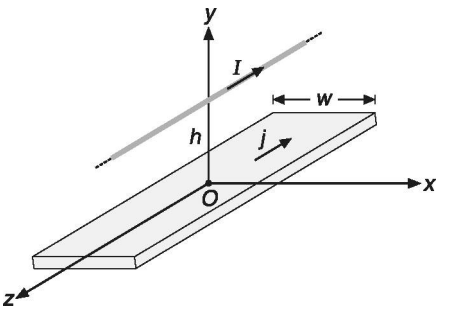
$$\Rightarrow \quad dF=\frac{\mu_0 I_1 I_2}{2\pi b}\frac{dx}{x}$$

$$\Rightarrow \quad F=\int dF=\frac{\mu_0 I_1 I_2}{2\pi b}\int_a^{a+b}\frac{dx}{x}$$

$$\Rightarrow \quad F=\frac{\mu_0 I_1 I_2}{2\pi}\log_e\left(\frac{a+b}{a}\right)$$

ILLUSTRATION 97

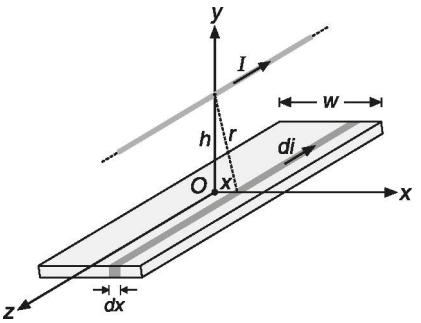
A very long straight conductor carries a current I is placed parallel to a current strip of width w having a uniform current per unit width j flowing throughout its width. The conductor is at a height h above the strip as shown in Figure.



Calculate the force per unit length on the conductor due to the strip. Also discuss the obtained result when the width w of the current sheet approaches infinity.

SOLUTION

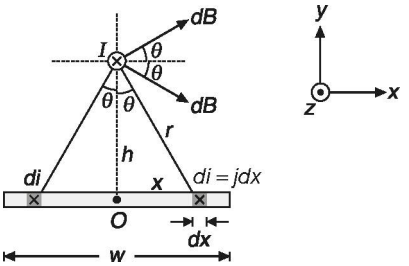
Let us consider an infinitesimal segment of sheet of thickness dx at a distance x to the right of origin as shown in Figure.



If di be the current flowing through this infinitesimal strip, then $di = jdx$. The force of attraction between the infinitesimal strip and the conductor of section length L is

$$dF = (dB)IL = \frac{\mu_0 (jdx)}{2\pi r} IL \text{, where } r = \sqrt{h^2 + x^2}$$

Let us consider another infinitesimal segment of sheet of thickness dx at a distance x to the left of origin as shown in Figure.



Due to symmetry, the x-components of the forces i.e. $dF\sin\theta$ and $dF\sin\theta$ cancel. So, we get

$$F_{net} = \int dF\cos\theta = \int \frac{\mu_0 IL(jdx)}{2\pi r}\cos\theta,$$

where $\cos\theta = \dfrac{h}{\sqrt{h^2 + x^2}}$

Integrating over the width of the strip i.e. from $x = -\dfrac{w}{2}$ to $x = \dfrac{w}{2}$, we get

$$\Rightarrow \quad F_{net} = \int_{-\frac{w}{2}}^{\frac{w}{2}} \left(\frac{\mu_0 IjLdx}{2\pi\sqrt{h^2 + x^2}}\right)\left(\frac{h}{\sqrt{h^2 + x^2}}\right)$$

$$\Rightarrow \quad F_{net} = \frac{\mu_0 IjLh}{2\pi}\int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{dx}{h^2 + x^2}$$

Since $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \quad F_{net} = \frac{\mu_0 IjL}{\pi}\tan^{-1}\left(\frac{w}{2h}\right)$$

So the force of attraction per unit length of the conductor is given by

$$\frac{F_{net}}{L} = \frac{\mu_0 Ij}{\pi}\tan^{-1}\left(\frac{w}{2h}\right) \qquad \ldots(1)$$

This force is attractive in nature as the currents in wire as well as the sheet are in the same direction.

When $w \to \infty$, then $\tan^{-1}\left(\frac{w}{2h}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$

So, from equation (1), we get

$$\frac{F}{L} = \left(\frac{\mu_0 Ij}{\pi}\right)\left(\frac{\pi}{2}\right) = \frac{\mu_0 Ij}{2}$$

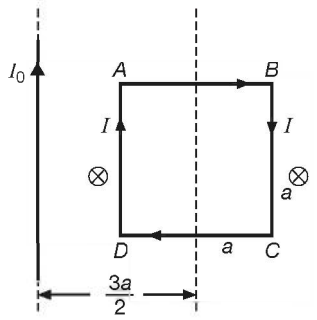
ILLUSTRATION 98

A square frame carrying a current I is located in the same plane as a long straight wire carrying a current I_0. The frame side has a length a. The axis of the frame passing the midpoints of opposite sides is parallel to the wire and is separated from it by the distance which is $\eta = 1.5$ times greater than the side of the frame. Find

(a) the ampere force acting on the frame.
(b) the mechanical work to be performed in order to turn the frame through $180°$ about its axis, with currents maintained constant.

SOLUTION

The force per unit length on the different sections of the square wire frame due to the straight current carrying wire is given by



$$\vec{F}_{AB} = \left(\frac{\mu_0 I I_0}{2\pi a}\right) a \qquad \{\text{attractive}\}$$

$$\vec{F}_{BC} = \frac{\mu_0 I I_0}{2\pi (2a)} u \qquad \{\text{repulsive}\}$$

On the sections AB and CD of the square frame, the magnetic force due to the wire has same magnitude but opposite direction, hence the net force on the square wire frame is given by

$$\Rightarrow \quad F_{\text{net}} = \frac{\mu_0 I I_0}{2\pi}\left(1 - \frac{1}{2}\right) \qquad \{\text{attractive}\}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{\mu_0 I I_0}{4\pi} \qquad \{\text{attractive}\}$$

Since, $W = U_f - U_i$

Ley us now find the initial and the final magnetic potential energy. For calculating this let us consider a very small strip element of length a, thickness dx i.e. area $dA = adx$, such that magnetic moment dM due to this small elemental strip is given by

$$dM = I(dA) = I(adx)$$

$$\Rightarrow \quad dU_i = -(dM)B\cos(0^\circ)$$

$$\Rightarrow \quad dU_i = -I(adx)\frac{\mu_0 I_0}{2\pi x}$$

$$\Rightarrow \quad U_i = -\frac{\mu_0 I I_0 a}{2\pi}\int_a^{2a}\frac{dx}{x}$$

$$\Rightarrow \quad U_i = -\frac{\mu_0 I I_0 a}{2\pi}\log_e(2)$$

Similarly, we get

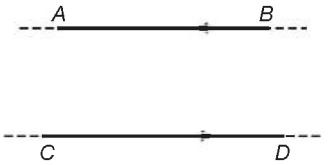
$$U_f = \int dU_f = -\int (dM)B\cos(180^\circ) = (dM)B$$

$$\Rightarrow \quad U_f = \frac{\mu_0 (I I_0) a}{2\pi}\log_e(2)$$

$$\Rightarrow \quad W = |U_f - U_i| = \frac{\mu_0 (I I_0) a}{\pi}\log_e(2)$$

ILLUSTRATION 99

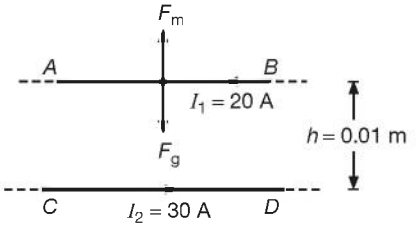
A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in Figure.



Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.

SOLUTION

Let λ be the mass per unit length of wire AB. At a height h above the wire CD, magnetic force per unit length on wire AB will be acting upwards as shown in Figure.



The magnitude of this magnetic fore is given by

$$F_m = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{h}, \text{ upwards} \qquad \ldots(1)$$

Also, weight per unit length of wire AB is

$$F_g = \left(\frac{m}{l}\right)g = \lambda g, \text{ downwards}$$

If the wire is in equilibrium, say at $x = d$, then the magnetic force must balance the weight of the wire. So,

$$F_m = F_g$$

$$\Rightarrow \quad \frac{\mu_0}{2\pi}\frac{I_1 I_2}{h} = \lambda g \qquad \ldots(2)$$

When AB is depressed through x, then h decreases therefore, F_m will increase, while F_g remains the same due to which the net restoring force F acting on a portion of length l of wire AB is given by

$$F=\frac{\mu_0 I_1 I_2 l}{2\pi(h-x)}-\lambda lg=\frac{\mu_0 I_1 I_2 l}{2\pi h}\left(1-\frac{x}{h}\right)^{-1}-\lambda lg$$

$$\Rightarrow \quad F=\frac{\mu_0 I_1 I_2 l}{2\pi h}\left(1+\frac{x}{h}\right)-\lambda lg=\frac{\mu_0 I_1 I_2 l}{2\pi h^2}x \qquad \{\because \text{ of (2)}\}$$

$$\Rightarrow \quad m\ddot{x}=-\left(\frac{\mu_0 I_1 I_2 l}{2\pi h^2}\right)x$$

Negative sign indicates the restoring nature of the force.

$$\Rightarrow \quad m\ddot{x}=-\left(\frac{\lambda g l}{h}\right)x \qquad \{\because \text{ of (2)}\}$$

However, we have $\lambda l = m$

$$\Rightarrow \quad \ddot{x}+\frac{g}{h}x=0$$

Comparing with standard equation of SHM, i.e.

$$\ddot{x}+\omega^2 x=0$$

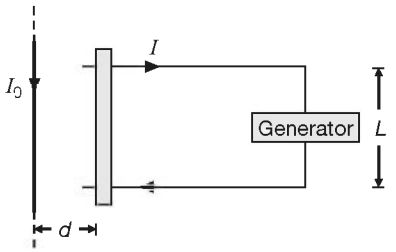
$$\Rightarrow \quad \omega=\sqrt{\frac{g}{h}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{h}{g}}=2\pi\sqrt{\frac{0.01}{9.8}}$$

$$\Rightarrow \quad T=0.2 \text{ s}$$

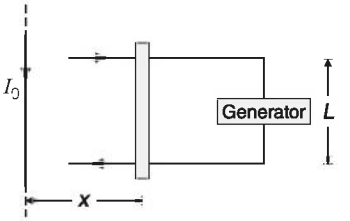
ILLUSTRATION 100

A bar of mass m slides on a smooth horizontal conducting rail. A constant current I is maintained in the rod using a constant current generator as shown. There is a long straight conductor carrying current I_0 parallel to the rod. Find the speed of the bar after it has travelled a distance D.



SOLUTION

Consider an instant when the sliding bar is at a distance x from the straight long conductor as shown in Figure.



The magnetic force acting on the sliding bar at this instant is given by

$$F=\frac{\mu_0 I I_0 L}{2\pi x}$$

Since $F=ma=m\left(v\frac{dv}{dx}\right)$

$$\Rightarrow \quad mv\frac{dv}{dx}=\frac{\mu_0 I I_0 L}{2\pi x}$$

$$\Rightarrow \quad v\,dv=\frac{\mu_0 I I_0 L}{2m}\frac{dx}{x}$$

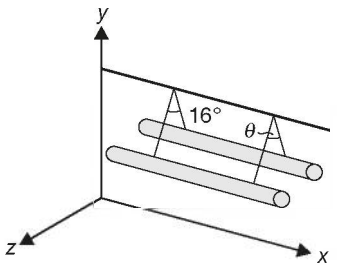
$$\Rightarrow \quad \int_0^v v dv=\frac{\mu_0 I I_0 L}{2\pi m}\int_d^{d+D}\frac{dx}{x}$$

$$\Rightarrow \quad \frac{v^2}{2}=\frac{\mu_0 I I_0 L}{2\pi m}\log_e\left(1+\frac{D}{d}\right)$$

$$\Rightarrow \quad v=\sqrt{\frac{\mu_0 I I_0 L\log_e\left(1+\frac{D}{d}\right)}{\pi m}}$$

ILLUSTRATION 101

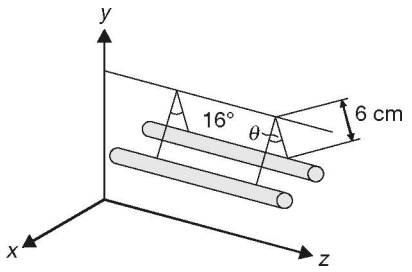
Two long, parallel wires, each having a mass per unit length of 40 gm^{-1}, are supported in a horizontal plane by strings 6 cm long. When both wires carry the same current I, the wires repel each other so that the angle θ between the supporting strings is $16°$. Find the magnitude and the directions of the currents.



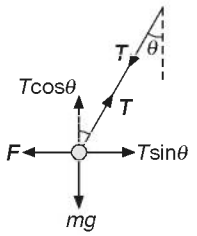
SOLUTION

The separation between the wires is

$$r = 2(6\text{ cm})\sin 8° = 1.67\text{ cm}$$



Since the wires repel so, the currents are in the opposite directions.



Cross Sectional View

The magnetic force F acts horizontally, so we get

$$T\cos\theta = mg \qquad \ldots(1)$$

$$T\sin\theta = F = \frac{\mu_0 I^2 \ell}{2\pi r}$$

$$\Rightarrow \quad \tan\theta = \frac{\mu_0 I^2 \ell}{(2\pi r)mg}$$

$$\Rightarrow \quad \tan(8°) = \frac{\mu_0 I^2 \ell}{2\pi r mg}$$

$$\Rightarrow \quad I^2 = \frac{mg2\pi r}{\ell\mu_0}\tan(8°)$$

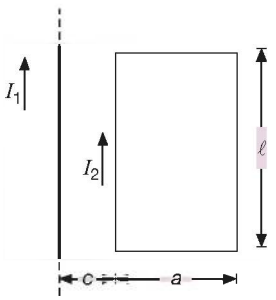
$$\Rightarrow \quad I = 67.8\text{ A}$$

Test Your Concepts-VIII

Based on Force between Current Carrying Conductors

(Solutions on page H.27)

1. In figure, the current in the long, straight wire is $I_1 = 5$ A and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10$ A. The dimensions are $c = 0.1$ m, $a = 0.15$ m, and $\ell = 0.45$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.



2. Two long, parallel conductors, separated by 10 cm, carry currents in the same direction. The first wire carries current $I_1 = 5$ A and the second carries $I_2 = 8$ A.

(a) What is the magnitude of the magnetic field created by I_1 at the location of I_2?

(b) What is the force per unit length exerted by I_1 on I_2?

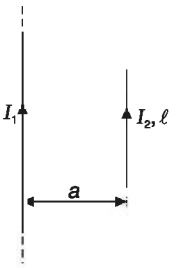
(c) What is the magnitude of the magnetic field created by I_2 at the location of I_1?

(d) What is the force per length exerted by I_2 on I_1?

3. Two long, parallel wires are attracted to each other by a force per unit length of 320 μNm^{-1} when they are separated by a vertical distance of 0.5 m. The current in the upper wire is 20 A to the right. Determine the location of the line in the plane of the two wires along which the total magnetic field is zero.

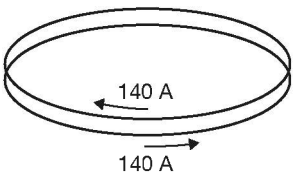
4. Three long wires (wire 1, wire 2, and wire 3) hang vertically. The distance between wire 1 and wire 2 is 20 cm. On the left, wire 1 carries an upward current of 1.5 A. To the right, wire 2 carries a downward current of 4 A. Wire 3 is located such that when it carries a certain current, each wire experiences no net force. Find the position of wire 3. Also find the magnitude and direction of the current in wire 3.

5. A fixed long straight conductor carrying current I_1 is placed at a distance a from another conductor of length l carrying current I_2 as shown in Figure.



Calculate the minimum work required to be done to increase the separation between the conductors through a distance b.

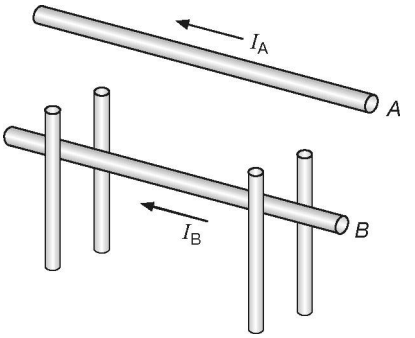
6. Two circular current carrying loops, each of radius 10 cm, are parallel, coaxial, and almost in contact, 1 mm apart. The top loop carries a clockwise current of 140 A. The bottom loop carries a counter clockwise current of 140 A.



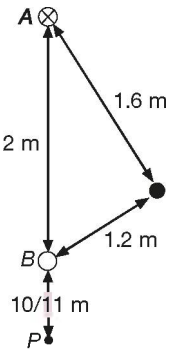
(a) Calculate the magnetic force exerted by the bottom loop on the top loop.

(b) The upper loop has a mass of 0.021 kg. Calculate its acceleration, assuming that the only forces acting on it are the magnetic force and the gravitational force. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } g = 10 \text{ ms}^{-2}\right)$

7. Two long, parallel conductors carry currents in the same direction as shown. Conductor A carries a current of 150 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. If the mass per unit length of conductor B is $0.1\,\text{gcm}^{-1}$, what value of current I_B will result in equilibrium when the distance between the two conductors is 2.5 cm?



8. Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 A, directed into the plane of the paper as shown in Figure.



The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $\frac{10}{11}$ m from the wire B, is zero. Calculate the force per unit length on the wire B.

SOLVED PROBLEMS

PROBLEM 1

A particle having charge $-q$ and mass $m = 2.58 \times 10^{-15}$ kg is travelling through a region containing a uniform magnetic field $\vec{B} = -(0.12\ \text{T})\hat{k}$. At a particular instant of time the velocity of the particle is $\vec{v} = (1.05\times10^{6}\ \text{ms}^{-1})(-3\hat{i} + 4\hat{j} + 12\hat{k})$ and the force $\vec{F}$ on the particle has a magnitude of 1.25 N.

(a) Calculate the charge q.
(b) Calculate the acceleration $\vec{a}$ of the particle.
(c) Explain why the path of the particle is a helix, and determine the radius of curvature R of the circular component of the helical path.
(d) Determine the cyclotron frequency of the particle.
(e) Although helical motion is not periodic in the full sense of the word, the x and y-coordinates do vary in a periodic way. If the coordinates of the particle at $t = 0$ are $(x, y, z) = (R, 0, 0)$, determine its coordinates at a time $t = 2T$, where T is the period of the motion in the xy-plane.

SOLUTION

(a) Given that,

$$\vec{B} = -(0.12\ \text{T})\hat{k},$$

$$\vec{v} = (1.05\times10^{6}\ \text{ms}^{-1})(-3\hat{i} + 4\hat{j} + 12\hat{k}),$$

$$F = 1.25\ \text{N}$$

Since $\vec{F} = q(\vec{v}\times\vec{B})$

$$\Rightarrow \vec{F} = q(1.05\times10^{6})\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 12 \\ 0 & 0 & -0.12 \end{vmatrix}$$

$$\Rightarrow \vec{F} = -q(1.26\times10^{5})(4\hat{i} + 3\hat{j}) \qquad \ldots(1)$$

The magnitude of the vector $(4\hat{i} + 3\hat{j})$ is

$$\sqrt{3^2 + 4^2} = 5$$

$$\Rightarrow F = -q(1.26\times10^{5}\ \text{NC}^{-1})(5)$$

$$\Rightarrow q = -\frac{F}{5(1.26\times10^{5}\ \text{NC}^{-1})}$$

$$\Rightarrow q = -\frac{1.25\ \text{N}}{5(1.26\times10^{5}\ \text{NC}^{-1})}$$

$$\Rightarrow q = -1.98\times10^{-6}\ \text{C} = -1.98\ \mu\text{C}$$

(b) Since, $\Sigma\vec{F} = m\vec{a}$, so $\vec{a} = \dfrac{\vec{F}}{m}$.

From equation (1), we have

$$\vec{F} = -q(1.26\times10^{5})(4\hat{i} + 3\hat{j})$$

$$\Rightarrow \vec{F} = -(-1.98\times10^{-6}\ \text{C})(1.26\times10^{5})(4\hat{i} + 3\hat{j})$$

$$\Rightarrow \vec{F} = +0.25(4\hat{i} + 3\hat{j})$$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \left(\frac{0.25}{2.58\times10^{-15}}\right)(4\hat{i} + 3\hat{j})$$

$$\Rightarrow \vec{a} = (9.69\times10^{13})(4\hat{i} + 3\hat{j})\ \text{ms}^{-2}$$

(c) Since, $\vec{F}$ is in the xy-plane, so in the z-direction the particle moves with constant speed $12.6\times10^{6}\ \text{ms}^{-1}$. In the xy-plane the force $\vec{F}$ causes the particle to move in a circle, with $\vec{F}$ directed in towards the centre of the circle.

$$\Sigma\vec{F} = m\vec{a}$$

$$\Rightarrow F = \frac{mv^2}{R}$$

$$\Rightarrow R = \frac{mv^2}{F}, \text{ where } v^2 = v_x^2 + v_y^2$$

$$\Rightarrow v^2 = (-3.15\times10^{6})^2 + (4.2\times10^{6})^2$$

$$\Rightarrow v^2 = 2.756\times10^{13}\ \text{m}^2\text{s}^{-2}$$

$$\Rightarrow v = 5.25\times10^{6}\ \text{ms}^{-1}$$

From equation (1), we have

$$F = \sqrt{F_x^2 + F_y^2} = (0.25)\sqrt{4^2 + 3^2} = 1.25\ \text{N}$$

$$\Rightarrow R = \frac{mv^2}{F} = \frac{(2.58\times10^{-15})(2.756\times10^{13})}{1.25}$$

$$\Rightarrow R = 0.0569\ \text{m} = 5.69\ \text{cm}$$

(d) Since the cyclotron frequency is $f = \dfrac{qB}{2\pi m}$

$$\Rightarrow f = 1.47\times10^{7}\ \text{Hz and}$$

$$\Rightarrow \omega = 2\pi f = 9.23\times10^{7}\ \text{rads}^{-1}$$

(e) Compare t to the period T of the circular motion in the xy-plane to find the x and y coordinates at this t. In the z-direction the particle moves with constant speed, so $z = z_0 + v_z t$.

The period of the motion in the xy-plane is given by

$$T = \frac{1}{f} = \frac{1}{1.47\times10^{7}\ \text{Hz}} = 6.8\times10^{-8}\ \text{s}$$

For $t = 2T$, the particle returns to the same x and y coordinates. The motion along the z-direction is the motion with constant velocity v_z given by

$$v_z = (1.05\times10^{6})(12) = 12.6\times10^{6}\ \text{ms}^{-1}$$

$$\Rightarrow\quad z = z_0 + v_z t$$

$$\Rightarrow\quad z = 0 + (12.6\times10^{6}\ \text{ms}^{-1})(2)(6.8\times10^{-8}\ \text{s})$$

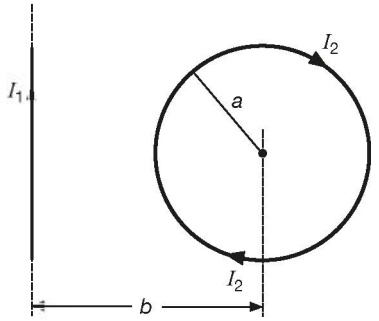
$$\Rightarrow\quad z = +1.71\ \text{m}$$

So, the coordinates at $t = 2T$ are $x = R$, $y = 0$, $z = +1.71$ m i.e., $(R,\ 0,\ 1.71)$ metre

Please note that the circular motion is in the plane perpendicular to $\vec{B}$. The radius of this motion gets smaller when B increases and it gets larger when v increases. There is no magnetic force in the direction of $\vec{B}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the xy-plane and constant speed motion in the z-direction is a helical path.

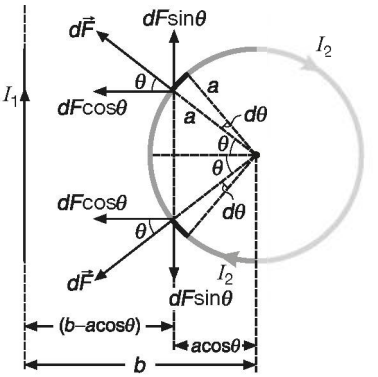
PROBLEM 2

A circular loop of radius a carrying a current I_2 is placed near an infinitely long straight wire carrying a current I_1 as shown. The wire and the loop are coplanar. The distance of the centre of the loop from the wire is b. Find the force of interaction between the wire and the loop.



SOLUTION

For this type of problem, let us divide the circular loop in two equal halves and then proceed. Figure here shows the one half of the loop. We observe that on taking the symmetrical locations of the infinitesimal elements, the sine components cancel. So, the net force on this left half is attractive and equals



$$F_{\text{left half}} = \int dF\cos\theta \qquad \{\text{towards the wire}\}$$

where, $dF = B_1 I_2\,(dl)$

Since $B_1 = \dfrac{\mu_0 I_1}{2\pi r_\perp}$, $r_\perp = b - a\cos\theta$ and $dl = ad\theta$

$$\Rightarrow\quad dF = B_1 I_2\,(dl) = \frac{\mu_0 I_1 I_2\,(ad\theta)}{2\pi(b - a\cos\theta)}$$

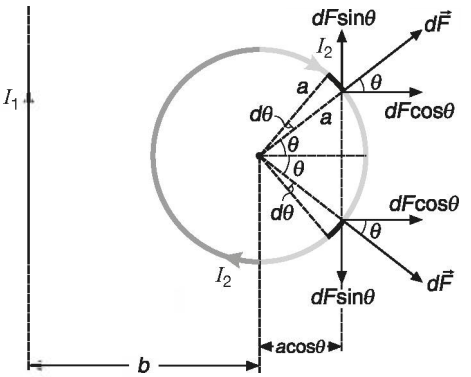
$$\Rightarrow\quad F_{\text{left half}} = F_1 = \frac{\mu_0 I_1 I_2 a}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b - a\cos\theta}$$

$$\Rightarrow\quad F_1 = \frac{\mu_0 I_1 I_2 a}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b - a\cos\theta}$$

$$\Rightarrow\quad F_{\text{left half}} = F_1 = \frac{\mu_0 I_1 I_2 a}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b - a\cos\theta} \qquad \ldots(1)$$

$\{\text{towards the wire}\}$

Similarly, we get for the other half (right half)



$$F_{\text{right half}} = \int dF' \cos\theta \qquad \{\text{away from the wire}\}$$

where $dF' = B_1' I_2 dl$

$$B_1' = \frac{\mu_0 I_1}{2\pi r_\perp'},\ r_\perp' = b + a\cos\theta \text{ and } dl = ad\theta$$

$$\Rightarrow \quad dF' = B_1' I_2 dl = \frac{\mu_0 I_1 I_2 (ad\theta)}{2\pi(b + a\cos\theta)}$$

$$\Rightarrow \quad F_{\text{right half}} = F_2 = \frac{\mu_0 I_1 I_2 a}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(b + a\cos\theta)}$$

$$\Rightarrow \quad F_2 = \frac{\mu_0 I_1 I_2 a}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b + a\cos\theta}$$

$$\Rightarrow \quad F_{\text{right half}} = F_2 = \frac{\mu_0 I_1 I_2 a}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(b + a\cos\theta)} \qquad \ldots(2)$$

{away from wire}

Please note that $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ when $f(x) = f(-x)$, has been used to simplify the integrals (1) and (2).

So, we have

$$\vec{F}_{\text{loop}} = \vec{F}_{\text{left half}} + \vec{F}_{\text{right half}}$$

$$\Rightarrow \quad F_{\text{loop}} = F_1 - F_2 \qquad \{\text{towards the wire}\}$$

where F_1 and F_2 are to be calculated from (1) and (2).

Now $$\int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b - a\cos\theta} = \frac{1}{a}\left(\frac{2b}{\sqrt{b^2 - a^2}}\tan^{-1}\left(\sqrt{\frac{b+a}{b-a}}\right) - \frac{\pi}{2}\right)$$

and $$\int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{b + a\cos\theta} = \frac{1}{a}\left(\frac{\pi}{2} - \frac{2b}{\sqrt{b^2 - a^2}}\tan^{-1}\left(\sqrt{\frac{b-a}{b+a}}\right)\right)$$

$$\Rightarrow \quad F_1 = \frac{\mu_0 I_1 I_2}{\pi}\left(\frac{2b}{\sqrt{b^2 - a^2}}\tan^{-1}\left(\sqrt{\frac{b+a}{b-a}}\right) - \frac{\pi}{2}\right)$$

and $$F_2 = \frac{\mu_0 I_1 I_2}{\pi}\left(\frac{\pi}{2} - \frac{2b}{\sqrt{b^2 - a^2}}\tan^{-1}\left(\sqrt{\frac{b-a}{b+a}}\right)\right)$$

$$\Rightarrow \quad F = F_1 - F_2$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{\pi}\left[\frac{2b}{\sqrt{b^2 - a^2}}\left(\tan^{-1}\sqrt{\frac{b+a}{b-a}} + \tan^{-1}\sqrt{\frac{b-a}{b+a}}\right) - \pi\right]$$

But we know that

$$\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$$

$$\Rightarrow \quad F = F_1 - F_2 = \frac{\mu_0 I_1 I_2}{\pi}\left(\frac{2b}{\sqrt{b^2 - a^2}}\frac{\pi}{2} - \pi\right)$$

$$\Rightarrow \quad F = \mu_0 I I_2\left(\frac{b}{\sqrt{b^2 - a^2}} - 1\right) \qquad \{\text{towards the wire}\}$$

PROBLEM 3

A parallel plate capacitor with area of each plate equal to A and the separation between them to d is put into a stream of conducting liquid with resistivity ρ. The liquid moves parallel to the plates with a constant velocity v. The whole system is located in a uniform magnetic field of induction B, parallel to the plates and perpendicular to the stream direction. The capacitor plates are interconnected by means of an external resistance R.

(a) What amount of power is generated in that resistance?

(b) At what value of R is the generated power the highest?

(c) What is this highest power equal to?

SOLUTION

Resistance of the liquid in between the plates is $\frac{\rho d}{A}$

Voltage applied across the plates is $V = Ed$

For constant velocity of the liquid to be maintained under the simultaneous influence of electric and magnetic field, we must have

$$v = \frac{E}{B}$$

$$\Rightarrow \quad V = (vB)d \qquad \ldots(1)$$

If I be the current through the plates then

$$I = \frac{V}{R_{\text{total}}} = \frac{vBd}{R + \frac{\rho d}{A}}$$

(a) So, power generated in the external resistance R is

$$P = I^2R = \frac{v^2B^2d^2R}{\left(R + \frac{\rho d}{A}\right)^2}$$

(b) $\frac{dP}{dR} = 0$, for P to be MAXIMUM

$$\Rightarrow \frac{d}{dR}\left\{R\left(R + \frac{\rho d}{A}\right)^{-2}\right\} = 0$$

$$\Rightarrow \left(R + \frac{\rho d}{A}\right)^{-2} - 2R\left(R + \frac{\rho d}{A}\right)^{-3} = 0$$

$$\Rightarrow 1 - \frac{2R}{R + \frac{\rho d}{A}} = 0$$

$$\Rightarrow 2R = R + \frac{\rho d}{A}$$

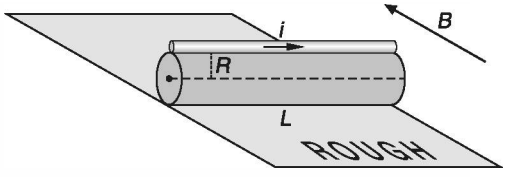
$$\Rightarrow R = \frac{\rho d}{A}$$

(c) $P_{\text{MAX}} = \frac{v^2B^2d^2R}{4R^2} = \frac{v^2B^2d^2}{4R}$ where $R = \frac{\rho d}{A}$

$$\Rightarrow P_{\text{MAX}} = \frac{v^2B^2d^2}{4\left(\frac{\rho d}{A}\right)} = \frac{v^2B^2Ad}{4\rho}$$

PROBLEM 4

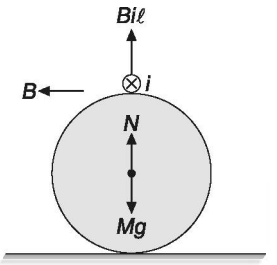
A wooden cylinder of mass M, length L and radius R is placed on a rough horizontal surface. A massless current carrying wire of length L is fixed on its curved surface parallel to its length. There exists a uniform magnetic field B along horizontal as shown in Figure.



Find the various forces acting on the cylinder and draw the free body diagram of cylinder. Assuming that the wooden cylinder does not slip on the ground, find the time period of resulting motion when it is slightly disturbed from the equilibrium position.

SOLUTION

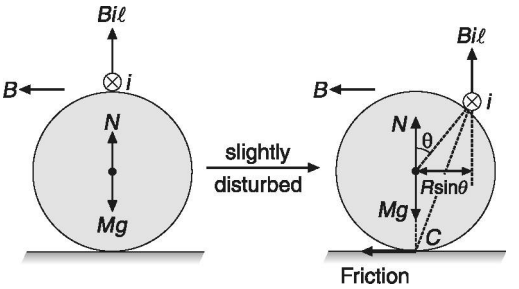
The forces acting on the cylinder are shown in figure.



For equilibrium, we have

$$Mg = N + Bil$$

$$\Rightarrow N = Mg - Bil$$



When there is no slipping at point of contact, C is instantaneous centre of rotation.

Torque about the point C is

$$\tau = (Bil)(R\sin\theta)$$

For small θ, $\sin\theta \approx \theta$

$$\Rightarrow \tau = BilR\theta$$

Since, $\alpha = \frac{\tau}{I}$, where $I = I_{\text{cm}} + MR^2$

$$\Rightarrow \alpha = \frac{BilR\theta}{\left(\frac{1}{2}MR^2 + MR^2\right)}$$

$$\Rightarrow \alpha = \frac{2Bil}{3MR}\theta$$

Also, we see that, the angular displacement θ is clockwise, whereas the angular acceleration $\alpha = \ddot{\theta}$ is counter-clockwise, so

$$\Rightarrow \quad \alpha = -\frac{2BiL}{3MR}\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{2BiL}{3MR}\theta$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{2BiL}{3MR}\right)\theta = 0$$

Comparing with standard equation of SHM i.e., $\ddot{\theta} + \omega^2\theta = 0$, we get

$$\omega^2 = \frac{2BiL}{3MR}$$

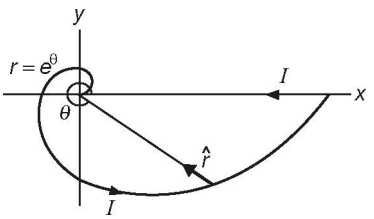
$$\Rightarrow \quad \omega = \sqrt{\frac{2BiL}{3MR}}$$

Since $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3MR}{2BiL}}$$

PROBLEM 5

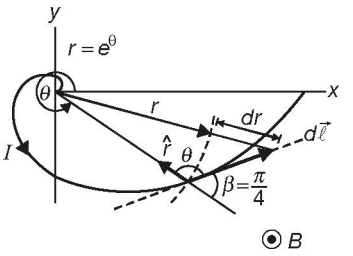
A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$, from $\theta = 0$ to $\theta = 2\pi$ as shown in figure. To complete a loop, the ends of the spiral are connected by a straight wire along the x-axis. Find the magnitude and direction of $\vec{B}$ at the origin.



SOLUTION

Before we start with the problem, we must know and keep in mind that the angle β between a radial line and its tangent line at any point on the curve $r = f(\theta)$ are related to the function as

$$\tan\beta = \frac{r}{\frac{dr}{d\theta}}$$



Thus in this case, we have $r = e^{\theta}$ and so we get $\tan\beta = 1$ and $\beta = \frac{\pi}{4}$. Therefore, the angle between $d\vec{l}$ and $\hat{r}$ is $(\pi - \beta) = \frac{3\pi}{4}$. Also

$$d\vec{l} = \frac{dr}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}dr$$

From Biot-Savart's Law, we know that there is no contribution from the straight portion of the wire since $d\vec{l} \times \vec{r} = 0$. For the field of the spiral, we have

$$dB = \frac{\mu_0 I}{4\pi}\frac{(d\vec{l} \times \hat{r})}{r^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{|d\vec{l}|\sin\theta|\hat{r}|}{r^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\sqrt{2}dr\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}dr = -\frac{\mu_0 I}{4\pi}(r^{-1})\Big|_{\theta=0}^{2\pi}$$

Substitute $r = e^{\theta}$ we get

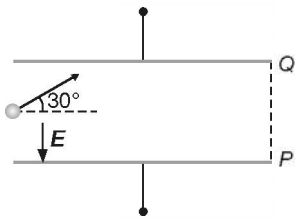
$$B = -\frac{\mu_0 I}{4\pi}(e^{-\theta})\Big|_0^{2\pi} = -\frac{\mu_0 I}{4\pi}(e^{-2\pi} - e^{0})$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}(1 - e^{-2\pi}) \text{ out of the page.}$$

PROBLEM 6

A charged particle having charge 10^{-6} C and mass 10^{-10} kg is fired from the middle of the plate making an angle 30° with plane of the plate. Length of the plate is 0.17 m and it is separated by 0.1 m. An electric

field $E = 10^{-3}$ NC^{-1} is present between the plates and just outside the plates a magnetic field is present. Find the velocity of projection of charged particle and magnitude of the magnetic field perpendicular to the plane of the figure, if it has to graze the plate at Q and P parallel to the surface of the plate. (Neglect gravity).



SOLUTION

The acceleration of the particle is given by

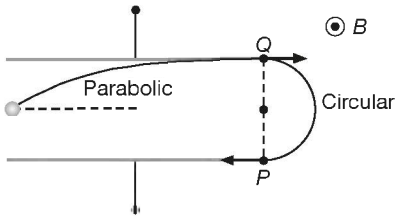
$$a = \frac{qE}{m} = \frac{(10^{-6})(10^{-3})}{(10^{-10})} = 10 \text{ ms}^{-2}, \text{ downwards}$$

Path of the particle is a projectile. The particle will graze at Q when length of the plates equals half the range i.e. $l = \frac{R}{2}$

$$\Rightarrow \quad l = \frac{R}{2} = \frac{u^2 \sin 2\theta}{2g}$$

$$\Rightarrow \quad 0.17 = \frac{u^2 \sin(2\theta)}{2g} = \frac{u^2 \sin 60°}{20}$$

$$\Rightarrow \quad u = 1.98 \text{ ms}^{-1}$$



Speed of particle at Q is,

$$v = u\cos\theta = 1.98\cos 30° = 1.7 \text{ ms}^{-1}$$

It will graze at P if PQ is twice the radius of circular path followed by the particle in magnetic field i.e.

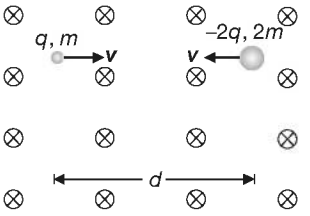
$$PQ = 2R = 2\left(\frac{mv}{qB}\right)$$

$$\Rightarrow \quad 0.1 = 2\left(\frac{mv}{qB}\right)$$

$$\Rightarrow \quad B = \frac{2\times 10^{-10} \times 1.7}{0.1 \times 10^{-6}} = 3.4 \times 10^{-3} \text{ T} = 3.4 \text{ mT}$$

PROBLEM 7

A charged particle $+q$ of mass m is placed at a distance d from another charged particle $-2q$ of mass 2 m in a uniform magnetic field B as shown in figure. If the particles are projected towards each other with equal speeds v.



(a) Find the maximum value of projection speed v_m so that the two particles do not collide.

(b) Find the time after which collision occurs between the particles if projection speed equals $2v_m$.

(c) Assuming the collision to be perfectly inelastic find the radius of particle in subsequent motion. (Neglect the electric force between the charges).

SOLUTION

(a) The particles will move in circular paths, as velocity vector is perpendicular to magnetic field. Time period of both the particles is same $\left(T = \frac{2\pi m}{qB}\right)$. So, for collision not to take place, we must have

$$r_1 + r_2 < d$$

$$\Rightarrow \quad \frac{mv}{qB} + \frac{2mv}{2qB} < d$$

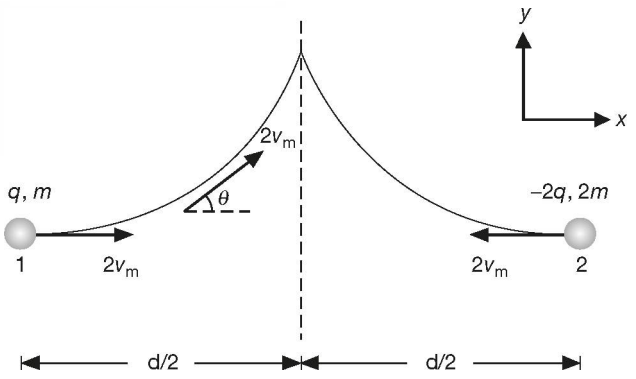
$$\Rightarrow \quad v < \frac{qBd}{2m}$$

Therefore, maximum speed should be $\frac{qBd}{2m}$

$$\Rightarrow \quad v_{\max} = v_m = \frac{qBd}{2m}$$

(b) From symmetry, we observe that they collide at $\frac{d}{2}$.

$$\Rightarrow \theta = \omega t = \left(\frac{qB}{m}\right)t$$



x-component of velocity at time t is,

$$v_x = 2v_m \cos\theta = 2v_m \cos\left(\frac{qB}{m}\right)t$$

$$\Rightarrow \frac{d}{2} = \int_0^t v_x dt = 2v_m \int_0^t \cos\left(\frac{qB}{m}t\right)dt$$

$$\Rightarrow \frac{d}{2} = \frac{2mv_m}{qB}\left[\sin\left(\frac{qB}{m}t\right)\right]\Bigg|_0^t$$

$$\Rightarrow \frac{d}{2} = \frac{2mv_m}{qB}\sin\left(\frac{qB}{m}t\right)$$

Substituting, $v_{max} = v_m = \frac{qBd}{2m}$, we get

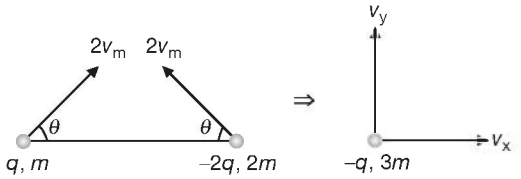
$$\sin\left(\frac{qB}{m}t\right) = \frac{1}{2}$$

$$\Rightarrow \frac{qBt}{m} = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{m\pi}{6qB}$$

(c) At the time of collision

$$\theta = \omega t = \left(\frac{qB}{m}\right)\left(\frac{m\pi}{6qB}\right) = \frac{\pi}{6} = 30^\circ$$



After the collision

$$v_x = \frac{2mv_m\cos 30^\circ - 4mv_m\cos 30^\circ}{3m} = -\frac{v_m}{\sqrt{3}}$$

$$v_y = \frac{2mv_m\sin 30^\circ + 4mv_m\sin 30^\circ}{3m} = v_m$$

So, speed of combined mass is

$$v = \sqrt{v_x^2 + v_y^2} = \frac{2}{\sqrt{3}}v_m$$

$$\Rightarrow R = \frac{(3m)(v)}{qB} = \frac{(3m)\left(\frac{2}{\sqrt{3}}v_m\right)}{qB} = \frac{2\sqrt{3}mv_m}{qB}$$

Substituting, $v_m = \frac{qBd}{2m}$, we get

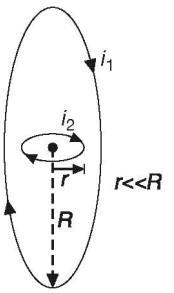
$$R = \sqrt{3}\,d$$

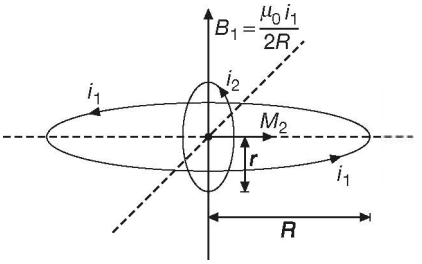
PROBLEM 8

A coil of radius R, mass m_1 carries a current i_1. Another concentric coil of radius $r(\ll R)$, mass m_2 carries a current i_2. The planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Calculate the maximum kinetic energy attained by the two coils, when both are released from rest.

SOLUTION

Let us first draw the arrangement described for a clear visualisation of the problem.





The magnetic field due to coil of bigger radius R is

$$B_1 = \frac{\mu_0 i_1}{2R}$$

This field exerts a torque on smaller coil due to which an equal and opposite torque is exerted by the shorter coil on the larger coil.

Since no external torque is acting on the system, so the angular momentum remains conserved. Also, the total mechanical energy of the system remains conserved.

Now, when the system is released, the magnetic dipole starts rotating to attain equilibrium position and the magnetic energy is converted into kinetic energy of both the coils.

The kinetic energy is maximum when it reaches the equilibrium position or the kinetic energies are maximum when both the coils become coplanar. The two coils rotate due to their mutual interaction only and if one coil rotates clockwise, then the other coil rotates anticlockwise.

Let I_1 and I_2 be the moment of inertia of the respective coils about the axis of rotation. Let ω_1 and ω_2 be the angular velocities of larger and smaller coils when they become coplanar, then by Law of Conservation of Angular Momentum, we have

$$I_1\omega_1 = I_2\omega_2 \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

where, $U = -MB\cos\theta$ and $K = \frac{1}{2}I\omega^2$

$$\Rightarrow \quad \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = U \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{1}{2}I_1\left(\frac{I_2\omega_2}{I_1}\right)^2 + \frac{1}{2}I_2\omega_2^2 = U$$

$$\Rightarrow \quad \frac{1}{2}I_2\omega_2^2\left(\frac{I_2}{I_1}+1\right) = U$$

$$\Rightarrow \quad \frac{1}{2}I_2\omega_2^2 = K_2 = \left(\frac{I_1}{I_1+I_2}\right)U$$

Since, $K_1 + K_2 = U$

$$\Rightarrow \quad \frac{1}{2}I_1\omega_1^2 = K_1 = \left(\frac{I_2}{I_1+I_2}\right)U$$

Also, we have $U = \left(\pi r^2 i_2\right)\left(\frac{\mu_0 i_1}{2R}\right)$

So, maximum kinetic energy of smaller coil is

$$K_2 = \left(\frac{\mu_0\pi r^2 i_1 i_2}{2R}\right)\left(\frac{\frac{1}{2}m_1R^2}{\frac{1}{2}m_1R^2 + \frac{1}{2}m_2r^2}\right)$$

$$\Rightarrow \quad K_2 = \frac{\mu_0\pi r^2 i_1 i_2 m_1 R}{2\left(m_1R^2 + m_2r^2\right)}$$

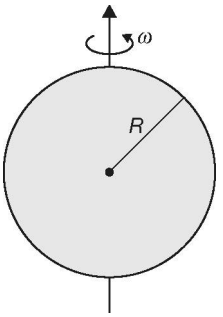
Also, $K_1 = \frac{1}{2}I_1\omega_1^2 = \left(\frac{I_2}{I_1+I_2}\right)U$

$$\Rightarrow \quad K_1 = \left(\frac{\mu_0\pi r^2 i_1 i_2}{2R}\right)\left(\frac{\frac{1}{2}m_2r^2}{\frac{1}{2}m_1R^2 + \frac{1}{2}m_2r^2}\right)$$

$$\Rightarrow \quad K_1 = \frac{\mu_0\pi r^4 i_1 i_2 m_2}{2R\left(m_1R^2 + m_2r^2\right)}$$

PROBLEM 9

A sphere of radius R has a uniform volume charge density ρ. Determine the magnetic field at the centre of the sphere when it rotates as a rigid object with angular speed ω about an axis through its centre. Also find the magnetic dipole moment of the sphere under the situation mentioned. Also express both the results in terms of the charge Q on the sphere. Now if the given sphere is completely embedded in an external magnetic field $\vec{B} = B_0\hat{i}$. Repeat your calculation for the cases when $\vec{B} = B_0\hat{j}$ and $\vec{B} = B_0\hat{k}$.

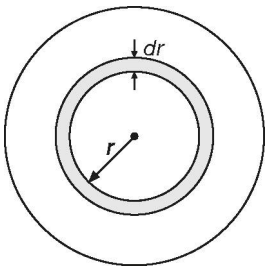


SOLUTION

The sphere can be assumed to be made up of a large number of concentric shells having radii between zero and R. Consider one such shell of radius r thickness dr having a charge dq. Then field contribution of such a shell at the centre is

$$dB = \left(\frac{\mu_0 \omega}{6\pi r}\right) dq$$

{as derived earlier in the case of a shell}



where $dq = (4\pi r^2 dr)\rho$

$$\Rightarrow \quad dB = \left(\frac{\mu_0 \omega}{6\pi r}\right)(4\pi r^2 dr)\rho$$

$$\Rightarrow \quad dB = \frac{2\mu_0 \omega \rho}{3} r\, dr$$

$$\Rightarrow \quad B = \int dB = \frac{2\mu_0 \omega \rho}{3} \int_0^R r\, dr$$

$$\Rightarrow \quad B = \left(\frac{2\mu_0 \omega \rho}{3}\right)\left(\frac{R^2}{2}\right)$$

$$\Rightarrow \quad B = \frac{1}{3}\left(\mu_0 \omega \rho R^2\right) \qquad \{\text{in terms of } \omega,\ \rho \text{ and } R\}$$

Since, we have

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \quad B = \frac{1}{3}\mu_0 \omega \left(\frac{Q}{\frac{4}{3}\pi R^3}\right) R^2$$

$$\Rightarrow \quad B = \frac{\mu_0 \omega Q}{4\pi R} \qquad \{\text{in terms of } \omega,\ Q \text{ and } R\}$$

For calculating the magnetic moment, we can directly take the help of Gyromagnetic Ratio, which is given by

$$\gamma = \frac{|\vec{M}|}{|\vec{L}|} = \frac{Q}{2m}, \text{ where } Q = \text{charge on sphere,}$$

m = mass of sphere

$$\Rightarrow \quad \gamma = \frac{|\vec{M}|}{\left(\frac{2}{5}mR^2\right)\omega} = \frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}| = \left(\frac{2}{5}mR^2\right)\omega\left(\frac{Q}{2m}\right)$$

$$\Rightarrow \quad |\vec{M}| = \frac{1}{5}Q\omega R^2 = \frac{4\pi\rho\omega R^5}{15} \qquad \left\{\because\ \rho = \frac{Q}{\frac{4}{3}\pi R^3}\right\}$$

So, please note that the GYROMAGNETIC RATIO (γ) is a strong tool in your hands that will always help you to calculate the magnetic moment of a body of mass m, charge Q rotating with angular velocity ω.

Now since $\vec{\tau} = \vec{M} \times \vec{B}$ where $\vec{M} = \left(\frac{1}{5}Q\omega R^2\right)\hat{j}$ and $\vec{B} = B_0 \hat{i}$

$$\Rightarrow \quad \vec{\tau} = \left(\frac{1}{5}QB_0\omega R^2\right)(-\hat{k}) \qquad \{\because\ \hat{j} \times \hat{i} = -\hat{k}\}$$

Similarly, for the other two cases, we get

$$\vec{\tau} = \vec{0} \text{ and } \vec{\tau} = \left(\frac{1}{5}QB_0\omega R^2\right)\hat{i}$$

PROBLEM 10

A very long straight conductor of circular cross-section with radius R has current density $J(r)$ directed into the plane of paper (along negative z direction) and varying with radial distance r as

$$J(r) = \begin{cases} \text{zero} & \text{for } r < \frac{R}{2} \\ \rho_0 \dfrac{r^2}{R^2} & \text{for } \frac{R}{2} \le r \le R \end{cases}$$

where ρ_0 is a positive constant. Assuming the origin to lie on the axis of the very long straight conductor, consider a point $P(0,a,0)$ outside the conductor i.e. $a > R$. If two infinitely long thin conducting wires each carrying current I_0 in the same direction and parallel to the axis of the conductor are placed at $(a,0,0)$ and $(-a,0,0)$ such that the magnetic field at P is zero, then calculate the current I_0 in the thin wires and the direction of current with respect to the very long straight conductor.

SOLUTION

Current through the very long straight conductor is given by

$$I = \int_0^R J dA \text{, where } dA = 2\pi r dr$$

$$\Rightarrow \quad I = \int_0^{\frac{R}{2}} 2\pi r J(r)\,dr + \int_{\frac{R}{2}}^{R} 2\pi r J(r)\,dr$$

$$\Rightarrow \quad I = \int_0^{\frac{R}{2}} 2\pi r(0)\,dr + \int_{\frac{R}{2}}^{R} 2\pi r \rho_0 \frac{r^2}{R^2} dr$$

$$\Rightarrow \quad I = 0 + \frac{2\pi\rho_0}{R^2}\int_{\frac{R}{2}}^{R} r^3 dr = \frac{2\pi\rho}{R^2}\left(\frac{r^4}{4}\right)\Bigg|_{\frac{R}{2}}^{R}$$

$$\Rightarrow \quad I = \frac{2\pi\rho_0}{4R^2}\left(R^4 - \frac{R^4}{16}\right) = \frac{15}{32}\pi\rho_0 R^2$$

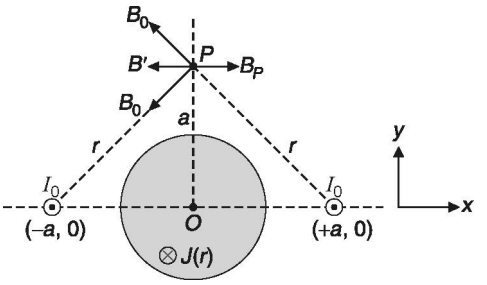
For calculating the magnetic field at P due to the very long straight conductor, let us consider an Amperian loop of radius a having centre at the axis of the conductor, then for all points on the Amperian loop, the magnetic field B has the same value. Applying Ampere's Circuital Law, we get

$$B(2\pi a) = \mu_0 I$$

$$\Rightarrow \quad B(2\pi a) = \mu_0\left(\frac{15}{32}\pi\rho_0 R^2\right)$$

$$\Rightarrow \quad B = B_P = \frac{15}{64}\frac{\mu_0\rho_0 R^2}{a}\text{, along } +x \text{ direction}$$

The visualisation to the situation given in the problem is shown in Figure.



At P, field due to the conductor is directed along positive x direction. Field due to wires A_1 and A_2 must cancel B, which is possible only when the current I_0 in each wire is outwards along positive z axis. Field at P due to wire A_1 is

$$|\vec{B}_1| = B_0 = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 I_0}{2\pi(a\sqrt{2})}$$

Field at P due to wire A_2 is

$$|\vec{B}_2| = B_0 = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 I_0}{2\pi(a\sqrt{2})}$$

Resultant of these two fields B_1 and B_2 (i.e. B') is along negative x direction (i.e. opposite to B_P). So, we have

$$B' = 2B_0\cos(45°) = 2\left(\frac{\mu_0 I_0}{2\pi(a\sqrt{2})}\right)\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad B' = \frac{\mu_0 I_0}{2\pi a}\text{, along } -x \text{ direction}$$

Since B' is acting opposite to B_P, so net field at P will be zero when we have both B' and B_P equal in magnitude.

$$\Rightarrow \quad B' = B_P$$

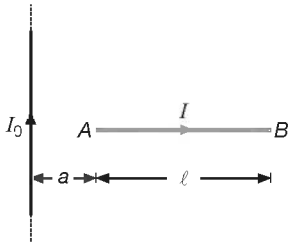
$$\Rightarrow \quad \frac{\mu_0 I_0}{2\pi a} = \frac{15}{64}\frac{\mu_0\rho_0 R^2}{a}$$

$$\Rightarrow \quad I_0 = \frac{15}{32}\pi\rho_0 R^2$$

PROBLEM 11

A finite conductor AB carrying current I is placed near a fixed very long current carrying wire I_0 as shown in the figure. Find the point of application and magnitude of the net ampere force on the conductor AB.

What happens to the conductor AB if it is free to move? (Neglect gravitational field).



SOLUTION

Consider an element of rod AB of length dx at a distance x from the wire. The field at this element is

$$B = \frac{\mu_0 I_0}{2\pi x}$$

$$|d\vec{F}| = BIdx = \frac{\mu_0 I_0 I}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \int dF = \frac{\mu_0 I I_0}{2\pi}\int_a^{a+l}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 I I_0}{2\pi}\log_e\left(\frac{a+l}{a}\right) \qquad \{\text{in the direction of } I_0\}$$

For finding the point of application of force we shall make use of the following mathematical formula, according to which

$$r = \frac{\int x\,dF}{F} \text{ where } F = \frac{\mu_0 I I_0}{2\pi}\log_e\left(\frac{a+l}{a}\right)$$

Consider an element of length dx at a distance x from wire I_0.

Then $dF = B(x)Idx$ where $B(x) = \dfrac{\mu_0 I_0}{2\pi x}$

$$\Rightarrow \quad r = \frac{\int_a^{a+l} x\left(\frac{\mu_0 I_0}{2\pi x}\right)Idx}{F}$$

$$\Rightarrow \quad r = \frac{\left(\frac{\mu_0 I I_0}{2\pi}\right)l}{\left(\frac{\mu_0 I I_0}{2\pi}\right)\log_e\left(\frac{a+l}{a}\right)}$$

$$\Rightarrow \quad r = \frac{l}{\log_e\left(\frac{a+l}{a}\right)}$$

Please note that the formula used $r = \dfrac{\int xdF}{F}$ is just another way of expressing the "Law of Conservation of Moments of force".

PROBLEM 12

A solid sphere of radius R carries a uniform charge density $+\rho$ from $r = 0$ to $\dfrac{R}{2}$ and an equal charge density of opposite sign $-\rho$, from $\dfrac{R}{2}$ to R. If the sphere rotates about its diameter with some uniform angular velocity ω, then calculate the magnetic moment of the sphere. Assume that the mass is distributed uniformly on the sphere.

SOLUTION

The solid sphere can be visualized to be made up of a number of infinitesimal concentric spherical shells. Consider one such shell of charge dq, thickness dr, radius r having magnetic moment dM, then due to the rotation of the sphere with uniform angular speed ω, the magnetic moment is given by

$$dM = \frac{1}{3}(dq)\omega r^2$$

where, $dQ = \rho dV = \rho(4\pi r^2 dr)$

$$\Rightarrow \quad dM = \frac{1}{3}(dq)\omega r^2 = \frac{1}{3}(4\pi r^2\rho dr)\omega r^2$$

Magnetic moment of solid portion between $r = 0$ to $r = \dfrac{R}{2}$ is obtained by integrating the above expression. So, we have

$$M_1 = \int dM = \int_0^{R/2}\frac{1}{3}(4\pi r^2\rho dr)\omega r^2$$

$$\Rightarrow \quad M_1 = \frac{4}{3}\pi\rho\omega\int_0^{R/2} r^4 dr = \frac{4}{3}\pi\rho\omega\left(\frac{r^5}{5}\right)\Bigg|_0^{\frac{R}{2}}$$

$$\Rightarrow \quad M_1 = \frac{4}{15}\pi\rho\omega\left(\frac{R^5}{32}\right) = \frac{1}{120}\pi\rho\omega R^5$$

Similarly, the magnetic moment of solid portion of charge density $-\rho$ from $r=\frac{R}{2}$ to $r=R$ is

$$M_2=\int d\mu=-\int_{R/2}^{R}\frac{1}{3}\left(4\pi r^2\rho dr\right)\omega r^2$$

$$\Rightarrow \quad \mu_2=-\frac{4}{3}\pi\rho\omega\left(\frac{r^5}{5}\right)\Bigg|_{\frac{R}{2}}^{R}$$

$$\Rightarrow \quad M_2=-\frac{4}{15}\pi\rho\omega R^5\left(1-\frac{1}{32}\right)$$

$$\Rightarrow \quad M_2=-\frac{31}{120}\pi\rho\omega R^5$$

So, net magnetic moment is given by

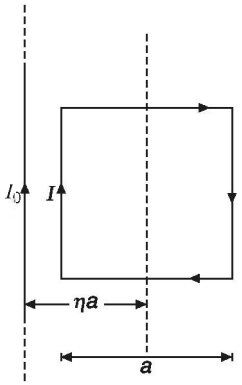
$$M_{\text{net}}=M_1+M_2$$

$$\Rightarrow \quad M_{\text{net}}=-\frac{1}{4}\pi\rho\omega R^5$$

Negative sign implies that the net magnetic moment is directed opposite to the direction of ω.

PROBLEM 13

A square frame carrying a current I is located in the same plane as a long straight wire carrying a current I_0. The frame side has a length a. The axis of the frame passing through the midpoints of the opposite sides is parallel to the wire and is separated from it by the distance which is η times greater than the side of the frame as shown in Figure.



Calculate the force acting on the frame. Also calculate the mechanical work to be performed in order to turn the frame through 180° about its axis, assuming the currents to be constant throughout the process of turning the frame.

SOLUTION

Force of attraction between wires carrying currents in the same direction is

$$F_1=\frac{\mu_0 I I_0}{2\pi\left(\eta a-\frac{a}{2}\right)}=\frac{\mu_0 I I_0}{\pi(2\eta-1)a}$$

Similarly, force of repulsion between wires carrying currents in the opposite direction is

$$F_2=\frac{\mu_0 I I_0}{2\pi\left(\eta a+\frac{a}{2}\right)}=\frac{\mu_0 I I_0}{\pi(2\eta+1)a}$$

Net force of attraction between the square frame and the long straight wire is

$$F=F_1-F_2$$

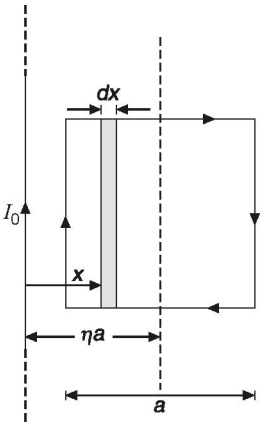
$$\Rightarrow \quad F=\frac{\mu_0 I I_0}{\pi(2\eta-1)a}-\frac{\mu_0 I I_0}{\pi(2\eta+1)a}$$

$$\Rightarrow \quad F=\frac{2\mu_0 I I_0}{\pi\left(4\eta^2-1\right)}$$

Since the work to be done in turning the frame through 180° is given by

$$W=-I\Delta\phi=-I(-\phi-\phi)=2I\phi \qquad \ldots(1)$$

where ϕ is the magnetic flux associated with the coil due to the magnetic field of the wire placed near it.
To calculate the magnetic flux associated with the square loop let us consider an infinitesimal strip element of length a, width dx at a distance x in the coil as shown in Figure.



The magnetic field B at the strip due to the infinite wire is

$$B = \frac{\mu_0 I_0}{2\pi x}$$

The magnetic flux associated with this infinitesimal strip is

$$d\phi = BdA$$

where $dA = adx$ is the area of the strip.

$$\Rightarrow \quad d\phi = \left(\frac{\mu_0 I_0}{2\pi x}\right) adx$$

Total magnetic flux associated with the square loop is obtained by integrating $d\phi$. So, we get

$$\phi = \int d\phi = \int_{\eta a - \frac{a}{2}}^{\eta a + \frac{a}{2}} \left(\frac{\mu_0 I_0}{2\pi x}\right) adx$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I_0 a}{2\pi} \int_{\eta a - \frac{a}{2}}^{\eta a + \frac{a}{2}} \frac{dx}{x}$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I_0 a}{2\pi} (\ln x)\Big|_{\eta a - \frac{a}{2}}^{\eta a + \frac{a}{2}}$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I_0 a}{2\pi}\left[\ln\left(\eta a + \frac{a}{2}\right) - \ln\left(\eta a - \frac{a}{2}\right)\right]$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I_0 a}{2\pi} \ln\left(\frac{2\eta + 1}{2\eta - 1}\right)$$

From equation (1), the work done to rotate the coil is given by

$$W = 2I\phi$$

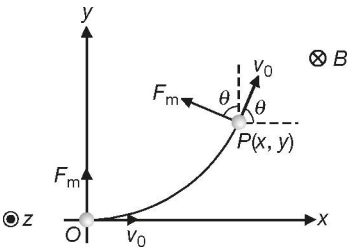
$$\Rightarrow \quad W = \frac{\mu_0 I I_0 a}{\pi} \ln\left(\frac{2\eta + 1}{2\eta - 1}\right)$$

PROBLEM 14

A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = v_0 \hat{i}$ in a nonuniform magnetic field $\vec{B} = -(B_0 x)\hat{k}$. Here v_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x coordinate of the particle during its motion.

SOLUTION

Magnetic field is along negative z-direction. So, in the coordinate axes shown in figure, it is perpendicular to paper inwards. $(\otimes)$ magnetic force on the particle at origin is along positive y-direction. So, it will rotate in x-y plane as shown. The path is not a perfect circle as the magnetic field is nonuniform. Speed of the particle in magnetic field remains constant.



Magnetic force is always perpendicular to velocity. Let at point $P(x, y)$ its velocity vector makes an angle θ with positive x-axis. Then magnetic force $\vec{F}_m$ will be at angle θ with positive y-direction. So,

$$a_y = \left(\frac{F_m}{m}\right)\cos\theta$$

$$\Rightarrow \quad \frac{dv_y}{dt} = \frac{(B_0 x)(qv_0 \cos\theta)}{m} \qquad \{\because F_m = Bqv_0 \sin 90^\circ\}$$

$$\Rightarrow \quad \left(\frac{dv_y}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{B_0 qx}{m}\right)(v_0 \cos\theta) \qquad \ldots(1)$$

where $\frac{dx}{dt} = v_x = v_0 \cos\theta$

From (1), we get

$$\frac{dv_y}{dx} = \left(\frac{B_0 q}{m}\right) x$$

$$\Rightarrow \quad \int_0^{v_0} dv_y = \left(\frac{B_0 q}{m}\right) \int_0^{x_{max}} x dx$$

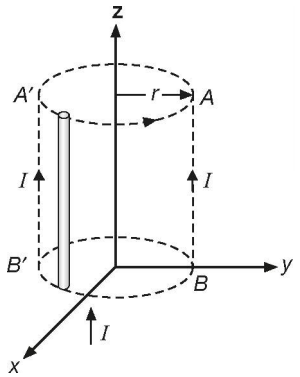
$$\Rightarrow \quad v_0 = \left(\frac{B_0 q}{m}\right)\left(\frac{x_{max}^2}{2}\right)$$

$$\Rightarrow \quad x_{max} = \sqrt{\frac{2mv_0}{B_0 q}}$$

Note that here, when the x displacement is maximum, then the velocity is along positive y direction.

PROBLEM 15

Find the work and power required to move the conductor of length l shown in the figure through one full turn in the clockwise direction at a rotational frequency of n revolutions per second if the magnetic field is of magnitude B_0 everywhere and points radially outwards from Z-axis. The figure shows the surface traced by the wire AB.



SOLUTION

Since we know that

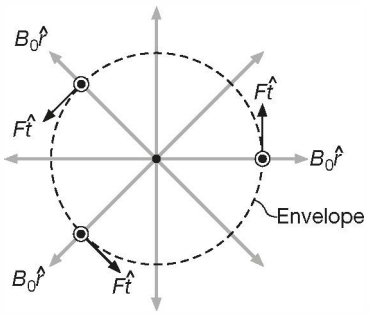
$$\vec{F} = I(\vec{l} \times \vec{B})$$

where $\vec{Il} = (Il)\hat{k}$ and $\vec{B} = B_0\hat{r}$

where $\hat{r}$ is a unit vector directed radially outwards.

$$\Rightarrow \quad \vec{F} = (IlB_0)\hat{t}$$

where we have $\hat{k} \times \hat{r} = \hat{t}$, with $\hat{t}$ directed towards the tangential direction.



View From the Top
$\hat{r}$ = Unit Radial Vector
$\hat{t}$ = Unit Tangential Vector

If dW is the work done by the external force to turn the conductor through angle $d\theta$ in the clockwise sense, then

$$dW = \vec{F} \cdot d\vec{x}$$

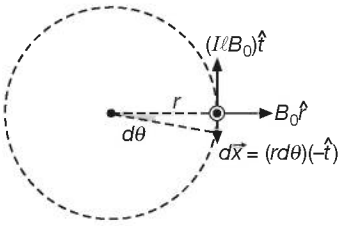
$$\Rightarrow \quad dW = (IlB_0)\hat{t} \cdot rd\theta(-\hat{t}) \qquad \{\because\ d\vec{x} = rd\theta(-\hat{t})\}$$

$$\Rightarrow \quad W = \int dW = -IlB_0 r \int_0^{2\pi} d\theta$$

$$\Rightarrow \quad W = -IlB_0(2\pi r) = -(2\pi r)(B_0 Il)$$

The negative sign shows that the field does work. Also, work done a round a closed loop is non-zero thus showing that the force doing this work is non-conservative in nature.

$$\text{Power} = \frac{\text{Work Done}}{\text{Time of one Revolution}} = \frac{W}{\left(\frac{2\pi}{\omega}\right)}$$



Since $\omega = n$ revolution per second

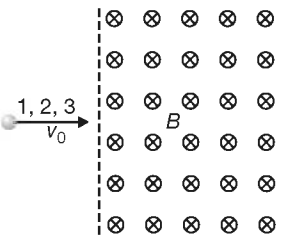
$$\Rightarrow \quad \omega = 2\pi n \text{ rads}^{-1}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = \left(\frac{1}{n}\right) \text{ second}$$

$$\Rightarrow \quad P = -\frac{2\pi r(B_0 Il)}{\left(\frac{1}{n}\right)} = -2\pi r n B_0 Il$$

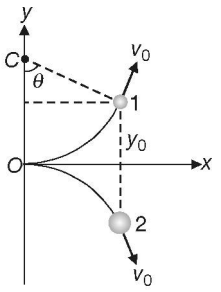
PROBLEM 16

Three charged particles 1, 2 and 3 of mass m, $2m$ and $3m$ having charges q, $-2q$ and $6q$ are projected into a magnetic field B as shown in figure. Find the distance between the particles 1, 2 and 1, 3 after time t, if at $t = 0$ they enter into the region of magnetic field. Ignore electric and magnetic effects due to charges on themselves.



SOLUTION

Distance between 1 and 2:



Since, $r_1 = \frac{mv_0}{qB}$

$$r_2 = \frac{2mv_0}{2qB} = \frac{mv_0}{qB}$$

Also, $\omega_1 = \frac{qB}{m}$ and $\omega_2 = \frac{2qB}{2m} = \frac{qB}{m}$

$$\Rightarrow \quad S_{12} = 2y_0 = 2r_1(1-\cos\theta) = 2r_1(1-\cos\omega_1 t)$$

$$\Rightarrow \quad S_{12} = \frac{2mv_0}{qB}\left(1-\cos\left(\frac{qBt}{m}\right)\right)$$

Distance between 1 and 3:

Since, $r_3 = \frac{3mv_0}{6qB} = \frac{mv_0}{2qB}$ and $\omega_3 = \frac{6qB}{3m} = \frac{2qB}{m}$

$$\Rightarrow \quad S_{13} = \sqrt{(x_1-x_3)^2+(y_1-y_3)^2}$$

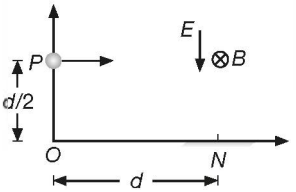
$$\Rightarrow \quad S_{13} = \sqrt{\left[r_1\sin(\omega_1 t)-r_3\sin(\omega_3 t)\right]^2 + \left[r_1(1-\cos(\omega_1 t))-r_3(1-\cos(\omega_3 t))\right]^2}$$

Substituting the values, we get

$$S_{13} = \frac{mv_0}{qB}\sqrt{\left\{\sin\left(\frac{qBt}{m}\right)-\frac{1}{4}\sin\left(\frac{2qBt}{m}\right)\right\}^2 + \left\{\frac{3}{4}+\cos\left(\frac{2qBt}{m}\right)-\cos\left(\frac{qBt}{m}\right)\right\}^2}$$

PROBLEM 17

A particle of mass m and charge q enters a region of electric field $\vec{E}$ as shown in the figure with some velocity at point P. At the moment the particle collides elastically with smooth surface at N, the electric field $\vec{E}$ is switched off and a magnetic field $\vec{B}$ perpendicular to the plane of paper automatically switched on. If the particle hits the surface at point O, then prove that $B = 2\sqrt{\frac{mE}{qd}}$.



SOLUTION

If t is the time taken to reach point N, then

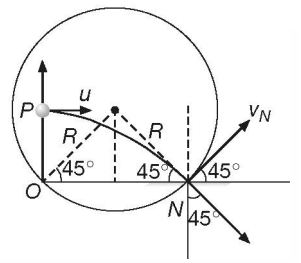
$$\frac{d}{2} = \frac{1}{2}\left(\frac{qE}{m}\right)\times t^2$$

$$\Rightarrow \quad t = \sqrt{\left(\frac{m}{qE}\right)d}$$

If u is the velocity while entering the field, then

$$d = u\times t$$

$$\Rightarrow \quad u = \frac{d}{t} = \sqrt{\frac{qEd}{m}}$$



also if $v_{N\perp}$ is the velocity of the particle in the direction of $\vec{E}$ at N, then

$$v_{N\perp} = \sqrt{2\times\left(\frac{qE}{m}\right)\times\frac{d}{2}} = \sqrt{\frac{qEd}{m}}$$

So, net velocity at N is

$$v_N = \sqrt{v_{N\perp}^2+u^2} = \sqrt{\frac{2qEd}{m}}$$

Since $\tan\theta = \frac{v_{N\perp}}{u} = 1$

$$\Rightarrow \quad \theta = 45^\circ$$

Hence, the collision is elastic and the particle is reflected back with the same speed v_N

The charge q now moves with a speed v_N in magnetic field $\vec{B}$. Radius of its path

$$R = \frac{mv_N}{qB} = \left(\sqrt{\frac{2mEd}{q}}\right)\times\frac{1}{B} \qquad \ldots(1)$$

From figure $R = \frac{d}{2}\times\sqrt{2} = \frac{d}{\sqrt{2}}$

Putting in (1), $\frac{d}{\sqrt{2}} = \frac{1}{B}\sqrt{\frac{2mEd}{q}}$

$$\Rightarrow \quad B = 2\sqrt{\frac{mE}{qd}}$$

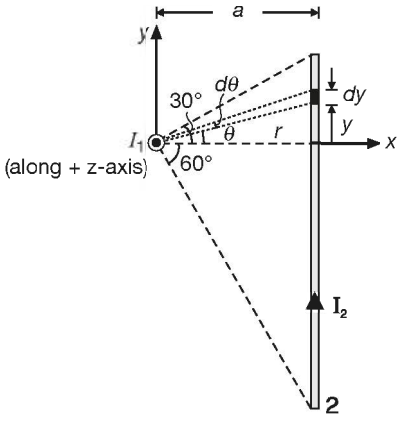
PROBLEM 18

An infinite wire 1, placed along z-axis, has current I_1 in positive z-direction. Another wire 2 is placed parallel to y-axis and carries a current I_2 in positive y-direction. The ends of the wire 2 subtend $+30°$ and $-60°$ at the origin with positive x-direction. The wire 2 is at a distance a from the origin. Find net force on the wire 2.

SOLUTION

The arrangement is shown here.
Please note that the direction of I_1 and the $+z$-axis, both are outwards towards the reader, denoted by $\odot$.
Let us consider an infinitesimal element of length dy at a distance y from x-axis. If $\vec{B}$ be the field due to the wire 1 at the element is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1}{2\pi\sqrt{a^2+y^2}}$$



The force on this element due to the field of wire 1 is

$$d\vec{F} = I\left(d\vec{l}\times\vec{B}\right)$$

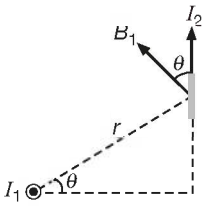
$$\Rightarrow \quad d\vec{F} = I_2\left(d\vec{y}\times\vec{B}_1\right)$$

$$\Rightarrow \quad |d\vec{F}| = dF = \left(I_2 dy\right)\left(\frac{\mu_0 I_1}{2\pi\sqrt{a^2+y^2}}\right)\sin\theta$$

$$\Rightarrow \quad dF = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{\sin\theta}{\sqrt{a^2+y^2}}\right)dy$$

{along positive z-direction, using Right Hand Thumb Rule}

But $\sin\theta = \dfrac{y}{\sqrt{a^2+y^2}}$



$$\Rightarrow \quad dF = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{ydy}{a^2+y^2}\right)$$

$$F = \frac{\mu_0 I_1 I_2}{2\pi}\int_{-\sqrt{3}a}^{\frac{a}{\sqrt{3}}}\frac{ydy}{a^2+y^2}$$

Since, $\displaystyle\int\frac{ydy}{a^2+y^2} = \frac{1}{2}\log_e\left(a^2+y^2\right)$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\left[\log_e\left(a^2+y^2\right)\right]_{-\sqrt{3}a}^{\frac{a}{\sqrt{3}}}$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left(\frac{a^2+\frac{a^2}{3}}{a^2+3a^2}\right)$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad F = -\frac{\mu_0 I_1 I_2}{4\pi}\log_e(3)$$

{Along negative z-direction}

In this particular problem, we must observe one thing that though the magnetic force on the infinitesimal element is along $+z$ axis, but the net force on the complete wire 2 is along $-z$ axis. This is quite obvious because the lower current carrying portion has a length more than the upper portion and so the extra unsymmetrical portion contributes to the net force along negative z-direction.

PROBLEM 19

A charged particle of mass m and charge q is projected on a rough horizontal X-Y plane with z-axis in vertically upward direction. Both electric and magnetic fields are acting in the region and given by $\vec{E} = -E_0\hat{k}$ and $\vec{B} = -B_0\hat{k}$ respectively. The particle enters into the field at $(a_0, 0, 0)$ with velocity $\vec{v} = v_0\hat{j}$. The particle starts moving into a circular path on the plane. If coefficient of friction between particle and the plane is μ. Then calculate the

(a) time when the particle will come to rest
(b) time when the particle will hit the centre
(c) distance travelled by the particle when it comes to rest.

SOLUTION

(a) $N = mg + qE_0$ ...(1)

$$qvB_0 = \frac{mv^2}{R} \quad \text{...(2)}$$

and $$-m\frac{dv}{dt} = \mu N \quad \text{...(3)}$$

From equation (2)

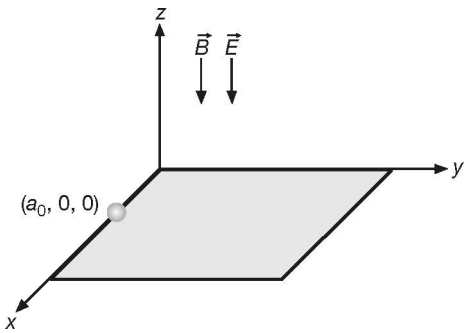
$$R = \frac{mv}{qB_0} \quad \text{...(4)}$$

From equation (1) and (3)

$$-m\frac{dv}{dt} = \mu(mg + qE_0)$$

$$\Rightarrow -m\int_{v_0}^{0} dv = -\mu(mg + qE_0)\int_0^t dt$$

$$t = \frac{mv_0}{\mu(mg + qE_0)}$$



(b) From equation (4)

$$dR = \frac{m}{qB_0}dv = -\frac{\mu(mg + qE_0)dt}{qB_0}$$

$$\Rightarrow \int_{R_i}^{0} dR = \frac{-\mu(mg + qE_0)}{qB_0}\int_0^t dt$$

$$\Rightarrow t = \frac{qB_0R_i}{\mu(mg + qE_0)}$$

Here, $R_i = \dfrac{mv_0}{qB_0}$

$$\Rightarrow t = \frac{mv_0}{\mu(mg + qE_0)}$$

We observe that both the times are equal.

(c) $$-mv\frac{dv}{dt} = \mu(mg + qE_0)$$

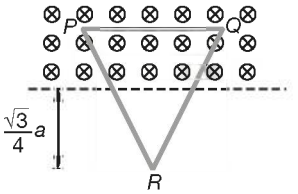
$$\Rightarrow -m\int_{v_0}^{0} vdv = \mu(mg + qE_0)\int_0^t dl$$

$$\Rightarrow \frac{mv_0^2}{2} = \mu(mg + qE_0)l$$

$$\Rightarrow l = \frac{mv_0^2}{2\mu(mg + qE_0)}$$

PROBLEM 20

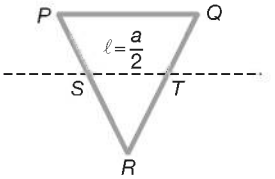
An equilateral triangular frame PQR of mass M and side a is at rest is under the influence of horizontal magnetic field B and gravitational field as shown in the figure.



(a) Find the magnitude and direction of current in the frame so that the frame remains at rest.
(b) If the frame is slightly displaced in its plane perpendicular to PQ, show that its motion is simple harmonic and find its period of oscillation.
(Neglect EMF induced due to motion of the loop)

SOLUTION

(a) The current I in the loop should be clockwise.



Now for the equilibrium of the loop

$$Mg = I(l)B$$

$$\Rightarrow \quad Mg = I \times \frac{a}{2} \times B$$

$$\Rightarrow \quad I = \frac{2Mg}{aB}$$

(b) Now if the loop is displaced downward by distance x

Restoring force

$$F = -\left[I(S'T')B - Mg \right]$$

$$\Rightarrow \quad F = -I\left[\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{4}a + x \right) \right] B + Mg$$

$$\Rightarrow \quad F = -\frac{IaB}{2} - \frac{2IB}{\sqrt{3}}x + Mg$$

$$\Rightarrow \quad F = -\left(\frac{2IB}{\sqrt{3}} \right) x$$

$$\Rightarrow \quad ma = -\left(\frac{2IB}{\sqrt{3}} \right) x$$

$$\Rightarrow \quad m\ddot{x} + \left(\frac{2IB}{\sqrt{3}} \right) x = 0$$

Hence motion is SHM comparing with standard equation of SHM, we get $\ddot{x} + \omega^2 x = 0$

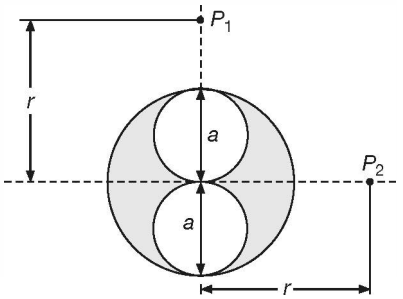
$$\Rightarrow \quad \ddot{x} + \left(\frac{2IB}{\sqrt{3}M} \right) x = 0$$

$$\Rightarrow \quad \omega^2 = \frac{2IB}{\sqrt{3}M}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\sqrt{3}M}{2IB}} = \pi\sqrt{\frac{\sqrt{3}a}{g}}$$

PROBLEM 21

A long cylindrical conductor of radius a has two cylindrical cavities of diameter a through its entire length, as shown in figure.



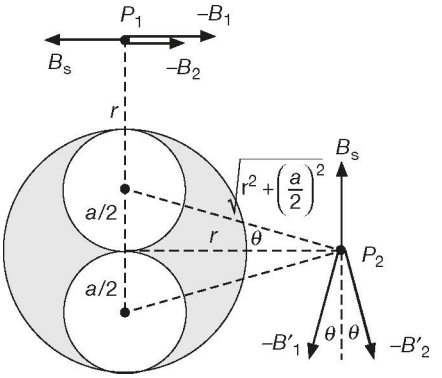
A current I is directed out of the page and is uniform through a cross section of the conductor. Find the magnitude and direction of the magnetic field in terms of μ_0, I, r and a at points P_1 and P_2.

SOLUTION

Note that the current I exists in the conductor with a current density $J = \frac{I}{A}$, where

$$A = \pi\left(a^2 - \frac{a^2}{4} - \frac{a^2}{4} \right) = \frac{\pi a^2}{2}$$

$$\Rightarrow \quad J = \frac{2I}{\pi a^2}$$



To find the field at either point P_1 or P_2 we follow the following steps.

STEP-1:

Find B_s which would exist if the conductor were solid, using Ampere's Law.

STEP-2:

Find B_1 and B_2 that would be due to the conductors of radius $\frac{a}{2}$ that could occupy the void where the holes exist.

STEP-3:

Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

At point P_1

$$B_s = \frac{\mu_0 J(\pi a^2)}{2\pi r}$$

$$B_1 = \frac{\mu_0 I \pi \left(\frac{a}{2}\right)^2}{2\pi\left(r - \frac{a}{2}\right)} \text{ and}$$

$$B_2 = \frac{\mu_0 I \pi \left(\frac{a}{2}\right)^2}{2\pi\left(r + \frac{a}{2}\right)}$$

$$\Rightarrow \quad B = B_s - B_1 - B_2$$

$$\Rightarrow \quad B = \frac{\mu I \pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4\left(r - \frac{a}{2}\right)} - \frac{1}{4\left(r + \frac{a}{2}\right)}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 (2I)}{2\pi}\left[\frac{4r^2 - a^2 - 2r^2}{4r\left(r^2 - \frac{a^2}{4}\right)}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right], \text{ directed to the left}$$

At point P_2

$$B_s = \frac{\mu_0 I (\pi a^2)}{2\pi r}$$

and $B_1' = B_2' = \dfrac{\mu_0 I \pi \left(\frac{a}{2}\right)^2}{2\pi\sqrt{r^2 + \left(\frac{a}{2}\right)^2}}$

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$\Rightarrow \quad B = B_s - B_1' \cos\theta - B_2' \cos\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 I (\pi a^2)}{2\pi r} - 2\left(\frac{\frac{\mu_0 I \pi a^2}{4}}{2\pi\sqrt{r^2 + \left(\frac{a^2}{4}\right)}}\right)\frac{r}{\sqrt{r^2 + \left(\frac{a^2}{4}\right)}}$$

$$\Rightarrow \quad B = \frac{\mu_0 I (\pi a^2)}{2\pi r}\left[1 - \frac{r^2}{2\left(r^2 + \left(\frac{a^2}{4}\right)\right)}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 (2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2 + a^2}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right]$$

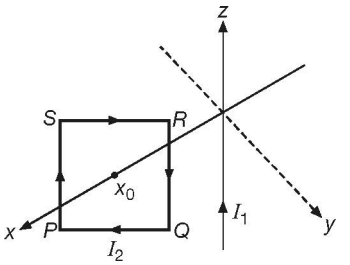
{directed toward the top of the page}

PROBLEM 22

Current I_1 flows along the positive z-direction in a long thin wire. Another current I_2, flows in a rectangularly shaped wire whose centre lies at $(x_0, 0, 0)$ and whose vertices are located at the points $P(x_0 + d, -a, -b)$, $Q(x_0 - d, a, -b)$, $R(x_0 - d, a, +b)$ and $S(x_0 + d, -a, +b)$ respectively as shown in the figure. Assume that a, b, $d \ll x_0$.

(a) Find the magnetic dipole moment vector of the rectangular wire.

(b) Find the force exerted on the rectangular coil by the magnetic field generated by I_1.



SOLUTION

(a) The magnetic moment of the rectangular coil is

$$\vec{M} = I_2 \vec{A}$$

where $\vec{A} = \overrightarrow{QP} \times \overrightarrow{PS}$

here $\overrightarrow{QP} = 2d\hat{i} - 2a\hat{j}$ and $\overrightarrow{PS} = 2b\hat{k}$

$$\Rightarrow \quad \vec{A} = 2\left(d\hat{i} - a\hat{j}\right) \times 2b\hat{k}$$

$$\Rightarrow \quad M = -4bI_2\left(d\hat{j} + a\hat{i}\right)$$

(b) The magnetic field caused by I_1 at the centre $(x_0, 0, 0)$ of rectangular coil is

$$\vec{B} = \left(\frac{\mu_0}{2\pi}\frac{I_1}{x_0}\right)\hat{j}$$

The potential energy of magnetic moment $\vec{M}$ in the field $\vec{B}$

$$U = -\vec{M} \cdot \vec{B}$$

$$\Rightarrow \quad U = (4bI_2 I_1)\left(\frac{\mu_0}{2\pi}\frac{d}{x_0}\right) = \frac{2\mu_0 bdI_1 I_2}{\pi x_0}$$

Since, $\vec{F} = -\left(\dfrac{\partial U}{\partial x}\right)\hat{i}$

$$\Rightarrow \quad \vec{F} = \left(\frac{2\mu_0 bdI_1 I_2}{\pi x_0^2}\right)\hat{i}$$

PROBLEM 23

A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = -B_0\hat{j}$ and $\vec{E} = E_0\hat{k}$. Find the components of the velocity and the speed of the particle as a function of its z-co-ordinate.

SOLUTION

Initially, when the particle is released, then magnetic force will do nothing, but the electrostatic force will make the particle gain some velocity. Let the velocity of the particle at time t be $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$. Then according to the Lorentz Force Formula, we have

$$\vec{F} = \vec{F}_m + \vec{F}_e, \text{ where}$$

$$\vec{F}_m = q(\vec{v}\times\vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & -B_0 & 0 \end{vmatrix}$$

$$\Rightarrow \quad \vec{F}_m = q\left[\hat{i}(v_zB_0) + \hat{k}(-v_xB_0)\right]$$

$$\Rightarrow \quad \vec{F}_m = qB_0\left(v_z\hat{i} - v_x\hat{k}\right) \text{ and } \vec{F}_e = q\vec{E} = qE_0\hat{k}$$

$$\Rightarrow \quad \vec{F} = q\left[B_0v_z\hat{i} + (E_0 - B_0v_x)\hat{k}\right]$$

$$\Rightarrow \quad ma_x = qB_0v_z,\ ma_y = 0,\ ma_z = q(E_0 - B_0v_x)$$

$$\Rightarrow \quad a_x = \left(\frac{qB_0}{m}\right)v_z$$

$$\Rightarrow \quad \frac{dv_x}{dt} = \left(\frac{qB_0}{m}\right)v_z$$

$$\Rightarrow \quad \frac{dv_x}{dt}\frac{dz}{dz} = \left(\frac{qB_0}{m}\right)v_z$$

$$\Rightarrow \quad \left(\frac{dv_x}{dz}\right)\left(\frac{dz}{dt}\right) = \left(\frac{qB_0}{m}\right)v_z$$

Since $v_z = \dfrac{dz}{dt}$, so we get

$$\frac{dv_x}{dz} = \frac{qB_0}{m}$$

$$\Rightarrow \quad \int_0^{v_x} dv_x = \frac{qB_0}{m}\int_0^z dz$$

$$\Rightarrow \quad v_x = \left(\frac{qB_0}{m}\right)z$$

$$v_y = 0$$

and $a_z = \dfrac{q}{m}(E_0 - B_0v_x) = \dfrac{qE_0}{m} - \left(\dfrac{qB_0}{m}\right)v_x$

But $v_x = \left(\dfrac{qB_0}{m}\right)z$

$$\Rightarrow \quad a_z = \frac{dv_z}{dt} = \frac{qE_0}{m} - \left(\frac{qB_0}{m}\right)^2 z$$

$$\Rightarrow \quad \frac{dv_z}{dt}\frac{dz}{dz} = \frac{qE_0}{m} - \left(\frac{qB_0}{m}\right)^2 z$$

$$\Rightarrow \quad v_z dv_z = \left(\frac{qE_0}{m}\right)dz - \left(\frac{qB_0}{m}\right)^2 z\,dz \qquad \left\{\because\ v_z = \frac{dz}{dt}\right\}$$

$$\Rightarrow \quad \int_0^{v_z} v_z dz = \frac{qE_0}{m}\int_0^z dz - \left(\frac{qB_0}{m}\right)^2\int_0^z z\,dz$$

$$\Rightarrow \quad \frac{v_z^2}{2} = \left(\frac{qE_0}{m}\right)z - \left(\frac{qB_0}{m}\right)^2\frac{z^2}{2}$$

$$\Rightarrow \quad v_z = \sqrt{\left(\frac{2qE_0}{m}\right)z - \left(\frac{qB_0}{m}\right)^2 z^2}$$

$$\Rightarrow \quad v_x = \frac{qB_0z}{m},\ v_y = 0 \text{ and}$$

$$v_z = \sqrt{\left(\frac{2qE_0}{m}\right)z - \left(\frac{qB_0}{m}\right)^2 z^2}$$

The speed of the particle is

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

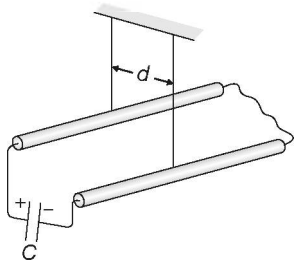
$$\Rightarrow \quad v = \sqrt{\left(\frac{qB_0}{m}\right)^2 z^2 + 0 + \left(\frac{2qE_0}{m}\right)z - \left(\frac{qB_0}{m}\right)^2 z^2}$$

$$\Rightarrow \quad v = \sqrt{\left(\frac{2qE_0}{m}\right)z}$$

PROBLEM 24

Two long, straight conducting wires with linear mass density λ are suspended from cords so that they are each horizontal, parallel to each other, and a distance d apart. The back ends of the wires are connected to each other by a slack, low resistance connecting wire. A charged capacitor (capacitance C) is now added to the system; the positive plate of the capacitor (initial charge $+Q_0$) is connected to the front end of one of the wires, and the negative plate of the capacitor

(initial charge $-Q_0$) is connected to the front end of the other wire. Both of these connections are also made by slack, low resistance wires. When the connection is made, the wires are pushed aside by the repulsive force between the wires, and each wire has an initial horizontal velocity of magnitude v_0. Assume that the time constant for the capacitor to discharge is negligible compared to the time it takes for any appreciable displacement in the position of the wires to occur.



(a) Show that the initial speed v_0 of either wire is given by

$$v_0 = \frac{\mu_0 Q_0^2}{4\pi\lambda RCd}$$

where R is the total resistance of the circuit.

(b) To what height h will each wire rise as a result of the circuit connection?

SOLUTION

(a) **METHOD I:**

The impulsive force, per unit length, due to (antiparallel) discharging currents is equal to the change in momentum. So,

$$\lambda v = \int_0^\infty F dt = \int_0^\infty \frac{\mu_0 I^2}{2\pi d} dt$$

where I = discharging current

Now $q = Q_0 e^{-t/RC}$

$$\Rightarrow |I| = \frac{Q_0}{RC} e^{-t/RC}$$

$$\Rightarrow \lambda v = \left| \int_0^\infty \frac{\mu_0}{2\pi d} \left(\frac{Q_0^2}{R^2C^2} e^{-\frac{2t}{RC}} \right) dt \right| = \frac{\mu_0 Q_0^2}{4\pi RCd}$$

$$\Rightarrow v = \frac{\mu_0 Q_0^2}{4\pi\lambda RCd}$$

METHOD II:

The current carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role

The magnetic force per unit length is $\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I^2}{d}$ and the acceleration obeys the equation $\left(\frac{F}{L}\right) = \left(\frac{m}{L}\right)a$

Also, the r.m.s. current over a short discharge time is $\frac{I_0}{\sqrt{2}}$

The force per unit length between the wires is given by

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I^2}{d} = \frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{F}{L} = \frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2 = \frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$$

According to Newton's Second Law, we have

$$\frac{F}{L} = \left(\frac{m}{L}\right)a = \lambda a = \frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$$

$$\Rightarrow a = \frac{\mu_0 Q_0^2}{4\pi\lambda R^2C^2d}$$

From the kinematics equation, we have

$$v_x = v_{0x} + a_x t$$

$$\Rightarrow v_0 = at = aRC = \frac{\mu_0 Q_0^2}{4\pi\lambda RCd}$$

(b) By Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2 = mgh$$

$$\Rightarrow h = \frac{v_0^2}{2g} = \frac{\left(\frac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g} = \frac{1}{2g}\left(\frac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2$$

Please note that, once the wires have swung apart, we would have to consider gravity in applying Newton's Second Law.

PROBLEM 25

A capacitor of capacitance C is connected to a battery of EMF E for a long time and then disconnected. The charged capacitor is then connected across a long solenoid having n turns per meter in its closely packed winding on its core. After connections it is found that the voltage across the capacitor drops to $\frac{E}{\eta}$ in a time Δt. In this period estimate the average magnetic induction at the centre of solenoid.

SOLUTION

Charge (q) on capacitor after time Δt is

$$q = \frac{CE}{\eta}$$

Charge flown (Δq) through the solenoid in this duration is

$$\Delta q = CE - \frac{CE}{\eta} = CE\left(1 - \frac{1}{\eta}\right)$$

Average current (i_{av}) in solenoid in this period is given by

$$i_{av} = \frac{\Delta q}{\Delta t} = \frac{CE}{\Delta t}\left(1 - \frac{1}{\eta}\right)$$

So, average magnetic field at the centre of solenoid is

$$B_{av} = \mu_0 n i_{av}$$

$$\Rightarrow \quad B_{av} = \frac{\mu_0 nCE}{\Delta t}\left(1 - \frac{1}{\eta}\right)$$

PROBLEM 26

A positively charged particle 1 having charge 1 C and mass 40 g, is revolving along a circle of radius 40 cm with velocity 5 ms^{-1} in a uniform magnetic field with centre of circle at origin O of a three dimensional system. At $t = 0$, the particle was at $(0, 0.4\text{ m}, 0)$ and velocity was directed along positive x-direction. Another particle 2 having charge 1 C and mass 10 g moving uniformly parallel to z-direction with velocity 40 ms^{-1} collides with the revolving particle at $t = 0$ and sticks to it. Neglecting gravitational force and coulomb force, calculate x, y, z coordinates of the combined particle at $t = \frac{\pi}{40}$ s.

SOLUTION

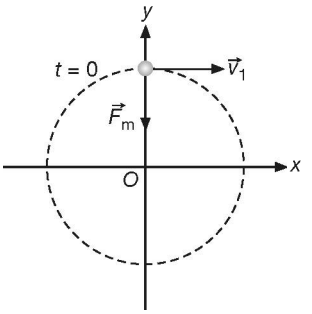
At time $t = 0$ particle 1 is at $(0, 0.4\text{ m}, 0)$ and its velocity is along positive x-direction and magnetic force $\vec{F}_m$ is towards origin or along negative y-direction as shown. Hence according to Fleming's left hand rule, the magnetic field $\vec{B}$ should be along positive z-direction. Since, the radius of the circle, r, is given by

$$r = \frac{m_1 v_1}{Bq_1}$$

$$\Rightarrow \quad B = \frac{m_1 v_1}{rq_1} = \frac{(0.04)(5)}{(0.4)(1)} = 0.5\text{ T}$$

Now by Law of Conservation of Linear Momentum, we get

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$$



$$\Rightarrow \quad (0.04)5\hat{i} + (0.01)(40\hat{k}) = (0.04 + 0.01)\vec{v}$$

$$\Rightarrow \quad 0.05\vec{v} = (0.04)5\hat{i} + (0.01)(40)\hat{k}$$

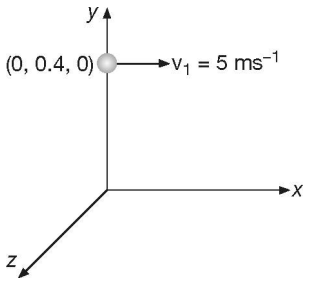
$$\Rightarrow \quad \vec{v} = 4\hat{i} + 8\hat{k}$$

Due to v_x, the combined particle tries to move clockwise along a circular path. However, v_z makes it move uniformly along z-axis so that its path becomes helical.

Also, we observe that this calculated velocity of the combined mass makes some angle $(\neq 0°, 180°\text{ or }90°)$ with the magnetic field. Hence the path of the combined mass is helical, say of radius R, given by

$$R = \frac{(m_1 + m_2)v_x}{(q_1 + q_2)B}$$

$$\Rightarrow \quad R = \frac{(0.05)(4)}{(2)(0.5)} = 0.2\text{ m}$$



Time period of revolution is

$$T = \frac{2\pi(m_1 + m_2)}{(q_1 + q_2)B}$$

$$\Rightarrow \quad T = \frac{2\pi(0.05)}{(2)(0.5)} = \frac{\pi}{10} \text{ s}$$

The time at which the coordinates are to be calculated is

$$t = \frac{\pi}{40} \text{ s} = \frac{T}{4}$$

During this time the combined mass will rotate through quarter circle (a part of Helix) or through an angle $\theta = \omega t = \frac{\pi}{2}$ in x-y plane.

Hence now, the radius of this circle will become 0.2 m

So, $x = R = 0.2$ m

$y = r - R = 0.2$ m and

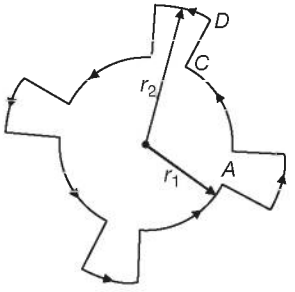
$$z = v_z t = (8)\left(\frac{\pi}{40}\right) = 0.628 \text{ m}$$

Hence, the position of combined mass at $t = \frac{\pi}{40}$ s is

$$(x,\ y,\ z) \equiv (0.2\ ,\ 0.2\ ,\ 0.628\ ) \text{ m}$$

PROBLEM 27

A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each subtends the same angle at the centre.



(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre?

SOLUTION

(a) Given $I = 10$ A, $r_1 = 0.08$ m and $r_2 = 0.12$ m. The straight portions, like CD, etc. will produce zero magnetic field at the centre. The remaining eight arcs will produce the magnetic field at the centre in the same direction, i.e., perpendicular to the paper outwards or vertically upwards and its magnitude is

$$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$$

$$\Rightarrow \quad B = \frac{1}{2}\left(\frac{\mu_0 I}{2r_1}\right) + \frac{1}{2}\left(\frac{\mu_0 I}{2r_2}\right)$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{4\pi}\right)(\pi I)\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

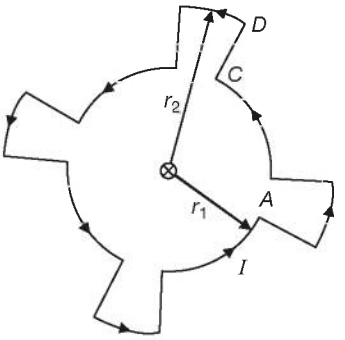
Substituting the values, we have

$$B = \frac{(10^{-7})(3.14)(10)(0.08 + 0.12)}{(0.08 \times 0.12)} \text{ T}$$

$$\Rightarrow \quad B = 6.54 \times 10^{-5} \text{ T}$$

{Vertically upward or outward normal to the paper}

(b) **Force on AC:** Force on circular portions of the circuit, i.e., AC etc. due to the wire at the centre will be zero because magnetic field due to the central wire at these arcs will be tangential ($\theta = 180°$) as shown.



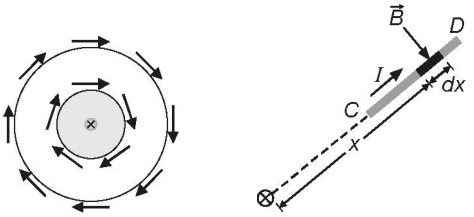
Force on *CD*: Current in central wire is also 10 A Magnetic field at P due to central wire,

$$B = \frac{\mu_0}{2\pi}\left(\frac{I}{x}\right)$$

Consider an infinitesimal element on CD at a distance x from wire. Let this element have a length dx. Then magnetic force on element dx due to this magnetic field is dF given by

$$dF = (I)\left(\frac{\mu_0}{2\pi}\frac{I}{x}\right)dx \qquad \{\because F = BIl\sin 90°\}$$

$$\Rightarrow \quad dF = \left(\frac{\mu_0 I^2}{2\pi}\right)\frac{dx}{x}$$



So, the net force on CD is given by

$$F = \int_{x=r_1}^{x=r_2} dF = \frac{\mu_0 I^2}{2\pi}\int_{0.08}^{0.12}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0}{2\pi} I^2 \log_e\left(\frac{3}{2}\right)$$

Substituting the values,

$$F = (2\times 10^{-7})(10)^2 \log_e(1.5)$$

$$\Rightarrow \quad F = 8.1\times 10^{-6}\ \text{N (inwards)}$$

Force on wire at the centre: Net magnetic field at the centre due to circuit is in vertical direction and current in the wire in centre is also in vertical direction. Therefore, net force on the wire at the centre will be zero. $(\theta = 180°)$. So, we have the force acting

(i) on the wire at the centre is zero.

(ii) on arc $AC = 0$

(iii) on segment CD is 8.1×10^{-6} N (inwards)

PROBLEM 28

A slightly divergent beam of charged particles accelerated by a potential difference V propagates from a point A along the axis of a solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2. Find the specific charge $\frac{q}{m}$ of the particles.

SOLUTION

The velocity of the particles accelerated by a potential V is given by

$$\frac{1}{2}mv^2 = Vq$$

$$\Rightarrow \quad v = \sqrt{\frac{2Vq}{m}}$$

Since the charged particles are slightly divergent so, they will follow a helical path.

If θ be the small angle made by a particle with B, then $\cos\theta \approx 1$.

So, pitch of the particle is

$$p = v_{\parallel} \times T$$

$$\Rightarrow \quad p = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \frac{2\pi v m}{qB}$$

Particles are focussed at a distance l, if l contains integral number of pitches i.e.,

$$l = np$$

$$\Rightarrow \quad p = \frac{l}{n} = l, \frac{l}{2}, \frac{l}{3}, \ldots\ldots$$

For two consecutive focussings (as B increases, p decreases), we get

$$l = \frac{2\pi m v}{qB_1} \quad \text{and} \quad \frac{l}{2} = \frac{2\pi m v}{qB_2}$$

$$\Rightarrow \quad B_1 = \frac{2\pi m v}{ql} \quad \text{and} \quad B_2 = \frac{4\pi m v}{ql}$$

$$\Rightarrow \quad B_2 - B_1 = \frac{2\pi m v}{ql}$$

$$\Rightarrow \quad B_2 - B_1 = \frac{2\pi m}{ql}\sqrt{\frac{2Vq}{m}}$$

$$\Rightarrow \quad \frac{q}{m} = \frac{8\pi^2 V}{l^2 (B_2 - B_1)^2}$$

PROBLEM 29

Three infinitely long thin wires, each carrying current I in the same direction, are in the X-Y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.

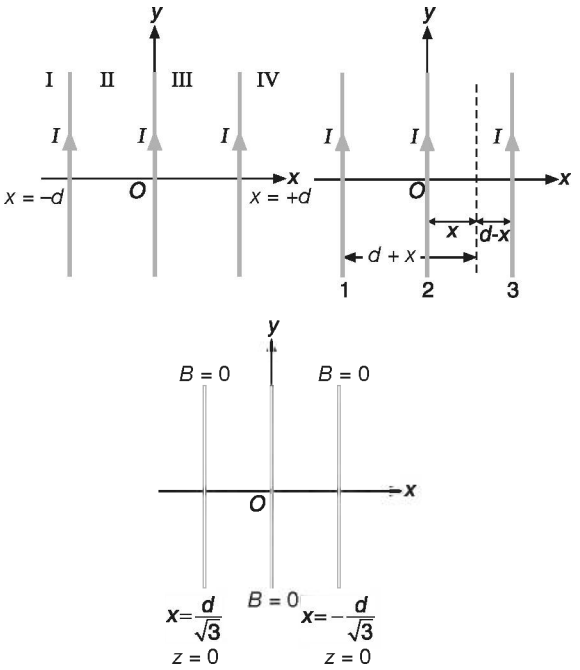
(a) Find the locus of the points for which the magnetic field B is zero.

(b) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation. '

SOLUTION

(a) Magnetic field will be zero on the y-axis, i.e., $x = 0 = z$.

Magnetic field cannot be zero in REGION I and REGION IV because in REGION I magnetic field will be along positive z-direction $\odot$ due to all the three wires, while in REGION IV magnetic field will be along negative z-axis $\otimes$ due to all the three wires. It can be zero only in REGIONS II and III.



Let magnetic field is zero on line $z = 0$ and $x = x$ (shown as dotted). The magnetic field on this line due to wires 1 and 2 will be along negative z-axis and due to wire 3 along positive z-axis. Thus,

$$B_1 + B_2 - B_3 = 0$$

$$\Rightarrow \frac{\mu_0}{2\pi}\frac{I}{(d+x)} + \frac{\mu_0 I}{2\pi x} = \frac{\mu_0}{2\pi}\frac{I}{(d-x)}$$

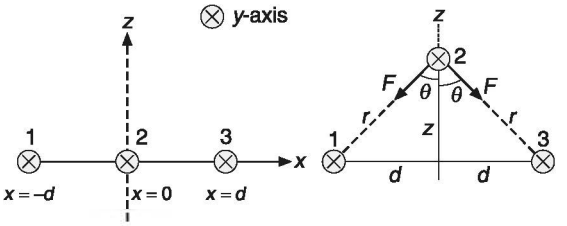
$$\Rightarrow \frac{1}{d+x} + \frac{1}{x} = \frac{1}{d-x}$$

Solving, we get, $x = \pm\frac{d}{\sqrt{3}}$

Hence, there will be two lines, where the magnetic field will be zero i.e.,

$$x = \frac{d}{\sqrt{3}} \text{ and } x = -\frac{d}{\sqrt{3}} \qquad \{z = 0\}$$

(b) For the sake of convenience and understanding the problem easily, let us change our coordinate axes system as shown.



Three wires 1, 2 and 3 are shown in figure. Let the wire 2 be displaced slightly towards the z-axis, then force of attraction per unit length between wires (1 and 2) and (2 and 3) is given by

$$F = \frac{\mu_0}{2\pi}\frac{I^2}{r}$$

The components of F along x-axis will be cancelled out. Net resultant force will be towards negative z-axis (or mean position) and is given by

$$F_{net} = 2F\cos\theta = 2\left(\frac{\mu_0}{2\pi}\frac{I^2}{r}\right)\left(\frac{z}{r}\right)$$

$$\Rightarrow F_{net} = \frac{\mu_0}{\pi}\frac{I^2}{(z^2+d^2)}z$$

For small displacements, we have $z \ll d$, then

$$z^2 + d^2 \approx d^2$$

$$\Rightarrow F_{net} = -\left(\frac{\mu_0}{\pi}\frac{I^2}{d^2}\right)z$$

Negative sign implies that F_{net} is restoring in nature. i.e., $F_{net} \propto -z$

Hence the wire will oscillate simple harmonically. Let a be the acceleration of wire in this position and λ the mass per unit length of wire then

$$F_{net} = \lambda a = -\left(\frac{\mu_0}{\pi}\frac{I^2}{d^2}\right)z$$

$$\Rightarrow \quad a = -\left(\frac{\mu_0 I^2}{\pi\lambda d^2}\right)z$$

Comparing with the standard equation of SHM i.e., $\ddot{z} + \omega^2 z = 0$

we get

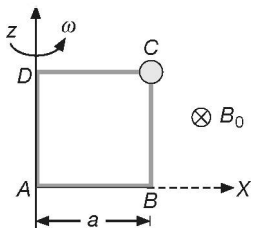
$$\omega = \sqrt{\frac{\mu_0 I^2}{\pi\lambda d^2}}$$

$$\Rightarrow \quad 2\pi f = \sqrt{\frac{\mu_0 I^2}{\pi\lambda d^2}}$$

$$\Rightarrow \quad f = \frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

PROBLEM 30

An insulated square frame $ABCD$ of side a is able to rotate about one of its side taken as positive z-axis. A magnetic field B is present in the region given by $\vec{B} = B_0\hat{j}$. A small bead of mass m and charge q movable alongside CB is initially near C, when frame lies in x-z plane. Now, frame is rotated by constant angular velocity ω about z-axis. Whole system lies in gravity free space. If after time t, the bead reaches point B, find B_0 in terms of t.



SOLUTION

Bead moves with speed $v = a\omega$ in x-y plane

Velocity of bead, when frame has rotated through angle $\theta = \omega t$

$$\vec{v} = a\omega\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right) + v_z\hat{k}$$

Force on bead

$$F = q\vec{v}\times\vec{B} = \left[a\omega\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right) + v_z\hat{k}\right]\times B_0\hat{j}$$

Force in z-direction $= a\omega q B_0\left(-\hat{k}\right)\sin\theta$

Acceleration of bead $\vec{a}_z = -\dfrac{a\omega q B_0 \sin\theta}{m}\left(-\hat{k}\right)$

$$\Rightarrow \quad \frac{dv_z}{dt} = -\frac{a\omega q B_0}{m}\sin\omega t$$

$$\Rightarrow \quad v_z = -\frac{a\omega q B_0}{m}\int_0^t \sin\omega t dt = -\frac{aB_0}{m}(1-\cos\omega t)$$

$$\Rightarrow \quad \frac{dz}{dt} = -\frac{aqB_0}{m}(1-\cos\omega t)$$

$$\Rightarrow \quad dz = -\frac{aqB_0}{m}\int_0^t (1-\cos\omega t)$$

$$\Rightarrow \quad z = -\frac{aqB_0}{m}\left[t - \frac{\sin\omega t}{\omega}\right]_0^1 = -\frac{aqB_0}{m}\left[t - \frac{\sin\omega t}{\omega}\right]$$

When bead reaches B, then we have $z = -a$

$$\Rightarrow \quad -a = -\frac{aqB_0}{m}\left(t - \frac{\sin\omega t}{\omega}\right)$$

$$\Rightarrow \quad B_0 = \frac{m\omega}{q(\omega t - \sin\omega t)}$$

PROBLEM 31

The force on a magnetic dipole $\vec{M}$ aligned with a non-uniform magnetic field in the x direction is given by $F_x = |\vec{M}|\dfrac{dB}{dx}$. Consider two flat loops of wire each have radius R and carry current I.

(a) The loops are arranged coaxially and separated by a variable distance x, large compared to R. Show that the magnetic force between them varies as $\dfrac{1}{x^4}$.

(b) Evaluate the magnitude of this force if $I = 10$ A, $R = 0.5$ cm and $x = 5$ cm.

SOLUTION

(a) The magnetic field produced by one loop at the center of the second loop for $x \gg R$ is given by

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I(\pi R^2)}{2\pi x^3} = \frac{\mu_0 M}{2\pi x^3}$$

where the magnetic moment of either loop is $M = I(\pi R^2)$. Therefore,

$$|F_x| = M\frac{dB}{dx} = \mu\left(\frac{\mu_0 M}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0(\pi R^2 I)^2}{2\pi x^4}$$

$$\Rightarrow |F_x| = \frac{3\pi}{2}\left(\frac{\mu_0 I^2 R^4}{x^4}\right)$$

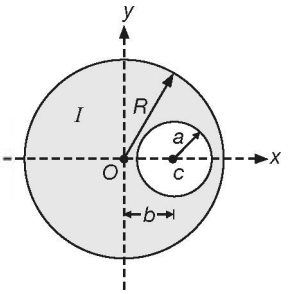
(b) $$|F_x| = \frac{3\pi}{2}\left(\frac{\mu_0 I^2 R^4}{x^4}\right)$$

$$\Rightarrow |F_x| = \frac{3\pi}{2}\frac{(4\pi\times10^{-7})(10)^2(5\times10^{-3})^4}{(5\times10^{-2})^4}$$

$$\Rightarrow |F_x| \cong 6\times10^{-8}\ \text{N}$$

PROBLEM 32

A very long straight conductor has a circular cross-section of radius R and carries a current I. Inside the conductor there is a cylindrical cavity of radius a whose axis is parallel to the axis of the conductor and a distance b from its centre. Let the z-axis be the axis of the conductor, and let the axis of the cavity be at $x = b$. Find the magnetic field



(a) on the x-axis at $x = 2R$

(b) on the y-axis at $y = 2R$

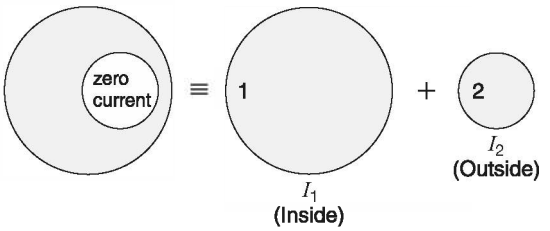
(c) at a point P inside the cylindrical cavity. Also show that the field at P is constant both in magnitude and direction.

SOLUTION

(a) Let us take help from the concept of current density. Since current density (current per unit area) in the wire is,

$$J = \frac{I}{\pi R^2 - \pi a^2} = \frac{I}{\pi(R^2 - a^2)}$$

The given arrangement is obtained by superimposing a uniform inward cylindrical current distribution of radius R and an inward cylindrical current distribution of radius a, so that net current in the cavity of radius a is zero.



Current void (zero current space) obtained from superposition of 1 and 2

$$I_1 = J(\pi R^2) \text{ and } I_2 = J(\pi a^2)$$

Magnetic field at $x = 2R$ due to current I_1 is,

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{2R} \quad \{\text{along negative } y\text{-direction}\}$$

Magnetic field at $x = 2R$ due to current I_2 is,

$$B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{(2R-b)} \quad \{\text{along positive } y\text{-direction}\}$$

Therefore, net magnetic field at $x = 2R$ is,

$$B = B_2 - B_1 \quad \{\text{along positive } y\text{-direction}\}$$

Substituting, the values, we get

$$B = \frac{\mu_0 J}{2\pi}\left[\frac{\pi a^2}{2R-b} - \frac{\pi R^2}{2R}\right] = \frac{\mu_0 J}{2}\left(\frac{a^2}{2R-b} - \frac{R}{2}\right)$$

$$B = \frac{\mu_0 I}{2\pi(R^2 - a^2)}\left(\frac{a^2}{2R-b} - \frac{R}{2}\right)$$

(b) At $y = 2R$, magnetic field due to current I_1 is,

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{2R} \quad \{\text{along positive } x\text{-direction}\}$$

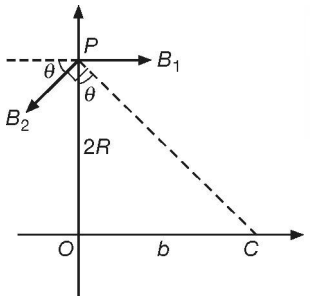
and due to current I_2, magnetic field is B_2 in a direction perpendicular to CP. Here

$$B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{\sqrt{(2R)^2+b^2}} = \frac{\mu_0}{2\pi}\frac{I_2}{\sqrt{4R^2+b^2}}$$

Now this has two components one along negative x-axis and another along negative y-axis. Thus,

$$B_{2x} = -B_2\cos\theta = -\frac{\mu_0}{2\pi}\frac{(I_2)(2R)}{(4R^2+b^2)}$$

and $B_{2y} = -B_2\sin\theta = -\frac{\mu_0}{2\pi}\frac{(I_2)(b)}{(4R^2+b^2)}$



Therefore, net magnetic field along x-axis is,

$$B_x = B_1 + B_{2x} = \frac{\mu_0}{2\pi}\frac{I_1}{2R} - \frac{\mu_0}{2\pi}\frac{(I_2)(2R)}{(4R^2+b^2)}$$

Substituting the value of I_1 and I_2, we have

$$B_x = \frac{\mu_0 J}{2}\left[\frac{R}{2} - \frac{2Ra^2}{4R^2+b^2}\right]$$

and $B_y = -\frac{\mu_0 J}{2}\left[\frac{a^2 b}{4R^2+b^2}\right]$

$\Rightarrow \vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2}\left[\left(\frac{R}{2} - \frac{2Ra^2}{4R^2+b^2}\right)\hat{i} - \left(\frac{a^2 b}{4R^2+b^2}\right)\hat{j}\right],$$

where $J = \frac{I}{\pi(R^2-a^2)}$

(c) Let the point P inside the cavity be at a distance r_1 from O and r_2 from C.

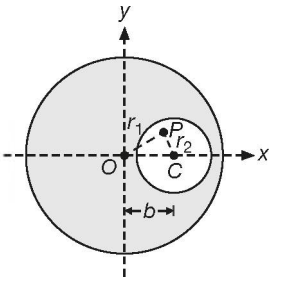
At point P magnetic field due to I_1 is B_1 (perpendicular to OP) and is $\vec{B}_2$ due to I_2 (perpendicular to CP) in the directions shown. Although $\vec{B}_1$ and $\vec{B}_2$ are actually at P, but for getting a clear picture and better understanding they are drawn at O and C respectively. Let B_x be the x component of resultant of $\vec{B}_1$ and $\vec{B}_2$ and B_y its y-component. Then,

$$B_x = B_1\sin\alpha - B_2\sin\beta$$

$$B_x = \left(\frac{\mu_0}{2\pi}\frac{I_1}{R^2}r_1\right)\sin\alpha - \left(\frac{\mu_0}{2\pi}\frac{I_2}{a^2}\cdot r_2\right)\sin\beta$$

$$B_x = \left(\frac{\mu_0}{2\pi}\frac{J\pi R^2}{R^2}\cdot r_1\sin\alpha\right) - \left(\frac{\mu_0}{2\pi}\frac{J\cdot\pi a^2}{a^2}r_2\sin\beta\right)$$

$$B_x = \frac{\mu_0 J}{2}(r_1\sin\alpha - r_2\sin\beta) \qquad \ldots(1)$$

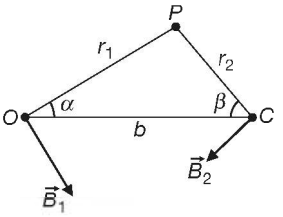


In ΔOPC, we have, from LAMI's Theorem

$$\frac{r_1}{\sin\beta} = \frac{r_2}{\sin\alpha}$$

$\Rightarrow r_1\sin\alpha - r_2\sin\beta = 0$

$\Rightarrow B_x = 0$



Now, $B_y = -(B_1\cos\alpha + B_2\cos\beta)$

$$B_y = -\frac{\mu_0 J}{2}(r_1\cos\alpha + r_2\cos\beta) \qquad \ldots(2)$$

From ΔOPC, we can see that

$$r_1\cos\alpha + r_2\cos\beta = b$$

$$\Rightarrow B_y = -\frac{\mu_0 Jb}{2} = -\frac{\mu_0 Ib}{2\pi(R^2-a^2)} = \text{constant}$$

Thus, we can see that net magnetic field at point P is along negative y direction and is constant in magnitude.

CHECK POINT

At point C magnetic field due to I_2 is zero $(\text{i.e. } B_2 = 0)$ while that due to I_1 is $\frac{\mu_0}{2\pi}\frac{I_1}{R^2}b$ in negative y-direction.

Substituting, $I_1 = J(\pi R^2)$ we get,

$$B = B_2 = \frac{\mu_0 Ib}{2\pi(R^2 - a^2)} \{\text{along negative } y\text{-direction}\}$$

This agrees with the result derived above.

PROBLEM 33

In a certain region of space there exists a uniform and constant electric field of magnitude E along the positive y-axis of a co-ordinate system. A charged particle of mass m and charge $-q(q>0)$ is projected with speed $2v_0$ at an angle of $60°$ with the positive x-axis in x-y plane from the origin. When the x-coordinate of the particle becomes $\frac{\sqrt{3}mv_0^2}{qE}$, a uniform and constant magnetic field of strength B is also switched on along the positive y-axis. Find the co-ordinate of the particle as a function of time t after its projection.

SOLUTION

According to the problem, we have

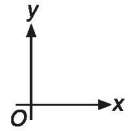
$$\vec{E} = E\hat{j}$$

$$\vec{v} = 2v_0\cos 60°\hat{i} + 2v_0\sin 60°\hat{j} = v_0\hat{i} + v_0\sqrt{3}\hat{j}$$

$$\vec{a} = -\frac{qE}{m}\hat{j}$$

$$\vec{R}(t) = v_0 t\hat{i} + \left(v_0\sqrt{3}t - \frac{1}{2}\frac{qE}{m}t^2\right)\hat{j}$$

Given, $x = v_0 t = \frac{\sqrt{3}mv_0^2}{qE}$



$$\Rightarrow \quad t = \frac{\sqrt{3}mv_0}{qE}$$

For $t \le \frac{\sqrt{3}mv_0}{qE}$

$$x(t) = v_0 t \text{ and } v_y(t) = v_0\sqrt{3} - \frac{qE}{m}t$$

$$\Rightarrow \quad v_y\left(t = \frac{\sqrt{3}mv_0}{2E}\right) = v_0\sqrt{3} - \left(\frac{qE}{m}\right)\frac{\sqrt{3}mv_0}{qE} = 0$$

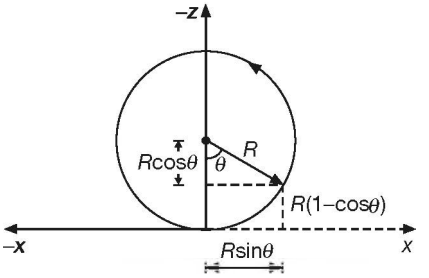
For $t \ge \frac{\sqrt{3}mv_0}{qE}$, magnetic field is also present

The particle will start moving on helical path. The cross-section of the helix will be in x-z plane

$$R = \frac{mv_0}{qB}$$

$$\omega = \frac{qB}{m}$$

$$\theta = \omega t = \frac{qBt}{m}$$



For $t \ge \frac{\sqrt{3}mv_0}{qE}$, we get

$$x(t) = \frac{\sqrt{3}mv_0^2}{qE} + R\sin\left(\frac{qB}{m}t\right),$$

$$y(t) = \frac{3mv_0^2}{2qE} - \frac{1}{2}\left(\frac{qE}{m}\right)t^2 \text{ and}$$

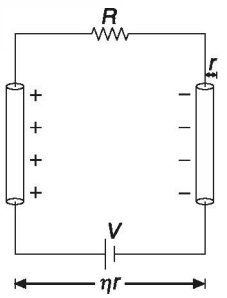
$$z(t) = -R\left[1 - \cos\left(\frac{qB}{m}t\right)\right]$$

PROBLEM 34

Two long parallel wires of negligible resistance are connected at one end to a resistance R and at the other end to a constant voltage source of voltage V. The distance between the axes of the wires is η times greater than the cross-sectional radius of each wire. At what value of resistance R, does the resultant force of interaction between the wires will become zero?

SOLUTION

The situation described in the question is shown in figure.



The two wires form a wire capacitor where capacitance is given by

$$C = \frac{\pi\varepsilon_0 l}{\ln \eta}$$

Charge (q) on the capacitor in steady state is

$$q = CV = \frac{\pi\varepsilon_0 lV}{\ln \eta} \quad \ldots(1)$$

Also, we observe that due to voltage source, a constant current flow in opposite directions through the wires which repel each other by a magnetic force. In this magnetic force gets nullified by the force of attraction between the two wires due to opposite charges on them, then the resultant force of interaction between the wires will become zero.
The force of attraction is given as

$$F_e = qE = q\left(\frac{q}{2\pi\varepsilon_0 l(\eta r)}\right) \quad \ldots(2)$$

Substituting the value of q from equation (1) in equation (2), we get

$$F_e = \frac{\left(\frac{\pi\varepsilon_0 lV}{\ln \eta}\right)^2}{2\pi\varepsilon_0 l(\eta r)} \quad \ldots(3)$$

Since the current flows is in the opposite direction in the two wires, so the net magnetic force of repulsion between these wires is given by

$$F_m = \frac{\mu_0 I^2 l}{2\pi(\eta r)} = \frac{\mu_0\left(\frac{V}{R}\right)^2 l}{2\pi\eta r} \quad \ldots(4)$$

Since the net force between the two wires is zero, so we have

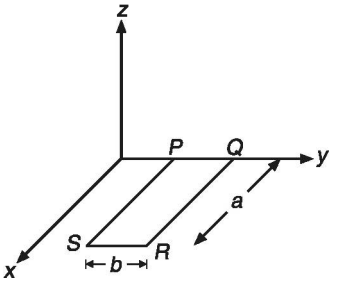
$$F_e = F_m$$

$$\Rightarrow \quad \frac{\mu_0\left(\frac{V}{R}\right)^2 l}{2\pi\eta r} = \frac{\left(\frac{\pi\varepsilon_0 lV}{\ln \eta}\right)^2}{2\pi\varepsilon_0 l(\eta r)}$$

$$\Rightarrow \quad R = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\ln \eta}{\pi}$$

PROBLEM 35

A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (shown in figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.



(a) What is the direction of the current I in PQ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

SOLUTION

(a) Let the direction of current in wire PQ is from P to Q and its magnitude be I.

The magnetic moment of the given loop is $\vec{M} = -Iab\hat{k}$

Torque on the loop due to magnetic forces is

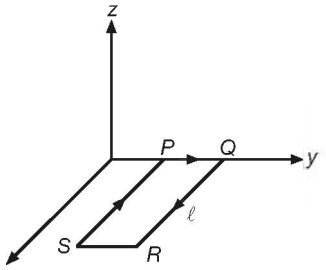
$$\vec{\tau}_1 = \vec{M} \times \vec{B} = \left(-Iab\hat{k}\right) \times \left(3\hat{i} + 4\hat{k}\right)B_o\hat{i}$$

$$\Rightarrow \quad \vec{\tau}_1 = -3IabB_o\hat{j}$$

Torque of weight of the loop about axis PQ is

$$\vec{\tau}_2 = \vec{r} \times \vec{F} = \left(\frac{a}{2}\hat{i}\right) \times \left(-mg\hat{k}\right) = \frac{mga}{2}\hat{j}$$

We see that when the current in the wire PQ is from P to Q, $\vec{\tau}_1$ and $\vec{\tau}_2$ are in opposite direction, so they can cancel each other and the loop may remain in equilibrium. So, the direction of current I in wire PQ is from P to Q.



(b) Further for equilibrium of the loop:

$$|\vec{\tau}_1| = |\vec{\tau}_2|$$

$$\Rightarrow \quad 3IabB_o = \frac{mga}{2}$$

$$\Rightarrow \quad I = \frac{mg}{6bB_o}$$

(c) Magnetic force on wire RS is

$$\vec{F} = I\left(\vec{l} \times \vec{B}\right) = I\left[\left(-b\hat{j}\right) \times \left\{\left(3\hat{i} + 4\hat{k}\right)B_o\right\}\right]$$

$$\Rightarrow \quad \vec{F} = IbB_o\left(3\hat{k} - 4\hat{i}\right)$$

PROBLEM 36

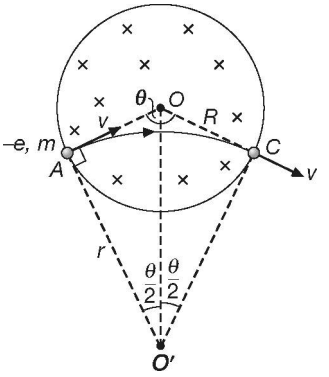
A direct current flowing through the winding of a long cylindrical solenoid of radius R produces a uniform magnetic induction B in it. An electron travelling at velocity v enters into the solenoid along the radial direction between its turns at right angles to the solenoid axis. After a certain time, the electron deflected by magnetic field leaves the solenoid. Calculate the time which the electron spends inside the solenoid.

SOLUTION

The magnetic field due to the solenoid is directed along its axis. The magnetic force on the electron at any instant will be in a plane perpendicular to solenoid axis and the trajectory of the electron in the solenoid will be an arc of a circle. The radius r of the circular arc is given by

$$r = \frac{mv}{qB} = \frac{mv}{eB} \qquad \ldots(1)$$

The trajectory followed by the electron is shown in figure.



If electron leaves the solenoid at point C after suffering a deviation θ, then from figure we have

$$\angle AO'O = \frac{1}{2}\angle AO'C = \frac{\theta}{2}$$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2}\right) = \frac{R}{r} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\tan\left(\frac{\theta}{2}\right) = \frac{eBR}{mv}$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{eBR}{mv}\right)$$

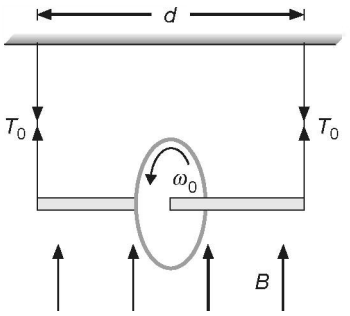
Since the magnitude of velocity remains constant over the entire trajectory, so the transit time of electron inside the solenoid is given by

$$t = \frac{r\theta}{v} = \left(\frac{m}{eB}\right)\theta$$

$$\Rightarrow \quad t = \frac{2m}{eB}\tan^{-1}\left(\frac{eBR}{mv}\right)$$

PROBLEM 37

A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on the ring is rotated at constant angular velocity w. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.



SOLUTION

In equilibrium, $2T_o = mg$

$$\Rightarrow \quad T_o = \frac{mg}{2} \qquad \ldots(1)$$

Magnetic moment, $M = IA = \left(\frac{\omega}{2\pi}Q\right)\left(\pi R^2\right)$

$$\Rightarrow \quad \tau = MB\sin 90^o = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on $(T_1 > T_2)$.

For translational equilibrium of ring in vertical direction.

$$T_1 + T_2 = mg \qquad \ldots(2)$$

For rotational equilibrium,

$$\left(T_1 - T_2\right)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2}$$

$$\Rightarrow \quad T_1 - T_2 = \frac{\omega BQR^2}{2} \qquad \ldots(3)$$

Solving equations (2) and (3), we have

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

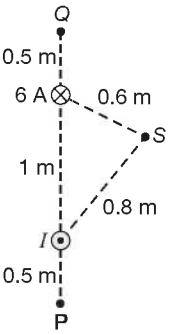
As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_o}{2}$, so we get

$$\frac{3T_o}{2} = T_o + \frac{\omega_{\max} BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_o\right)$$

$$\Rightarrow \quad \omega_{\max} = \frac{DT_o}{BQR^2}$$

PROBLEM 38

Two long, straight, parallel wires are 1 m apart. The wire on the left carries a current I_1 of 6 A into the plane of the paper.



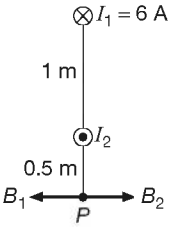
(a) What must the magnitude and direction of the current I_2 be for the net field at point P to be zero?

(b) What are the magnitude and direction of the net field at Q?

(c) What is the magnitude of the net field at S?

SOLUTION

(a) The directions of the fields at point P due to the two wires are shown in figure $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at P to be zero. $\vec{B}_2$ is to the right so I_2 is out of the page.



$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6\text{ A}}{1.5\text{ m}}\right) \text{ and}$$

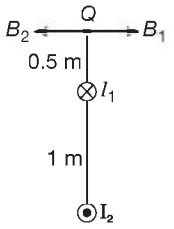
$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.5\text{ m}}\right)$$

$$B_1 = B_2$$

$$\Rightarrow \quad \frac{\mu_0}{2\pi}\left(\frac{6\ A}{1.5\text{ m}}\right) = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.5\text{ m}}\right)$$

$$\Rightarrow \quad I_2 = \left(\frac{0.5\text{ m}}{1.5\text{ m}}\right)(6\text{ A}) = 2\text{ A}$$

(b) The directions of the fields at point Q are shown in figure



$$B_1 = \frac{\mu_0 I_1}{2\pi r_1}$$

$$\Rightarrow B_1 = (2\times10^{-7}\ \text{T}\cdot\text{mA}^{-1})\left(\frac{6\ \text{A}}{0.5\ \text{m}}\right)$$

$$\Rightarrow B_1 = 2.4\times10^{-6}\ \text{T} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r_2}$$

$$\Rightarrow B_2 = (2\times10^{-7}\ \text{T}\cdot\text{mA}^{-1})\left(\frac{2\ \text{A}}{1.5\ \text{m}}\right)$$

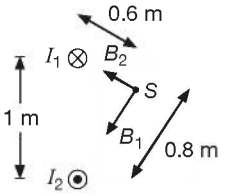
$$\Rightarrow B_2 = 2.67\times10^{-7}\ \text{T}$$

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$. Hence

$$B = B_1 - B_2 = 2.4\times10^{-6}\ \text{T} - 2.67\times10^{-7}\ \text{T}$$

$$B = 2.13\times10^{-6}\ \text{T}, \text{ and } \vec{B} \text{ is to the right}$$

(c) The directions of the fields at point S are shown in figure



$$B_1 = \frac{\mu_0 I_1}{2\pi r_1}$$

$$\Rightarrow B_1 = (2\times10^{-7}\ \text{T}\cdot\text{mA}^{-1})\left(\frac{6\ \text{A}}{0.6\ \text{m}}\right) = 2\times10^{-6}\ \text{T}$$

and $B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$$\Rightarrow B_2 = (2\times10^{-7}\ \text{TmA}^{-1})\left(\frac{2\ \text{A}}{0.8\ \text{m}}\right) = 5\times10^{-7}\ \text{T}$$

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2\times10^{-6}\ \text{T})^2 + (5\times10^{-7}\ \text{T})^2}$$

$$B = 2.06\times10^{-6}\ \text{Ta}$$

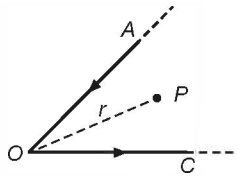
PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Two observers moving with different velocities see that a point charge produces same magnetic field at the same point A. Their relative velocity must be parallel to $\vec{r}$, where $\vec{r}$ is the position vector of point A with respect to point charge. This statement is
(A) true
(B) false
(C) nothing can be said
(D) true only if the charge is moving perpendicular to the $\vec{r}$

2. Two semi-infinite wires AO and OC coinciding at O carry equal currents I as shown in Figure.



Angle AOC is α. The magnitude of magnetic field at a point P on the bisector of these two wires at a distance r from point O is

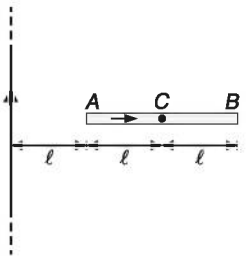
(A) $\frac{\mu_0 I}{2\pi r}\cot\left(\frac{\alpha}{2}\right)$

(B) $\frac{\mu_0 I}{4\pi r}\cot\left(\frac{\alpha}{2}\right)$

(C) $\frac{\mu_0 I}{2\pi r}\text{cosec}\left(\frac{\alpha}{2}\right)+\cot\left(\frac{\alpha}{2}\right)$

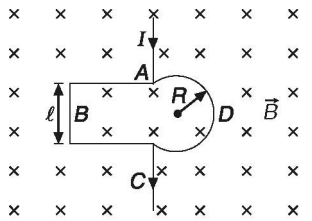
(D) $\frac{\mu_0 I}{4\pi r}\sin\left(\frac{\alpha}{2}\right)$

3. A current carrying rod AB is placed perpendicular to an infinitely long current carrying wire as shown in Figure.



The point at which the conductor should be hinged so that it will not rotate ($AC = CB$)
(A) A
(B) somewhere between B and C
(C) C
(D) somewhere between A and C

4. Figure shows a conducting loop $ABCDA$ placed in a uniform magnetic field (strength B) perpendicular to its plane. The part ABC is the $\left(\frac{3}{4}\right)$ the portion of the square of side length l. The part ADC is a circular arc of radius R. The point A and C are connected to a battery which supplies a current I to the circuit. The magnetic force on the loop due to the field B is



(A) ZERO (B) BIl
(C) $2BIR$ (D) $\frac{BIlR}{I+R}$

5. A conducting gas is in the form of a long cylinder. Current flows through the gas along the length of the cylinder. The cylinder is distributed uniformly across the cross-section of the gas. Disregard thermal and electrostatic forces among the gas molecules. Due to the magnetic fields set up inside the gas and the forces which they exert on the moving ions, the gas will tend to
(A) expand
(B) contract
(C) expand and contract alternately
(D) None of the above

6. Two very long straight parallel wires carry steady currents I and $2I$ in opposite directions. The distance between the wires is d. At a certain instant of time a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of

the force due to the magnetic field acting on the charge at this instant is

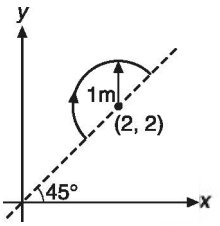
(A) $\frac{\mu_0 Iqv}{\pi d}$ (B) $\frac{\mu_0 Iqv}{2\pi d}$

(C) $\frac{3\mu_0 Iqv}{2\pi d}$ (D) ZERO

7. An infinitely long wire has linear charge density λ. It moves along its axis with a non-relativistic speed v. A point P lies at a distance r from the wire. The ratio of the electric field to the magnetic field at this point is

(A) proportional to r
(B) inversely proportional to r
(C) proportional to λ
(D) independent of r

8. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{j} + \hat{k})$ exists in region of space. A semi-circular wire of radius 1 m carrying current 1 A having its centre at (2, 2, 0) is placed in x-y plane as shown in Figure.

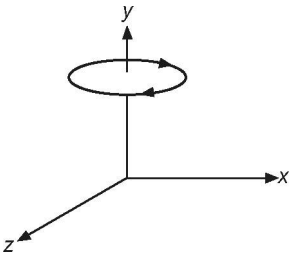


The force on semi-circular wire will be

(A) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$ (B) $\sqrt{2}(\hat{i} - \hat{j} + \hat{k})$

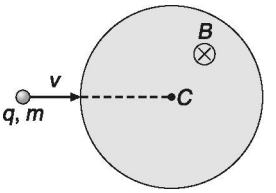
(C) $\sqrt{2}(\hat{i} + \hat{j} - \hat{k})$ (D) $\sqrt{2}(-\hat{i} + \hat{j} + \hat{k})$

9. A circular coil carrying a current I (in the direction shown), having mass m is kept above the ground (x-z plane) at some height. For the magnetic force to balance the weight of the coil, a uniform magnetic field $\vec{B}$ must be applied along the



(A) positive x-direction
(B) negative x-direction
(C) positive z-direction
(D) None of the above

10. A charged particle of mass 2 g and charge $-5\ \mu C$ enters a circular region of radius 10 cm, in which there is a uniform magnetic field of strength 4 T and directed perpendicular to the plane of circular region as shown in Figure.



If the particle's velocity vector rotates through an angle of 90° in passing through this region, then its speed is

(A) 0.25 mms^{-1} (B) 4 mms^{-1}

(C) 1 cms^{-1} (D) 1 mms^{-1}

11. A charged particle enters a uniform magnetic field with velocity that makes an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p. The radius of the helix is

(A) $\sqrt{2}p$ (B) $\frac{p}{\sqrt{2}\pi}$

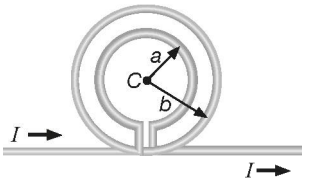
(C) $\frac{\sqrt{2}p}{\pi}$ (D) $\frac{p}{2\pi}$

12. A wire lying along y-axis from $y = 0$ to $y = 1$ m carries a current of 2 mA in the negative y-direction. The wire lies in a non-uniform magnetic field given by $\vec{B} = (0.3\ Tm^{-1})y\hat{i} + (0.4\ Tm^{-1})y\hat{j}$. The magnetic force on the entire wire, in newton, is

(A) $(-3\times10^{-4})\hat{j}$ (B) $(6\times10^{-3})\hat{k}$

(C) $(-3\times10^{-4})\hat{k}$ (D) $(3\times10^{-4})\hat{k}$

13. An otherwise infinite, straight wire has two concentric loops of radii a and b carrying equal currents in opposite directions as shown. The magnetic field at the common centre is zero for

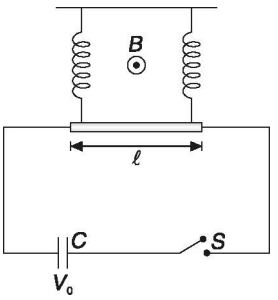


No cross-contacts Exist

(A) $\frac{a}{b} = \frac{\pi+1}{\pi}$ (B) $\frac{a}{b} = \frac{\pi}{\pi+1}$

(C) $\frac{a}{b} = \frac{\pi-1}{\pi+1}$ (D) $\frac{a}{b} = \frac{\pi+1}{\pi-1}$

14. A straight conducting bar of mass m and length l is suspended horizontally with two non-conducting springs of stiffness k as shown in Figure.



The capacitor is initially charged to the potential difference V_0. At time $t = 0$ the switch S is closed and the capacitor discharges. The bar start oscillating in vertical plane. The amplitude of these oscillations (assuming the time of discharge of capacitor is much smaller than the period T of the mechanical oscillation of the bar) is

(A) $\frac{BlCV_0}{\sqrt{2km}}$ (B) $\frac{BlCV_0}{\sqrt{km}}$

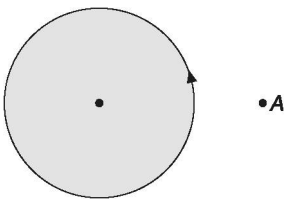
(C) $\frac{BlCV_0}{\sqrt{8km}}$ (D) $\frac{BlCV_0}{4\sqrt{km}}$

15. A uniform magnetic field of intensity 1 T is applied in a circular region of radius 0.1 m, directed into the plane of paper. A charged particle of mass 5×10^{-5} kg and charge $q = 5\times10^{-4}$ C enters the field with velocity $\frac{1}{\sqrt{3}}$ ms^{-1} making an angle of ϕ with a radial line of circular region in such a way that it passes through centre of applied field the angle ϕ is

(A) 30° (B) 45°

(C) 60° (D) 90°

16. The negatively and uniformly charged nonconducting disc as shown is rotated clockwise as shown in Figure.



The direction of the magnetic field at point A in the plane of the disc is

(A) into the page (B) out of the page

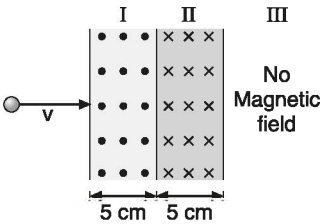
(C) up to the page (D) down the page

17. A particle of mass m and charge q starts moving from the origin under the action of an electric field $\vec{E} = E\hat{i}$ and magnetic field $\vec{B} = B\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle becomes $2v_0$ after a time t. Then t equals

(A) $2\left(\frac{mv_0}{qE}\right)$ (B) $2\left(\frac{m}{qB}\right)$

(C) $\sqrt{3}\left(\frac{m}{qB}\right)$ (D) $\sqrt{3}\left(\frac{mv_0}{qE}\right)$

18. Two uniform magnetic fields exist in two regions each consists of width 5 cm as shown in Figure. In the first region I, the magnetic induction is 0.001 T outwards and in region II the magnetic induction is 0.002 T inwards. The minimum speed of an electron entering region I as shown in Figure so that it can come out from region II to enter region III is (Take mass of electron to be 9×10^{-31} kg)



(A) $\frac{8}{9}\times10^7$ ms^{-1} (B) $\frac{4}{9}\times10^7$ ms^{-1}

(C) $\frac{16}{9}\times10^7$ ms^{-1} (D) $\frac{4}{7}\times10^7$ ms^{-1}

19. If W_E is the work done by an electric force in displacing a charged particle and W_M is the work done by a magnetic force associated with a steady magnetic field to displace the charged particle, then

(A) $W_E = W_M = 0$ (B) $W_E = 0;\ W_M \neq 0$

(C) $W_E \neq 0;\ W_M = 0$ (D) $W_E \neq 0;\ W_M \neq 0$

20. A straight section PQ of a circuit lies along the x-axis from $x = -\left(\frac{a}{2}\right)$ to $x = +\left(\frac{a}{2}\right)$ and carries a steady current I. The magnetic field due to the section PQ at a point $x = +a$ will be

(A) proportional to a (B) proportional to a^2

(C) proportional to $\left(\frac{1}{a}\right)$ (D) equal to zero

21. A charge q moves with a velocity 2 ms^{-1} along x-axis in a uniform magnetic field $\vec{B}=(\hat{i}+2\hat{j}+3\hat{k})T$. The charge will experience a force that lies

(A) along $-y$ axis (B) along $-z$ axis

(C) along $+z$ axis (D) in yz plane

22. A wire of mass 100 g is carrying a current of 2 A towards increasing x in the form of $y=x^2$ where $-2\text{ m}\le x\le +2\text{ m}$. This wire is placed in a magnetic field $\vec{B}=-0.02\hat{k}$ tesla. The acceleration of the wire (in ms^{-2}) is

(A) $-1.6\hat{j}$ (B) $-3.2\hat{j}$

(C) $1.6\hat{j}$ (D) zero

23. A particle is moving with velocity $\vec{v}=\hat{i}+3\hat{j}$ and at a point it produces an electric field given by $\vec{E}=2\hat{k}$. The magnetic field produced by the particle at the same point is

(A) $\dfrac{6\hat{i}-2\hat{j}}{c^2}$

(B) $\dfrac{6\hat{i}+2\hat{j}}{c^2}$

(C) zero

(D) true only if the charge is moving perpendicular to the $\vec{r}$

24. A proton experiences a force $\vec{F}_1=e(-\hat{j}+\hat{k})$ N in a magnetic field $\vec{B}$ when it has a velocity $\vec{v}_1=1\hat{i}$ ms^{-1}. The force becomes $\vec{F}=e(\hat{i}-\hat{k})$ N when the velocity is changed to $\vec{v}_2=1\hat{j}$ ms^{-1}. The magnetic induction vector at that point is

(A) $(\hat{i}-\hat{j}-\hat{k})T$ (B) $(-\hat{i}-\hat{j}+\hat{k})T$

(C) $(\hat{i}+\hat{j}-\hat{k})T$ (D) $(\hat{i}+\hat{j}+\hat{k})T$

25. A tightly wound, long solenoid carries a current of 2.00 A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of 1.00×10^8 $revs^{-1}$. The number of turns per metre in the solenoid will be

(A) 142 turns/m (B) 1420 turns/m

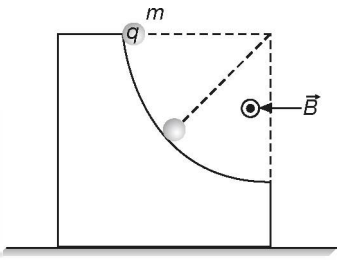
(C) 152 turns/m (D) 1520 turns/m

26. The dimensional formula of magnetic flux is

(A) $MLT^{-3}A^2$ (B) $ML^{-2}T^2A$

(C) $ML^2T^{-2}A^{-1}$ (D) $ML^{-2}TA^{-1}$

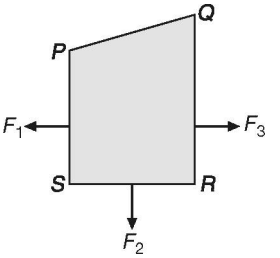
27. In the figure a charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position shown. There exists a uniform and constant horizontal magnetic field of induction B. The maximum force exerted by the track on the sphere is



(A) $mg+qB\sqrt{2gR}$ (B) $3mg-qB\sqrt{2gR}$

(C) $3mg+qB\sqrt{2gR}$ (D) $mg-qB\sqrt{2gR}$

28. A closed loop $PQRS$ carrying a current is placed in a uniform magnetic field. If the magnetic forces on segments PS, SR and RQ are F_1, F_2 and F_3 respectively and are in the plane of the paper and along the directions shown, the force on the segment QP is



(A) $\sqrt{(F_3-F_1)^2-F_2^2}$ (B) $F_3-F_1+F_2$

(C) $F_3-F_1-F_2$ (D) $\sqrt{(F_3-F_1)^2+F_2^2}$

29. In a region of space that extents from $x=0$ to $x=L$, a uniform magnetic field of magnitude B directed along the negative z-axis. A charged particle is projected with velocity v_0 at $x=0$ along positive x-axis. The particle emerges at $x=L$ after suffering a deviation of 60°. The velocity with which the same particle is projected at $x=0$ along positive x-axis so that when it emerges at $x=L$, the deviation suffered by it is 30° is

(A) $2v_0$ (B) $\dfrac{v_0}{2}$

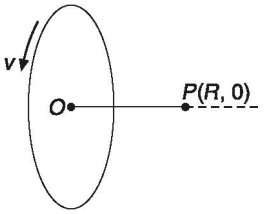
(C) $\dfrac{v_0}{\sqrt{3}}$ (D) $v_0\sqrt{3}$

30. A charged particle moving along positive x-direction with a velocity v enters a region where there is a uniform magnetic field $\vec{B}=-B\hat{k}$ from $x=0$ to $x=d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is

(A) $\frac{Bd}{v\cos\theta}$ (B) $\frac{v\tan\theta}{Bd}$

(C) $\frac{B\sin\theta}{vd}$ (D) $\frac{v\sin\theta}{Bd}$

31. A uniformly charged ring of radius R is rotated about its axis with constant linear speed v of each of its particles. The ratio of electric field to magnetic field at a point P on the axis of the ring distant $x = R$ from centre of ring is (c is speed of light)



(A) $\frac{c^2}{v}$ (B) $\frac{v^2}{c}$

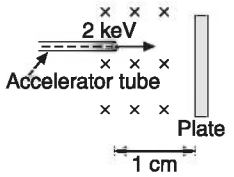
(C) $\frac{v}{c}$ (D) $\frac{c}{v}$

32. A particle of specific charge α is released from origin with a velocity $\vec{v} = v_0\hat{i}$ in a uniform magnetic field $\vec{B} = -B_0\hat{k}$. If the particle passes through the point $(0, y, 0)$, then y equals

(A) $-\frac{v_0}{B_0\alpha}$ (B) $-\frac{2v_0}{B_0\alpha}$

(C) $\frac{v_0}{B_0\alpha}$ (D) $\frac{2v_0}{B_0\alpha}$

33. A charge particle having mass of 1.6×10^{-26} kg comes out of accelerator tube with kinetic energy 2 keV. The smallest magnitude of magnetic field that should be applied in vertically downwards direction to just prevent the charge particle from colliding the plate (assuming charge on particle to be equal to charge of proton) is



(A) 2 T (B) 4 T

(C) 0.02 T (D) 0.04 T

34. A particle of mass m and charge q is projected towards negative x-axis with speed v from a point $(d, 0, 0)$ in a region of space where a uniform magnetic field $\vec{B} = B_0\hat{j}$ exists. The maximum value of v for which the particle does not hit the y-z plane is v_0. Then v_0 equals

(A) $\frac{qB}{2md}$ (B) $\frac{qBd}{2m}$

(C) $\frac{qBd}{m}$ (D) $\frac{2qB}{md}$

35. A charged particle enters a uniform magnetic field with a velocity vector at an angle of 45° with the magnetic field. The pitch of helical path followed by the particle is p. The radius of the helix will be

(A) $\frac{p}{\sqrt{2}\pi}$ (B) $\sqrt{2}p$

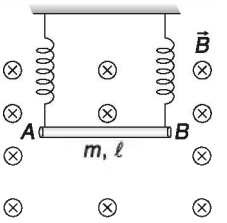
(C) $\frac{p}{2\pi}$ (D) $\frac{\sqrt{2}p}{\pi}$

36. The ratio of the magnetic field at the centre of a current carrying coil of radius a to the field at a distance $3a$ on its axis is

(A) $20\sqrt{10}$ (B) $2\sqrt{10}$

(C) $10\sqrt{10}$ (D) $\sqrt{10}$

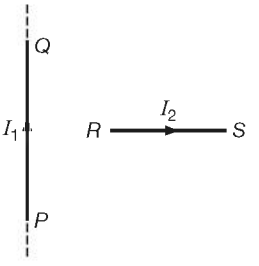
37. In the arrangement shown, if the extension in both the springs each of spring constant k increases from x to x_0 on flowing current I in the rod (of mass m, length l) from B to A, then the value of magnetic field will be



(A) $\frac{mg}{Il}$ (B) $\frac{mgx}{Il}$

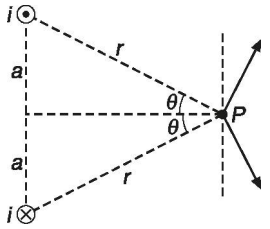
(C) $\frac{mgI}{Ix_0}$ (D) $\frac{mg}{Il}\left(\frac{x_0 - x}{x}\right)$

38. A current carrying wire RS is placed near an another long current carrying wire PQ. If free to move, wire RS will have



(A) translational motion only
(B) rotational motion only
(C) translational as well as rotational motion
(D) neither translational nor rotational motion

39. Two parallel wires situated at a distance $2a$ are carrying equal current i in opposite direction as shown in Figure.

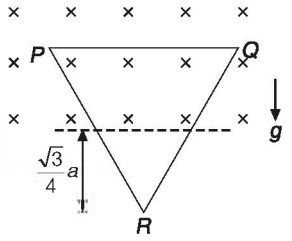


The value of magnetic field at a point P situated at equal distances from both the wires will be

(A) $\frac{\mu_0 ia}{\pi r}$ (B) $\frac{\mu_0 ia^2}{\pi r}$

(C) $\frac{\mu_0 ia^2}{\pi r^2}$ (D) $\frac{\mu_0 ia}{\pi r^2}$

40. An equilateral triangle frame PQR of mass M and side a is kept under the influence of magnetic force due to inward perpendicular magnetic field B and gravitational field as shown in Figure.



The magnitude and direction of current in the frame so that the frame remains at rest, is

(A) $I=\frac{2Mg}{aB}$, counter-clockwise

(B) $I=\frac{2Mg}{aB}$, clockwise

(C) $I=\frac{Mg}{aB}$, counter-clockwise

(D) $I=\frac{Mg}{aB}$, clockwise

41. Three infinitely long wires arranged along positive x, y and z directions have equal currents I flowing through them. The magnetic field at a point $P(0, 0, -a)$ is

(A) $\frac{\mu_0 I}{2\pi a}(\hat{i}+\hat{j})$ (B) $\frac{\mu_0 I}{2\pi a}(\hat{i}-\hat{j})$

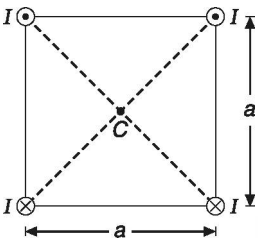
(C) $\frac{\mu_0 I}{2\pi a}(-\hat{i}+\hat{j})$ (D) $\frac{\mu_0 I}{2\pi a}(\hat{i}+\hat{j}+\hat{k})$

42. Current i_0 is being carried by an infinite wire passing through origin along the direction $\hat{i}+\hat{j}+\hat{k}$. The magnetic field due to the wire at point $(1\text{ m}, 1\text{ m}, 1\text{ m})$ is

(A) $\frac{\mu_0 i}{2\pi}$ T (B) $\frac{\mu_0 i}{\sqrt{2}\pi}$ T

(C) $\frac{\mu_0 i}{\sqrt{3}\pi}$ T (D) ZERO

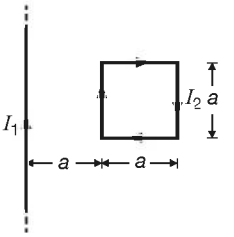
43. Four long and parallel wires each carrying current I are kept at the corners of a square having side a. Magnetic field produced at centre C is



(A) $\frac{2\mu_0 I}{\pi a}$ (B) $\sqrt{2}\frac{\mu_0 I}{\pi a}$

(C) $\frac{\mu_0 I}{2\pi a}$ (D) $\frac{4\mu_0 I}{\pi a}$

44. A current carrying square loop is placed near an infinitely long current carrying wire as shown in Figure.



The torque acting on the loop is τ. Then τ equals

(A) ZERO (B) $\frac{\mu_0 I_1 I_2 a}{2\pi}$

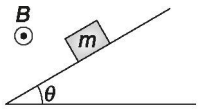
(C) $\frac{\mu_0}{2\pi}\left(\frac{I_1 I_2 a}{2}\right)$ (D) $\frac{\mu_0 I_1 I_2 a}{2\pi}\log_e(2)$

45. Four very long straight wires carry equal electric currents in the $+z$- direction. They intersect the x-y plane at $(x, y)=(-a, 0)$, $(0, a)$ $(a, 0)$ and $(0, -a)$. The magnetic force exerted on the wire at position $(-a, 0)$ is along

(A) $+y$-axis (B) $-y$-axis

(C) $+x$-axis (D) $-x$-axis

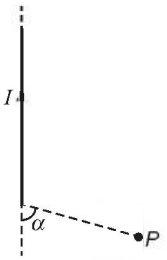
46. A block of mass m and charge q is released from rest on a long smooth inclined plane in the presence of a magnetic field B which is constant, uniform, horizontal and parallel to the surface as shown in Figure. The time from start when block loses contact with the surface is



(A) $\frac{m\cos\theta}{qB}$ (B) $\frac{m\operatorname{cosec}\theta}{qB}$

(C) $\frac{m\cot\theta}{qB}$ (D) $\frac{m\sin\theta}{qB}$

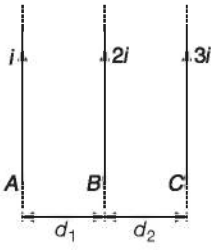
47. For the semi-infinite wire shown, a point P is taken at a distance r from its finite end as shown. Assume that the current in the wire is I, the field at P is



(A) $\frac{\mu_0 I}{4\pi r}$ (B) $\frac{\mu_0 I}{4\pi r}\sin\alpha$

(C) $\frac{\mu_0 I}{4\pi r}\cot\left(\frac{\alpha}{2}\right)$ (D) $\frac{\mu_0 I}{4\pi r}\tan\left(\frac{\alpha}{2}\right)$

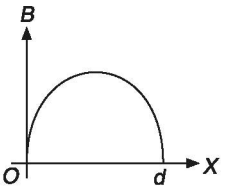
48. In the figure shown all the wires are long and parallel, if net force per unit length on wire B is zero, then $\frac{d_1}{d_2}$ will be

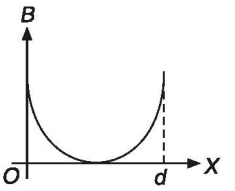


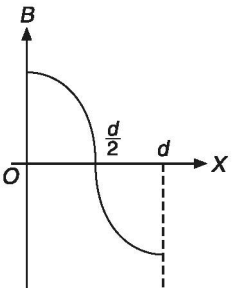
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{4}$

49. Two parallel current carrying wires carrying equal currents I in the same direction are fixed at a separation d. P is a point on a line joining the wires, at a distance x from any one of them. The magnetic field at P is B. The plot of B vs x is best represented by

(A)


(B)


(C)


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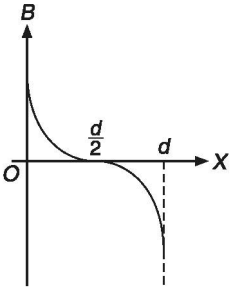
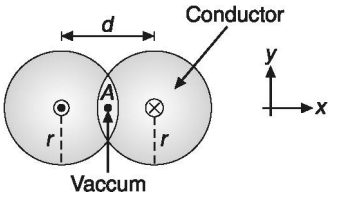


50. Two long conductors are arranged as shown in Figure to form overlapping cylinders, each of radius r, having their centres separated by a distance d. A current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown in Figure.



The magnitude and direction of the magnetic field at point A is given by

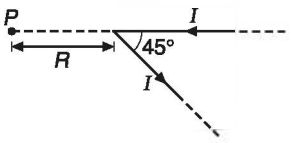
(A) $\left(\frac{\mu_0}{2\pi}\right)\pi dJ$, along the $+y$- direction

(B) $\left(\frac{\mu_0}{2\pi}\right)\frac{d^2}{r}$, along the $+y$- direction

(C) $\left(\frac{\mu_0}{2\pi}\right)\frac{4d^2J}{r}$, along the $-y$- direction

(D) $\left(\frac{\mu_0}{2\pi}\right)\frac{Jr^2}{d}$, along the $-y$- direction

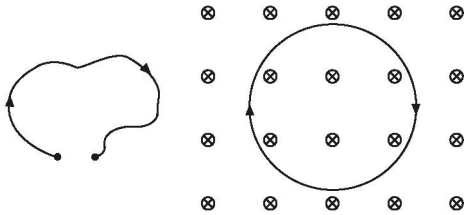
51. A long straight wire carrying current I, is bent at its midpoint to form an angle of $45°$ as shown in Figure. Magnetic induction at point P, distance R from point of bending is equal to



(A) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi R}$ (B) $\frac{(\sqrt{2}+1)\mu_0 I}{4\pi R}$

(C) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$ (D) $\frac{(\sqrt{2}-1)\mu_0 I}{2\sqrt{2}\pi R}$

52. A thin flexible wire of length L is connected to two adjacent fixed points carries a current I in the clockwise direction, as shown in Figure.

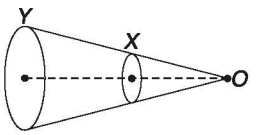


When system is put in a uniform magnetic field of strength B going into the plane of paper, the wire takes the shape of a circle. The tension in the wire is

(A) IBL (B) $\frac{IBL}{\pi}$

(C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

53. Two circular coils X and Y have equal number of turn and carry equal currents in the same sense and subtend same solid angle at point O. If the smaller coil X is midway between O and Y, then if we represent the magnetic induction due to bigger coil Y at O as B_Y and that due to smaller coil X at O as B_X, then



(A) $\frac{B_Y}{B_X}=1$ (B) $\frac{B_Y}{B_X}=2$

(C) $\frac{B_Y}{B_X}=\frac{1}{2}$ (D) $\frac{B_Y}{B_X}=\frac{1}{4}$

54. A particle of charge q and mass m is released from origin with velocity $\vec{v}=v_0\hat{i}$ in a magnetic field given by

$$\vec{B}=\begin{cases}-B_0\hat{k} & \text{for } x\le\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}\\ 0 & \text{for } x>\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}\end{cases}$$

The x-co-ordinates of the particle at time $t\left(>\frac{m\pi}{3qB_0}\right)$ is

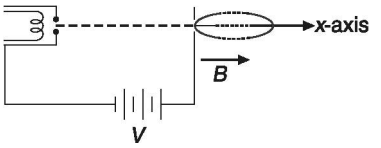
(A) $\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+\frac{\sqrt{3}}{2}v_0\left(t-\frac{m\pi}{qB_0}\right)$

(B) $\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+v_0\left(t-\frac{m\pi}{3qB_0}\right)$

(C) $\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+\frac{v_0 t}{2}$

(D) $\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+\frac{v_0}{2}\left(t-\frac{m\pi}{3qB_0}\right)$

55. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference V along the x-axis. These electrons emerge from a narrow hole into a uniform magnetic field of strength B directed along x-axis. Some electrons emerging at slightly divergent angles as shown in Figure.

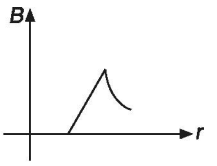


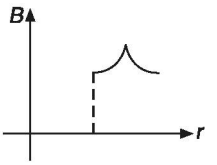
These paraxial electrons are refocused on the x-axis at a distance

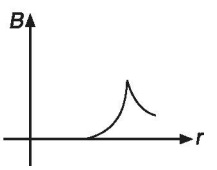
(A) $\sqrt{\frac{8\pi^2 mV}{eB^2}}$ (B) $\sqrt{\frac{2\pi^2 mV}{eB}}$

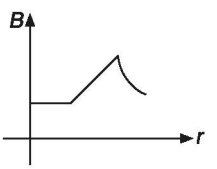
(C) $\sqrt{\frac{4\pi^2 mV}{eB^2}}$ (D) $\sqrt{\frac{2\pi^2 mV}{eB^2}}$

56. A current I is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius a and outer radius b. The variation of the magnetic field B with distance r from the axis of the cylinder is given by

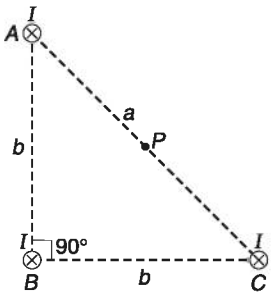
(A) 

(B) 

(C) 

(D) 

57. Three infinitely long straight wires A, B and C are arranged as shown in Figure.

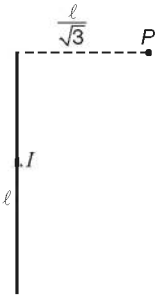


If the wires A, B and C each carry a current I into the plane of the paper then magnetic induction at mid-point (P) of AC is

(A) $\frac{\mu_0 I}{\pi a}$ (B) $\frac{3\mu_0 I}{\pi a}$

(C) $\frac{\sqrt{2}\mu_0 I}{\pi a}$ (D) ZERO

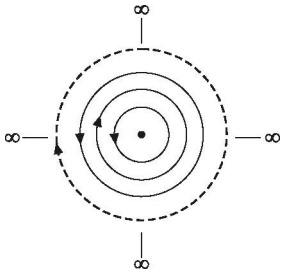
58. The magnitude of magnetic field produced by the current carrying straight wire of length l carrying a current I, at point P is B. Then B equals



(A) $\frac{\mu_0 I}{4\pi l}$ (B) $\frac{\sqrt{2}\mu_0 I}{\pi l}$

(C) $\frac{\mu_0 I}{2\sqrt{2}\pi l}$ (D) $\frac{3\mu_0 I}{8\pi l}$

59. In figure, infinite conducting rings each having current I in the direction shown are placed concentrically in the same plane as shown in Figure.

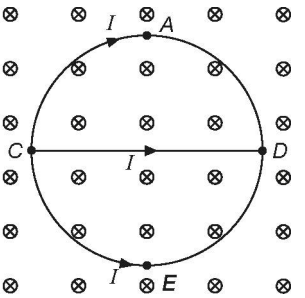


The radii of rings are r, $2r$, $2^2 r$, $2^3 r$,..., ∞. The magnetic field at the centre of rings will be

(A) ZERO (B) $\frac{\mu_0 I}{r}$

(C) $\frac{\mu_0 I}{2r}$ (D) $\frac{\mu_0 I}{3r}$

60. Consider a wireframe shown in figure. Equal currents I flow through all the three branches of the wire frame. The frame is a combination of two semicircles ACD and CDE of radius a. It is placed in uniform magnetic field B acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(A) $6BIa$ (B) $3BIa$

(C) BIa (D) ZERO

61. An electron is projected with velocity v_0 in a uniform electric field E perpendicular to the field. Again, it is projected with velocity v_0 perpendicular to a uniform magnetic field B. If r_1 is initial radius of curvature just after entering in the electric field and r_2 is the final radius of curvature just after entering in magnetic field, then the ratio $\frac{r_1}{r_2}$ is equal to

(A) $\frac{Bv_0^2}{E}$ (B) $\frac{B}{E}$

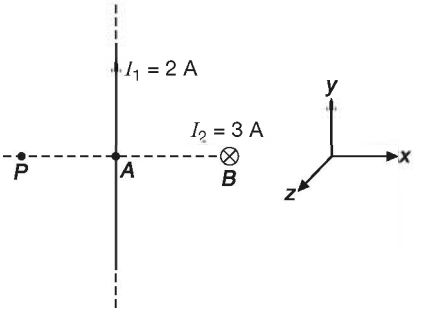
(C) $\frac{Ev_0}{B}$ (D) $\frac{Bv_0}{E}$

62. A charged particle enters a magnetic field at right angles to the magnetic field. The field exists in space for a length equal to 1.5 times the radius of the circular path of the particle. The particle will be deviated from its path by

(A) $\frac{\pi^c}{2}$ (B) $\sin^{-1}\left(\frac{2}{3}\right)$

(C) $\frac{\pi^c}{6}$ (D) π^c

63. Two infinitely long straight wires are arranged perpendicular to each other and are in mutually perpendicular planes as shown in Figure.



If $I_1 = 2$ A along the y-axis and $I_2 = 3$ A along negative z-axis and $AP = AB = 1$ cm. The magnetic induction $\vec{B}$ at point P is

(A) $(3\times10^{-5}\ \text{T})\hat{j}+(-4\times10^{-5}\ \text{T})\hat{k}$

(B) $(3\times10^{-5}\ \text{T})\hat{j}+(4\times10^{-5}\ \text{T})\hat{k}$

(C) $(4\times10^{-5}\ \text{T})\hat{j}+(3\times10^{-5}\ \text{T})\hat{k}$

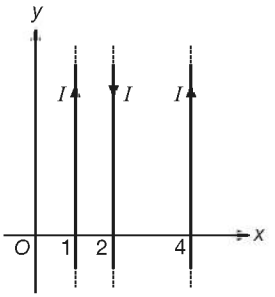
(D) $(-3\times10^{-5}\ \text{T})\hat{j}+(4\times10^{-5}\ \text{T})\hat{k}$

64. A small circular coil of radius r and number of turns n is placed at the centre of another big circular coil of radius R and number of turns N. Initially the two coils are coplanar and the same current I flows through both the coils. Then the amount of work done in rotating the small coil about any of its diameter by an angle π will be

(A) $\dfrac{2\mu_0\pi nNr^2I^2}{R}$ (B) $\dfrac{\mu_0\pi nNr^2I^2}{2R}$

(C) $\dfrac{\mu_0\pi nNr^2I^2}{R}$ (D) ZERO

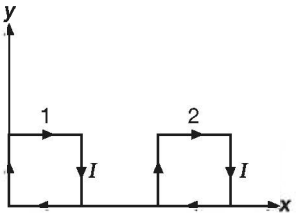
65. A considerably large number of very long wires are placed parallel to the y-axis at locations with x values given by 1 m, 2 m, 4 m, 8 m and so on. Each wire carries a current of 1 A and every consecutive wire having current in opposite direction. The magnetic field at origin (in tesla) is



(A) $-1.33\times10^{-7}\hat{k}$ (B) $-2.67\times10^{-7}\hat{k}$

(C) $1.33\times10^{-7}\hat{k}$ (D) $2.67\times10^{-7}\hat{k}$

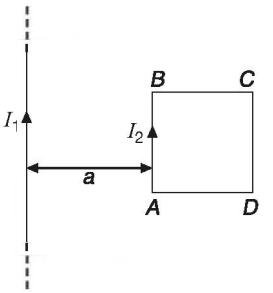
66. Magnetic field in a region is given by $\vec{B} = B_0x\hat{k}$. Two loops each of side a is placed in this magnetic region in the xy-plane with one of its sides on x-axis. If F_1 is the force on loop 1 and F_2 be the force on loop 2, then



(A) $F_1 = F_2 = 0$ (B) $F_1 > F_2$

(C) $F_2 > F_1$ (D) $F_1 = F_2 \neq 0$

67. A square loop of side a is placed at a distance a away from a long wire carrying a current I_1. If the loop carries a current I_2 as shown in Figure, then the nature of the force and its magnitude is



(A) $\dfrac{\mu_0I_1I_2}{2\pi a}$, attractive (B) $\dfrac{\mu_0I_1I_2}{4\pi}$, attractive

(C) $\dfrac{\mu_0I_1I_2}{4\pi}$, repulsive (D) $\dfrac{\mu_0I_1I_2}{4\pi a}$, repulsive

68. An electron is moving along positive x-axis. A uniform electric field exists towards negative y-axis. The direction of magnetic field of suitable magnitude so that net force on electron is zero is along

(A) positive z-axis (B) positive y-axis

(C) negative z-axis (D) negative y-axis

69. A metallic wire is folded to form a square loop of side a. It carries a current I and is kept perpendicular to the region of uniform magnetic field B. If the shape of the loop is changed from square to an equilateral triangle without changing the length of the wire and current, the amount of work done in doing so is

(A) $BIa^2\left(1-\dfrac{4\sqrt{3}}{9}\right)$ (B) $BIa^2\left(1-\dfrac{\sqrt{3}}{9}\right)$

(C) $\dfrac{2}{3}BIa^2$ (D) ZERO

70. If E and B denote electric and magnetic fields respectively, which of the following is dimensionless?

(A) $\sqrt{\mu_0\varepsilon_0}\left(\frac{B}{E}\right)$ (B) $(\mu_0\varepsilon_0)\left(\frac{E}{B}\right)$

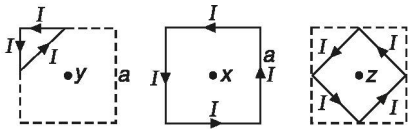
(C) $\frac{1}{\mu_0\varepsilon_0}\left(\frac{B}{E}\right)^2$ (D) $\left(\frac{\mu_0}{\varepsilon_0}\right)\left(\frac{E}{B}\right)$

71. A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of induction 4×10^{-4} T. The magnetic field is acting parallel to the direction of current. The magnitude of the resultant magnetic induction in tesla at a point 2.0 cm away from the wire is

(A) 10^{-4} (B) 3×10^{-4}

(C) 5×10^{-4} (D) 6×10^{-4}

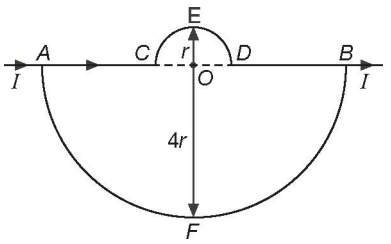
72. The magnetic flux density in vacuum at the centre of any square coil (of one turn) of side a and carrying a current I is $\frac{KI}{a}$ where K is independent of a. The magnitude of induction at x, y, z are B_1, B_2 and B_3 respectively. Then



(A) $B_3 > B_1 > B_2$ (B) $B_2 > B_3 > B_1$

(C) $B_2 > B_1 > B_3$ (D) $B_1 > B_2 > B_3$

73. In the shown in Figure AC and BD are straight lines and CED and AFB are semi-circular with radii r and $4r$, respectively.



The entire setup is lying in the same plane. If I is current entering at A what fraction of I will flow in the $ACEDB$ such that resultant magnetic field at O is zero

(A) 1 (B) $\frac{3}{5}$

(C) $\frac{4}{5}$ (D) $\frac{1}{5}$

74. A uniform magnetic field exists in region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicularly with speed v along perpendicular bisector to one side and comes out along perpendicular bisector to the other side. The magnetic field in the triangle is

(A) $\frac{mv}{qa}$ (B) $\frac{2mv}{qa}$

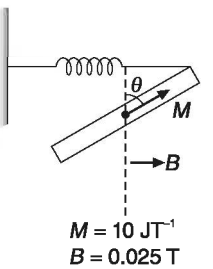
(C) $\frac{mv}{2qa}$ (D) $\frac{mv}{4qa}$

75. A wire of length l is bent in the form of a circular coil of some turns. A current I flows through the coil. The coil is placed in a uniform magnetic field B. The maximum torque on the coil can be τ_0. Then τ_0 equals

(A) $\frac{BIl^2}{2\pi}$ (B) $\frac{2BIl^2}{\pi}$

(C) $\frac{BIl^2}{4\pi}$ (D) $\frac{BIl^2}{\pi}$

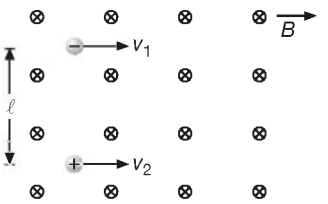
76. A spring is connected to the end of a magnetic dipole having magnetic moment of 10 JT^{-1} which is pivoted about its mid-point and is placed in a uniform external field of 0.025 T as shown in Figure. The torque exerted on the dipole by the spring can be represented by $\tau = C\theta$, where $C = 1$ $Nmrad^{-1}$ and θ is a small angle in radian. Under equilibrium, the angle θ is



(A) 0.24 radian (B) 4.12 radian

(C) 0.36 radian (D) 0.18 radian

77. Two identical particles having the same mass m and charges $+q$ and $-q$ separated by a distance l enter in a uniform magnetic field B directed perpendicular to paper inwards with speeds v_1 and v_2 as shown in Figure.

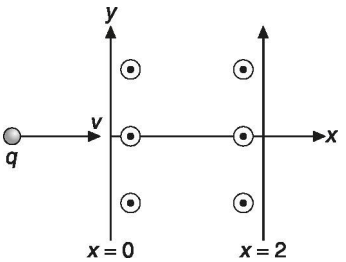


The particles will not collide when (Ignore electrostatic force between the particles)

(A) $v_1 = v_2$ (B) $l > \frac{m}{qB}(v_1 + v_2)$

(C) $l < \frac{m}{qB}(v_1 + v_2)$ (D) $l > \frac{2m}{qB}(v_1 + v_2)$

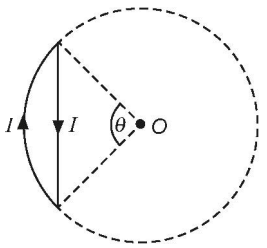
78. A charge $q = +2\ \mu C$ enters a uniform magnetic field $B = 2\sqrt{3} \times 10^{-3}\hat{k}$ tesla while travelling along positive x-axis. The magnetic field exists in the region $x = 0$ and $x = 2$ m. The particle emerges from magnetic field region such that its deviation from initial direction is $60°$. Magnitude of linear momentum of particle is



(A) 2.4×10^{-9} kgms^{-1} (B) 1.6×10^{-8} kgms^{-1}

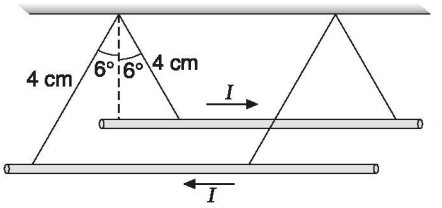
(C) 1.2×10^{-6} kgms^{-1} (D) 3.2×10^{-9} kgms^{-1}

79. For the current carrying loop shown, the net magnetic field at the centre of the circle O due to it, for $\theta < 180°$ is B. Then B is



(A) perpendicular to paper inwards
(B) perpendicular to paper outwards
(C) perpendicular to paper inwards if $\theta \le 90°$ and perpendicular to paper outwards if $90° \le \theta < 180°$
(D) ZERO

80. Two long parallel wires are hung by 4 cm long cords from a common axis. The wires have mass 0.0125 kgm^{-1} and carry equal currents in opposite direction. The current in each wire if the cords hang at $6°$ with the vertical (as shown in Figure) is (Take $g = 10$ ms^{-2})



(A) 11.2 A (B) 22.3 A

(C) 717 A (D) 71.7 A

81. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c, respectively. The inner wire carries a current I and outer shell carries an equal and opposite current. The magnetic field at a distance x from the axis where $b < x < c$ is

(A) $\frac{\mu_0 I}{2\pi x}\left(\frac{c^2 - b^2}{c^2 - a^2}\right)$ (B) $\frac{\mu_0 I}{2\pi x}\left(\frac{c^2 - x^2}{c^2 - a^2}\right)$

(C) $\frac{\mu_0 I}{2\pi x}\left(\frac{c^2 - x^2}{c^2 - b^2}\right)$ (D) ZERO

82. A charged particle of specific charge α is released from origin at time $t = 0$ with velocity $\vec{v} = v_0(\hat{i} + \hat{j})$ in a uniform magnetic field $\vec{B} = B_0\hat{i}$. At time $t = \frac{\pi}{B_0\alpha}$ the particle is at the point. $P(x, y, z)$. Then we have the coordinates of the point P as

(A) $\left(\frac{-v_0}{2B_0\alpha}, 0, 0\right)$

(B) $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$

(C) $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

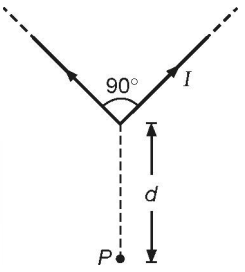
(D) $\left(\frac{v_0}{2B_0\alpha}, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$

83. A straight wire of length L can slide on two parallel plastic rails kept in horizontal plane with a separation d. The coefficient of friction between the wire and the rails is μ. If the wire carries current i what minimum magnetic field should exist in the space in order to slide the wire on the rails?

(A) $\frac{\mu mg}{iL\sqrt{1+\mu^2}}$ (B) $\frac{\mu m^2 g}{iL}$

(C) $\frac{\mu mg^2}{iL}$ (D) $\frac{\mu mg}{L}$

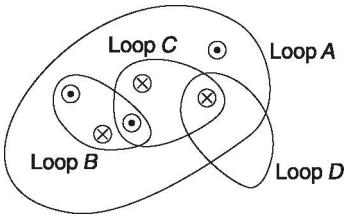
84. In the current carrying arrangement shown, the magnetic field at the point P is



(A) $\frac{\mu_0 I}{\sqrt{2}\pi d}\left(1+\frac{1}{\sqrt{2}}\right), \otimes$ (B) $\frac{\mu_0 I}{\sqrt{2}\pi d}\left(1-\frac{1}{\sqrt{2}}\right), \otimes$

(C) $\frac{\sqrt{2}\mu_0 I}{\pi d}, \otimes$ (D) $\frac{\mu_0 I}{\sqrt{2}\pi d}, \otimes$

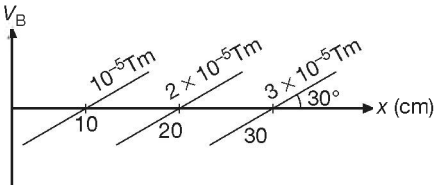
85. Consider six wires coming into or out of the page as shown in figure, all with the same current. Rank the line integral of the magnetic field from most positive to most negative taken counter clockwise around each loop shown in Figure.



(A) $B > C > D > A$ (B) $B > C = D > A$

(C) $B > A > C = D$ (D) $C > B = D > A$

86. Figure shows some of the equipotential surfaces drawn for the magnetic scalar potential. The magnetic field at a point in the region can then be given by



(A) 2×10^{-5} T (B) 10^{-5} T

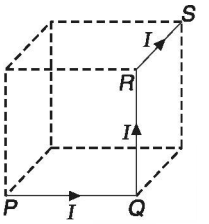
(C) 10^{-4} T (D) 2×10^{-4} T

87. For a current carrying circular coil, the ratio of the magnetic field at the centre of a current carrying circular coil to its magnetic moment is x. On doubling the radius and the current, the x values

(A) decreases by $\frac{x}{8}$ (B) increases by $\frac{x}{8}$

(C) decreases by $\frac{7x}{8}$ (D) increases by $\frac{7x}{8}$

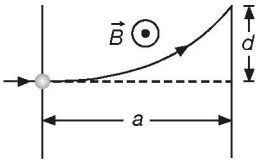
88. A wire $PQRS$ carrying a current I run along three edges of a cube of side l as shown. There exists a uniform magnetic field of magnitude B along one of the sides of the cube. The magnitude of the force acting on the wire is



(A) ZERO (B) BIl

(C) $\sqrt{2}BIl$ (D) $2BIl$

89. A charged particle q enters a region of uniform $\vec{B}$ (out of the page) and is deflected a distance d after travelling a horizontal distance a. The magnitude of the momentum of the particle is



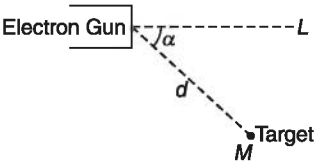
(A) $\frac{1}{2}qB\left[\frac{a^2}{d}+d\right]$

(B) $\frac{1}{2}qBa$

(C) ZERO

(D) not possible to be determined as it keeps changing

90. An electron gun T emits an electron of mass m that is accelerated by a potential difference V in a vacuum in the direction of the line L as shown in Figure. A target M is placed at a distance d as shown in Figure.

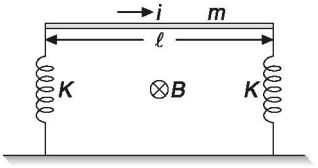


The magnetic field perpendicular to the plane determined by line L and the point M so that the electron hits the target M is

(A) $2\sqrt{\frac{2Vm}{e}}\frac{\sin\alpha}{d}$ (B) $\sqrt{\frac{2Vm}{e}}\frac{\sin\alpha}{2d}$

(C) $\sqrt{\frac{2Vm}{e}}\frac{\sin\alpha}{d}$ (D) $8\sqrt{\frac{2Vm}{e}}\frac{\sin\alpha}{d}$

91. A horizontal metallic rod of mass m and length l is supported by two vertical identical springs of spring constant k each and natural length l_0. A current i is flowing in the rod in the direction shown. If the rod is in equilibrium, then the length of each spring in this state is



(A) $l_0+\frac{ilB-mg}{k}$ (B) $l_0+\frac{ilB-mg}{2k}$

(C) $l_0+\frac{mg-ilB}{2k}$ (D) $l_0+\frac{mg-ilB}{k}$

92. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in the same directions. Let B_1 and B_2 be the magnetic fields in the regions between the conductors, and outside the conductor, respectively

(A) $B_1=B_2=0$ (B) $B_1\neq 0,\ B_2\neq 0$

(C) $B_1=0,\ B_2\neq 0$ (D) $B_1\neq 0,\ B_2=0$

93. The magnetic field B due to a current carrying circular loop of radius 12 cm at its centre is 0.5×10^{-4} T. The magnetic field due to this loop at a point on the axis at a distance of 5 cm from the centre

(A) 3.9×10^{-5} T (B) 5.2×10^{-5} T

(C) 2.1×10^{-5} T (D) 9×10^{-5} T

94. The pressure experienced by the lateral surface of a long straight solenoid having n turns per unit length carrying a current I through it is P. Then

(A) $P=\mu_0n^2I^2$ (B) $P=\frac{\mu_0n^2I^2}{2}$

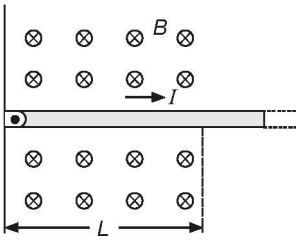
(C) $P=\frac{\mu_0n^2I^2}{3}$ (D) $P=\frac{\mu_0n^2I^2}{4}$

95. A proton, a deuteron and an α particle with the same kinetic energy enter in a region of uniform magnetic field, moving at right angles to B. What is the ratio of the radii of their circular paths?

(A) $1:\sqrt{2}:1$ (B) $1:\sqrt{2}:\sqrt{2}$

(C) $\sqrt{2}:1:1$ (D) $\sqrt{2}:\sqrt{2}:1$

96. A straight conductor of mass m and carrying a current I is hinged at one end and placed in a plane perpendicular to the magnetic field of intensity $\vec{B}$ as shown in Figure.

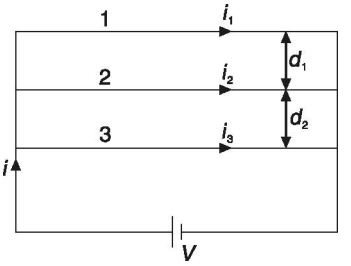


At any moment if the conductor is let free, then the angular acceleration of the conductor will be (neglect gravity)

(A) $\frac{IB}{2m}$ (B) $\frac{2IB}{3m}$

(C) $\frac{3IB}{2m}$ (D) $\frac{IB}{3m}$

97. Three long wires of resistances in the ratio $3:4:5$ are connected in parallel to each other as shown in Figure.



If net force on middle wire is zero, then $\frac{d_1}{d_2}$ will be

(A) $9:25$ (B) $5:3$

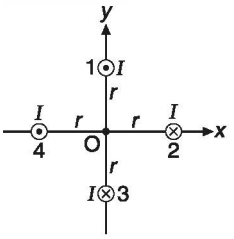
(C) $\sqrt{5}:\sqrt{3}$ (D) $3:5$

98. O^{++}, C^{+}, He^{++} and H^{+} ions are projected on the photographic plate with same velocity in a mass spectrograph. Which one will strike farthest?

(A) O^{++} (B) C^{+}

(C) He^{++} (D) H^{+}

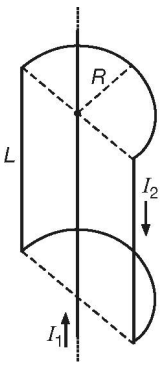
99. The magnitude of magnetic field at origin due to four infinite wires, if each wire produces a magnetic field B at origin is



(A) $4B$ (B) $\sqrt{2}B$

(C) $2\sqrt{2}B$ (D) ZERO

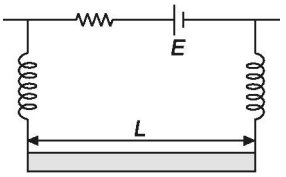
100. An infinitely long straight wire carrying a current I_1 is partially surrounded by a loop of length L, radius R and carrying a current I_2 as shown, with its axis coinciding with the wire. The force exerted on the loop is



(A) $\dfrac{\mu_0 \pi I_1 I_2}{R}$ (B) $\dfrac{\mu_0 I_1 I_2 L}{R}$

(C) $\dfrac{\mu_0 I_1 I_2 L}{\pi R}$ (D) $\dfrac{\mu_0 I_1 I_2 R}{\pi L}$

101. A straight rod of mass m and length L is suspended from the identical springs as shown in Figure.

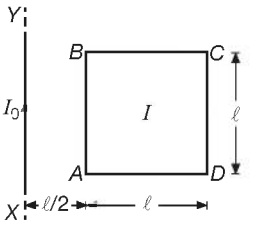


The spring stretched a distance x_0 due to the weight of the wire. The circuit has total resistance R. When the magnetic field perpendicular to the plane of paper is switched on, springs are observed to extend further by the same distance. The magnetic field strength is

(A) $\dfrac{2mgR}{EL}$ (B) $\dfrac{mgR}{EL}$

(C) $\dfrac{mgR}{2EL}$ (D) $\dfrac{mgR}{E}$

102. A square loop $ABCD$, carrying a current I, is placed near and coplanar with a long straight conductor XY carrying a current I_0. The net force on the loop is



(A) $\dfrac{\mu_0 I I_0}{2\pi}$ (B) $\dfrac{\mu_0 I I_0}{2\pi}$

(C) $\dfrac{2\mu_0 I I_0}{3\pi}$ (D) $\dfrac{2\mu_0 I I_0}{3\pi}$

103. A wire along x-axis carries a current 3.5 A. The force, in newton, on a 1 cm section of the wire exerted by a magnetic field of $\vec{B} = \left(0.74\hat{j} - 0.3\hat{k}\right)$ T is

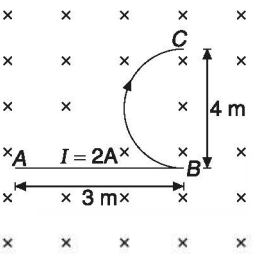
(A) $\left(2.59\hat{k} + 1.26\hat{j}\right) \times 10^{-2}$

(B) $\left(1.26\hat{k} - 2.59\hat{j}\right) \times 10^{-2}$

(C) $\left(-2.59\hat{k} - 1.26\hat{j}\right) \times 10^{-2}$

(D) $\left(-1.26\hat{k} + 2.59\hat{j}\right) \times 10^{-2}$

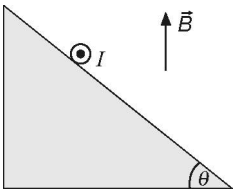
104. In the figure, the force on the wire ABC in the given uniform magnetic field will be $(B = 2$ tesla$)$



(A) $4(3 + 2\pi)$ N (B) 20 N

(C) 30 N (D) 40 N

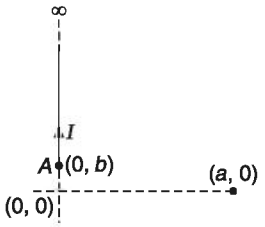
105. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current I is flowing in the conductor in a direction perpendicular to paper inwards and a vertically upward magnetic field $\vec{B}$ exists in space. Then



(A) $|\vec{B}| = \frac{mg}{Il}\sin\theta$ (B) $|\vec{B}| = \frac{mg}{Il}\tan\theta$

(C) $|\vec{B}| = \frac{mg\cos\theta}{Il}$ (D) $|\vec{B}| = \frac{mg}{Il\sin\theta}$

106. An infinitely long wire carrying current I is along y-axis such that its one end is at point $A(0, b)$ while the wire extends up to $+\infty$. The magnitude of magnetic field strength at point $(a, 0)$ is



(A) $\frac{\mu_0 I}{4\pi a}\left(1+\frac{b}{\sqrt{a^2+b^2}}\right)$ (B) $\frac{\mu_0 I}{4\pi a}\left(1-\frac{b}{\sqrt{a^2+b^2}}\right)$

(C) $\frac{\mu_0 I}{4\pi a}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ (D) None of these

107. The magnetic field at the centre of an equilateral triangular loop of side $2L$ carrying a current I is

(A) $\frac{3\sqrt{3}\mu_0 I}{4\pi L}$ (B) $\frac{2\sqrt{3}\mu_0 I}{\pi L}$

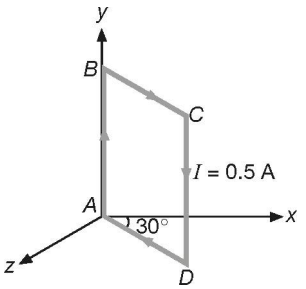
(C) $\frac{3\mu_0 I}{4\pi L}$ (D) $\frac{9\mu_0 I}{4\pi L}$

108. An electron enters into a homogeneous magnetic field perpendicular to the force lines. The velocity of the electron is $v = 4\times10^7$ ms^{-1}. The induction of the field is 10^{-3} T. The tangential acceleration of electron in the magnetic field is

(A) 7×10^{15} ms^{-2} (B) 7×10^{13} ms^{-2}

(C) 7×10^{14} ms^{-2} (D) ZERO

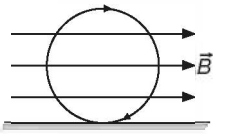
109. Figure shows a square current carrying loop $ABCD$ of side 2 m and current $I = 0.5$ A. The magnetic moment $\vec{M}$ of the loop is



(A) $(\hat{j}-\hat{k})$ Am^2 (B) $(\sqrt{3}\hat{i}+\hat{k})$ Am^2

(C) $(\hat{i}-\sqrt{3}\hat{k})$ Am^2 (D) $(\hat{i}+\hat{k})$ Am^2

110. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current $I = 4$ A. A horizontal magnetic field $B = 10$ T is switched on at time $t = 0$ as shown in Figure.



The initial angular acceleration of the ring is

(A) 5π rads^{-2} (B) 15π rads^{-2}

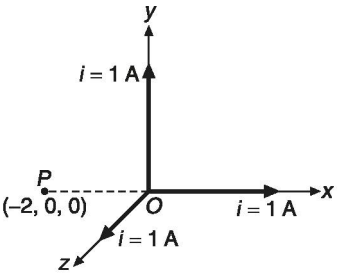
(C) 20π rads^{-2} (D) 40π rads^{-2}

111. A long coaxial cable consists of two hollow concentric cylinders of radii a and b. The central conductor of the cable carries a steady current I and outer conductor provides the return path of the same current. Calculate the energy stored in the magnetic field l of such a cable

(A) $\frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{a}{b}\right)$

(C) $\frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{2b}{a}\right)$ (D) $\frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{a}{2b}\right)$

112. Three infinitely long wires each carrying a current 1 A are placed such that one end of each wire is at origin and one of these wires are along x-axis, y-axis and z-axis. Magnetic induction at point $P(-2, 0, 0)$ m is



(A) $\frac{\mu_0}{4\pi}(\hat{j}+\hat{k})$ (B) $\frac{\mu_0}{4\pi}(\hat{j}-\hat{k})$

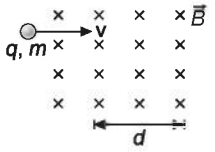
(C) $\frac{\mu_0}{8\pi}(-\hat{j}+\hat{k})$ (D) $\frac{\mu_0}{8\pi}(\hat{j}+\hat{k})$

113. A particle of mass m and charge q starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x_0, 0, 0)$ is $(4\hat{i}+3\hat{j})$. Then

(A) $x_0 = \dfrac{13qE_0}{2mB_0}$ (B) $x_0 = \dfrac{16qB_0}{mE_0}$

(C) $x_0 = \dfrac{25m}{2qE_0}$ (D) $x_0 = \dfrac{5q}{2B_0m}$

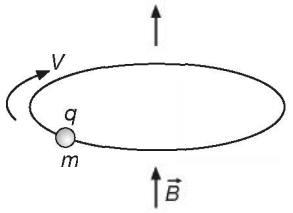
114. A charged particle of mass m and charge q is projected into a uniform magnetic field of induction B with speed v which is perpendicular to B. The width of the magnetic field is d. The impulse imparted to the particle by the field when $qBd \ll mv$ is



(A) qBv (B) $\dfrac{mv}{qB}$

(C) qBd (D) $\dfrac{2mv^2}{qB}$

115. A circular wire loop of radius r can withstand a radial force T before breaking. A particle of mass m and charge $q(q>0)$ is sliding over the wire. A magnetic field B is applied normal to the plane of the wire. The maximum speed V_{max} the particle can have before the loop breaks is



(A) ZERO

(B) $\sqrt{\dfrac{rT}{m}}$

(C) $\dfrac{r}{2}\left[qB + \sqrt{q^2B^2 + \dfrac{4T}{mr}}\right]$

(D) $\dfrac{r}{2}\left[\dfrac{qB}{m} + \sqrt{\dfrac{q^2B^2}{m^2} + \dfrac{4T}{mr}}\right]$

116. A cyclotron is accelerating proton, where the applied magnetic field is 2 T and the potential gap is 100 keV . To acquire a kinetic energy of 20 MeV , the number of turns, the proton has to move between the dees is

(A) 200 (B) 300

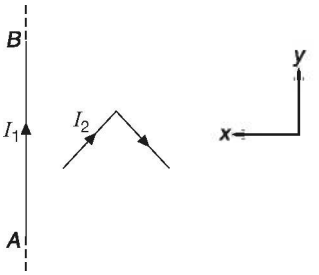
(C) 150 (D) 100

117. The field normal to the plane of coil of N turns, radius r carrying a current I at a point lying a small distance h $(\ll r)$ from the centre of coil is B_a and the field at the centre of coil is B_c.

(A) $\dfrac{B_c - B_a}{B_c} = \dfrac{2h^2}{3r^2}$ (B) $\dfrac{B_c - B_a}{B_c} = \dfrac{3r^2}{2h^2}$

(C) $\dfrac{B_c - B_a}{B_c} = \dfrac{3h}{2r}$ (D) $\dfrac{B_c - B_a}{B_c} = \dfrac{3h^2}{2r^2}$

118. In the figure shown a current I_1 is established in the long straight wire AB. Another wire CD carrying current I_2 is placed in the plane of the paper. The line joining the ends of this wire is perpendicular to the wire AB. The resultant force on the wire CD is



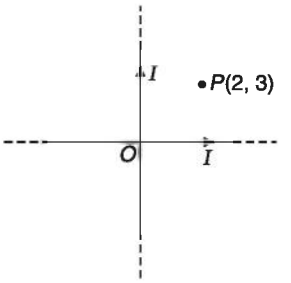
(A) ZERO
(B) towards negative x-axis
(C) towards positive y-axis
(D) in the xy plane

119. The dimensional formula of $\dfrac{B^2R^2C^2}{2\mu_0}$ (where B is magnetic field, and μ_0 is permeability of free space, R is resistance and C is capacitance) is same as that of

(A) qvB (B) $\dfrac{qB}{v}$

(C) $\dfrac{qB}{v^2}$ (D) qv^2B

120. Two mutually perpendicular insulated very conducting wires carrying equal currents I, intersect at origin. Then the resultant magnetic induction at point $P(2, 3)$ m will be



(A) $\frac{\mu_0 I}{5a}$ (B) $\frac{5\mu_0 I}{2\pi}$

(C) $\frac{\mu_0 I}{12\pi}$ (D) 0

121. A wire of length 1 m carrying a current of 1 A, placed in x-z plane. The coefficient of friction between the wire and the surface is 0.2 and mass of the wire is 2 kg. The magnetic field of strength 2 T exists along positive y- direction. Then choose the correct statement.

(A) Acceleration of wire is 0.5 ms^{-2}

(B) Acceleration of wire is 1 ms^{-2}

(C) Acceleration of wire is 2 ms^{-2}

(D) Wire will not move at all

122. A circular coil carrying a certain current produces a magnetic field B_0 at its centre. The coil is now rewound so as to have 3 turns and the same current is passed through it. The new magnetic field at the centre is

(A) $\frac{B_0}{9}$ (B) $9B_0$

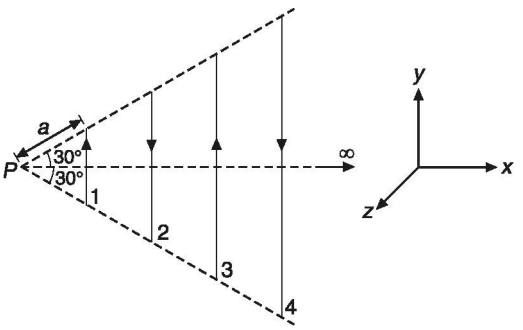
(C) $\frac{B_0}{3}$ (D) $3B_0$

123. A rigid circular loop of radius r and mass m lies in the x-y plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $\vec{B} = B_x\hat{i} + B_z\hat{k}$. The value of I so that one edge of the loop lifts from the table is

(A) $I = \frac{mg}{\pi r B_z}$ (B) $I = \frac{mg}{\pi r B_x}$

(C) $I = \frac{mg}{\pi r\sqrt{B_x^2 + B_z^2}}$ (D) $I = \frac{mg}{\pi r\sqrt{B_x B_z}}$

124. Infinite number of straight wires each carrying current I are equally spaced as shown in Figure.

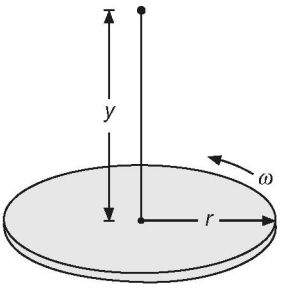


Adjacent wires have current in opposite direction. Net magnetic induction at point P is

(A) $\frac{\mu_0 I}{4\pi}\frac{\ln 2}{\sqrt{3}a}\hat{k}$ (B) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{k}$

(C) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}(-\hat{k})$ (D) ZERO

125. The magnetic field at distance y from the centre of the axis of a disc of radius r and uniform surface charge density σ, if the disc spins with angular velocity ω is



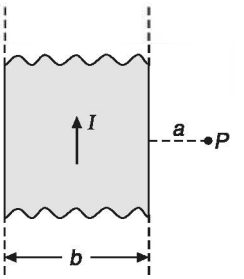
(A) $\frac{\mu_0\sigma\omega}{3}\left(\frac{r^2 - 2y^2}{\sqrt{r^2 - y^2}} + 2y\right)$

(B) $\frac{\mu_0\sigma\omega}{2}\left(\frac{r^2 + y^2}{\sqrt{r^2 + y^2}}\right)$

(C) $\frac{\mu_0\sigma\omega}{2}\left(\frac{r^2 + 2y^2}{\sqrt{r^2 + y^2}} - 2y\right)$

(D) $\frac{2\mu_0\sigma\omega}{3}\left(\frac{r^2 + 2y^2}{\sqrt{r^2 + y^2}} - 2y\right)$

126. A very long thin strip of metal of width b carries a current I along its length as shown in Figure.



The magnitude of magnetic field in the plane of the strip at a distance a from the edge nearest to the point is

(A) $\frac{\mu_0}{4\pi}\frac{2I}{b}\log_e\left(1 - \frac{b}{a}\right)$ (B) $\frac{\mu_0}{4\pi}\frac{2I}{b}\log_e\left(1 + \frac{b}{a}\right)$

(C) $\frac{\mu_0}{2\pi}\frac{2I}{b}\log_e\left(1 - \frac{b}{a}\right)$ (D) $\frac{\mu_0 I}{\pi b}\left(1 + \frac{b}{a}\right)$

127. A non-conducting disc of radius R, having charge q distributed uniformly over its surface is rotating about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. The magnetic moment of the disc is M. Then M equals

(A) $q\omega R^2$ (B) $\frac{1}{2}q\omega R^2$

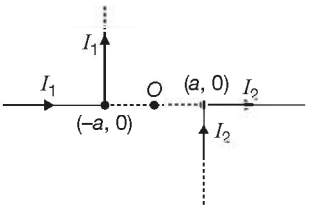
(C) $\frac{1}{8}q\omega R^2$ (D) $\frac{1}{4}q\omega R^2$

128. A particle of mass m and charge q is launched from origin at $t=0$ with velocity $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ ms^{-1} in a region with uniform magnetic field $\vec{B}=2\hat{i}$ tesla. After time $t=\frac{\pi m}{qB}$, an electric field $\vec{E}$ is switched on such that particle moves on a straight line with constant speed. $\vec{E}$ may be

(A) $5\hat{k}-10\hat{j}$ units (B) $-6\hat{k}-9\hat{j}$ units

(C) $-6\hat{k}+8\hat{j}$ units (D) $6\hat{k}+8\hat{j}$ units

129. Currents I_1 and I_2 flow in the wires shown in Figure.

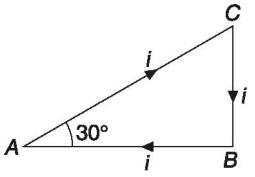


The field is zero at distance x to the right of O. Then,

(A) $x=\left(\frac{I_1}{I_2}\right)a$ (B) $x=\left(\frac{I_2}{I_1}\right)a$

(C) $x=\left(\frac{I_1-I_2}{I_1+I_2}\right)a$ (D) $x=\left(\frac{I_1+I_2}{I_1-I_2}\right)a$

130. A wire bent in the form of a right-angled triangle ABC as shown in Figure carries a current 1 A. It is placed in the region of a uniform magnetic induction field $B=0.2$ T as shown in Figure.

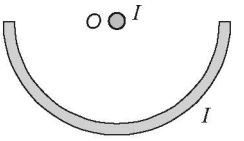


If $AC=1$ m. The net force on the wire is

(A) 1.73 N (B) 3.46 N

(C) 2.732 N (D) 0

131. A direct current I flows in a long straight conductor whose cross-section has the form of a thin half-ring of radius R. The same current flows in the opposite direction along a thin conductor located on the axis of the first conductor. The magnetic interaction force between the given conductors reduced to a unit of their length is



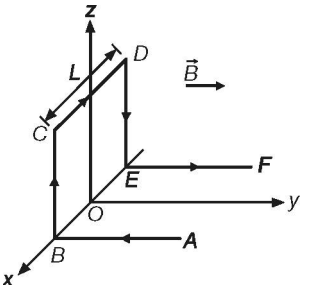
(A) $\frac{\mu_0 I^2}{2\pi R}$ (B) $\frac{\mu_0 I^2}{\pi R^2}$

(C) $\frac{\mu_0 I^2}{\pi^2 R}$ (D) $\frac{\mu_0 I}{\pi^2 R}$

132. A point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a point P on the axis of the circle. Then this magnetic field is

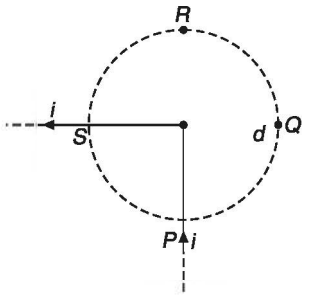
(A) constant in magnitude only.
(B) constant in direction only.
(C) constant both in direction and magnitude.
(D) not constant both in magnitude and direction.

133. A conductor $ABCDEF$, with each side of length L, is bent as shown. It is carrying a current I in a uniform magnetic induction B, parallel to the positive y direction. The force experienced by the wire is



(A) BIL in the positive y-direction
(B) BIL in the positive z-direction.
(C) $3BIL$ in the positive y-direction
(D) ZERO

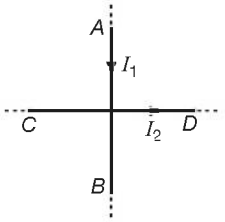
134. A long wire is bent at the middle to form a right angle as shown in Figure. The magnitude of the magnetic field at the points Q and R is



(A) $\frac{\mu_0 i}{4\pi d}, \frac{\mu_0 i}{4\pi d}$ (B) $\frac{\mu_0 i}{2\pi d}, \frac{\mu_0 i}{4\pi d}$

(C) $\frac{\mu_0 i}{\pi d}, \frac{\mu_0 i}{2\pi d}$ (D) $\frac{\mu_0 i}{d}, \frac{\mu_0 i}{d}$

135. Two long wires AB and CD carry currents I_1 and I_2 in the directions shown in Figure.



If $\vec{F}$ be the net force on the wire and $\vec{\tau}$ be net torque on it, then

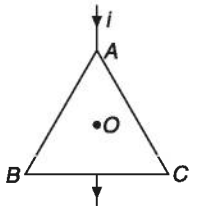
(A) $\vec{F}=\vec{0},\ \vec{\tau}=\vec{0}$

(B) $\vec{F}=\vec{0},\ \vec{\tau}\neq\vec{0}$ (CCW)

(C) $\vec{F}=\vec{0},\ \tau\neq\vec{0}$ (CW)

(D) $\vec{F}\neq\vec{0},\ \vec{\tau}=\vec{0}$

136. The wire ABC shown in Figure forms an equilateral triangle.

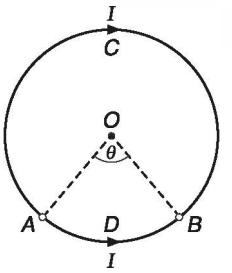


The magnetic field B at the centre O of the triangle assuming the wire to be uniforms, will be

(A) ∞ (B) 0

(C) $\frac{\mu_0 i}{AB}$ (D) $\frac{\mu_0 i}{BC}$

137. In two segments of a circular loop of radius a, equal currents I flow in the directions shown in Figure.



Magnetic field at the centre of the loop is

(A) $\frac{\mu_0 I}{2a}$ (B) $\frac{\mu_0 I}{2\pi a}(2\pi-\theta)$

(C) $\frac{\mu_0 I}{2\pi a}(\pi-\theta)$ (D) ZERO

138. In a cyclotron, if a deuteron can gain an energy of 40 MeV, then a proton can gain an energy of

(A) 40 MeV (B) 80 MeV

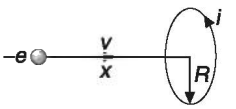
(C) 20 MeV (D) 60 MeV

139. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in opposite directions. Let B_1 and B_2 be the magnetic fields in the regions between the conductors, and outside the conductor, respectively

(A) $B_1=B_2=0$ (B) $B_1\neq 0,\ B_2\neq 0$

(C) $B_1=0,\ B_2\neq 0$ (D) $B_1\neq 0,\ B_2=0$

140. An electron moving with a velocity v along the axis approaches a circular current carrying loop as shown in Figure.

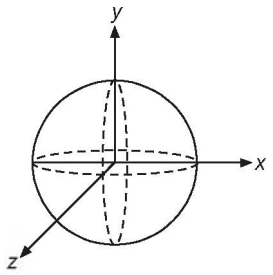


The magnitude of magnetic force on electron at this instant is

(A) $\frac{\mu_0 eviR^2x}{2(x^2+R^2)^{\frac{3}{2}}}$ (B) $\frac{\mu_0 eviR^2x}{\pi(x^2+R^2)^{\frac{3}{2}}}$

(C) $\frac{\mu_0 eviR^2x}{4\pi(x^2+R^2)^{\frac{3}{2}}}$ (D) 0

141. Three rings, each having equal radius R and carrying a current I are placed mutually perpendicular to each other such that each has its centre at the origin of co-ordinate system. The magnitude of the magnetic field at the common centre is



(A) ZERO (B) $\sqrt{3}\left(\frac{\mu_0 I}{2R}\right)$

(C) $(\sqrt{3}-\sqrt{2})\left(\frac{\mu_0 I}{2R}\right)$ (D) $(\sqrt{2}-1)\left(\frac{\mu_0 I}{2R}\right)$

142. A current carrying flat coil has a magnetic moment M. It is initially in equilibrium, with its plane perpendicular to a magnetic field of magnitude B. If the coil is now rotated through an angle θ, then the work done is

(A) $MB\sin\theta$ (B) $MB\cos\theta$

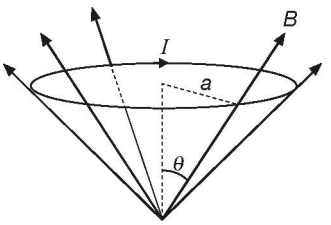
(C) $MB\theta$ (D) $MB(1-\cos\theta)$

143. A neutral atom of atomic mass number 100 is lying at the origin in gravity free space. It emits an α-particle A in x-direction and the product ion formed is P. A uniform magnetic field exists in the $-z$-direction. Neglecting the electromagnetic interaction between A and P, the angle of rotation of A (in radian) after which A and P meet for the first time is

(A) $\frac{12\pi}{25}$ (B) $\frac{24\pi}{25}$

(C) $\frac{36\pi}{25}$ (D) $\frac{48\pi}{25}$

144. A circular current loop of radius a is placed in a radial field B as shown. The net force acting on the loop is



(A) ZERO (B) $2\pi BIa$

(C) $2\pi aIB\sin\theta$ (D) πaIB

145. A wire of length l carries a current I along the x-axis. A magnetic field exists which is given as $\vec{B}=B_0\left(\hat{i}+\hat{j}+\hat{k}\right)$ T. The magnitude of the magnetic force acting on the wire will be

(A) $B_0 Il$ (B) $3B_0 Il$

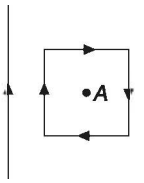
(C) $\sqrt{3}B_0 Il$ (D) $\sqrt{2}B_0 Il$

146. A metallic wire carrying a current I, kept perpendicular to a uniform magnetic field, is folded to form a square loop of side a. Now the shape of the loop is changed from square to a circle without changing the length of the wire and current. The amount of work done in doing so is

(A) $BIa^2\left(\frac{4}{\pi-1}\right)$ (B) $BIa^2(\pi+2)$

(C) $BIa^2\left(1-\frac{4}{\pi}\right)$ (D) $BIa^2(\pi-2)$

147. A wire is parallel to a square coil. The coil and wire carries currents in the directions shown. Then, at any point A within the coil the magnetic field will be



(A) less than the magnetic field produced due to coil only

(B) more than the magnetic field produced due to coil only

(C) equal to the field by coil alone

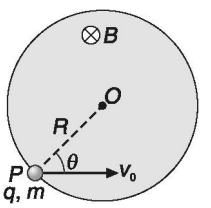
(D) zero

148. A copper wire of diameter 3.2 mm carries a current I. The maximum magnetic field due to this wire is 5×10^{-3} T. The value of I is

(A) 40 A (B) 10 A

(C) 20 A (D) 80 A

149. A particle of charge q and mass m is projected with a velocity v_0 towards a circular region of radius R having uniform magnetic field B perpendicular and into the plane of paper from point P as shown in Figure.



If O is the centre of the circular region and the line OP makes angle θ with the direction of v_0, then the value of v_0 so that particle passes through O is

(A) $\frac{qBR}{m\sin\theta}$ (B) $\frac{qBR}{2m\sin\theta}$

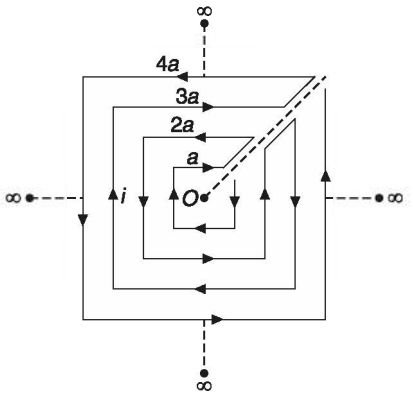
(C) $\frac{2qBR}{m\sin\theta}$ (D) $\frac{3qBR}{2m\sin\theta}$

150. The magnetic field at the centre of a circular current carrying loop of area A is B. The magnetic moment of the loop is M. Then

(A) $M=\frac{BA}{\mu_0}\sqrt{A}$ (B) $M=\frac{BA^2}{\mu_0\pi}$

(C) $M=\frac{2BA}{\mu_0}\sqrt{\frac{A}{\pi}}$ (D) $M=\frac{BA\sqrt{A}}{\mu_0\pi}$

151. Determine the magnetic field at the centre of the current carrying wire arrangement shown in figure.

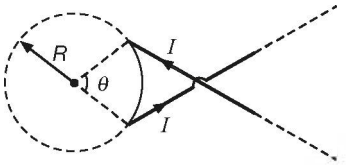


The arrangement extends to infinity. (The wires joining the successive squares are along the line passing through the centre).

(A) $\frac{\mu_0 i}{\sqrt{2}\pi a}$ (B) $\frac{2\sqrt{2}\mu_0 i}{\pi a}\ln(2)$

(C) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$ (D) $\frac{2\sqrt{2}\mu_0 i}{\pi a}\ln(3)$

152. A current carrying wire has the configuration shown in Figure.



Two semi-infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle θ along the circumference of the circle, with all sections lying in the same plane. What must be θ in order for B (magnetic field) to be zero at the centre of the circle?

(A) $\theta=4$ rad (B) $\theta=3$ rad

(C) $\theta=2$ rad (D) $\theta=1$ rad

153. A length L of wire carrying current I is bent into a circle of one turn. The field at the centre of the coil is B_1. A similar wire of length L carrying current I is bent into a square of one turn. The field at its centre is B_2. Then

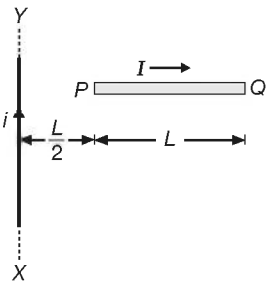
(A) $B_1 > B_2$

(B) $B_1 < B_2$

(C) $B_1 = B_2$

(D) nothing can be predicted

154. A conductor PQ of length L, carries a current I. PQ is placed perpendicular to a long straight conductor XY carrying a current i as shown. The force acting on PQ is F.



(A) $F=\frac{\mu_0 Ii}{2\pi}\ln 2$; upwards

(B) $F=\frac{\mu_0 Ii}{2\pi}\ln 2$; downwards

(C) $F=\frac{\mu_0 Ii}{2\pi}\ln 3$; upwards

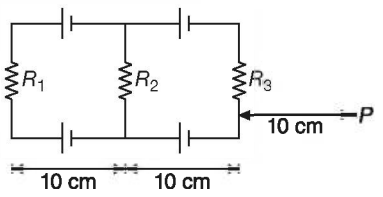
(D) $F=\frac{\mu_0 Ii}{2\pi}\ln 3$; downwards

155. On a smooth horizontal surface, a conducting rod of mass m and length l is placed. A uniform magnetic field B is acting perpendicular to the rod. A charge q is suddenly passed through the rod and it acquires a velocity v on the surface, then

(A) $q=\frac{mv}{2Bl}$ (B) $q=\frac{mv}{Bl}$

(C) $q=\frac{2mv}{Bl}$ (D) $q=\frac{Blv}{2m}$

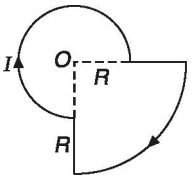
156. In the Figure shown, each battery has emf of 5 V. Then the magnetic field at P is



(A) ZERO (B) $\frac{10\mu_0}{R_1(4\pi)(0.2)}$

(C) $\frac{20\mu_0}{(R_1+R_2)(4\pi)(0.2)}$ (D) None of these

157. A current I flowing through the loop as shown in the adjoining figure. The magnetic field at centre O is



(A) $\frac{7\mu_0 I}{16R}\otimes$ (B) $\frac{7\mu_0 I}{16R}\odot$

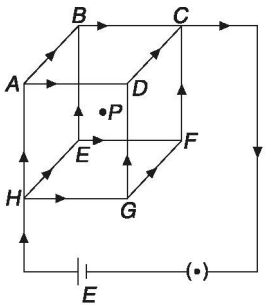
(C) $\frac{5\mu_0 I}{16R}\otimes$ (D) $\frac{5\mu_0 I}{16R}\odot$

158. The magnetic field near a large metal sheet that carries an electric current of current per unit length λ, along its surface is

(A) $\frac{\mu_0\lambda}{2\pi}$ (B) $\frac{\mu_0\lambda}{2}$

(C) $\mu_0\lambda$ (D) $\frac{\mu_0}{2\lambda\pi}$

159. A steady current is set up in a network composed of wires of equal resistance and length d as shown in Figure. The magnetic field at the centre P is



(A) $\frac{\mu_0}{4\pi}\frac{2I}{d}$ (B) $\frac{\mu_0}{4\pi}\frac{3I}{\sqrt{2}d}$

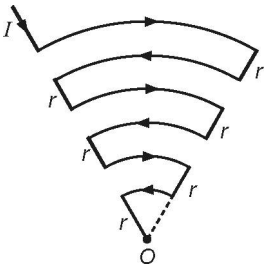
(C) ZERO (D) $\frac{\mu_0}{4\pi}\frac{I\theta}{d}$

160. A particle of charge Q and of negligible initial speed is accelerated through a potential difference of U. The particle reaches a region of uniform magnetic field of induction B, where it undergoes circular motion. If potential difference is doubled and B is also doubled then magnetic moment of the circular current due to circular motion of charge Q will become.

(A) double (B) half

(C) four times (D) remain same

161. Shown in Figure is a conductor carrying a current I.

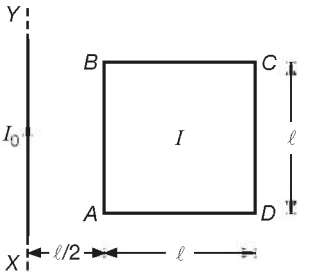


The magnetic field intensity at the point O (common centre of all the three arcs) is

(A) $\frac{37\mu_0 I\theta}{120\pi r}$ (B) $\frac{37\mu_0 I\theta}{240\pi r}$

(C) $\frac{11\mu_0 I\theta}{240\pi r}$ (D) ZERO

162. A square loop $ABCD$, carrying a current I, is placed near and coplanar with a long straight conductor XY carrying a current I_0



(A) There is no net force on the loop.

(B) The loop will always be attracted by the conductor.

(C) The loop will be attracted by the conductor only if the current in the loop flows anticlockwise.

(D) The loop will be attracted by the conductor only if the current in the loop flows clockwise.

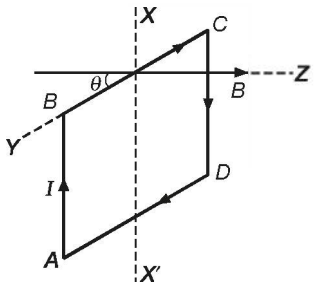
163. Two long parallel wires separated by a distance r have equal currents I flowing in each. Either wire experiences a magnetic force $F\ \text{Nm}^{-1}$. If the distance r is increased to $3r$ and current in each wire

is reduced to $\frac{I}{3}$, the force per unit length between them will now be

(A) $3F$ (B) $9F$

(C) $\frac{F}{9}$ (D) $\frac{F}{27}$

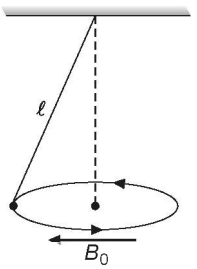
164. The square loop $ABCD$, carrying a current I, is placed in a uniform magnetic field B, as shown. The loop can rotate about the axis XX', as shown. The plane of the loop makes an angle $\theta(\theta < 90°)$ with the direction of B. The angle through which the loop will rotate all by itself before the torque on it becomes zero is



(A) $(\pi - \theta)$ (B) $\left(\frac{\pi}{2} + \theta\right)$

(C) $\left(\frac{\pi}{2} - \theta\right)$ (D) θ

165. A uniform current carrying ring of mass m and radius R is connected by a massless string of length l as shown in Figure.



A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is

(A) $\frac{mg}{\pi RB_0}$ (B) $\frac{mg}{RB_0}$

(C) $\frac{mg}{3\pi RB_0}$ (D) $\frac{mg}{\pi R^2 B_0}$

166. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in the same directions. Let B_1 be the magnetic field at a point between the two conductors, at a distance x from the axis. Let B_2 be the magnetic field at a point outside the outer conductor, at a distance $2x$ from the axis

(A) $B_1 = 2B_2$ (B) $B_2 = 2B_1$

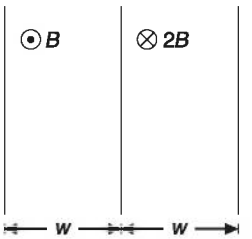
(C) $B_2 = B_1$ (D) $B_2 = 4B_1$

167. A current I flows through a lengthy thin walled tube of radius R having a longitudinal slit of width w. The magnetic field inside the tube at the axis under the condition $w \ll R$ is

(A) ZERO (B) $B = \frac{\mu_0 wI}{2\pi R^2}$

(C) $B = \frac{\mu_0 wI}{4\pi^2 R^2}$ (D) $B = \frac{\mu_0 wI}{4\pi R^2}$

168. The magnetic field shown in the figure consist of the two magnetic fields as shown.



If v is the velocity just required for a charge particle of mass m and charge q to pass through the magnetic field and the particle is projected with this velocity v, then, the time spend by the charged particle in the magnetic field is

(A) $\frac{\pi m}{2qB}$ (B) $\frac{\pi m}{qB}$

(C) $\frac{\pi m}{4qB}$ (D) $\frac{3\pi m}{2qB}$

169. The magnetic moment of an electron in nth orbit of hydrogen atom is M. If m be the mass of electron, then M equals

(A) $\frac{neh}{2\pi m}$ (B) $\frac{neh}{4\pi m}$

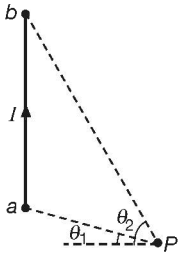
(C) $\frac{neh}{\pi m}$ (D) $\frac{2neh}{\pi m}$

170. Two coaxial long solenoids of equal lengths have current i_1, i_2, number of turns per unit length n_1, n_2 and radius r_1, $r_2 (> r_1)$ respectively. If $n_1 i_1 = n_2 i_2$ and the two solenoids carry currents in opposite sense, the magnetic energy stored per unit length e is

(A) $\frac{\mu_0 \pi}{2} n_1^2 i_1^2 (r_2^2 - r_1^2)$ (B) $\mu_0 n_1^2 i_1^2 \pi (r_2^2 - r_1^2)$

(C) $\frac{\mu_0}{2} n_1^2 i_1^2 \pi r_1^2$ (D) $\frac{\mu_0 \pi}{2} n_2^2 i_2^2 r_2^2$

171. P is a point at perpendicular distance d from a straight wire ab carrying a current I. The magnetic field at P due to ab has magnitude



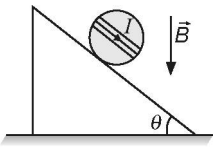
(A) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

(B) $\frac{\mu_0 I}{4\pi d}(\cos\theta_1 - \cos\theta_2)$

(C) $\frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

(D) $\frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)$

172. In the Figure shown, a coil of single turn, carrying a current I is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the incline and lies in the equatorial plane of the sphere. The sphere is in equilibrium, then $|\vec{B}|$ equals



(A) $\frac{mg\tan\theta}{\pi IR}$ (B) $\frac{mg\sin\theta}{\pi IR}$

(C) $\frac{mg\cos\theta}{\pi IR}$ (D) $\frac{mg}{\pi IR}$

173. A rectangular loop of metallic wire is of length a and breadth b and carries a current i. The magnetic field at the centre of the loop is

(A) $\frac{\mu_0 i}{4\pi}\left(\frac{8\sqrt{a^2+b^2}}{ab}\right)$ (B) $\frac{\mu_0 i}{4\pi}\left(\frac{4\sqrt{a^2+b^2}}{ab}\right)$

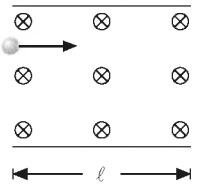
(C) $\frac{\mu_0 i}{4\pi}\left(\frac{2\sqrt{a^2+b^2}}{ab}\right)$ (D) $\frac{\mu_0 i}{4\pi}\left(\frac{\sqrt{a^2+b^2}}{ab}\right)$

174. An α particle is moving along a circle of radius R with a constant angular velocity ω. A point A lies in the same plane at a distance $2R$ from the centre. The magnetic field produced by α particle is measured at the point A. If the minimum time interval between two successive times at which A records zero magnetic field is t, then the angular speed ω in terms of t is

(A) $\frac{2\pi}{t}$ (B) $\frac{2\pi}{3t}$

(C) $\frac{\pi}{3t}$ (D) $\frac{\pi}{t}$

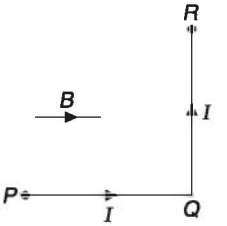
175. An electron moves straight inside a charged parallel plate capacitor of uniform surface charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. The time of straight-line motion of the electron in the capacitor is



(A) $\frac{e\sigma}{\varepsilon_0 lB}$ (B) $\frac{\varepsilon_0 lB}{\sigma}$

(C) $\frac{e\sigma}{\varepsilon_0 B}$ (D) $\frac{\varepsilon_0 B}{e\sigma}$

176. A wire PQR is bent as shown in Figure and is placed in a region of uniform magnetic field B.



The length of $PQ = QR = l$. A current I ampere flows through the wire as shown. The magnitude of force on PQ and QR will be

(A) $BIl, 0$ (B) $2BIl, 0$

(C) $0, BIl$ (D) $0, 0$

177. Two parallel wires carrying equal currents in opposite directions are placed at $x = \pm a$ parallel to y-axis with $z = 0$. If B_0 be the field at the origin and B be the field at point $P(2a, 0, 0)$, then B equals

(A) $\frac{B_0}{2}, \otimes$ (B) $\frac{B_0}{2}, \odot$

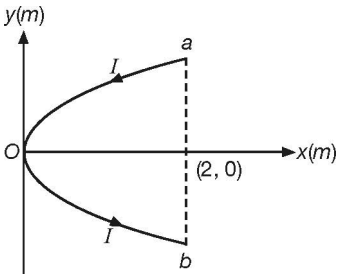
(C) $\frac{B_0}{3}, \otimes$ (D) $\frac{B_0}{3}, \odot$

178. A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\hat{r}$, where $\hat{r}$ denotes the unit vector along the radial direction. A circular loop of radius a carrying a current i, is placed with its plane parallel to the x-y plane and centre at $(0, 0, d)$. The magnitude of magnetic force acting on the loop is

(A) $\frac{2\pi a^2 iB_0}{d}$ (B) $\frac{2\pi a^2 iB_0}{\sqrt{a^2+d^2}}$

(C) $\frac{\pi a^2 iB_0}{d}$ (D) $\frac{\pi a^2 iB_0}{\sqrt{a^2+d^2}}$

179. A conducting wire, carrying a current $I = 2$ A is bent in the form of a parabola having an equation given by $y^2 = 2x$ as shown in Figure.



This wire is placed in a uniform magnetic field $\vec{B} = -4\hat{k}$ tesla. The magnetic force on the wire, in newton, is

(A) $-16\hat{i}$ (B) $32\hat{i}$

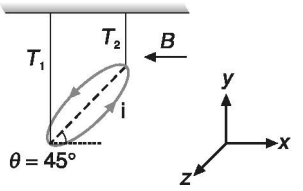
(C) $-32\hat{i}$ (D) $16\hat{i}$

180. A particle of charge $q = 4\ \mu$C and mass $m = 10$ mg starts moving from the origin under the action of an electric field $\vec{E} = 4\hat{i}$ and magnetic field $\vec{B} = (0.2T)\hat{k}$. Its velocity at $(x, 3, 0)$ is $(4\hat{i} + 3\hat{j})$. The value of x is

(A) $\frac{115}{16}$ m (B) $\frac{125}{16}$ m

(C) $\frac{135}{16}$ m (D) $\frac{145}{16}$ m

181. A current carrying circular coil of single turn of mass m is suspended in xz plane with the help of two ideal strings as shown in Figure.



A constant magnetic field $\vec{B}$ is set up in the horizontal direction in the region. The ratio of tension $\frac{T_1}{T_2}$ in the string will be [Take $\pi BIR = \frac{mg}{4}$ and $\theta = 45°$ in Figure shown]

(A) 2 : 1 (B) 5 : 3

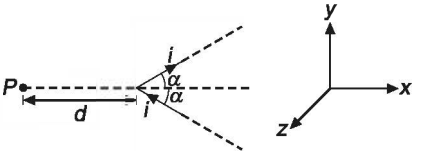
(C) 4 : 1 (D) 1 : 2

182. Two long concentric cylindrical conductors of radii a and b $(b < a)$ are maintained at a potential difference V and carry equal and opposite currents I. An electron with a particular velocity v_0 parallel to the axis will travel undeviated in the evacuated region between the conductors. Then v_0 equals

(A) $\frac{4\pi V}{\mu_0 I \ln\left(\frac{b}{a}\right)}$ (B) $\frac{2\pi V}{\mu_0 I \ln\left(\frac{a}{b}\right)}$

(C) $\frac{2\pi V}{\mu_0 I \ln\left(\frac{b}{a}\right)}$ (D) $\frac{8\pi V}{\mu_0 I \ln\left(\frac{a}{b}\right)}$

183. V shaped wire is in x-y plane. The direction of the field B at P is along



(A) $+x$ axis (B) $+z$ axis

(C) $-x$ axis (D) $+y$ axis

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A co-axial cable consists of a thin inner current carrying conductor fixed along the axis of a hollow current carrying conductor. Let B_1 and B_2 be the magnetic fields in the region between the conductors and outside the conductor, respectively

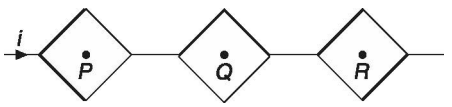
(A) $B_1 \neq 0$, $B_2 \neq 0$ for conductors carrying equal currents in opposite directions

(B) $B_1 \neq 0$, $B_2 = 0$ for conductors carrying equal currents in opposite directions

(C) $B_1 \neq 0$, $B_2 \neq 0$ for conductors carrying equal currents in the same direction

(D) $B_1 = B_2$ for conductors carrying equal currents in same directions at distances x and $2x$ from the axis respectively

2. Two thick wires and two thin wires, all of same material and same length form a square in the three different ways P, Q and R as shown in Figure. The current flowing through the arrangement is i. Let B_P, B_Q and B_R represent the magnetic field at the centre of squares in the three cases, then



(A) $B_P = 0$ (B) $B_Q = 0$

(C) $B_R = 0$ (D) $B_P \neq 0$

3. A particle having specific charge α moves with a velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = \frac{B_0}{\sqrt{2}}(\hat{j} + \hat{k})$. It is observed that the

(A) path of the particle is a helix

(B) path of the particle is a circle

(C) distance moved by particle in time $t = \frac{\pi}{\alpha B_0}$ is $\frac{\pi v_0}{\alpha B_0}$

(D) velocity of particle after time $t = \frac{\pi}{3\alpha B_0}$ is $\left(\frac{v_0}{\sqrt{2}}\hat{i} + \frac{v_0}{\sqrt{2}}\hat{j}\right)$

4. Two identical charged particles enter a uniform magnetic field with same speed but at angles 30° and 60° with field. The ratio of their time periods, radii and pitches of the helical paths are a, b and c respectively. Then

(A) $abc < 1$ (B) $c = 3ab$

(C) $abc = 1$ (D) $a = bc$

5. A thin wire carrying current i is bent to form a closed loop of one turn. The loop is placed in y-z plane with centre at origin. If R is the radius of the loop, B_x be the magnetic field at a point $(x, 0)$ on the x-axis, then

(A) $B_x = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$ (B) $\int_0^\infty B_x dx = \frac{\mu_0 i}{2}$

(C) $\int_{-\infty}^{\infty} B_x dx = \mu_0 i$ (D) $\int_{-\infty}^{\infty} B_x dx = 0$

6. Which of the following statement(s) is/are correct?

(A) Units of magnetic field B can be written as $NA^{-1}m^{-1}$

(B) Units of magnetic permeability μ_0 can be written as NA^{-2}

(C) Units of magnetic flux ϕ_B can be written as $NA^{-1}m^{-2}$

(D) Units of $\sqrt{\frac{L}{C}}$ can be VA^{-1}

7. Two circular coils A and B with their centres lying on the same axis have same number of turns and carry equal currents in the same sense. They are separated by a distance, have different diameters but subtend same angle at a point P lying on their common axis. The coil B lies exactly midway between coil A and the point P. The magnetic field at point P due to coils A and B is B_1 and B_2 respectively

(A) $B_1 > B_2$ (B) $B_1 < B_2$

(C) $\frac{B_1}{B_2} = 2$ (D) $\frac{B_1}{B_2} = \frac{1}{2}$

8. If the acceleration and velocity of a charged particle moving in a constant magnetic region is given by $\vec{a} = a_1\hat{i} + a_2\hat{k}$, $\vec{v} = b_1\hat{i} + b_2\hat{k}$, where a_1, a_2, b_1 and b_2 are constants, then select the correct statement(s).

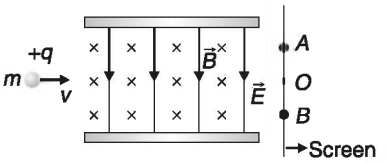
(A) Magnetic field may be along y-axis

(B) $a_1b_1 + a_2b_2 = 0$

(C) Magnetic field is along x-axis

(D) Kinetic energy of particle is always constant

9. A region of space has electric field $\vec{E}$ and magnetic field $\vec{B}$ as shown. A particle of charge $+q$, mass m enters this region with velocity $\vec{v}$ perpendicular to $\vec{E}$ and $\vec{B}$ both.



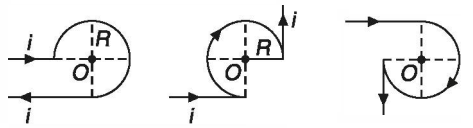
(A) The charged particle will hit the screen at A, experiencing a net force $q(vB-E)$.

(B) The charged particle will hit the screen at B, experiencing a net force $q(E-vB)$.

(C) The charged particle will hit the plate at O, under the state of equilibrium.

(D) The velocity v is the ratio of the magnitude of electric field to that of magnitude of magnetic field.

10. Three long straight current carrying conductors are shown in Figure. The straight parts are long and the circular part in each case in three fourth of a complete circle. Let B_a, B_b and B_c represents the strength of field at the centre O in the three cases, then



(A) $B_a=\dfrac{\mu_0 i}{4R}\left(\dfrac{3}{2}+\dfrac{1}{\pi}\right)$ (B) $B_b=\dfrac{\mu_0 i}{2R}\left(\dfrac{3}{4}-\dfrac{1}{\pi}\right)$

(C) $B_a=\dfrac{\mu_0 i}{4R}\left(\dfrac{3}{2}-\dfrac{1}{\pi}\right)$ (D) $B_c=\dfrac{3\mu_0 i}{8R}$

11. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will

(A) never move parallel to the x-axis

(B) move parallel to the x-axis once during every rotation for all values of θ

(C) move parallel to the x-axis at least once during every rotation if $\theta=45°$

(D) never move perpendicular to the x-direction

12. A charged particle of charge q, mass m is projected in a plane perpendicular to uniform magnetic field. The areal velocity (area swept per unit time), kinetic energy, momentum, field strength are denoted by $\dfrac{dA}{dt}$, K, p and $|\vec{B}|$ respectively. Then

(A) $\dfrac{dA}{dt}\propto K$ (B) $\dfrac{dA}{dt}\propto\dfrac{1}{|\vec{B}|}$

(C) $\dfrac{dA}{dt}\propto p^2$ (D) $\dfrac{dA}{dt}\propto q$

13. A proton, moving with velocity $\vec{v}$, enters a region having simultaneous, uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$ respectively such that $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular the proton is deflected along positive x-axis when either of the fields or both are switched on simultaneously. Which of the following supports the argument given above?

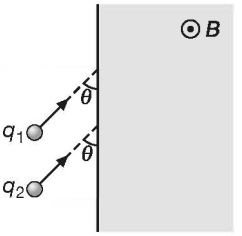
(A) $\vec{B}$ may be along negative y-axis

(B) $\vec{B}$ may be along positive z-axis

(C) $\vec{E}$ is along positive x-axis

(D) $\vec{v}$ may be along positive y-axis

14. Two charges q_1 and q_2 having same magnitude and equal mass are moving parallel to each other and they enter into a region of uniform magnetic field as shown. If the time spent by them in the magnetic field are t_1 and t_2 respectively, then select the correct statement(s).



(A) For $q_1=-q_2$, $t_1=t_2$

(B) For $q_1=q_2$, $t_1=t_2$

(C) For $q_1>0$, $q_2<0$, $t_1<t_2$

(D) For $q_1<0$, $q_2>0$, $t_1<t_2$

15. Current flows through a straight cylindrical conductor of radius r. The current is distributed uniformly over its cross-section. The magnetic field at a distance x from the axis of the conductor has magnitude B

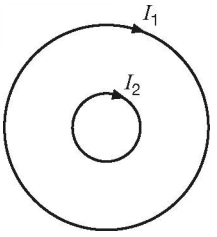
(A) $B=0$ at the axis

(B) $B\propto x$ for $0\le x\le r$

(C) $B\propto\dfrac{1}{x}$ for $x>r$

(D) B is maximum for $x=r$

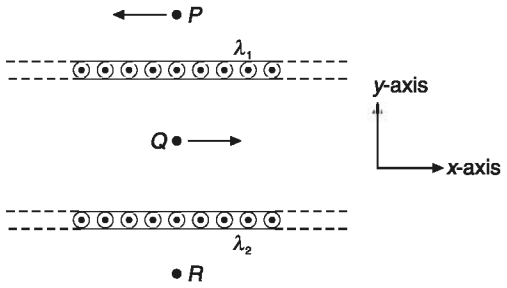
16. Two concentric circular coils of radii R and $r(\ll R)$ carry currents of I_1 and I_2 respectively. When the smaller coil, of mass m, is rotated slightly about one of its diameter, it starts oscillating. If T is the period of oscillation, then



(A) $T \propto \dfrac{1}{\sqrt{I_1 I_2}}$ (B) $T \propto r^0$

(C) $T \propto \dfrac{1}{\sqrt{m}}$ (D) $T \propto \sqrt{R}$

17. The figure shows two infinite parallel sheets of current, with current per unit length λ_1 and λ_2. If B_P, B_Q and B_R represent the magnetic field at the points P, Q and R respectively, then



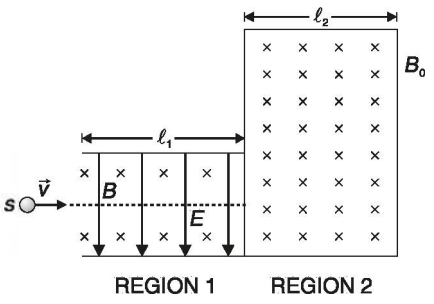
(A) $B_P = \left(\dfrac{\lambda_1 + \lambda_2}{2}\right)\mu_0$, along x-axis

(B) $B_Q = \left(\dfrac{\lambda_1 - \lambda_2}{2}\right)\mu_0$, along x-axis

(C) $B_R = \left(\dfrac{\lambda_1 + \lambda_2}{2}\right)\mu_0$, along x-axis

(D) $B_Q = \left(\dfrac{\lambda_1 - \lambda_2}{2}\right)\mu_0$, along $-x$-axis

18. A charged particle of specific charge s passes through a region of space shown.



(A) Velocity of the particle in REGION 1 is $v = \dfrac{E}{B}$.

(B) Work done to move the charged particle in REGION 1 and REGION 2 is ZERO.

(C) The radius of the trajectory of the charged particle in REGION 2 is $\dfrac{E}{sBB_0}$.

(D) The particle emerges from REGION 2 with a velocity $\vec{v}'$ where $\vec{v}' = -\vec{v}$ for $\ell_2 > \dfrac{E}{sBB_0}$.

19. A charged particle of unit mass and unit charge at some instant has velocity $\vec{v} = \left(8\hat{i} + 6\hat{j}\right)$ ms^{-1} in magnetic field $\vec{B} = \left(2\hat{k}\right)$ tesla. (Neglect all other forces). Select the correct option(s).

(A) The path of particle may be $x^2 + y^2 - 4x - 21 = 0$

(B) The path of particle may be $x^2 + y^2 = 25$

(C) The path of particle may be $y^2 + z^2 = 25$

(D) Time period of particle will be 3.14 s

20. Two long thin parallel conductors of mass density m_1 and m_2 carry currents I_1 and I_2 respectively. They are placed at a distance d. The force acting on a unit length of the first conductor is F. Assuming second conductor to be fixed.

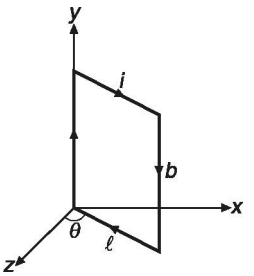
(A) $F \propto I_1 I_2$ and attractive for I_1 and I_2 flowing in the same direction.

(B) $F \propto I_1 I_2$ and repulsive for I_1 and I_2 flowing in the opposite direction.

(C) $F \propto \dfrac{1}{d}$

(D) The first conductor moves towards/away from the second with an acceleration $a = \dfrac{\mu_0 I_1 I_2}{2\pi m_1 d}$

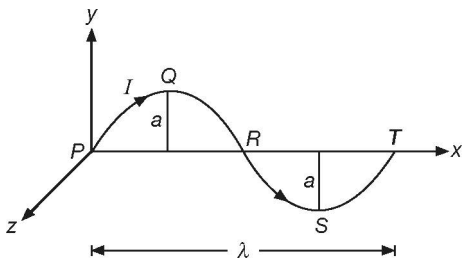
21. A rectangular loop of dimensions $l \times b$ carries a current i. A uniform magnetic field $\vec{B} = B_0\hat{i}$ exists in space. Then select the correct statement(s).



(A) Torque on the loop is $ilbB_0 \sin\theta$
(B) Torque on the loop is in negative y-direction
(C) When free to move, the loop turns so as to increase θ
(D) When free to move, the loop turns so as to decrease θ

22. A charged particle moves in a gravity free space where an electric field of strength E and a magnetic field of induction B exist. Which of the following statements are correct?
(A) If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant
(B) If $E \neq 0$, particle cannot trace a circular path
(C) If $E \neq 0$, kinetic energy of the particle remains constant
(D) None of the above

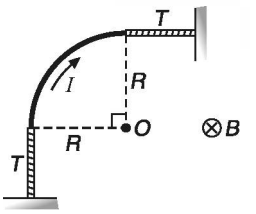
23. A conductor $PQRST$, shaped as shown, carries a current I. It is placed in the xy plane with the ends P and T on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be



(A) zero, if B is in the x-direction
(B) λIb in the z-direction, if B is in the y-direction
(C) λIb in the negative y-direction, if B is in the z-direction
(D) $2BIa$ if B is in the x-direction

24. The magnetic field due to a current carrying toroidal solenoid
(A) is independent of the radius of the solenoid
(B) depends on the number of turns and the current in the solenoid
(C) is constant in magnitude inside the solenoid
(D) is always radial inside the solenoid

25. A wire carrying current I, bent in the form of a quarter circle is held at rest on a smooth table with two threads as shown. A uniform magnetic field exists in the region directed into the plane select the correct alternatives



(A) Net force on wire is zero
(B) Net magnetic force on wire is $\sqrt{2}BIR$
(C) Tension in each thread is BIR
(D) Tension in each thread is $\sqrt{2}BIR$

26. Two long, thin, parallel conductors are kept very close to each other, without touching. One carries a current I, and the other has charge λ per unit length. An electron moving parallel to the conductors is undeflected. Let c = velocity of light.
(A) $v = \dfrac{\lambda c^2}{I}$
(B) $v = \dfrac{I}{\lambda}$
(C) $c = \dfrac{I}{\lambda}$
(D) The electron may be at any distance from the conductor

27. The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$. Which pairs of vectors are at right angles to each other?
(A) $\vec{F}$ and $\vec{v}$ (B) $\vec{F}$ and $\vec{B}$
(C) $\vec{B}$ and $\vec{v}$ (D) $\vec{F}$ and $(\vec{v} \times \vec{B})$

28. Two circular coils of radii 5 cm and 10 cm carry currents of 2 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as their centres coincide. Magnitude of magnetic field at the common centre of coils is
(A) $8\pi \times 10^{-4}$ T, if currents in the coils are in same sense
(B) $4\pi \times 10^{-4}$ T, if currents in the coils are in opposite sense
(C) Zero, if currents in the coils are in opposite sense
(D) $8\pi \times 10^{-4}$ T, if currents in the coils are in opposite sense

29. A charged particle is fired at an angle θ with the uniform field directed along x-axis. If the pitch of the helical path is equal to the maximum distance of the

particle from the x-axis, then which of the following are not correct?

(A) $\cos\theta = \frac{1}{\pi}$ (B) $\sin\theta = \frac{1}{\pi}$

(C) $\tan\theta = \frac{1}{\pi}$ (D) $\tan\theta = \pi$

30. Consider three physical quantities $x = \frac{E}{B}$, $y = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$ and $z = \frac{l}{CR}$. Here, l is the length of a wire, C is capacitance and R is resistance. All other symbols have standard meanings.

(A) x, y have the same dimensions

(B) y, z have the same dimensions

(C) z, x have the same dimensions

(D) None of the three pairs have the same dimensions

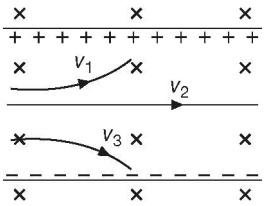
31. A circular coil of radius R, carries a current I. The magnetic field at the centre of the coil is B. Then B equals.

(A) $\frac{I}{2c^2\varepsilon_0 R}$ (B) $\frac{\mu_0 I}{2\pi R}$

(C) $\frac{\mu_0 I}{2R}$ (D) $\frac{Ic^2}{2\varepsilon_0 R}$

where c is the speed of light in vacuum.

32. In a region of crossed fields as shown, the strength of electric field is E and that of magnetic field is B. Three positively charged particles with speeds v_1, v_2 and v_3 are projected and their paths are shown. From this we conclude that



(A) $v_1 > \frac{E}{B}$ (B) $v_2 = \frac{E}{B}$

(C) $v_3 < \frac{E}{B}$ (D) None of these

33. The velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are given by $\vec{v} = 6\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$. Then

(A) $x = -1.5$

(B) $x = -3$

(C) Magnetic field is along z-direction

(D) Kinetic energy of the particle is constant

34. A straight thin walled tube of radius a has a current I flowing through it. If $B(r)$ is the magnitude of the magnetic field at a distance r from the axis of the tube then,

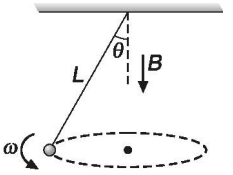
(A) $B(r) = 0$ for $0 \le r < a$

(B) $B(r) \propto \frac{1}{r}$ for $0 \le r < a$

(C) $B(r) \propto \frac{1}{r}$ for $r > a$

(D) $B(r) = 0$ for $r > a$

35. A conical pendulum of length L making an angle θ with vertical rotating with angular velocity ω has a bob whose charge to mass ratio is β. If a magnetic field B is directed vertically downwards as shown in Figure, then select the correct statement(s).



(A) $B = \frac{1}{\beta}\left(\omega - \frac{g}{\omega L\cos\theta}\right)$

(B) Angular momentum of the bob about the point of suspension remains constant

(C) If the direction of B were reversed maintaining same ω and L, then θ will remain unchanged

(D) Rate of change of angular momentum of the bob about the point of suspension is not a constant vector.

36. The cyclotron frequency of a particle in a magnetic field is independent of

(A) the mass of the particle

(B) the charge of the particle

(C) the speed of the particle

(D) the radius of the circular path of the particle

37. The current density in a wire of radius a varies with radial distance r as $J = kr^2$, where k is a constant.

(A) Total current passing through the cross section of the wire is $I = \frac{\pi k a^4}{2}$

(B) Total current passing through the cross section of the wire is $I = \frac{3\pi k a^3}{2}$

(C) The field at a distance $r > a$ is $B = \frac{\mu_0 k a^4}{4r}$

(D) The field at a distance $r < a$ is $B = \frac{\mu_0 k r^3}{4}$

38. A particle of specific charge α is projected from origin at $t=0$ with a velocity $\vec{v}=v_0(\hat{i}+\hat{k})$ in a magnetic field $\vec{B}=-B_0\hat{k}$. Then select the correct statement(s).

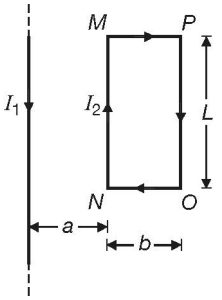
(A) At $t=\dfrac{\pi}{\alpha B_0}$, velocity of the particle is $\vec{v}'=-v_0(\hat{i}-\hat{k})$

(B) At $t=\dfrac{\pi}{4\alpha B_0}$, speed of the particle is v_0

(C) At $t=\dfrac{2\pi}{\alpha B_0}$, magnitude of displacement of the particle is more than $\dfrac{2v_0}{\alpha B_0}$

(D) At $t=\dfrac{2\pi}{\alpha B_0}$, distance travelled by the particle is less than $\dfrac{2\sqrt{2}\pi v_0}{\alpha B_0}$

39. For the situation shown, the correct statement(s) is/are



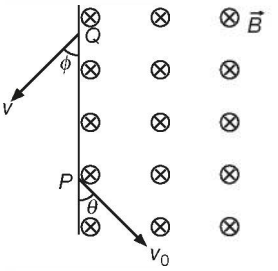
(A) The net force on the long straight wire is $F=\dfrac{\mu_0 I_1 I_2 L}{2\pi}\left[\dfrac{1}{a}-\dfrac{1}{a+b}\right]$ and is repulsive.

(B) The loop will be compressed.

(C) No torque will be acting on the loop as the forces experienced by the wires of the loop lie in the plane of the loop.

(D) No torque will be acting on the loop as the forces experienced by the wires of the loop are normal to the plane of the loop.

40. A particle of mass m, charge $-q$ enters a uniform magnetic field $\vec{B}$ (perpendicular to paper inwards) at P with velocity v_0 at an angle θ and leaves the field at Q with velocity v at angle ϕ as shown in Figure. Then



(A) $v=v_0$

(B) $\theta=\phi$

(C) particle remains in the field for time $t=\dfrac{2m(\pi-\theta)}{qB}$

(D) $PQ=\dfrac{2mv_0\sin\theta}{qB}$

41. A long, straight wire of radius R carries a current distributed uniformly over its cross-section. The magnitude of the magnetic field is

(A) maximum at the axis of the wire

(B) minimum at the axis of the wire

(C) maximum at the surface of the wire

(D) minimum at the surface of the wire

42. A charged particle P leaves the origin with speed $v=v_0$ at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B=B_0$. P will also strike T if

(A) $B=2B_0$, $v=2v_0$ (B) $B=2B_0$, $v=v_0$

(C) $B=B_0$, $v=2v_0$ (D) $B=\dfrac{B_0}{2}$, $v=2v_0$

43. In a region having a uniform field $\vec{B}=B_0\hat{j}$, a proton is fired from origin with velocity $\vec{v}=v_0\hat{j}+v_0\hat{k}$. The subsequent motion of the proton has the best description given by the following statement(s).

(A) Its x and z co-ordinates cannot be zero at the same time

(B) Its y co-ordinate will be proportional to its time of flight

(C) Its z co-ordinate can never be negative

(D) Its x co-ordinate can never be positive

44. Non-zero uniform electric field and magnetic field coexist in a region of space. The path of a charged particle in this region

(A) may be a circle (B) cannot be a circle
(C) may be a helix (D) may be a straight line

45. A charged particle having mass m, charge q is moving with velocity $\vec{v} = \alpha\hat{i} + \beta\hat{j}$ in a magnetic field $\vec{B} = \beta\hat{i} + \alpha\hat{j}$ and experiences a force $\vec{F}$. Which one of the following statement(s) is/are correct?

(A) $\vec{F} = \vec{0}$ for $\alpha = \beta$

(B) $|\vec{F}| \propto \alpha^2 - \beta^2$ for $\alpha > \beta$

(C) $\vec{F}$ acts along z-axis for $\alpha > \beta$

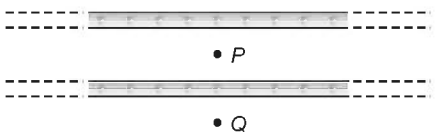
(D) $\vec{F}$ acts along y-axis for $\alpha < \beta$

46. A uniform linear charge density λ exists over a straight wire of length $2a$ having a charge Q. The wire is rotating about an axis passing through its centre and perpendicular to its length with an angular velocity 3ω. The equivalent dipole moment is

(A) $m = \lambda\omega a^3$ (B) $m = \frac{1}{2}Q\omega a^2$

(C) $m = \frac{3}{2}\lambda\omega a^3$ (D) $m = \frac{3}{2}Q\omega a^2$

47. Two infinite plates carry j ampere of current out of the page per unit width of the plate as shown. B_P and B_Q represent magnitude of field at points P and Q respectively.



(A) $B_P = 0$ (B) $B_P = \mu_0 j$

(C) $B_Q = 0$ (D) $B_Q = \mu_0 j$

48. A cylindrical rod of radius a carries a current I distributed uniformly over the entire wire. If $B(r)$ is the magnetic field due to the rod at distance r from the centre, then

(A) $B(r) \propto r$ for $0 < r \le a$

(B) $B(r) = 0$ at the axis

(C) $B(r) \propto \frac{1}{r}$ for $r > a$

(D) $B(r)$ is maximum at $r = a$ i.e., the surface.

49. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field

(A) increases linearly from the axis to the surface
(B) is constant inside the tube
(C) is zero at the axis
(D) is zero outside the tube

50. An electron (mass $= m_e$) and a proton (mass $= m_p$) initially at rest move through a certain distance in a uniform electric field in times t_1 and t_2. Neglect the effect of gravity.

(A) The acceleration of electron is much greater than that of proton

(B) The acceleration of proton is much greater than that of electron

(C) $\frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$

(D) $\frac{t_1}{t_2} = \left(\frac{m_p}{m_e}\right)^{1/2}$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If a charged particle passes through a region without getting any change in its velocity implies that the region is free from electric field as well as magnetic field.
Statement-2: Whenever a charged particle is placed in magnetic field or (and) electric field it may experience a net force.

2. **Statement-1:** Acceleration of a moving charged particle in a magnetic field is non-zero.
Statement-2: Inside magnetic field region, the particle may be moving on curved path.

3. A charge is projected in a region of magnetic field. (no other field is present)

Statement-1: Kinetic energy of charge particle will remain constant.
Statement-2: Work done by magnetic force on moving charge particle is zero.

4. **Statement-1:** A beam of electron passes undeflected through a region.
Statement-2: In the region, $\vec{E}$ and $\vec{B}$, both are present and perpendicular to each other and the particle is moving perpendicular to both of them.

5. **Statement-1:** Magnetic monopoles do not exist.
Statement-2: $\oint \vec{B} \cdot d\vec{A} = 0$
Symbols have their usual meanings.

6. **Statement-1:** Magnitude of $\vec{B}$ is constant along a magnetic field line.
Statement-2: $\vec{B}$ is tangent to a magnetic field line.

7. **Statement-1:** A rectangular current loop is in an arbitrary orientation in an external uniform magnetic field. No work is required to rotate the loop about an axis perpendicular to its plane.
Statement-2: All positions represent the same level of energy.

8. **Statement-1:** For a charged particle describing uniform circular motion in a magnetic field $T^2 \propto r^3$ (symbols have their usual meanings).
Statement-2: The relation $T^2 \propto r^3$ is valid only when $F \propto \frac{1}{r^2}$.

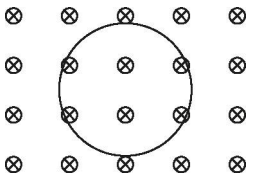
9. **Statement-1:** A loosely bound helix made of stiff wire is suspended vertically with the lower end just touching a dish of mercury. When a current is passed though the wire, the wire executes oscillatory motion with the lower end jumping out of and into the mercury.
Statement-2: When electric current is passed through helix, a magnetic field is produced both inside and outside the helix.

10. **Statement-1:** Two parallel wires carrying current in same direction, attract each other while two similar charges moving parallel to each other repel each other.
Statement-2: Electric force is stronger than magnetic force.

11. **Statement-1:** Magnetic field at a point on the surface of long cylindrical wire is maximum.
Statement-2: For any other point, closed loop perpendicular to the wire and of radius equal to the distance between axis of wire and given point will enclose less current.

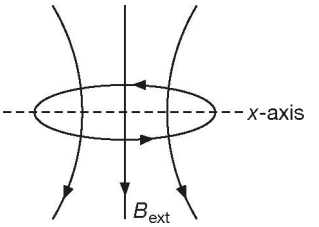
12. **Statement-1:** When radius of a circular loop carrying current is doubled its magnetic moment becomes four times.
Statement-2: Magnetic moment depends on area of the loop.

13. **Statement-1:** A charged particle is moving in a circular path under the action of a uniform magnetic field as shown in the figure. During motion kinetic energy of charged particle is constant.



Statement-2: During the motion magnetic force acting on the particle is perpendicular to instantaneous velocity.

14. **Statement-1:** A closed current carrying loop behave like a magnetic dipole.
Statement-2: Force and torque on the loop is zero as shown in figure.



15. **Statement-1:** A linear solenoid carrying current is equivalent to a bar magnet.
Statement-2: The magnetic field lines of both are same.

16. **Statement-1:** The Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$ is a non-conservative force.
Statement-2: The work done by the Lorentz force is always zero.

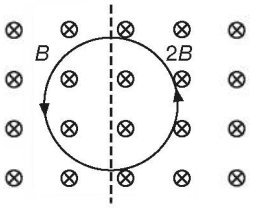
17. **Statement-1:** Any current carrying loop is considered as a magnetic dipole.
Statement-2: The net force on a current loop in a uniform magnetic field is zero.

18. **Statement-1:** If a proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle is double than that of proton.
Statement-2: In a magnetic field, the period of revolution of a charged particle is directly proportional to the mass of the particles and inversely proportional to charge of particle.

19. **Statement-1:** In electric circuits, wires carrying currents in opposite directions are often twisted together.
Statement-2: If the wires are not twisted together, the combination of wires forms a current loop. The magnetic field generated by the loop might affect adjacent circuits or components.

20. **Statement-1:** A charged particle moves perpendicular to magnetic field. Its kinetic energy will remain constant but momentum charges.
Statement-2: Force acts perpendicular to velocity of particle.

21. **Statement-1:** The magnetic field on the closed loop in figure is zero.
Statement-2: Force (magnetic) on the wire is $\int dF = \int Id\vec{l} \times \vec{B}$



22. **Statement-1:** The current constituted by electrons in a metallic wire creates only electric field while electron beam creates both, electric and magnetic fields.
Statement-2: The electron beam contains only electrons while metallic wire carries both positive and negative charges and also the wire is electrically neutral.

23. A semicircular ring is present in the uniform magnetic field. Magnetic field is perpendicular to loop of ring.
Statement-1: Force $\vec{F}$ on each element of ring is different.
Statement-2: Net force on ring must be perpendicular to magnetic field.

24. **Statement-1:** A current I flows along the length of an infinitely long straight and thick-walled pipe. Then the magnetic field at any point inside the pipe is zero.
Statement-2: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

25. **Statement-1:** Magnitude of force acting on a current carrying loop placed in uniform magnetic field will be equal to zero whether magnetic field is in the plane or perpendicular to the plane of loop.
Statement-2: Magnitude of force does not depend upon the direction of magnetic field.

26. **Statement-1:** The poles of magnet cannot be separated by breaking into two pieces.
Statement-2: The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

27. **Statement-1:** The magnetic field at the ends of a very long current carrying solenoid is half of that at the centre.
Statement-2: If the solenoid is sufficiently long, the field with in it is uniform.

28. **Statement-1:** A magnetic field independent of time can change the velocity of a charged particle.
Statement-2: It is not possible to change the velocity of a particle in a magnetic field as magnetic field does not work on charged particle.

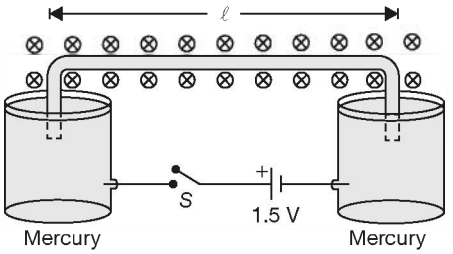
29. **Statement-1:** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.
Statement-2: Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension I

An insulated wire with mass $m = 5.4 \times 10^{-5}$ kg is bent into the shape of an inverted U such that the horizontal part has a length $l = 15$ cm. The bent ends of the wire are partially immersed in two pools of mercury, with 2.5 cm of each end below the mercury's surface. The entire structure is in a region containing a uniform 0.0065 T magnetic field directed into the page. An electrical connection from the mercury pools is made through the ends of the wires. The mercury pools are connected to a 1.5 V battery and a switch S. When switch S is closed, the wire jumps 35 cm into the air, measured from its initial position. Based on the facts provided, answer the following questions.



1. The speed v of the wire as it leaves the mercury is

(A) $5.2\ \text{ms}^{-1}$ (B) $2.5\ \text{ms}^{-1}$

(C) $70\ \text{cms}^{-1}$ (D) $7\ \text{cms}^{-1}$

2. Assuming that the current I through the wire was constant from the time the switch was closed until the wire left the mercury, then I equals.

(A) 7.6 A (B) 6.5 A

(C) 6.7 A (D) 5.6 A

3. Ignoring the resistance of the mercury and the circuit wires, the resistance of the moving wire is

(A) 0.4 Ω (B) 0.3 Ω

(C) 0.2 Ω (D) 0.1 Ω

Comprehension 2

Consider a particle, having charge of magnitude q, mass 4×10^{-15} kg, to be moving in a region containing a uniform magnetic field $\vec{B}=-(0.4\hat{k})$ tesla. At any instant, velocity of the particle is $\vec{v}=(8\hat{i}-6\hat{j}+4\hat{k})\times10^{6}\ \text{ms}^{-1}$ and force acting on it has a magnitude 1.6 N. Based on the information given answer the following questions.

4. Magnitude of charge on the particle, q is

(A) 0.7 μC (B) 0.5 μC

(C) 0.4 μC (D) 0.25 μC

5. Motion of charged particle will be along helical path with

(A) a translational component along x-direction and a circular component in the y-z plane

(B) a translational component along y-direction and a circular component in the x-z plane.

(C) a translational component along z-axis and a circular component in the x-y plane.

(D) direction of translational component and plane of circular component will be uncertain.

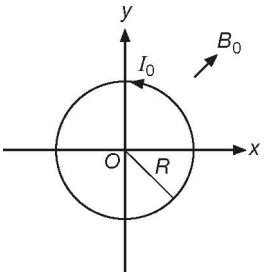
6. If the coordinate of the particle at $t=0$ are $(2\text{ m}, 1\text{ m}, 0)$, then the coordinates at a time $t=3\ T$ (where T is the time period of circular component of motion) will be

(A) $(2, 1, 400)$ m (B) $(0.142, 130, 0)$ m

(C) $(2, 1, 1.884)$ m (D) $(142, 130, 628)$ m

Comprehension 3

A uniform constant magnetic field $\vec{B}_0$ is directed at an angle of 45° to the x-axis in xy plane. A circular ring of mass M, radius R is carrying a steady current I_0, with its centre at the origin O. At time $t=0$, the ring is at rest in the position shown in the figure with the plane of the ring in xy plane. Based on the facts provided, answer the following questions.



7. Torque $|\vec{\tau}|$ about O acting on the ring is

(A) $I_0\pi R^2B_0$ (B) $\dfrac{I_0\pi R^2B_0}{2}$

(C) $\dfrac{I_0\pi R^2B_0}{\sqrt{2}}$ (D) $\sqrt{2}I_0\pi R^2B_0$

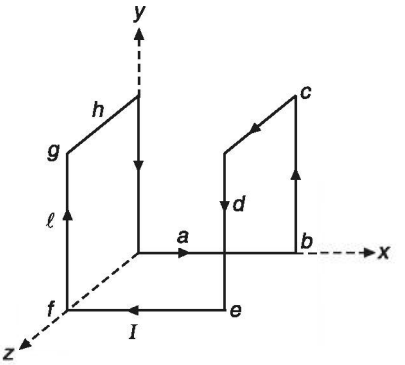
8. The angle by which the ring rotates in a very short interval of time Δt is

(A) $\dfrac{\pi I_0B_0}{2M}(\Delta t)^2$ (B) $\dfrac{\pi I_0B_0}{\sqrt{2}M}(\Delta t)^2$

(C) $\dfrac{2\pi I_0B_0}{M}(\Delta t)^2$ (D) $\dfrac{\pi I_0B_0}{M}(\Delta t)^2$

Comprehension 4

A current I flows through a loop *abcdefgha* along the edge of a cube of length l as shown in Figure.



One corner a of the loop lies at origin. Based on the facts provided, answer the following questions.

9. The current path $abcdefgha$ can be treated as a superposition of three-square loops carrying current I. Select the correct option.

(A) $fghaf$, $fabef$, $ebcde$ (B) $fghaf$, $fabef$, $fgdef$

(C) $fghaf$, $abcha$, $ebcde$ (D) $fgdef$, $fabef$, $ebcde$

10. The unit vector in the direction of magnetic field at the centre of cube $abcdefgh$ is

(A) $\hat{i}$ (B) $-\hat{j}$

(C) $\dfrac{2\hat{i}-\hat{j}}{\sqrt{5}}$ (D) $\hat{k}$

11. Now if a uniform external magnetic field $\vec{B}=B_0\hat{j}$ is switched on, then the torque due to external magnetic field $(\vec{B})$ acting on the current carrying loop $(abcdefgha)$ is

(A) $(Il^2B_0)\hat{k}$ (B) $-(Il^2B_0)\hat{i}$

(C) $-(Il^2B_0)\hat{k}$ (D) $\vec{0}$

Comprehension 5

A very large parallel-plate capacitor caries charge with uniform charge per unit area $+\sigma$ on the upper plate and $-\sigma$ on the lower plate. The plates are horizontal and both move horizontally with speed v to the right. Based on the information given answer the following questions.

12. The magnetic field between the plates is

(A) $\mu_0\sigma v$ (B) $2\mu_0\sigma v$

(C) $\dfrac{\mu_0\sigma v}{2}$ (D) ZERO

13. The magnetic field close to the plates but outside of the capacitor is

(A) $\mu_0\sigma v$ (B) $2\mu_0\sigma v$

(C) $\dfrac{\mu_0\sigma v}{2}$ (D) ZERO

14. The magnetic force per unit area acting on the upper plate is

(A) $\mu_0\sigma v^2$ (B) $\mu_0\sigma^2v^2$

(C) $\dfrac{1}{2}\mu_0\sigma v^2$ (D) $\dfrac{1}{2}\mu_0\sigma^2v^2$

15. The electric force per unit area acting on the upper plate is

(A) $\dfrac{\sigma^2}{\varepsilon_0}$ (B) $\dfrac{\sigma^2}{2\varepsilon_0}$

(C) $\dfrac{\sigma}{\varepsilon_0}$ (D) $\dfrac{\sigma^2}{3\varepsilon_0}$

16. At some hypothetical speed v (say), the magnetic force on a plate balances the electric force on the plate. Then v equals

(A) $\dfrac{c}{3}$ (B) $\dfrac{c}{2}$

(C) c (D) $\dfrac{c}{4}$

Comprehension 6

A particle having charge $q=10\ \mu C$ moves in uniform magnetic field with velocity $v_1=10^6\ ms^{-1}$ at angle $45°$ with x-axis in the x-y plane and experiences a force $F_1=5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2=10^6\ ms^{-1}$ along the z-axis it experiences a force F_2 in y direction. Based on the information given, answer the following questions.

17. The magnetic field $\vec{B}$ (in tesla) is

(A) $(10^{-3})(\hat{i}+\hat{j})$ (B) $(2\times10^{-3})\hat{i}$

(C) $(10^{-3})\hat{i}$ (D) $(2\times10^{-3})(\hat{i}+\hat{j})$

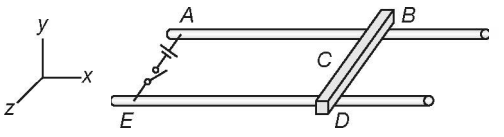
18. The magnitude of the force F_2 (in newton) is

(A) 10^{-2} (B) 10^{-3}

(C) 10^{-4} (D) 10^{-5}

Comprehension 7

Rail guns have been suggested for launching projectiles into space without chemical rockets, and for ground-to-air antimissile weapons of war. A tabletop model rail gun consists of two long parallel horizontal rails 2.5 cm apart, bridged by a bar BD of mass 5 g. The bar is originally at rest at the midpoint of the rails and is free to slide without friction. When the switch is closed, electric current is quickly established in the circuit $ABCDEA$. The rails and bar have low electric resistance, and the current is limited to a constant 25 A by the power supply. Based on the facts provided, answer the following questions.



19. The magnitude of the magnetic field, 1.25 cm from a single very long straight wire carrying current 24 A is

(A) 100 μT (B) 200 μT

(C) 300 μT (D) 400 μT

20. The magnitude of the magnetic field at point C in the diagram, the midpoint of the bar, immediately after the switch is closed.

(A) 400 μT, along $-y$ axis

(B) 300 μT, along $-x$ axis

(C) 200 μT, along x axis

(D) 400 μT, along $+y$ axis

21. At other points along the bar BD, the field is in the same direction as at point C, but larger in magnitude. Assume that the average effective magnetic field along BD is five times larger than the field at C. With this assumption, the magnitude and direction of the force on the bar is

(A) 1.25×10^{-4} N, along $-x$ axis

(B) 1.25×10^{-3} N, along $+x$ axis

(C) 1.25×10^{-6} N, along $+y$ axis

(D) 1.25×10^{-5} N, along $-y$ axis

22. The acceleration of the bar when it is in motion is

(A) 0.25 cms^{-2} (B) 0.125 cms^{-2}

(C) 25 cms^{-2} (D) 12.5 cms^{-2}

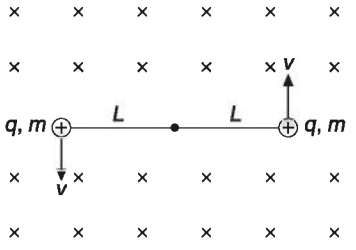
23. The velocity of the bar after it has travelled 120 cm to the end of the rails is

(A) 30 cms^{-1} (B) 40 cms^{-1}

(C) 50 cms^{-1} (D) 60 cms^{-1}

Comprehension 8

Two charge particles each of mass m, carrying charge $+q$ and connected with each other by a massless inextensible string of length $2L$ are describing circular path in the plane of paper, each with speed $v=\dfrac{q\alpha L}{m}$ (where α is constant) about their centre of mass in the region in which an uniform magnetic field $\vec{B}$ exists into the plane of paper as shown in Figure.



Neglect any effect of electrical and gravitational forces, answer the following questions.

24. The magnitude of the magnetic field, such that no tension is developed in the string is

(A) $\dfrac{\alpha}{2}$ (B) α

(C) 2α (D) 0

25. If the actual magnitude of magnetic field is half the value calculate above, then tension in the string will be

(A) $\dfrac{3}{4}\dfrac{q^2\alpha^2L}{m}$ (B) ZERO

(C) $\dfrac{q^2\alpha^2L}{2m}$ (D) $\dfrac{2q^2\alpha^2L}{m}$

26. If the string breaks when the tension is $T=\dfrac{3}{4}\dfrac{q^2\alpha^2L}{m}$ and the magnetic field is reduced to a value such that the string just breaks, then the maximum separation between the two particles during their motion is

(A) $16L$ (B) $4L$

(C) $14L$ (D) $2L$

Comprehension 9

A current carrying ring with its centre at origin and moment of inertia 2×10^{-2} kgm^2 about an axis passing through its centre and perpendicular to its plane has magnetic moment $\vec{M}=\left(3\hat{i}-4\hat{j}\right)$ Am^2. At time $t=0$ a magnetic field $\vec{B}=\left(4\hat{i}+3\hat{j}\right)$ T is switched on. Based on the facts provided, answer the following questions.

27. The initial angular acceleration of the ring, in rads^{-2} is

(A) ZERO (B) 1250

(C) 2500 (D) 5000

28. The maximum angular velocity of the ring in rads^{-1} is

(A) $25\sqrt{2}$ (B) $50\sqrt{2}$

(C) $100\sqrt{2}$ (D) $200\sqrt{2}$

Comprehension 10

The cyclotron is a device which is used to accelerate charged particles such as protons, deuterons, alpha particles, etc. to very high energy. The principle on which a cyclotron works is based on the fact that an electric field can accelerate a charged particle and a magnetic field can throw it into a circular orbit. A particle of charge $+q$ experiences a force qE in an electric field E and this force is independent of velocity of particle. The particle is accelerated in the direction of the magnetic field. On the other

hand, a magnetic field at right angles to the direction of motion of the particle throws the particle in a circular orbit in which the particle revolves with a frequency that does not depend on its speed. A modest potential difference is used as a source of electric field. If a charged particle is made to pass through this potential difference a number of times, it will acquire an enormous by large velocity and hence kinetic energy.

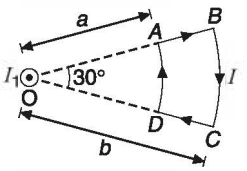
29. Which of the following cannot be accelerated in a cyclotron?
(A) Protons (B) Deuterons
(C) Alpha particles (D) Neutrons

30. The working of a cyclotron is based on the fact that
(A) The force experienced by a charged particle in an electric field is independent of its velocity.
(B) The radius of the circular orbit of a charged particle in a magnetic field increase with increase in its speed.
(C) At a given speed, the radius of the circular orbit is the same for particles having same charge to mass ratio.
(D) The frequency of revolution of the particle along the circular path does not depend on its speed.

31. Cyclotron is not suitable for accelerating
(A) Electrons (B) Protons
(C) Deuterons (D) Alpha particles

Comprehension 11

A current loop $ABCD$ is held fixed on the plane of the paper as shown in Figure. The arcs BC (radius $= b$) and DA (radius $= a$) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin. Based on the information given answer the following questions.



32. The magnitude of the magnetic field (B) due to the loop $ABCD$ at the origin (O) is
(A) ZERO
(B) $\dfrac{\mu_0 I(b-a)}{24ab}$
(C) $\dfrac{\mu_0 I}{4\pi}\left[\dfrac{b-a}{ab}\right]$
(D) $\dfrac{\mu_0 I}{4\pi}\left[2(b-a)+\dfrac{\pi}{3}(a+b)\right]$

33. Due to the presence of the current I_1 at the origin
(A) The forces on AB and DC are zero
(B) The forces on AD and BC are zero
(C) The magnitude of the net force on the loop is given by $\dfrac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\dfrac{\pi}{3}(a+b)\right]$
(D) The magnitude of the net force on the loop is given by $\dfrac{\mu_0 I I_1}{24ab}(b-a)$

34. Force on wire AB is
(A) ZERO (B) $\dfrac{\mu_0 I I_1}{2\pi}\log_e\left(\dfrac{b}{a}\right)$
(C) $\dfrac{\mu_0 I I_1}{2\pi}\log_e\left(\dfrac{b+a}{a}\right)$ (D) $\dfrac{\mu_0 I I_1}{\pi}\log_e\left(\dfrac{b}{a}\right)$

Comprehension 12

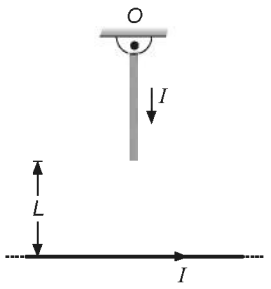
Consider a particle having specific charge α released from the origin with velocity $\vec{v}=v_0\hat{i}$ in a region of uniform electric and magnetic field both parallel to y-axis given by $\vec{E}=E_0\hat{j}$ and $\vec{B}=B_0\hat{j}$. Based on the facts provided, answer the following questions.

35. The path followed by the particle is
(A) a circle
(B) a straight line
(C) a helix with non-uniform pitch
(D) a helix with uniform pitch

36. The y-coordinate of particle when it crosses the y-axis for the nth time is
(A) $\dfrac{n^2E_0}{B_0^2\alpha}$ (B) $\dfrac{2n^2\pi^2E_0}{B_0^2\alpha}$
(C) $\dfrac{B_0^2\alpha}{n^2E_0}$ (D) $\dfrac{B_0^2\alpha}{2n^2\pi^2E_0}$

Comprehension 13

A wire of length L, mass m and carrying a current I is suspended from point O as shown. An another infinitely long wire carrying the same current I is at a distance L below the lower end of the wire. Given $I=2$ A, $L=1$ m and $m=0.1$ kg$(\log_e 2=0.7)$



37. The angular acceleration of the wire just after it is released from the position shown is

(A) 6.4×10^{-5} rads^{-2} (B) 3.2×10^{-4} rads^{-2}

(C) 1.6×10^{-6} rads^{-2} (D) 9.6×10^{-6} rads^{-2}

38. It is desired to keep the suspended wire stationary by placing a third infinitely long wire carrying an upward current. Then this wire should be placed

(A) to the right of suspended wire

(B) to the left of suspended wire

(C) we can keep it either to the right or to the left. It will depend on the magnitude of the current in the third wire

(D) we can't keep suspended wire stationary be placing a third wire to the right or to the left of it

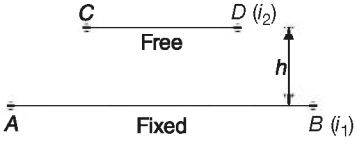
39. The distance r, from the suspended wire where the new wire (having the same current) should be placed to keep it stationary is

(A) 2.5 m (B) 2 m

(C) 1.25 m (D) 3 m

Comprehension 14

The force per unit length between two parallel current carrying wires $=\frac{\mu_0 i_1 i_2}{2\pi r}$. The force is attractive when the current is in same direction and repulsive, when they are in opposite directions. The force between the wires depends on the distance between them. An arrangement of two parallel wires is shown. We can determine the equilibrium position. Then we displace upper wire by a small distance, keeping lower wire fixed. If the wire returns to or tries to return to its equilibrium position, its equilibrium is stable. We can thus show that upper wire can execute linear simple harmonic motion or not. The length of wire AB is large as compared to separation between the wires.



40. If wire CD is in equilibrium position then which of the following may represent the directions of current in the wires

(A) $A\to B$ in AB and $C\to D$ in CD

(B) $A\to B$ in AB and $D\to C$ in CD

(C) $B\to A$ in AB and $D\to C$ in CD

(D) None of these

41. If λ is mass per unit length of wire CD, then the equilibrium separation h is given by

(A) $h=\frac{\mu_0 i_1 i_2}{2\pi\lambda g}$ (B) $h=\frac{2\mu_0 i_1 i_2}{2\pi\lambda g}$

(C) $h=\frac{2\pi\lambda g}{\mu_0 i_1 i_2}$ (D) $h=\frac{4\pi\lambda g}{\mu_0 i_1 i_2}$

42. If wire CD is displaced upward to increase the separation by dh, the magnitude of net force per unit length acting on the wire CD becomes

(A) $\frac{\mu_0 i_1 i_2}{2\pi(h+dh)}$ (B) $\frac{\mu_0 i_1 i_2}{2\pi h^2}$

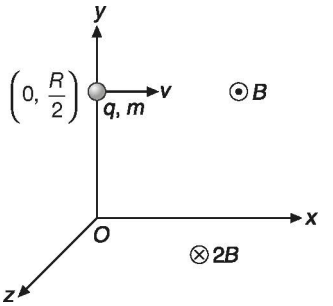
(C) $\frac{\mu_0 i_1 i_2 dh}{2\pi}$ (D) $\frac{\lambda g dh}{h}$

Comprehension 15

Magnetic field in a region is given by

$$\vec{B}=\begin{cases} B\hat{k}, \text{ for } x>0,\ y>0 \\ -(2B)\hat{k}, \text{ for } x>0,\ y<0 \\ \text{zero, for all other cases} \end{cases}$$

A particle of charge q, mass m, moving with a speed v parallel to x-axis enters in the region at $\left(0, \frac{R}{2}\right)$, where $R=\frac{mv}{qB}$ as shown in Figure.



Based on the information given, answer the following questions.

43. Time taken by the particle to hit the x-axis first time is

(A) $\frac{\pi m}{qB}$ (B) $\frac{\pi m}{3qB}$

(C) $\frac{\pi m}{2qB}$ (D) $\frac{4\pi m}{3qB}$

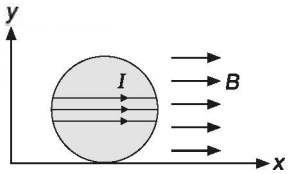
44. Time taken by the particle when its velocity becomes parallel to x-axis first time after entering the magnetic field is

(A) $\frac{\pi m}{qB}$ (B) $\frac{\pi m}{3qB}$

(C) $\frac{\pi m}{2qB}$ (D) $\frac{4\pi m}{3qB}$

Comprehension 16

A person wants to roll a solid non-conducting spherical ball of mass m and radius r on a surface whose coefficient of static friction is μ. He placed the ball on the surface wrapped with n turns of closely packed conducting coils of negligible mass at the diameter. By some arrangement he makes a current I to pass through the coils either in the clockwise direction or in the anti-clockwise direction. A constant horizontal magnetic field $\vec{B}$ is present throughout the space as shown in the figure. Assume μ is sufficient enough to ensure pure rolling motion. Based on the facts provided, answer the following questions.



45. The maximum torque in the coil is

(A) $-(\pi nIr^2B)\hat{k}$ (B) $(\pi nIr^2B)\hat{j}$

(C) $-(\pi nIr^2B)\hat{j}$ (D) $(\pi nIr^2B)\hat{k}$

46. Angular acceleration of the ball after it has rotated through an angle $\theta(\theta<180°)$, is

(A) $\frac{5}{7}\left(\frac{\pi nIB}{m}\right)\cos\theta$ (B) $\frac{2}{5}\left(\frac{\pi nIB}{m}\right)\cos\theta$

(C) $\frac{7}{5}\left(\frac{\pi nIB}{m}\right)\cos\theta$ (D) $\frac{5}{2}\left(\frac{\pi nIB}{m}\right)\cos\theta$

47. The angular velocity of the ball when it has rotated through an angle θ is $(\theta<180°)$, is

(A) $\sqrt{\frac{10}{7}\left(\frac{\pi nIB}{m}\right)\sin\theta}$ (B) $\sqrt{\frac{5}{14}\left(\frac{\pi nIB}{m}\right)\sin\theta}$

(C) $\sqrt{\frac{5}{14}\left(\frac{\pi nIB}{m}\right)\cos\theta}$ (D) $\sqrt{\frac{5}{7}\left(\frac{\pi nIB}{m}\right)\sin\theta}$

48. The minimum value of μ for which the rolling motion is possible, is

(A) $\left(\frac{14\pi}{5g}\right)\left(\frac{nIB}{m}\right)r$ (B) $\left(\frac{5\pi}{7g}\right)\left(\frac{nIB}{m}\right)r$

(C) ZERO (D) $\left(\frac{7\pi}{5g}\right)\left(\frac{nIB}{m}\right)r$

49. If the surface is frictionless, then the sphere will

(A) only translate
(B) only rotate
(C) undergo pure rolling
(D) undergo rolling with sliding

Comprehension 17

A uniform and constant magnetic field $\vec{B}=\left(6\hat{i}-8\hat{j}+24\hat{k}\right)$ T exists in space. A particle with charge to mass ratio $\frac{1}{13}\times10^3$ Ckg^{-1} enters this region at time $t=0$ with a velocity $\vec{v}=\left(6\hat{i}+24\hat{j}+8\hat{k}\right)$ ms^{-1}. Assuming that the charged particle always remains in space having the given magnetic field, answer the following questions.

50. During the further motion of the particle in the magnetic field, the angle between the magnetic field and velocity of the particle

(A) remains constant
(B) increases
(C) decreases
(D) may increase or decrease

51. The frequency of the revolution of the particle is

(A) 636 Hz (B) 318 Hz

(C) 159 Hz (D) 1272 Hz

52. The pitch of the helical path followed by the particle is approximately

(A) $\frac{1}{100}$ m (B) $\frac{1}{120}$ m

(C) $\frac{1}{260}$ m (D) $\frac{1}{130}$ m

Comprehension 18

Each of two long parallel wires carries a current I along the same direction. The wires are placed at $(-L, 0)$ and $(L, 0)$. Both wires carry current along $+z$ direction. Based on the information given answer the following questions.

53. The magnetic field midway between the wires at the point $(0, y, 0)$ is given by

(A) $\frac{\mu_0 I}{2\pi}\left(\frac{y}{\sqrt{L^2+y^2}}\right)$ (B) $\frac{\mu_0 I}{\pi}\left(\frac{y}{L^2+y^2}\right)$

(C) $\frac{2\mu_0 I}{\pi}\left(\frac{y}{\sqrt{L^2+y^2}}\right)$ (D) $\frac{\mu_0 I}{4\pi}\left(\frac{y}{\sqrt{L^2+y^2}}\right)$

54. The value of y for which magnitude of magnetic induction in the symmetry plane of the system located midway between the wires is maximum is

(A) $\frac{L}{2}$ (B) $\frac{L}{3}$

(C) $\frac{L}{4}$ (D) L

55. The maximum magnitude of magnetic field in the symmetry plane of the system located between the wires is

(A) $\frac{\mu_0 I}{\pi L}$ (B) $\frac{\mu_0 I}{2\pi L}$

(C) $\frac{\mu_0 I}{4\pi L}$ (D) $\frac{2\mu_0 I}{\pi L}$

Comprehension 19

A particle having unit mass and charge (in SI system) is released from rest from the origin of the coordinate system. There are electric and magnetic fields given by

$\vec{E} = (10\hat{i})\ \text{NC}^{-1}$ for $0 \le x \le 1.8m$ and

$\vec{B} = (-5\hat{k})T$ for $1.8m \le x \le 2.4m$

A screen is placed parallel to yz plane at $x = 3$ m. Neglecting forces due to gravity answer the following questions.

56. The speed with which the particle will collide the screen in ms^{-1} is

(A) 12 (B) 9
(C) 6 (D) 3

57. The particle collides the screen at $P(x, y)$, where x and y are in metre. Then

(A) $x = 3$, $y = 1.9$ (B) $x = 3$, $y = 0.5$

(C) $x = 3$, $y = 0$ (D) $x = 2.4$, $y = 1.5$

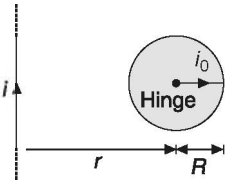
58. The time after which the particle will collide the screen is

(A) $\frac{1}{5}\left(3+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ s (B) $\frac{1}{5}\left(6+\frac{\pi}{3}+\sqrt{3}\right)$ s

(C) $\frac{1}{5}\left(5+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ s (D) $\frac{1}{5}\left(6+\frac{\pi}{18}+\sqrt{3}\right)$ s

Comprehension 20

A long straight conductor carrying current i and a wooden disc of mass m, radius R is placed near each other such that conductor lies in the plane of the disc as shown in Figure.



The distance between the wire and centre of disc is $r(>R)$. A wire of length R and carrying current i_0 is pasted on the disc such that the wire is initially perpendicular to the long straight conductor. The disc is hinged at the centre. Neglecting mass of conducting wire, answer the following questions.

59. Angular acceleration of the disc at that instant is

(A) $\frac{\mu_0 i i_0}{\pi m r^2}\left[R - r\log_e\left(\frac{r}{R+r}\right)\right]$

(B) $\frac{\mu_0 i i_0}{\pi m R^2}\left[R - r\log_e\left(1+\frac{R}{r}\right)\right]$

(C) $\frac{\mu_0 i i_0}{\pi m r^2}\left[R - r\log_e\left(1+\frac{R}{r}\right)\right]$

(D) $\frac{\mu_0 i i_0}{\pi m R^2}\left[R - r\log_e\left(1+\frac{R^2}{r^2}\right)\right]$

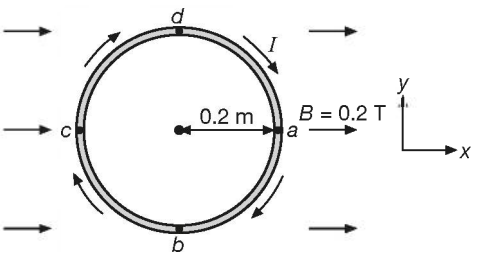
60. Reaction at the hinge at that instant is

(A) $\frac{\mu_0 i i_0}{2\pi}\log_e\left(1+\frac{r}{R}\right)$ (B) $\frac{\mu_0 i i_0}{2\pi}\log_e\left(1+\frac{R^2}{r^2}\right)$

(C) $\frac{\mu_0 i i_0}{4\pi}\log_e\left(1+\frac{r}{R}\right)$ (D) $\frac{\mu_0 i i_0}{2\pi}\log_e\left(1+\frac{R}{r}\right)$

Comprehension 21

A rigid circular loop has a radius of 0.2 m and is in the x-y plane. A clockwise current I is carried by the loop, as shown. The magnitude of the magnetic moment of the loop is $0.75\ \text{Am}^2$. A uniform external magnetic field, $B = 0.2$ T in the positive x-directions is present



61. In figure, the magnitude of the magnetic torque exerted on the loop is closest to

(A) 0.55 Nm (B) 0.15 Nm
(C) 0.45 Nm (D) 0.35 Nm

62. In figure the loop is released from rest. The initial motion of the loop is described by

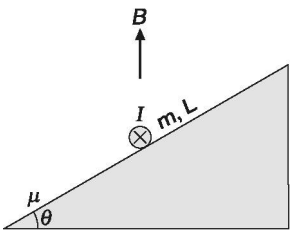
(A) point a move out of the plane, point c moves into the plane
(B) points a, b, c and d move counter clockwise
(C) points a, b, c and d move clockwise
(D) point c moves out of the plane, point a move into the plane.

63. In figure, an external torque changes the orientation of loop from one of lowest potential energy to one of highest potential energy. The work done by the external torque is closest to

(A) 0.5 J (B) 0.2 J
(C) 0.3 J (D) 0.4 J

Comprehension 22

On an inclined plane, a conducting rod of mass m, length L is placed so that it is horizontal. A vertically upward uniform magnetic field B is present in the region as shown in Figure.



A current I flows through the rod. The coefficient of friction between the rod and the incline is μ. Assuming that $\mu > \tan\theta$, answer the following questions

64. The minimum value of B for which the rod stays in equilibrium is

(A) $\frac{mg}{IL}\left(\frac{\sin\theta-\mu\cos\theta}{\sin\theta+\mu\cos\theta}\right)$ (B) $\frac{mg}{IL}\left(\frac{\sin\theta-\mu\cos\theta}{\cos\theta+\mu\sin\theta}\right)$

(C) $\frac{mg}{IL}\left(\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right)$ (D) $\frac{mg}{IL}\left(\frac{\cos\theta+\mu\sin\theta}{\cos\theta-\mu\sin\theta}\right)$

65. The maximum value of B for which the rod stays in equilibrium is

(A) $\frac{mg}{IL}\left(\frac{\cos\theta-\mu\sin\theta}{\sin\theta+\mu\cos\theta}\right)$ (B) $\frac{mg}{IL}\left(\frac{\sin\theta-\mu\cos\theta}{\cos\theta+\mu\sin\theta}\right)$

(C) $\frac{mg}{IL}\left(\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right)$ (D) $\frac{mg}{IL}\left(\frac{\cos\theta+\mu\sin\theta}{\cos\theta-\mu\sin\theta}\right)$

Comprehension 23

A particle of mass m, charge q is released from the origin O with velocity $\vec{v}=v_0\hat{i}$ in a uniform magnetic field $\vec{B}=\frac{B_0}{2}\hat{i}+\frac{\sqrt{3}B_0}{2}\hat{j}$. Assuming that the field extends all over the space, answer the following questions.

66. The path followed by the particle is a/an

(A) circle (B) helix
(C) ellipse (D) cycloid

67. Pitch of the helical path described by the particle is

(A) $\frac{2\pi mv_0}{qB_0}$ (B) $\frac{\pi mv_0}{qB_0}$

(C) $\frac{2\sqrt{3}\pi mv_0}{qB_0}$ (D) $\frac{\sqrt{3}\pi mv_0}{2qB_0}$

68. z-component of velocity is $\frac{\sqrt{3}v_0}{2}$ after time $t=$ .....

(A) $\frac{\pi m}{4qB_0}$ (B) $\frac{\pi m}{2qB_0}$

(C) $\frac{\pi m}{qB_0}$ (D) $\frac{2\pi m}{qB_0}$

69. Maximum z-coordinate of the particle is

(A) $\frac{mv_0}{qB_0}$ (B) $\frac{2mv_0}{qB_0}$

(C) $\frac{\sqrt{3}mv_0}{qB_0}$ (D) $\frac{2\sqrt{3}mv_0}{qB_0}$

70. When z-co-ordinate has its maximum value

(A) $v_y=\frac{\sqrt{3}}{2}v_0$

(B) $v_x=-\frac{v_0}{2}$

(C) Both (A) and (B) are correct
(D) Both (A) and (B) are incorrect

Comprehension 24

In the region between the plane $z=0$ and $z=a$ $(a>0)$, the uniform electric and magnetic fields are given by $\vec{E}=E_0\left(\hat{j}-\hat{k}\right)$, $\vec{B}=B_0\hat{i}$. The region defined by $a\le z\le b$ contains only magnetic field $\vec{B}=-B_0\hat{i}$. Beyond $z>b$ no field exits. A positive point charge q is projected from the origin with velocity $v_0\hat{k}$ such that the particle moves undeviated up to the plane $z=a$. Assuming the mass of the particle to be $\frac{2}{3}\left(\frac{qE_0a}{v_0^2}\right)$ and ignoring gravitational force everywhere, answer the following questions.

71. The value of E_0 is

(A) $\frac{B_0 v_0}{\sqrt{2}}$ (B) $B_0 v_0$

(C) $\sqrt{2} B_0 v_0$ (D) $\frac{B_0 v_0}{2}$

72. The minimum value of b such that the particle reverses its direction completely is

(A) $\frac{4E_0 a}{3v_0 B_0}$ (B) $a\left(1+\frac{4E_0}{3v_0 B_0}\right)$

(C) $\frac{2E_0 a}{2v_0 B_0}$ (D) $a\left(1+\frac{2E}{3v_0 B_0}\right)$

73. When the particle just reverses its direction, all the fields are switched off. Time taken by the particle to touch the plane $z = 0$ from that instant is

(A) $\frac{a}{v_0}$ (B) $\frac{a}{2v_0}$

(C) $\frac{a}{3v_0}$ (D) $\frac{a}{4v_0}$

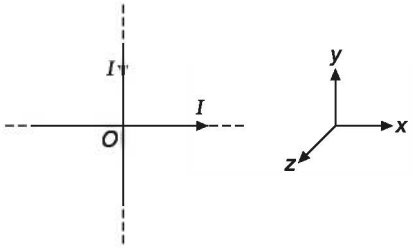
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	○	○	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Equal currents are flowing in two infinitely long wires lying along x and y-axes in the directions shown in Figure. Match the following two columns.



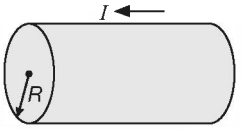
COLUMN-I	COLUMN-II
(A) Magnetic field at (a, a)	(p) along positive y-axis
(B) Magnetic field at $(-a, -a)$	(q) along positive z-axis
(C) Magnetic field at $(a, -a)$	(r) along negative z-axis
(D) Magnetic field at $(-a, a)$	(s) ZERO

2. Match the force/field in **COLUMN-I** to the best respective properties in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Electric field	(p) Stationary charge
(B) Magnetic field	(q) Moving charge
(C) Electric force	(r) Changes the kinetic energy
(D) Magnetic force	(s) Does not change kinetic energy

(*Continued*)

3. A long straight conductor of radius R carries a current I as shown. The current density j varies as a function of radius according to $j = br$ where b is a constant. Match the quantities in **COLUMN-I** with those in **COLUMN-II**

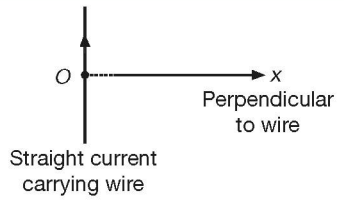
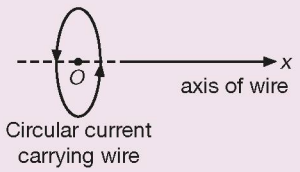
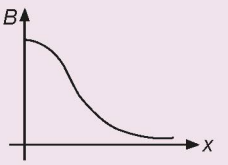


COLUMN-I	COLUMN-II
(A) B_{inside}	(p) Maximum
(B) $B_{surface}$	(q) $\frac{\mu_0 br^2}{3}$
(C) $B_{outside}$	(r) $\frac{2\pi br^3}{3}$
(D) I_{inside}	(s) $\frac{\mu_0 bR^3}{3r}$
	(t) Continuous

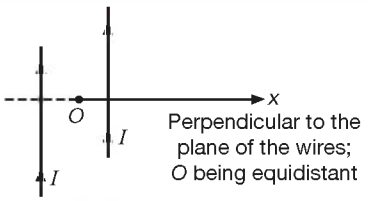
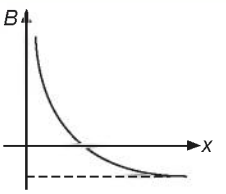
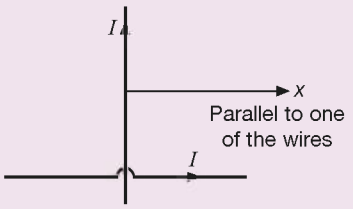
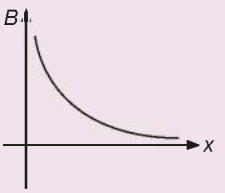
4. An electron is moving along positive x-direction. Match the following two columns for deflection of electron just after the electric field E_0 (in magnitude) and magnetic field B_0 (in magnitude) are switched on.

COLUMN-I	COLUMN-II
(A) When only magnetic field $\vec{B} = B_0\hat{j}$ is switched on	(p) Negative y-axis
(B) When only magnetic field $\vec{B} = B_0\hat{k}$ is switched on	(q) Positive y-axis
(C) When both magnetic field $\vec{B} = B_0\hat{i}$ and electric field $\vec{E} = E_0\hat{j}$ are switched on	(r) Negative z-axis
(D) When only electric field $\vec{E} = E_0\hat{k}$ is switched on	(s) Positive z-axis

5. The entries in **COLUMN-I** depict certain current distributions, while the entries in **COLUMN-II** depict the variation of the magnetic field (B) as one moves along the x-axis for each of these distributes, but in a different order. Match the entries in **COLUMN-I** with the proper entries in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) 	(p)
(B) 	(q) 

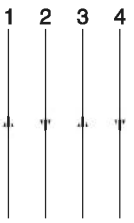
(Continued)

COLUMN-I	COLUMN-II
(C)  Parallel current carrying wires in the same plane	(r) 
(D)  Two perpendicular current carrying wires in the same plane	(s) 

6. Match the following

COLUMN-I	COLUMN-II
(A) Magnetic field inside a long straight solenoid is	(p) Not constant
(B) Magnetic field inside a toroidal solenoid is	(q) ZERO
(C) Magnetic field inside a conducting hollow pipe having current parallel to its axis	(r) Constant
(D) Magnetic field due to current carrying wire on its surface is	(s) Maximum

7. Equal currents are flowing in four infinitely long wires. Distance between two wires is same and directions of currents are shown in Figure. Match the forces in **COLUMN-I** to respective match in **COLUMN-II**.

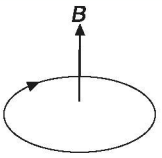
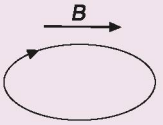
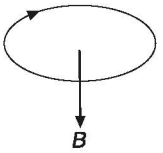
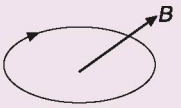


COLUMN-I	COLUMN-II
(A) Force on wire-1	(p) inwards
(B) Force on wire-2	(q) leftwards
(C) Force on wire-3	(r) rightwards
(D) Force on wire-4	(s) ZERO

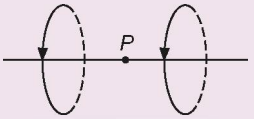
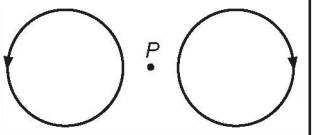
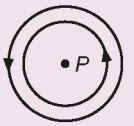
8. For a charged particle of charge q, mass m that enters a region of space the field options are given in **COLUMN-I**. Match those to the trajectories in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Electric field	(p) Straight line path
(B) Magnetic field	(q) Circular path
(C) In crossed field for $v = \frac{E}{B}$	(r) Helical path
(D) In mutually perpendicular electric and magnetic field, charge initially at rest	(s) Cycloidal path
	(t) Parabolic path

9. In **COLUMN-I**, a current carrying loop and a uniform external applied magnetic field are shown. Match the situations in **COLUMN-I** with conclusions in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) Force = 0
(B) 	(q) Maximum torque
(C) 	(r) Minimum potential energy
(D) 	(s) Positive potential energy

10. Two wires each carrying a steady current I are shown in four configurations in **COLUMN-I**. Some of the resulting effects are described in **COLUMN-II**. Match the statements in **COLUMN-I** with the statements in **COLUMN-II**.

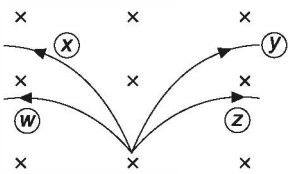
COLUMN-I	COLUMN-II
(A) Point P is situated midway between the wires.	(p) The magnetic field (B) at P due to the currents in the wires are in the same direction.
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	(q) The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	(r) There is no magnetic field at P.
(D) Point P is situated at the common center of the wires. 	(s) The wires repel each other.

(Continued)

11. Match the physical quantities in **COLUMN-II** with the respective SI units in **COLUMN-I**

COLUMN-I	COLUMN-II
(A) $NA^{-1}m^{-1}$	(p) Magnetic permeability (μ_0)
(B) Am^2	(q) Magnetic flux
(C) NmA^{-1}	(r) Magnetic potential energy
(D) NA^{-2}	(s) Magnetic flux density
	(t) Magnetic dipole moment

12. Four charged particles, $(-q, m)$, $(-3q, 4m)$, $(+q, m)$ and $(+2q, m)$ enter in uniform magnetic field directed inwards (away from the reader) with same kinetic energy as shown in Figure. Their paths are shown inside the magnetic field. Match the particles in **COLUMN-I** with their respective paths (w, x, y, z) in **COLUMN-II**

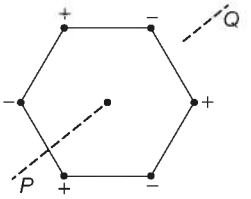
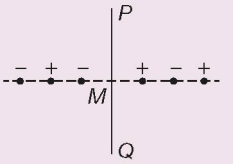
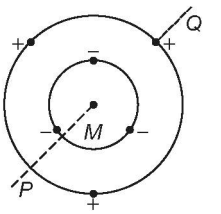
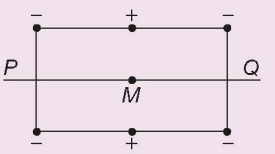
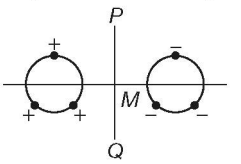


COLUMN-I	COLUMN-II
(A) Particle $(-q, m)$	(p) w
(B) Particle $(-3q, 4m)$	(q) x
(C) Particle $(+q, m)$	(r) y
(D) Particle $(+2q, m)$	(s) z

13. A current flow along the length of a long thin cylindrical shell of radius R. Match the fields in different regions given in **COLUMN-I** to their respective matches in **COLUMN-II**.

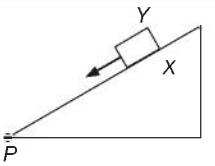
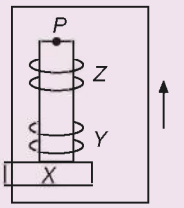
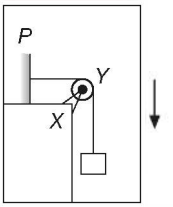
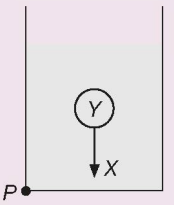
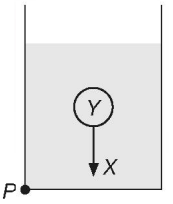
COLUMN-I	COLUMN-II
(A) $B(r < R)$	(p) $B \propto \frac{1}{r}$
(B) $B(r > R)$	(q) ZERO
(C) $B(r = R)$	(r) $B \propto r$
(D) $B(r = 0)$	(s) Maximum
	(t) Discontinuous

14. Six-point charges, each of the same magnitude q, are arranged in different manners as shown in **COLUMN-II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

COLUMN-I	COLUMN-II
(A) $E = 0$	(p) Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon. 
(B) $V \neq 0$	(q) Charges are on a line perpendicular to PQ at equal intervals. M is the midpoint between the two innermost charges. 
(C) $B = 0$	(r) Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of rings. 
(D) $\mu \neq 0$	(s) Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangular. PQ is parallel to the longer sides. 
	(t) Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings. 

(Continued)

15. **COLUMN-II** shows five systems in which two objects are labelled as X and Y. Also, in each case a point P is shown. **COLUMN-I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **COLUMN-II**.

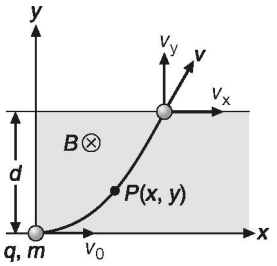
COLUMN-I	COLUMN-II
(A) The force exerted by X on Y has a magnitude Mg.	(p) Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity. 
(B) The gravitational potential energy of X is continuously increasing.	(q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 
(C) Mechanical energy of the system $X+Y$ is continuously decreasing.	(r) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity. 
(D) The torque of the weight of Y about point P is zero.	(s) A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid. 
	(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container. 

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

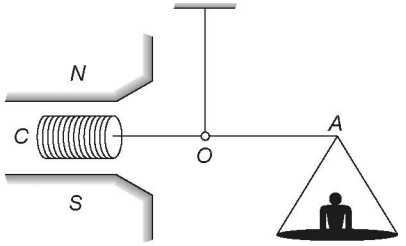
1. In a certain region surrounding the origin of the coordinates, $\vec{B}=5\times10^{-4}$ T and $\vec{E}=5\hat{k}$ Vm^{-1}. A proton enters the fields at the origin with an initial velocity $\vec{v}_0=2.5\times10^{5}\hat{i}$ ms^{-1}. Describe the proton's motion and give its position, in metre, after three complete revolution.

2. A non-uniform magnetic field $\vec{B}=B_0\left(1+\frac{y}{d}\right)\hat{k}$ is present in region of space in between $y=0$ and $y=d$. The lines are shown in Figure.



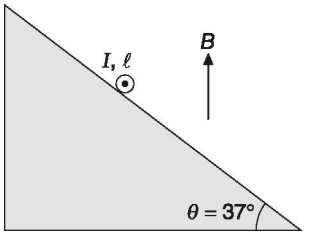
A particle of mass m and positive charge q enters the field with an initial velocity $v = v_0\hat{i}$. The x-component of velocity of the particle is $v_x = v_0 - \frac{kqB_0d}{m}$ when it leaves the field. Then find the value of k.

3. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in Figure.



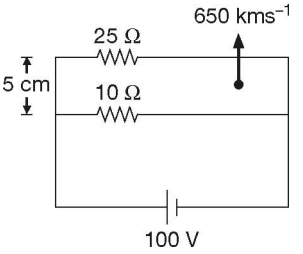
The cross-sectional area of the coil is $A = 1\ \text{cm}^2$, the length of the arm OA of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil the equilibrium is restored by putting the additional counterweight of mass $\Delta m = 60$ mg on the balance pan. Calculate the magnetic induction, in millitesla, at the spot where the coil is located.

4. Two conducting rails are connected to a source of emf and form an incline as shown in Figure. A rod of mass 50 g slides without friction down the incline through a vertical uniform magnetic field B as shown in Figure.



If the length of the rod is 50 cm and a current of 2.5 A is flowing through it, then calculate the value of B in millitesla for which the bar slides at a constant velocity. Take $g = 10\ \text{ms}^{-2}$.

5. The battery branch of the circuit is very far from the two horizontal segments containing two resistors as shown in Figure.



These horizontal segments are separated by 5 cm, and they are much longer than 5 cm. A proton (charge $+e$) is fired at $650\ \text{kms}^{-1}$ from a point midway between the upper two horizontal segments of the circuit. The initial velocity of the proton is in the plane of the circuit and is directed towards the upper wire. It is calculated that the initial magnetic force on the proton comes to be $x \times 10^{-18}$ N, where x happens to be a single digit integer. Find x. Take $e = 1.6 \times 10^{-19}$ C.

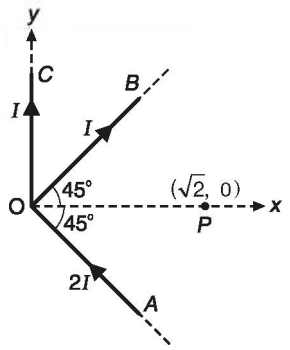
6. A charged particle is accelerated through a potential difference of 12 kV and acquires a speed of $10^6\ \text{ms}^{-1}$. It is projected perpendicularly into the magnetic field of strength 0.2 T. Calculate the radius of circle described by the particle in cm.

7. The magnitude of the earth's magnetic field at either pole is approximately 7×10^{-5} T. If this field is obtained from a current loop around the equator (without considering any contribution from the magnetic materials inside the earth). Calculate the current, in mega ampere, that would generate the field. Take the radius of the earth to be 6.37×10^6 m.

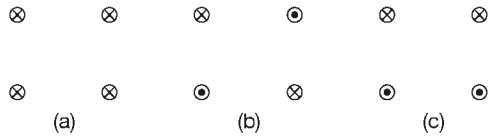
8. A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find B (in tesla) at which the ring is broken.

9. A wire having a linear mass density of $1\ \text{gcm}^{-1}$ is placed on a horizontal surface that has a coefficient of kinetic friction of 0.75. The wire carries a current of 2 A toward the east and slides horizontally to the north. What is the magnitude and direction of the smallest magnetic field that enables the wire to move in this fashion? $\left(\text{Take } g = 10\ \text{ms}^{-2},\ \sin(37) = \frac{3}{5}\right)$. Give the value of field in millitesla.

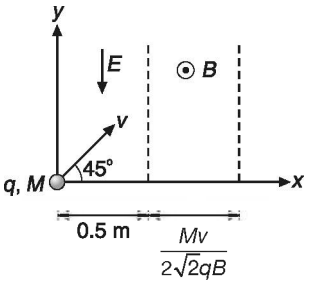
10. Figure shows three sections of a long current carrying wire. At origin, the current is divided in two equal parts. All section lies in xy plane. If the net magnetic field at P is $\frac{\mu_0 Ix}{8\pi}(\sqrt{2}+1)^2$, then calculate x.



11. Four long, parallel power lines each carry 100 A currents. A cross-sectional diagram of these lines is a square, 20 cm on each side. For each of the three cases shown in figure, calculate the magnetic field at the centre of the square for all the situations (a), (b) and (c) shown. Give all the answers in microtesla.

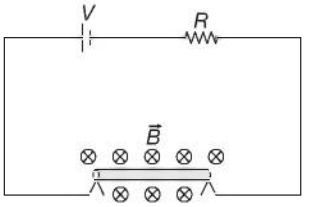


12. A charge particle of charge q and mass M is projected in a region which contains electric (from $x=0$ to $x=0.5$ m) and magnetic field from $x=0.5$ m to $x=\left(0.5+\frac{Mv}{2\sqrt{2}qB}\right)$ m as shown in Figure with velocity v making at an angle of 45° with x-direction.



If $v=\sqrt{\frac{qE}{M}}$, then the total deviation for the particles motion will be (neglect the effect of gravity) in clockwise direction in radian is $\frac{x\pi}{y}$. Calculate x and y.

13. A thin, 50 cm long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a uniform 0.5 T magnetic field, as shown in Figure.

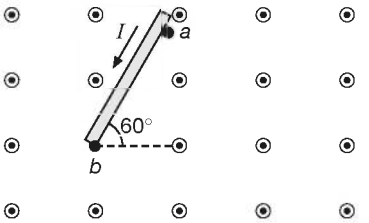


A battery and a 25 Ω resistor in series are connected to the supports. Take $g=10\ \text{ms}^{-2}$

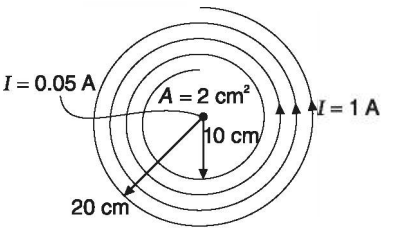
(a) What is the highest voltage, in volt, that the battery can have without breaking the circuit at the supports?

(b) The battery voltage has the maximum value calculated in part (a). If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2Ω, find the initial acceleration, in ms^{-2}, of the bar.

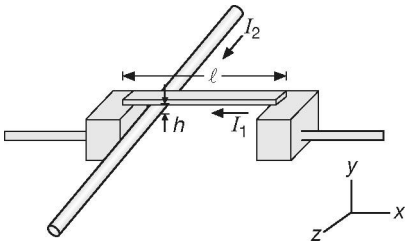
14. A uniform, 500 g metal bar 100 cm long carries a current I in a uniform, horizontal, 0.5 T magnetic field as shown in figure. The bar is hinged at b but rests unattached at a. What is the largest current, in ampere, that can flow from a to b without breaking the electrical contact at a? Assume that the wire be inclined to the horizontal at an angle of 60° as shown. (Take $g=10\ \text{ms}^{-2}$).



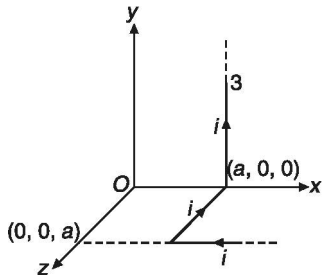
15. A spiral loop has inner radius 10 cm and outer radius 20 cm and carries current 1 A in each turn. The total number of turn in loop is 100. A small rectangular loop of area 2 cm^2 carries current 0.05 A is placed at the centre of loop. If magnitude of potential energy of rectangular loop is $\frac{\mu_0 \ln 2}{x}$ J, Calculate x.



16. A thin copper bar of length $l = 10$ cm is supported horizontally by two (non-magnetic) contacts. The bar carries current $I_1 = 100$ A in the $-x$ direction, as shown in figure. At a distance $h = 0.5$ cm below one end of the bar, a long straight wire carries a current $I_2 = 200$ A in the z-direction. Determine the magnetic force exerted on the bar, in milli newton. Take $\log_e(401) = 0.6$.

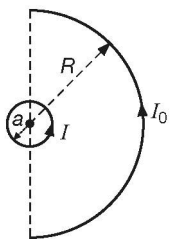


17. An infinite current carrying conductor is bent into three segments as shown in Figure. If it carries current i, the magnetic field at the origin is found to be $\frac{\mu_0 i}{4\pi a}\left[(\sqrt{x}-1)\hat{j}+\hat{k}\right]$. Calculate x.



18. Two protons move parallel to each other with an equal velocity $v = 300$ kms^{-1}. The ratio of forces of magnetic and electrical interaction of the protons is found to be $x \times 10^{-6}$. Find x.

19. A small current carrying loop having current I_0 is placed in the plane of paper as shown. Another semi-circular loop having current I_0 is placed concentrically in the same plane as that of small loop, the radius of semi-circular loop is $R(R \gg a)$. Find the force applied by the smaller ring on bigger ring in newton. (Given $R = 1$ m, $I = I_0 = \frac{40}{\sqrt{\mu_0}}$ A, $a = 0.1$ m)



20. Consider a wire to be placed in a magnetic field which varies with distance x from the origin as

$$\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$$

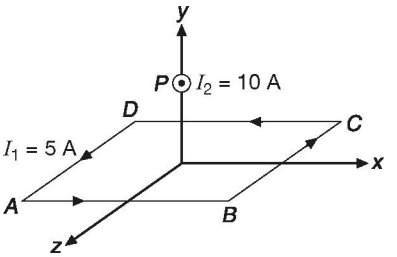
The ends of wire are at $(a, 0)$ and $(2a, 0)$ and it carries a current I. If force on wire is calculated to be

$$\vec{F} = -\left(\frac{kB_0aI}{2}\right)\hat{j}$$

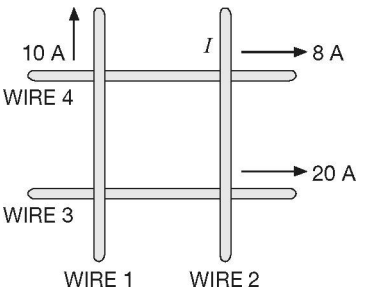
then find the value of k.

21. A current I flows in a rectangular shaped wire whose centre lies at $(x_0, 0, 0)$ and whose vertices are located at the points $A(x_0 + d, -a, -b)$, $B(x_0 - d, a, -b)$, $C(x_0 - d, a, +b)$ and $D(x_0 + d, -a, +b)$ respectively. Assume that $a, b, d \ll x_0$. Find the magnitude of magnetic dipole moment vector of the rectangular wire frame in SI units. (Given $b = 10$ m, $I = 0.01$ A, $d = 4$ m, $a = 3$ m)

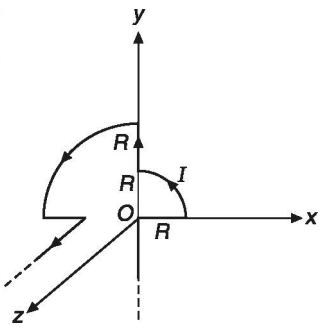
22. A square loop, 16 cm on each side lies in the x-z plane with its centre at the origin. The loop carries a current of 5 A. Above the loop at $y = 6$ cm is an infinitely long straight wire (P) parallel to the z-axis carrying a current of 10 A. The net force on the loop is $F \times 10^{-7}$ N. Find F.



23. Four very long, current carrying wires 1, 2, 3 and 4 in the same plane intersect to form a square 40 cm on each side, as shown in figure. Find the magnitude and direction of the current I in wire 2, so that the magnetic field at the centre of the square is zero. Assume that there are no cross-contacts between any pair of wires. Give your answer in ampere.

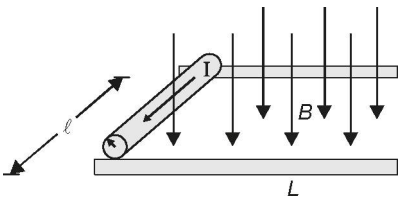


24. In the arrangement shown, if magnetic field at origin is given by $\frac{\mu_0 I}{4R}\left(\frac{a}{4}\hat{k}+\frac{b}{\pi}\hat{j}\right)$.

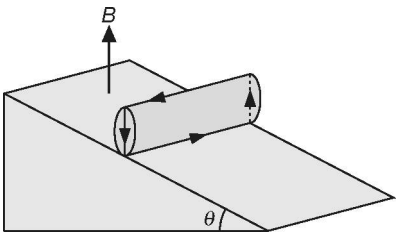


Calculate ab.

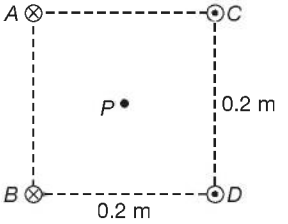
25. A rod of mass 0.72 kg and radius 6 cm rests on two parallel rails that are $l = 12$ cm apart and $L = 45$ cm long. The rod carries a current of $I = 48$ A (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field of magnitude 0.24 T is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod, in cms^{-1}, it leaves the rails?



26. A wooden cylinder of mass $m = 0.250$ kg and length $L = 0.100$ m, has $N = 10$ turns of the wire wrapped around it longitudinally, so that the plane of the coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the inclined plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current i through the coil that prevents the cylinder from rolling down the plane.

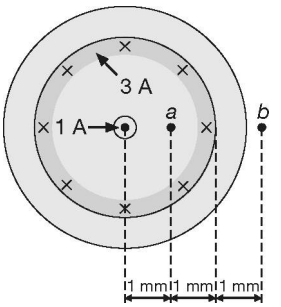


27. Four long, parallel conductors carry equal currents of $I = 5$ A. The figure is an end view of the conductors. The current direction is into the page at points A and B (indicated by the crosses) and out of the page at C and D (indicated by the dots). Calculate the magnitude, in μT and direction of the magnetic field at point P, located at the centre of the square of edge length $l = 0.2$ m.



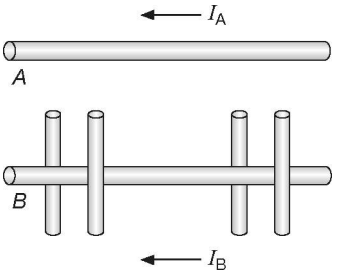
28. A constant homogeneous electric field of $100\ \text{Vm}^{-1}$ exists in the region $x = 0$ and $x = 0.167$ m directed along the positive x-direction. A constant homogeneous magnetic field B exists in the region from $x = 0.167$ m to $x = 0.334$ m directed along the z-direction. A proton lying at rest at the origin $(0, 0)$ is released. Calculate the minimum strength of the magnetic field in millitesla so that the proton will strike the point $(0,\ 0.167\ \text{m})$. Given that the mass of proton is 1.67×10^{-27} kg.

29. Figure is a cross-sectional view of a coaxial cable. The centre conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1 A out of the page and the current in the outer conductor is 3 A into the page. Determine the magnitude and direction of the magnetic field at points a and b. Give your answer in μT.



30. A long conducting cylinder or radius R carries a current I. The current density J varies with radial distance r as $J = br$, where b is a constant. The magnetic field at a distance $r > R$ measured from the axis of cylinder is $B = \frac{\mu_0 b R^x}{y r_2}$. Calculate the ratio of x and y.

31. Two long, parallel conductors are carrying currents in the same direction as in figure. Conductor A carries a current of 100 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. If the linear density of conductor B is 0.15 gcm^{-1}, what value of current I_B, in A, will result in equilibrium when the distance between the two conductors is 2 cm?



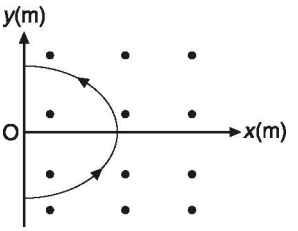
32. The rotor in a certain electric motor is a flat rectangular coil with 1000 turns of wire and dimensions 2.5 cm by 4 cm. The rotor rotates in a uniform magnetic field of 0.8 T. When the plane of the rotor is perpendicular to 10 mA. In this orientation, the magnetic moment of the rotor is directed opposite the magnetic field. The rotor then turns through one-half revolution. This process is repeated to cause the rotor to turn steadily at 600 rev min^{-1}.

(a) Find the maximum torque, in milli newton metre, acting on the rotor.
(b) Find the peak power output of the motor, in watt.
(c) Determine the amount of work performed, in milli joule, by the magnetic field on the rotor in every full revolution.
(d) What is the average power of the motor, in milli watt?
(e) Also, we have that Peak

$$\text{Power} = \frac{(*)^2}{16}(\text{Average Power}),$$

where $*$ is not readable. Find $*$.

33. A wire carrying a current of 3 A is bent in the form of a parabola $y^2 = 4 - x$ as shown in Figure, where x and y are in metre.



The wire is placed in a uniform magnetic field $\vec{B} = 5\hat{k}$ tesla. Calculate the force acting on the wire in newton.

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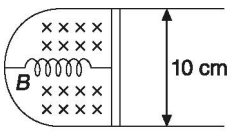
1. **[Online April 2019]**
A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 g, its resistance 10 Ω and air drag negligible, N will be close to

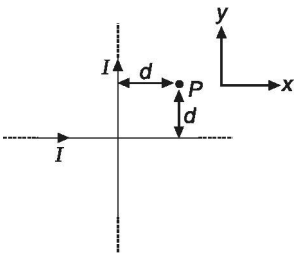


(A) 1000 (B) 5000
(C) 50000 (D) 10000

2. **[Online April 2019]**
A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is

(A) $\frac{B\pi r^2 I}{N}$ (B) $\frac{Br^2 I}{\pi N}$
(C) $B\pi r^2 IN$ (D) ZERO

3. **[Online April 2019]**
Two very long, straight and insulated wires are kept at 90° angle from each other in xy-plane as shown in Figure.



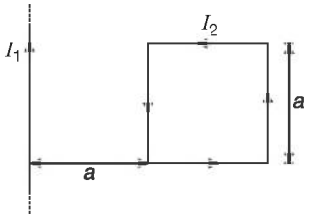
These wires carry current of equal magnitude I, whose directions are shown in Figure. The net magnetic field at point P will be

(A) ZERO (B) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

(C) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ (D) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

4. [Online April 2019]

A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in Figure. The net force on the loop due to the wire will be



(A) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(B) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$

(C) ZERO

(D) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

5. [Online April 2019]

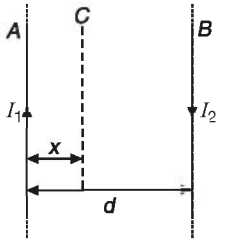
A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} Nm/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately

(A) 10^{-4} (B) 10^{-2}

(C) 10^{-1} (D) 10^{-3}

6. [Online April 2019]

Two wires A and B are carrying currents I_1 and I_2 as shown in Figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are



(A) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

(B) $x = \left(\frac{I_1}{I_1+I_2}\right)d$ and $x = \left(\frac{I_2}{I_1-I_2}\right)d$

(C) $x = \left(\frac{I_2}{I_1+I_2}\right)d$ and $x = \left(\frac{I_2}{I_1-I_2}\right)d$

(D) $x = \left(\frac{I_1}{I_1-I_2}\right)d$ and $x = \left(\frac{I_2}{I_1+I_2}\right)d$

7. [Online April 2019]

A proton, an electron and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(A) $r_e < r_p < r_{He}$ (B) $r_e > r_p = r_{He}$

(C) $r_e < r_p = r_{He}$ (D) $r_e > r_p > r_{He}$

8. [Online April 2019]

The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is [Take $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}]

(A) 18 μT (B) 1 μT

(C) 3 μT (D) 9 μT

9. [Online April 2019]

A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be

(A) $\frac{2m}{\pi}$ (B) $\frac{4m}{\pi}$

(C) $\frac{m}{\pi}$ (D) $\frac{3m}{\pi}$

10. [Online April 2019]

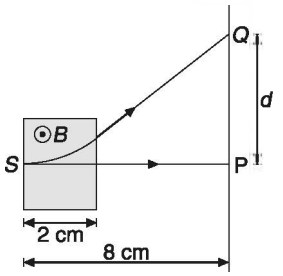
A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π $rads^{-1}$ about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to $\left(\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}\right)$.

(A) 4×10^{-5} C (B) 3×10^{-5} C

(C) 7×10^{-6} C (D) 2×10^{-6} C

11. [Online April 2019]

An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}\ \text{T})\hat{k}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is (electron's charge $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg)

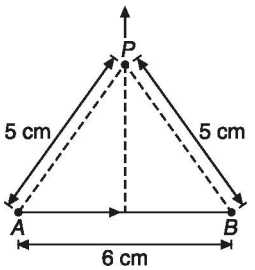


(A) 11.65 cm (B) 12.87 cm

(C) 2.25 cm (D) 1.22 cm

12. [Online April 2019]

Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (see figure) $(\mu_0 = 4\pi \times 10^{-7}\ \text{NA}^{-2})$

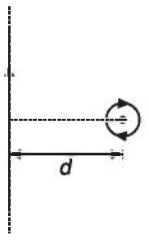


(A) 2.5×10^{-5} T (B) 1.5×10^{-5} T

(C) 3.0×10^{-5} T (D) 2.0×10^{-5} T

13. [Online January 2019]

An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d (d \gg a)$. If the loop applies a force F on the wire then

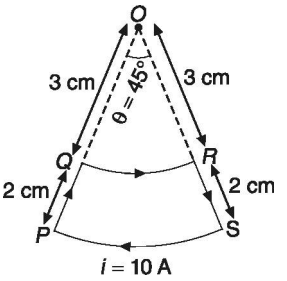


(A) $F = 0$ (B) $F \propto \left(\frac{a}{d}\right)^2$

(C) $F \propto \left(\frac{a}{d}\right)$ (D) $F \propto \left(\frac{a^2}{d^3}\right)$

14. [Online January 2019]

A current loop, having two circular arcs joined by two radial lines is shown in Figure. It carries a current of 10 A. The magnetic field at point O will be close to



(A) 1.5×10^{-7} T (B) 1.0×10^{-5} T

(C) 1.5×10^{-5} T (D) 1.0×10^{-7} T

15. [Online January 2019]

One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be

(A) $\frac{1}{N}$ (B) N^2

(C) N (D) $\frac{1}{N^2}$

16. [Online January 2019]

A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 Vm^{-1} makes it to move in a straight path, then the mass of the particle is (Given charge of electron $= 1.6 \times 10^{-19}$ C)

(A) 9.1×10^{-31} kg (B) 1.6×10^{-27} kg

(C) 1.6×10^{-19} kg (D) 2.0×10^{-24} kg

17. [Online January 2019]

A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 ms^{-1}. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is

(A) 12 mV (B) 2 mV
(C) 6 mV (D) 1 mV

18. [Online January 2019]
In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron $=1.6\times10^{-19}$ C, Mass of the electron $=9.1\times10^{-31}$ kg]

(A) 7.5×10^{-3} m (B) 7.5 m
(C) 7.5×10^{-2} m (D) 7.5×10^{-4} m

19. [Online January 2019]
The region between $y=0$ and $y=d$ contains a magnetic field $\vec{B}=B\hat{k}$. A particle of mass m and charge q enters the region with a velocity $\vec{v}=v\hat{i}$. If $d=\frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is

(A) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$ (B) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$
(C) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ (D) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

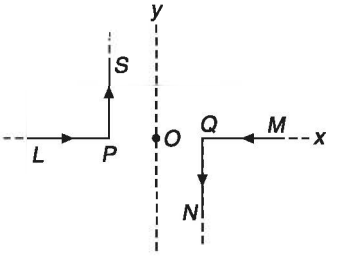
20. [Online January 2019]
A particle of mass m and charge q is in an electric and magnetic field given by $\vec{E}=2\hat{i}+3\hat{j}$; $\vec{B}=4\hat{j}+6\hat{k}$
The charged particle is shifted from the origin to the point $P(x=1,\ y=1)$ along a straight path. The magnitude of the total work done is

(A) $(0.15)q$ (B) $5q$
(C) $(0.35)q$ (D) $(2.5)q$

21. [Online January 2019]
A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be

(A) 1 : 3 (B) 1 : 2
(C) $1:\sqrt{3}$ (D) $1:\sqrt{2}$

22. [Online January 2019]
As shown in Figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If $OP=OQ=4$ cm and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the currents in each wire and the direction of the magnetic field at O will be $\left(\mu_0=4\pi\times10^{-7}\ \text{NA}^{-2}\right)$



(A) 40 A, perpendicular into the page
(B) 20 A, perpendicular into the page
(C) 40 A, perpendicular out of the page
(D) 20 A, perpendicular out of the page

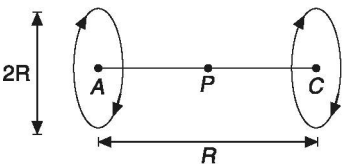
23. [2018]
An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(A) $r_e > r_p = r_\alpha$ (B) $r_e < r_p = r_\alpha$
(C) $r_e < r_p < r_\alpha$ (D) $r_e < r_\alpha < r_p$

24. [2018]
The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(A) 2 (B) $\sqrt{3}$
(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

25. [Online 2018]
A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by [Refer to given figure]



(A) $\frac{4N\mu_0 I}{5^{\frac{1}{2}}R}$ (B) $\frac{4N\mu_0 I}{5^{\frac{3}{2}}R}$

(C) $\frac{8N\mu_0 I}{5^{\frac{3}{2}}R}$ (D) $\frac{8N\mu_0 I}{5^{\frac{1}{2}}R}$

26. [Online 2018]

A current of 1 A is flowing in the sides of an equilateral triangle of side 4.5×10^{-2} m. The magnetic field at the centre of the triangle will be

(A) 4×10^{-5} Wbm^{-2} (B) 8×10^{-5} Wbm^{-2}

(C) 2×10^{-5} Wbm^{-2} (D) Zero

27. [Online 2018]

A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with respect to normal axis then the magnetic moment of the loop is

(A) $\frac{1}{2}q\omega r^2$ (B) $q\omega r^2$

(C) $\frac{3}{2}q\omega r^2$ (D) $\frac{4}{3}q\omega r^2$

28. [Online 2018]

A galvanometer with its coil resistance 25 Ω requires a current of 1 mA for its full deflection. In order to construct an ammeter to read up to a current of 2A, the approximate value of the shunt resistance should be

(A) 1.25×10^{-3} Ω (B) 1.25×10^{-2} Ω

(C) 2.5×10^{-3} Ω (D) 2.5×10^{-2} Ω

29. [2017]

When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is

(A) 1.985×10^{3} Ω (B) 2.045×10^{3} Ω

(C) 2.535×10^{3} Ω (D) 4.005×10^{3} Ω

30. [Online 2017]

In a certain region static electric and magnetic fields exist. The magnetic field is given by $\vec{B}=B_0\left(\hat{i}+2\hat{j}-4\hat{k}\right)$. If a test charge moving with a velocity $\vec{v}=v_0\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ experiences no force in that region, then the electric field in the region, in SI units, is

(A) $\vec{E}=-v_0B_0\left(14\hat{j}+7\hat{k}\right)$

(B) $\vec{E}=v_0B_0\left(14\hat{j}+7\hat{k}\right)$

(C) $\vec{E}=-v_0B_0\left(\hat{i}+\hat{j}+7\hat{k}\right)$

(D) $\vec{E}=-v_0B_0\left(3\hat{i}-2\hat{j}-4\hat{k}\right)$

31. [Online 2017]

A magnetic dipole in a constant magnetic field has

(A) zero potential energy when the torque is maximum.

(B) minimum potential energy when the torque is maximum.

(C) maximum potential energy when the torque is maximum.

(D) zero potential energy when the torque is minimum.

32. [Online 2017]

A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is

(A) parallel to the wire opposite to the current

(B) parallel to the wire along the current

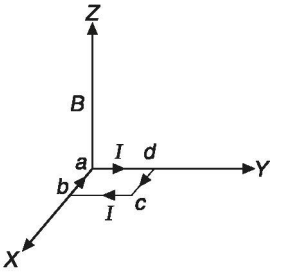
(C) away from the wire

(D) towards the wire

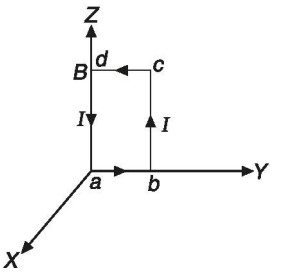
33. [Online 2017]

A uniform magnetic field B of 0.3 T is along the positive Z-direction. A rectangular loop $(abcd)$ of sides 10 cm × 5 cm carries a current I of 12 A. Out of the following different orientations which one corresponds to stable equilibrium?

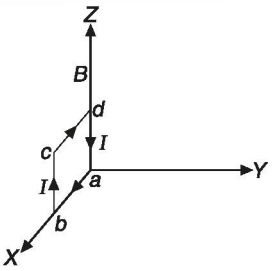
(A)



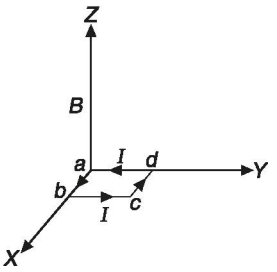
(B)



(C)



(D)



34. [2016]

Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(A) $\frac{\pi^2}{8}$ (B) $\frac{\pi^2}{16\sqrt{2}}$

(C) $\frac{\pi^2}{16}$ (D) $\frac{\pi^2}{8\sqrt{2}}$

35. [2016]

A galvanometer having a coil resistance of $100\ \Omega$ gives a full-scale deflection, when a current of $1\ \text{mA}$ is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full-scale deflection for a current of $10\ \text{A}$, is

(A) $0.01\ \Omega$ (B) $2\ \Omega$

(C) $0.1\ \Omega$ (D) $3\ \Omega$

36. [Online 2016]

To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S are related by the equation

(A) $S(R+G)=RG$ (B) $2S(R+G)=RG$

(C) $2G=S$ (D) $2S=G$

37. [Online 2016]

A $50\ \Omega$ resistance is connected to a battery of $5\ \text{V}$. A galvanometer of resistance $100\ \Omega$ is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit?

(A) $r_s = 0.5\ \Omega$ in series with the galvanometer

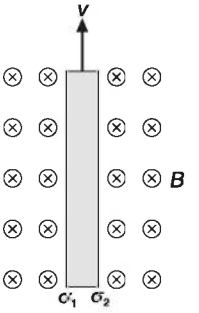
(B) $r_s = 1\ \Omega$ in series with the galvanometer

(C) $r_s = 1\ \Omega$ in parallel with the galvanometer

(D) $r_s = 0.5\ \Omega$ in parallel with the galvanometer

38. [Online 2016]

Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed v in a uniform magnetic field B going into the plane of the paper (see figure). If charge densities σ_1 and σ_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects)



(A) $\sigma_1 = \frac{-\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{\varepsilon_0 vB}{2}$

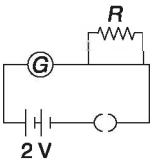
(B) $\sigma_1 = \varepsilon_0 vB$, $\sigma_2 = -\varepsilon_0 vB$

(C) $\sigma_1 = \frac{\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{-\varepsilon_0 vB}{2}$

(D) $\sigma_1 = \sigma_2 = \varepsilon_0 vB$

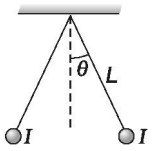
39. [Online 2016]

A galvanometer has a 50 division scale. Battery has no internal resistance. It is found that there is deflection of 40 divisions when $R = 2400\ \Omega$. Deflection becomes 20 divisions when resistance taken from resistance box is $4900\ \Omega$. Then we can conclude



(A) current sensitivity of galvanometer is 20 μA/division
(B) resistance of galvanometer is 200 Ω
(C) resistance required on R.B. for a deflection of 10 divisions is 9800 Ω
(D) full scale deflection current is 2 mA

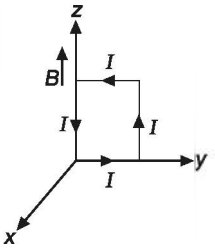
40. [2015]
Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in Figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is
(g = gravitational acceleration)

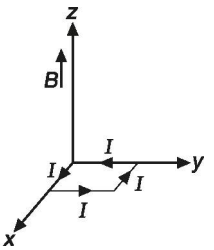


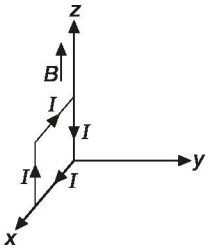
(A) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (B) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

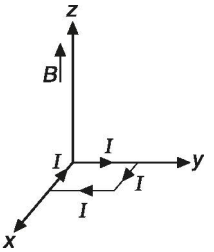
(C) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (D) $2\sin\theta\sqrt{\frac{\pi\lambda gL.}{\mu_0\cos\theta}}$

41. [2015]
A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figure below.

(i)


(ii)


(iii)


(iv)


If there is a uniform magnetic field of 0.3 T in the positive z direction, then the orientations in which the loop would be in stable and unstable equilibrium respectively are

(A) (ii) and (iv), respectively
(B) (ii) and (iii), respectively
(C) (i) and (ii), respectively
(D) (i) and (iii), respectively

42. [2015]
Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then

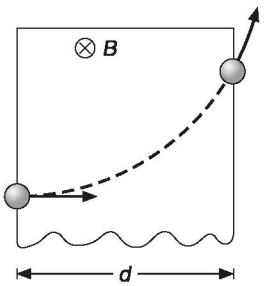
(A) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$

(B) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

(C) $\vec{F}_1 = \vec{F}_2 = 0$

(D) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$ is radially outwards

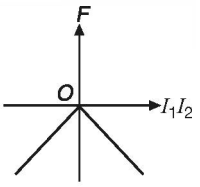
43. [Online 2015]
A Proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width d. If α be the angle of deviation of proton from initial direction of motion (see figure), the value of $\sin\alpha$ will be

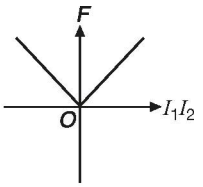


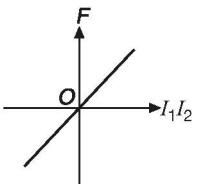
(A) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$ (B) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$

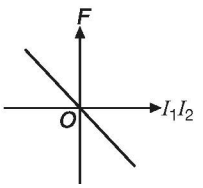
(C) $Bd\sqrt{\frac{q}{2mV}}$ (D) $qV\sqrt{\frac{Bd}{2m}}$

44. [Online 2015]
Two long straight parallel wires, carrying (adjustable) currents I_1 and I_2, are kept at a distance d apart. If the force F between the two wires is taken as positive when the wires repel each other and negative when the wires attract each other, the graph showing the dependence of F, on the product I_1I_2, would be

(A) 

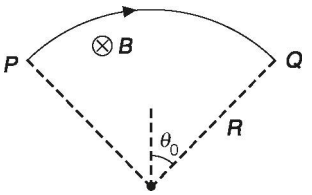
(B) 

(C) 

(D) 

45. [Online 2015]

A wire carrying current I is tied between points P and Q and is in the shape of a circular arc of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, inwards) in the vicinity of the wire. If the wire subtends an angle $2\theta_0$ at the centre of the circle (of which it forms an arc) then the tension in the wire is

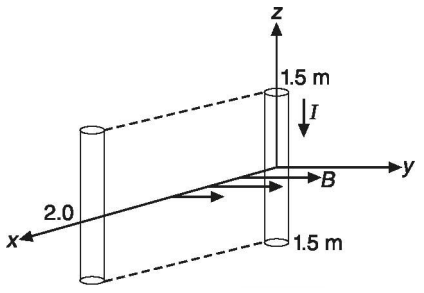


(A) IBR (B) $\dfrac{IBR}{\sin\theta_0}$

(C) $\dfrac{IBR}{2\sin\theta_0}$ (D) $\dfrac{IBR\theta_0}{\sin\theta_0}$

46. [2014]

A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction shown in Figure.



For a field $\vec{B} = 3.0\times10^{-4}e^{-0.2x}\hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis.

(A) 29.7 W (B) 1.57 W

(C) 2.97 W (D) 14.85 W

47. [2013]

This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-I: Higher the range, greater is the resistance of ammeter.

Statement-II: To increase the range of ammeter, additional shunt needs to be used across it.

(A) Statement-I is false, Statement-II is true.

(B) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.

(C) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(D) Statement-I is true, Statement-II is false.

48. [2012]

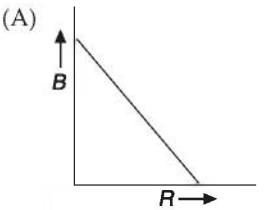
Proton, deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct?

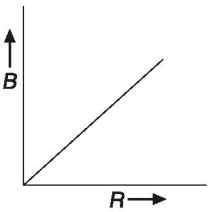
(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha > r_d > r_p$

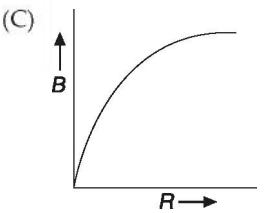
(C) $r_\alpha = r_d > r_p$ (D) $r_\alpha = r_p = r_d$

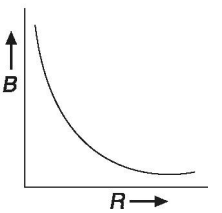
49. [2012]

A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure

(A) 

(B) 

(C) 

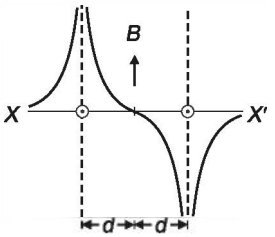
(D) 

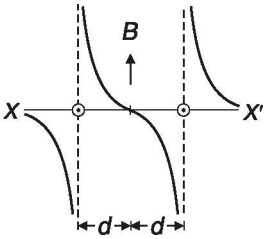
50. **[2011]**
A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

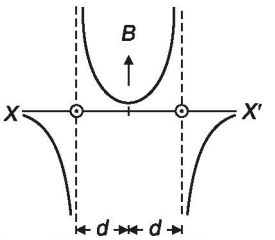
(A) $\frac{\mu_0 I}{\pi^2 R}$ (B) $\frac{\mu_0 I}{2\pi^2 R}$

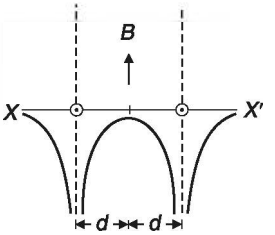
(C) $\frac{\mu_0 I}{2\pi R}$ (D) $\frac{\mu_0 I}{4\pi R}$

51. **[2010]**
Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(A)


(B)


(C)


(D)


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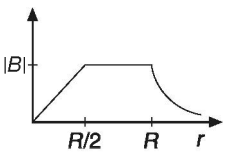
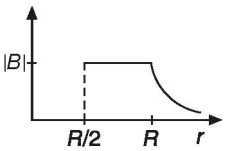
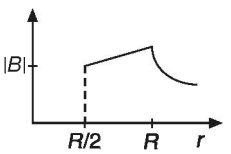
Single Correct Choice Type Problems

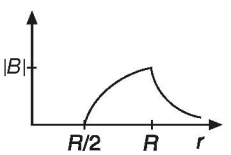
This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. **[IIT-JEE 2012]**
An infinitely long hollow conducting cylinder with inner radius $\frac{R}{2}$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

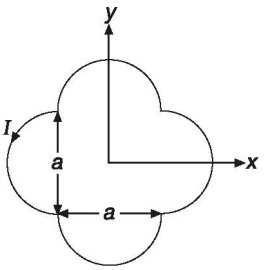
(A)


(B)


(C)


(D)


2. **[IIT-JEE 2012]**
A loop carrying current I lies in the x-y plane as shown in Figure.

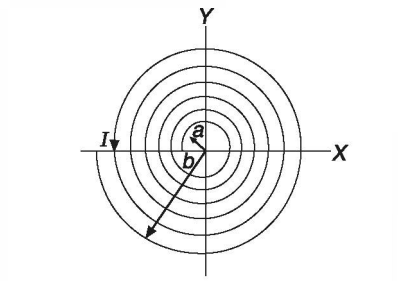


The unit vector $\vec{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

(A) $a^2 I\vec{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\vec{k}$

(C) $-\left(\frac{\pi}{2}+1\right)a^2 I\vec{k}$ (D) $(2\pi+1)a^2 I\vec{k}$

3. **[IIT-JEE 2011]**
A long-insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is

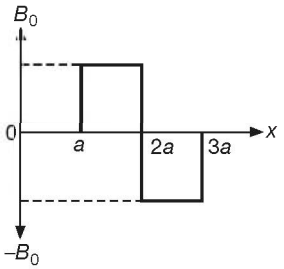


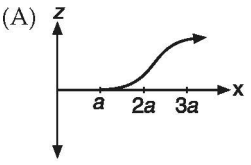
(A) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

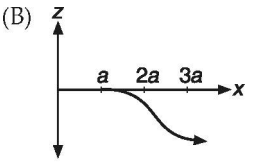
(C) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

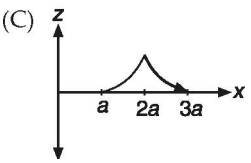
4. [IIT-JEE 2007]

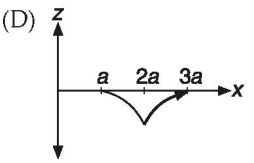
A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_0\hat{j}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like



(A) 

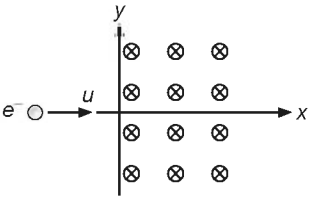
(B) 

(C) 

(D) 

5. [IIT-JEE 2004]

An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $\vec{B} = -B_0\hat{k}$ which exists to the right of y-axis. The electron exits from the region after some time with the speed v at co-ordinate y, then

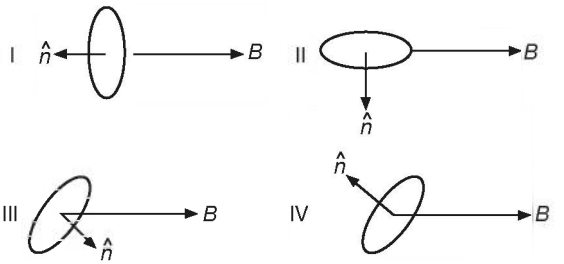


(A) $v > u$, $y < 0$ (B) $v = u$, $y > 0$

(C) $v > u$, $y > 0$ (D) $v = u$, $y < 0$

6. [IIT-JEE 2003]

A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III & IV, arrange them in the decreasing order of Potential Energy.

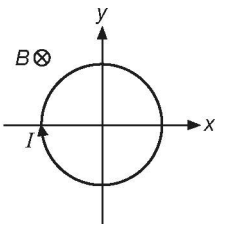


(A) I > III > II > IV (B) I > II > III > IV

(C) I > IV > II > III (D) III > IV > I > II

7. [IIT-JEE 2003]

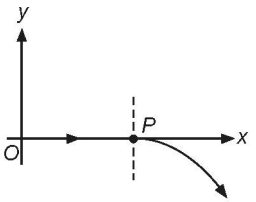
A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to



(A) contract
(B) expand
(C) move towards +ve x-axis
(D) move towards −ve x-axis

8. [IIT-JEE 2003]

For a positively charged particle moving in a x-y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x-y plane and is found to be non-circular. Which one of the following combinations is possible?



(A) $\vec{E}=0\ ;\ \vec{B}=b\hat{j}+c\hat{k}$ (B) $\vec{E}=a\hat{i}\ ;\ \vec{B}=c\hat{k}+a\hat{i}$

(C) $\vec{E}=0\ ;\ \vec{B}=c\hat{j}+b\hat{k}$ (D) $\vec{E}=a\hat{i}\ ;\ \vec{B}=c\hat{k}+b\hat{j}$

9. [IIT-JEE 2002]

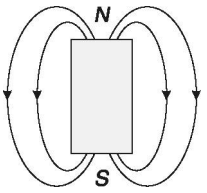
A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from $x=a$ to $x=b$. The minimum value of v required so that the particle can just enter the region $x>b$ is

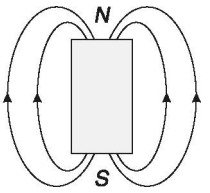
(A) $\frac{qbB}{m}$ (B) $\frac{q(b-a)B}{m}$

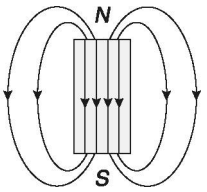
(C) $\frac{qaB}{m}$ (D) $\frac{q(b+a)B}{2m}$

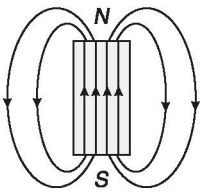
10. [IIT-JEE 2002]

The magnetic field lines due to a bar magnet are correctly shown in

(A) 

(B) 

(C) 

(D) 

11. [IIT-JEE 2002]

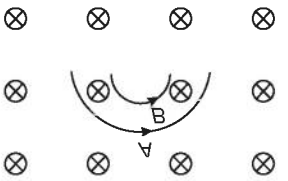
A long straight wire along the z-axis carries a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) on the $z=0$ plane is

(A) $\frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i}-x\hat{j}}{x^2+y^2}\right)$ (B) $\frac{\mu_0 I}{2\pi}\left(\frac{x\hat{i}+y\hat{j}}{x^2+y^2}\right)$

(C) $\frac{\mu_0 I}{2\pi}\left(\frac{x\hat{j}-y\hat{i}}{x^2+y^2}\right)$ (D) $\frac{\mu_0 I}{2\pi}\left(\frac{x\hat{i}-y\hat{j}}{x^2+y^2}\right)$

12. [IIT-JEE 2001]

Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speed of the particles are v_A and v_B respectively and the trajectories are as shown in Figure. Then



(A) $m_A v_A < m_B v_B$

(B) $m_A v_A > m_B v_B$

(C) $m_A < m_B$ and $v_A < v_B$

(D) $m_A = m_B$ and $v_A = v_B$

13. [IIT-JEE 2001]

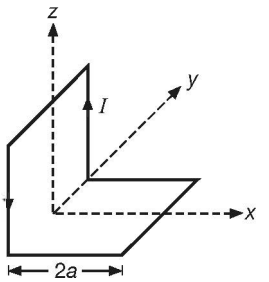
A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre will be

(A) $\frac{\mu_0 NI}{b}$ (B) $\frac{2\mu_0 NI}{a}$

(C) $\frac{\mu_0 NI}{2(b-a)}\log_e\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{(b-a)}\log_e\left(\frac{b}{a}\right)$

14. [IIT-JEE 2001]

A non-planar loop of conducting wire carrying a current I is placed as shown in Figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ points in the direction



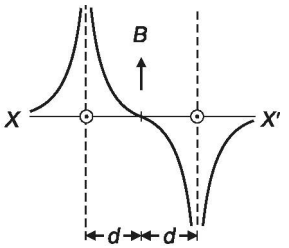
(A) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (B) $\frac{1}{\sqrt{3}}\left(-\hat{i}-\hat{j}+\hat{k}\right)$

(C) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (D) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

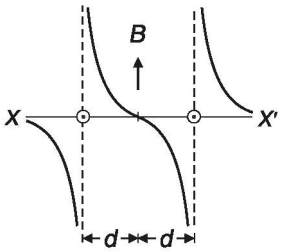
15. [IIT-JEE 2000]

Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

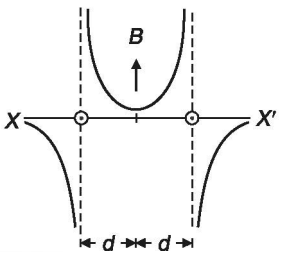
(A)



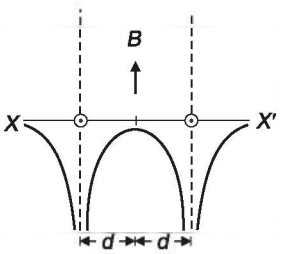
(B)



(C)



(D)



16. [IIT-JEE 2000]

A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(A) ω and q (B) ω, q and m

(C) q and m (D) ω and m

17. [IIT-JEE 2000]

An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$ direction and a magnetic field along the $+z$ direction then,

(A) positive ions deflect towards $+y$ direction and negative ions towards $-y$ direction.

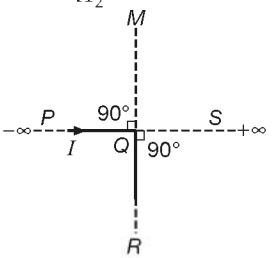
(B) all ions deflect towards $+y$ direction.

(C) all ions deflect towards $-y$ direction.

(D) positive ions deflect towards $-y$ direction and negative ions towards $+y$ direction.

18. [IIT-JEE 2000]

An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current carrying conductor at the point M is H_1. Now, another infinitely long straight conductor QS, is connected at Q so that the current is $\frac{1}{2}I$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio $\frac{H_1}{H_2}$ is given by



(A) $\frac{1}{2}$ (B) 1

(C) $\frac{2}{3}$ (D) 2

19. [IIT-JEE 1999]

A circular loop of radius R, carrying current I, lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is

(A) directly proportional to I

(B) directly proportional to R

(C) inversely proportional to R

(D) zero

20. [IIT-JEE 1999]

A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a

(A) straight line (B) circle

(C) helix (D) cycloid

21. [IIT -JEE 1998]

Two very long, straight, parallel wires carry steady currents I and $-I$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane

of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) 0

22. [IIT-JEE 1998]

Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$

(C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$

23. [IIT-JEE 1997]

A proton, a deuteron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then

(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha > r_d > r_p$

(C) $r_\alpha = r_d > r_p$ (D) $r_p = r_d = r_\alpha$

24. [IIT-JEE 1995]

A battery is connected between two points A and D on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AD of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is

(A) proportional to $(180° - \theta)$

(B) inversely proportional to r

(C) zero, only if $\theta = 180°$

(D) zero for all values of θ

25. [IIT-JEE 1993]

A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then

(A) the magnetic field at all points inside the pipe is the same, but not zero

(B) the magnetic field at any point inside the pipe is zero

(C) the magnetic field is zero only on the axis of the pipe

(D) the magnetic field is different at different points inside the pipe

26. [IIT-JEE 1988]

Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

(A) $\left(\frac{R_1}{R_2}\right)^{\frac{1}{2}}$ (B) $\frac{R_2}{R_1}$

(C) $\left(\frac{R_1}{R_2}\right)^2$ (D) $\frac{R_1}{R_2}$

27. [IIT-JEE 1986]

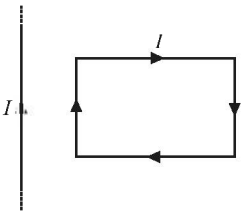
Two thin long parallel wires separated by a distance b are carrying a current I ampere each. The magnitude of the force per unit length exerted by one wire on the other is

(A) $\frac{\mu_0 I^2}{b^2}$ (B) $\frac{\mu_0 I^2}{2\pi b}$

(C) $\frac{\mu_0 I}{2\pi b}$ (D) $\frac{\mu_0 I}{2\pi b^2}$

28. [IIT-JEE 1985]

A rectangular loop carrying a current I is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I is established in the wire as shown in Figure, the loop will



(A) rotate about an axis parallel to the wire

(B) move away from the wire

(C) move towards the wire

(D) remain stationary

29. [IIT-JEE 1983]

A conducting circular loop of radius r carries a constant current I. It is placed in a uniform magnetic field $\vec{B}_0$ such that $\vec{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is

(A) $2\pi Ir\vec{B}_0$ (B) $Ir\vec{B}_0$

(C) $\pi Ir\vec{B}_0$ (D) ZERO

30. [IIT-JEE 1982]

A magnetic needle is kept in a non-uniform magnetic field. It experiences

(A) a force and a torque

(B) a force but not a torque

(C) a torque but not a force

(D) neither a force nor a torque

Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2018]

Two infinitely long straight wires lie in the xy-plane along the lines $x=\pm R$. The wire located at $x=+R$ carries a constant current I_1 and the wire located at $x=-R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

(A) If $I_1=I_2$, then $\vec{B}$ cannot be equal to zero at the origin $(0, 0, 0)$

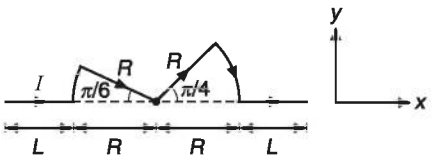
(B) If $I_1>0$ and $I_2<0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

(C) If $I_1<0$ and $I_2>0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

(D) If $I_1=I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

2. [JEE (Advanced) 2015]

A conductor (shown in Figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are



(A) if $\vec{B}$ is along $\hat{z}$, $F\propto(L+R)$

(B) if $\vec{B}$ is along $\hat{x}$, $F=0$

(C) if $\vec{B}$ is along $\hat{y}$, $F\propto(L+R)$

(D) if $\vec{B}$ is along $\hat{z}$, $F=0$

3. [JEE (Advanced) 2013]

A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1=4\hat{i}$ ms^{-1}, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x=0$ to $x=L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2=2\left(\sqrt{3}\hat{i}+\hat{j}\right)$ ms^{-1}. The correct statement(s) is/are

(A) the direction of the magnetic field is $-z$ direction.

(B) the direction of the magnetic field is $+z$ direction.

(C) the magnitude of the magnetic field is $\frac{50\pi M}{3Q}$ units.

(D) the magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

4. [JEE (Advanced) 2013]

A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)

(A) In the region $0<r<R$, the magnetic field is non-zero

(B) In the region $R<r<2R$, the magnetic field is along the common axis

(C) In the region $R<r<2R$, the magnetic field is tangential to the circle of radius r, centred on the axis

(D) In the region $r>2R$, the magnetic field is non-zero

5. [IIT-JEE 2012]

Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E}=E_0\hat{j}$ and $\vec{B}=B_0\hat{j}$. At time $t=0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t>0$?

(A) If $\theta=0°$, the charge moves in a circular path in the x-z plane.

(B) If $\theta=0°$, the charge undergoes helical motion with constant pitch along the y-axis

(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

6. [JEE (Advanced) 2011]

An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(A) They will never come out of the magnetic field region

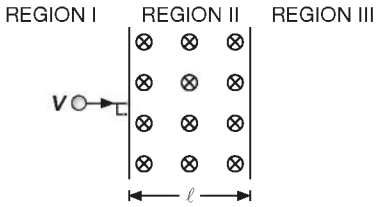
(B) They will come out travelling along parallel paths

(C) They will come out at the same time

(D) They will come out at different times

7. [IIT-JEE 2008]

A particle of mass m and charge q, moving with velocity V enters REGION II normal to the boundary as shown in the figure. REGION II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the REGION II is l. Choose the correct choice(s)



(A) The particle enters REGION III only if its velocity $V > \frac{qlB}{m}$

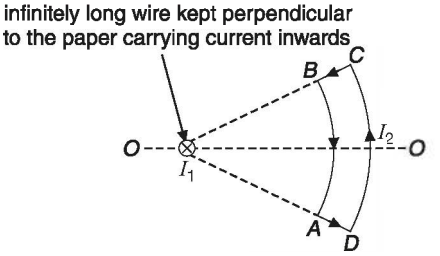
(B) The particle enters REGION III only if its velocity $V < \frac{qlB}{m}$

(C) Path length of the particle in REGION II is maximum when velocity $V = \frac{qlB}{m}$

(D) Time spent in REGION II is same for any velocity V as long as the particle returns to REGION I

8. [IIT-JEE 2006]

Which of the following statement is correct in the given Figure?



(A) net force on the loop is zero

(B) net torque on the loop is zero

(C) loop will rotate clockwise about axis OO' when seen from O

(D) loop will rotate anticlockwise about OO' when seen from O

9. [IIT-JEE 1994]

H^+, He^+ and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{++} are 1 u, 4 u and 16 u respectively.

(A) H^+ will be deflected the most.

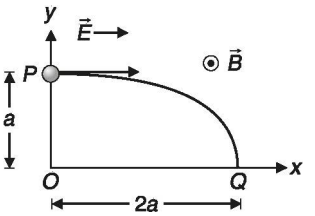
(B) O^{++} will be deflected the most.

(C) He^+ and O^{++} will be deflected equally.

(D) All will be deflected equally.

10. [IIT-JEE 1991]

A particle of mass m, charge q is moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$. It follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$.



(A) $E = \frac{3}{4}\frac{mv^2}{qa}$

(B) The rate of work done by the electric field at P is $\frac{3}{4}\frac{mv^3}{a}$

(C) The rate of work done by the electric field at P is zero

(D) The rate of work done by both the fields at Q is zero

11. [IIT-JEE 1985]

A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively, this region of space may have

(A) $E=0$, $B=0$ (B) $E=0$, $B\neq 0$

(C) $E\neq 0$, $B=0$ (D) $E\neq 0$, $B\neq 0$

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2008]

Statement-1: The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

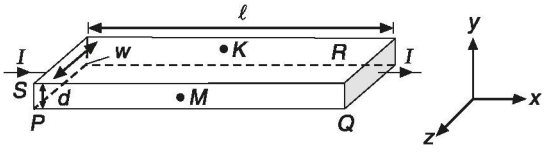
Statement-2: Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in Figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



1. [JEE (Advanced) 2015]

Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statements is/are

(A) If $w_1=w_2$ and $d_1=2d_2$, then $V_2=2V_1$

(B) If $w_1=w_2$ and $d_1=2d_2$, then $V_2=V_1$

(C) If $w_1=2w_2$ and $d_1=d_2$, then $V_2=2V_1$

(D) If $w_1=2\omega_2$ and $d_1=d_2$, then $V_2=V_1$

2. [JEE (Advanced) 2015]

Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct options is/are

(A) If $B_1=B_2$ and $n_1=2n_2$, then $V_2=2V_1$

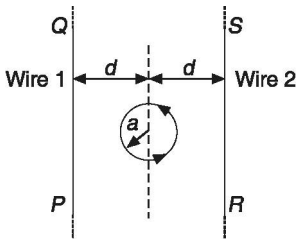
(B) If $B_1=B_2$ and $n_1=2n_2$, then $V_2=V_1$

(C) If $B_1=2B_2$ and $n_1=n_2$, then $V_2=0.5V_1$

(D) If $B_1=2B_2$ and $n_1=n_2$, then $V_2=V_1$

Comprehension 2

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counter-clockwise direction if seen from above.



3. [JEE (Advanced) 2014]

When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$

(B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$

(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

4. [JEE (Advanced) 2014]

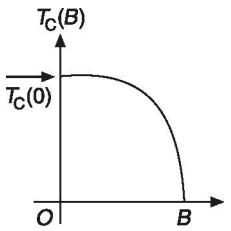
Consider $d \gg a$ and the loop is rotated about its diameter parallel to the wires by $30°$ from the position shown in Figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(A) $\dfrac{\mu_0 I^2 a^2}{d}$ (B) $\dfrac{\mu_0 I^2 a^2}{2d}$

(C) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

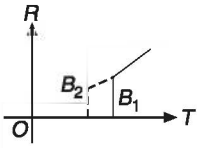
Comprehension 3

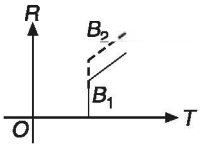
Electrical resistance of certain materials, known as superconductors, changes abruptly from a non-zero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field i.e. the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in Figure.

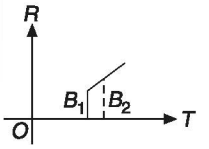


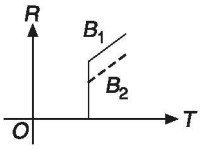
5. [IIT-JEE 2010]

In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

(A) 

(B) 

(C) 

(D) 

6. [IIT-JEE 2010]

A superconductor has $T_C(0) = 100$ K. When a magnetic field of 75 T is applied, its T_C decreases to 75 K. For this material one can definitely say that when (Note T = Tesla)

(A) $B = 5$ T, $T_C(B) = 80$ K

(B) $B = 5$ T, $75\text{ K} < T_C(B) < 100$ K

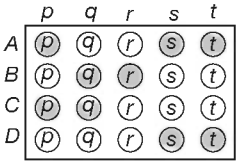
(C) $B = 10$ T, $75\text{ K} < T_C(B) < 100$ K

(D) $B = 10$ T, $T_C(B) = 70$ K

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



Directions (Question Numbers 1-4):

Matching the information given in the three columns of the following table. A charged particle (electron or proton) is

introduced at the origin $(x=0,\ y=0,\ z=0)$ with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns I, II and III, respectively. The quantities E_0, B_0 are positive in magnitude.

COLUMN-I	COLUMN-II	COLUMN-III
(I) Electron with $\vec{v}=2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E}=E_0\hat{z}$	(p) $\vec{B}=-B_0\hat{x}$
(II) Electron with $\vec{v}=\frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E}=-E_0\hat{y}$	(q) $\vec{B}=B_0\hat{x}$
(III) Proton with $\vec{v}=0$	(iii) $\vec{E}=-E_0\hat{x}$	(r) $\vec{B}=B_0\hat{y}$
(IV) Proton with $\vec{v}=2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E}=E_0\hat{x}$	(s) $\vec{B}=B_0\hat{z}$

1. **[JEE (Advanced) 2017]**
In which case would the particle move in a straight line along the negative direction of Y-axis (i.e. move along $-\hat{y}$)?

(A) (IV) (ii) (s) (B) (II) (iii) (q)
(C) (III) (ii) (r) (D) (III) (ii) (p)

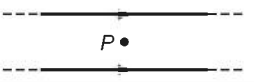
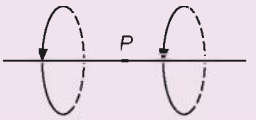
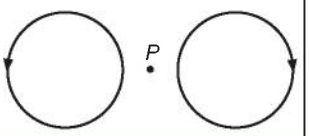
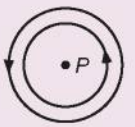
2. **[JEE (Advanced) 2017]**
In which case will the particle move in a straight line with constant velocity?

(A) (II) (iii) (s) (B) (III) (iii) (p)
(C) (IV) (i) (s) (D) (III) (ii) (r)

3. **[JEE (Advanced) 2017]**
In which case will the particle describe a helical path with axis along the positive z-direction ?

(A) (II) (ii) (r) (B) (III) (iii) (p)
(C) (IV) (i) (s) (D) (IV) (ii) (r)

4. **[IIT-JEE 2007]**
Two wires each carrying a steady current I are shown in four configurations in **COLUMN-I**. Some of the resulting effects are described in **COLUMN-II**. Match the statements in **COLUMN-I** with the statements in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Point P is situated midway between the wires. 	(p) The magnetic field (B) at P due to the currents in the wires are in the same direction.
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	(q) The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	(r) There is no magnetic field at P.
(D) Point P is situated at the common center of the wires. 	(s) The wires repel each other.

5. **[IIT-JEE 2007]**
COLUMN-I gives certain situations in which a straight metallic wire of resistance R is used and **COLUMN-II** gives some resulting effects. Match the statements in **COLUMN-I** with the statements in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) A charged capacitor is connected to the ends of the wire.	(p) A constant current flow through the wire.

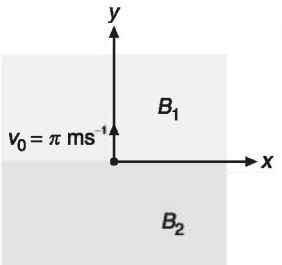
(*Continued*)

COLUMN-I	COLUMN-II
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion.	(q) Thermal energy is generated in the wire.
(C) The wire is placed in a constant electric field that has a direction along the length of the wire.	(r) A constant potential difference develops between the ends of the wire.
(D) A battery of constant emf is connected to the ends of the wire.	(s) Charges of constant magnitude appear at the ends of the wire.

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

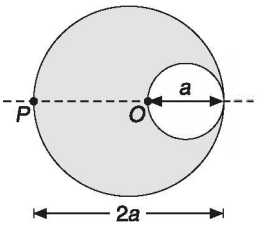
1. **[JEE (Advanced) 2018]**
In the xy-plane, the region $y>0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y<0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0=\pi$ ms^{-1} at $t=0$, as shown in Figure. Neglect gravity in this problem. Let $t=T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2=4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is_____.



2. **[JEE (Advanced) 2018]**
A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} Nmrad^{-1}. When a current flow through the galvanometer, a full-scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is_____.

3. **[JEE (Advanced) 2014]**
Two parallel wires in the plane of the paper are distance x_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance x_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1}=3$ and value of $\frac{R_1}{R_2}$ is_____.

4. **[IIT-JEE 2012]**
A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



5. **[IIT-JEE 2009]**
A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ=3x$, $PR=4x$ and $QR=5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Force and Fleming's Left Hand Rule)

1. LT^{-1}
2. (a) 0
   (b) 0
   (c) $v^2\vec{B}$
3. 4
4. $0.33\ \text{ms}^{-2}$, along $-z$ axis
5. (a) $v_y = -48.6\ \text{ms}^{-1}$
   $v_x = -106\ \text{ms}^{-1}$
   (b) Yes, $v_z = 0$
   (c) $\frac{\pi}{2}$
6. 3.7 T, perpendicular to the initial velocity of the coin inwards.
7. $1.25\ \text{ms}^{-2}$
8. $\vec{B} = \left(-0.4\hat{j}\right)$ T
9. (a) $\frac{F_2}{qv_1}$, along $-y$ -direction
   (b) $F_1 = \frac{F_2}{\sqrt{2}}$
10. $3.8\hat{k}\ \text{ms}^{-2}$
11. 1.5 mT, 30° ccw from $+x$-axis
12. $F_x = 0$, $F_y = 2.49\times10^{-3}$ N, $F_z = 1.32\times10^{-3}$ N
13. $\sqrt{\ell^2 - \frac{1}{\left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2}}$

Test Your Concepts-II (Based on Charged Particle in a Magnetic Field)

2. Along $+z$ -axis.
3. Yes, No
4. (a) electron
   (b) electron
5. (a) 0.501 m
   (b) 45°
6. (a) 5.14 m
   (b) 1.72×10^{-6} s
   (c) 6.08×10^{-3} m
   (d) 0.0304 m
7. $\vec{v} = v_x\hat{i} + v_y\cos\left(\frac{eBt}{m}\right)\hat{j} - v_y\sin\left(\frac{eBt}{m}\right)\hat{k}$
8. 8
9. $\sin^{-1}\left(\frac{qBd}{mv}\right)$
10. $\frac{2\pi}{d}\left(\frac{2m_e\Delta V}{e}\right)^{1/2}$
11. $r_\alpha = r_d = \sqrt{2}r_p$
12. 115 keV
13. $\left(\sqrt{\frac{2qV}{m}}\right)\cos\theta,\ \frac{2\pi m}{qB},\ \frac{2\pi\cos\theta}{B}\sqrt{\frac{2mV}{q}}$
14. $\frac{eB\left(b^2-a^2\right)}{2mb}$
15. (a) 5 cm, (b) $9\times10^6\ \text{ms}^{-1}$
16. $\left(\frac{m}{qB}\right)\sin^{-1}\left(Bd\sqrt{\frac{q}{2mV}}\right)$
17. 0.6 cm, 2.2 cm
18. (a) 3.7 mT, (b) 6.7 mT

Test Your Concepts-III (Based on Charged Particle in Magnetic and Electric Field)

1. (a) $\left(3.52\hat{i} - 1.6\hat{j}\right)\times10^{-18}$ N
   (b) 24.4°
2. $244\ \text{kVm}^{-1}$
4. $\frac{25m}{2qE_0}$
5. $\sqrt{3}\left(\frac{mv_0}{qE}\right)$
6. $\frac{\varepsilon_0\ell B}{\sigma}$
8. $\frac{2\pi m}{qB_0}\left(v_0 + \frac{\pi E_0}{B_0}\right)$
9. $\frac{3}{4}\left(\frac{mv^2}{qa}\right)$, zero and $\frac{3}{4}\left(\frac{mv^3}{a}\right)$, zero and zero
10. $E=0$, $B=0$ OR $E=0$, $B\neq0$ OR $E\neq0$, $B\neq0$

Test Your Concepts-IV (Based on Biot Savart's Law and Applications)

1. (a) $6.3\ \mu T$
   (b) $2.3\ \mu T$
2. $n\left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right)$, $\frac{\mu_0 I}{2R}$
3. $2\sqrt{3}\times10^{-5}$ T, 2×10^{-5} T
4. $\frac{\mu_0 I}{2\pi R}(\pi-\theta+\tan\theta)$, $\odot$
5. $\frac{\mu_0 I}{8}\left(\frac{3}{a}+\frac{1}{b}\right)$, $\otimes$
6. $\frac{\mu_0 I}{4\pi}\left(\frac{3\pi}{2a}+\frac{\sqrt{2}}{b}\right)$, $\otimes$
7. $\left(\frac{\mu_0 I}{2\pi a}\right)\hat{k}$, $\frac{\mu_0 I}{8\pi a}\left(-\hat{j}+\hat{k}\right)$, $\frac{\mu_0 I}{4\pi a}\left(1-\frac{1}{\sqrt{2}}\right)\hat{k}$
8. $\frac{\mu_0 I}{r}\left(\frac{1}{2}-\frac{1}{\pi}\right)$, inwards
9. $\frac{\mu_0 I}{4R}$, $\otimes$
10. $\frac{\mu_0 I}{4\pi R}\left(1+\frac{3\pi}{2}\right)$, $\otimes$
11. $\frac{\mu_0 I}{4\pi R}(2+\pi)$, $\otimes$
12. $\sqrt{2}\left(\frac{\mu_0 I}{4\pi\ell}\right)$
13. $-\left(\frac{\mu_0 I}{4\pi R}\right)(\pi\hat{i}+2\hat{k})$, $\frac{\mu_0 I}{4\pi R}\sqrt{\pi^2+4}$
14. $-\left(\frac{\mu_0 I}{4\pi R}\right)\left[(1+\pi)\hat{i}+\hat{k}\right]$, $\frac{\mu_0 I}{4\pi R}\sqrt{2+2\pi+\pi^2}$
15. $-\left(\frac{\mu_0 I}{4\pi R}\right)\left[\left(\frac{3\pi}{2}\right)\hat{i}+\hat{j}+\hat{k}\right]$, $\frac{\mu_0 I}{4\pi R}\sqrt{\frac{9\pi^2}{4}+2}$
16. 13 μT, along $-y$ axis
17. 12.5 T
18. (a) $I_2 = 2I_1$, out of the paper
    (b) $I_2 = 6I_1$, into the paper
19. (a) $y = 1$ m, below wire 1
    (b) $14.4\times10^{-3}\ \text{N}\left(-\hat{j}\right)$
20. $\frac{2\mu_0 Q\omega}{5\sqrt{5}\pi R}$
21. $\frac{qv\mu_0 I}{2\pi mg}$
22. $\frac{9\mu_0 I}{32R}$
23. $\frac{4}{\pi^2}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)$
24. $y=\left(\frac{I_1}{I_2}\right)x$
25. $I=\left(\frac{\pi D}{R}\right)I_0 a$

Test Your Concepts-V (Based on Ampere's Circuital Law and Applications)

1. (a) $2\pi b\delta\left(1-e^{-a/\delta}\right)$
   (b) $B=\frac{\mu_0 I_0}{2\pi r}$
   (c) $I(r)=I_0\frac{\left(e^{r/\delta}-1\right)}{\left(e^{a/\delta}-1\right)}$
   (d) $B=\frac{\mu_0 I_0\left(e^{r/\delta}-1\right)}{2\pi r\left(e^{a/\delta}-1\right)}$
2. (a) $B=0$
   (b) $B=\frac{\mu_0 I\left(r^2-a^2\right)}{2\pi r\left(b^2-a^2\right)}$
   (c) $B=\frac{\mu_0 I}{2\pi r}$
3. (a) $6.34\times10^{-3}\ \text{Nm}^{-1}$, inwards
   (b) greatest at the outer surface.
4. $\mu_0\left(I_1+I_2-I_3\right)$
5. (a) $\frac{\mu_0 b r_1^2}{3}$
   (b) $\frac{\mu_0 b R^3}{3r_2}$
6. (a) $\mu_0\sigma v$
   (b) 0
   (c) $\frac{1}{2}\mu_0\sigma^2 v^2$, up
   (d) $3\times10^8\ \text{ms}^{-1}$
7. (a) 0
   (b) $\frac{\mu_0 NI}{2\pi r_2}$
   (c) 4 mT
8. (a) $\frac{\mu_0 Ir}{2\pi a^2}$, (b) $\frac{\mu_0 I}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)$

9. ZERO, $\frac{\mu_0 I}{2\pi R}$, $\frac{\mu_0 I^2}{4\pi^2 R^2}$ (inwards)

10. $\frac{1}{2}\mu_0 n^2 I^2$

Test Your Concepts-VI (Based on Force on Current Carrying Conductor)

1. (a) $I(\vec{h}\times\vec{B})$, in the direction of $\vec{\ell}$
   (b) $j\ell B$
2. (a) a
   (b) 3.21 kg
3. 39.2 mT
4. $\frac{\lambda g}{I}\tan\theta$
5. 0.245 T, eastward
6. Force on ab is 0
   Force on bc is $BI\ell(-\hat{i})$
   Force on cd is $BI\ell(-\hat{k})$
   Force on da is $BI\ell(\hat{k}+\hat{i})$
7. $(2BIR)\hat{j}$, BIR
8. 0.588 T
9. 0.109 A, to the right.
10. $\frac{2\lambda g}{\sqrt{3}I}$
11. $\frac{\mu_0 I_0 l}{4\pi^2 r}$
12. $-3aB_0 I\hat{j}$
13. $\frac{\mu mg}{B\ell(\cos\phi-\mu\sin\phi)}$

Test Your Concepts-VII (Based on Magnetic Moment and Torque)

1. 3.7×10^{-24} Nm
2. $MB\left(1-\frac{1}{\sqrt{2}}\right)$
3. (a) 5.41×10^{-3} Am^2
   (b) 4.33×10^{-3} Nm
4. 9.98 Nm, CW sense
5. (a) 4°
   (b) 3.39×10^{-3} Nm
6. (a) 80×10^{-3} Nm
   (b) 0.104 Nm
   (c) 0.132 Nm
   (d) The circular loop experiences the largest torque
7. (a) 118 μNm
   (b) $-118\ \mu J \le U \le +118\ \mu J$
8. $\sqrt{\frac{4\pi iB}{m}}$
9. $I\left(\frac{\pi R^2}{4}\right)\hat{i}+I\left(\frac{R^2}{2}\right)\hat{j}+I(R^2)\hat{k}$
10. 1.71×10^{-7} Nmrad^{-1}
11. $\frac{neh}{4\pi m}$, $\frac{\mu_0\pi m^2 e^7}{8\varepsilon_0 h^5 n^5}$
12. $\vec{F}_{AB}=-\vec{F}_{CD}=-N(BI\ell\sin\theta)\hat{k}$
    $\vec{F}_{BC}=-\vec{F}_{DA}=N(BIb\sin\theta)\hat{j}$
    $\vec{\tau}=(\sqrt{2}NI\ell bB\cos\theta\sin\theta)\hat{k}$
13. $-(0.04\pi)\hat{k}$ Am^2, $0.18(\hat{i}-\hat{j})$ Nm
14. (a) $\mu_0\pi nII_0 r^2$
    (b) $2\pi\sqrt{\frac{2m}{5\pi\mu_0 nI_0 I}}$
15. Zero, $(0.1\hat{i}+0.05\hat{j}+0.05\hat{k})$ Am^2, $(-0.1\hat{i}-0.4\hat{k})$ Nm

Test Your Concepts-VIII (Based on Force Between Current Carrying Conductors)

1. 2.7×10^{-5} N, towards left
2. (a) 1×10^{-5} T, out of the page
   (b) 8×10^{-5} N, towards first wire
   (c) 1.6×10^{-5} T, into the page
   (d) 8×10^{-5} N, towards second wire
3. 0.167 m, below the upper wire
4. 12 cm, left of wire 1
   2.4 A, down
5. $\frac{\mu_0 I_1 I_2 \ell}{2\pi}\ln\left(\frac{a+b}{a}\right)$
6. (a) 2.46 N, upwards
   (b) 107 ms^{-2}, upwards
7. 82 A
8. 3 A, 2.88×10^{-6} Nm^{-1}

Single Correct Choice Type Questions

1. A	2. C	3. D	4. B	5. B	6. D	7. D	8. B	9. D	10. D
11. D	12. D	13. B	14. A	15. C	16. A	17. D	18. A	19. C	20. D
21. D	22. C	23. A	24. D	25. B	26. C	27. C	28. D	29. D	30. D
31. A	32. D	33. A	34. C	35. C	36. C	37. D	38. C	39. D	40. B
41. C	42. D	43. A	44. A	45. C	46. C	47. D	48. A	49. D	50. A
51. A	52. C	53. C	54. D	55. A	56. C	57. A	58. D	59. D	60. A
61. D	62. D	63. B	64. C	65. C	66. D	67. B	68. C	69. A	70. C
71. C	72. A	73. D	74. B	75. C	76. A	77. D	78. B	79. B	80. B
81. C	82. C	83. A	84. B	85. C	86. D	87. C	88. C	89. A	90. A
91. B	92. B	93. A	94. B	95. A	96. C	97. B	98. B	99. C	100. C
101. B	102. D	103. A	104. B	105. B	106. B	107. D	108. D	109. C	110. D
111. A	112. C	113. C	114. C	115. D	116. D	117. D	118. D	119. B	120. C
121. D	122. B	123. B	124. B	125. C	126. B	127. D	128. C	129. C	130. D
131. C	132. A	133. A	134. A	135. B	136. B	137. C	138. B	139. D	140. D
141. B	142. D	143. D	144. C	145. D	146. C	147. B	148. A	149. B	150. C
151. B	152. C	153. B	154. C	155. B	156. A	157. A	158. B	159. C	160. D
161. B	162. D	163. D	164. B	165. A	166. C	167. C	168. B	169. B	170. A
171. D	172. D	173. A	174. B	175. B	176. C	177. C	178. B	179. B	180. B
181. B	182. B	183. B							

Multiple Correct Choice Type Questions

1. B, C, D	2. A, C	3. B, C	4. B, C, D	5. A, B, C
6. A, B, D	7. B, D	8. A, B, D	9. C, D	10. A, B, D
11. A, D	12. A, B, C, D	13. B, C, D	14. B, D	15. A, B, C, D
16. A, B, D	17. B, C	18. A, B, C, D	19. A, B, D	20. A, B, C, D
21. A, B, D	22. A, C	23. A, C	24. A, B, C	25. A, B, C
26. A, D	27. A, B	28. A, C	29. A, B, C	30. A, B, C
31. A, C	32. A, B, C	33. A, C, D	34. A, C	35. A, D
36. C, D	37. A, C, D	38. A, C	39. A, B, C	40. A, B, C, D
41. B, C	42. A, B	43. B, D	44. B, C, D	45. A, B, C
46. A, B	47. A, D	48. A, B, C, D	49. B, C,	50. A, C

Reasoning Based Questions

1. D	2. A	3. A	4. A	5. A	6. D	7. A	8. D	9. B	10. A
11. C	12. A	13. A	14. B	15. A	16. B	17. B	18. A	19. A	20. A
21. D	22. D	23. B	24. D	25. C	26. B	27. B	28. C	29. C	

Linked Comprehension Type Questions

1. B	**2.** A	**3.** C	**4.** C	**5.** C	**6.** C	**7.** A	**8.** D	**9.** A	**10.** B
11. D	**12.** A	**13.** D	**14.** D	**15.** B	**16.** C	**17.** C	**18.** A	**19.** D	**20.** A
21. B	**22.** C	**23.** D	**24.** B	**25.** C	**26.** C	**27.** C	**28.** B	**29.** D	**30.** D
31. A	**32.** B	**33.** B	**34.** B	**35.** C	**36.** B	**37.** D	**38.** B	**39.** C	**40.** B
41. A	**42.** D	**43.** B	**44.** C	**45.** A	**46.** A	**47.** A	**48.** B	**49.** A	**50.** A
51. B	**52.** C	**53.** B	**54.** D	**55.** B	**56.** C	**57.** B	**58.** A	**59.** B	**60.** D
61. B	**62.** A	**63.** C	**64.** B	**65.** C	**66.** B	**67.** B	**68.** B	**69.** C	**70.** C
71. B	**72.** B	**73.** B							

Matrix Match/Column Match Type Questions

1.	A → (q)	B → (r)	C → (q)	D → (r)
2.	A → (p, q)	B → (q)	C → (p, q, r)	D → (q, s)
3.	A → (q)	B → (p, t)	C → (s)	D → (r)
4.	A → (r)	B → (q)	C → (p)	D → (r)
5.	A → (s)	B → (q)	C → (p)	D → (r)
6.	A → (r)	B → (p)	C → (q, r)	D → (s)
7.	A → (q)	B → (r)	C → (q)	D → (r)
8.	A → (p, t)	B → (p, q, r)	C → (p)	D → (s)
9.	A → (p, s)	B → (p, q)	C → (p, r)	D → (p, s)
10.	A → (q, r)	B → (p)	C → (q, r)	D → (q)
11.	A → (s)	B → (t)	C → (q)	D → (p)
12.	A → (r)	B → (s)	C → (q)	D → (p)
13.	A → (q)	B → (p)	C → (s, t)	D → (q)
14.	A → (p, r, s)	B → (r, s)	C → (p, q, t)	D → (r, s)
15.	A → (p, t)	B → (q, s, t)	C → (p, r, t)	D → (q)

Integer/Numerical Answer Type Questions

1. 37	**2.** 1.5	**3.** 400	**4.** 300	**5.** 5
6. 12	**7.** 2000	**8.** 1	**9.** 300	**10.** 2
11. 400	**12.** $x = 5, y = 12$	**13.** (a) 750, (b) 115	**14.** 5	**15.** 200
16. 12	**17.** 2	**18.** 1	**19.** 8	**20.** 5
21. 2	**22.** 256	**23.** 2	**24.** 3	**25.** 107
26. 25	**27.** 20	**28.** 7	**29.** 133	**30.** 1
31. 147	**33.** (a) 8, (b) 3, (c) 32, (d) 1920, (e) 5		**33.** **60**	

ARCHIVE: JEE MAIN

1. B	**2.** C	**3.** A	**4.** A	**5.** D	**6.** A	**7.** C	**8.** A	**9.** B	**10.** B
11. B	**12.** B	**13.** B	**14.** D	**15.** D	**16.** D	**17.** A	**18.** D	**19.** *	**20.** B
21. D	**22.** B	**23.** B	**24.** C	**25.** C	**26.** A	**27.** A	**28.** B	**29.** A	**30.** A
31. A	**32.** D	**33.** D	**34.** D	**35.** A	**36.** A	**37.** D	**38.** B	**39.** A	**40.** D
41. A	**42.** C	**43.** C	**44.** D	**45.** A	**46.** C	**47.** A	**48.** A	**49.** D	**50.** A
51. B									

* No given option is correct

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. D	**2.** B	**3.** A	**4.** A	**5.** D	**6.** C	**7.** B	**8.** B	**9.** B	**10.** D
11. A	**12.** B	**13.** C	**14.** D	**15.** B	**16.** C	**17.** C	**18.** C	**19.** D	**20.** A
21. D	**22.** A	**23.** A	**24.** D	**25.** B	**26.** C	**27.** B	**28.** C	**29.** D	**30.** A

Multiple Correct Choice Type Problems

1. A, B, D	**2.** A, B, C	**3.** A, C	**4.** A, D	**5.** C, D
6. B, D	**7.** A, C, D	**8.** A, C	**9.** A, C	**10.** A, B, D
11. A, B, D				

Reasoning Based Questions

1. C

Linked Comprehension Type Questions

1. A, D	**2.** A, C	**3.** C	**4.** B	**5.** A	**6.** B

Matrix Match/Column Match Type Questions

1. C	**2.** A	**3.** C	
4. A → (q, r)	B → (p)	C → (q, r)	D → (q)
5. A → (q)	B → (r, s)	C → (s)	D → (p, q, r)

Integer/Numerical Answer Type Questions

1. 2	**2.** 5.5.6	**3.** 3	**4.** 5	**5.** 7

CHAPTER 2

Magnetism and Matter

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Bar Magnet, Magnetic Poles and Properties
(b) Magnetic Moment of an Orbital Electron
(c) Magnetic Dipole and Properties
(d) Earth's Magnetism
(e) Tangent Law and Tangent Galvanometer
(f) Vibration Magnetometer
(g) Properties of Magnetic Materials

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION TO BAR MAGNET AND MAGNETIC POLES

Moving charges or current loops cause magnetism. This phenomenon was known long before the magnetic effect of current was discovered. Bar magnets are the simplest source of magnetic field (apart from electric current). A bar magnet possesses the following properties.

(a) A freely suspended bar magnet always orients itself (approximately) along the North-South direction. It is important to note here that the end which points towards geographical north is called the **North pole** (not the South pole) and the end towards geographical South is called the **South pole** (and not the north pole).

(b) Like poles repel each other and unlike poles attract each other with a force which obeys the inverse square law.

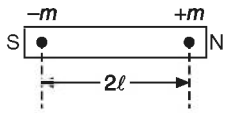
(c) A magnet attracts certain substances e.g., small pieces of iron, iron filings sprinkled on a sheet of paper held over a bar magnet form characteristic patterns similar to the lines of force of an electric dipole.

Conceptual Note(s)

It was thought that, like two types of electric charges, there are two types of magnetic charges (or poles). However, every effort to find such magnetic charges, or to isolate the poles of a magnet have failed. If we break a magnet into two parts, the two pieces become two new magnets, each having both N-pole and S-pole. Even if we break up a magnet into the electrons and nuclei, it will be found that even these are magnetic dipoles. Thus, **a magnetic monopole does not exist.**

A bar magnet consists of two equal and opposite magnetic poles separated by a distance, hence the magnet is also called the **magnetic dipole**.

If m is pole strength and $2l$ the separation between poles, then



Magnetic moment of bar magnet,

$$M = m(2l)$$

M is a vector directed from South Pole to the North Pole of magnet.

If I is the current flowing in a coil of area A, then magnetic moment of current loop

$$M = IA \ (\text{in Am}^2)$$

If the coil contains N turns, then

$$M = NIA$$

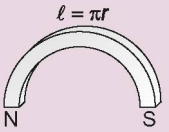
The unit of pole strength m is ampere-metre (Am).

Conceptual Note(s)

(a) On bending a magnet its pole strength remains unchanged whereas its magnetic moment changes.

(b) The unit of magnetic moment is Am^2 and its dimensional formula is $M^0L^2T^0A$

(c) If a magnet of length ℓ and magnetic moment M is bent in the form of a semi-circular arc then its new magnetic moment will be $M' = \frac{2M}{\pi}$



$$M' = m_p(2r) = \frac{m_p 2\ell}{\pi} = \frac{2M}{\pi}$$

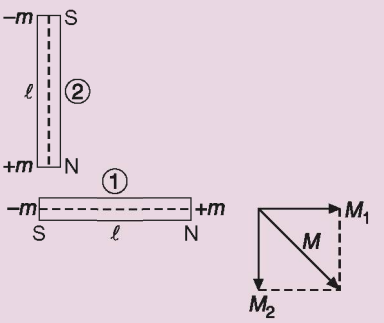
(d) The property of magnetism in materials is on account of magnetic moment in that material.

(e) In para magnetic and ferro magnetic materials the direction of M is in the direction of H whereas in diamagnetic materials it is opposite to that of H.

(f) If a magnet is cut along its length then the magnetic moment decreases i.e.

$$m_p \propto A$$

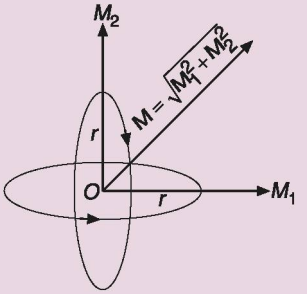
(g) When two bar magnets are lying mutually perpendicular to each other, then the resultant magnetic moment is



$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}m\ell$$

(h) When two coils, each of radius r and carrying current I, are lying concentrically with their planes at right angles to each other, then the resultant magnetic moment is

$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}I\pi r^2 \text{ for } M_1 = M_2$$



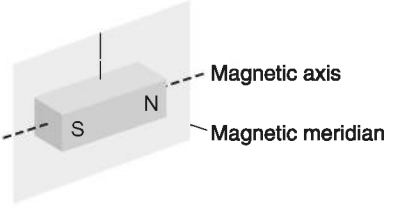
(i) The magnetic moment of a magnet decreases on increasing temperature.

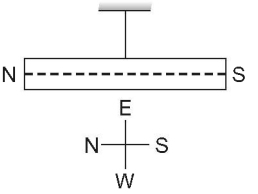
Properties of a Magnet

A magnet has following properties.

(a) It attracts ferromagnetic materials towards it.

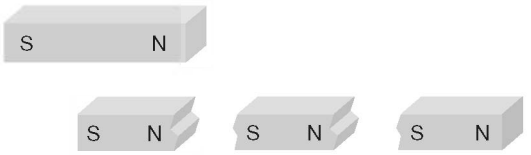
(b) When suspended freely it rests in a particular direction i.e. North-South direction.





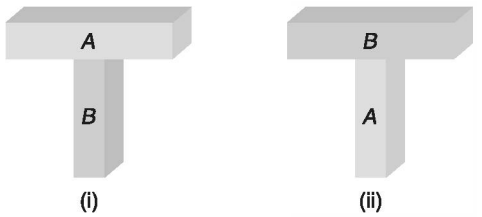
(c) The pole strength of its two poles is the same.

(d) The two poles of a magnet cannot be isolated i.e. separated out *i.e. magnetic monopoles do not exist.* If a bar magnet is broken into a number of pieces, then each piece behaves a s an individual magnet rather than behaving like an isolated pole. So, magnetic monopoles do not exist



(e) Like poles repel and unlike poles attract each other.

(f) For two rods *A* and *B* as shown, if both the rods attract in case (i) and do not attract in case (ii) then,
B is a magnet and *A* is a simple iron rod showing that repulsion is the sure test of magnetism.



(f) It can be magnetically saturated.

(g) It can be demagnetized by beating, mechanical jerks, heating and with lapse of time.

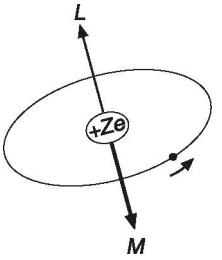
(h) It produces magnetism in other materials by induction.

MAGNETIC MOMENT OF AN ORBITAL ELECTRON

The magnetic moment of an electron due to its orbital motion is $l\mu_B$ whereas that due to its spin motion it is $\frac{\mu_B}{2}$.

So, $M_{\text{orbital}} = l\left(\frac{eh}{4\pi m}\right) = l\left(\frac{e\hbar}{2m}\right)$

and $M_{\text{spin}} = s\left(\frac{eh}{4\pi m}\right) = s\left(\frac{e\hbar}{2m}\right) = s\mu_B = \frac{\mu_B}{2}$



where μ_B is the Bohr magneton. The value of Bohr magneton is

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} \qquad \left\{\text{where } \hbar = \frac{h}{2\pi}\right\}$$

$$\Rightarrow \quad \mu_B = 0.93 \times 10^{-23}\ \text{Am}^2$$

The magnetic moment associated with the electron revolving in the first Bohr orbit is known as Bohr magneton.

Problem Solving Technique(s)

(a) Other formulae for magnetic moment M

(i) $M = ni\pi r^2$

(ii) $M = \frac{evr}{2} = \frac{er^2\omega}{2} = \frac{er^2 2\pi f}{2} = \frac{er^2\pi}{T}$

(iii) $M = \frac{eL}{2m}$, where L = Angular Momentum of the Electron

(iv) $M = n\mu_B$

(b) About 90% of magnetic moment is due to spin motion of electrons whereas 10% part is due to their orbital motion.

(c) The net magnetic moment of an atom is equal to the vector sum of magnetic moments of all its electrons.

ILLUSTRATION 1

The electron in hydrogen atom moves with a speed of 2.2×10^6 ms^{-1} in an orbit of radius 5.3×10^{-11} cm. Calculate the magnetic moment of the orbiting electron.

SOLUTION

Frequency of revolution of electron is

$$f = \frac{v}{2\pi r}$$

Since the moving charge is equivalent to a current loop, so

$$I = qf$$

$$\Rightarrow \quad I = \frac{ev}{2\pi r}$$

If A be the area of the orbit, then the magnetic moment of the orbiting electron is,

$$M = IA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Substituting the values, we get

$$M = \frac{(1.6\times10^{-19})(2.2\times10^{6})(5.3\times10^{-11}\times10^{-2})}{2}$$

$$\Rightarrow \quad M = 9.3\times10^{-26}\ \text{Am}^2$$

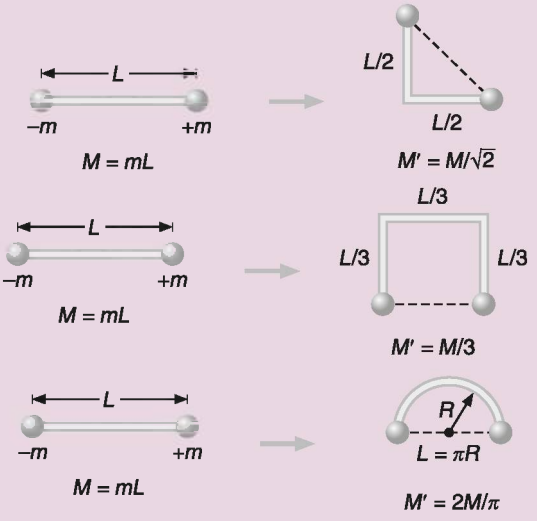
Conceptual Note(s)

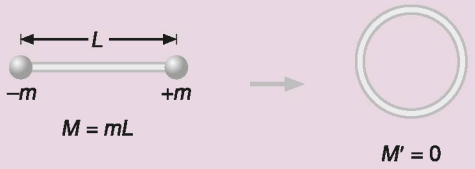
(a) Bohr magneton $\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\ \text{Am}^{-2}$. It serves as natural unit of magnetic moment. Bohr magneton can be defined as the orbital magnetic moment of an electron circulating in inner most orbit.

(b) Magnetic moment of straight current carrying wire is zero.

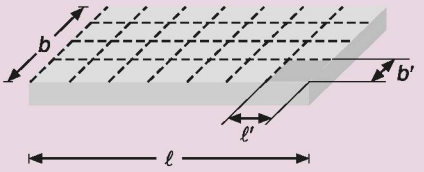
(c) Magnetic moment of toroid is zero.

(d) If a magnetic wire of magnetic moment (M) is bent into any shape then it's magnetic moment M decreases as it's length (L) always decreases but pole strength remains constant.





(e) Cutting of a bar magnet: Consider a rectangular bar magnet having length ℓ, breadth b and mass w, then if the magnet is cut in n equal parts along its length as well as perpendicular to its length simultaneously as shown in the figure, then



(i) length of each part is $\ell' = \frac{\ell}{\sqrt{n}}$

(ii) breadth of each part $b' = \frac{b}{\sqrt{n}}$ and

(iii) Mass of each part $w' = \frac{w}{n}$,

(iv) The pole strength of each part is $m' = \frac{m}{\sqrt{n}}$

(v) Magnetic moment of each part is $M' = m'L' = \frac{m}{\sqrt{n}} \times \frac{\ell}{\sqrt{n}} = \frac{M}{n}$

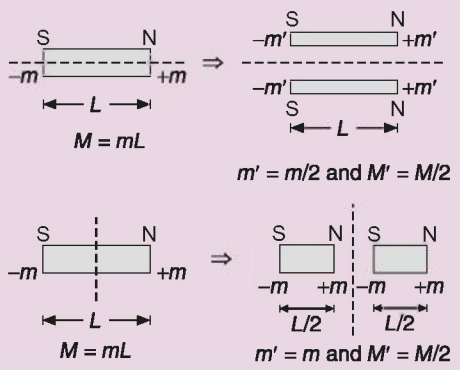
(vi) If initially moment of inertia of bar magnet about the axes passing from centre and perpendicular to its length is $I = w\left(\frac{L^2+b^2}{12}\right)$ then moment of inertia of each part $I' = \frac{I}{n^2}$

(f) For short bar magnet i.e. for a magnet of small width, we have

$b = 0$, so $\ell' = \frac{\ell}{n}$, $w' = \frac{w}{n}$, $m' = m$, $M' = \frac{M}{n}$ and $I' = \frac{I}{n^3}$

(g) If a magnet of monopole strength m and magnetic moment M is cut parallel to its length in two equal halves or is cut parallel to width in two equal haves then the divided pieces of magnet

will have monopole strength m' and magnetic moment M' as shown.



MAGNETISM

(a) That property of a magnet, by virtue of which it attracts ferromagnetic materials towards it and rests itself in North-South direction, is known as magnetism.
(b) The magnetism produced by induction depends upon the distance between inducing pole and induced pole.
(c) The magnetism of materials is mainly due to the spin motion of its electrons.

MAGNETIC POLES

The two points at the ends of a magnet at which magnetism is maximum, are defined as magnetic poles. The magnetic poles are of two types.

(a) North pole
(b) South pole

North Pole

(a) The pole of a magnet which when freely suspended always points towards geographical north, is known as north pole.
(b) The pole, from which the magnetic lines of force emerge out, is known as north pole.
(c) This is also called as the **positive pole**.
(d) Here north direction means geographical north.

South Pole

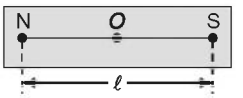
(a) The pole of the magnet, which when freely suspended always points towards south, is known as south pole.
(b) The magnetic pole into which the lines of force merge into, is known as south pole.
(c) This is also called as **negative pole**.
(d) It is not necessary that magnetic poles are exactly in the middle of its ends rather these can be situated away from the middle.

MAGNETIC AXIS

The imaginary line joining the two poles of a magnet is defined as magnetic axis.

EFFECTIVE LENGTH OF A MAGNET

The distance between the two poles of a magnet is defined as its **effective length**. This length is measured along the axis of the magnet.



The distance of magnetic pole from the centre of the magnet is known as **half-length of magnet**. Effective length of magnet is $\frac{5}{6}$ times its geometric length.

MAGNETIC MERIDIAN

An imaginary plane passing through the axis of a freely suspended stationary magnet is defined as **magnetic meridian**. All imaginary planes drawn parallel to this plane are also called **magnetic meridian**. It is not a particular plane rather it is the direction of earth's magnetic field. This plane cuts the surface of earth in a line, hence it is represented by line on the piece of paper.

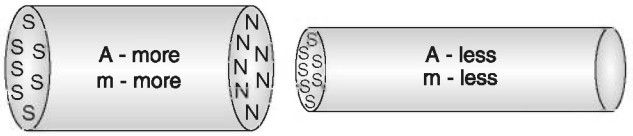
POLE STRENGTH (M)

The strength of a magnetic pole to attract magnetic materials towards it, is known as pole strength. Greater the number of unit poles in a magnetic pole, greater will be its strength.

$$m = \frac{F}{B} = \frac{\text{Magnetic force}}{\text{Magnetic induction}}$$

Its unit is ampere-metre (Am)

(a) It is a scalar quantity (just like charge)
(b) Pole strength of N and S pole of a magnet is conventionally represented by $+m$ and $-m$ respectively.
(c) Pole strength of the magnet depends on the nature of material of magnet and area of cross section. It doesn't depend upon length

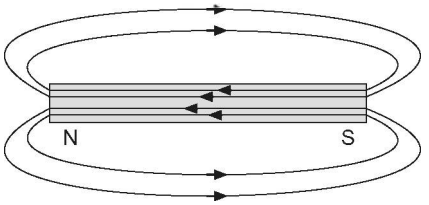


MAGNETIC LINES OF FORCE

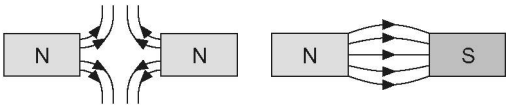
The imaginary lines straight or curved which represent the direction of magnetic field, are known as **magnetic lines of force**. The imaginary path traced by an isolated (imaginary) unit north pole is defined as a **line of force**.

Properties

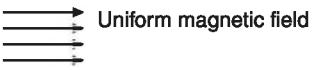
(a) Magnetic lines of force are closed curves. Outside the magnet their direction is from north pole to south pole and inside the magnet these are from south to north pole.



(b) They neither have an origin nor an end.
(c) These lines do not intersect, because if they do so then it would mean two directions of magnetic field at a single point, which is not possible.
(d) The tangent drawn at any point to the line of force indicates the direction of magnetic field at that point.
(e) The number of magnetic lines of force passing through unit normal area is defined as **magnetic induction** (B) whereas the number of lines of force passing through any area is known as **magnetic flux**.
(f) The larger the number of field lines crossing a unit area, the stronger is the magnetic field.
(g) At the poles of the magnet the magnetic field is stronger because the lines of force there are crowded together and away from the poles the magnetic field is week. So, we conclude that Magnetic Field Intensity $\propto$ Number of Lines of Force.
(h) The lines of force can emerge out of the north pole of magnet at any angle and these can merge into the south pole at any angle.
(i) Magnetic field lines have tendency to contract longitudinally indicating attraction between unlike magnetic poles. The lines also have tendency to dilate laterally, indicating repulsion between like magnetic poles as shown below.



(j) Uniform magnetic field lines are shown below.



(k) Non-uniform magnetic field lines are shown below.

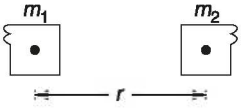
Conceptual Note(s)

The Electrostatic Analog: We can write most of the results for magnetism using analogy with electrostatics by making the following replacements:

$$q \to m,\ \vec{E} \to \vec{B},\ \vec{p} \to \vec{M},\ \frac{1}{4\pi\varepsilon_0} \to \frac{\mu_0}{4\pi}$$

COULOMB'S LAW IN MAGNETISM

The force between two magnetic poles of strength m_1 and m_2 lying at a distance r in vacuum is given by



$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$$

ILLUSTRATION 2

Two magnetic poles, one of which is four times stronger than the other, exert a force of 10 gf on each other when placed at a distance of 20 cm. Find the strength of each pole.

SOLUTION

Let the pole strength of the two dipoles be m and $4m$

Here, $F = 10 \text{ gf} = 10\times10^{-3} \text{ Kgf} = 10\times10^{-3}\times9.8 \text{ N}$

and $r = 20 \text{ cm} = 0.2 \text{ m}$

Using Coulomb's law of magnetism

$$F = \frac{\mu_0}{4\pi}\left(\frac{m_1 m_2}{r^2}\right)$$

Substituting the values,

$$10\times10^{-3}\times9.8 = \frac{10^{-7}\times m\times 4m}{(0.2)^2}$$

$$\Rightarrow \quad m^2 = \frac{10\times9.8\times(0.2)^2\times10^4}{4} = 9800$$

$$\Rightarrow \quad m_1 = m = 98.9 \text{ Am}$$

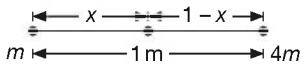
$$\Rightarrow \quad m_2 = 4m = 4\times98.9 = 396 \text{ Am}$$

ILLUSTRATION 3

Two similar magnetic poles, having pole strengths in the ratio $1:3$ and placed 1 m apart. Find the point where a unit pole experiences no net force due to these two poles.

SOLUTION

Let the pole strengths of the two magnetic poles be m and $3m$. Suppose the required point is located at distance x from the first pole. Then at this point,



Force on unit pole due to first pole

= Force on unit pole due to second pole

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\left(\frac{m\times1}{x^2}\right) = \frac{\mu_0}{4\pi}\left(\frac{3m\times1}{(1-x)^2}\right)$$

$$\Rightarrow \quad 3x^2 = (1-x)^2 \text{ or } \sqrt{3}x = 1-x$$

$$\Rightarrow \quad x = \frac{1}{1+\sqrt{3}} = 0.366 \text{ m}$$

STRENGTH OF MAGNETIC FIELD

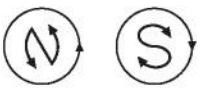
It is defined as the force experienced by a unit north pole or unit text monopole (m_0) placed at the given point in the magnetic field.

Hence, magnetic field due to an imaginary magnetic pole with pole strength m is

$$B = \frac{F}{m_0} = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

CURRENT LOOP AS A MAGNETIC DIPOLE

If we compare the field produced by a bar magnet and a current carrying solenoid, we find that they are similar. Thus, a solenoid behaves as a dipole. Since, a solenoid is a collection of current loops, therefore we can assert that a single current loop is the most elementary magnetic dipole with its one face behaving as a north pole and the other as a south pole.



The face in which the current is anticlockwise acts as N pole because the lines of force emerge out of this face, and the face in which the current is clockwise acts as S pole.

For a loop of n turns or for a solenoid,

$$m = nIA$$

The S.I. unit of m is Am^2 or JT^{-1}

ILLUSTRATION 4

Each atom of an iron bar $(5 \text{ cm}\times1 \text{ cm}\times1 \text{ cm})$ has a magnetic moment 1.8×10^{-23} Am^2. Knowing that the density of iron is 7.78×10^3 kgm^{-3}, atomic weight is 56 and Avogadro's number is 6.02×10^{23}. Calculate the magnetic moment of bar in the state of magnetic saturation

SOLUTION

The number of atoms per unit volume in a specimen,

$$n = \frac{\rho N_A}{A}$$

For iron,

$$\rho = 7.8\times10^3 \text{ kgm}^{-3},$$

$$N_A = 6.02\times10^{26}/\text{kg mol}, \quad A = 56$$

$$\Rightarrow \quad n = \frac{7.8\times10^3 \times 6.02\times10^{26}}{56}$$

$$n = 8.38\times10^{28} \text{ m}^{-3}$$

Total number of atoms in the bar is

$$N_0 = nV = 8.38\times10^{28}\times\left(5\times10^{-2}\times1\times10^{-2}\times1\times10^{-2}\right)$$

$$N_0 = 4.19\times10^{23}$$

The saturated magnetic moment of bar is

$$M = 4.19\times10^{23}\times1.8\times10^{-23} = 7.54 \text{ Am}^2$$

ILLUSTRATION 5

The moment of a magnet is 0.1 Am^2 and the force acting on each pole in a uniform magnetic field of strength 0.36 oersted is 1.224×10^{-4} N. Find the distance between the poles of the magnet.

SOLUTION

$$F = mB$$

$$\Rightarrow \quad m = \frac{F}{B} = \frac{1.224\times10^{-4}}{0.36\times10^{-4}}$$

$$\Rightarrow \quad m = 4 \text{ Am}$$

Since, $M = m(2l)$

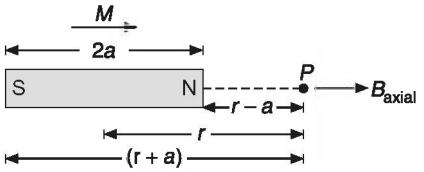
$$\Rightarrow \quad 2l = \frac{M}{m} = \frac{0.1}{4} \text{ m} = \frac{10}{4} \text{ cm}$$

$$\Rightarrow \quad 2l = 2.50 \text{ cm}$$

MAGNETIC FIELD STRENGTH

At Point P Lying on Axial Line (End on Position)

The intensity of magnetic field due to a magnetic dipole of length $2a$ at a point P distant r on its axis from the centre of the magnet is given by



$$B_{\text{axial}} = B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2-a^2\right)^2} \text{ (along } \vec{M}\text{)}$$

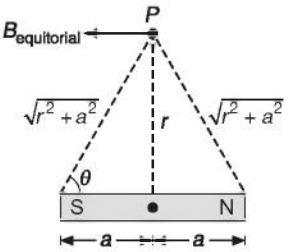
In case of a magnetic dipole if the two poles are situated very close to each other, then

$$a \ll r$$

$$\Rightarrow \quad B_{\text{axial}} \cong \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

At Point P Lying on Equitorial Line (Broad-side on Position)

The intensity of magnetic field due to a magnetic dipole at a point P distant r on its equatorial line from the centre of dipole is given by



$$B_{\text{equitorial}} = B_e = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2+a^2\right)^{3/2}} \text{ (opposite to } \vec{M}\text{)}$$

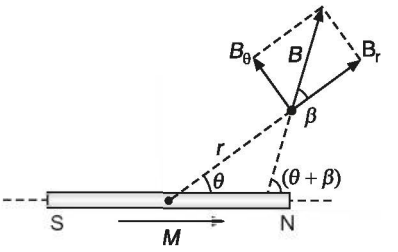
If $a \ll r$, then

$$B_{\text{equitorial}} \cong \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

At any Point P

The intensity of magnetic field at any point P with polar co-ordinates (r,θ) i.e. point P is and inclined at angle θ with dipole moment M of the magnet distant r from the centre of magnetic dipole (magnet) is given by

$$B = \sqrt{B_r^2 + B_\theta^2}$$



where $B_r = \begin{pmatrix}\text{Field in the direction}\\ \text{of increasing } r\end{pmatrix} = \frac{\mu_0}{4\pi}\left(\frac{2M\cos\theta}{r^3}\right)$

and $B_\theta = \begin{pmatrix}\text{Field in the direction}\\ \text{of increasing } \theta\end{pmatrix} = \frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{r^3}\right)$

$$\Rightarrow \quad B=\frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

The direction of this intensity is given by

$$\tan\beta=\frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \beta=\tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

and net field B makes an angle $(\theta+\beta)$ with the magnetic moment (M) of the magnetic dipole.

ILLUSTRATION 6

A short bar magnet has a magnetic moment of $0.48\ \text{JT}^{-1}$. Find the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on

(a) the axis
(b) the equatorial lines (normal bisector) of the magnet.

SOLUTION

(a) When the point lies on the axis, then let B_1 be the magnetic field at P on the axial line.
So, for $r=10\ \text{cm}=0.1\ \text{m}$, we have

$$B_1=\frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)=10^{-7}\times\frac{2\times0.48}{(0.1)^3}$$

$$\Rightarrow \quad B_1=0.96\times10^{-4}\ \text{T}$$ from S pole to N pole

(b) Let B_2 be the magnetic field at point P on the equatorial line, then

$$B_2=\frac{\mu_0}{4\pi}\frac{M}{r^3}=10^{-7}\times\frac{0.48}{(0.1)^3}$$

$$\Rightarrow \quad B_2=0.48\times10^{-4}\ \text{T}=0.48\ \text{G}$$ along from N pole to S pole.

ILLUSTRATION 7

What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5.0 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is $0.40\ \text{Am}^2$.

SOLUTION

$$B_{\text{eq}}=\frac{\mu_0}{4\pi}\frac{M}{r^3}=\frac{10^{-7}\times0.4}{(0.5)^3}=3.2\times10^{-7}\ \text{T}$$

$$B_{\text{axial}}=\frac{\mu_0}{4\pi}\frac{2M}{r^3}=\frac{10^{-7}\times2\times0.4}{(0.5)^3}=6.4\times10^{-7}\ \text{T}$$

ILLUSTRATION 8

Calculate the magnetic induction at a point 1 Å away from a proton, measured along its axis of spin. The magnetic moment of the proton is $1.4\times10^{-26}\ \text{Am}^2$.

SOLUTION

On the axis of a magnetic dipole, magnetic induction is given by

$$B=\frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$$

Substituting the values, we get

$$B=\frac{\left(10^{-7}\right)(2)\left(1.4\times10^{-26}\right)}{\left(10^{-10}\right)^3}=2.8\times10^{-3}\ \text{T}=2.8\ \text{mT}$$

ILLUSTRATION 9

A bar magnet of length 0.1 m has pole strength of 50 Am. Calculate the magnetic field at distance of 0.2 m from its centre on

(a) its axial line and
(b) its equatorial line.

SOLUTION

Here, $m=50\ \text{Am}$, $r=0.2\ \text{m}$, $2l=0.1\ \text{m}$

or $\quad l=0.05\ \text{m}$

So, Magnetic dipole moment is

$$M=m(2l)=50\times0.1=5\ \text{Am}^2$$

Since r and l are comparable, so

(a) $$B_{\text{axial}}=\frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2-l^2\right)^2}=\frac{10^{-7}\times2\times5\times0.2}{\left(0.2^2-0.05^2\right)^2}$$

$$\Rightarrow \quad B_{\text{axial}}=1.42\times10^{-4}\ \text{T}$$

(b) $$B_{\text{eq}}=\frac{\mu_0}{4\pi}\frac{M}{\left(r^2+l^2\right)^{3/2}}=\frac{10^{-7}\times5}{\left(0.2^2+0.05^2\right)^{3/2}}$$

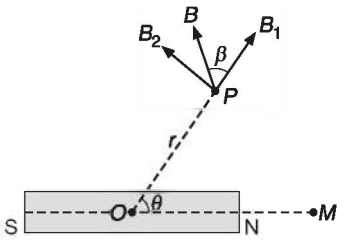
$$\Rightarrow \quad B_{\text{eq}}=5.71\times10^{-5}\ \text{T}$$

ILLUSTRATION 10

Find the magnetic field due to a dipole of magnetic moment $3\ \text{Am}^2$ at a point 5 m away from it in the direction making angle of 45° with the dipole axis.

SOLUTION

The condition given in the figure can be drawn as



So, the magnetic field at point P is,

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

Here, $M = 3\ \text{Am}^2$, $r = 5\ \text{m}$, $\theta = 45°$

$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{(5)^3}\sqrt{1+3\cos^2(45°)}$$

$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{125} \times \sqrt{1+1.5}$$

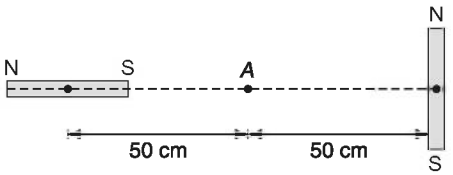
$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{125} \times 1.58 = 3.79 \times 10^{-9}\ \text{T}$$

Also, $\tan\beta = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 45° = \frac{1}{2}$

$$\Rightarrow \quad \beta = \tan^{-1}(0.5)$$

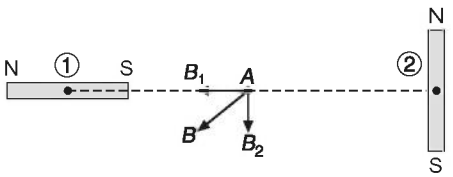
ILLUSTRATION 11

Calculate the magnetic field at point A due to the arrangement of two small dipoles each having a dipole moment of $1\ \text{Am}^2$ as shown in Figure.



SOLUTION

Point A lies on the axial line of first magnet and the field produced by the first magnet is



$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = \frac{10^{-7} \times 2 \times 1}{(0.5)^3} = 16 \times 10^{-7}\ \text{T}$$

Point A lies on the equatorial line of second magnet and the field produced by the second magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{r^3} = \frac{10^{-7} \times 1}{(0.5)^3} = 8 \times 10^{-7}\ \text{T}$$

Therefore, net magnetic field at A is

$$B = \sqrt{B_1^2 + B_2^2} = 8\sqrt{5} \times 10^{-7}\ \text{T}$$

MAGNETIC FORCE BETWEEN TWO SHORT MAGNETS

(a) When they are coaxial, the force between the dipoles is

$$F = \frac{\mu_0}{4\pi}\left(\frac{6M_1M_2}{r^4}\right)$$

(b) When their axes are mutually perpendicular, then the force between the dipoles is

$$F = \frac{\mu_0}{4\pi}\left(\frac{3M_1M_2}{r^4}\right)$$

ILLUSTRATION 12

Two small magnets each of magnetic moment $10\ \text{Am}^2$ are placed in end on position 0.1 m apart from their centres. Calculate the force acting between them.

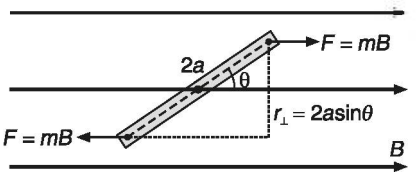
SOLUTION

$$F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4} = (10^{-7})\frac{6(10\times 10)}{(0.1)^4}$$

$$\Rightarrow \quad F = 0.06\ \text{N}$$

TORQUE ON A DIPOLE IN A UNIFORM MAGNETIC FIELD

Consider a bar magnet or a small compass needle of known magnetic moment $\vec{M}$, moment of inertia I is placed in a uniform magnetic field $\vec{B}$ at an angle θ as shown in Figure



We observe that, both of its poles experience equal and opposite forces due to which net force on it is always zero. Hence the magnetic dipole is always in translational equilibrium in a uniform magnetic field.

However, since these forces do not have the same line of action so it experiences a non-zero torque on it. This torque τ is given by

$$\tau = Fr_{\perp} = (mB)(2a\sin\theta)$$

$$\Rightarrow \quad \tau = m(2a)(B\sin\theta)$$

$$\Rightarrow \quad \tau = MB\sin\theta \qquad \{\because\ m(2a) = M\}$$

In vector form, $\vec{\tau} = \vec{M}\times\vec{B}$

Maximum torque is obtained when

$$\sin\theta = 1 \text{ or } \theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \tau_{max} = MB$$

$$\Rightarrow \quad M = \frac{\tau_{max}}{B}$$

If $B = 1$ tesla, then $M = \tau_{max}$

So, the magnetic moment of a magnetic dipole is numerically equal to the torque acting on the dipole placed perpendicular to a magnetic field of strength 1 tesla.

SPECIAL CASES

(a) When $\theta = 0°$, $\vec{\tau} = 0$
The dipole is in rotational equilibrium also.

(b) When $\theta = 180°$, $\vec{\tau} = 0$ again.

(c) When $\theta = 90°$, $|\vec{\tau}| = \tau_{max} = MB$

ILLUSTRATION 13

A magnetic dipole is placed at an angle of $30°$ with a magnetic field of intensity 10^4 T. It experiences a torque equal to 5 Nm. Calculate the pole strength of the dipole, if dipole length is 1 cm.

SOLUTION

As $\tau = MB\sin\theta$, $M = \dfrac{\tau}{B\sin\theta}$

$$\Rightarrow \quad M = \frac{5}{\left((10^4)(0.5)\right)} = 10^{-3} \text{ Am}^2$$

Since $M = md$

$$\Rightarrow \quad m = \frac{M}{d} = \frac{10^{-3}}{10^{-2}} = 10^{-1} \text{ Am}$$

ILLUSTRATION 14

A bar magnet when placed at an angle of $30°$ to the direction of magnetic field of 5×10^{-2} T, experiences a moment of couple 2.5×10^{-6} Nm. If the length of the magnet is 5 cm, then calculate its pole strength.

SOLUTION

Since torque acting on the magnet is

$$\tau = MB\sin\theta = m(2l)B\sin\theta, \text{ where}$$

$$\theta = 30°,\ B = 5\times10^{-2} \text{ T},\ \tau = 2.5\times10^{-6} \text{ Nm}$$

$$2l = 5 \text{ cm} = 0.05 \text{ m} \text{ and } m = ?$$

$$\Rightarrow \quad m = \frac{\tau}{B(2l)\sin\theta} = \frac{2.5\times10^{-6}}{5\times10^{-2}(0.05)\sin30°}$$

$$\Rightarrow \quad m = 2\times10^{-3} \text{ Am}$$

ILLUSTRATION 15

A bar magnet of length 10 cm and having the pole strength equal to 10^{-3} Wb is kept in a magnetic field having magnetic induction (B) equal to $4\pi\times10^{-3}$ T. It makes an angle of $30°$ with the direction of magnetic induction (B). Calculate the value of the torque acting on the magnet. Given that $(\mu_0 = 4\pi\times10^{-7} \text{ WbA}^{-1}\text{m}^1)$

SOLUTION

The unit of pole strength in S.I. system is Am, but here it is given in weber and since the unit of $\mu_0 m$ is weber, so we have

$$\mu_0 m = 10^{-3} \text{ weber}$$

$$\Rightarrow \quad m = \frac{10^{-3}}{\mu_0}$$

Magnetic moment $M = m(2\ell)$

$$\Rightarrow \quad M = \frac{10^{-3}}{\mu_0}\times0.10 = \frac{10^{-4}}{\mu_0}$$

Further $\tau = MB\sin\theta$

$$\Rightarrow \quad \tau = \frac{10^{-4}}{\mu_0}\times(4\pi\times10^{-3})\sin30°$$

$$\Rightarrow \quad \tau = \frac{4\pi}{\mu_0}\times10^{-7}\times0.5 = 0.5 \text{ Nm}$$

ILLUSTRATION 16

A magnet of magnetic moment $50\hat{i}\ \text{Am}^2$ is placed along the x-axis in a magnetic field $\vec{B}=\left(0.5\hat{i}+3\hat{j}\right)\ \text{T}$. Calculate the torque acting on the magnet.

SOLUTION

Since torque, $\vec{\tau}=\vec{M}\times\vec{B}$

where, $\vec{M}=50\hat{i}\ \text{Am}^2$, $\vec{B}=\left(0.5\hat{i}+3\hat{j}\right)\ \text{T}$

$\Rightarrow \quad \vec{\tau}=50\hat{i}\times\left(0.5\hat{i}+3\hat{j}\right)$

$\Rightarrow \quad \vec{\tau}=150\left(\hat{i}\times\hat{j}\right)=150\hat{k}\ \text{Nm}$

WORK DONE IN ROTATING A DIPOLE IN A UNIFORM MAGNETIC FIELD

Since the torque acting on a dipole placed in a uniform magnetic field is given by

$$\tau=MB\sin\theta$$

If dipole is rotated by a small angle $d\theta$, then work is done against torque which is given by

$$dW=\tau d\theta=MB\sin\theta\, d\theta$$

Total work done is given by

$$W=\int dW=\int_{\theta_1}^{\theta_2}MB\sin\theta\, d\theta$$

$$\Rightarrow \quad W=-MB\left(\cos\theta_2-\cos\theta_1\right)$$

The work done in rotating the dipole from equilibrium position $\theta_1=0^\circ$ through an angle θ is

$$W=-MB\left(\cos\theta-1\right)=MB\left(1-\cos\theta\right)$$

The work done in rotating the magnetic dipole from $\theta_1=90^\circ$ to $\theta_2=\theta$ is

$$W=-MB\left(\cos\theta-\cos 90^\circ\right)=-MB\cos\theta$$

This work done is stored in the form of potential energy U of system, so the potential energy of magnetic dipole is given by

$$U=-\vec{M}\cdot\vec{B}=-MB\cos\theta$$

POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM MAGNETIC FIELD

The work done by external torque in rotating the dipole from an angle θ_1 to an angle θ_2 is

$$W=-MB\left(\cos\theta_2-\cos\theta_1\right) \qquad \ldots(1)$$

Since we know that work done by an external force or torque equals the change in potential energy, so from equation (1), we have

$$W=U_f-U_i=-MB\cos\theta_2-\left(-MB\cos\theta_1\right)$$

This makes us conclude that

$$U_f=-MB\cos\theta_2 \text{ and } U_i=-MB\cos\theta_1$$

and while drawing this conclusion, we have simply assumed that the potential energy of the dipole is zero at $\theta=90^\circ$. So, we have

$$U_{\text{initial}}=0, \text{ when } \theta_1=90^\circ$$

and $U_{\text{final}}=U=-MB\cos\theta$, when $\theta_2=\theta$

$$\Rightarrow \quad U=-MB\cos\theta=-\vec{M}.\vec{B}$$

SPECIAL CASES

(a) When $\theta=0^\circ$, $\vec{\tau}=0$, $U=U_{\text{min}}=-MB$

The dipole is in total equilibrium and its potential energy is minimum, so its equilibrium is stable.

(b) When $\theta=180^\circ$, $\vec{\tau}=0$, $U=U_{\text{max}}=MB$

(c) When $\theta=90^\circ$, $|\vec{\tau}|=\tau_{\text{max}}=MB$, $U=0$

ILLUSTRATION 17

A magnetic dipole of length 5 cm having pole strength $\pm 2\times10^{-3}$ Am, placed at 53° with the uniform magnetic field, experiences a torque of 8 Nm. Calculate the

(a) magnitude of magnetic field.
(b) potential energy of the dipole.

Given that $\tan(53^\circ)=\dfrac{4}{3}$

SOLUTION

(a) Since $\tau=MB\sin\theta$

$$\Rightarrow \quad B=\frac{\tau}{M\sin\theta}$$

$$\Rightarrow \quad B=\frac{8}{\left(2\times10^{-3}\right)(0.05)(0.8)}=10^5\ \text{T}$$

(b) Since $U=-MB\cos\theta$

$$\Rightarrow \quad U=-\left(2\times10^{-3}\right)(0.05)\left(10^5\right)(0.6)=-6\ \text{J}$$

TIME PERIOD OF SMALL OSCILLATIONS OF MAGNETIC DIPOLE/BAR MAGNET IN UNIFORM MAGNETIC FIELD

Let a bar magnet of dipole moment M and moment of inertia I is slightly disturbed by a small angle θ from its stable equilibrium position in a uniform magnetic field B. Restoring torque acting on it is

$$\tau = -MB\sin\theta$$

For small angle $\sin\theta \approx \theta$

$$\Rightarrow \quad \tau = -MB\theta \qquad \ldots(1)$$

$$\text{Since, } \tau = I_{CM}\alpha = I_{CM}\left(\frac{d^2\theta}{dt^2}\right) = I_{CM}\ddot{\theta} \qquad \ldots(2)$$

From (1) and (2), we get

$$I_{CM}\left(\frac{d^2\theta}{dt^2}\right) = -MB\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \left(\frac{MB}{I_{CM}}\right)\theta = 0 \qquad \ldots(3)$$

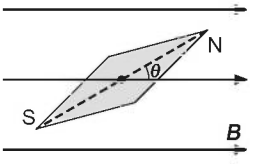
Comparing equation (3), with the standard equation of angular SHM i.e. $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$, we get

$$\omega = \sqrt{\frac{MB}{I_{CM}}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I_{CM}}{MB}}$$

ILLUSTRATION 18

A magnetic needle is free to oscillate in a uniform magnetic field as shown in Figure.



The magnetic moment of magnetic needle is 7.2 Am^2 and moment of inertia $I = 6.5\times10^{-6}$ kgm^2. The number of oscillations performed in 5 s is 10. Calculate the magnitude of magnetic field.

SOLUTION

Given that

$$T = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{5}{10} = 0.5 \text{ s}$$

$$M = 7.2 \text{ Am}^2, \; I = 6.5\times10^{-6} \text{ kgm}^2$$

Since $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{I}{MB}\right)$$

So, the magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4\times(3.14)^2\times6.5\times10^{-6}}{7.2\times(0.5)^2} = 1.42\times10^{-4} \text{ T}$$

PROPERTIES OF ELECTRIC AND MAGNETIC DIPOLES

Property	Electric dipole	Magnetic dipole
Torque in an external field	$\vec{\tau} = \vec{p}\times\vec{E}$	$\vec{\tau} = \vec{M}\times\vec{B}$
Field at distant point along axis (end-on position)	$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{x^3}$	$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$
Field at distant point along perpendicular bisector (broad-side-on position)	$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{x^3}$	$\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{M}}{x^3}$
Work done in rotating the dipole in an external field from the equilibrium position	$W = pE(1-\cos\theta)$ $\Rightarrow W = pE - \vec{p}\cdot\vec{E}$	$W = MB(1-\cos\theta)$ $\Rightarrow W = MB - \vec{M}\cdot\vec{B}$
Potential energy in an external field	$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{M}\cdot\vec{B}$

ILLUSTRATION 19

The work done in turning a magnet of magnetic moment M through an angle of $90°$ from the meridian, is n times the corresponding work done to turn it through an angle of $60°$. Calculate n.

SOLUTION

Since, we know that

$$W = MB(1-\cos\theta)$$

According to the problem, we have

$$W_{0°\to 90°} = n\left(W_{0°\to 60°}\right)$$

$$\Rightarrow \quad MB(1-\cos(90°)) = nMB(1-\cos(60°))$$

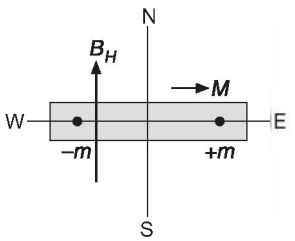
$$\Rightarrow \quad n = 2$$

ILLUSTRATION 20

A bar magnet of magnetic moment $2\ \text{Am}^2$ is free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. If horizontal component of earth's field is $25\ \mu\text{T}$ and it is directed from south to north, then calculate the kinetic energy of the magnet as it takes north-south position.

SOLUTION

Initially the magnet is along east-west direction, so the angle between M and B_H is $90°$ i.e. $\theta_1 = 90°$ as shown in Figure.



So, initial potential energy of the magnet is

$$U_1 = -MB_H\cos 90° = 0$$

Finally, when the magnet takes the north-south position, then it aligns along the earths magnetic field, so the angle between M and B_H is $0°$ i.e. $\theta_2 = 0°$.
So, the final potential energy of the magnet is

$$U_2 = -MB_H\cos 0° = -MB_H$$

According to Law of Conservation of Energy, we know that the loss in potential energy of the magnet equals the gain in its kinetic energy, so we have

$$\Delta K = U_i - U_f = MB_H$$

$$\Rightarrow \quad \Delta K = 2\left(25\times 10^{-6}\right) = 50\times 10^{-6}\ \text{J} = 50\ \mu\text{J}$$

GAUSS'S LAW FOR MAGNETISM

The law states that

$$\oint \vec{B}\cdot d\vec{A} = 0$$

for all closed surfaces. This is a precise expression of the fact that magnetic monopoles do not exist.

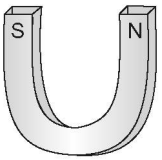
The flux emanating out of a surface is taken positive and the flux entering the surface is to be taken as negative. So, we conclude that total magnetic flux entering a closed surface equals the total magnetic flux leaving. This can also be attributed to the fact that magnetic field lines are continuous and form closed loops unlike electrostatic field lines which begin at a positive charge and end at a negative charge.

Test Your Concepts-I

Based on Bar Magnet and Properties

(Solutions on page H.107)

1. The work done in turning a magnet of magnetic moment M by an angle 90° from the meridian is two times the corresponding work done to turn it through an angle of θ. Calculate θ.
2. Calculate the geometric length of a bar magnet that has a magnetic length of 10 cm.
3. The distance between the poles of a horse shoe magnet is 0.1 m and its pole strength is 0.01 Am. Calculate the induction of magnetic field at a point midway between the poles.



4. A current I is flowing in a conducting wire of length L. If the wire is bent to form a circular loop, then calculate its magnetic dipole moment.
5. The length of a magnetised steel wire is l and its magnetic moment is M. It is bent into an L shaped wire with two sides equal. Calculate the new magnetic moment of the wire.
6. A magnetised wire of moment M is bent into an are of a circle subtending an angle of 60° at the centre. Calculate the new magnetic moment.
7. A short bar magnet of magnetic moment $m = 0.32\ \text{JT}^{-1}$ is placed in a uniform magnetic field 0.15 T. If the bar magnet is free to rotate in the plane of the field, which orientation would correspond to its stable and unstable equilibrium. Also calculate the potential energy of the magnet in each case.
8. A thin bar magnet of length $2L$ is bent at the mid-point, so that the angle between them is 60°. Calculate the new length of the magnetic dipole.
9. A wire of length ℓ is bent in the form a circular coil of some turns. A current i flows through the coil. The coil is placed in a uniform magnetic field B. Calculate the maximum torque acting on the coil.
10. A bar magnet of magnetic moment 4 Am^2 is free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. If horizontal component of earth's field is 25 μT and it is directed from south to north, then calculate the kinetic energy of the magnet as it takes north-south position.

EARTH'S MAGNETISM

The earth behaves like a magnet. When a bar magnet is suspended freely in the earth's magnetic field, it stays along north-south direction. The north and south poles of magnet stay along South and North poles of earth respectively. It is observed that the magnetic poles are at some distance from geographical poles. The latest theories providing explanation to earth's magnetism are given below.

(a) The earth rotates about its axis and has a surrounding ionised region due to interaction of cosmic rays. Due to rotation of earth, the surrounding ionised region gives rise to strong electric currents which cause magnetisation.

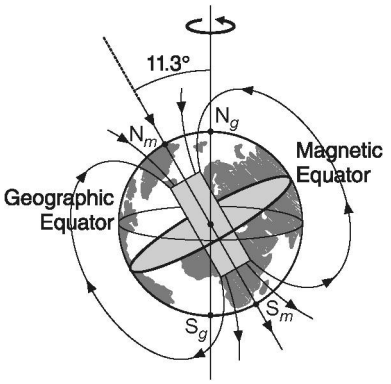
(b) There exists molten iron and nickel within the core of earth and when the earth rotates, then due to the convective motion of these metallic fluids, a magnetic field is produced which causes magnetisation and this effect is called as *"the dynamo effect"*.

Both theories given above, show that earth's magnetism will be destroyed if the earth stops rotating.

PROPERTIES OF EARTH'S MAGNETIC FIELD

Originally, it was assumed that the magnetic field of earth is similar to one which would be obtained if a huge magnet is assumed to be buried deep inside the earth at its centre. The strength of the earth's magnetic field is not constant and changes irregularly from place to place at the earth's surface and even at a given place, it varies with time too. The order of magnitude of the earth's magnetic field is 10^{-5} T. The following things must also be kept in mind while understanding the earth's magnetic field.

(a) The axis of rotation of earth is called geographic axis and the points where it cuts the surface of earth are called geographical poles i.e. geographical north N_g and geographical south S_g. The circle on the earth's surface perpendicular to the geographical axis is called equator.
(b) A vertical plane passing through the geographical axis is called Geographical Meridian (GM).
(c) The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth. The points where the magnetic axis cuts the surface of earth are called magnetic poles. The circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.



(d) Magnetic axis and Geographical axis do not coincide but they make an angle of $11.3°$ with each other.
(e) A vertical plane passing through the magnetic axis is called Magnetic Meridian (MM).
(f) Direction of earth's magnetic field is from S (geographical south) to N (Geographical north).

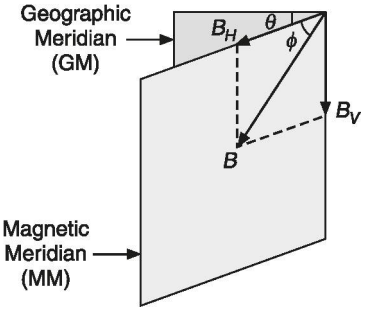
COMPONENTS OF EARTH'S MAGNETIC FIELD

It has been observed that the magnitude and direction of the earth's magnetic field at a place can be completely calculated by the three quantities known as magnetic elements. These magnetic elements also called as components of earth's magnetic field. These essential components of earth's magnetic field are

(a) Angle of Declination or Magnetic Declination (θ)
(b) Angle of Dip or Magnetic Dip (ϕ)
(c) Horizontal component of earth's magnetic field (B_H).

ANGLE OF DECLINATION (θ)

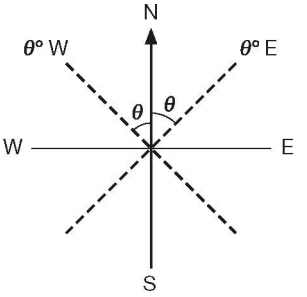
The vertical plane passing through the line joining magnetic north and south poles is called the **magnetic meridian** while that passing through the line joining the geographical north and south poles is called the **geographical meridian** figure.



The angle between magnetic and geographical meridian is called the **angle of declination** and is denoted by θ.

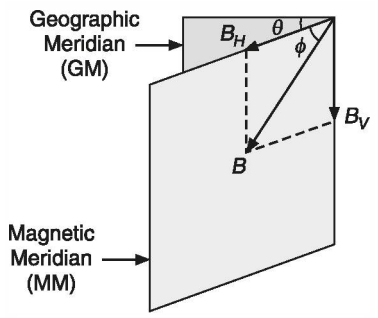
If, at a place, the north pole of the compass needle lies $\theta°$ to the east of geographical axis, then declination at that place is expressed at $\theta° E$.

If, at a place, the north pole of the compass needle lies $\theta°$ to the west of geographical axis, then declination at that place is expressed at $\theta° W$.



ANGLE OF DIP OR ANGLE OF INCLINATION (ϕ)

The magnetic field of earth varies in magnitude and direction. The angle of dip is the angle made by resultant earth's magnetic field (B) and the horizontal line in the magnetic meridian as shown in Figure.



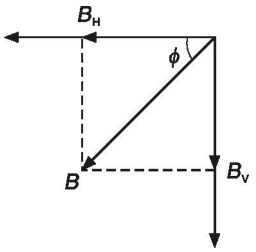
It is also the angle which the axis of a freely suspended magnet makes with the horizontal. The angle of dip is measured by dip circle.

At poles $\phi = 90°$ and at equator $\phi = 0°$.

In the northern hemisphere the north pole of the magnet points below the horizontal and hence the angle is called as angle of dip, whereas in the southern hemisphere the North Pole points above the horizontal and hence the angle is called as the Angle of Inclination.

HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD (B_H)

The earth's magnetic field strength (B) makes an angle ϕ with the horizontal as shown in Figure.



It can be resolved into two components, the

(a) horizontal component B_H, which is from south to north and

(b) vertical component B_V

From figure, we observe that

$$B_H = B\cos\phi \text{ and } B_V = B\sin\phi$$

$$\Rightarrow \quad B = \sqrt{B_H^2 + B_V^2} \text{ and } \tan\phi = \frac{B_V}{B_H}$$

Conceptual Note(s)

At equator $\phi = 0$

$\Rightarrow$ $B_H = B$ and $B_V = 0$

while at poles $\phi = 90°$

$\Rightarrow$ $B_H = 0$ and $B_V = B$

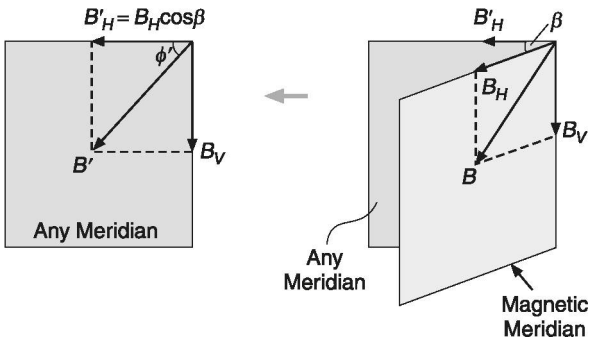
APPARENT DIP (ϕ')

The value of dip at a place is determined with the help of an instrument known as a **dip circle**. It consists of a magnetised needle capable of rotating in the vertical plane about a horizontal axis. The ends of the needle move over a vertical scale graduated in degrees.

When the plane of the scale of the dip circle is in the magnetic meridian, the needle rests in the direction of the earth's magnetic field. The angle made by the needle with the horizontal is called **true dip**.

If the plane of the scale of the dip circle is not in the magnetic meridian, then the needle will not indicate the correct direction of the earth's magnetic field. The angle made by the needle with the horizontal is called the **apparent dip**.

So **apparent dip** is the angle of dip shown by the dip circle when its plane is placed in any general meridian at some angle with the magnetic meridian as shown in Figure.



Suppose the dip circle is set at an angle β to the magnetic meridian. In this new vertical plane inclined at an angle β to the magnetic meridian, the vertical component of the earth's magnetic field remains the same whereas in this new plane the horizontal

component of the earth's magnetic field will have a value

$$B'_H = B_H \cos\beta$$

Now, the apparent dip ϕ' is given by

$$\tan\phi' = \frac{B_V}{B'_H} = \frac{B_V}{B_H \cos\beta}$$

But $\tan\phi = \frac{B_V}{B_H}$, where ϕ is the true dip, so we get

$$\tan\phi' = \frac{\tan\phi}{\cos\beta}$$

Problem Solving Technique(s)

(a) When the magnetic needle oscillates in the vertical east-west plane at right angle to magnetic meridian, then only vertical component of earth's magnetic (B_V) field acts on it.

(b) When the dip needle oscillates at right angles to the magnetic meridian in a horizontal plane, then only horizontal component of earth's magnetic field (B_H) acts on it.

(c) When the dip needle oscillates in the vertical plane in magnetic meridian then both the components B_V and B_H act on it.

(d) The horizontal component of earth's magnetic field is from S to N

(e) If at any place the angle of dip is θ and magnetic latitude is λ then,

$$\tan\theta = 2\tan\lambda$$

(f) The total intensity of earth's magnetic field

$$I = I_0\sqrt{1+3\sin^2\lambda}$$

here $I_0 = \frac{M}{R^3}$

Here M and R are the magnetic moment of bar magnet of earth and radius of earth respectively.

(g) At magnetic equator of earth $\lambda = 0°$ and at poles $\lambda = 90°$

$$I_{pole} = 2I_{equator}$$

(h) At the poles and equator of earth, the values of total intensity are 0.66 oersted and 0.33 oersted respectively.

ILLUSTRATION 21

The magnetic field of the earth at the equator is approximately 0.4 G. Taking the radius of earth to be 6400 km, estimate the dipole moment of the earth.

SOLUTION

The equatorial magnetic field for the earth can be approximated as

$$B_e = \frac{\mu_0 M}{4\pi R_e^3}$$

where, $B_e \approx 0.4\text{ G} = 4\times10^{-5}\text{ T}$, $R_e = 6.4\times10^6\text{ m}$

$$\Rightarrow \quad M = \frac{4\pi B_e R_e^3}{\mu_0} = \frac{(4\times10^{-5})(6.4\times10^6)^3}{\mu_0/4\pi}$$

$$\Rightarrow \quad M = \frac{(4\times10^{-5})(6.4\times10^6)^3}{10^{-7}} = 1.05\times10^{23}\text{ Am}^2$$

This value is close to the value $8\times10^{22}\text{ Am}^2$ quoted in texts related to geomagnetism.

ILLUSTRATION 22

The plane of the dip circle is set in the geographical meridian and the apparent dip is θ_1. It is then set in a vertical plane perpendicular to the geographical meridian, the apparent dip becomes θ_2. Find the angle of declination α in terms of θ_1 and θ_2 at that place.

SOLUTION

Since, $\tan\theta_1 = \frac{\tan\theta}{\cos\alpha}$

and $\tan\theta_2 = \frac{\tan\theta}{\cos(90°-\alpha)} = \frac{\tan\theta}{\sin\alpha}$

$$\Rightarrow \quad \cos\alpha = \frac{\tan\theta}{\tan\theta_1} \qquad \ldots(1)$$

$$\text{and } \sin\alpha = \frac{\tan\theta}{\tan\theta_2} \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$$

ILLUSTRATION 23

A magnet 10 cm long and having a pole strength 2 Am is deflected through 30° from the magnetic meridian. The horizontal component of earth's induction is 0.32×10^{-4} T. Find the value of deflecting couple.

SOLUTION

Magnetic moment $M = m(2l)$

$\Rightarrow \quad M = 2\times(10\times10^{-2}) = 0\cdot20 \text{ Am}^2$

Since, $\tau = MB_H \sin\theta$

$\Rightarrow \quad \tau = (0.20)(0.32\times10^{-4})\sin30°$

$\Rightarrow \quad \tau = 32\times10^{-7} \text{ Nm}$

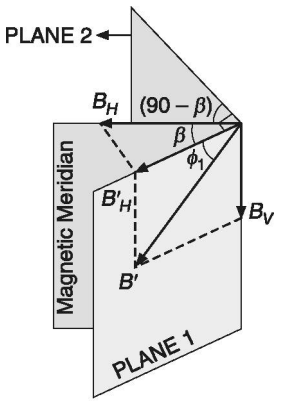
ILLUSTRATION 24

If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then prove that

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

SOLUTION

Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90°-\beta)$ with it. The components of B_H in these planes will be $B_H\cos\alpha$ and $B_H\sin\alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan\phi_1 = \frac{B_V}{B_H\cos\beta}$$

$$\Rightarrow \quad \cos\beta = \frac{B_V}{B_H\tan\phi_1} \qquad \ldots(1)$$

$$\tan\phi_2 = \frac{B_V}{B_H\sin\beta}$$

$$\Rightarrow \quad \sin\beta = \frac{B_V}{B_H\tan\phi_2} \qquad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$\cos^2\beta + \sin^2\beta = \left(\frac{B_V}{B_H}\right)^2\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

$$\Rightarrow \quad 1 = \frac{B_V^2}{B_H^2}\left(\cot^2\phi_1 + \cot^2\phi_2\right)$$

$$\Rightarrow \quad \frac{B_H^2}{B_V^2} = \cot^2\phi_1 + \cot^2\phi_2$$

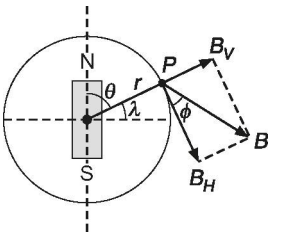
$$\Rightarrow \quad \cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

ILLUSTRATION 25

Consider the earth as a short magnetic with its centre coinciding with the centre of earth and dipole moment M. Calculate the angle of dip ϕ at a place where the magnetic latitude is λ.

SOLUTION

For a short magnet/dipole, the radial (or vertical) component of magnetic field is found by taking the field due to the small dipole at any general point $P(r, \theta)$



The vertical and horizontal components of earth's magnetic field are

$$B_V = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ and } B_H = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

where $\theta = (90°-\lambda)$

$$\Rightarrow \quad B_V = \frac{\mu_0}{4\pi}\frac{2M\cos(90°-\lambda)}{r^3} = \frac{\mu_0}{4\pi}\frac{2M\sin\lambda}{r^3}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{2M\sin(90°-\lambda)}{r^3} = \frac{\mu_0}{4\pi}\frac{M\cos\lambda}{r^3}$$

Angle of dip ϕ is the angle made by resultant magnetic field intensity with the horizontal. So,

$$\tan\phi = \frac{B_V}{B_H}$$

$$\Rightarrow \quad \tan\phi = \frac{\frac{\mu_0}{4\pi}\left(\frac{2M\sin\lambda}{r^3}\right)}{\frac{\mu_0}{4\pi}\left(\frac{M\cos\lambda}{r^3}\right)} = 2\tan\lambda$$

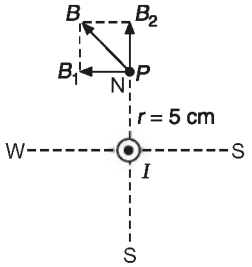
$$\Rightarrow \quad \phi = \tan^{-1}(2\tan\lambda)$$

ILLUSTRATION 26

A long vertical wire carries a steady current of 10 A flowing upwards through it at a place where horizontal component of earth's magnetic field is 0.3 G. Calculate the total magnetic induction at a point 5 cm from the wire due magnetic north of wire.

SOLUTION

The magnetic field B_1 due to current carrying wire at distance $r = 5\text{ cm} = 5\times10^{-2}$ m is shown in Figure and is given by



$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times10^{-7}\times10}{2\pi\times5\times10^{-2}}$$

$$\Rightarrow \quad B_1 = 0.4\times10^{-4}\text{ T}$$

$$\Rightarrow \quad B_1 = 0.4\text{ G, due west}$$

The earth's field is

$$B_2 = B_H = 0.4\text{ G, towards north}$$

So, net magnetic field is given by

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\Rightarrow \quad B = \sqrt{(0.4)^2 + (0.3)^2} = 0.5\text{ G}$$

ILLUSTRATION 27

A compass needle whose magnetic moment is 60 Am^2 pointing geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μT, experiences a torque of 1.2×10^{-3} Nm. Calculate the declination of the place.

SOLUTION

Torque acting on compass needle is

$$\tau = MB_H\sin\theta$$

$$\Rightarrow \quad 1.2\times10^{-3} = 60\times(40\times10^{-6})\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1.2\times10^{-3}}{50\times40\times10^{-6}} = \frac{1}{2}$$

So, declination at that place is

$$\theta = 30^\circ$$

ILLUSTRATION 28

A magnet is suspended in the magnetic meridian using an untwisted wire. The upper end of wire is rotated through 180° to deflect the magnet by 30° from magnetic meridian. When this magnet is replaced by another magnet, the upper end of wire is rotated through 270° to deflect the magnet 30° from magnetic meridian. Calculate the ratio of magnetic moments of magnets.

SOLUTION

Let M_1 and M_2 be the magnetic moments of magnets and H the horizontal component of earth's field. Since

$$\tau = MB_H\sin\theta$$

If ϕ is the twist of wire, then $\tau = C\phi$, C being restoring couple per unit twist of wire also called as torsional constant. At equilibrium, we have

$$C\phi = MB_H\sin\theta \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{\phi_1}{\phi_2} = \frac{M_1}{M_2}$$

where $\phi_1 = (180^\circ - 30^\circ) = 150^\circ$

$$\Rightarrow \quad \phi_1 = 150\times\frac{\pi}{180}\text{ radian}$$

and $\phi_2 = (270^\circ - 30^\circ) = 240^\circ$

$$\Rightarrow \quad \phi_2 = 240\times\frac{\pi}{180}\text{ radian}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{\phi_1}{\phi_2} = \frac{150\times\left(\frac{\pi}{180}\right)}{240\times\left(\frac{\pi}{180}\right)} = \frac{15}{24} = \frac{5}{8}$$

ABOUT MAGNETIC MAPS

It is observed that dip, declination and horizontal component of earth's magnetic field over the earth's surface varies from place to place. However, there are many places on the earth which possess the same value of magnetic elements. Lines are drawn joining all the places on the earth having the same value of magnetic elements and these lines form magnetic maps.

Isogonic Lines

The lines joining the equal angle of declination are called **isogonic lines**.

Agonic Lines

The lines joining the zero angle of declination are called **agonic lines**.

Isoclinic Lines

The lines joining the equal angle of dip are called **isoclinic lines**.

Aclinic Lines (or Magnetic Equator)

The lines joining the zero angle of dip are called **aclinic lines**.

Isodynamic Lines

The lines joining equal horizontal component of earth's magnetic field are called **isodynamic lines**.

NEUTRAL POINT

A neutral point is obtained when horizontal component of earth's field is balanced by the field produced by the magnet. i.e. at neutral point the intensity of magnetic field is equal to the horizontal component of earth's magnetic field but its direction is opposite to that of B_H.

NEUTRAL POINT WHEN BAR MAGNET IS PLACED HORIZONTALLY ON A HORIZONTAL PLANE/TABLE

Consider a bar magnet that is placed horizontally on a horizontal plane/table such that

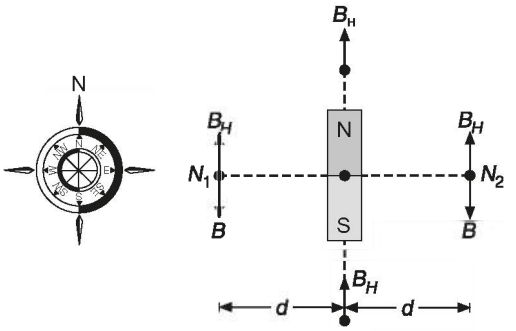
(a) the north pole of magnet is facing geographical north of earth

(b) the north pole of magnet is facing geographical south of earth

In both these cases two neutral points are obtained as discussed below.

CASE-1:

When the bar magnet is placed horizontally in the horizontal plane such that the north pole of magnet is facing geographical north of earth, then two neutral points N_1 and N_2 are obtained in the plane of the magnet on the equatorial line of bar magnet at a distance d from the centre of the magnet as shown.



This is because the field of the earth B_H is from geographical south to geographical north and the field of the magnet B is from north pole to south pole of magnet, outside it.

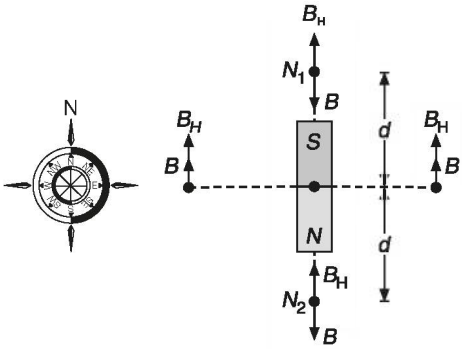
At Neutral points, we have

$$B_H = B_{\text{equitorial}}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

CASE-2:

When the bar magnet is placed horizontally in the horizontal plane such that the south pole of magnet is facing geographical north of earth, then two neutral points N_1 and N_2 are obtained in the plane of the magnet on the equatorial line of bar magnet at a distance d from the centre of the magnet as shown.



This is because the field of the earth B_H is from geographical south to geographical north and the field of the magnet B is from north pole to south pole of magnet, outside it.

At neural point, in this case, we have

$$B_H = B_{\text{axial}}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

NEUTRAL POINT WHEN BAR MAGNET IS PLACED VERTICALLY ON A HORIZONTAL PLANE/TABLE

Consider a magnet placed vertically in a horizontal plane just like a magnet placed vertically on a horizontal table surface, such that

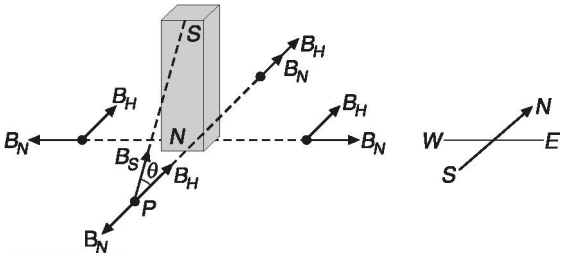
(a) the north pole of the magnet lies in the plane of the table

(b) the south pole of the magnet lies in the plane of the table.

In both the said cases, only one neutral point is obtained as discussed below.

CASE-1:

North pole of the magnet lies in the horizontal plane i.e. the plane of the table.



B_H = Magnetic field due to N-pole

B_H = Magnetic field due to S-pole

M = Pole strength of each pole of the magnet

At neutral point P we have

$$B_N - B_S\cos\theta = B_H \qquad \{B_S < B_N\}$$

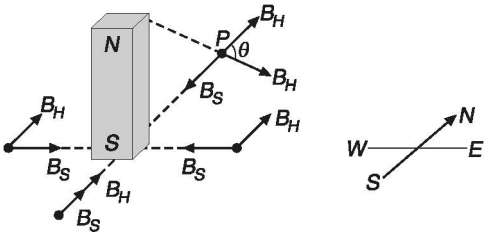
If we neglect the effect of South pole, then too only one neutral point is obtained (as seen from above) as shown and then at the neutral point, we have

$$B_N = B_H$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$

CASE-2:

South pole of the magnet lies in the horizontal plane i.e. the plane of the table.



At neutral point P, we have

$$B_S - B_N\cos\theta = B_H \qquad \{B_S > B_N\}$$

Here too, if we neglect the effect of North pole, then also only one neutral point is obtained (as seen from above) as shown and then at the neutral point, we have

$$B_S = B_H$$

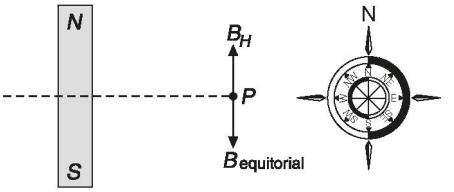
$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$

ILLUSTRATION 29

A short bar magnet with its north pole facing north forms a neutral point at P in the horizontal plane. If the magnet is rotated by $90°$ in the horizontal plane, calculate the net magnetic induction at P, assuming that the horizontal component of earth's magnetic field is B_H.

SOLUTION

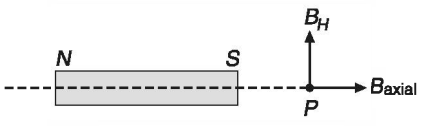
Initially the magnet is placed as shown in Figure.



So, neutral point obtained equitorial line and at neutral point, we have

$$|B_H| = |B_{\text{equitorial}}|$$

where B_H is the horizontal component of earth's magnetic field, $B_{\text{equitorial}}$ is the magnetic field due to bar magnet on its equitorial line.
Finally, we have



Now the point P lies on axial line of the magnet and at P, net magnetic field is given by

$$B = \sqrt{B_{\text{axial}}^2 + B_H^2}$$

$$\Rightarrow \quad B = \sqrt{(2B_e)^2 + (B_H)^2}$$

Since $B_e = B_{\text{equitorial}} = B_H$

$$\Rightarrow \quad B = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}B_H$$

Test Your Concepts-II

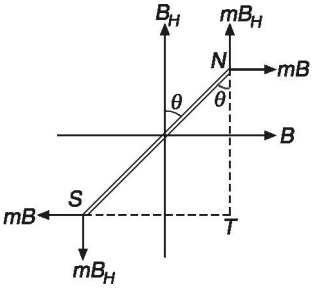
Based on Earth's Magnetism

(Solutions on page H.108)

1. A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.
2. A short magnet of moment 6.75 Am^2 produces a neutral point on its axis. If horizontal component of earth's magnetic field is 5×10^{-5} Wbm^{-2}, then calculate the distance of the neutral point.
3. A dip circle shows an apparent dip of 45° at a place where the true dip is 30°. If the dip circle is rotated through 90°, what apparent dip will it show?
4. A compass needle of magnetic moment 60 Am^2 is pointing geographical north at a certain place. It experiences a torque of 1.2×10^{-3} Nm. The horizontal component of earth's magnetic field at that place is 40 μWbm^{-2}. Calculate the angle of declination at that place.
5. At a certain location on earth, a compass point 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.
6. A very small magnet is placed in the magnetic meridian with its south pole pointing north. The null point is obtained 20 cm away from the centre of the magnet. If the earth's magnetic field (horizontal component) at this point be 0.3 G, then calculate the magnetic moment of the magnet.
7. A ship is to reach a place 10° south of west. In which direction should it be steered if the declination at the place is 18° west of north?
8. The horizontal and vertical component of earth's field at a place are 0.22 G and 0.38 G respectively. Calculate the angle of dip and resultant intensity of earth's field.
9. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 60°. Now the dip circle is rotated so that the plane in which the needle moves makes and angle of 30° with the magnetic meridian. In this new position, calculate the angle of dip.
10. In the magnetic meridian of a certain place, the horizontal component of earth's magnetic field is 0.26 G and the dip angle is 60°. Calculate the vertical component of earth's magnetic field and the net magnetic field of earth at this place.

TANGENT LAW

It is a relation which states the condition of equilibrium of a magnet subjected to two uniform magnetic fields at right angle to each other.



Consider a magnet NS of length $2l$ suspended in two uniform fields, B and B_H, at right angles to each other. Ordinarily field B_H is the horizontal component of earth's magnetic field and B is due to some magnet. Let it come to rest making an angle θ with the lines of force of one of the fields, say B_H. Two couples act on the magnet.

Deflecting Couple (mB – mB)

This couple acts on the magnet due to the magnetic field B. It tries to increase the angle θ and hence the name deflecting couple. Its torque τ_d (clockwise) is given by

$$\tau_d = mB \times NT = mB \times 2l\cos\theta$$

$$\Rightarrow \quad \tau_d = MB\cos\theta \text{ (clockwise)}$$

Restoring Couple (mB_H – mB_H)

This couple acts on the magnet due to the field B_H. It tries to decrease the value of θ and hence the name restoring couple. Its torque τ_r (anti-clockwise) is given by

$$\tau_r = mB_H \times ST = 2mlB_H \sin\theta$$

$$\Rightarrow \quad \tau_r = MB_H \sin\theta \text{ (anti-clockwise)}$$

At equilibrium, we have

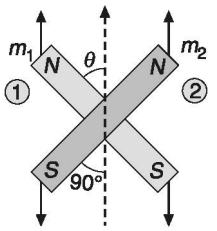
$$\tau_r = \tau_d$$

$$\Rightarrow \quad MB\cos\theta = MB_H \sin\theta$$

$$\Rightarrow \quad B = B_H \tan\theta$$

ILLUSTRATION 30

Two magnets of equal mass are joined at right angles to each other as shown the magnet 1 has a magnetic moment 3 times that of magnet 2. This arrangement is pivoted so that it is free to rotate in the horizontal plane. Calculate the angle which the magnet 1 makes with the magnetic meridian in equilibrium.



SOLUTION

For equilibrium of the system, torques on M_1 and M_2 due to B_H must counter balance each other i.e.,

$$\vec{M}_1 \times \vec{B}_H = \vec{M}_2 \times \vec{B}_H$$

If θ is the angle between M_1 and B_H, then angle between $\vec{M}_2$ and $\vec{B}_H$ is $(90-\theta)$

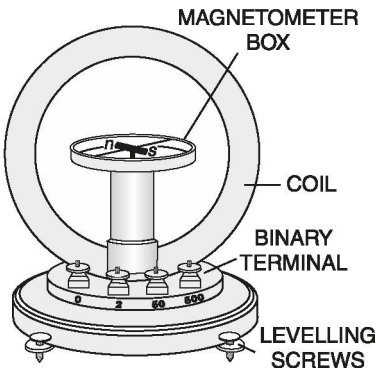
$$\Rightarrow \quad M_1 B_H \sin\theta = M_2 B_H \sin(90-\theta)$$

$$\Rightarrow \quad \tan\theta = \frac{M_2}{M_1} = \frac{M}{3M} = \frac{1}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

TANGENT GALVANOMETER

It is an instrument based on Tangent Law, and used for detection of electric current in a circuit.
The vertical coil of the galvanometer is placed in the magnetic meridian.



When a current is passed through the coil having N turns, radius R, then a magnetic field (B) is produced at right angles to the plane of the coil i.e. at right angles to the horizontal component of earth's magnetic field (B_H).

The magnetic needle of the galvanometer undergoes a deflection θ under the influence of two crossed magnetic fields B and B_H, such that at equilibrium, according to Tangent Law, we have

$$B = B_H \tan\theta$$

Since, $B = \dfrac{\mu_0 NI}{2r}$

$$\Rightarrow \quad \frac{\mu_0 NI}{2r} = B_H \tan\theta$$

$$\Rightarrow \quad I = \frac{2rB_H}{\mu_0 N}\tan\theta$$

$$\Rightarrow \quad I = \frac{B_H}{G}\tan\theta$$

where $G = \frac{\mu_0 N}{2r}$, which is known as the **Galvanometer Constant**.

$$\Rightarrow \quad I \propto \tan\theta$$

Reduction Factor (K)

The quantity $\frac{B_H}{G}$ in the above equation is known as the reduction factor of tangent galvanometer.

$$\Rightarrow \quad K = \frac{B_H}{G} = \frac{2rB_H}{\mu_0 n}$$

$$\Rightarrow \quad I = K\tan\theta$$

If $\theta = 45°$, then $K = I$

Thus, the reduction factor of tangent galvanometer is defined as the amount of current required to produce a deflection of $45°$ in it.

Sensitivity and Accuracy of Tangent Galvanometer

Sensitivity is the measure of change in deflection produced by a unit current. Mathematically it is given by $\frac{d\theta}{dI}$. Since we have

$$I = K\tan\theta$$

Differentiating, $dI = K\sec^2\theta\, d\theta$

$$\Rightarrow \quad \frac{d\theta}{dI} = \frac{1}{K\sec^2\theta} = \frac{1}{K(1+\tan^2\theta)}$$

$$\Rightarrow \quad \frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

A tangent galvanometer is both sensitive and accurate if the change in its deflection is large for a given fractional change in current.

Since, $I = K\tan\theta$

Differentiating, $dI = K\sec^2\theta\, d\theta$

$$\Rightarrow \quad \frac{dI}{I} = \frac{\sec^2\theta\, d\theta}{\tan\theta} = \frac{\cos\theta\, d\theta}{\cos^2\theta\sin\theta} = \frac{d\theta}{\sin\theta\cos\theta}$$

Since $2\sin\theta\cos\theta = \sin(2\theta)$

$$\Rightarrow \quad \frac{dI}{I} = \frac{2d\theta}{\sin 2\theta}$$

$$\Rightarrow \quad d\theta = \frac{\sin 2\theta}{2}\frac{dI}{I}$$

Clearly, $d\theta$ would be maximum if $\sin 2\theta$ is maximum i.e., 1. This is possible if $2\theta = 90°$ or $\theta = 45°$. So, tangent galvanometer has maximum sensitivity when the deflection is $45°$.

ILLUSTRATION 31

The coil of a tangent galvanometer of radius 12 cm is having 200 turns. If the horizontal component of earth's magnetic field is $25\ \mu$T. Find the current which gives a deflection of $60°$.

SOLUTION

Since, $I = K\tan\theta$

where, $K = \frac{2rB_H}{\mu_0 N}$

$$\Rightarrow \quad I = \frac{2rB_H}{\mu_0 N}\tan\theta$$

It is given that

$r = 12\text{ cm} = 0.12\text{ m}$, $N = 200$,

$B_H = 25\ \mu\text{T} = 25\times10^{-6}$ T and $\phi = 60°$

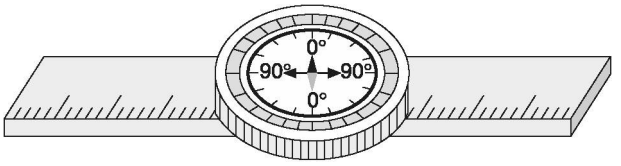
$$\Rightarrow \quad I = \frac{2\times0.12\times25\times10^{-6}\times\tan 60°}{4\pi\times10^{-7}\times200}$$

$$\Rightarrow \quad I = 0.042\text{ A}$$

DEFLECTION MAGNETOMETER

It's working is based on the principle of tangent law.

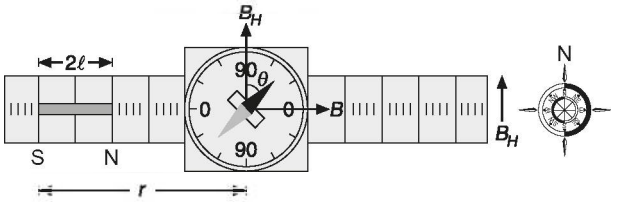
It consists of a small compass needle, pivoted at the centre of a circular box. The box is kept in a wooden frame having two-meter scale fitted on it's two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.



Different position of deflection magnetometer: Deflection magnetometer can be used according to two following positions.

tan *A* Position

For this position, the arms of magnetometer are placed along E-W direction such that magnetic needle is acted upon by only horizontal component of earth's magnetic field (B_H) as shown.



If a bar magnet of magnetic moment M is placed on one arm with its length parallel to arm, then the magnetic needle comes under the influence of two mutual perpendicular magnetic fields

(a) Horizontal component of the earth's magnetic field, B_H and

(b) The axial magnetic field of experimental bar magnet.

At equilibrium, we have $B = B_H \tan\theta$ (Based on Tangent Law)

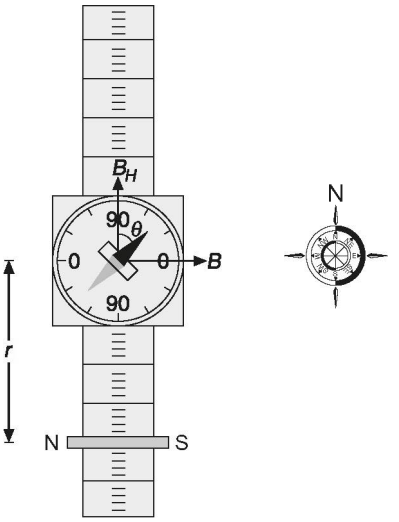
$$B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2 - l^2\right)^2}$$

where r is the distance of needle from centre of magnet and $2l$ is the length of magnet.
If the magnet is small, then we have

$$\Rightarrow \quad B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

tan *B* Position

Arms of magnetometer are placed along N-S direction such that magnetic needle align itself in the direction of earth's magnetic field (*i.e.* B_H) as shown.



If a bar magnet of magnetic moment M is placed on one arm with its length perpendicular to arm, so that the magnetic needle comes under the influence of two mutual perpendicular magnetic fields

(a) Horizontal component of the earth's magnetic field, B_H and

(b) The equatorial magnetic field of experimental bar magnet.

At equilibrium, we have $B = B_H \tan\theta$ (Based on Tangent Law)

$$\Rightarrow \quad B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2 + l^2\right)^{3/2}}$$

where r is the distance of needle from centre of magnet and $2l$ is the length of magnet.
If the magnet is small, then we have

$$B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

Problem Solving Technique(s)

Deflection magnetometer also used to compare the magnetic moments either by deflection method or by null deflection method.

(a) By Deflection method, we have $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$

(b) By Null deflection method, we have $\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$

where d_1 and d_2 are the position of two bar magnets placed simultaneously on each arm.

ILLUSTRATION 32

The needle of a deflection galvanometer shows a deflection of $60°$ due to a short bar magnet at a certain distance in $\tan A$ position. Find the deflection when the distance is doubled.

SOLUTION

For short bar magnet in $\tan A$-position

$$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H \tan\theta \qquad \ldots(1)$$

When distance is doubled, then new deflection θ' is given by

$$\frac{\mu_0}{4\pi}\frac{2M}{(2d)^3} = B_H \tan\theta' \qquad \ldots(2)$$

Dividing (2) by (1), we get

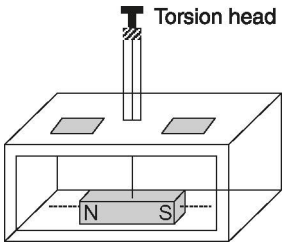
$$\frac{\tan\theta'}{\tan\theta} = \frac{1}{8}$$

$$\Rightarrow \quad \tan\theta' = \frac{\tan\theta}{8} = \frac{\tan 60°}{8} = \frac{\sqrt{3}}{8}$$

$$\Rightarrow \quad \theta' = \tan^{-1}\left(\frac{\sqrt{3}}{8}\right)$$

VIBRATION MAGNETOMETER

Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle, that whenever a freely suspended magnet in a uniform magnetic field, is disturbed from its equilibrium position, it starts vibrating about the mean position.



Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

where, I is the moment of inertia of short bar magnet of mass w and is given by $I = \frac{wL^2}{12}$.

TIME PERIOD OF A MAGNETIC NEEDLE IN EARTH'S MAGNETIC FIELD

Let a small magnetic needle of moment M be in earth field. When it is given a displacement such that it makes an angle θ with the magnetic field B_H, then restoring couple on needle is

$$\tau = -mB_H(2l\sin\theta)$$

$$\Rightarrow \quad \tau = -[m(2l)]B_H\sin\theta = -MB_H\sin\theta$$

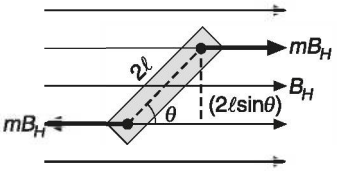
If I is moment of inertia of needle and $\alpha\left(=\ddot{\theta}\right)$ its angular acceleration, then

$$I\alpha = -MB_H\sin\theta$$

For small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB_H}{I}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} = -\left(\frac{MB_H}{I}\right)\theta \qquad \left\{\because \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}\right\}$$



This is condition of angular SHM.

Since, $T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Conceptual Note(s)

When a dip needle oscillates in a vertical plane in the magnetic meridian, it oscillates under the action of total intensity B_e, of earth's field, but if the needle oscillates in a vertical plane at right angles to the magnetic meridian (i.e., vertical E-W plane) it oscillates under the action of vertical component V only, when it oscillates in horizontal plane it oscillates under horizontal component H.

USES OF VIBRATION MAGNETOMETER

Determination of Magnetic Moment of a Magnet

The given magnet is put into vibration magnetometer and its time period T is determined.

Now $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$$\Rightarrow \quad M = \frac{4\pi^2 I}{B_H T^2}$$

The moment of inertia I can be determined by the geometry of magnet.

Comparison of Horizontal Components of Earth's Magnetic Field at Two Places

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Since, I and M of the magnet are constant, so

$$T^2 \propto \frac{1}{B_H}$$

$$\Rightarrow \quad \frac{B_{H_1}}{B_{H_2}} = \frac{T_2^2}{T_1^2}$$

Comparison of Magnetic Moments of Two Magnets of Same Size and Same Mass

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

where I and B_H are constants

$$\Rightarrow \quad M \propto \frac{1}{T^2}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

If two magnets have same magnetic length then

$$2ml \propto \frac{1}{T^2}$$

$$\Rightarrow \quad m \propto \frac{1}{T^2} \qquad \{\because M = m(2l)\}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{T_2^2}{T_1^2}$$

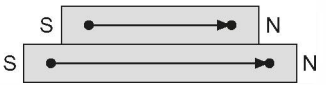
Comparison of Magnetic Moments of Two Magnets of Unequal Sizes and Masses

(a) When magnetic moments of two magnets align. Then

$$I = I_1 + I_2$$

$$M = M_1 + M_2$$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}}$$

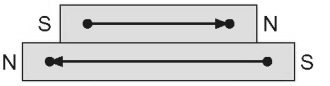


(b) When magnetic moments of two magnets do not align. Then

$$I = I_1 + I_2$$

$$M = M_1 - M_2$$

$$\Rightarrow \quad T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)H}}$$



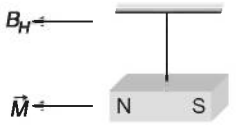
$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

So, by calculating the time periods in the two cases M_1 and M_2 can be compared.

To Find the Ratio of Magnetic Field with the Horizontal Component of Earth's Field

Suppose it is required to find the ratio $\dfrac{B}{B_H}$ where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.



To determine $\dfrac{B}{B_H}$ a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and it's time period of oscillation (T) is noted.

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

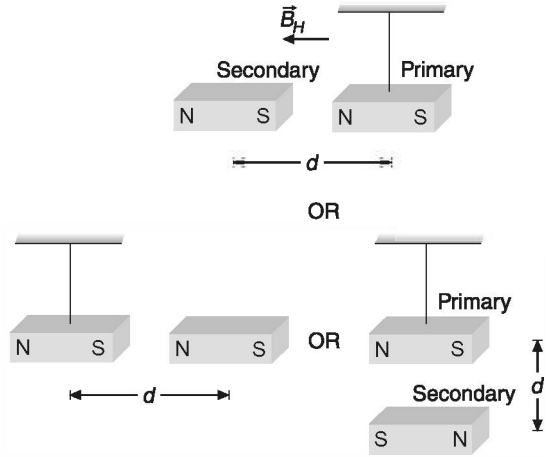
and frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

Now a secondary magnet placed near the primary magnet so primary magnet oscillate in a new field with is the resultant of B and B_H and now time period, is noted again.

There are two important possibilities for placing secondary magnet.

Possibility 1:

New field increases so time period of oscillation of primary magnet decreases.



Now time period $T = 2\pi\sqrt{\frac{I}{M(B+B_H)}}$

or new frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{M(M+B_H)}{I}}$

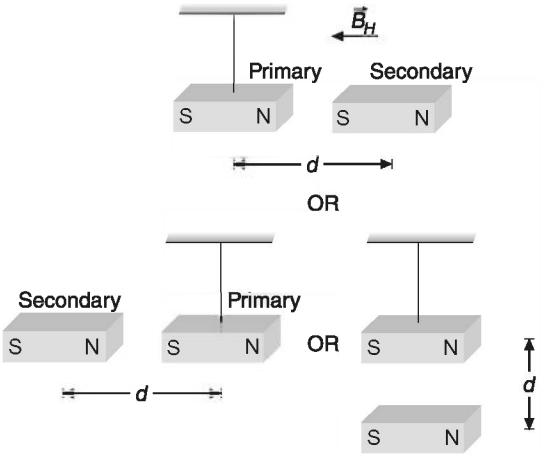
Also $\frac{\nu'}{\nu} = \sqrt{\frac{B+B_H}{B_H}}$

$\Rightarrow \quad \left(\frac{\nu'}{\nu}\right)^2 = \frac{B}{B_1} + 1$

$\Rightarrow \quad \frac{B}{B_H} = \left(\frac{\nu'}{\nu}\right)^2 - 1$

Possibility 2:

Net field decreases so time period of oscillation of primary magnet increases.



$$T = 2\pi\sqrt{\frac{I}{M(B_H - B)}}\,(B_H > B)$$

and $\nu' = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$

Also $\frac{\nu'}{\nu} = \sqrt{\frac{B_H - B}{B_H}}$

$\Rightarrow \quad \left(\frac{\nu'}{\nu}\right)^2 = 1 - \left(\frac{B}{B_1}\right)$

$\Rightarrow \quad \frac{B}{B_H} = 1 - \left(\frac{\nu'}{\nu}\right)^2$

Conceptual Note(s)

If a rectangular bar magnet is cut in n equal parts then time period of each part will be $\frac{1}{\sqrt{n}}$ times that of complete magnet (i.e., $T = \frac{T}{\sqrt{n}}$) while for short magnet $T = \frac{T}{n}$. If nothing is said then bar magnet is treated as short magnet.

ILLUSTRATION 33

A freely suspended magnet oscillates with period T in earth's horizontal magnetic field. When a bar magnet is brought near it, the period decreases to $\frac{T}{2}$. Calculate the ratio of the field of the magnet B to the earth's magnetic field (B_H).

SOLUTION

For freely suspended magnet of magnetic moment M oscillating in earth's magnetic field

$$T = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots(1)$$

Let bar magnet produce magnetic field F in the vicinity of the oscillations of magnet.

As time period decreases, the net magnetic field must increase, so that magnetic field is $(B_H = B)$

$$\Rightarrow \quad \frac{T}{2} = 2\pi\sqrt{\frac{I}{M(B_H + B)}} \quad \ldots(2)$$

Dividing (1) by (2)

$$2 = \sqrt{\frac{B_H + B}{H}}$$

$$\Rightarrow \quad 4 = 1 + \frac{B}{B_H}$$

$$\Rightarrow \quad \frac{B}{B_H} = 3$$

ILLUSTRATION 34

A dip needle vibrates in the vertical plane perpendicular to the magnetic meridian. The time period of vibration is found to be 2 s. The same needle is then allowed to vibrate in the horizontal plane and the time period is again found to be 2 s. Calculate the angle of dip.

SOLUTION

In vertical plane perpendicular to magnetic meridian, we have

$$T = 2\pi\sqrt{\frac{I}{MB_V}} \quad \ldots(1)$$

In horizontal plane, we have

$$T' = 2\pi\sqrt{\frac{I}{MB_H}} \quad \ldots(2)$$

Since $T' = T$, so from (1) and (2), we get

$$B_V = B_H$$

$$\Rightarrow \quad \tan\phi = \frac{B_V}{B_H}$$

$$\Rightarrow \quad \tan\phi = 1$$

$$\Rightarrow \quad \phi = 45^\circ$$

ILLUSTRATION 35

A small bar magnet having a magnetic moment of 9×10^{-3} Am^2 is suspended at its centre of gravity by a light torsion less string at a distance of 10^{-2} m vertically above a long, straight horizontal wire carrying a current of 1.0 A from east to west. Find the frequency of oscillation of the magnet about its equilibrium position. The moment of inertia of the magnet is 6×10^{-9} kgm^2 and horizontal component of earth's magnetic field is 3×10^{-5} T.

SOLUTION

The magnetic moment of the bar magnet is

$$M = 9\times10^{-9} \text{ Am}^2$$

The magnitude of the magnetic field at the location of the magnet due to current carrying wire is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(2\times10^{-7})(1.0)}{10^{-2}}$$

$$\Rightarrow \quad B = 2\times10^{-5} \text{ Wbm}^{-2}, \text{ from } S \text{ to } N$$

The earth's horizontal magnetic field is,

$$B_H = 3\times10^{-5} \text{ T} = 3\times10^{-5} \text{ Wbm}^{-2}, \text{ from } S \text{ to } N$$

$$\Rightarrow \quad B_{\text{net}} = B + B_H = 5\times10^{-5} \text{ Wbm}^{-2}$$

The frequency of oscillation will be,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{M(B + B_H)}{I}}$$

where I is moment of inertia of the magnet

$$\Rightarrow \quad \nu = \frac{1}{2\pi}\sqrt{\frac{(9\times10^{-9})\times(5\times10^{-5})}{6\times10^{-9}}}$$

$$\Rightarrow \quad \nu = 1.38\times10^{-3} \text{ Hz}$$

Test Your Concepts-III

Based on Tangent Law, Tangent Galvanometer and Vibration Magnetometer

(Solutions on page H.110)

1. A magnet performs 15 oscillations per minute in a horizontal plane, where angle of dip is 60° and earth's total field is 0.5 G. At another place, where total field is 0.6 G, the magnet performs 20 oscillations per minute. What is the angle of dip at this place?
2. For what deflection, error in measuring the current using a tangent galvanometer is minimum?
3. A bar magnet of length 5 cm, width 3 cm and height 2 cm takes 5 s to complete an oscillation in vibration magnetometer placed in a horizontal magnetic field of 20 μT. The mass of this bar magnet is 250 g.
   (a) Find the magnetic moment of the magnet.
   (b) If the magnet is put in the magnetometer with its 0.5 cm edge horizontal, what would be the new time period?
4. A magnet is suspended in such a way that it oscillates in the horizontal plane. If it makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. Calculate the ratio of total earth's magnetic field at the two places.
5. Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produced deflections of 60° and 45° respectively. Calculate the ratio of the number of turns in the coils.
6. In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31°. By what percentage, the current has to be increased, so as to produce a deflection of 42°?
7. A compass needle placed at a distance r from a short magnet in tan$A$ position shows a deflection of 60°. If the distance is increased to $r(3)^{\frac{1}{3}}$, then calculate the deflection of the compass needle.
8. The time period of vibration of two magnets in sum position (magnets placed with similar poles on one side one above the other) is 3 s. When polarity of weaker magnet is reversed the combination makes 12 oscillations per minute. What is the ratio of magnetic moments of two magnets?
9. The ratio of magnetic moments of two bar magnet is 13 : 5. These magnets are held together in a vibration magnetometer are allowed to oscillate in earth's magnetic field with like poles together 15 scillations per minutes are made. Calculate the frequency of oscillations of system if unlike poles are together.

MAGNETIC PROPERTIES OF MATERIALS

Magnetic Induction (B)

Magnetic induction inside a magnetic substance is the number of magnetic lines of force crossing a unit area normal to their direction. It is also called as magnetic flux density (MFD). Its SI unit is tesla or weber/m^2 and cgs unit is gauss.

$$1\ \text{T} = 1\ \text{Wbm}^{-2} = 10^4\ \text{G}$$

INTENSITY OF MAGNETISATION ($\vec{I}$)

When a material is placed in a magnetising field, it acquires magnetic moment M. The intensity of magnetisation is defined as the magnetic moment per unit volume (V).

$$\Rightarrow \quad I = \frac{M}{V}$$

If the material is in the form of a bar magnet of cross-sectional area A, length $2l$ and pole strength m, then $M = m(2l)$ and $V = A(2l)$

$$\Rightarrow \quad I = \frac{M}{V} = \frac{m(2l)}{A(2l)} = \frac{m}{A}$$

So, the intensity of magnetisation may also be defined as the pole strength per unit cross-sectional area.
Its SI unit is amperemetre^{-1} $(= \text{Am}^{-1})$.

It is a vector quantity. The magnetic field due to the magnetisation of the material (B_m) is proportional to the intensity of magnetisation (I) i.e. $B_m \propto I$

$$\Rightarrow \quad B_m = \mu_0 I$$

MAGNETISING FIELD ($\vec{H}$)

It is the degree or extent to which a magnetic field can magnetise a substance. In order to magnetise a material, it has to be kept in an external field B. So, it is defined as

$$H = \frac{B}{\mu_0} - I$$

In vacuum or in the absence of any material, $I = 0$, so we have

$$H = \frac{B}{\mu_0} \text{ (in vacuum)}$$

It's SI unit is amperemetre^{-1} $(= \text{Am}^{-1})$

It's CGS unit is oersted and 1 oersted $= 80\ \text{Am}^{-1}$

For a solenoid having n turns per unit length, carrying a current i, we have $B = \mu_0 ni$

$$\Rightarrow \quad H = \frac{B}{\mu_0} = ni$$

So, the magnetising field is independent of the material of the core of the solenoid.
It is a vector quantity.

Conceptual Note(s)

(a) The Unit of intensity of magnetisation is ampere/metre $\left(\text{Am}^{-1}\right)$ and its dimensional formula is $M^0L^{-1}T^0A^1$.

(b) $\vec{I}$ is a vector quantity whose direction is along the magnetic field.

(c) In paramagnetic and ferromagnetic materials its direction is in the direction of H and in diamagnetic materials it is opposite to that of H.

(d) I is produced in materials due to spin motion of electrons.

(e) The value of I and its direction in a material depend on the nature of that material.

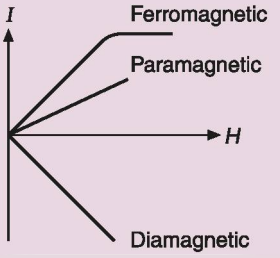
(f) The value of $\vec{I}$ depends on temperature.

(g) $\vec{I}$ is produced on account of induction in a material. For low magnetising field $I \propto H$.

$\Rightarrow \quad I \propto H$

$\Rightarrow \quad I = \chi_m H$

(i) I-H curve



MAGNETIC SUSCEPTIBILITY (χ_m)

It is the property of substance which tells how easily a substance can be magnetized.

The magnetic susceptibility is defined as the intensity of magnetisation per unit magnetising field or it is defined as the ratio of I to H, so mathematically,

$$\chi_m = \frac{I}{H}$$

When $H = 1$ oersted then $\chi_m = I$.

So, the intensity of magnetisation induced in a material by unit magnetising field is defined as magnetic susceptibility.
It is a scalar quantity and has no dimensional formula.

Problem Solving Technique(s)

(a) χ_m has no unit and is dimensionless.

(b) χ_m is a measure of ease with which a material can be magnetised by a magnetising field (H).

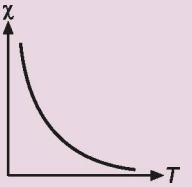
(c) Magnetic susceptibility of various materials

(i) For diamagnetic materials-χ_m is low and negative.

(ii) For paramagnetic materials-χ_m is low but positive.

(iii) For ferromagnetic materials-χ_m is high and positive.

(d) For paramagnetic substances it is inversely proportional to temperature i.e. $\chi_m \propto \frac{1}{T}$



(e) For low magnetising field the value of χ_m is constant.

MAGNETIC PERMEABILITY

The magnetic permeability of a material is the measure of degree to which the material can be permeated by a magnetic field and is defined as the ratio of magnetic induction (B) in the material to the magnetising field (H).

$$\Rightarrow \quad \mu = \frac{B}{H}$$

In other words, the extent to which magnetic lines of force can enter a medium is known as **magnetic permeability** of that medium. It is the characteristic property of a magnetic material because it represents the amplification of magnetising field in that material. It is always positive and is different for different materials.

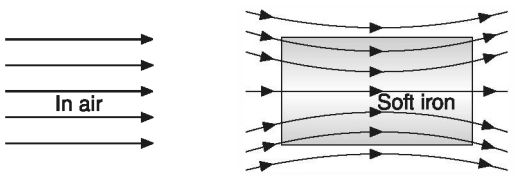
RELATIVE PERMEABILITY (μ_r)

The ratio of magnetic permeability of medium (μ) to the magnetic permeability of free space (μ_0) is defined as **relative permeability** (μ_r). So,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0} = \frac{\text{Magnetic flux density in material}}{\text{Magnetic flux density in vacuum}}$$

$\mu = \mu_0\mu_r$, where μ_0 is the absolute permeability of air or free space is $4\pi \times 10^{-7}$ TmA^{-1}

Permeability is the characteristic of a medium which allows magnetic flux to pass through the medium e.g. permeability of soft iron is 1000 times greater than that of air.



So, μ_r for soft iron is 1000

Conceptual Note(s)

(a) $\mu_r = \dfrac{\left(\begin{array}{c}\text{Number of magnetic lines of force}\\ \text{passing through unit area in medium}\end{array}\right)}{\left(\begin{array}{c}\text{Number of magnetic lines of force}\\ \text{passing through unit area in vacuum}\end{array}\right)}$

(b) $\mu_r = \dfrac{\text{Magnetic flux density in material}}{\text{Magnetic flux density in vacuum}}$

(c) The limit upto which a magnetic field penetrates matter, is known as relative permeability of that material.

(d) It has no units and is dimensionless.

(e) For Diamagnetic substances, $\mu_r < 1$

For Paramagnetic substances, $\mu_r > 1$

For Ferromagnetic substances, $\mu_r \gg 1$

RELATION BETWEEN MAGNETIC SUSCEPTIBILITY (χ_m) AND RELATIVE PERMEABILITY (μ_r)

When a magnetic material is placed in a magnetic field B_0, then the material gets magnetised and produces a magnetic field B_m of its own inside it. Due to this the total magnetic field in the material is

$$B = B_0 + B_m$$

where

$$B = \mu H, \; B_0 = \mu_0 H \text{ and } B_m = \mu_0 I$$

$$\Rightarrow \quad B = \mu_0 H + \mu_0 I$$

$$\Rightarrow \quad B = \mu_0 H\left(1 + \frac{I}{H}\right) = \mu_0 H(1 + \chi_m)$$

$$\Rightarrow \quad \frac{B}{H} = \mu_0(1 + \chi_m)$$

Since $\dfrac{B}{H} = \mu$

$$\Rightarrow \quad \mu = \mu_0(1 + \chi_m) \qquad \ldots(1)$$

Also, we know that $\dfrac{\mu}{\mu_0} = \mu_r$

So, from equation (1), the relative magnetic permeability is given by

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$$

Conceptual Note(s)

Comparison Table of Relative permeability and susceptibility of various substances

MATERIAL	μ_r	χ_m
Diamagnetic	slightly less than unity	constant, small and negative
Paramagnetic	slightly more than unity	constant, small and positive
Ferromagnetic	much greater than unity ($\sim 10^3$)	variable, large and positive

ILLUSTRATION 36

The magnetising field of 20 CGS units produces a flux of 2400 CGS units in a bar of iron of cross-section 0.2 cm^2. Calculate the

(a) permeability and
(b) susceptibility of the bar.

SOLUTION

Since Magnetic field,

$$B = \frac{\phi}{A} = \frac{2400 \times 10^{-8}}{0.2 \times 10^{-4}} = 1.20 \text{ Wbm}^{-2}$$

(a) The permeability of the bar material is

$$\mu = \frac{B}{H} = \frac{1.2}{\frac{20}{4\pi \times 10^{-3}}} = 7.54 \times 10^{-4} \text{ Hm}^{-1}$$

(b) The magnetic susceptibility and permeability of a material are related with each other as,

$$\mu = \mu_0 (1 + \chi_m)$$

$$\Rightarrow \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.54 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 599$$

MAGNETIC SHIELDING PROCESS

The process of protecting any apparatus from the effect of earth's magnetic field is known as **magnetic shielding**.

When an iron box is placed in a magnetic field then the value of magnetic field inside the box is zero, whereas if a magnet is enclosed in an iron box then magnetic field outside the box is zero. So, to protect any instrument from external magnetic field, it is put inside an iron box. This is why, passengers travelling by aeroplane are allowed to carry magnets inside iron box only.

MAGNETOMOTIVE FORCE (F_M)

The line integral of magnetising field around a closed path in a magnetic field is defined as magnetomotive force. So

$$F_m = \oint \vec{H} \cdot d\vec{l} = \frac{1}{\mu} \oint \vec{B} \cdot d\vec{l} = \Sigma I$$

Its unit is ampere turns. Its value around the closed path is equal to the algebraic sum of electric currents enclosed by the closed path.

ILLUSTRATION 37

An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then find the magnetic moment of the rod.

SOLUTION

Since, we have, $\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m = 1 + \frac{I}{H}$

$$\Rightarrow \quad I = (\mu_r - 1)H$$

For a solenoid having n turns per unit length and current i, we have

$$H = ni$$

$$\Rightarrow \quad I = (\mu_r - 1)ni$$

$$\Rightarrow \quad I = (1000 - 1) \times 500 \times 0.5$$

$$\Rightarrow \quad I = 2.5 \times 10^5 \text{ Am}^{-1}$$

Since, magnetic moment is $M = IV$

$$\Rightarrow \quad M = 2.5 \times 10^5 \times 10^{-4}$$

$$\Rightarrow \quad M = 25 \text{ Am}^2$$

ILLUSTRATION 38

The space within a current carrying solenoid is filled with magnesium having magnetic susceptibility $\chi = 1.2 \times 10^{-5}$. Calculate the percentage increase in magnetic field.

SOLUTION

Magnetic field without magnesium is

$$B_0 = \mu_0 H$$

Magnetic field with magnesium is

$$B = \mu H = \mu_0 (1+\chi) H$$

Fractional increase in magnetic field is given by

$$\frac{\Delta B}{B_0} = \frac{B - B_0}{B_0}$$

$$\Rightarrow \quad \frac{\Delta B}{B_0} = \frac{\mu_0 (1+\chi) H - \mu_0 H}{\mu_0 H} = \chi$$

Percentage increase is

$$\frac{\Delta B}{B_0} \times 100\% = \chi_{Mg} \times 100\% = 1.2 \times 10^{-5} \times 100$$

$$\Rightarrow \quad \frac{\Delta B}{B_0} \times 100\% = 1.2 \times 10^{-3}$$

ILLUSTRATION 39

A solenoid having 2000 turns/m has a core of a material with relative permeability 220. The area of core is 4 cm^2 and carries a current of 5 A. Calculate

(a) magnetic intensity
(b) magnetic field
(c) magnetisation (I) of the core. Also calculate the pole strength developed.

SOLUTION

Given that $n = 2000$ turns, $I = 5$ A, $\mu_r = 220$ and $A = 4$ cm^2

(a) Magnetic intensity

$$H = nI = 2000 \times 5 = 10000 \text{ Am}^{-1}$$

(b) Magnetic field, $B = \mu H = \mu_0 \mu_r H$

$$\Rightarrow \quad B = 4\pi \times 10^{-7} \times 220 \times 10,000 = 88\pi \times 10^{-2} \text{ T}$$

(c) As we know, $B = B_0 (H + I)$

$$\Rightarrow \quad 88\pi \times 10^{-2} = 4\pi \times 10^{-7} (10000 + I)$$

$$\Rightarrow \quad 2.20 \times 10^6 = 10^4 + I$$

$$\Rightarrow \quad I = \left(2.20 \times 10^6 - 10^4\right) = 2.19 \times 10^6 \text{ Am}^{-1}$$

(d) Pole strength,

$$m = IA = \left(2.19 \times 10^6\right)\left(4 \times 10^{-4}\right) = 876 \text{ Am}$$

ILLUSTRATION 40

Consider a bar magnet having pole strength 2 Am, magnetic length 4 cm and area of cross-section 1 cm^2. Find

(a) the magnetisation I
(b) the magnetic intensity H and
(c) the magnetic field at the centre of magnet.

SOLUTION

Given that $m = 2$ Am, $2l = 4$ cm and $A = 1$ cm^2

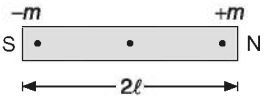
(a) Magnetisation

$$I = \frac{M}{V} = \frac{m}{A}\frac{(2l)}{(2l)} = \frac{m}{A} = \frac{2}{1 \times 10^{-4}}$$

$$\Rightarrow \quad I = 2 \times 10^4 \text{ Am}^{-1}$$

The direction will be from S to N-pole

(b)



At centre, $H_N = \dfrac{\frac{\mu_0}{4\pi}\frac{m}{l^2}}{\mu_0} = \dfrac{m}{4\pi l^2}$, along north pole

$$H_S = \frac{m}{4\pi l^2}, \text{ along south pole}$$

$$H = H_N + H_S = \frac{m}{2\pi l^2} = \frac{2}{2\pi\left(2 \times 10^{-2}\right)^2}$$

$$\Rightarrow \quad H = \frac{1}{4\pi} \times 10^4 \text{ Am}^{-1}, \text{ along south pole}$$

(c) Magnetic field at the centre of magnet, $B = \mu_0 (H + I)$

$$\Rightarrow \quad B = 4\pi \times 10^{-7}\left(-\frac{1}{4\pi} \times 10^4 + 2 \times 10^4\right)$$

$$\Rightarrow \quad B = -10^{-3} + 8\pi \times 10^{-3} = (8\pi - 1) \times 10^{-3}$$

$$\Rightarrow \quad B = 2.4 \times 10^{-2} \text{ T}, \text{ along north pole}$$

MAGNETISATION

The phenomenon of magnetising an unmagnetised substance by the process of magnetic induction is defined as **magnetisation** or it is the phenomenon of increasing the pole strength of a magnet.

DEMAGNETISATION

The phenomenon of decreasing or spoiling magnetic strength of a material is known as **demagnetisation**. It occurs due to mechanical jerks, thermal changes and temporal variations. When a magnet is beaten with a hammer then its magnetism gets spoiled. When a magnet is heated and then cooled then too its magnetism is spoiled. Keeping a magnet at a place for a long time also spoils its magnetism.

MAGNETIC SATURATION

The state of a material after which the increase in its magnetic strength stops is known as **magnetic saturation**.

CLASSIFICATION OF MAGNETIC MATERIALS

The root cause of magnetism in matter is the motion of electric charges. The motion of electrons and protons in atoms is responsible for their magnetic properties. The variation in the number of fundamental charged particles and variation in their arrangement in different materials are responsible for differences in their magnetic properties.

Curie and Faraday observed that almost all substances have certain magnetic properties. On the basis of mutual interactions or magnetic behaviour of various materials in an external magnetic field, the materials are divided in three main categories.

(a) Diamagnetic Substances
(b) Paramagnetic Substances
(c) Ferromagnetic Substances

MAGNETIC PROPERTIES OF MATERIALS

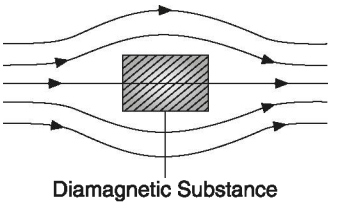
Diamagnetic Materials

Diamagnetic materials are repelled by magnetic field. In other words, as we know that a bar magnet attracts metals like iron, however, it is observed that a bar magnet will repel a diamagnetic material. Following are the properties of diamagnetic materials.

(a) Diamagnetic materials have a tendency to move from the stronger to weaker part of external magnetic field.

(b) They develop this tendency because they are feebly magnetized in a direction opposite to that of external magnetising field.

(c) Some of diamagnetic materials are copper, lead, bismuth, silicon, nitrogen (at STP), water and sodium chloride.



(d) The magnetic field lines are expelled by these materials.

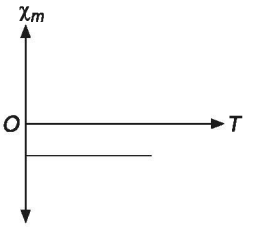
(e) Magnetic field inside diamagnetic material (B) is less than in free space (B_0), therefore for a diamagnetic material, we have

$$\frac{B}{B_0}<1,\ \frac{\mu}{\mu_0}<1,\ \mu_r<1.$$

(f) Relative permeability of diamagnetic material is less than one.

(g) Since $\mu_r=(1+\chi_m)$ and $\mu_r<1$, for diamagnetic material, so χ_m is negative and small for a diamagnetic material.

(h) Magnetic susceptibility χ_m of diamagnetic material is independent of temperature.



(i) Diamagnetism is a universal property i.e. it is present in all substances. However, the effect is so weak in most cases that it gets dominated by other effects like paramagnetism, ferromagnetism etc.

EXPLANATION OF DIAMAGNETISM

Diamagnetism is the intrinsic property of every material and it is generated due to mutual interaction between the applied magnetic field and orbital motion of electrons.

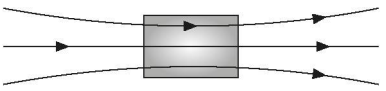
Electrons in an atom orbiting around nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are those in which net magnetic dipole moment of an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in opposite direction speed up. This is due to induced current in accordance with Lenz's law. Thus, the substance develops a net magnetic moment opposite to applied field due to which it is repelled from stronger field to weaker field.

Conceptual Note(s)

(a) In these materials the electron number is even and every two electrons get coupled.
(b) In these materials all the orbitals or atoms are completely filled.
(c) The resultant magnetic moment is zero for diamagnetic substances.
(d) Although diamagnetism is an inherent property of all materials, even then due to other properties like paramagnetism and ferromagnetism being much stronger, the property of diamagnetism is suppressed.
(e) Diamagnetism is the result of small variations in the velocity of electrons moving in atomic orbits.

Paramagnetic Materials

(a) These materials have tendency to move from region of weak magnetic field to strong magnetic field i.e. they get weakly attracted to a magnet.
(b) This is because these substances get feebly magnetized in the direction of applied external magnetic field.
(c) Examples of some paramagnetic substances are aluminum, sodium, calcium, oxygen (at STP) and copper chloride.
(d) Magnetic field lines tend to pass through these substances therefore magnetic field inside substance is more than the outside.

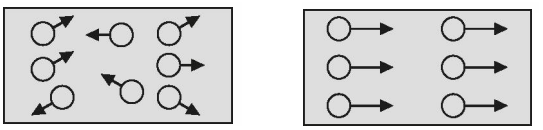


(e) $B > B_0$, $\frac{B}{B_0} > 1$, $\mu > \mu_0$, $\frac{\mu}{\mu_0} > 1$, $\mu_r > 1$
(f) The relative permeability of paramagnetic substances is greater than one.
(g) Since $\mu_r = 1 + \chi_m$, so χ_m is positive. (The magnetic susceptibility of paramagnetic substance is small and positive).

Explanation of Paramagnetism

In paramagnetic substances, vector sum of orbital magnetic moment of electrons is not zero, therefore, each atom behaves like tiny magnetic dipole and has some finite dipole moment. Due to thermal agitations the atomic dipoles in substance are randomly oriented hence total net dipole moment becomes zero. In paramagnetic substances, the inner orbits of atoms are incomplete.

Since the electron spins are uncoupled, consequently on applying a magnetic field the magnetic moment generated due to spin motion align in the direction of magnetic field and induces magnetic moment in its direction due to which the material gets feebly magnetised. In these materials the electron number is odd.



(a) When no field is applied (b) On application of field (B)

NOTE: The magnetic susceptibility of paramagnetic substances is around hundred times higher than that of diamagnetic substances.

CURIE LAW AND CURIE TEMPERATURE

This law states that the intensity of magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T.

$$I \propto \frac{B_0}{T}$$

$$\Rightarrow \quad I = \frac{CB_0}{T}$$

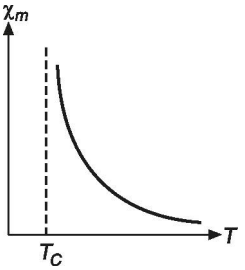
Dividing both sides of this equation by H, we get

$$\Rightarrow \quad \frac{I}{H} = \frac{C(B_0/H)}{T}$$

Since $\chi_m = \frac{I}{H}$ and $\frac{B_0}{H} = \mu_0$

$$\Rightarrow \quad \chi_m = \frac{C\mu_0}{T}$$

where C is called the Curie's constant.



Thus, for a paramagnetic substance both χ_m and μ_r depend not only on the material but also on the sample temperature. As the field is increased or the temperature is lowered, then the magnetisation increases until it reaches the saturation value (m_s) and at this instant all atomic dipoles are aligned with the applied field. Beyond this Curie's Law is not applicable.

Ferromagnetic Substances

(a) These are the substances which get strongly magnetized when placed in an external magnetic field.
(b) They have strong tendency to move from a region of weak magnetic field to strong magnetic field.
(c) They get strongly attracted to the magnet.
(d) Some of the ferromagnetic substances are as follows: iron, cobalt, nickel, alloys like alnico etc.
(e) Magnetic field lines tend to crowd into ferromagnetic material.
(f) Magnetic susceptibility χ_m of ferromagnetic substance is very high, therefore, they can be magnetized easily and strongly.
(g) With rise in temperature, susceptibility of ferromagnetic materials decreases. At a certain temperature ferromagnetic substance is converted into paramagnetic substance. This transition temperature is called Curie temperature of Curie point T_C.

ILLUSTRATION 41

A solenoid having 5000 turns/m carries a current of 2 A. An aluminium ring at temperature 300 K inside the solenoid provides the core.

(a) If the magnetisation I is 2×10^{-2} Am^{-1}, find the susceptibility of aluminium at 300 K.
(b) If temperature of the aluminium ring is 320 K, what will be the magnetisation?

SOLUTION

(a) $H = I = 5000\times2 = 10^4$ Am^{-1}

Since, $I = \chi H$

$$\Rightarrow \quad \chi = \frac{I}{H} = \frac{2\times10^{-2}}{10^4} = 2\times10^{-6}$$

(b) According to Curie Law, we have

$$\chi = \frac{C}{T} \Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad \chi_2 = \frac{T_2}{T_1}\chi_1 = \frac{320}{300}\times2\times10^{-6} = 2.13\times10^{-6}$$

Magnetisation at 320 K

$$I = \chi_2 H = 2.13\times10^{-6}\times10^4 = 2.13\times10^{-2} \text{ Am}^{-1}$$

CURIE-WEISS LAW

At temperature above the Curie temperature, a ferromagnetic substance becomes an ordinary paramagnetic substance whose magnetic susceptibility obeys the **Curie-Weiss law.** At temperatures above Curie Temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_c)$ i.e.

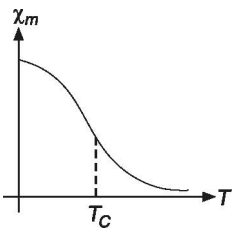
$$\chi \propto \frac{1}{T - T_C}$$

$$\Rightarrow \quad \chi = \frac{C}{(T - T_C)}$$

where T_c = Curie Temperature

according to which

$$\chi_m = \frac{C}{T - T_C}$$

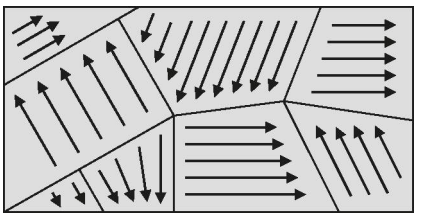


EXPLANATION OF FERROMAGNETISM

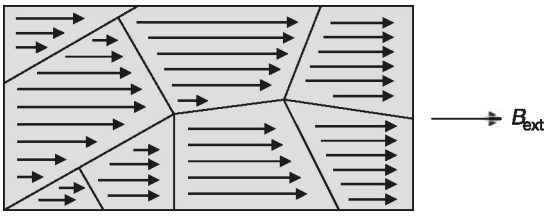
The individual atom in a ferromagnetic substance has net dipole moment. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Each domain has net magnetisation but magnetisation of the whole sample is zero (in absence of an external field).

At ordinary temperature, every piece of iron is not a magnet because at this temperature the distribution of domains is random as a result of which the resultant magnetic moment is zero.

So, every ferromagnetic material is made of a very large number of miniature regions called as **domains**. The linear dimension of domains ranges from 10^{-2} m to 10^{-5} m. Each domain contains 10^{17} to 10^{21} atoms whose axes are aligned in the same direction. All spin magnetic moments are in the same direction in a particular domain but it is different than that in any other domain.
At ordinary temperatures these domains do not align, rather these are scattered randomly as shown.



When external field B_{ext} is applied, the domains the domains orient themselves in the direction of B_{ext} and simultaneously the domains oriented in direction of B_{ext} grow in size as shown.



Curie Temperature (T_C)

The temperature above which a ferromagnetic material behaves like a paramagnetic material is defined as **Curie Temperature** (T_c).

OR

The minimum temperature at which a ferromagnetic substance is converted into paramagnetic substance is defined as **Curie Temperature**.
For various ferromagnetic materials its values are different. e.g. for Ni, $T_{C_{Ni}} = 358°C$

for Fe, $T_{C_{Fe}} = 770°C$

for CO, $T_{C_{CO}} = 1120°C$

At this temperature the ferromagnetism of the substances suddenly vanishes.

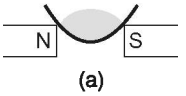
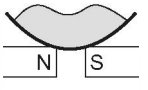
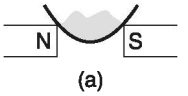
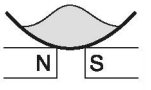
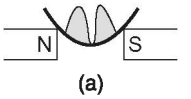
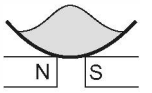
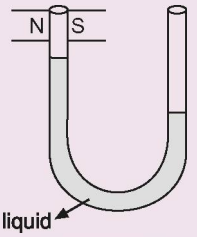
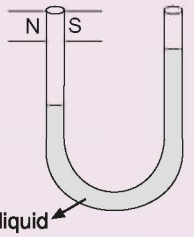
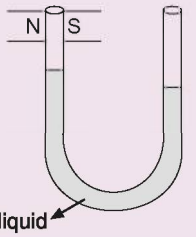
Materials and their Curie Temperature

Material	$T_C(K)$
Cobalt	1394
Iron	1043
Fe_2O_3	893
Nickel	631
Godolinium	317

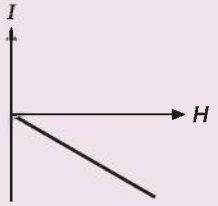
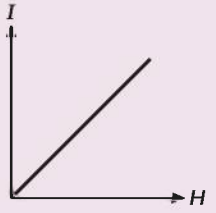
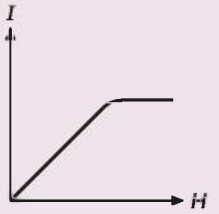
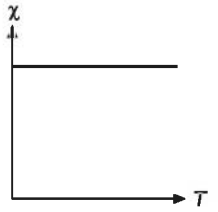
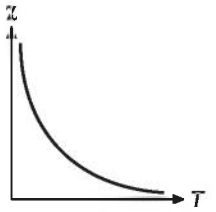
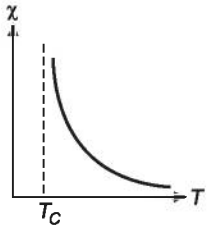
COMPARATIVE STUDY OF THESE MATERIALS

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Cause of magnetism	Orbital motion of electrons.	Spin motion of electrons.	Formation of domains.
Explanation of magnetism	On the basis of orbital motion of electrons.	On the basis of spin and orbital motion of electrons.	On the basis of domains formed.

(*Continued*)

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Behaviour In a non-uniform magnetic field	These are repelled in an external magnetic field i.e. have a tendency to move from high to low field region.	These are feebly attracted in an external magnetic field i.e. have a tendency to move from low to high field region	These are strongly attracted in an external magnetic field. i.e. they easily move from low to high field region.
State of magnetisation	These are weekly magnetised in a direction opposite to that of applied magnetic field	These get weekly magnetised in the direction of applied magnetic field	These get strongly magnetised in the direction of applied magnetic field
Liquid or powder in a watch glass when placed between the pole pieces (a) when poles are far apart (b) when poles are close to each other	(a) The liquid gets bulged at the middle  (a) (b) The liquid gets depressed at the middle  (b)	(a) The liquid gets depressed at the middle  (a) (b) The liquid gets bulged at the middle  (b)	(a) The liquid is very much depressed at the middle  (a) (b) The liquid gets very much bulged at the middle  (b)
When the material in the form of liquid is filled in the U-tube and placed between pole pieces.	Liquid level in that limb gets depressed 	Liquid level in that limb rises up 	Liquid level in that limb rises up very much 
On placing the gaseous materials between pole pieces	The gas expands at right angles to the magnetic field.	The gas expands in the direction of magnetic field	The gas rapidly expands in the direction of magnetic field
The value of magnetic induction B	$B < B_0$	$B > B_0$	$B >> B_0$
	where B_0 is the magnetic induction in vacuum		
Magnetic susceptibility χ	Low and negative $\lvert\chi\rvert \approx 1$	Low but positive $\chi \approx 1$	Positive and high $\chi \approx 10^2$
Dependence of χ on temperature	Does not depend on temperature (except *Bi* at low temperature)	Inversely proportional to temperature $\chi \propto \frac{I}{T}$ or $\chi = \frac{C}{T}$. This is called Curie Law, where C = Curie constant.	$\chi \propto \frac{1}{T - T_C}$ or $\chi = \frac{C}{T - T_C}$ This is called Curie-Weiss Law. T_C = Curie Temperature.

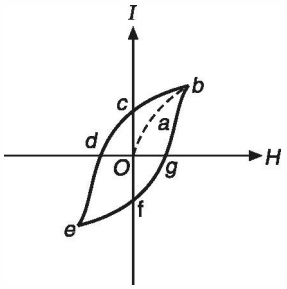
(*Continued*)

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Dependence of χ on H	Does not depend Independent	Does not depend Independent	Does not depend Independent
Relative Permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$ $\mu_r \approx 10^2$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low.	I is in the direction of H but value is low.	I is in the direction of H and value is very high.
I-H Curves			
Magnetic moment (M)	The value of M is very low (≈ 0 and is in a direction opposite to H.)	The value of M is very low and is in the direction of H	The value of M is very high and is in the direction of H.
Transition of materials (at Curie temperature)	These do not change.	On cooling, these get converted to ferro-magnetic materials at Curie temperature.	These get converted into paramagnetic materials above Curie temperature.
χ-T Curve			
The property of magnetism	Diamagnetism is found in those materials the atoms of which have even number electrons.	Paramagnetism is found in those materials the atoms of which have majority of electron spins in the same direction.	Ferro-magnetism is found in those materials which when placed in an external magnetic field are strongly magnetised.
Examples	$Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O$ air and diamond etc.	$Al, Mn, Pt, Na, CuCl_2, O_2$, and crown glass.	Fe, Co, Ni, Cd, Fe_3O_4 etc.
Nature of effect	Distortion effect	Orientation effect	Hysteresis effect

HYSTERESIS

When a bar of ferromagnetic material is magnetised by a varying magnetic field and the intensity of magnetisation I induced is measured for different values of magnetising field H, the graph of I versus H is shown in figure. From graph, it is observed that

(a) when magnetising field is increased from O, the intensity of magnetisation I increases and becomes maximum. This maximum value is called the **saturation value**.
The state of magnetic material in which the value of I becomes maximum and does not increase further on increasing the value of H is called the state of **Magnetic saturation**.



(b) when H is reduced, I reduces but is not zero when $H=0$. The remainder value Oc of magnetisation when $H=0$ is called the residual magnetism or retentivity.
The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called **Retentivity** or **Residual Magnetism** or **Remnant Magnetism**.

(c) when magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = Od$ when $I=0$. This value of H is called the **coercivity**.
So, the process of demagnetising a material completely by applying magnetising field in a negative direction is defined as **Coercivity**.
Coercivity assesses the magnetic softness or hardness of a magnetic material. If the ***coercivity*** of a magnetic material is ***low*** then it is ***magnetically soft*** and if the ***coercivity*** is ***high*** then material is ***magnetically hard***. So,
Magnetic hard substance (steel) $\rightarrow$ High coercivity
Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

(d) when field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e., point e).

(e) when H is decreased to zero and changed direction in steps, we get the part $efgb$.
Thus, complete cycle of magnetisation and demagnetisation is represented by $bcdefgb$.

In the complete cycle the intensity of magnetisation I is lagging behind the applied magnetising field. This is called **hysteresis** and the closed loop $bcdefgb$ is called **hysteresis cycle**.

The energy loss in magnetising and demagnetising a specimen is proportional to the area of hysteresis loop.

Problem Solving Technique(s)

$$\begin{pmatrix}\text{Hysteresis}\\ \text{Energy}\\ \text{Loss}\end{pmatrix} = \begin{pmatrix}\text{Area bound by}\\ \text{the hysteresis}\\ \text{loop}\end{pmatrix} = VAnt \text{ joule}$$

where,
V is the volume of Ferromagnetic sample,
A is the area of B-H
n is the frequency of alternating magnetic field and
t is the time for which the material is being magnetised

ILLUSTRATION 42

A ferromagnetic substance of volume $10^{-3}\ \text{m}^3$ is placed in an alternating field of frequency $50\ \text{Hz}$. If the area of the hysteresis curve obtained is $0.1\ \text{m}^2$, then calculate the heat produced due to energy loss per second in the substance.

SOLUTION

Since we know that

$$(\text{Heat Loss}) = VAnt$$

where $V = 10^{-3}\ \text{m}^3$, $A = 0.1\ \text{m}^2$, $n = 50\ \text{Hz}$ and $t = 1\ \text{s}$

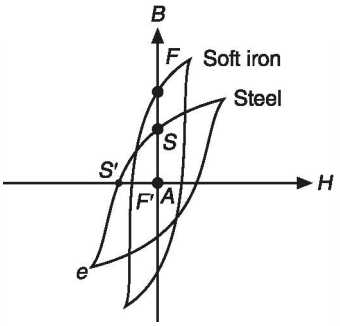
$$\Rightarrow \begin{pmatrix}\text{Heat}\\ \text{Loss}\end{pmatrix} = (10^{-3})(0.1)(50)(1) = 5\times10^{-3}\ \text{J} = 5\ \text{mJ}$$

PROPERTIES OF SOFT IRON AND STEEL

For soft iron the susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel.

For permanent magnet, the substance must have high retentivity and high coercivity while for electromagnet the substance must have low retentivity and high permeability.

Permanent magnets are made of steel and cobalt while electromagnets are made of soft iron. Hysteresis curves for soft iron and steel are shown.



Comparison Chart

Soft Iron	Steel
The hysteresis loss for this is low.	The hysteresis loss for this is high.
The area of hysteresis curve for this is less.	The area of hysteresis curve for this is more.
For this the value of remnant magnetism is high (AF in the diagram).	For this value of remnant magnetism is low (AS in the diagram).
For this the coercivity is low (AF′ in the diagram).	For this the coercivity is high (AS′ in the diagram).
For this magnetic permeability (μ) is high.	For this magnetic permeability (μ) is low.
For this the magnetic susceptibility (χ) is high.	For this the magnetic susceptibility (χ) is low.
For this intensity of magnetisation (I) is high.	For this intensity of magnetisation (I) is low.
For this the crystal arrangement is simple.	For this the crystal arrangement is complicated.
Its magnetisation and demagnetisation is easy.	Its magnetisation and demagnetisation is complicated and takes place with difficulty.

(*Continued*)

Selection of Materials

(a) **For permanent magnets: Permanent magnets should have high retentivity, so that the magnet is strong and high coercivity so that the magnetisation is not easily erased by stray magnetic fields, temperature fluctuations or minor mechanical damages.** An alloy Alnico (*Al + Ni + Co*) is used for permanent magnets. Also, it is observed that steel has slightly smaller retentivity than soft iron but this fact is dominated by the fact that steel has much smaller coercivity than soft iron and due to this reason steel is preferred over soft iron to make permanent magnets.

(b) **For electromagnets: Since an electromagnet must magnetise and demagnetise easily so it should have high permeability and low retentivity.** Soft iron is used for electromagnets. If a soft iron rod is placed in a solenoid and current is passed through the solenoid, then the magnetic field in the solenoid is increased 1000 times as $\mu_r = 1000$ for soft iron. As soon as the current is switched off, the magnetic field quickly becomes zero as the soft iron core has low retentivity.

(c) **For transformer, dynamo and tape recorder tapes:** Soft iron is used for these because its permeability is high, hysteresis loss is less and coercivity is low. For transformer core a special alloy permalloy is used. For soft materials μ, K and I are high and hysteresis loss or the area of I-H or B-H curve is low.

Hard and Soft Magnets

(a) The ferromagnetic material which retain magnetization for a long period of time are called hard magnetic material or hard ferromagnets. Some

hard-magnetic materials are Alnico (an alloy of iron, aluminum, nickel, cobalt and copper) and naturally occurring lodestone. They are used for permanent magnets. For permanent magnet material should have high retentivity and high coercivity.

(b) The ferromagnetic material which retain magnetization as long as the external field persists are called soft magnetic materials or soft ferromagnets. Soft ferromagnets is soft iron. Such material is used for making electromagnets. For electromagnets material should have low retentivity and low coercivity. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms.

Conceptual Note(s)

(a) Area under *B*-*H* loop is equal to the energy loss per cycle per unit volume, because unit of $B \times H$ is Jm^{-3}

(b) Area of *I*-*H* loop is $I \times H$

So, unit of Area of *I*-*H* loop is A^2m^{-2}

$$\because [IH] = \left(\frac{A}{m}\right)\left(\frac{A}{m}\right) = A^2m^{-2}$$

(c) Unit of area of *B*-*H* loop is μ_0 times the unit of area of *I*-*H* loop.

Test Your Concepts-IV

Based on Magnetic Properties of Materials

(Solutions on page H.112)

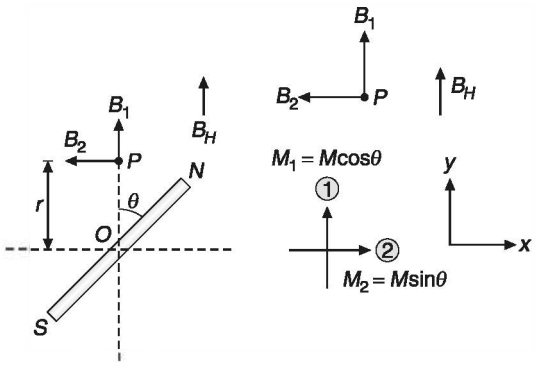
1. A bar magnet has coercivity 4×10^3 Am^{-1}. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. Calculate the current that should be sent through the solenoid.
2. The magnetic moment of a magnet $(15\text{ cm} \times 2\text{ cm} \times 1\text{ cm})$ is 1.2 Am^2. Calculate its intensity of magnetisation.
3. The dipole moment of each molecule of a paramagnetic gas is 1.5×10^{-23} Am^2. The temperature of gas is 27°C and the number of molecules per unit volume in it is 2×10^{26} m^{-3}. Calculate the maximum possible intensity of magnetisation in the gas.
4. The coercivity of a certain permanent magnet is 4.0×10^4 Am^{-1}. The magnet is placed inside a solenoid 20 cm long and having 700 turns and a current is passed in the solenoid to demagnetise it completely. Find the current.
5. An iron rod of 0.2 cm^2 cross-sectional area is subjected to a magnetizing field of 1200 Am^{-1}. The susceptibility of iron is 599. Calculate the magnetic flux produced in the rod.
6. The magnetic susceptibility of a paramagnetic material at –73 °C is 0.0075. Find its value at –173 °C.
7. The hysteresis loss for a specimen of iron weighing 15 kg is equivalent to 300 Jm^{-3} $cycle^{-1}$. Calculate the loss of energy per hour at 25 $cycles^{-1}$. Density of iron is 7500 kgm^{-3}.
8. A cylindrical rod magnet has a length of 5 cm and a diameter of 1 cm. It has a uniform magnetisation of 5.30×10^3 Am^{-3}. Find the magnetic dipole moment of the rod.
9. Relative permeability of iron is 5500, calculate its magnetic susceptibility.

SOLVED PROBLEMS

PROBLEM 1

A short magnet $(M = 4\times10^{-2}\ \text{Am}^2)$ lying in a horizontal plane with its north-pole points $37°$ east of north. Find the net horizontal field at a point of the magnet $0.1\ \text{m}$ away from it $(B_H = 11\ \mu\text{T})$ $\left(\sin 37° = \frac{3}{5},\ \cos 37° = \frac{4}{5}\right)$

SOLUTION



Due to magnet, magnetic field at P is

$$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M_1}{r^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M\cos\theta}{r^3}\right)$$

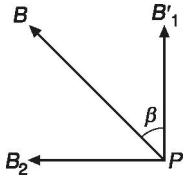
$$\Rightarrow \quad B_1 = \frac{10^{-7}\times 2\times 4\times 10^{-2}\times\frac{4}{5}}{(0.1)^3} = 6.4\times10^{-6}\ \text{T}$$

and $B_2 = \frac{\mu_0}{4\pi}\left(\frac{M_2}{r^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{r^3}\right)$

$$\Rightarrow \quad B_2 = \frac{10^{-7}\times 4\times 10^{-2}\times\frac{3}{5}}{(0.1)^3} = 2.4\times10^{-6}\ \text{T}$$

Since, B_1 and B_H are in same direction, so we get

$$B_1' = B_1 + B_H = 6.4\times10^{-6} + 11\times10^{-6} = 17.4\times10^{-6}\ \text{T}$$



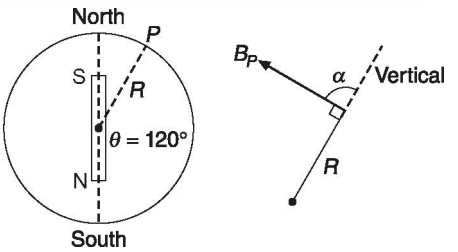
$$\Rightarrow \quad B_P = \sqrt{B_1'^2 + B_2^2} = \sqrt{(17.4)^2 + (2.4)^2}\times10^{-6}$$

$$\Rightarrow \quad B_P = \sqrt{308.52}\times10^{-6}\ \text{T}$$

$$\tan\beta = \frac{B_2}{B_1'} = \frac{2.4\times10^{-6}}{17.4\times10^{-6}} = 0.14°$$

PROBLEM 2

The earth's magnetic field at geomagnetic poles has a magnitude 8×10^{-5} T. Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of $120°$ with the axis of the earth's assumed magnetic dipole. What is the inclination (dip) at this point?



SOLUTION

The geomagnetic poles are lying in end on position. The magnetic field at geomagnetic poles is

$$B' = \frac{\mu_0}{4\pi}\left(\frac{2M}{R^3}\right) = 8\times10^{-5}\ \text{T}$$

The magnetic field at point P is

$$B_P = \frac{\mu_0 M}{4\pi R^3}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \quad B_P = \frac{B'}{2}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \quad B_P = \frac{8\times10^{-5}}{2}\sqrt{1+3\cos^2 120°}$$

$$\Rightarrow \quad B_P = 4\times10^{-5}\sqrt{1+\frac{3}{4}} = 2\sqrt{7}\times10^{-5}\ \text{T}$$

Also, $\tan\alpha = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 120°$

$$\Rightarrow \quad \tan\alpha = \frac{1}{2}\times(-\sqrt{3}) = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Dip ϕ is an angle made by the earth's magnetic field with the horizontal plane, so we have

$$\phi = \alpha - 90^\circ = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 90^\circ$$

PROBLEM 3

A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given horizontal component of earth's field = 0.34 G.

SOLUTION

When magnet is placed with its north pole pointing south, then neutral point is obtained on its axial line.

$$\Rightarrow \quad B_{\text{axial}} = B_H$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\times\frac{2Mr}{(r^2-l^2)^2} = B_H$$

$$\Rightarrow \quad M = \frac{4\pi B_H (r^2-l^2)^2}{2\mu_0 r}$$

Since $2l = 30$ cm

$\Rightarrow \quad l = 15$ cm $= 0.15$ m, $r = 30$ cm $= 0.30$ m

and $B_H = 0.34$ G $= 0.34\times10^{-4}$ T

$$\Rightarrow \quad M = \frac{1}{10^{-7}}\left[\frac{0.34\times10^{-4}\times(0.30^2-0.15^2)^2}{2\times0.30}\right]$$

$$\Rightarrow \quad M = \frac{0.34\times10^{-4}\times(0.0675)^2}{10^{-7}\times2\times0.30} = 2.582 \text{ Am}^2$$

The pole strength of the magnet is

$$m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.606 \text{ Am}$$

PROBLEM 4

A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If $H = 0.4$ G. Calculate the magnetic moment of the magnet.

SOLUTION

When north pole of the magnet points towards magnetic north, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of earth's magnetic field B_H. So, we have

$$B_H = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{r^3}$$

$$\Rightarrow \quad M = \frac{B_H r^3}{\left(\frac{\mu_0}{4\pi}\right)}$$

Substituting the values, we get

$$M = \frac{(0.4\times10^{-4})(10\times10^{-2})^3}{10^{-7}} = 0.4 \text{ Am}^2$$

PROBLEM 5

A magnetic needle performs 20 oscillations per minute in a horizontal plane. if the angle of dip be 30°, then how many oscillations per minute will this needle perform in vertical north-south plane and in vertical east-west plane?

SOLUTION

In horizontal plane, the magnetic needle oscillates in horizontal component of earths magnetic field. So,

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

In the vertical north-south plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e and in vertical east-west plane (plane perpendicular to the magnetic meridian) it oscillates only in earth's vertical component B_V. If its time period be T_1 and T_2, then

$$T_1 = 2\pi\sqrt{\frac{I}{MB_e}} \text{ and } T_2 = 2\pi\sqrt{\frac{I}{MB_V}}$$

From above equations, we get

$$\frac{T_1^2}{T^2} = \frac{B_H}{B_e}$$

$$\Rightarrow \quad \frac{n_1^2}{n^2} = \frac{B_e}{B_H}$$

Similarly, $\frac{n_2^2}{n^2} = \frac{B_V}{B_H}$

Further, $\frac{B_e}{B_H} = \sec\phi = \sec(30°) = \frac{2}{\sqrt{3}}$

and $\frac{B_V}{B_H} = \tan\phi = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad n_1^2 = n^2\left(\frac{B_e}{B_H}\right) = (20)^2\left(\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \quad n_1 = 21.5 \text{ oscillations/min}$

and $n_2^2 = n^2\left(\frac{B_V}{B_H}\right) = (20)^2\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad n_2 = 15.2 \text{ oscillations/min}$

PROBLEM 6

A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T'. Then, find the ratio T'/T.

SOLUTION

When magnet is divided into two equal parts the pole strength remains same, so, the magnetic dipole moment is

$$M' = (\text{Pole Strength})\frac{l}{2} = \frac{M}{2}$$

Also, the mass of magnet becomes half i.e., $m' = \frac{m}{2}$

Moment of inertia of magnet is

$$I = \frac{ml^2}{12}$$

New moment of inertia is

$$I' = \frac{1}{12}\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12\times 8}$$

$\Rightarrow \quad I' = \frac{I}{8}$

Since, $T = 2\pi\sqrt{\frac{I}{MB_H}}$

and $T' = 2\pi\sqrt{\frac{I'}{M'B_H}} = 2\pi\sqrt{\frac{I/8}{(M/2)B_H}}$

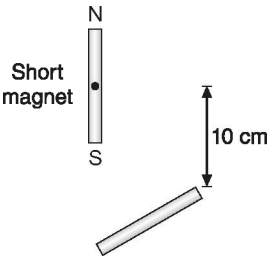
$\Rightarrow \quad T' = \frac{T}{2}$

$\Rightarrow \quad \frac{T'}{T} = \frac{1}{2}$

PROBLEM 7

The time period of the magnetic in an oscillation magnetometer in the earth magnetic field is 2 s. A short bar magnet is placed to the north of the magnetometer, at a separation 10 cm from the oscillating magnet, with its north pole pointing towards north. The time period becomes half. Calculate the magnetic moment of this short magnet.

$B_H = 12\ \mu\text{T}$.



SOLUTION

Since time period $T = 2\pi\sqrt{\frac{I}{MB}}$

$\Rightarrow \quad T \propto \frac{1}{\sqrt{H}}$

$\Rightarrow \quad B \propto \frac{1}{T^2}$

Let M is magnet moment due to short magnet and B' be the magnetic field due to short magnet, along south to north, then

$$\frac{(B+B')}{B} = \frac{T_1^2}{T_2^2}$$

where $T_1 = 2$ s, $B_H = 12\ \mu$T

$T_2 = 1$ s, $B_H = B + B' = 12 + B'$

$$\frac{12+B'}{12} = \left(\frac{2}{1}\right)^2 = 4$$

$\Rightarrow \quad B' = 36\ \mu\text{T}$

Since, $B' = \dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$

$\Rightarrow \quad 36\times10^{-6} = 10^{-7}\times\dfrac{2M}{(0.10)^3}$

$\Rightarrow \quad M = 0.18\ \text{Am}^2$

PROBLEM 8

Each atom of an iron bar $(5\ \text{cm}\times1\ \text{cm}\times1\ \text{cm})$ has a magnetic moment $1.8\times10^{-23}\ \text{Am}^2$. Knowing that the density of iron is $7.78\times10^3\ \text{kg}^{-3}\text{m}$, atomic weight is 56 and Avogadro's number is 6.02×10^{23}, calculate the magnetic moment of bar in the state of magnetic saturation.

SOLUTION

The number of atoms per unit volume in a specimen is

$$n = \frac{N_A}{V} = \frac{N_A}{\left(\dfrac{m_{Fe}}{\rho_{Fe}}\right)} = \frac{N_A\rho_{Fe}}{m_{Fe}}$$

For iron, $\rho = 7.8\times10^{-3}\ \text{kgm}^{-3}$

$$N_A = 6.02\times10^{23},\ m_{Fe} = 56\ \text{g} = \frac{56}{1000}\ \text{kg}$$

$$\Rightarrow \quad n = \frac{7.8\times10^3\times6.02\times10^{23}}{(56/1000)} = 8.38\times10^{28}\ \text{m}^{-3}$$

The volume of the iron bar is

$$V = (5\times10^{-2})(1\times10^{-2})(1\times10^{-2}) = 5\times10^{-6}\ \text{m}^3$$

Total number of atoms in the bar is

$$N_0 = nV = (8.38\times10^{28})(5\times10^{-6})$$

$$\Rightarrow \quad N_0 = 4.19\times10^{23}$$

The saturated magnetic moment of bar is

$$M_{\text{saturated}} = N_0 M_{\text{each atom}}$$

$$\Rightarrow \quad M_{\text{saturated}} = (4.19\times10^{23})(1.8\times10^{-23})$$

$$\Rightarrow \quad M_{\text{saturated}} = 7.54\ \text{Am}^2$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Relative permeability of iron is 5500, then its magnetic susceptibility will be

(A) 5500×10^{7} (B) 5500×10^{-7}

(C) 5501 (D) 5499

2. In sum and difference method in vibration magnetometer, the time period is more if

(A) Similar poles of both magnets are on same sides

(B) Opposite poles of both magnets are on same sides

(C) Both magnets are perpendicular to each other

(D) Nothing can be said

3. A magnet of magnetic moment 20 C.G.S. units are freely suspended in a uniform magnetic field of intensity 0.3 C.G.S units. The amount of work done in deflecting it by an angle of 30° in C.G.S. units is

(A) 6 (B) $3\sqrt{3}$

(C) $3(2-\sqrt{3})$ (D) 3

4. The direction of the null points is on the equatorial line of a bar magnet, when the north pole of the magnet is pointing

(A) North (B) South

(C) East (D) West

5. Which of the following statements is false

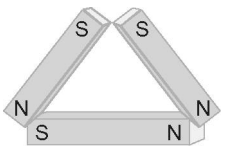
(A) magnetic intensity is a measure of magnetic lines of force passing through unit area held normal to it

(B) magnetic lines of force form a closed curve

(C) due to magnet magnetic lines of force never cut each other

(D) inside a magnet the magnetic lines of force run from north pole of a magnet towards south pole

6. Three identical bar magnets each of magnetic moment M are placed in the form of an equilateral triangle as shown.

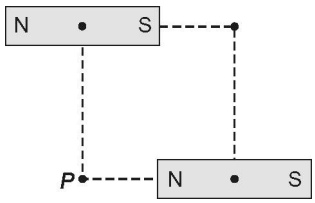


The net magnetic moment of the system is

(A) zero (B) $2M$

(C) $M\sqrt{3}$ (D) $\frac{3M}{2}$

7. Two short magnets of magnetic moment 1000 Am^2 are placed as shown at the corners of a square of side 10 cm . The net magnetic induction at P is



(A) 0.1 T (B) 0.2 T

(C) 0.3 T (D) 0.4 T

8. Rate of change of torque τ with deflection θ is maximum for a magnet suspended freely in a uniform magnetic field of induction B, when

(A) $\theta = 0°$ (B) $\theta = 45°$

(C) $\theta = 60°$ (D) $\theta = 90°$

9. The area of hysteresis loop of a material is equivalent to 250 J . When 10 kg material is magnetised by an alternating field of 50 Hz then energy lost in one hour will be if the density of material is 7.5 gcm^{-3}

(A) 6×10^{4} J (B) 6×10^{4} erg

(C) 3×10^{2} J (D) 3×10^{2} erg

10. Two magnets of same size and mass make respectively 10 and 15 oscillations per minute at certain place. The ratio of their magnetic moments is

(A) 4 : 9 (B) 9 : 4

(C) 2 : 3 (D) 3 : 2

11. The sensitivity of a tangent galvanometer is increases if

(A) Number of turn decreases

(B) Number of turn increases

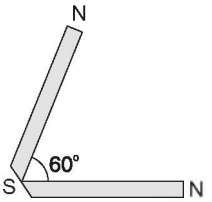
(C) Field increases

(D) None of the above

12. Force between two identical bar magnets whose centres are r metre apart is 4.8 N, when their axes are in the same line. If separation is increased to $2r$, the force between them is reduced to

(A) 2.4 N (B) 1.2 N
(C) 0.6 N (D) 0.3 N

13. The net magnetic moment of two identical magnets each of magnetic moment M_0, inclined at 60° with each other is



(A) M_0 (B) $\sqrt{2}M_0$
(C) $\sqrt{3}M_0$ (D) $2M_0$

14. The magnetic needle of an oscillation magnetometer makes 10 oscillations per minute under the action of earth's magnetic field. When a bar magnet is placed at some distance along the axis of the needle it makes 14 oscillations per minute. If the bar magnet is turned so that its poles interchange their position, then the new frequency of oscillation of the needle is

(A) 10 vibrations per minute
(B) 14 vibrations per minute
(C) 4 vibrations per minute
(D) 2 vibrations per minute

15. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is $2^{5/4}$ seconds. One of the magnets is removed and if the other magnet oscillates in the same field, then the time period in seconds is

(A) $2^{\frac{1}{4}}$ (B) $2^{\frac{1}{2}}$
(C) 2 (D) $2^{\frac{3}{4}}$

16. A small bar magnet A oscillates in a horizontal plane with a period T at a place where the angle of dip is 60°. When the same needle is made to oscillate in a vertical plane coinciding with the magnetic meridian, its period will be

(A) $\frac{T}{\sqrt{2}}$ (B) T
(C) $\sqrt{2}T$ (D) $2T$

17. The distance between the poles of horse shoe magnet is 10 cm and its pole strength is 10^{-4} Am. The magnetic field induction at point P midway between the poles is

(A) ZERO (B) 2×10^{-9} T
(C) 4×10^{-9} T (D) 8×10^{-9} T

18. A straight wire carrying current i is turned into a circular loop. If the magnitude of magnetic moment associated with it in M.K.S. unit is M, the length of wire will be

(A) $4\pi iM$ (B) $\sqrt{\frac{4\pi M}{i}}$
(C) $\sqrt{\frac{4\pi i}{M}}$ (D) $\frac{M\pi}{4i}$

19. The distance of two points on the axis of a magnet from its centre is 10 cm and 20 cm respectively. The ratio of magnetic intensity at these points is 12.5 : 1. The length of the magnet will be

(A) 5 cm (B) 25 cm
(C) 10 cm (D) 20 cm

20. A magnet is parallel to a uniform magnetic field. If it is rotated by 60°, the work done is 0.8 J. The work done in rotating it further by 30° is

(A) 0.8×10^{7} erg (B) 0.4 J
(C) 8 J (D) 0.8 erg

21. At a certain place, the horizontal component B_0 and the vertical component of the earth's magnetic field are equal in magnitude. The total intensity at the place will be

(A) B_0 (B) B_0^2
(C) $2B_0$ (D) $\sqrt{2}B_0$

22. The radius of the coil of a tangent galvanometer, which has 10 turns, is 0.1 m. The current required to produce a deflection of 60° (if horizontal component of earth's field is 4×10^{-4} T) is

(A) 3 A (B) 1.1 A
(C) 2.1 A (D) 1.5 A

23. The moment of a magnet is 0.1 Am^2 and the force acting on each pole in a uniform magnetic field of strength 0.36 oersted is 1.224×10^{-4} N. The distance between the poles of the magnet is

(A) 1.56 cm (B) 0.78 cm
(C) 2.50 cm (D) 1.17 cm

24. If the earth's field induction at a place is 0.36 G and the angle of dip is $60°$. The horizontal and vertical components of the field are B_H and B_V respectively.

(A) $B_H = 0.18$ G, $B_V = \frac{0.18}{\sqrt{3}}$ G

(B) $B_H = 0.18\sqrt{3}$ G, $B_V = 0.18$ G

(C) $B_H = 0.18$ G, $B_V = 0.36$ G

(D) $B_H = 0.18$ G, $B_V = 0.18\sqrt{3}$ G

25. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is $40°$. Now the dip circle is rotated so that the plane in which the needle moves makes and angle of $30°$ with the magnetic meridian. In this position the needle will dip by an angle

(A) $40°$ (B) $30°$

(C) More than $40°$ (D) Less than $40°$

26. The magnetic field due to a short magnet at a point on its axis at distance X cm form the middle point of the magnet is 200 G. The magnetic field at a point on the neutral axis at a distance X cm from the middle of the magnet is

(A) 100 G (B) 400 G

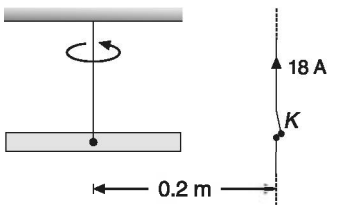
(C) 50 G (D) 200 G

27. A bar magnet A of magnetic moment M_A is found to oscillate at a frequency twice that of magnet B of magnetic moment M_B when placed in a vibrating magnetometer. We may say that

(A) $M_A = 2\,M_B$ (B) $M_A = 8\,M_B$

(C) $M_A = 4\,M_B$ (D) $M_B = 8\,M_A$

28. Figure shows a short magnet executing small oscillations in a vibration magnetometer in earth's magnetic field having horizontal component 24 μT. The period of oscillation is 0.1 s. When the key K is closed, and upward current of 18 A is established as shown. The new time period is



(A) 0.1 s (B) 0.2 s

(C) 0.3 s (D) 0.4 s

29. The values of the apparent angles of dip in two planes at right angles to each other are $30°$ and $45°$. Then the true value of the angle of dip at the place is

(A) $\tan^{-1} 1$ (B) $\tan^{-1} 2$

(C) $\cot^{-1} 2$ (D) $\cot^{-1} 1$

30. The magnet of vibration magnetometer is heated so as to reduce its magnetic moment by 36%. By doing this the periodic time of the magnetometer will

(A) Increases by 36%

(B) Increases by 25%

(C) Decreases by 25%

(D) Decreases by 64%

31. At a certain place the angle of dip is $30°$ and the horizontal component of earth's magnetic field is 0.50 oersted. The earth's total magnetic field (in oersted) is

(A) $\sqrt{3}$ (B) 1

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}$

32. A bar magnet of length 10 cm and having the pole strength equal to 10^{-3} Wb is kept in a magnetic field having magnetic induction (B) equal to $4\pi \times 10^{-3}$ T. It makes an angle of $30°$ with the direction of magnetic induction. The value of the torque acting on the magnet is

(A) $2\pi \times 10^{-7}$ Nm (B) $2\pi \times 10^{-5}$ Nm

(C) 0.5 Nm (D) 0.5×10^{2} Nm

$\left(\mu_0 = 4\pi \times 10^{-7}\ \text{WbA}^{-1}\text{m}^{-1}\right)$

33. A dip circle lying initially in the magnetic meridian is rotated through angle θ in the horizontal plane. The tangent of angle of dip is increased in the ratio

(A) $\cos\theta : 1$ (B) $\sin\theta : 1$

(C) $1 : \cos\theta$ (D) $1 : \sin\theta$

34. Magnetic A and B are geometrically similar but the magnetic moment of A is twice that of B. If T_1 and T_2 be the time period of the oscillation when their like poles and unlike poles are kept together respectively, then $\frac{T_1}{T_2}$ will be

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

35. The dip at a place is ϕ. For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes angle θ with the magnetic meridian, the apparent dip will be given $\tan\phi'$ which is equal to:

(A) $\tan\phi\cos\theta$ (B) $\tan\phi\sec\theta$

(C) $\tan\phi\sin\theta$ (D) $\tan\phi\operatorname{cosec}\theta$

36. A dip needle in a plane perpendicular to magnetic meridian will remain

(A) vertical
(B) horizontal
(C) in any direction
(D) inclined at $45°$ with horizontal

37. Two like magnetic poles of strength 10 and 40 SI units are separated by a distance 30 cm. The intensity of magnetic field is zero on the line joining them

(A) At a point 10 cm from the stronger pole
(B) At a point 20 cm from the stronger pole
(C) At the mid-point
(D) At infinity

38. A thin magnet of magnetic moment M is divided into two equal parts by cutting it along its length. The new magnetic moment of each part is M'. If the time period of each part is T' and the time period of original magnet is T for oscillations in the same magnetic field, then

(A) $M'=\frac{M}{2}, T'=\frac{T}{2}$ (B) $M'=\frac{M}{2}, T'=T$

(C) $M'=M, T'=T$ (D) $M'=M, T'=2T$

39. A bar magnet is oscillating in earth's magnetic field with a period T. What happens to its period and motion if its mass is quadrupled?

(A) motion remains simple harmonic with a new period $4T$
(B) motion remains simple harmonic with a new period $\frac{T}{2}$
(C) motion does not remain simple harmonic and the period stays nearly constant
(D) motion remains simple harmonic with a new period $2T$

40. If the angular momentum of an electron is J, then the magnitude of the magnetic moment will be

(A) $\frac{eJ}{m}$ (B) $\frac{eJ}{2m}$

(C) $eJ2m$ (D) $\frac{2m}{eJ}$

41. In a vibration magnetometer, the time period of a bar magnet oscillating in a horizontal component of earth's magnetic field is 2 s. When the magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio of the horizontal component of earth's magnetic field to the magnetic field due to magnet is

(A) 3 (B) $\frac{1}{3}$

(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

42. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio of the horizontal component of earth's field to the field due to magnet is

(A) 3 (B) $\frac{1}{3}$

(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

43. The magnetic field at a point x on the axis of a small bar magnet is equal to the field at a point y on the equator of the same magnet. The ratio of the distance of x and y from the centre of the magnet is

(A) 2^{-3} (B) $2^{-\frac{1}{3}}$

(C) 2^{3} (D) $2^{\frac{1}{3}}$

44. The real angle of dip, if magnet is suspended at an angle of $30°$ to the magnetic meridian and the dip needle makes an angle of $45°$ with horizontal is

(A) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\tan^{-1}(\sqrt{3})$

(C) $\tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$ (D) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

45. Which of the following statement is incorrect about hysteresis?

(A) This effect is common to all ferromagnetic substance
(B) The hysteresis loop area is proportional to the thermal energy developed per unit volume of the material
(C) The hysteresis loop area is independent of the thermal energy developed per unit volume of material
(D) The shape of the hysteresis loop is characteristic of the material

46. Two short magnets have equal to pole strengths but one is twice as long as the other. The shorter magnet is placed 20 cm in $\tan A$ position from the compass needle. The longer magnet must be placed on the other side of the magnetometer for no deflection at a distance that equals

(A) 20 cm (B) $20\times(2)^{\frac{1}{3}}$ cm

(C) $20\times(2)^{\frac{4}{3}}$ cm (D) $20\times(2)^{\frac{2}{3}}$ cm

47. A thin magnet of magnetic moment M is divided into two equal parts by cutting it perpendicular to its length. The new magnetic moment of each part is M'. If the time period of each part is T' and the time period of original magnet is T for oscillations in the same magnetic field, then

(A) $M'=\frac{M}{2}, T'=\frac{T}{2}$ (B) $M'=\frac{M}{2}, T'=T$

(C) $M'=M, T'=T$ (D) $M'=M, T'=2T$

48. A short magnet oscillates with a time period 0.1 s at a place where horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire 20 cm east of the magnet. The new time period of oscillator

(A) 0.1 s (B) 0.089 s

(C) 0.076 s (D) 0.057 s

49. Two small identical magnetic dipoles of magnetic moments 1.0 Am^2 each, placed at a separation of 2 m with their axis perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

(A) 5×10^{-7} T (B) $\sqrt{5}\times10^{-7}$ T

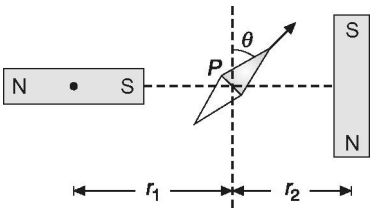
(C) 10^{-7} T (D) 2×10^{-7} T

50. A certain amount of current when flowing in a properly set tangent galvanometer, produces a deflection of 45°. If the current be reduced by a factor of $\sqrt{3}$, the deflection would

(A) Decrease by 30° (B) Decrease by 15°

(C) Increase by 15° (D) Increase by 30°

51. Two magnets A and B are identical and these are arranged as shown in Figure. Their length is negligible in comparison to the separation between them. A magnetic needle is placed between the magnets at point P which gets deflected through an angle θ under the influence of magnets. The ratio of distance r_1 and r_2 will be



(A) $(2\tan\theta)^{\frac{1}{3}}$ (B) $(2\tan\theta)^{-\frac{1}{3}}$

(C) $(2\cot\theta)^{\frac{1}{3}}$ (D) $(2\cot\theta)^{-\frac{1}{3}}$

52. A bar magnet A of magnetic moment M_A is found to oscillate at a frequency twice that of magnet B of magnetic moment M_B when placed in a vibrating magnetometer. We may say that

(A) $M_A=2M_B$ (B) $M_B=4M_A$

(C) $M_A=4M_B$ (D) $M_B=8M_A$

53. A tangent galvanometer shown a deflection 45° when 10 mA current pass through it. If the horizontal component of the earth's field is 3.6×10^{-5} T and radius of the coil is 10 cm. The number of turns in the coil is

(A) 5700 turns (B) 57 turns

(C) 570 turns (D) 5.7 turns

54. In a deflection magnetometer experiment the deflections produced separately by two short bar magnets kept at the same distance are 45° and 30°. Then the ratio of the magnetic moments of the two magnets is

(A) $\sqrt{3}:2$ (B) $\sqrt{3}:1$

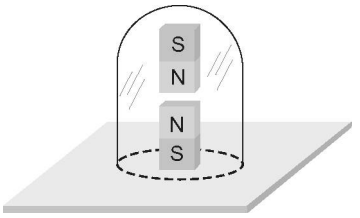
(C) $\sqrt{2}:1$ (D) $1:\sqrt{3}$

55. Two tangent galvanometer have radii 7.5 cm and 10 cm, number of turns are 15 and 10 and resistances are 8 Ω and 12 Ω. They are joined in parallel in circuit. If deflection in one is 60°, the deflection in second galvanometer is

(A) 45° (B) 30°

(C) 40° (D) 35°

56. Two identical bar magnets with a length 10 cm and weight 50 gwt are arranged freely with their like poles facing in an inverted vertical glass tube. The upper magnet hangs in the air above the lower one so that the distance between the nearest pole of the magnet is 3 mm. Pole strength of the poles of each magnet will be



(A) 6.64 Am (B) 2 Am

(C) 10.25 Am (D) None of these

57. If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then

(A) $\cos^2\phi = \cos^2\phi_1 + \cos^2\phi_2$

(B) $\sec^2\phi = \sec^2\phi_1 + \sec^2\phi_2$

(C) $\tan^2\phi = \tan^2\phi_1 + \tan^2\phi_2$

(D) $\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$

58. A thin rectangular magnet suspended freely has a period of oscillation of $4\ \text{s}$. If it is broken into two halves (each having half the original length) and one of the pieces is suspended similarly. The period of its oscillation will be

(A) 4 s (B) 2 s

(C) 0.5 s (D) 0.25 s

59. Points A and B are situated perpendicular to the axis of a $2\ \text{cm}$ long bar magnet at large distances x and $3x$ from its centre on opposite sides. The ratio of the magnetic fields at A and B will be approximately equal to

(A) 1 : 9 (B) 2 : 9

(C) 27 : 1 (D) 9 : 1

60. A magnetic needle vibrates in a vertical plane parallel to the magnetic meridian about a horizontal axis passing through its centre. Its frequency is n. If the plane of oscillation is turned about a vertical axis by $90°$, the frequency of its oscillation in vertical plane will be

(A) n (B) ZERO

(C) less than n (D) more than n

61. The angle of dip at a place is $40.6°$ and the intensity of the vertical component of the earth's magnetic field $V = 6\times10^{-5}\ \text{T}$. The total intensity of the earth's magnetic field (B) at this place is

(A) $7\times10^{-5}\ \text{T}$ (B) $6\times10^{-5}\ \text{T}$

(C) $5\times10^{-5}\ \text{T}$ (D) $9.2\times10^{-5}\ \text{T}$

62. A needle of a deflection magnetometer shows a deflection of $60°$ due to a short bar magnet at a distance in $\tan A$ position. The deflection under same conditions in $\tan B$ position will be (Given $\tan 74° = 3.464$ and $\tan 41° = 0.866$).

(A) 74° (B) 41°

(C) 47° (D) 33°

63. The earth's magnetic field at a certain place has a horizontal component $0.3\ \text{G}$ and the total strength $0.5\ \text{G}$. The angle of dip is

(A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{3}{4}\right)$

(C) $\tan^{-1}\left(\frac{4}{3}\right)$ (D) $\sin^{-1}\left(\frac{3}{5}\right)$

64. A copper bar on suspension in a magnetic field orients itself across the field lines. This shows that copper is a

(A) non-magnetic substance

(B) paramagnetic substance

(C) ferromagnetic substance

(D) diamagnetic substance

65. Susceptibility of Mg at $300\ \text{K}$ is 1.2×10^{-5}. The temperature at which susceptibility will be 1.8×10^{-5} is

(A) 450 K (B) 200 K

(C) 375 K (D) None of these

66. Which of the following is most suitable for the core of the electromagnets?

(A) Air (B) Steel

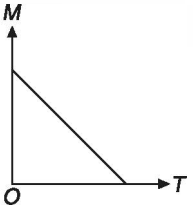
(C) Soft iron (D) *Cu-Ni* alloy

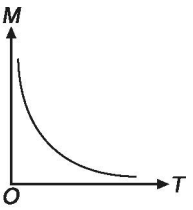
67. Two short magnets with their axes horizontal and perpendicular to the magnetic meridian are placed with their centres $40\ \text{cm}$ east and $50\ \text{cm}$ west of magnetic needle. If the needle remains undeflected, the ratio of their magnetic moments $M_1 : M_2$ is

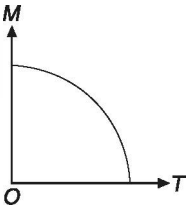
(A) 4 : 5 (B) 16 : 25

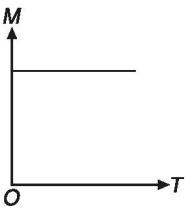
(C) 64 : 125 (D) $2:\sqrt{5}$

68. A curve between magnetic moment and temperature of magnet is

(A) 

(B) 

(C) 

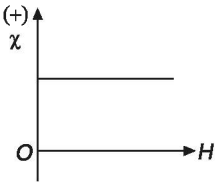
(D) 

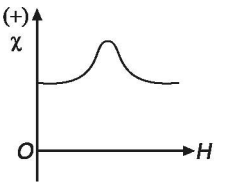
69. A compass needle whose magnetic moment is $60\ \text{Am}^2$ pointing geographical north at a certain place, where the horizontal component of earth's field is $40\mu\ \text{Wbm}^{-2}$, experience a torque 1.2×10^{-3} Nm. The declination at this place is

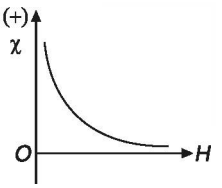
(A) 30° (B) 45°

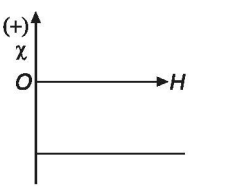
(C) 60° (D) 25°

70. The variation of magnetic susceptibility (χ) with magnetising field for a paramagnetic substance is

(A) 

(B) 

(C) 

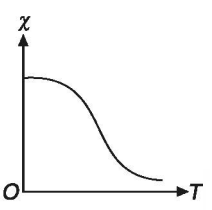
(D) 

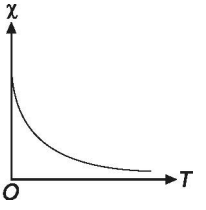
71. A tangent galvanometer has a coil of 25 turns and radius of 15 cm. The horizontal component of the earth's magnetic field is 3×10^{-5} T. The current required to produce a deflection of 45° in it, is

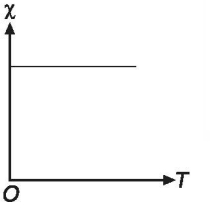
(A) 0.29 A (B) 1.2 A

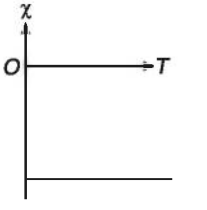
(C) 3.6×10^{-5} A (D) 0.14 A

72. The variation of magnetic susceptibility (χ) with absolute temperature T for a ferromagnetic material is

(A) 

(B) 

(C) 

(D) 

73. Iron is ferromagnetic

(A) at all temperatures (B) at NTP only

(C) above 770 °C (D) below 770 °C

74. The most suitable metal for making permanent magnets is

(A) iron (B) copper

(C) steel (D) aluminium

75. In a tangent galvanometer a current of 0.1 A produces a deflection of 30°. The current required to produce a deflection of 60° is

(A) 0.2 A (B) 0.3 A

(C) 0.4 A (D) 0.5 A

76. All the magnetic materials lose their magnetic properties when

(A) dipped in water

(B) dipped in oil

(C) heated

(D) brought near a piece of iron

77. When all the molecules in a magnet arrange themselves in the direction of the magnetic field lines, the condition is called

(A) saturation (B) reluctance

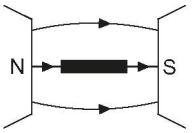
(C) retentivity (D) permeability

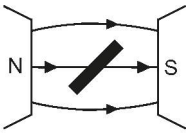
78. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque required to maintain the needle in this position will be

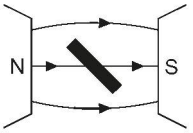
(A) $\sqrt{3}W$ (B) W

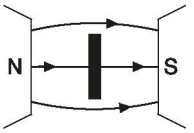
(C) $\frac{\sqrt{3}}{2}W$ (D) $2W$

79. A rod of a diamagnetic material is placed in a non-uniform strong magnetic field. Which of the figure below shows the alignment of the rod in the field?

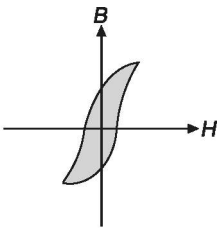
(A) 

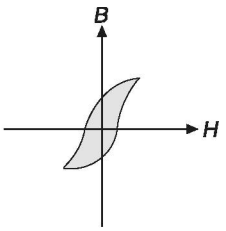
(B) 

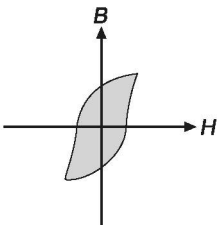
(C) 

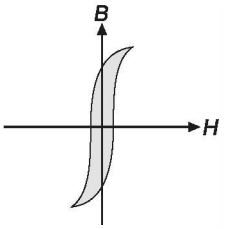
(D) 

80. For substance hysteresis $(B-H)$ curves are given as shown in Figure. For making temporary magnet which of the following is the best?

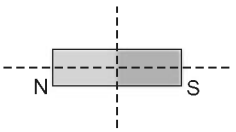
(A) 

(B) 

(C) 

(D) 

81. Time period for magnet is T. If it is divided in four equal parts along its axis and perpendicular to its axis as shown, then time period for each part will be



(A) $4T$ (B) $\frac{T}{4}$

(C) $\frac{T}{2}$ (D) T

82. A bar magnet has coercivity 4×10^3 Am^{-1}. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current that should be sent through the solenoid is

(A) 2 A (B) 4 A

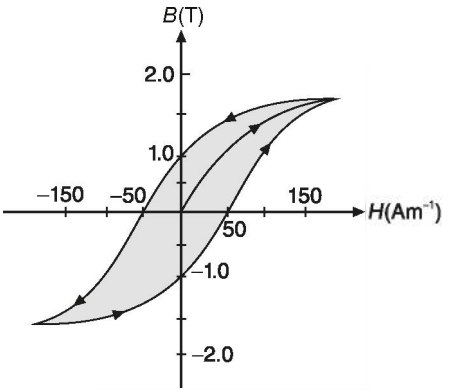
(C) 6 A (D) 8 A

83. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is

(A) 10 Am^2 (B) 15 Am^2

(C) 20 Am^2 (D) 25 Am^2

84. The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are



(A) 150 Am^{-1}, 1.0 T and 1.5 T

(B) 1.0 T, 50 Am^{-1} and 1.5 T

(C) 1.5 T, 50 Am^{-1} and 1.0 T

(D) 1.5 T, 50 Am^{-1} and 1.0 T

85. At a place the earth's horizontal component of magnetic field is 0.36×10^{-4} Wbm^{-2}. If the angle of dip at that place is 60°, then the vertical component of earth's field at that place in Wbm^{-2} will be approximately

(A) 0.12×10^{-4} (B) 0.24×10^{-4}

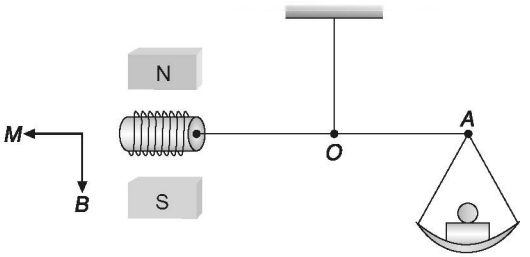
(C) 0.40×10^{-4} (D) 0.62×10^{-4}

86. A watch glass containing some powdered substance is placed between the pole pieces of a magnet. Deep concavity is observed at the centre. The substance in the watch glass is

(A) iron (B) chromium

(C) carbon (D) wood

87. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in Figure. The cross sectional area of coil is $A = 1.0\ \text{cm}^2$, length of arm OA of the balance beam is $l = 30\ \text{cm}$. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22\ \text{mA}$ through the coil the equilibrium is restored by putting the additional counter weight of mass $\Delta m = 60\ \text{mg}$ on the balance pan. Fine the magnetic induction at the spot where coil is located.



(A) 0.4 T (B) 0.3 T

(C) 0.2 T (D) 0.1 T

88. A current carrying coil is placed with its axis perpendicular to NS direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop be H. If a magnet is suspended inside the loop, it makes angle θ with H. Then $\theta =$

(A) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (B) $\tan^{-1}\left(\frac{H}{H_0}\right)$

(C) $\text{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (D) $\cot^{-1}\left(\frac{H_0}{H}\right)$

89. A bar magnet has coercivity $4\times10^3\ \text{Am}^{-1}$. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current that should be sent through the solenoid is

(A) 2 A (B) 4 A

(C) 6 A (D) 8 A

90. The magnetic moment produced in a substance of mass 1 g is $6\times10^{-7}\ \text{Am}^2$. If its density is $5\ \text{gcm}^{-3}$, then the intensity of magnetisation in Am^{-1} will be

(A) 8.3×10^6 (B) 3.0

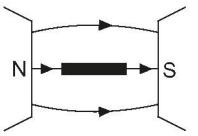
(C) 1.2×10^{-7} (D) 3×10^{-6}

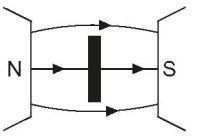
91. A magnet freely suspended in a vibration magnetometer makes 10 oscillations per minute at a place A and 20 oscillation per minute at a place B. If the horizontal component of earth's magnetic field at A is 36×10^{-6} T, then its value at B is

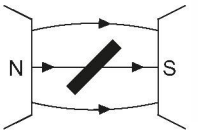
(A) 36×10^{-6} T (B) 72×10^{-6} T

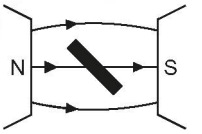
(C) 144×10^{-6} T (D) 288×10^{-6} T

92. A rod of a paramagnetic substance is placed in a non-uniform magnetic field. Which of the following figure shows its alignment in the field?

(A) 

(B) 

(C) 

(D) 

93. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is T. The magnet is cut along its length into six parts and these parts are then placed together as shown in Figure. The time period of this combination will be

N	S
N	S
S	N
S	N
S	N
S	N

(A) T (B) $\frac{T}{\sqrt{3}}$

(C) $\frac{T}{2\sqrt{3}}$ (D) ZERO

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1. [Online January 2019]

A paramagnetic material has 10^{28} atoms m^{-3}. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is

(A) 3.726×10^{-4} (B) 3.672×10^{-4}

(C) 2.672×10^{-4} (D) 3.267×10^{-4}

2. [Online January 2019]

A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} JT^{-1} when a magnetic intensity of 60×10^{3} Am^{-1} is applied. Its magnetic susceptibility is

(A) 3.3×10^{-2} (B) 2.3×10^{-2}

(C) 3.3×10^{-4} (D) 4.3×10^{-2}

3. [Online January 2019]

A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then

(A) $T_h = 0.5T_c$ (B) $T_h = T_c$

(C) $T_h = 2T_c$ (D) $T_h = 1.5T_c$

4. [Online January 2019]

At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is

(A) 1.3×10^{-5} N (B) 1.8×10^{-5} N

(C) 6.5×10^{-5} N (D) 3.6×10^{-5} N

5. [Online January 2019]

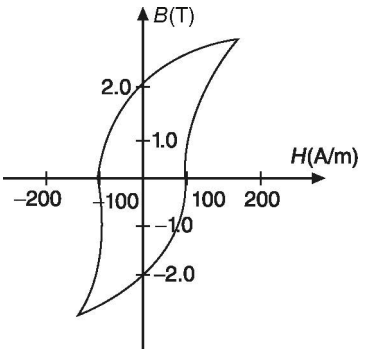
A magnet of total magnetic moment $10^{-2}\hat{i}$ Am^2 is placed in a time varying magnetic field, $B\hat{i}(\cos\omega t)$ where $B = 1$ Tesla and $\omega = 0.125$ $rads^{-1}$. The work done for reversing the direction of the magnetic moment at $t = 1$ second, is

(A) 0.028 J (B) 0.007 J

(C) 0.014 J (D) 0.01 J

6. [Online 2018]

The B-H curve for a ferromagnet is shown in Figure.



The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is

(A) 1 mA (B) 20 μA

(C) 2 mA (D) 40 μA

7. [2017]

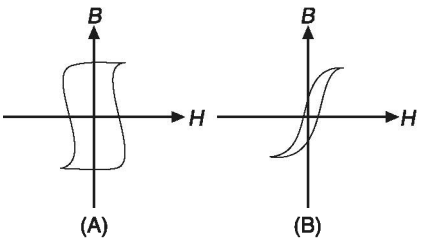
A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kgm^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(A) 6.65 s (B) 8.89 s

(C) 6.98 s (D) 8.76 s

8. [2016]

Hysteresis loops for two magnetic materials A and B are given below



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use

(A) A for electric generators and transformers.

(B) A for electromagnets and B for electric generators.

(C) A for transformers and B for electric generators.

(D) B for electromagnets and transformers.

9. [Online 2016]

A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of 30° with this field. The magnitude of the other field (in mT) is close to

(A) 1 (B) 11

(C) 36 (D) 1060

10. [Online 2015]

A 25 cm long solenoid has radius 2 cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetisation $\vec{M}$ (magnetic moment/volume), then $|\vec{M}|$ is

(A) 3π Am^{-1} (B) 30000 Am^{-1}

(C) 300 Am^{-1} (D) 30000π Am^{-1}

11. [Online 2015]

A short bar magnet is placed in the magnetic meridian of the earth with north pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East – West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in Am^2 is close to

(Given that $\frac{\mu_0}{4\pi} = 10^{-7}$ in SI units and B_H is the horizontal component of earth's magnetic field equal to 3.6×10^{-5} tesla)

(A) 9.7 (B) 4.9

(C) 19.4 (D) 14.6

12. [2014]

The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is

(A) 6 A (B) 30 mA

(C) 60 mA (D) 3 A

13. [2013]

Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to

(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wbm^{-2})

(A) 5.80×10^{-4} Wbm^{-2}

(B) 3.6×10^{-5} Wbm^{-2}

(C) 2.56×10^{-4} Wbm^{-2}

(D) 3.50×10^{-4} Wbm^{-2}

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Bar Magnet and Properties)

1. $60°$
2. 12 cm
3. 8×10^{-7} T
4. $\frac{IL^2}{4\pi}$
5. $\frac{M}{\sqrt{2}}$
6. $\frac{3M}{\pi}$
7. For stable equilibrium
   $\theta=0°$ and $U_{min}=-0.048$ J
   For unstable equilibrium
   $\theta=180°$ and $U_{max}=+0.048$ J
8. L
9. $\frac{iBl^2}{4\pi}$
10. 100 μJ

Test Your Concepts-II (Based on Earth's Magnetism)

1. $41°$
2. 30 cm
3. $35.2°$
4. $30°$
5. 32 μT, in a vertical plane $12°$ west of geographic meridian at an angle of $60°$ above the horizontal line.
6. 1.2×10^{3} e.m.u.
7. $82°$ west of magnetic north
8. $59°\ 56'$, 0.427 G
9. $\tan^{-1}(2)$
10. 0.45 G, 0.52 G

Test Your Concepts-III (Based on Tangent Law, Tangent Galvanometer and Vibration Magnetometer)

1. $42.2°$
2. $45°$
3. (a) 5.58 Am^2, (b) 4.30 s
4. $\frac{16}{9\sqrt{3}}$
5. $\sqrt{3}:1$
6. 50%
7. $30°$
8. 17 : 8
9. 10 oscillations/min

Test Your Concepts-IV (Based on Magnetic Properties of Materials)

1. 8.0 A
2. 4×10^{4} Am^{-1}
3. 3×10^{3} Am^{-1}
4. 11.5 A
5. 1.81×10^{-5} Wb
6. 0.015
7. 54000 J
8. 2.08×10^{-2} JT^{-1}
9. 5499

Single Correct Choice Type Questions

1. D	2. B	3. C	4. A	5. D	6. B	7. A	8. A	9. A	10. A
11. B	12. D	13. C	14. D	15. C	16. A	17. D	18. B	19. C	20. A
21. D	22. B	23. C	24. D	25. C	26. A	27. C	28. B	29. C	30. B
31. C	32. A	33. C	34. C	35. B	36. A	37. B	38. B	39. D	40. B

41. B	**42.** B	**43.** D	**44.** A	**45.** C	**46.** B	**47.** A	**48.** C	**49.** B	**50.** B
51. C	**52.** C	**53.** C	**54.** B	**55.** B	**56.** A	**57.** D	**58.** B	**59.** C	**60.** C
61. D	**62.** B	**63.** C	**64.** D	**65.** B	**66.** C	**67.** C	**68.** C	**69.** A	**70.** A
71. A	**72.** A	**73.** D	**74.** C	**75.** B	**76.** C	**77.** A	**78.** A	**79.** D	**80.** D
81. C	**82.** D	**83.** D	**84.** B	**85.** D	**86.** A	**87.** A	**88.** A	**89.** D	**90.** B
91. C	**92.** A	**93.** C							

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1. D	**2.** C	**3.** B	**4.** C	**5.** *	**6.** A	**7.** A	**8.** D	**9.** B	**10.** B
11. A	**12.** D	**13.** C							

* No given option is correct

CHAPTER 3

Electromagnetic Induction

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Magnetic Flux and Applications
(b) Faraday's Laws of EMI, Lenz's Law and Applications
(c) Motional EMF and Applications
(d) AC Generator
(e) Induced Electric Field and Applications
(f) Eddy Currents
(g) Inductor, Self-Inductance and Applications
(h) Growth and Decay of Current in Series LR Circuits
(i) Energy Stored in Inductor as Magnetic Field
(j) Mutual Inductance and Applications
(k) Inductors in Series and Parallel
(l) Oscillations in an LC Circuit

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION TO ELECTROMAGNETIC INDUCTION AND FARADAY'S LAWS

INTRODUCTION

Till now, we have studied that the electric fields and magnetic fields have been produced by stationary charges and moving charges (currents), respectively. Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field. One could then think whether or not an electric field could be produced by a magnetic field. In 1831, Michael Faraday discovered that, by varying magnetic field with time, an electric field could be generated. The phenomenon is known as **electromagnetic induction**. The term electromagnetic induction constitutes two phenomenon.

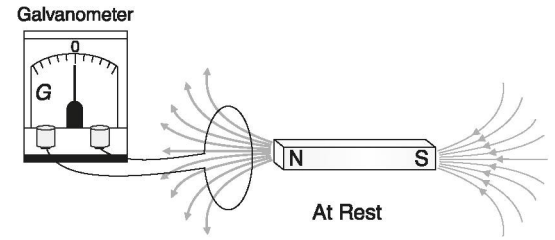
Phenomenon I: Involving the current induced in a conductor that moves relative to the field lines.

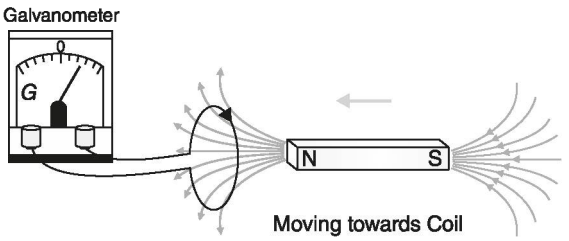
Phenomenon II: Involving the generation of an electric field associated with a time varying magnetic field.

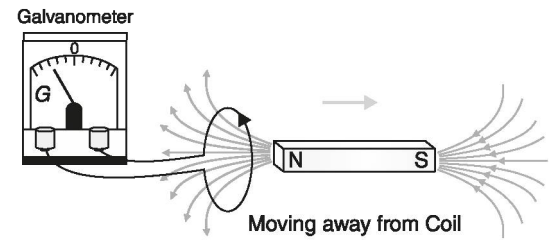
FARADAY'S EXPERIMENTS

Faraday observed for the set up shown that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop.

However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop. In particular, the galvanometer deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away.



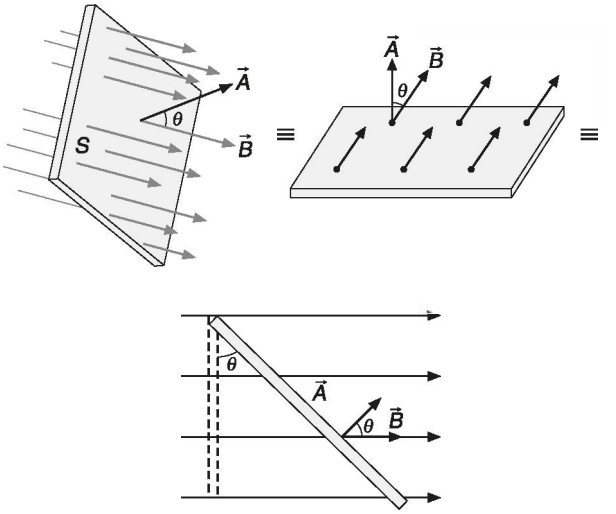




Faraday's experiment demonstrates that an electric current is induced in the loop by changing the magnetic field. The coil behaves as if it were connected to an emf source. Experimentally it is found that the induced emf depends on the rate of change of magnetic flux through the coil. So, let us first understand the concept of magnetic flux which is very similar to the concept of electric flux (discussed earlier).

MAGNETIC FLUX

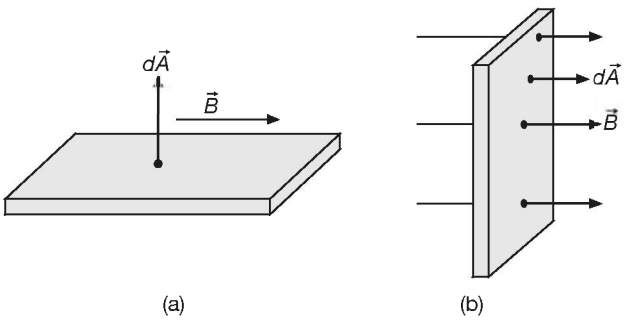
Consider a uniform magnetic field passing through a surface S, as shown in Figure. Then magnetic flux is defined in a manner similar to the one used to define electric flux.



Let the area vector be $\vec{A} = A\hat{n}$, where A is the area of the surface and $\hat{n}$ its unit normal. If θ be the angle between $\vec{B}$ and $\hat{n}$, then the magnetic flux through the surface is given by

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

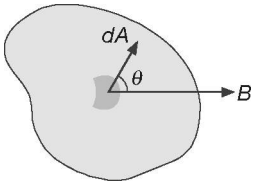
If the magnetic field is parallel to the plane, as in Figure, then $\theta = 90°$ and the flux through the plane is zero. If the field is perpendicular to the plane, as in Figure, then $\theta = 0$ and the flux through the plane is BA (the maximum value).



(a) The flux through the plane is zero when the magnetic field is parallel to the plane surface.
(b) The flux through the plane is maximum when the magnetic field is perpendicular to the plane.

The unit of magnetic flux is Tm^2, which is defined as a weber (Wb), so $1\ Wb = 1\ Tm^2$. The flux is maximum, when $\theta = 0°$ i.e., $\phi_{max} = BA$ and the flux is minimum when $\theta = 180°$ i.e., $\phi_{min} = -BA$.

For an arbitrarily shaped surface consider an infinitesimal element of area dA on it, as shown in Figure.

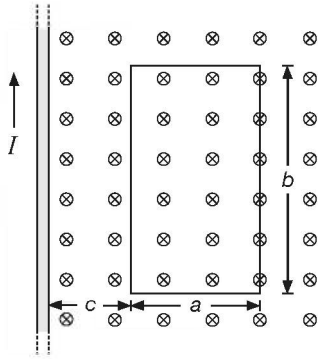


If the magnetic field at this element is $\vec{B}$, the infinitesimal magnetic flux through the element is $d\phi_B = \vec{B}\cdot d\vec{A}$, where $d\vec{A}$ is a vector that is outward normal to the surface and has a magnitude equal to the area dA. Therefore, the total magnetic flux ϕ_B through the surface is

$$\phi_B = \int \vec{B}\cdot d\vec{A}$$

ILLUSTRATION 1

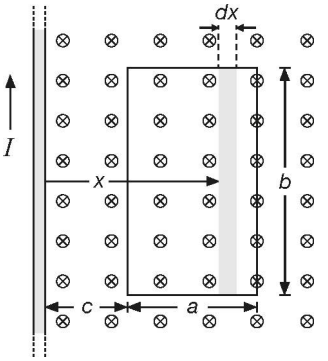
A rectangular loop of width a and length b is located near a long wire carrying a current I. The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire. Suppose we move the loop in Figure very far away from the wire. What happens to the magnetic flux?



SOLUTION

We know that the magnitude of the magnetic field created by the wire at a distance x from the wire is

$$B = \frac{\mu_0 I}{2\pi x}$$



The factor $\frac{1}{x}$ indicates that the field varies over the loop, and Figure shows that the field is directed into the page at the location of the loop. Since $\vec{B}$ is parallel to $d\vec{A}$ at any point within the loop, the magnetic flux through an area element dA is

$$\phi_B = \int B\,dA\cos(0^\circ) = \int\left(\frac{\mu_0 I}{2\pi x}\right)dA \quad \left\{\because\ B = \frac{\mu_0 I}{2\pi r}\right\}$$

Consider an infinitesimal strip of length b, width dx at a distance x from the wire then the area of the strip is $dA = bdx$. If $d\phi_B$ is the flux associated with the strip due to the field of wire, then

$$d\phi_B = BdA$$

$$\phi_B = \frac{\mu_0 Ib}{2\pi}\int_c^{a+c}\frac{dx}{x} = \frac{\mu_0 Ib}{2\pi}\log_e x\Big|_c^{a+c}$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ib}{2\pi}\log_e\left(\frac{a+c}{c}\right) = \frac{\mu_0 Ib}{2\pi}\log_e\left(1+\frac{a}{c}\right) \quad \ldots(1)$$

The flux should become smaller as the loop moves into weaker and weaker fields.

As the loop moves far away, the value of c is much larger than that of a, so that $\frac{a}{c}\to 0$. So, the natural logarithm in equation (1) approaches zero value, because

$$\log_e\left(1+\frac{a}{c}\right)\longrightarrow \log_e(1+0) = \log_e(1) = 0$$

and we find that $\phi_B \to 0$, as expected.

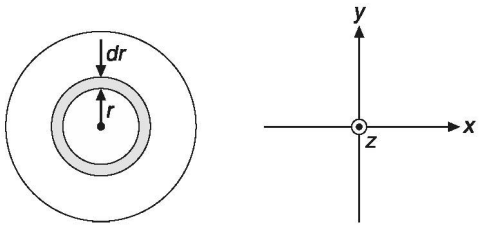
ILLUSTRATION 2

In a region of space varies, the magnetic field varies with radial distance r as $\vec{B}=(B_0 r)\hat{k}$ where r is the perpendicular distance of a point (x, y, z) from z-axis. Calculate the magnetic flux associated with a circle of radius a, centred at origin and lying in x-y plane.

SOLUTION

Since the magnetic field varies with radial distance r, so we divide the circular area into small concentric rings. Consider one such ring of radius r thickness dr. Area of this infinitesimal ring is

$$dA = 2\pi r dr$$



Magnetic flux $d\phi$ associated with this infinitesimal ring is

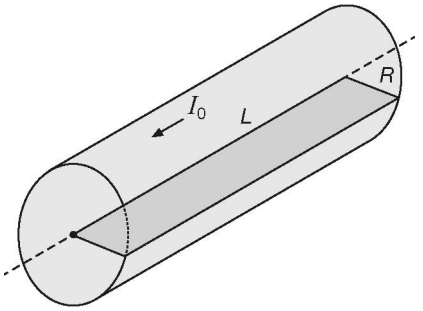
$$d\phi = \vec{B}.d\vec{A} = (B_0 r)(2\pi r dr) = 2\pi B_0 r^2 dr$$

So, total flux associated with the loop is

$$\phi = \int d\phi = 2\pi B_0 \int_0^a r^2 dr = \frac{2\pi B_0 a^3}{3}$$

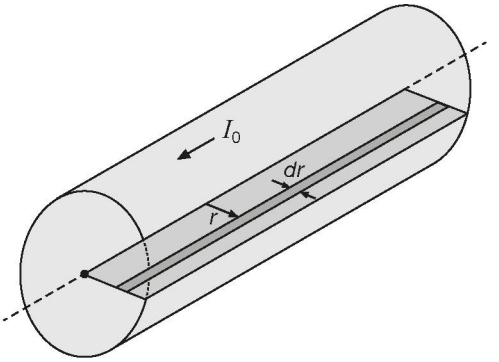
ILLUSTRATION 3

A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire. Calculate the magnetic flux through a rectangle that has one side of length L running down the centre of the wire and another side of length R, as shown in Figure.



SOLUTION

Let us find the magnetic field at a distance r from the centre of the wire and divide the rectangle into narrow infinitesimal strips of width dr. The magnetic flux through each infinitesimal strip will then be integrated to get the total flux.



From Ampere's Law, the field inside the wire is

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

The field B is normal to the area of the strip Ldr, so angle between $\vec{B}$ and $d\vec{A}$ is $0°$. Hence,

$$d\phi_B = \vec{B}\cdot d\vec{A} = \left(\frac{\mu_0 Ir}{2\pi R^2}\right)(Ldr)\cos(0°)$$

$$\phi_B = \int d\phi = \frac{\mu_0 IL}{2\pi R^2}\int_0^R r\,dr$$

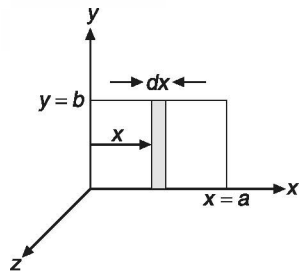
$$\Rightarrow \quad \phi_B = \frac{\mu_0 IL}{4\pi}$$

ILLUSTRATION 4

The magnetic field in a region of space varies with x as $\vec{B}=(\alpha+\beta x)\hat{k}$ where a is a constant. Calculate the magnetic flux associated with a surface lying in x-y plane and bound by the lines $x=0$, $y=0$ and $x=a$, $y=b$.

SOLUTION

We observe that the magnetic field varies over the surface in magnitude. So, let us first divide the area into infinitesimal elements. Consider a shaded strip so chosen that the field does not vary over the strip. The magnetic flux associated with the infinitesimal strip is



$$d\phi = \vec{B}\cdot d\vec{A} = (\alpha + \beta x)\,b\,dx$$

$$\phi = \int d\phi = \int_0^a (\alpha + \beta x)\,b\,dx$$

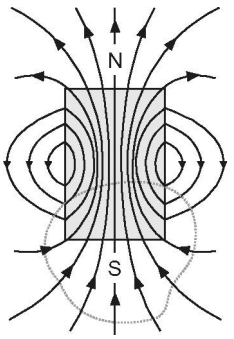
$$\Rightarrow \quad \phi = \alpha b a + \beta b \frac{a^2}{2}$$

$$\Rightarrow \quad \phi = \frac{ab}{2}(2\alpha + \beta a)$$

GAUSS'S LAW IN MAGNETISM

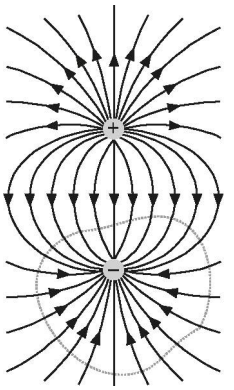
Since we know that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's Law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges.

The situation is quite different for magnetic fields, which are continuous and form closed loops. In other words, magnetic field lines do not begin or end at any point, as illustrated in Figure, which shows the magnetic field lines of a bar magnet.



The magnetic field lines of a bar magnet from closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dotted line represents the intersection of the surface with the page).

Note that for any closed surface, such as the one outlined by the dotted line in Figure, the number of lines entering the surface equals the number leaving the surface and thus, the net magnetic flux is zero. In contrast, for a closed surface surrounding one charge of an electric dipole, the net electric flux is not zero.



The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge. The electric flux through a closed surface surrounding one of the charges is not zero.

Gauss's Law in magnetism states that the net magnetic flux through any closed surface is always zero, i.e.,

$$\oint \vec{B}\cdot d\vec{A} = 0 \qquad \{\text{Gauss's Law in Magnetism}\}$$

This statement is based on the experimental fact, mentioned at the beginning of the previous chapter, that isolated magnetic poles (monopoles) have never been detected and perhaps do not exist.

FARADAY'S LAWS OF ELECTRO-MAGNETIC INDUCTION

On the basis of experimental observations Faraday summarised the phenomenon of electromagnetic induction by giving following laws.

(a) Whenever magnetic flux linked with a closed coil changes, an induced emf (or induced current) is set up in the coil.

(b) This induced emf (or induced current) lasts as long as the change in magnetic flux continues.

(c) The magnitude of induced emf is proportional to the rate of change of magnetic flux linked with the circuit. If ϕ_B is magnetic flux linked with the circuit at any instant t, then induced emf

$$\xi \propto \frac{d\phi_B}{dt} \qquad \ldots(1)$$

(d) The nature of induced emf (or induced current, if circuit is closed) is such that it opposes the change in flux that produces it. This law is also called **Lenz's Law**. So, according to Lenz's Law **"the induced current always produces a magnetic field which tends to oppose the change in magnetic flux that produces the induced current".**

In view of Lenz's Law, equation (1) takes the form

$$\xi = -\frac{d\phi_B}{dt}$$

where the 'negative' sign indicates the opposing nature of induced emf

Conventionally, the change in flux is given with one turn and if the coil contains N turns, then

$$\xi = -N\frac{d\phi_B}{dt}$$

Now, since we know that $\phi_B = BA\cos\theta$, so we have

$$\xi = -\frac{d}{dt}(BA\cos\theta)$$

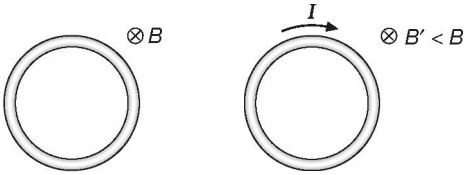
$$\xi = -(A\cos\theta)\left(\frac{dB}{dt}\right) - (B\cos\theta)\left(\frac{dA}{dt}\right) - (AB)\frac{d}{dt}(\cos\theta)$$

where all of B, A and θ are assumed to be varying with time.

$$\Rightarrow \quad \xi = -A\left(\frac{dB}{dt}\right)\cos\theta - B\left(\frac{dA}{dt}\right)\cos\theta + (BA\sin\theta)\frac{d\theta}{dt}$$

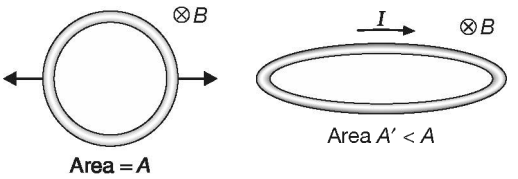
Thus, we see that an emf may be induced in the following ways, by varying the

(a) magnitude of $\vec{B}$ with time (illustrated in Figure)



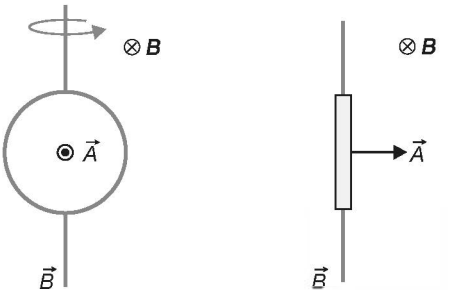
Inducing emf by changing the magnetic field strength

(b) magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)



Inducing emf by changing the area of the loop

(c) angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)



Inducing emf by varying the angle between $\vec{B}$ and $\vec{A}$

LENZ'S LAW: REVISITED

According to **Lenz's Law,** the nature of induced emf (or direction of induced current, if circuit is closed) is such that it opposes the change in flux that produces it. So, according to Lenz's Law "the induced current always produces a magnetic field which tends to oppose the change in magnetic flux that produces the induced current".

Mathematically, we have

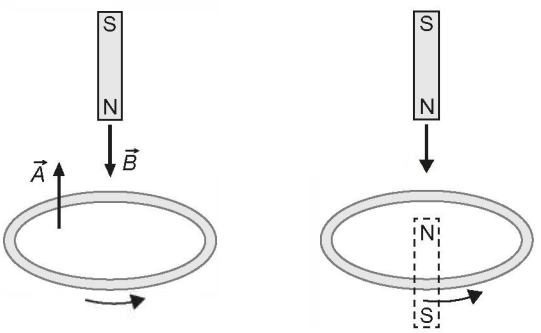
$$\xi = -\frac{d\phi_B}{dt}$$

where the 'negative' sign indicates the opposing nature of induced emf

Conventionally, the change in flux is given with one turn and if the coil contains N turns, then

$$\xi = -N\frac{d\phi_B}{dt}$$

As an example, to illustrate how Lenz's Law may be applied, consider the situation where a bar magnet is moving toward a conducting loop with its north pole down, as shown in Figure.



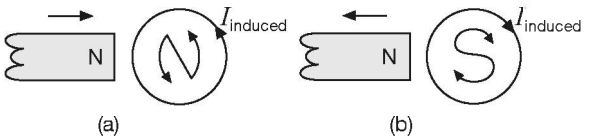
(a) A bar magnet moving toward current loop.
(b) Determination of the direction of induced current by considering the magnetic force between the bar magnet and the loop.

The direction of the induced current can be determined from the point of view of magnetic force. Lenz's Law states that the induced emf must be in the direction that opposes the change. Therefore, as the bar magnet approaches the loop, it experiences a repulsive force due to the induced emf. Since like poles repel, the loop must behave as if it were a bar magnet with its north pole pointing up. Using the right-hand thumb rule, the direction of the induced current is counter clockwise, as seen from above.

LENZ'S LAW IN ACCORDANCE WITH LAW OF CONSERVATION OF ENERGY

Lenz's Law is based on Law of Conservation of Energy and it gives the nature of induced emf or direction of induced current in the coil.

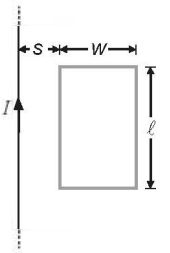
When north pole of a magnet is moved towards the coil, the induced current flows in a direction so as to oppose the motion of the magnet towards the coil. This is only possible when nearer face of the coil acts as a magnetic north pole which makes an anticlockwise current to flow in the coil. Then the repulsion between two similar poles opposes the motion of the magnet towards the coil.



Similarly, when the magnet is moved away from the coil, the direction of induced current is such as to make the nearer face of the coil as a south pole which makes a clockwise induced current to flow in the coil. Then the attraction between two opposite poles opposes the motion of the magnet away from the coil. In either case, therefore work has to be done in moving the magnet. It is this mechanical work, which appears as electrical energy in the coil. Hence the production of induced emf or induced current in the coil is in accordance with the Law of Conservation of Energy.

ILLUSTRATION 5

An infinite straight wire carrying a current I, varying with time t as $I(t) = a + bt$, is placed to the left of a rectangular loop of wire with width w and length l, as shown in Figure.



Assuming a and b to be positive constants, calculate the induced emf in the loop and the direction of the induced current?

SOLUTION

For calculating the induced emf we must first calculate the flux associated with the loop due to field of current carrying wire.

The magnetic field due to a current-carrying wire at a distance r away is

$$B = \frac{\mu_0 I}{2\pi r}$$

The total magnetic flux ϕ_B through the loop can be obtained by summing over contributions from all differential area elements $dA = ldr$. So,

$$\phi_B = \int d\phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Il}{2\pi} \int_s^{s+w} \frac{dr}{r} = \frac{\mu_0 Il}{2\pi} \log_e \left(\frac{s+w}{s} \right)$$

According to Faraday's Law, the induced emf is

$$\xi = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Il}{2\pi} \log_e \left(\frac{s+w}{s} \right) \right]$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 l}{2\pi}\log_e\left(\frac{s+w}{s}\right)\frac{dI}{dt}$$

Given that, $I = I(t) = a + bt$

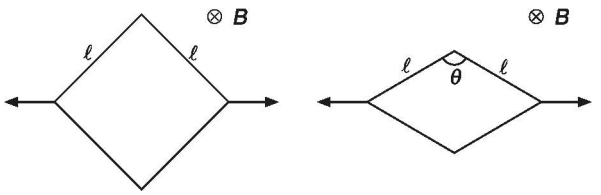
$$\Rightarrow \quad \frac{dI}{dt} = b$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 bl}{2\pi}\log_e\left(\frac{s+w}{s}\right)$$

The straight wire carrying a current I produces a magnetic flux into the page through the rectangular loop. By Lenz's Law, the induced current in the loop must be flowing *counterclockwise* in order to produce a magnetic field out of the page to counteract the increase in inward flux.

ILLUSTRATION 6

A square loop with length l on each side is placed in a uniform magnetic field pointing into the page. During a time, interval, Δt, the loop is pulled from its two edges and turned into a rhombus, as shown in Figure.



Conducting loop changing area

Assuming that the total resistance of the loop is R, find the average induced current in the loop and its direction.

SOLUTION

Using Faraday's Law, we have

$$\xi = -\frac{\Delta\phi_B}{\Delta t} = -B\left(\frac{\Delta A}{\Delta t}\right)$$

Since the initial and the final areas of the loop are $A_i = l^2$ and $A_f = l^2\sin\theta$, respectively (recall that the area of a parallelogram defined by two vectors $\vec{l}_1$ and $\vec{l}_2$ is $A = |\vec{l}_1 \times \vec{l}_2| = l_1 l_2 \sin\theta$), the average rate of change of area is

$$\frac{\Delta A}{\Delta t} = \frac{A_f - A_i}{\Delta t} = -\frac{l^2(1-\sin\theta)}{\Delta t} < 0$$

$$\Rightarrow \quad \xi = \frac{Bl^2(1-\sin\theta)}{\Delta t} > 0$$

Thus, the average induced current is

$$I = \frac{\xi}{R} = \frac{Bl^2(1-\sin\theta)}{\Delta t R}$$

Since $\left(\frac{\Delta A}{\Delta t}\right) < 0$, the magnetic flux into the page decreases. Hence, the current flows in the clockwise direction to compensate the loss of flux.

ILLUSTRATION 7

A conducting loop of resistance R, has its radius varying with time t as $r = r_0 + \alpha t$, where α is a positive constant. This loop is placed in a region of uniform magnetic field $B\hat{j}$ such that its plane is perpendicular to the direction of magnetic field. Calculate the induced current flowing through the loop as a function of time.

SOLUTION

Since $\phi = \vec{B}.\vec{A} = BA\cos\theta$

$$\Rightarrow \quad \phi = B(\pi r^2)\cos 0°$$

$$\Rightarrow \quad \phi = B\pi(r_0 + \alpha t)^2$$

According to Lenz's Law, we have

$$\xi = -\frac{d\phi}{dt}$$

where, $\frac{d\phi}{dt} = B\pi 2(r_0 + \alpha t)\alpha$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = -2\alpha\pi B(r_0 + \alpha t)$$

The induced current is given by

$$i = \frac{|\xi|}{R}$$

$$\Rightarrow \quad i = \frac{2\alpha\pi B(r_0 + \alpha t)}{R}$$

As area increases, flux also increases, so the induced current tries to decrease the flux by setting up a magnetic field opposite to the applied field and hence induced current in the loop is clockwise when seen from above.

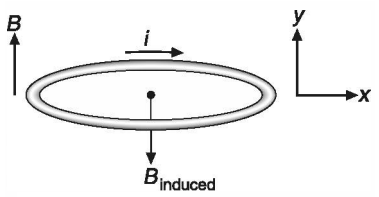
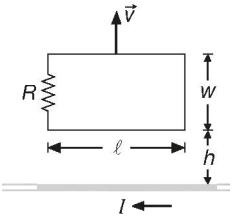


ILLUSTRATION 8

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{v}$ away from an infinitely long straight wire carrying a current I in the plane of the loop, as shown in Figure. Let the total resistance of the loop be R. What is the current in the loop at the instant the near side is a distance h from the wire? Can you derive the same result using $\xi = \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$?



SOLUTION

The magnetic field at a distance x from the straight wire is

$$B = \frac{\mu_0 I}{2\pi x}$$

The magnetic flux through a differential area element $dA = ldx$ of the loop is

$$d\phi_B = \vec{B}\cdot d\vec{A} = \frac{\mu_0 I}{2\pi x} ldx$$

where we have chosen the area vector to point into the page, so that $\phi_B > 0$. Integrating over the entire area of the loop, the total flux is

$$\phi_B = \frac{\mu_0 Il}{2\pi}\int_h^{h+w}\frac{dx}{x} = \frac{\mu_0 Il}{2\pi}\log_e\left(\frac{h+w}{h}\right)$$

Since, this loop is moving away from the wire so the flux when the lower end is at a distance y from the wire is

$$\phi_B = \frac{\mu_0 Il}{2\pi}\log_e\left(\frac{y+w}{y}\right)$$

Differentiating with respect to t, we obtain the induced emf as

$$\xi = -\frac{d\phi_B}{dt} = -\frac{\mu_0 Il}{2\pi}\frac{d}{dt}\left[\log_e\left(\frac{y+w}{y}\right)\right]$$

Since $\frac{d}{dt}\left[\log_e\left(\frac{y+w}{y}\right)\right] = \frac{y}{y+w}\frac{d}{dt}\left(\frac{y+w}{y}\right)$

$$= \frac{y}{y+w}\left(-\frac{w}{y^2}\right)\frac{dy}{dt} = \frac{w}{y(y+w)}\left(\frac{dy}{dt}\right)$$

Since $\frac{dy}{dt} = v \Rightarrow \xi = \frac{\mu_0 Il}{2\pi}\left(\frac{wv}{y(y+w)}\right)$

Using

$$\xi = \oint (\vec{v}\times\vec{B})\cdot d\vec{l} = vl\left[B(y) - B(y+w)\right]$$

where $B(y)$ is field at distance y and $B(y+w)$ is field at distance $(y+w)$, both from the wire

So, $B(y) = \frac{\mu_0 I}{2\pi y}$ and $B(y+w) = \frac{\mu_0 I}{2\pi(y+w)}$

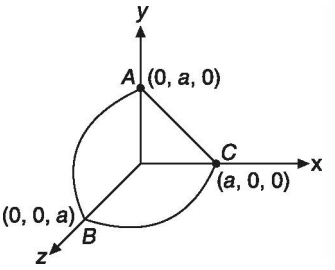
$$\Rightarrow \quad \xi = vl\left[\frac{\mu_0 I}{2\pi y} - \frac{\mu_0 I}{2\pi(y+w)}\right] = \frac{\mu_0 Il}{2\pi}\frac{vw}{y(y+w)}$$

The induced current is

$$I = \frac{|\xi|}{R} = \frac{\mu_0 Il}{2\pi R}\left(\frac{vw}{y(y+w)}\right)$$

ILLUSTRATION 9

A conducting loop $ABCA$ of resistance R_0 is lying in a region of space having magnetic field varying with time as $\vec{B} = (\hat{i}+\hat{j}+\hat{k})e^{-2t}$ shown in Figure.



The arc AB is a semicircle in y-z plane, arc BC is again a semicircle but in x-z plane and AC is a straight line lying in x-y plane. Calculate the flux associated with the loop and the induced current in the loop.

SOLUTION

Area of the loop $\vec{A} = \frac{\pi a^2}{4}\hat{i} + \frac{\pi a^2}{4}\hat{j} + \frac{a^2}{2}\hat{k}$

Flux associated with the loop is

$$\phi = \vec{B}.\vec{A}$$

$$\Rightarrow \quad \phi = \left(\hat{i}+\hat{j}+\hat{k}\right)e^{-2t}\cdot\left(\frac{\pi a^2}{4}\hat{i} + \frac{\pi a^2}{4}\hat{j} + \frac{a^2}{2}\hat{k}\right)$$

$$\Rightarrow \quad \phi = \left(\frac{\pi}{4}+\frac{\pi}{4}+\frac{1}{2}\right)e^{-2t}a^2 = \left(\frac{\pi}{2}+\frac{1}{2}\right)a^2e^{-2t}$$

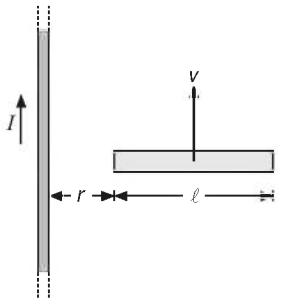
$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = 2a^2\left(\frac{\pi}{2}+\frac{1}{2}\right)e^{-2t} = a^2(\pi+1)e^{-2t}$$

So, induced current is

$$i = \frac{\xi}{R_0} = \frac{2a^2}{R_0}\left(\frac{\pi}{2}+\frac{1}{2}\right)e^{-2t} = \frac{a^2}{R_0}(\pi+1)e^{-2t}$$

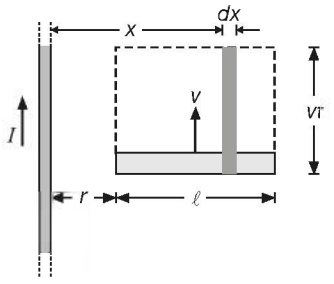
ILLUSTRATION 10

A conducting rod of length l moves with velocity $\vec{v}$ parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away, as shown in Figure. Find the magnitude of the emf induced in the rod.



SOLUTION

Let us find an expression for the flux through a rectangular area swept out by the bar in time t.



The magnetic field at a distance x from wire is

$$B = \frac{\mu_0 I}{2\pi x}$$

Since $\phi_B = \int BdA$

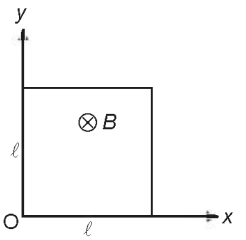
$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ivt}{2\pi}\int_r^{r+l}\frac{dx}{x}$$

where vt is the distance the bar has moved in time t. So, we get

$$|\xi| = \frac{d\phi_B}{dt} = \frac{\mu_0 Iv}{2\pi}\log_e\left(1+\frac{l}{r}\right)$$

ILLUSTRATION 11

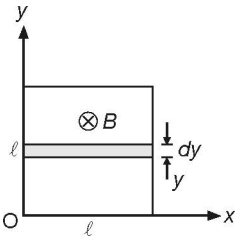
The Figure shows a square loop of wire with sides of length $l = 2$ cm.



A magnetic field points into the page and its magnitude is given by $B = at^2y$ where $a = 4\ \text{Tm}^{-1}\text{s}^{-2}$, B is in tesla, t is in second and y is in metre. Determine the emf induced in the loop at $t = 2.5$ s.

SOLUTION

Here we note that the field is a simultaneous function of y and t. Let us first calculate the flux associated with the loop. For that let us consider a rectangular strip of length l, thickness dy at a distance y from the x-axis.



Then the flux associated with this strip is

$$d\phi = BdA = Bldy$$

$$\Rightarrow \quad d\phi = at^2 y l dy$$

$$\Rightarrow \quad \phi = \int d\phi = at^2 l \int_0^l y dy$$

$$\Rightarrow \quad \phi = \frac{at^2 l^3}{2} \qquad \left\{ \because \int_0^l y dy = \frac{l^2}{2} \right\}$$

Now, according to Faraday's Laws, we have

$$\xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = -\frac{al^3}{2}(2t)$$

$$\Rightarrow \quad \xi = -al^3 t$$

$$\Rightarrow \quad |\xi| = (4)(8\times 10^{-6})(2.5)$$

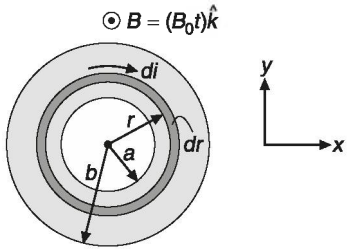
$$\Rightarrow \quad |\xi| = 80\ \mu V$$

ILLUSTRATION 12

In a region of space, there exists a uniform magnetic field $\vec{B} = (B_0 t)\hat{k}$. A ring with square cross-section made of a conducting material of conductivity ρ is placed such that it lies in the x-y plane. The thickness of the ring is h and its inner and outer radii are equal to a and b respectively. Neglecting the inductance of the ring, calculate the current induced in the loop.

SOLUTION

Consider a thin ring of radius r and thickness dr as shown in Figure.



The resistance of this ring is $R = \frac{\rho l}{A}$

$$\Rightarrow \quad R = \frac{\rho(2\pi r)}{hdr}$$

The magnitude of induced emf in this ring is

$$|\xi| = \frac{d\phi}{dt} = A\frac{dB}{dt} = \pi r^2 B_0 \frac{dt}{dt} = \pi r^2 B_0$$

The current flowing is

$$di = \frac{\xi}{R} = \frac{\pi r^2 B_0}{\rho(2\pi r)} \times hdr$$

$$\Rightarrow \quad di = \frac{B_0 h}{2\rho} r dr$$

$$\Rightarrow \quad i = \frac{B_0 h}{2\rho} \int_a^b r dr$$

$$\Rightarrow \quad i = \frac{B_0 h}{4\rho}(b^2 - a^2)$$

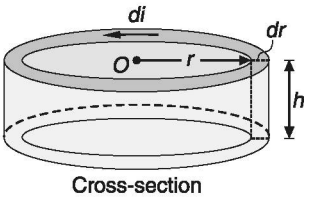


ILLUSTRATION 13

In a coil of resistance R, the magnetic flux due to an external magnetic field varies with time as $\phi = k(4 - t^2)$, where k is a positive constant. Calculate the total heat produced in the coil till the time flux becomes zero.

SOLUTION

EMF induced in coil is given as

$$\xi = \left|\frac{d\phi}{dt}\right| = 2kt$$

Induced current in coil is given as

$$i = \frac{\xi}{R} = \frac{2kt}{R}$$

Flux in the coil is zero at $t = 2$ s

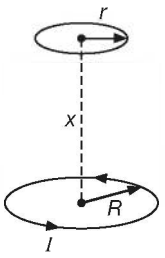
So, heat produced in coil is

$$H = \int_0^2 i^2 R dt = \int_0^2 \frac{4k^2 t^2}{R} dt$$

$$\Rightarrow \quad H = \frac{4k^2}{R}\left(\frac{t^3}{3}\Big|_0^2\right) = \frac{32k^2}{3R}$$

ILLUSTRATION 14

The Figure shows two parallel and co-axial loops. The smaller loop (radius r) is above the large loop (radius R) at a distance $x \gg R$. The magnetic field due to current I in the larger loop is nearly constant near the smaller loop. Suppose that x is increasing at the constant rate $\frac{dx}{dt} = v$.



(a) Determine the magnetic flux through the smaller loop as a function of x.

(b) Find the induced emf and the direction of induced current in the smaller loop at the instant when separation between them is X.

SOLUTION

(a) Since the magnetic field at the axis of the coil at a point lying a distance x from the centre is

$$B = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{3/2}}$$

Due to this field of the bigger loop, the flux associated with the smaller loop is

$$\phi = BA = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{3/2}}\left(\pi r^2\right)$$

Since $x \gg R$, so

$$\phi \simeq \frac{\mu_0 \pi IR^2 r^2}{2x^3}$$

(b) When x increases at a constant rate, then let its value at some later time t be X. So, we have

$$X = x + vt$$

$$\Rightarrow \quad \phi = BA = \frac{\mu_0 \pi IR^2 r^2}{2X^3}$$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 \pi IR^2 r^2}{2}\frac{d}{dt}\left(X^{-3}\right) = \frac{3}{2}\frac{\mu_0 IR^2 r^2}{X^4}\left(\frac{dX}{dt}\right)$$

$$\Rightarrow \quad \xi = \frac{3}{2}\frac{\mu_0 IR^2 r^2 v}{X^4} \qquad \left\{\because \frac{dX}{dt} = v\right\}$$

$$\Rightarrow \quad \xi = \frac{3}{2}\frac{\mu_0 IR^2 r^2 v}{(x+vt)^4}$$

The direction of the induced current is clockwise when seen from the bigger loop.

ILLUSTRATION 15

A long straight solenoid of cross-sectional diameter d and with n turns per unit of its length has a round turn of copper wire of cross-sectional area A and density ρ is tightly put on its winding. Find the current flowing in the turn if the current in the solenoid winding is increased with a constant rate I ampere per second.

SOLUTION

The magnetic field inside the solenoid is given as

$$B = \mu_0 ni$$

The flux through its cross-section of copper winding is given as

$$\phi = BA = \left(\mu_0 ni\right)\frac{\pi d^2}{4}$$

EMF induced in the copper wire is given as

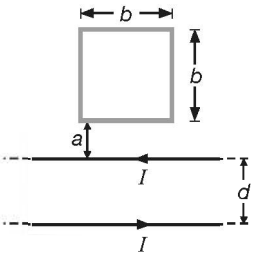
$$e = \left|\frac{d\phi}{dt}\right| = \mu_0 n\pi \frac{d^2}{4}\frac{di}{dt}$$

Current in the copper wire is given as

$$I_C = \frac{e}{R} = \frac{\mu_0 n\pi d^2 I}{4\rho\left(\frac{\pi d}{A}\right)} = \frac{\mu_0 ndAI}{4\rho}$$

ILLUSTRATION 16

Two fixed long straight wires carry the same current I in opposite directions as shown in Figure. A square loop of side b is fixed in the plane of the wires with its length parallel to one wire at a distance a as shown in Figure.



Calculate the induced emf in the loop if the current in both wires is changing at the rate $\frac{dI}{dt}$. What is the direction of force on the loop if $\frac{dI}{dt}$ is positive?

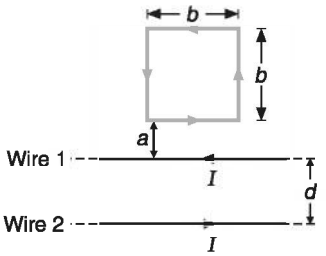
SOLUTION

(a) Flux associated with the square loop due to wire 1 is

$$\phi_{\text{wire 1}} = \frac{\mu_0 Ib}{2\pi}\int_a^{a+b}\frac{dx}{x} = \frac{\mu_0 Ih}{2\pi}\log_e\left(\frac{a+b}{a}\right)$$

Flux associated with the square loop due to wire 2 is

$$\phi_{\text{wire 2}} = \frac{\mu_0 Ib}{2\pi}\int_{a+d}^{a+d+b}\frac{dx}{x} = \frac{\mu_0 Ib}{2\pi}\log_e\left(\frac{a+d+b}{a+d}\right)$$



Net flux associated with the square loop is

$$\phi = \phi_{\text{wire 1}} - \phi_{\text{wire 2}}$$

$$\Rightarrow \phi = \frac{\mu_0 Ib}{2\pi}\log_e\left(\frac{a+b}{a}\right) - \frac{\mu_0 Ib}{2\pi}\log_e\left(\frac{a+d+b}{a+d}\right)$$

$$\Rightarrow \phi = \frac{\mu_0 Ib}{2\pi}\log_e\left[\frac{(a+b)(a+d)}{a(a+d+b)}\right]$$

$$\Rightarrow \xi = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 b}{2\pi}\left[\log_e\left(\frac{(a+b)(a+d)}{a(a+d+b)}\right)\right]\left(\frac{dI}{dt}\right)$$

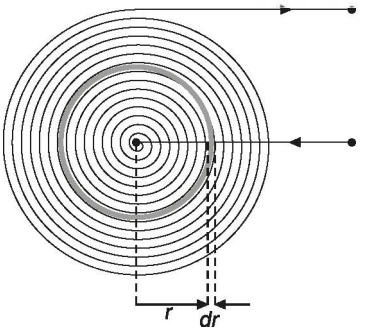
(b) Net field through the loop is in the inward ($\otimes$) direction. If $\frac{dI}{dt}$ is positive, the induced current in the loop is anticlockwise as I is increasing. So, loop will be repelled from the wires, because wires 1 and 2 are close to each other and current in these two wires are in opposite direction.

ILLUSTRATION 17

A plane spiral wound tightly, having large number of turns N, is placed in a uniform magnetic field perpendicular to its plane. The outer radius of the spiral's turns is equal to a. The magnetic induction varies with time as $B = B_0 \sin\omega t$, where B_0 and ω are constants. Calculate the amplitude of emf induced in the spiral.

SOLUTION

We consider an elemental circular strip of radius r and radial width dr in the spiral as shown in Figure.



If dN be the number of turns in this infinitesimal spiral strip, then

$$dN = \frac{N}{a}dr$$

The emf induced across this elemental strip is

$$d\xi = \left|\frac{d\phi}{dt}\right| = \pi r^2 (dN)\frac{dB}{dt}$$

$$\Rightarrow d\xi = \pi r^2 (dN)\frac{d}{dt}(B_0 \sin\omega t)$$

$$\Rightarrow d\xi = \pi r^2 (dN) B_0 \omega \cos(\omega t)$$

$$\Rightarrow d\xi = \left(\frac{N}{a}dr\right)\pi r^2 B_0 \omega \cos(\omega t)$$

$$\Rightarrow \xi = \int d\xi = \int_0^a \left(\frac{N}{a}dr\right)\pi r^2 B_0 \omega \cos(\omega t)$$

$$\Rightarrow \xi = \int d\xi = \frac{N\pi B_0 \omega \cos(\omega t)}{a}\int_0^a r^2 dr$$

$$\Rightarrow \xi = \frac{1}{3}N\pi B_0 a^2 \omega \cos(\omega t) = \xi_0 \cos\omega t$$

Hence the amplitude of induced emf in the spiral is

$$\xi_0 = \frac{1}{3}\pi N a^2 B_0 \omega$$

Conceptual Note(s)

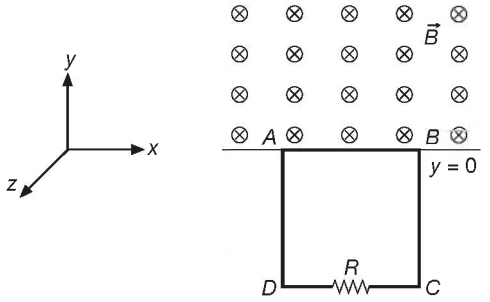
Please understand that, the statement says that the coil is placed in uniform field and at the same time the problem says that the field is varying with time as $B = B_0 \sin\omega t$. This creates a lot of confusion amongst the students. Please don't get confused and let me take the privilege to explain this thing to you.

Actually, according to the problem the field is constant in space but varying with time.

We should understand that for a conducting loop placed in a field, the emf is induced in the loop when the field is time varying (space variation i.e. uniform or non-uniform in space doesn't matter) and emf will not be induced in the loop if it is placed in constant time independent field (uniform or non-uniform in space doesn't matter).

ILLUSTRATION 18

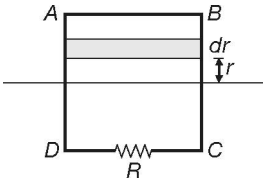
A square loop $ABCD$ of side a is moving in the xy-plane with velocity $\vec{v} = \beta t\hat{j}$.



A non-uniform magnetic field $\vec{B} = -B_0\left(1+\alpha y^2\right)\hat{k}$ exists in the space $y > 0$, where B_0 and α are constants. Initially, the upper wire of the loop is at $y = 0$. Find the induced voltage across the resistance R as a function of time. Neglect the magnetic force due to the induced current.

SOLUTION

Consider an infinitesimal strip shown in Figure.



Flux associated with the strip is

$$d\phi = B(ldr) = B_0\left(1+\alpha r^2\right)l\,dr$$

Total flux in the loop is

$$\phi = \int_0^y B_0\left(1+\alpha r^2\right)ldr$$

$$\Rightarrow \quad \phi = B_0 l\left(y + \frac{\alpha y^3}{3}\right)$$

Induced emf is given by

$$\xi = \frac{-d\phi}{dt} = -B_0 l\left(1 + \frac{3\alpha y^2}{3}\right)\frac{dy}{dt}$$

$$\Rightarrow \quad \xi = -B_0 l\left(1+\alpha y^2\right)\frac{dy}{dt} \qquad \ldots(1)$$

Given, $v = \beta t$

$$\Rightarrow \quad \frac{dy}{dt} = \beta t$$

$$\Rightarrow \quad \int_0^y dy = \int_0^t \beta t\,dt$$

$$\Rightarrow \quad y = \frac{\beta t^2}{2}$$

$$\Rightarrow \quad E = -B_0 l\left(1 + \frac{\alpha\beta^2 t^4}{4}\right)\beta t$$

$$\Rightarrow \quad E = -B_0 l\beta\left(t + \frac{\alpha\beta^2 t^5}{4}\right)$$

CHARGE INDUCED IN THE CIRCUIT

Since $\xi = -N\frac{d\phi}{dt}$

If R is resistance of circuit, then current induced is

$$I = \frac{\xi}{R} = -\frac{N}{R}\frac{d\phi}{dt}$$

The charge induced in time dt is given by

$$dq = Idt = -\frac{N}{R}\frac{d\phi}{dt}dt = -\frac{N}{R}d\phi$$

$$\Rightarrow \quad q = \int_0^q dq = -\frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi$$

$$\Rightarrow \quad |q| = \left(\frac{N}{R}\right)\Delta\phi$$

$$\Rightarrow \quad q = \frac{\text{Net Change in flux}}{\text{Resistance}}$$

So, we observe that the charge induced is independent of time.

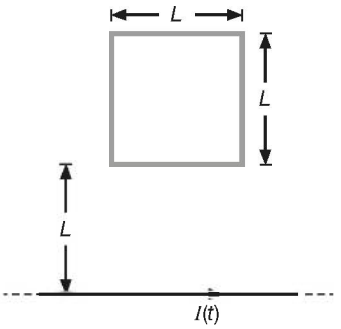
ILLUSTRATION 19

A square loop of side L, resistance R, lies at a distance L from the wire carrying a current $I(t)$ which decreases gradually with time t as

$$I(t) = \begin{cases} (1-\alpha t)I_0 & \text{for } 0 \le t \le \dfrac{1}{\alpha} \\ 0 & \text{for } t > \dfrac{1}{\alpha} \end{cases}$$

(a) In what direction does the induced current in the square loop flow?

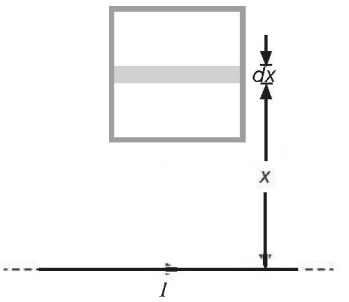
(b) What total charge passes through a given point in the loop during the time this current flow?



SOLUTION

(a) Since we observe that I decreases with t, hence B also decreases with time. So, the induced current must be set up in the sense which does not allow B to decrease (i.e. the induced current must give an outward pointing field), so I_{induced} must be in the counter clockwise direction.

(b) Consider an infinitesimal strip shown in Figure.



Flux associated with the strip is

$$d\phi = \frac{\mu_0}{2\pi}\frac{IL}{x}dx$$

$$\Rightarrow \quad \phi = \int_{L}^{2L} \frac{\mu_0}{2\pi}\frac{IL}{x}dx$$

$$\Rightarrow \quad \phi = \frac{\mu_0 IL}{2\pi}\log_e(2)$$

Please note here that ϕ depends on I which is a function of t (but not x).

Since, $I_i = I_0$

$$\Rightarrow \quad \phi_i = \frac{\mu_0 I_0 L}{2\pi}\log_e(2)$$

and $I_f = 0$

$$\Rightarrow \quad \phi_f = 0$$

$$\Rightarrow \quad |\Delta\phi| = \frac{\mu_0 I_0 L}{2\pi}\log_e(2)$$

Since, the charge flown through the circuit is given by

$$|\Delta q| = \frac{1}{R}|\Delta\phi|$$

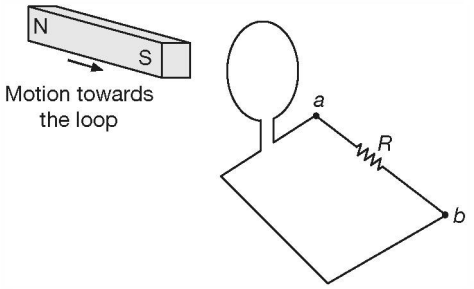
$$\Rightarrow \quad \Delta q = \frac{|\Delta\phi|}{R} = \frac{\mu_0 I_0 L}{2\pi R}\log_e(2)$$

Test Your Concepts-I

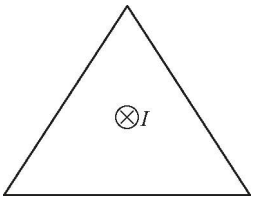
Based on Magnetic Flux, Faraday's Laws and Induced EMF

(Solutions on page H.127)

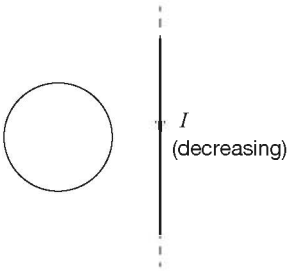
1. In Figure, the bar magnet is being moved towards the loop. Will $V_a - V_b$ be positive, negative, or zero? Explain.



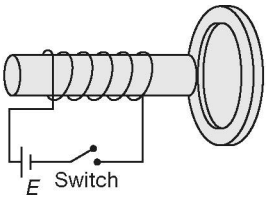
2. A current carrying straight wire passes through a triangular coil as shown in Figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop if current in the wire is increased.



3. A circular loop is placed near a current carrying conductor as shown. Find the direction of induced current if the current in the wire is decreasing.



4. A metal ring is placed near a solenoid, as shown in Figure. Find the direction of the induced current in the ring

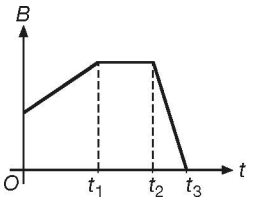


(a) at the instant the switch in the circuit containing the solenoid is thrown closed.

(b) after the switch has been closed for several seconds, and

(c) at the instant the switch is just opened.

5. A flat coil is oriented with the plane of its area at right angles to a uniform magnetic field. The magnitude of this field varies with time according to the graph in Figure. Draw a graph of the emf induced in the coil as a function of time. Please do not forget to plot and identify the times t_1, t_2 and t_3 on your graph.



6. Two bulbs are connected to opposite sides of a circular loop of wire, as shown in Figure. A changing magnetic field (confined to the smaller circular area shown in the Figure) induces an emf in the loop that causes the two bulbs to light. Now, when the switch is closed, the resistance-free wires connected to the switch short out bulb 2 and it goes off. What will happen, if the switch remain connected at points *a* and *b*, but the switch and the wires are lifted up and moved to the other side of the field, as in Figure. The wire is still connected to bulb 2 as it was before, so does it continue to stay dark?

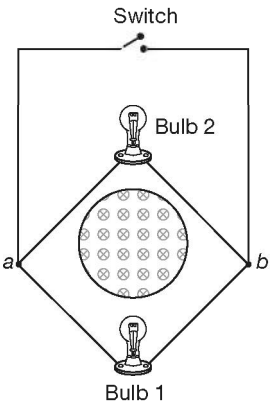


Figure (a)

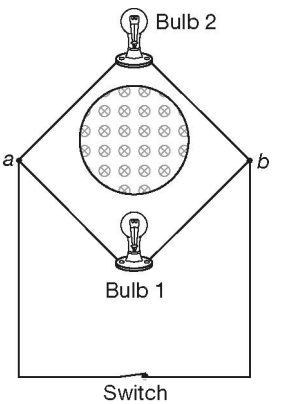
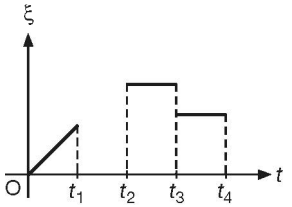


Figure (b)

7. A coil is stationary in a region of uniform, external, time varying magnetic field. The emf induced in this coil as a function of time is shown in Figure. Sketch a clear qualitative graph of the external magnetic field as a function of time, given that it started from zero. Include the points t_1, t_2, t_3 and t_4 on your graph.

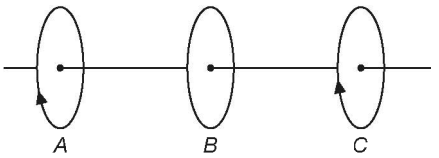


8. A particle having mass 2×10^{-16} kg and charge 30 nC starts from rest, is accelerated by a strong electric field, and is fired from a small source inside a region of uniform constant magnetic field 0.6 T. The velocity of the particle is perpendicular to the field. The circular orbit of the particle encloses a magnetic flux of 15 μWb.

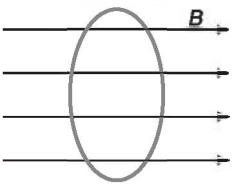
(a) Calculate the speed of the particle.

(b) Calculate the potential difference through which the particle accelerated inside the source.

9. Three identical closed coils *A*, *B* and *C* are placed with their planes parallel to one another. Coils *A* and *C* carry equal currents as shown in Figure. Coils *B* and *C* are fixed in position and coil *A* is moved towards *B* with uniform motion. Is there any induced current in *B*? If no, give reasons. If yes, mark the direction of the induced current in the diagram.



10. A circular loop of wire of radius *r* is in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field. The magnetic field varies with time according to $B(t)=a+bt$, where *a* and *b* are constants.



(a) Calculate the magnetic flux through the loop at $t=0$.

(b) Calculate the emf induced in the loop.

(c) If the resistance per unit length of the loop is λ, what is the induced current?

(d) At what rate is energy being delivered to the resistance of the loop?

11. A closed coil consists of 500 turns on a rectangular frame of area 4 cm^2 and has a resistance of 50 Ω. The coil is kept with its plane perpendicular to a uniform magnetic field of 0.2 Wbm^{-2}. Calculate the amount of charge flowing through the coil if it is rotated through 180°.

12. A wire in the form of a circular loop of radius 10 cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s, find the average induced emf in the loop.

13. In a region of space, a non-uniform magnetic field $B=\left(B_0y^2\right)\hat{i}$ is present. The field is restricted to positive *y*-*z* plane only. A rectangular loop of sides *a* and 2*a* is placed in *y*-*z* plane with one vertex at origin, smaller side along *y*-axis and longer side along *z*-axis. The loop starts moving along +*y*-axis with a uniform speed *v*. Find the induced emf as a function of time.

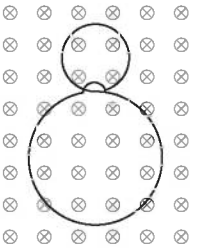
14. A circular coil of radius 4 cm, 30 turns and resistance 1 Ω is placed in a magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies with time according to the expression $B=(0.01)t+(0.04)t^2$, where *t* is in seconds and *B* is in tesla. Calculate the emf induced in the coil at $t=5$ s.

15. A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.8 s, what is the magnitude of the induced emf in the coil while the field is changing?

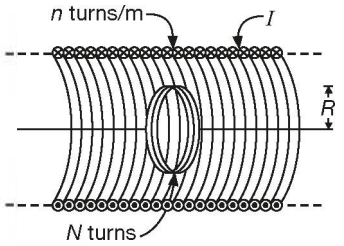
16. A loop of wire enclosing an area *A* is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{B}$ varies in time according to the expression $B=B_0e^{-at}$, where B_0 and *a* are constants. Find the induced emf in the loop as a function of time.

17. A piece of insulated wire is shaped into a digit 8, as shown in Figure. The radius of the upper circle is 5 cm and that of the lower circle is 9 cm. The wire

has a uniform resistance per unit length of $3\ \Omega m^{-1}$. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown. The magnetic field is increasing at a constant rate of $2\ Ts^{-1}$. Find the magnitude and direction of the induced current in the wire.

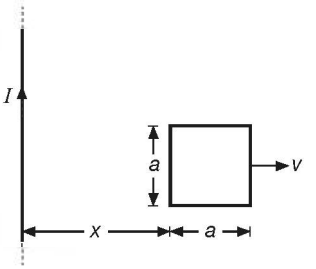


18. A long solenoid having 400 turns per metre carrying a current given by $I=(30\ A)(1-e^{-1.6t})$. Inside the solenoid and coaxial with it is a coil that has a radius of 6 cm and consists of a total of $N=250$ turns of fine wire as shown. Calculate the emf induced in the coil by the changing current?

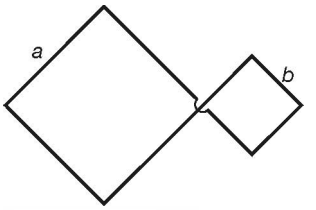


19. In order to impart an angular velocity to an earth satellite the geomagnetic field can be used. Find the maximum possible angular velocity about its own axis gained by the satellite if a storage battery with a capacity of $Q=5$ Ahr is discharged suddenly through a coil of $N=20$ turns wound around the satellite's surface along the circumference of the largest circle. The satellite has a mass of $m=1000$ kg and is assumed to be a thin walled sphere. The geomagnetic field is parallel to the winding plane and its flux density is $B=0.5$ G. (1 gauss $=10^{-4}$ tesla).

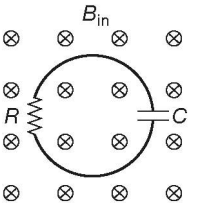
20. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Figure. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



21. A plane loop shown in Figure is shaped as two squares with sides $a=20$ cm and $b=10$ cm and is introduced into a uniform magnetic field at right angles to the loop's plane. The magnetic induction varies with time as $B=B_0\sin(\omega t)$, where $B_0=10$ mT and $\omega=100\ s^{-1}$. Find the amplitude of the current induced in the loop if its resistance per unit length is $\lambda=50\ m\Omega m^{-1}$. The inductance of the loop is to be neglected.



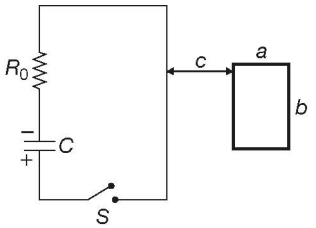
22. In Figure, a uniform magnetic field decreases at a constant rate $\frac{dB}{dt}=-K$, where K is a positive constant. A circular loop of wire of radius a containing a resistance R and a capacitance C is placed with its plane normal to the field.



(a) Find the charge Q on the capacitor when it is fully charged.
(b) Which plate is at the higher potential?
(c) Discuss the force that causes the separation of charges.

23. In the circuit a capacitor having capacitance $C=20\ \mu F$ initially charged to 100 V with the polarity is shown. The resistor R_0 has resistance $10\ \Omega$.

At time $t = 0$ the switch is closed. The small circuit is not connected in any way to the large one. The wire of the small circuit has a resistance of $1\,\Omega m^{-1}$ and contains 25 loops. The large circuit is a rectangle 2 m by 4 m, while the small one has dimensions $a = 10$ cm and $b = 20$ cm. The distance c is 5 cm. Both circuits are held stationary. Assume that only the wire nearest the small circuit produces an appreciable magnetic field through it.



(a) Find the current in the large circuit 200 μs after S is closed.

(b) Find the current in the small circuit 200 μs after S is closed.

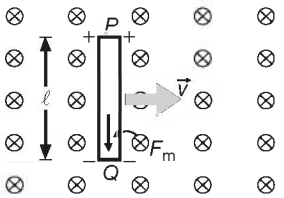
(c) Find the direction of the current in the small circuit.

(d) Justify why we can ignore the magnetic field from all the wires of the large circuit except for the wire closest to the small circuit.

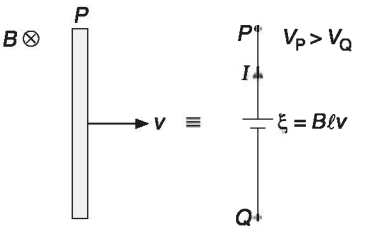
24. A uniform field of induction B is changing in magnitude at a constant rate $\frac{dB}{dt}$. You are given a mass m of copper which is to be drawn into a wire of radius a and formed into a circular loop of radius r. Show that the induced current in the loop does not depend on the size of the wire or of the loop and is given by $I = \left(\frac{\sigma m}{4\pi d}\right)\frac{dB}{dt}$, where σ is the conductivity and d the density of copper. Assume B perpendicular to loop.

INDUCED EMF IN CONDUCTING ROD MOVING THROUGH A UNIFORM MAGNETIC FIELD: MOTIONAL EMF

Let a thin conducting rod PQ of length l move in a uniform magnetic field B directed perpendicular to plane of paper, inwards. Let the velocity v of rod be in the plane of paper towards right.



Using Fleming's Left Hand Rule, we see that a positive charge (q) in the rod suffers magnetic force qvB directed from Q to P along the rod while an electron will experience a force evB directed from P to Q along the length of the rod. Due to this force the free electrons of rod move from P to Q, thus making end Q negative and end P positive as shown.



This causes a potential difference along the ends of rod. This potential difference developed is called **induced emf** ξ. If E is electric field developed in the rod, then

$$E = \frac{\xi}{l}$$

For equilibrium of charges, we must have

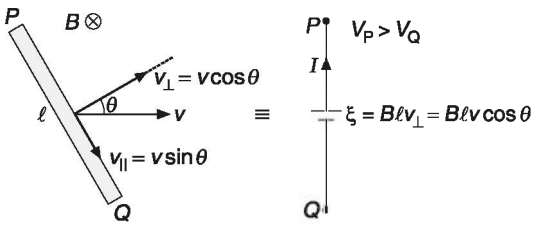
Electrical force = Magnetic force

$\Rightarrow \quad eE = evB$

$\Rightarrow \quad E = vB$

So, Induced emf $\xi = El = Blv$

If the rod moves in the magnetic field as shown in Figure, then the induced emf is given by



$$\xi = Blv_\perp = Blv\cos\theta$$

where $v_\perp$ is component of velocity perpendicular to the length of the rod. So, we have

$$V_P - V_Q = \xi = Bvl\cos\theta$$

Please note that the equivalent replacement of motional emf by a battery is shown the above figures.

Do not confuse that for the direction of induced current shown, we have $V_P < V_Q$ (as we start thinking that induced current will go from higher potential to lower potential). Of course, this is true but for the external circuit (excluding battery). So, note that for the rod in motion (acting as the source of emf), the induced conventional current is going from lower potential to higher potential (inside the source).

Just think this way that in the external circuit current goes from positive terminal to the negative terminal and inside the battery it goes from negative terminal to positive terminal.

Conceptual Note(s)

(a) If $\vec{\ell}$ is a vector directed along the direction of induced current, then a general notation for induced emf ξ is

$$\xi = \vec{B}\cdot(\vec{\ell}\times\vec{v}) = (\vec{v}\times\vec{B})\cdot\vec{\ell}$$

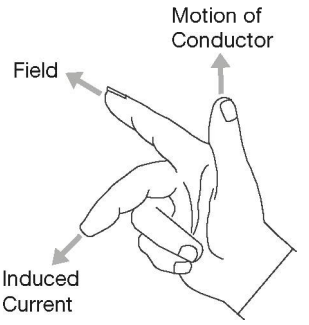
So, for emf or induced current to exist we must make sure that $\vec{B}$, $\vec{\ell}$ and $\vec{v}$ must never be coplanar.

(b) In general, the motional emf around a closed conducting loop can also be written as

$$\xi = \oint(\vec{v}\times\vec{B})\cdot d\vec{\ell} = \oint\vec{B}\cdot(d\vec{\ell}\times\vec{v})$$

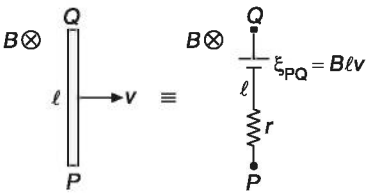
Fleming's Right Hand Rule

The direction of induced current is given by Fleming's Right Hand Rule according to which, *"Stretch the First Finger, Middle Finger and the Thumb of Right Hand in such a way that all three are mutually perpendicular to each other. First Finger points in the direction of field. Thumb points in the direction of motion of conductor, then Middle Finger points along the direction of Induced Conventional Current".*

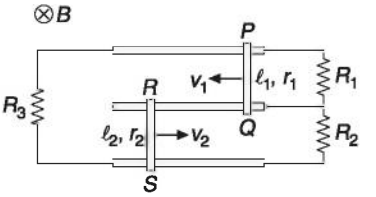


MOTIONAL EMF REPRESENTED AS AN EQUIVALENT BATTERY

When a conductor moves in a magnetic field, it can be considered like an equivalent battery or a source of potential difference with internal resistance equal to the resistance of the conductor. This conductor can act as a current source and supply current to the circuit. Consider a conductor of length l, resistance r moving with a velocity v in a uniform magnetic field B. Since B, l and v are perpendicular to each other, so this conductor can be replaced by an equivalent battery of emf Blv and internal resistance r as shown in Figure.



To understand the above situation better, let us consider two conductors PQ and RS having resistances r_1 and r_2 respectively, to be sliding on three conducting guide rails connected to resistances R_1, R_2 and R_3 as shown in Figure.



Applying Fleming's Right Hand Rule, we see that the induced current in the conductor PQ is from P to Q, whereas the induced current in the conductor RS is from S to R. So, the battery equivalents and the equivalent circuit for the arrangement of conductors is shown in Figure.

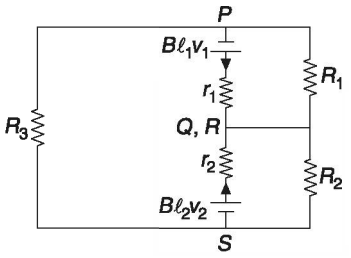


ILLUSTRATION 20

A straight segment of a rod is moving with velocity $\left(\hat{i}-2\hat{j}+\hat{k}\right)$ ms^{-1} and magnetic field in the region is $\left(4\hat{i}+2\hat{j}-\hat{k}\right)$ T. Calculate the emf induced across the ends of the rod when at an instant the end points are located between $(1,1,1)$ m and $(3,3,3)$ m.

SOLUTION

Since emf induced is

$$\xi=\vec{B}\cdot(\vec{l}\times\vec{v})=\begin{vmatrix}B_x & B_y & B_z\\ l_x & l_y & l_z\\ v_x & v_y & v_z\end{vmatrix}$$

where, $\vec{l}=\vec{l}_2-\vec{l}_1$

In the given case, we have

$$\vec{B}=\left(4\hat{i}+2\hat{j}-\hat{k}\right)\text{ T}$$

$$\vec{l}=\vec{l}_2-\vec{l}_1=\left(2\hat{i}+2\hat{j}+2\hat{k}\right)\text{ m}$$

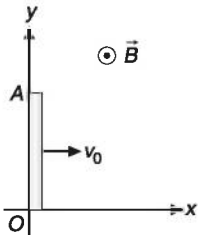
$$\vec{v}=\left(\hat{i}-2\hat{j}+\hat{k}\right)\text{ ms}^{-1}$$

$$\Rightarrow\quad \xi=\vec{B}\cdot(\vec{l}\times\vec{v})=\begin{vmatrix}4 & 2 & -1\\ 2 & 2 & 2\\ 1 & -2 & 1\end{vmatrix}$$

$$\Rightarrow\quad \xi=4(2+4)-2(2-2)-1(-4-2)=30\text{ V}$$

ILLUSTRATION 21

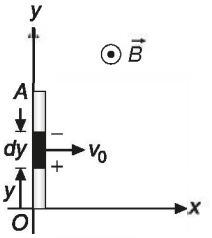
A conductor OA of length l is placed along the y-axis with one end at origin as shown in Figure.



In this region, a non-uniform magnetic field exists along z direction whose magnitude depends on the y coordinate as $B=B_0\left(1+\frac{y^2}{l^2}\right)$ T. If the conductor OA starts translating with a velocity $\vec{v}=v_0\hat{i}$, find the EMF induced across the ends of the conductor.

SOLUTION

Let us consider an element of width dy at a distance y from the origin as shown in Figure.



The motional emf induced across the element dy is

$$d\xi=Bv_0dy=B_0v_0\left(1+\frac{y^2}{l^2}\right)dy$$

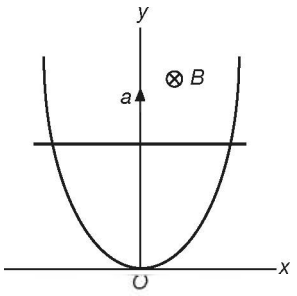
$$\Rightarrow\quad \xi_{OA}=\int d\xi=\int_0^l B_0v_0\left(1+\frac{y^2}{l^2}\right)dy$$

$$\Rightarrow\quad \xi_{OA}=B_0v_0\left(y+\frac{y^3}{3l^2}\right)\Bigg|_0^l=B_0v_0\left(l+\frac{l}{3}\right)$$

$$\Rightarrow\quad \xi_{OA}=\frac{4}{3}B_0v_0l$$

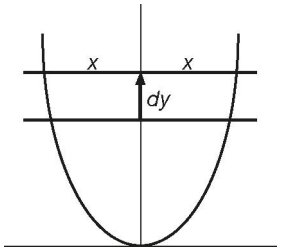
ILLUSTRATION 22

A wire bent as a parabola $y=kx^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the plane x, y. At the moment $t=0$ a connector starts sliding translation wise from the parabola apex with a constant acceleration a thus formed as a function of y.



SOLUTION

Let the connector be displaced through dy as shown in Figure.



The flux associated with the closed loop is then given by

$$d\phi = BdA = B(2x)dy$$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = B(2x)\left(\frac{dy}{dt}\right)$$

$$\Rightarrow \quad |\xi| = 2B\sqrt{\frac{y}{k}}\left(\frac{dy}{dt}\right) \qquad \left\{\because\ x = \sqrt{\frac{y}{k}}\right\}$$

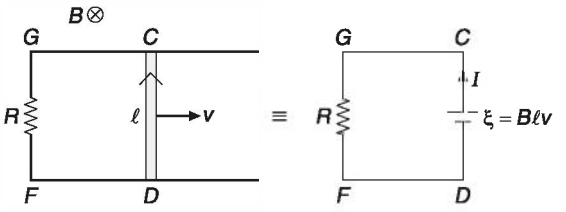
$$\Rightarrow \quad |\xi| = 2Bv\sqrt{\frac{y}{k}} \qquad \left\{\because\ \frac{dy}{dt} = v = \sqrt{2ay}\right\}$$

$$\Rightarrow \quad |\xi| = 2B\sqrt{2ay}\sqrt{\frac{y}{k}}$$

$$\Rightarrow \quad |\xi| = By\sqrt{\frac{8a}{k}}$$

INDUCED EMF IN A LOOP BY CHANGING ITS AREA

(a) Consider a straight conductor CD moving with velocity v towards right on a U-shaped conducting guide placed in a uniform magnetic field B directed into the page as shown in Figure.



As the conductor moves, the area of the loop $CDFG$ increases, causing a change in flux and hence an emf is induced in the coil. Let the loop move through dx in a time dt. Since

$$\xi = \frac{d\phi}{dt} = \frac{d}{dt}(BA) \qquad \text{(in magnitude)}$$

$$\Rightarrow \quad \xi = B\frac{dA}{dt} = Bl\left(\frac{dx}{dt}\right) \qquad \{\because\ A = l\,dx\}$$

$$\Rightarrow \quad \xi = Blv$$

If R is the resistance of the loop then the induced current is $I = \frac{Blv}{R}$. The direction of induced current is given by Fleming's Right Hand Rule and it will flow anticlockwise.

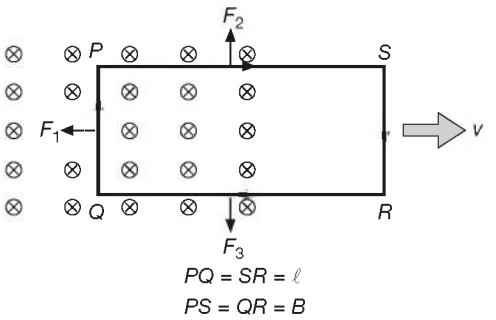
If instead of resistor a capacitor is used, then charge Q on the capacitor will be given by

$$Q = C\xi, \text{ where } \xi = B\ell v$$

$$\Rightarrow \quad I = \frac{dQ}{dt} = \frac{d}{dt}(B\ell Cv) = (B\ell C)\left(\frac{dv}{dt}\right)$$

i.e., **for the current to be induced in the circuit, the conductor must move with some acceleration, not with constant velocity.**

(b) Consider the situation shown in the Figure in which let the current flows clockwise.



Further, the current in the loop will cause forces F_1, F_2 and F_3 to act on the three arms PQ, PS and QR respectively. F_2 and F_3, being equal and opposite, will cancel out, F_1 is given by

$$F_1 = BIl = \frac{B^2l^2v}{R} \qquad \left\{\because\ I = \frac{Blv}{R}\right\}$$

The power required to pull the loop is

$$P = F_1v = \frac{B^2l^2v^2}{R}$$

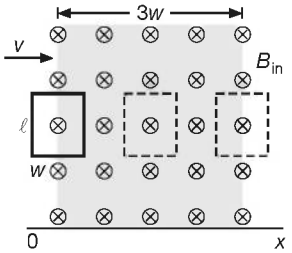
The rate at which Joule's heat is produced in the loop is

$$H = I^2R = \frac{B^2l^2v^2}{R}$$

which is same as P, as expected, according to the Law of Conservation of Energy.

ILLUSTRATION 23

A rectangular metallic loop of length l, width w and resistance R moves with constant speed v to the right, as in Figure.

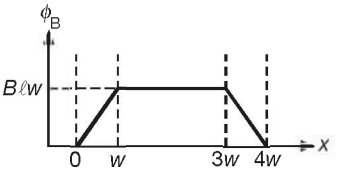


The loop passes through a uniform magnetic field $\vec{B}$ directed into the page and extending a distance $3w$ along the x-axis. Defining x as the position of the right side of the loop along the x-axis, plot as functions of x

(a) the magnetic flux through the area enclosed by the loop,
(b) the induced motional emf, and
(c) the external applied force necessary to counter the magnetic force and keep v constant.

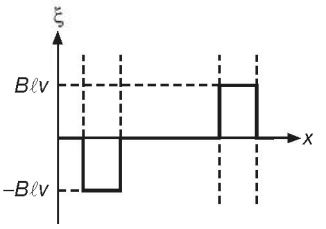
SOLUTION

(a) The flux through the area enclosed by the loop as a function of x is shown in Figure.



Before the loop enters the field, the flux is zero. As the loop enters the field, the flux increases linearly with position until the left edge of the loop is just inside the field and is given by the function $\phi_B = Bl(vt)$. Finally, the flux through the loop decreases linearly to zero as the loop leaves the field and is given by the function $\phi_B = Bl(w - vt)$.

(b) Before the loop enters the field, no motional emf is induced in it because no field is present. As the right side of the loop enters the field, the magnetic flux directed into the page increases. Hence, according to Lenz's Law, the induced current is counterclockwise because it must produce its own magnetic field directed out of the page. The motional emf $\xi = -\frac{d\phi_B}{dt} = -Blv$ arises from the magnetic force experienced by charges in the right side of the loop. When the loop is entirely in the field, the change in magnetic flux is zero, and hence the motional emf vanishes. This happens because, once the left side of the loop enters the field, the motional emf induced in it cancels the motional emf present in the right side of the loop. As the right side of the loop leaves the field, the flux begins to decrease, a clockwise current is induced, and the induced emf is $\xi = \frac{d\phi_B}{dt} = Blv$. As soon as the left side leaves the field, the emf decreases back to zero.

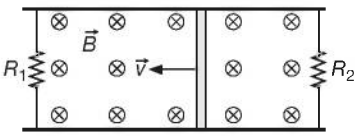


(c) The external force that must be applied to the loop to maintain this motion is plotted in Figure.

Before the loop enters the field, no magnetic force acts on it; hence, the applied force must be zero if v is constant. When the right side of the loop enters the field, the applied force necessary to maintain constant speed must be equal in magnitude and opposite in direction to the magnetic force exerted on that side. When the loop is entirely in the field, the flux through the loop is not changing with time. Hence, the net emf induced in the loop is zero, and the current also is zero. Therefore, no external force is needed to maintain the motion. Finally, as the right side leaves the field, the applied force must be equal in magnitude and opposite in direction to the magnetic force acing on the left side of the loop.
From this we conclude that power is supplied only when the loop is either entering or leaving the field. Also, we observe that the motional emf induced in the loop can be zero even when there is motion through the field. A motional emf is induced only when the magnetic flux through the loop changes with time.

ILLUSTRATION 24

A conducting rod of length l is free to side on two parallel conducting bars as in Figure.



Sliding Rod

In addition, two resistors R_1 and R_2 are connected across the ends of the bars. There is a uniform magnetic field pointing into the page. Suppose an external agent pulls the bar to the left at a constant speed v. Find the

(a) current through both resistors.
(b) total power delivered to the resistors.
(c) applied force required to maintain a constant velocity of the rod.

SOLUTION

(a) The emf induced between the ends of the moving rod is

$$\xi = \frac{d\phi_B}{dt} = -Blv$$

The currents through the resistors are

$$I_1 = \frac{|\xi|}{R_1} \text{ and } I_2 = \frac{|\xi|}{R_2}$$

Since the flux into the page for the left loop is decreasing, I_1 flows clockwise to produce a magnetic field pointing into the page. On the other hand, the flux into the page for the right loop is increasing. To compensate the change, according to Lenz's Law, I_2 must flow counterclockwise to produce a magnetic field pointing out of the page.

(b) The total power dissipated in the two resistors is

$$P_R = I_1|\xi| + I_2|\xi| = (I_1 + I_2)|\xi| = \xi^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$P_R = B^2l^2v^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(c) The total current flowing through the rod is $I = I_1 + I_2$. Thus, the magnetic force acting on the rod is

$$F_B = BIl = |\xi| = \xi^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$F_B = B^2l^2v^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

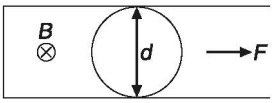
and the direction is to the right. Thus, an external agent must apply an equal but opposite force $\vec{F}_{ext} = -\vec{F}_B$ to the left in order to maintain a constant speed.
Alternatively, we note that since the power dissipated in the resistors must be equal to P_{ext}, the mechanical power supplied by the external agent. The same result is obtained since

$$P_{ext} = \vec{F}_{ext} \cdot \vec{v} = F_{ext}v$$

ILLUSTRATION 25

Two long parallel conducting horizontal rails are connected by a conducting wire at one end. A uniform magnetic field B directed vertically downwards

exists in the region of space. A light uniform ring of diameter d (which is practically equal to separation between the rails), resistance per unit length λ, is placed over the rails as shown in Figure.



Calculate force required to pull the ring with uniform velocity.

SOLUTION

When the ring moves to the right, then due to its motion the emf induced in each of the two semicircles is

$$\xi = Bvl_{eq} = Bvd \qquad \ldots(1)$$

Due to the rightward motion of the ring, the inward flux associated with the rails increases, due to which an induced counter-clockwise current flows in the ring. These two semi-circular rings behave like two identical sources in parallel, each of emf $\xi = Bvd$ having internal resistance r given by

$$r = \left(\frac{\pi d}{2}\right)\lambda$$

The equivalent resistance of parallel combination of these two semi-circular rings is $r_{eq} = \frac{r}{2}$, so we have

$$r_{eq} = \frac{r}{2} = \frac{\pi\lambda d}{4}$$

Since the rails have negligible resistance, hence the equivalent resistance of the circuit is

$$R = \frac{r}{2} = \frac{\pi\lambda d}{4}$$

The induced current in circuit is

$$i = \frac{\xi}{R} = \frac{Bvd}{\left(\frac{\pi\lambda d}{4}\right)} = \frac{4Bv}{\pi\lambda}$$

Current through each semi-circular ring is

$$i' = \frac{i}{2} = \frac{2Bv}{\pi\lambda}$$

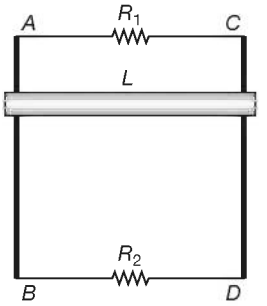
Force required to maintain the constant velocity of ring is equal to the opposing magnetic force experienced by the two semi-circular parts of the ring. So, we have

$$F_{ext} = 2(Bi'd) = 2B\left(\frac{i}{2}\right)d = Bid$$

$$\Rightarrow \quad F_{ext} = \frac{4B^2vd}{\pi\lambda}$$

ILLUSTRATION 26

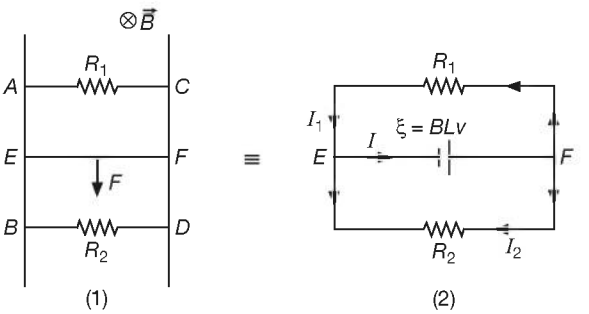
Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Figure.



A horizontal metallic bar of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.65 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2.

SOLUTION

Let us assume the magnetic field to be perpendicular to the plane of rails and inwards $\otimes$. If v be the terminal velocity of the rails, then potential difference across E and F would be BLv with E at lower potential and F at higher potential. The equivalent circuit is shown in Figure (2).



$$I_1 = \frac{\xi}{R_1} \qquad \ldots(1)$$

$$I_2 = \frac{\xi}{R_2} \qquad \ldots(2)$$

Power dissipated in R_1 is 0.76 W

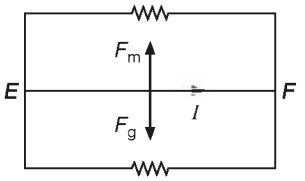
$$\Rightarrow \quad \xi I_1 = 0.76 \text{ W} \qquad \ldots(3)$$

Similarly, $\xi I_2 = 1.2$ W $\qquad \ldots(4)$

Now the total current in bar EF is

$$I = I_1 + I_2 \qquad \text{(from } E \text{ to } F\text{)} \qquad \ldots(5)$$

Under equilibrium condition i.e., when the terminal velocity is attained, magnetic force (F_m) on bar EF balances the weight (F_g) of bar EF.



$$\Rightarrow \quad F_m = F_g$$

$$\Rightarrow \quad BIL = mg \qquad \ldots(6)$$

$$\Rightarrow \quad I = \frac{mg}{LB} = \frac{(0.2)(9.8)}{(1)(0.6)} \text{ A} = 3.27 \text{ A}$$

Multiplying equation (5) by ξ, we get

$$\xi I = \xi I_1 + \xi I_2 = (0.76 + 1.2) \text{ W}$$

{from equation 3 and 4}

$$\Rightarrow \quad \xi I = 1.96 \text{ W}$$

$$\Rightarrow \quad \xi = \frac{1.96}{I} \text{ V} = \frac{1.96}{3.27} \text{ V} = 0.6 \text{ V}$$

Further since $\xi = BLv$

$$\Rightarrow \quad V = \frac{\xi}{BL} = \frac{(0.6)}{(0.6)(1)} \text{ ms}^{-1} = 1 \text{ ms}^{-1}$$

Hence, terminal velocity of bar is 1 ms^{-1}

Power in R_1 is 0.76 W

$$\Rightarrow \quad 0.76 = \frac{\xi^2}{R_1}$$

$$\Rightarrow \quad R_1 = \frac{\xi^2}{0.76} = \frac{(0.6)^2}{0.76} \ \Omega = 0.47 \ \Omega$$

Similarly, $R_2 = \dfrac{\xi^2}{1.2} = \dfrac{(0.6)^2}{1.2} \ \Omega = 0.3 \ \Omega$

ILLUSTRATION 27

An infinitesimally small bar magnet of dipole moment $\vec{M}$ is pointing and moving with the speed v in the $\vec{X}$-direction. A small closed circular conducting loop of radius a and negligible self-inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

SOLUTION

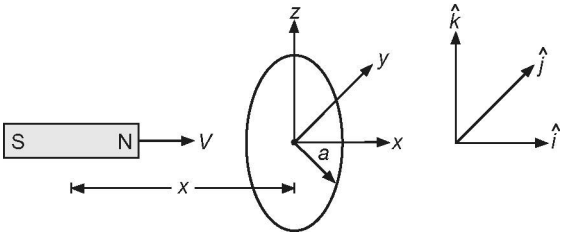
Since $B = \dfrac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}} = \dfrac{\mu_0 \left(I(\pi a^2)\right)}{2\pi (a^2 + x^2)^{3/2}}$

$$B = \frac{\mu_0}{2\pi} \frac{M}{(a^2 + x^2)^{3/2}} \qquad \left\{\because M = I(\pi a^2)\right\}$$

It is given that $x \gg a$

Magnetic field at the centre of the coil due to the bar magnet is, then

$$B = \frac{\mu_o}{2\pi} \frac{M}{x^3}$$



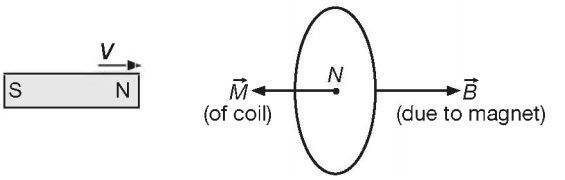
Due to this, magnetic flux linked with the coil is

$$\phi = BA = \frac{\mu_o}{2\pi} \frac{M}{x^3} (\pi a^2) = \frac{\mu_o M a^2}{2x^3}$$

Induced emf in the coil, due to motion of the magnet is,

$$\xi = -\frac{d\phi}{dt} = -\left(\frac{\mu_o M a^2}{2}\right) \frac{d}{dt}\left(\frac{1}{x^3}\right) = \frac{\mu_o M a^2}{2}\left(\frac{3}{x^4}\right)\frac{dx}{dt}$$

$$\xi = \frac{3}{2} \frac{\mu_o M a^2}{x^4} v \qquad \left\{\because \frac{dx}{dt} = v\right\}$$



Therefore, induced current in the coil is given by

$$I = \frac{\xi}{R} = \frac{3}{2}\frac{\mu_o M a^2}{R x^4} v$$

Magnetic moment of the coil due to this induced current is

$$M' = IA = \frac{3}{2}\frac{\mu_o M a^2}{R x^4} v(\pi a^2)$$

$$M' = \frac{3}{2}\frac{\mu_o \pi M a^4 v}{R x^4}$$

Potential energy of $\vec{M}'$ in $\vec{B}$ is given by

$$U = -\vec{M}' \cdot \vec{B} = -M' B \cos 180^o$$

$$\Rightarrow \quad U = M'B = \frac{3}{2}\frac{\mu_o \pi M a^4 v}{R x^4}\left(\frac{\mu_o}{2\pi}\frac{M}{x^3}\right)$$

$$\Rightarrow \quad U = \frac{3}{4}\frac{\mu_o^2 M^2 a^4 v}{R}\frac{1}{x^7}$$

Since $F = -\dfrac{dU}{dx}$

$$\Rightarrow \quad F = -\frac{dU}{dx} = \frac{21}{4}\frac{\mu_o^2 M^2 a^4 v}{R x^8}$$

Positive sign of F implies that there will be a repulsion between the magnet and the coil.

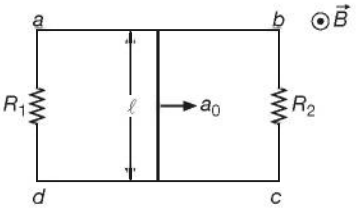
Conceptual Note(s)

Here we cannot apply $F = \dfrac{\mu_o}{4\pi}\dfrac{6MM'}{x^4}$ directly, because M' is a function of x and this equation can only be applied at places where M and M' both are constants.

ILLUSTRATION 28

A rectangular loop with a sliding conductor of mass m, length l is located in a uniform magnetic field perpendicular to the plane of loop. The magnetic induction perpendicular to the plane of loop is equal to B. The part ad and bc has electric resistance R_1 and R_2 respectively. The conductor starts moving with constant acceleration a_0 at time $t = 0$. Neglecting the self-inductance of the loop and resistance of conductor. Find

(a) the current through the conductor during its motion.
(b) the polarity of $abcd$ terminal.
(c) external force required to move the conductor with the given acceleration.



SOLUTION

(a) $I = \dfrac{\xi}{R} = \dfrac{Blv}{R}$

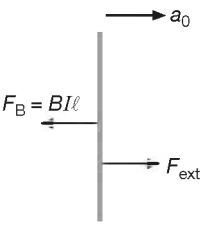
where $v = a_0 t$ and $R = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\Rightarrow \quad I = \frac{Bla_0 t(R_1 + R_2)}{R_1 R_2} \qquad \ldots(1)$$

(b) Flux linked with the left loop is increasing, so the induced current must set up a field that does not allow the flux to increase i.e., an inward field is set up, so the induced current goes from d to a. Hence a has negative polarity.
Similarly, flux linked with the right loop is decreasing, so the induced current must be in clockwise sense so that outward flux does not decrease. Hence b is also having negative polarity.

(c) According to Newton's Second Law,

$$F_{ext} - F_B = ma_0 \qquad \ldots(2)$$



where F_B is the magnetic force acting on the conductor due to the induced current I. So,

$$F_B = BIl = B\left[\frac{Bla_0 t(R_1 + R_2)}{R_1 R_2}\right] l$$

Using equation (2), we get

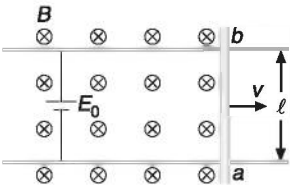
$$F_{ext} = F_B + ma_0$$

$$\Rightarrow F_{ext} = \frac{B^2l^2a_0t(R_1+R_2)}{R_1R_2} + ma_0$$

$$\Rightarrow F_{ext} = a_0\left(m + \frac{B^2l^2(R_1+R_2)t}{R_1R_2}\right)$$

ILLUSTRATION 29

A bar of mass m, length l and resistance R slides without friction in a horizontal plane, moving on parallel rails as shown in Figure. A battery that maintains a constant emf E is connected between the rails, and a constant magnetic field $\vec{B}$ is directed perpendicularly to the plane of the page. Assuming the bar starts from rest, find the velocity of the rod as a function of time. Also find the current in the loop.



SOLUTION

(a) Let at time t velocity of rod be v (towards right) and current in the circuit is I (from b to a). The magnetic force on it is BIl (towards right). Writing the equation of motion of the rod, we get

$$m\left(\frac{dv}{dt}\right) = BIl = B\left(\frac{E_0 - Blv}{R}\right)l$$

$$\Rightarrow \int_0^v \frac{dv}{\frac{E_0Bl}{mR} - \frac{B^2l^2}{mR}v} = \int_0^t dt$$

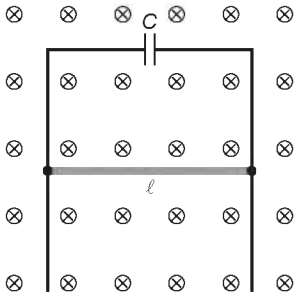
$$\Rightarrow v = \frac{E_0}{Bl}\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$$

(b) $$I = \frac{E + E_{ind}}{R} = \frac{E_0 - Blv}{R} = \frac{E_0 - E_0\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)}{R}$$

$$\Rightarrow I = \frac{E_0}{R}e^{-\left(\frac{B^2l^2}{mR}\right)t}$$

ILLUSTRATION 30

Two metal bars are fixed vertically and are connected on the top by a capacitor C. A sliding conductor length slides with its ends in contact with the bars. The arrangement is placed in a uniform horizontal magnetic field directed normal to the plane of the Figure. The conductor is released from rest. Find the displacement $x(t)$ of the conductor as a function of time t.



SOLUTION

The motion of the conductor in the magnetic field induces an emf in it. As a result of this, an induced current, say I, flows through the conductor. This induced current will be responsible for the magnetic force BIl which opposes the motion of conductor. At any instant, if v is the velocity of the conductor, then the net downward force F acting on the conductor is

$$F = mg - BIl \quad \ldots(1)$$

The charge Q on the capacitor is given by

$$Q = C\xi = C(Blv) = (BlC)v$$

So, we get $$I = \frac{dQ}{dt} = BlC\left(\frac{dv}{dt}\right) \quad \ldots(2)$$

From (1), we get

$$m\left(\frac{dv}{dt}\right) = mg - B\left[BlC\left(\frac{dv}{dt}\right)\right]l$$

$$\Rightarrow m\left(\frac{dv}{dt}\right) = mg - B^2l^2C\left(\frac{dv}{dt}\right)$$

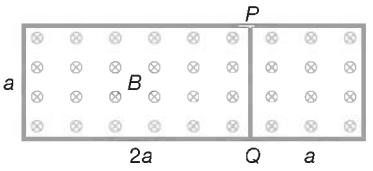
$$\Rightarrow a = \frac{dv}{dt} = \frac{mg}{m + B^2l^2C} = \text{constant}$$

So, we have $x(t) = ut + \frac{1}{2}at^2 = (0)t + \frac{1}{2}at^2 \quad \{\because u = 0\}$

$$\Rightarrow x(t) = \frac{1}{2}\left(\frac{mg}{m + B^2l^2C}\right)t^2$$

ILLUSTRATION 31

Find the current through section PQ of length a in Figure. The circuit is located in a time varying magnetic field $B = B_0 t$. Assume the resistance per length of the wire is λ.



SOLUTION

First of all, we must understand how to get the current direction in the left loop $PRSQP$ and the right loop $PTUQP$. Since B is increasing with t, so the induced current in both the loops must not allow B to increase, as a result of which the branches must have induced current which opposes the increase in the value of B inwards. So, an outward B must be set up due to currents in both the loops. The directions taken satisfy the argument supplied. Now, applying Kirchhoff's Loop Law for loops $PRSQP$ and $PTUQP$, we get

$$E_1 - (5\lambda a)I_1 - (\lambda a)I_{PQ} = 0 \qquad \ldots(1)$$

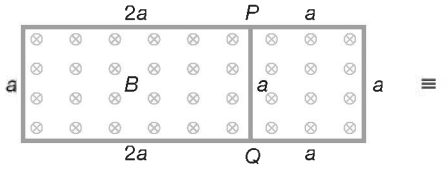
and $$-E_2 + (3\lambda a)I_2 - (\lambda a)I_{PQ} = 0$$

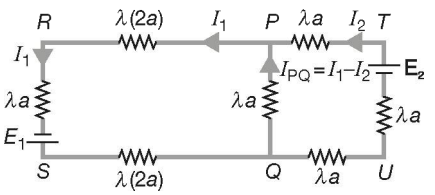
$$\Rightarrow \quad E_2 - (3\lambda a)I_2 + (\lambda a)I_{PQ} = 0 \qquad \ldots(2)$$

where $E_1 = \left(A_{\text{left loop}}\right)\dfrac{dB}{dt} = (2a^2)B_0$,

$$E_2 = \left(A_{\text{right loop}}\right)\frac{dB}{dt} = a^2 B_0$$

and $I_2 + I_{PQ} = I_1$ i.e., $I_{PQ} = I_1 - I_2$





So, equations (1) and (2) are rewritten as

$$2a^2 B_0 - (5\lambda a)I_1 - (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow \quad 2a^2 B_0 - (6\lambda a)I_1 + (\lambda a)I_2 = 0 \qquad \ldots(3)$$

$$a^2 B_0 - (3\lambda a)I_2 + (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow \quad a^2 B_0 + (\lambda a)I_1 - (4\lambda a)I_2 = 0 \qquad \ldots(4)$$

$6(4)+(3)$, gives

$$8a^2 B_0 - (23\lambda a)I_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{8aB_0}{23\lambda}$$

Similarly, doing $4(3)+(4)$, we get

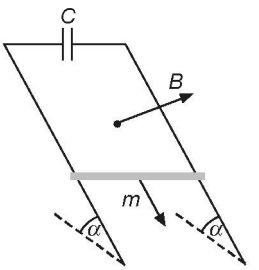
$$9a^2 B_0 - (23\lambda a)I_1 = 0$$

$$\Rightarrow \quad I_1 = \frac{9aB_0}{23\lambda}$$

$$\Rightarrow \quad I_{PQ} = I_1 - I_2 = \frac{aB_0}{23\lambda}$$

ILLUSTRATION 32

A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity. At the top the bars are interconnected through a capacitor C. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the acceleration of the connector.



SOLUTION

Applying Newton's Second Law to the connector, we get

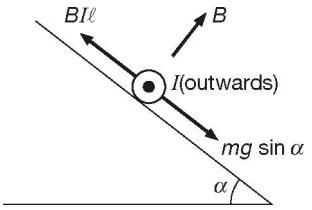
$$mg\sin\alpha - BIl = ma \qquad \ldots(1)$$

The emf induced across the connector is $\xi = Blv$

Potential difference across the capacitor plates is

$$\xi = \frac{q}{C}$$

$$\Rightarrow \quad q = C\xi = C(Blv)$$



The induced current in the circuit is then given by

$$I = \frac{dq}{dt} = BlC\left(\frac{dv}{dt}\right) \qquad \ldots(2)$$

Substituting (2) in (1), we get

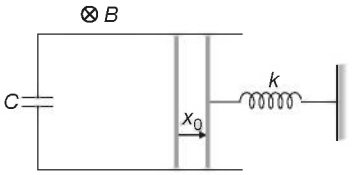
$$mg\sin\alpha - B\left(BlC\frac{dv}{dt}\right)l = ma$$

$$\Rightarrow \quad mg\sin\alpha = (m + B^2l^2C)a \qquad \left\{\because\ a = \frac{dv}{dt}\right\}$$

$$\Rightarrow \quad a = \frac{mg\sin\alpha}{m + B^2l^2C}$$

ILLUSTRATION 33

In the arrangement shown in the Figure there is a uniform magnetic field B_0 normal to the plane of paper. The connector is smooth and conducting and it has a mass m and length l. The connector is pushed against the spring so that the spring has compression x_0. The connector is released at $t = 0$. Find the time it will take to come to its original position again. The spring is non-conducting. The resistance of the rails is zero and neglect its self-inductance.

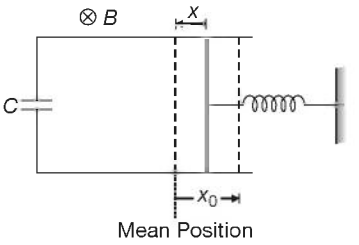


SOLUTION

Let at any time t, the rod be at a distance x from its original position. Then

$$v = \frac{dx}{dt}$$

$$\Rightarrow \quad \xi = Blv = Bl\left(\frac{dx}{dt}\right)$$



At any instant, the charge on the capacitor is

$$Q = C\xi = BlC\left(\frac{dx}{dt}\right)$$

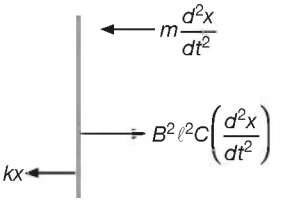
$$\Rightarrow \quad I = \frac{dQ}{dt} = BlC\left(\frac{d^2x}{dt^2}\right)$$

Force on the conductor due to this current is

$$F = BIl$$

$$\Rightarrow \quad F = B\left(BlC\frac{d^2x}{dt^2}\right)l$$

$$\Rightarrow \quad F = B^2l^2C\left(\frac{d^2x}{dt^2}\right)$$



So, net force on the rod is

$$m\frac{d^2x}{dt^2} = kx - B^2l^2C\left(\frac{d^2x}{dt^2}\right)$$

$$\Rightarrow \quad (m + B^2l^2C)\frac{d^2x}{dt^2} = kx$$

So, we observe that the acceleration of the rod is proportional to x and is directed opposite to it towards the mean position. Hence the motion of the rod is simple harmonic about the mean (initial) position, with period

$$T = 2\pi\sqrt{\frac{x}{\left|\frac{d^2x}{dt^2}\right|}}$$

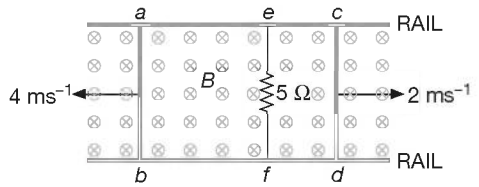
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m + B^2l^2C}{k}}$$

If t is the time taken by the rod to come to its initial position, then

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m + B^2l^2C}{k}}$$

ILLUSTRATION 34

Two parallel rails with negligible resistance are 10 cm apart and are connected by a 5 Ω resistor. The circuit also contains two metal rods 1 and 2 having resistances of 10 Ω and 15 Ω sliding along the rails. The rods are pulled away from the resistor at constant speeds of 4 ms^{-1} and 2 ms^{-1}, respectively. A uniform magnetic field of magnitude 10 mT is applied perpendicular to the plane of the rails. Determine the current in the 5 Ω resistor.



SOLUTION

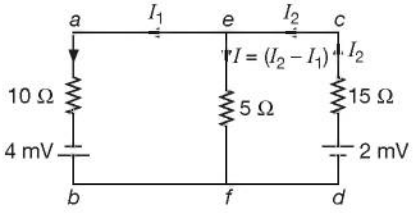
Let us first apply Fleming's Right Hand Rule to find the direction of induced current in Rod ab and Rod cd. Let I_1 and I_2 be the induced current in rod ab and cd respectively. Then I_1 is from a to b and I_2 is from d to c. Also, these two moving rods will produce an induced emf E_1 and E_2 whose magnitudes are given by

$$E_1 = Blv_1 \text{ (for } ab\text{) and } E_2 = Blv_2 \text{ (for } cd\text{)}$$

$$\Rightarrow \quad E_1 = (0.01)\left(\frac{10}{100}\right)(4) = 4\times10^{-3} \text{ V} = 4 \text{ mV}, \text{ and}$$

$$\Rightarrow \quad E_2 = (0.01)\left(\frac{10}{100}\right)(2) = 2\times10^{-3} \text{ V} = 2 \text{ mV}$$

So, the equivalent circuit for the above arrangement is shown below



If I be the current in the branch ef or the 5 Ω resistor, then

$$I = I_2 - I_1 \qquad \ldots(1)$$

Applying Kirchhoff's Loop rule to the two loops we get

For loop, $abfea$

$$-10I_1 + 4 + 5(I_2 - I_1) = 0$$

$$\Rightarrow \quad 15I_1 - 5I_2 = 4 \qquad \ldots(2)$$

For loop, $cdfec$

$$15I_2 - 2 + 5(I_2 - I_1) = 0$$

$$-5I_1 + 20I_2 = 2 \qquad \ldots(3)$$

$3\times$ ③ + ②, gives

$$60I_2 - 5I_2 = 10$$

$$55I_2 = 10$$

$$\Rightarrow \quad I_2 = \frac{10}{55} \text{ mA}$$

Similarly, $4\times$ ② + ③ gives

$$60I_1 - 5I_1 = 18$$

$$\Rightarrow \quad I_1 = \frac{18}{55} \text{ mA}$$

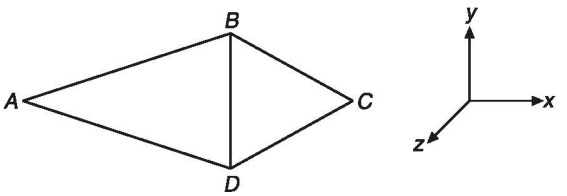
$$\Rightarrow \quad I = I_2 - I_1 = -\frac{8}{55} \text{ mA}$$

$$\Rightarrow \quad I = \frac{8}{55} \text{ mA from } f \text{ to } e \text{ (not from } e \text{ to } f \text{ as taken)}$$

$$\Rightarrow \quad I \cong 145\ \mu\text{A}$$

ILLUSTRATION 35

A wire loop $ABCD$ is divided into two parts. $AB = AD = 2$ m, $BC = CD = BD = 1$ m. The wire loop is lying on X-Y plane. The resistance per unit length of the wire is 2 Ωm^{-1}. There exists a time dependent magnetic field $\vec{B} = (2.5t)\hat{k}$ T. Find the current flowing through each part of the loop.



SOLUTION

Since $|\vec{B}| = B = 2.5t$

$$\Rightarrow \quad \frac{dB}{dt} = 2.5 \text{ Ts}^{-1}$$

Area of triangle ABD is

$$A_1 = \frac{1}{2}\left(\frac{1}{2}\right)\left(\sqrt{4-\frac{1}{4}}\right) = \frac{\sqrt{15}}{4}\ \text{m}^2$$

Area of triangle BCD is

$$A_2 = \frac{\sqrt{3}}{4}\ \text{m}^2$$

Induced emf in ABD is

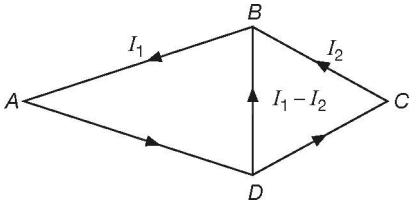
$$\xi_1 = \left|\frac{d\phi}{dt}\right| = A_1\left|\frac{dB}{dt}\right| = \frac{5\sqrt{15}}{8}\ \text{V}$$

Induced emf in BCD is

$$\xi_2 = \frac{5\sqrt{3}}{8}\ \text{V}$$

Resistance of $AB = AD = 4\ \Omega$

Resistance of $BC = CD = BD = 2\ \Omega$



Applying Kirchhoff's Loop Law to loop $BADB$, we get

$$\frac{5\sqrt{15}}{8} = 4I_1 + 4I_1 + 2(I_1 - I_2) \qquad \ldots(1)$$

Applying Kirchhoff's Loop Law to loop $BDCB$, we get

$$\frac{5\sqrt{3}}{8} = 4I_2 - 2(I_1 - I_2) \qquad \ldots(2)$$

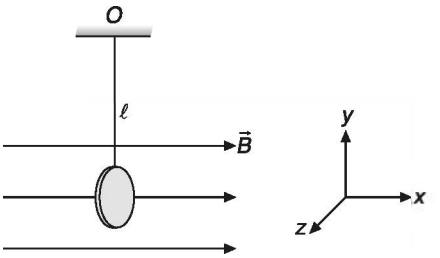
From equations (1) and (2), we get

$$I_1 = 0.3\ \text{A} \quad \text{and} \quad I_2 = 0.28\ \text{A}$$

So, $I_{BA} = I_{AD} = 0.3$ A, $I_{DC} = I_{CB} = 0.28$ A and $I_{DB} = 0.02$ A

ILLUSTRATION 36

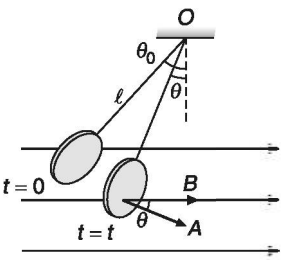
A small circular coil of area A is suspended from a point O by a string of length l in a uniform magnetic induction B acting the horizontal direction as shown in Figure.



Let the coil be set into oscillations at $t = 0$ just like a simple pendulum by displacing it through a small angle θ_0. Calculate the emf induced in the coil as a function of time. Assume that initially the coil lies in yz plane and it does not change its plane during the entire course of its motion.

SOLUTION

At time $t = 0$ and at time t after being released from the angular position θ_0, the positions of the coil are shown in Figure.



The angular position of the coil is given by

$$\theta = \theta_0 \cos \omega t\ \text{, where}\ \omega = \sqrt{\frac{g}{l}}$$

At time t, the magnetic flux associated with the coil is

$$\phi = BA\cos\theta$$

EMF induced in the coil is

$$\xi = \left|\frac{d\phi}{dt}\right| = (BA\sin\theta)\frac{d\theta}{dt}$$

Since $\frac{d\theta}{dt} = \frac{d}{dt}(\theta_0 \cos \omega t) = -\theta_0 \omega \sin \omega t$

According to the problem, θ_0 is small, so θ is also small an hence, we have $\sin\theta \approx \theta$

$$\Rightarrow \quad \xi = BA\theta\frac{d\theta}{dt} = BA(\theta_0 \cos \omega t)\frac{d}{dt}(\theta_0 \cos \omega t)$$

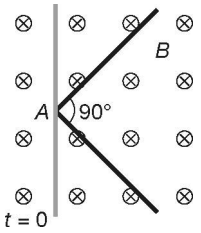
$$\Rightarrow \quad \xi = BA\left(\theta_0 \cos\sqrt{\frac{g}{l}}t\right)\left(\theta_0 \omega \sin\sqrt{\frac{g}{l}}t\right)$$

$$\Rightarrow \quad \xi = BA\omega\theta_0^2 \cos\left(\sqrt{\frac{g}{l}}t\right)\sin\left(\sqrt{\frac{g}{l}}t\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}BA\omega\theta_0^2 \sin\left(2\sqrt{\frac{g}{l}}t\right)$$

ILLUSTRATION 37

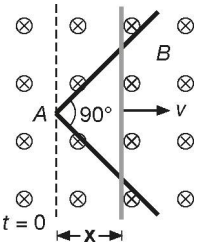
Figure shows a wire bent into the shape of a right angle that is fixed on a horizontal plane. A very long rod of mass m initially starts with a velocity v_0 from the apex A of the bent wire. The resistance of the wire and the rod is $\lambda\ \Omega m^{-1}$. The whole arrangement is placed in a region of uniform magnetic induction B. Find the distance travelled by the rod before it coming to rest.



SOLUTION

Let x be the distance travelled by the rod in time t, then the resistance of the circuit is

$$R = (2\sqrt{2}x + 2x)\lambda$$



The instantaneous induced emf is

$$E = B(2x)v$$

Induced current $I = \dfrac{E}{R} = \dfrac{Bv}{\lambda(1+\sqrt{2})}$

Magnetic force on the rod is

$$F = BI(2x) = \frac{2B^2vx}{\lambda(1+\sqrt{2})}$$

From Newton's Second Law, we have

$$F = -m\frac{dv}{dt} = \frac{2B^2vx}{\lambda(1+\sqrt{2})} \qquad \ldots(1)$$

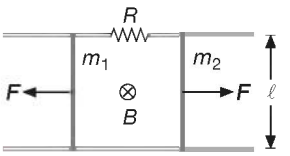
Eliminating t from the equation (1) and then integrating, we get

$$-\int_{v_0}^{0} dv = \left(\frac{2B^2}{m\lambda(1+\sqrt{2})}\right)\int_0^x x\,dx$$

$$\Rightarrow \quad x = \frac{\sqrt{mv_0\lambda(1+\sqrt{2})}}{B}$$

ILLUSTRATION 38

The arrangement shown is placed in a vertical uniform magnetic field. Two metal rods of length l and masses m_1 and m_2 are pulled apart from rest by a constant force F. Find the current in the resistor R as a function of time.



SOLUTION

Let v_1 and v_2 be the instantaneous velocity force the rods m_1 and m_2 respectively. The emf's across the rod will be

$$E_1 = Blv_1 \text{ and } E_2 = Blv_2$$

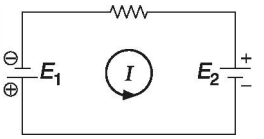
The instantaneous current is

$$I = \frac{E_1 + E_2}{R} = \frac{Bl(v_1 + v_2)}{R} \qquad \ldots(1)$$

The acceleration of each rod is given by

$$\frac{dv_1}{dt} = \frac{F - BIl}{m_1} = \frac{F}{m_1} - \frac{B^2l^2}{m_1R}(v_1 + v_2)$$

and $\dfrac{dv_2}{dt} = \dfrac{F}{m_2} - \dfrac{B^2l^2}{m_2R}(v_1 + v_2)$



Adding the above two equations, we get

$$\frac{d}{dt}(v_1 + v_2) = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left[F - \frac{B^2l^2}{R}(v_1 + v_2)\right]$$

Using equation (1), we get

$$v_1 + v_2 = \frac{IR}{Bl}$$

$$\Rightarrow \quad \frac{d}{dt}(v_1 + v_2) = \left(\frac{R}{Bl}\right)\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad \frac{R}{Bl}\frac{dI}{dt} = \frac{F}{\mu} - \frac{BlI}{\mu}$$

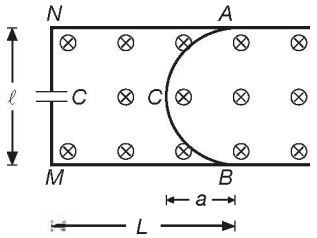
where, $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the system

$$\Rightarrow \quad \frac{dI}{F - BlI} = \frac{Bl}{\mu R}dt$$

$$\Rightarrow \quad I = \frac{F}{Bl}\left[1 - e^{-B^2l^2t/\mu R}\right]$$

ILLUSTRATION 39

Two long parallel conducting rods with negligible resistance are connected by capacitor C and semi-circular conducting wire in the same plane as shown. This system lies in a perpendicular magnetic field. Now loop ACB is moved on rails with its displacement given by $x = a\sin(\omega t)$ from the initial position. Find the maximum current in the capacitor.

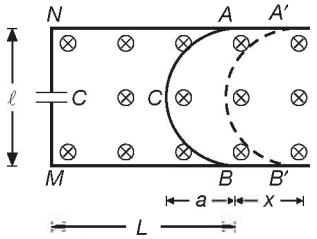


SOLUTION

Let in initial position $BM = AN = L$. At any time area of closed loop

$$A'NMB' = A$$

$$A = (L + x)d - \pi\frac{d^2}{8}$$



Flux through the loop, $\phi = BA = B\left(Ll - \dfrac{\pi l^2}{8} + xl\right)$

emf induced,

$$\xi = \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}\left(Ll - \frac{\pi l^2}{8}\right) + \frac{d}{dt}(Blx)$$

$$\Rightarrow \quad \xi = Bl\frac{dx}{dt} = Bl\frac{d}{dt}(a\sin(\omega t)) = Bal\omega\cos(\omega t)$$

As net potential drop in the loop is zero,

$$\xi = \frac{q}{C}$$

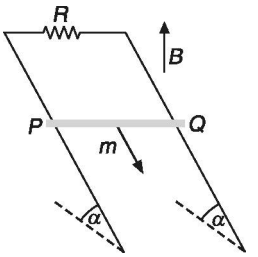
$$\Rightarrow \quad Q = CBal\omega\cos(\omega t)$$

$$\Rightarrow \quad I = \frac{dQ}{dt} = -CBal\omega^2\sin(\omega t)$$

$$\Rightarrow \quad I_{max} = CBal\omega^2$$

ILLUSTRATION 40

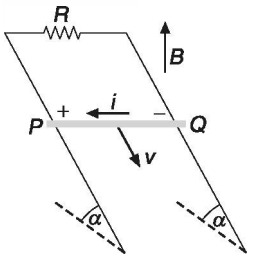
A copper rod PQ of mass m slides down two smooth copper bars which are set at an angle α to the horizontal as shown in Figure.



At the top of the bars these are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B directed vertically upwards. The resistances of the bars, the rod and the sliding contacts are considered to be negligible. If the rod is released from rest, calculate the velocity of rod as a function of time and its steady velocity attained.

SOLUTION

When the rod is released from rest, then is starts moving down the incline due to which it cuts the magnetic flux and an emf is induced across it. The induced current in the rod flows from Q to P as shown in Figure.



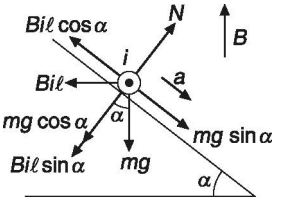
At any instant, the conductor velocity v is down the incline. The magnetic field component $B\sin\alpha$ acts opposite to the velocity and hence will not be responsible for producing the induced emf. However, the component of magnetic field $B\cos\alpha$ is perpendicular to the velocity as well as the length of the conductor, so the emf induced across the conductor is

$$\xi = B_{\perp} lv = (B\cos\alpha)lv = Blv\cos\alpha$$

The current induced i in the loop containing the resistance is

$$i = \frac{Blv\cos\alpha}{R}$$

Due to this induced current in the rod, the rod experiences a magnetic force $F = Bil$, leftwards as shown in Figure.



If the rod slides down the incline with an acceleration a, then by applying Newton's Second law, we get

$$mg\sin\alpha - Bil\cos\alpha = ma$$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = mg\sin\alpha - \frac{B^2l^2v\cos^2\alpha}{R} \qquad \ldots(1)$$

$$\Rightarrow \quad dv = \left(g\sin\alpha - \frac{B^2l^2v\cos^2\alpha}{mR}\right)dt$$

$$\Rightarrow \quad \int_0^v \frac{dv}{mgR\sin\alpha - (B^2l^2\cos^2\alpha)v} = \int_0^t \frac{dt}{mR}$$

$$\Rightarrow \quad -\frac{\ln\left(mgR\sin\alpha - B^2l^2v\cos^2\alpha\right)}{B^2l^2\cos^2\alpha}\Bigg|_0^v = \frac{t}{mR}$$

$$\Rightarrow \quad \ln\left(\frac{mgR\sin\alpha - B^2l^2v\cos^2\alpha}{mgR\sin\alpha}\right) = -\left(\frac{B^2l^2\cos^2\alpha}{mR}\right)t$$

$$\Rightarrow \quad v = \frac{mgR\sin\alpha}{B^2l^2\cos^2\alpha}\left(1 - e^{-\frac{B^2l^2\cos^2\alpha}{mR}t}\right)$$

When steady velocity (also called Terminal velocity) is attained, then acceleration of the rod is zero, so from equation (1), we get

$$v_S = \frac{mgR\sin\alpha}{B^2l^2\cos^2\alpha}$$

Theoretically, steady velocity is obtained after a long time i.e. when $t \to \infty$.

EMF INDUCED ACROSS THE ENDS OF A CONDUCTING ROD ROTATING IN A UNIFORM MAGNETIC FIELD

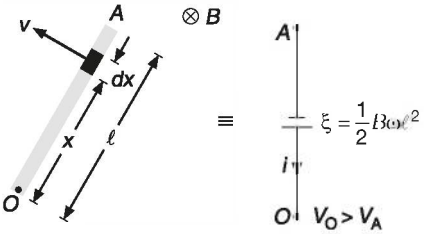
METHOD I

Consider a conducting rod of length l rotating about the point O (at one end of the rod) in a uniform magnetic field B. To find the emf induced across the ends of the rod, let us consider an infinitesimal element of length dx at a distance x from O, having a velocity v, as shown. If $d\xi$ be the induced emf across the element, then

$$d\xi = B(dx)v, \text{ where } v = x\omega$$

$$\Rightarrow \quad d\xi = B\omega x dx$$

$$\Rightarrow \quad \xi = B\omega\int_0^l x dx = B\omega\frac{x^2}{2}\Big|_0^l = \frac{1}{2}B\omega l^2$$



METHOD II

When the rod is rotating in the field with angular velocity ω, then the induced emf is

$$\xi = B\left(\frac{\text{Area swept by the Rod}}{\text{Time to complete one Revolution}}\right)$$

$$\Rightarrow \quad \xi = \frac{B(\pi l^2)}{\frac{2\pi}{\omega}}$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega l^2$$

ILLUSTRATION 41

A thin wire AC shaped as a semi-circle of diameter d rotates with a constant angular velocity ω in a uniform magnetic field of induction B, with $\omega \| B$. The rotation axis passes through the end A of the wire and is perpendicular to the diameter AC. Find the value of a line integral $\int \vec{E}\cdot d\vec{l}$ along the wire from point A to point C. Generalize the obtained result for an arbitrary shape of wire between A and C.

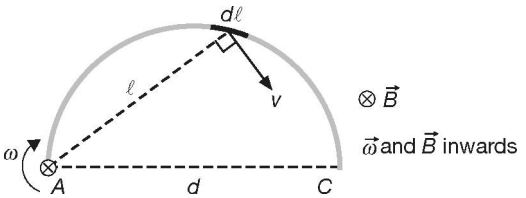
SOLUTION

Since $d\xi = (\vec{v}\times\vec{B})\cdot d\vec{l}$ ...(1)

Also $d\xi = -\vec{E}\cdot d\vec{l}$ ...(2)

So, from (1) and (2), we get

$$\int \vec{E}\cdot d\vec{l} = -\int (\vec{v}\times\vec{B})\cdot d\vec{l} = -\int vBdl$$

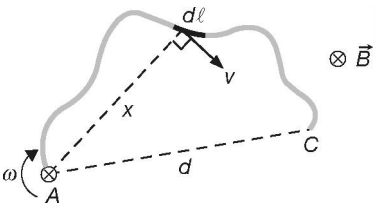


But $v = l\omega$, where l is the perpendicular distance of the element from A

$$\Rightarrow \quad \int_A^C \vec{E}\cdot d\vec{l} = -B\omega\int_0^d ldl = -\frac{1}{2}B\omega d^2$$

The above result can be generalised and extended to a wire AC of any arbitrary shape.
Since, from above we have

$$\int \vec{E}\cdot d\vec{l} = -\int (\vec{v}\times\vec{B})\cdot d\vec{l}$$



Further from our knowledge of rotational kinematics, we know that

$$\vec{v} = \vec{\omega}\times\vec{l}$$

$$\Rightarrow \quad \int \vec{E}\cdot d\vec{l} = -\int ((\vec{\omega}\times\vec{l})\times\vec{B})\cdot d\vec{l} = \int (\vec{B}\times(\vec{\omega}\times\vec{l}))\cdot d\vec{l}$$

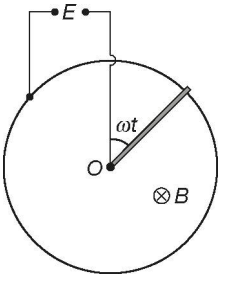
Also, we know that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$$\Rightarrow \quad \int \vec{E}\cdot d\vec{l} = \int ((\vec{B}\cdot\vec{l})\vec{\omega} - (\vec{B}\cdot\vec{\omega})\vec{l})\cdot d\vec{l}$$

$$\Rightarrow \quad \int_A^C \vec{E}\cdot d\vec{l} = 0 - B\omega\int_0^d ldl = -\frac{1}{2}B\omega d^2$$

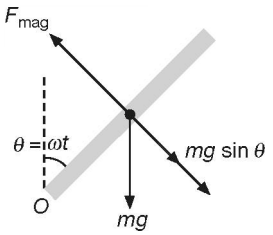
ILLUSTRATION 42

A metal rod of mass m can rotate about a horizontal axis O, sliding along a circular conductor of radius r. The arrangement is located in a uniform magnetic field of induction B directed perpendicular to the ring plane. The axis and the ring are connected to a source of emf E to form a circuit of resistance R. Neglecting the friction, circuit inductance, and ring resistance, find the expression according to which the source emf must vary to make the rod rotate with a constant angular velocity ω.



SOLUTION

Assume the rod to rotate in the clockwise sense. Due to rotation, the emf induced across the ends of the rod is $\xi = \frac{1}{2}B\omega r^2$



Net Emf in the rod-ring circuit is

$$E_{net} = E - \xi$$

The net current induced in the rod is

$$I = \frac{E_{net}}{R} = \frac{E-\xi}{R}$$

$$\Rightarrow \quad I = \frac{E - \frac{1}{2}B\omega r^2}{R} \qquad \ldots(1)$$

Due to this induced current, the magnetic force $F_m = BIr$ acts on the rod as shown. Simultaneously mg also acts on the rod. Now we want the rod to move with a constant angular velocity. Hence for this

$$\sum \vec{\tau}_{mag} = \sum \vec{\tau}_{gravity}$$

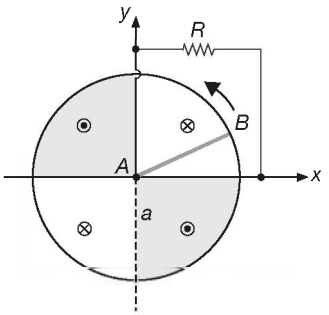
$$\Rightarrow \quad (mg\sin\theta)\frac{r}{2} = (BIr)\frac{r}{2}$$

$$\Rightarrow \quad mg\sin(\omega t) = B\left(\frac{E - \frac{1}{2}B\omega r^2}{R}\right)r$$

$$\Rightarrow \quad E = \frac{1}{2rB}\left(r^3B^2\omega + 2mgR\sin(\omega t)\right)$$

ILLUSTRATION 43

In a cylindrical region of radius a, magnetic field exists along its axis but the direction of magnetic field is opposite in the four quadrants of the region as shown in Figure.



A rod AB rotates with its end A at the centre of magnetic field and other end B slides on a smooth wire at the periphery of the region of magnetic field. At $t = 0$ the rod was situated along the $+x$ direction. Find and plot the time dependence of the current and thermal power in the resistance R, if the rod rotates with a

(a) constant angular velocity ω.
(b) constant angular acceleration α.

SOLUTION

(a) When the rod rotates with constant angular speed ω

$$\xi = \pm\frac{1}{2}B(a)(a\omega) = \pm\frac{1}{2}Ba^2\omega$$

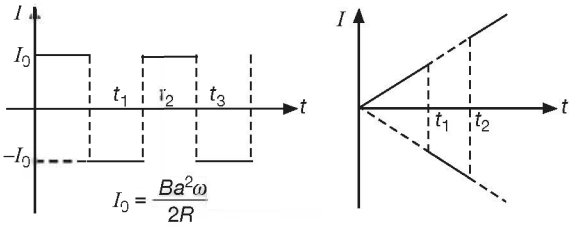
$$\Rightarrow \quad I = \frac{\xi}{R} = \pm\frac{Ba^2\omega}{2R}$$

So, for $t = 0$ to $\frac{T}{4}$, $\frac{T}{2}$ to $\frac{3T}{4}$ and so on $I = \frac{Ba^2\omega}{2R}$

and for $t = \frac{T}{4}$ to $\frac{T}{2}$, $\frac{3T}{4}$ to T, etc. $I = -\frac{Ba^2\omega}{2R}$

The I-t graph is as shown in Figure. The I-t equation can be written as,

$$I = \frac{1}{2R}(-1)^{n+1}Ba^2\omega$$



where $n = 1, 2, 3, \ldots$ and $t_n = \frac{n\pi}{2\omega}$

Here positive current means current from left to right through the resistance and vice-versa.

(b) When the rod rotates with constant angular acceleration α, $\omega = \alpha t$

and $I = (-1)^{n+1}\frac{Ba^2\alpha t}{2R}$ $\qquad \{\because \omega = \omega_0 + \alpha t\}$

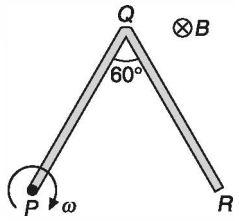
where $n = 1, 2, \ldots$ is the number of quarter revolution that the loop performs at the given instant.

The plot I-t is shown in Figure, where $t_n = \sqrt{\frac{n\pi}{\alpha}}$

$$\left\{\because\ n\pi = \frac{1}{2}\alpha t_n^2\right\}$$

ILLUSTRATION 44

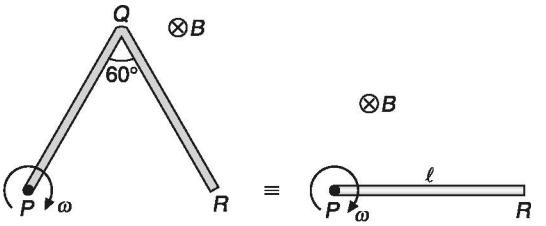
A rod PQR is bent such that $(PQ = QR = l)$ as shown in Figure.



The arrangement rotates about its end P with angular speed ω in a region of transverse magnetic field of strength B. Calculate the emf induced across the rod and the potential difference between points Q and R on the rod.

SOLUTION

The rod is equivalent to an equivalent rod joining the ends P to R and rotating in the same sense.



Since the point R is at a higher potential, so we get

$$V_R - V_P = \frac{B\omega l^2}{2} \qquad \ldots(1)$$

Also, we note that for the rod PQ, we have

$$V_Q - V_P = \frac{B\omega l^2}{2} \qquad \ldots(2)$$

Subtracting (1) from (2), we get

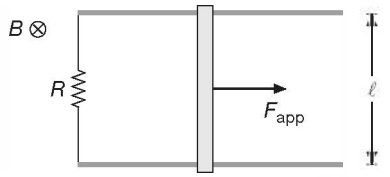
$$V_Q - V_R = 0$$

Test Your Concepts-II

Based on Faraday's Laws: Motional EMF

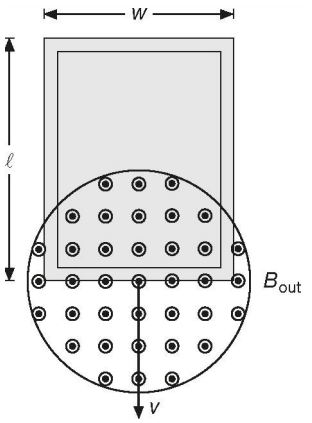
(Solutions on page H.132)

1. A conducting rod of length ℓ moves on two horizontal, frictionless rails, as shown in Figure. If a constant force of 1 N moves the bar at 2 ms^{-1} through a magnetic field $\vec{B}$ that is directed into the page



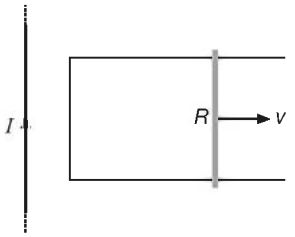
(a) what is the current through the 8 Ω resistor R?
(b) what is the rate at which energy is delivered to the resistor?
(c) what is the mechanical power delivered by the force F_{app}?

2. A conducting rectangular loop of mass M, resistance R and dimensions w by ℓ falls from rest into a magnetic field $\vec{B}$ as shown in Figure. During the time interval before the top edge of the loop reaches the field, the loop approaches a terminal speed v_T. Find v_T.

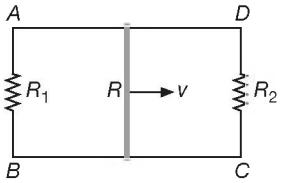


3. A long straight wire carrying a current I and a π-shaped conductor with sliding connector are located in the same plane as shown in Figure. The connector of length ℓ and resistance R slides to

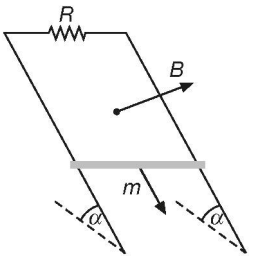
the right with a constant velocity v. Find the current induced in the loop as a function of separation r between the connector and the straight wire. The resistance of the π-shaped conductor and the self-inductance of the loop are assumed to be negligible.



4. A rectangular loop with a sliding connector of length ℓ is located in a uniform magnetic field perpendicular to the loop plane. The magnetic induction is equal to B. The connector has an electric resistance R, the sides AB and CD have resistances R_1 and R_2 respectively. Neglecting the self-inductance of the loop, find the current flowing in the connector during its motion with a constant velocity v.

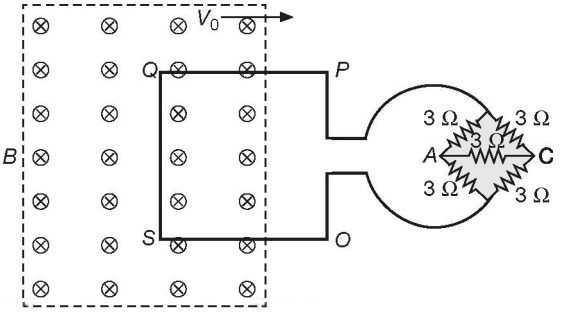


5. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity. At the top the bars are interconnected through a resistance R. The separation between the bars is equal to ℓ. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady state velocity of the connector.

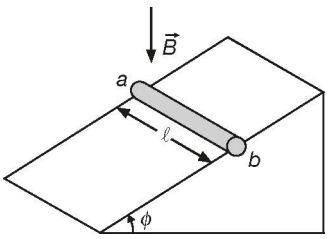


6. The two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmeter. What is the reading of the millivoltmeter when a train travels at a speed of 18 kmh^{-1} along the track given that the vertical components of earth's magnetic field is 0.2×10^{-4} weber m^{-2} and the rails are separated by 1 m? Track is south to north.

7. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2$ Wbm^{-2} as shown in the Figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 Ω. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in the loop? Give the direction of current in the loop.



8. A metal bar with length ℓ, mass m, and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. There is a uniform magnetic field of magnitude B directed downward in Figure. The bar is released from rest and slides down the rails.



(a) Is the direction of the current induced in the bar from a to b or from b to a?

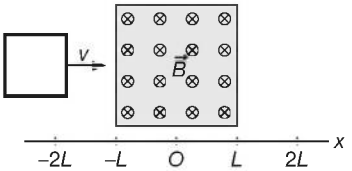
(b) What is the terminal speed of the bar?

(c) What is the induced current in the bar when the terminal speed has been reached?

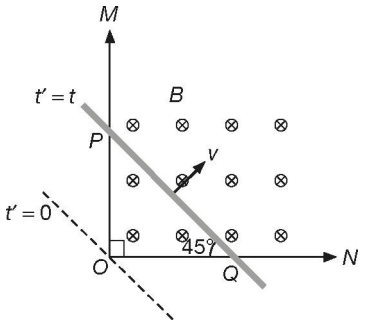
(d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar?

(e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

9. A square loop of wire of total length $4L$ and resistance R is moving at a constant speed v across a uniform magnetic field confined to a square region of area $4L^2$ (as shown). Assuming all the positive quantities to be along either $+x$ axis or along $+y$ axis, plot the graph of external force F required to move the loop at a constant speed as a function of coordinate x from $x=-2L$ to $x=+2L$, where x is to be measured from the centre of the magnetic field region to the centre of the loop. Also plot the graph of the induced current in the loop as a function of x. Take currents in the counter clockwise sense to be positive.

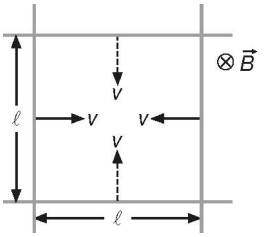


10. An electric circuit is composed of the three conducting rods MO, ON and PQ, as shown in Figure. The resistance of the rods per unit length is λ. The rod PQ slides, as shown in Figure, at a constant velocity v, keeping its tilt angle relative to ON and MO fixed at 45°. At each instant the circuit is closed. The whole system is embedded in a uniform magnetic field B, which is directed perpendicularly into the page.



Compute the time dependent induced electric current.

11. In the Figure shown the four rods have λ resistance per unit length. The arrangement is kept in a magnetic field of constant magnitude B and directed perpendicular to the plane of the Figure and directed inwards. Initially the sides as shown from a square. Now each wire starts moving with constant velocity v towards opposite wire. Find, as a function of time, the



(a) induced emf in the circuit.

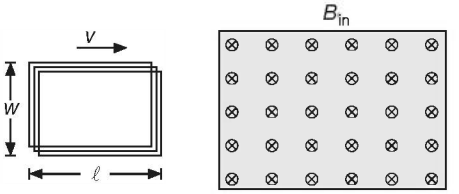
(b) induced current in the circuit with direction.

(c) force required on each wire to keep its velocity constant.
Neglect the interaction between the wires.

(d) total power required to maintain constant velocity.

(e) thermal power developed in the circuit.

12. A rectangular coil with resistance R has N turns, each of length ℓ and width w as shown in Figure. The coil moves into a uniform magnetic field $\vec{B}$ with constant velocity $\vec{v}$. What are the magnitude and direction of the total magnetic force on the coil.



(a) as it enters the magnetic field,

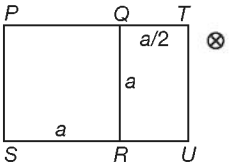
(b) as it moves within the field, and

(c) as it leaves the field?

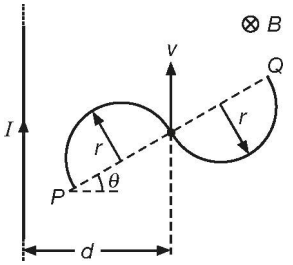
13. A helicopter has blades of length 3 m, extending out from a central hub and rotating at 2 revs^{-1}. If the vertical component of the Earth's magnetic field is 50 μT, what is the emf induced between the blade tip and the center hub?

14. Determine the current in the conductors of the circuit shown in Figure, if the intensity of a homogeneous magnetic field is perpendicular to the plane of the paper, inwards and changes in time

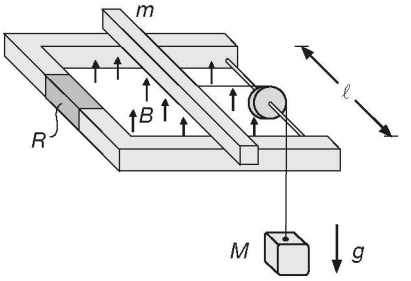
according to the law $B = kt$. The resistance of unit length of the conductors is λ.



15. An infinite wire carries a current I. An S-shaped conducting rod of two semicircles each of radius r is placed as shown. The centre of the conductor is at a distance d from the wire. If the rod translates parallel to the wire with a velocity v, calculate the emf induced across the ends PQ of the rod.

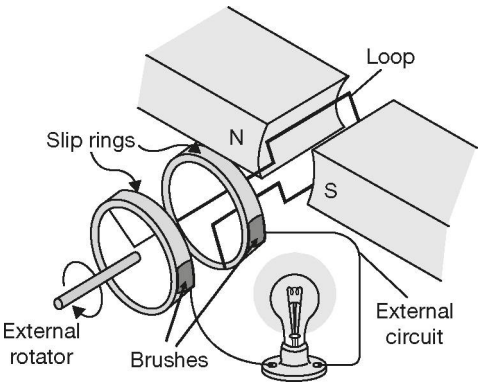


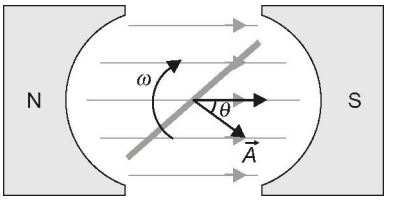
16. A bar of mass m is pulled horizontally across parallel rails by a massless string that passes over an ideal pulley and is attached to a suspended object of mass M as shown in Figure. The uniform magnetic field has a magnitude B and the distance between the rails is ℓ. The rails are connected at one end by a load resistor R. Derive an expression that gives the horizontal speed of the bar as a function of time, assuming that the suspended object is released with the bar at rest at $t = 0$. Assume no friction between rails and bar. Also find the terminal velocity obtained by the bar.



PRODUCTION OF INDUCED EMF BY ROTATING THE COIL IN A MAGNETIC FIELD: AN AC GENERATOR

One of the most important applications of Faraday's Law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electric energy into mechanical energy.





The rotating loop as seen from above

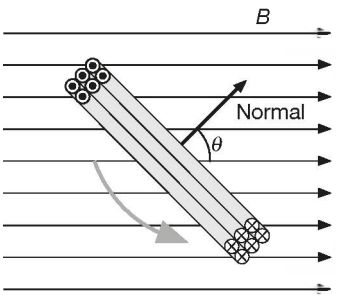


Figure is a simple illustration of a generator.

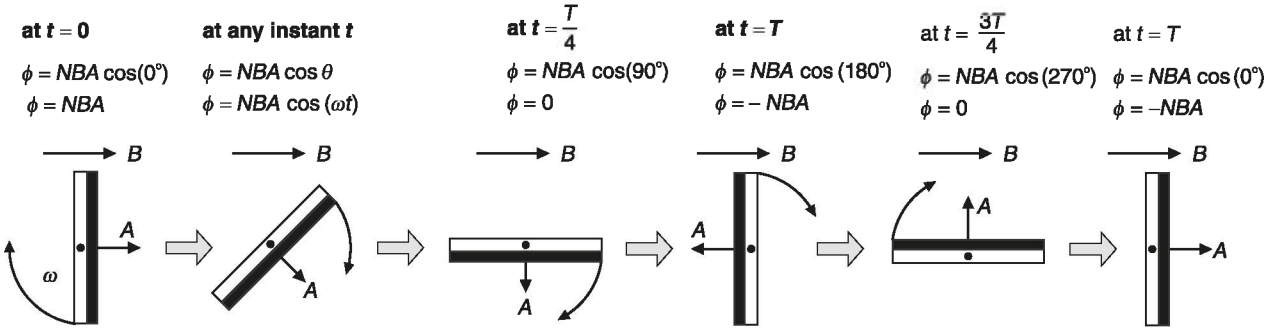


Figure Showing Cross-sectional view of coil rotating clockwise in a uniform magnetic field with uniform angular velocity ω.

It consists of an N-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure, we see that the magnetic flux through the loop may be written as

$$\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos(\omega t)$$

Suppose a coil of N turns, and area A is rotated in a uniform magnetic field B with angular velocity ω. As the coil rotates, the flux through it changes. Due to this change in flux an induced emf is set up in the coil.

Since, $\xi = -\frac{d\phi}{dt}$

At any instant, since we have

$$\phi = N\vec{B}\cdot\vec{A} = NBA\cos\theta = NBA\cos(\omega t)$$

So, $\xi = -\frac{d}{dt}(NBA\cos(\omega t))$

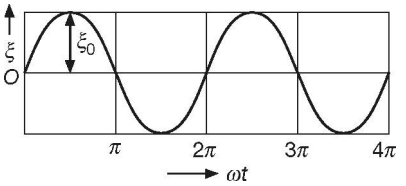
$\Rightarrow \quad \xi = NBA\omega\sin(\omega t)$

$\Rightarrow \quad \xi = \xi_0\sin(\omega t)$

where $\xi_0 = NBA\omega =$ Peak value of AC voltage developed.

We note that the induced emf ξ has a sinusoidal variation, having the peak value $NBA\omega$.

This forms the basic principle of the Alternating Current generator, which is a device converting mechanical energy to electrical energy.



If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by

$$I = \frac{|\xi|}{R} = \frac{NBA\omega}{R}\sin(\omega t)$$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = \frac{NBA\omega}{R}$. The power delivered to this circuit is

$$P = I|\xi| = \frac{(NBR\omega)^2}{R}\sin^2(\omega t)$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B\sin\theta = \mu B\sin(\omega t)$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\omega\sin(\omega t)$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin(\omega t)$$

the above expression becomes

$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin(\omega t)\right)B\omega\sin(\omega t)$$

$$\Rightarrow \quad P_m = \frac{(NAB\omega)^2}{R}\sin^2(\omega t)$$

As expected, the mechanical power input is equal to the electrical power output.

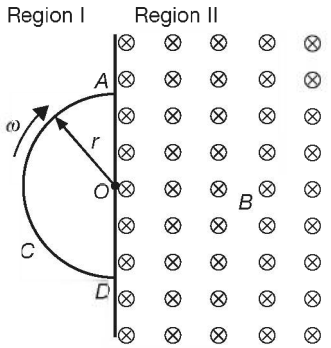
ILLUSTRATION 45

Space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semi-circular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is R.

(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into the region II.

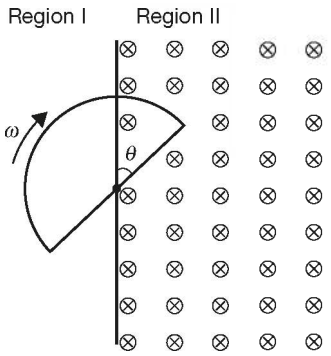
(c) Plot a graph between the induced emf and the time of rotation for two periods of rotation.



SOLUTION

(a) At time t, we have $\theta = \omega t$

Flux passing through coil is $\phi = BA\cos(0°)$



$$\Rightarrow \quad \phi = B\left(\frac{\theta}{2\pi}\right)(\pi r^2) \qquad \left\{\because A = \frac{1}{2}r^2\theta\right\}$$

$$\Rightarrow \quad \phi = \left(\frac{Br^2}{2}\right)\theta = \left(\frac{Br^2}{2}\right)\omega t$$

Magnitude of induced emf is given by

$$\xi = \frac{d\phi}{dt} = \frac{B\omega r^2}{2}$$

So, the induced current is

$$I = \frac{\xi}{R} = \frac{B\omega r^2}{2R}$$

(b) When the loop enters the Region II, the inward magnetic field passing through the loop is increasing. Hence, from Lenz's Law induced current will produce an outward magnetic field, which will be produced by a counter clockwise current.

(c) For half rotation i.e., at $t = \frac{T}{2} = \frac{\pi}{\omega}$, current in the loop will be of constant magnitude given by $I = \frac{B\omega r^2}{2R}$ and counter clockwise.

In the next half rotation, when loop comes out of Region II, current will be clockwise, but having the same magnitude. So, taking counter clockwise current as positive, I-t graph for two rotations will be as shown in Figure.

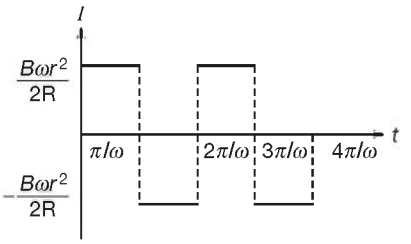
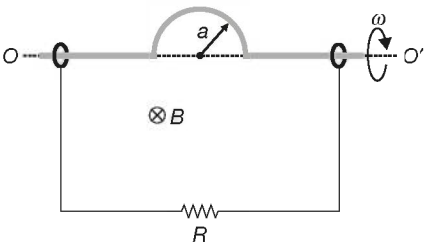


ILLUSTRATION 46

A wire shaped as a semi-circle of radius a rotates about an axis OO' with an angular velocity ω in a uniform magnetic field of induction B. The rotation axis is perpendicular to the field direction. The total resistance of the circuit is equal to R. Neglecting the magnetic field of the induced current, find the mean amount of thermal power being generated in the loop during a rotation period.



SOLUTION

Since, we know that

$$\phi = \vec{B}\cdot\vec{A}$$

$$\Rightarrow \quad \phi = B\left(\frac{\pi a^2}{2}\right)\cos(\omega t)$$

From Faraday's Laws, we have

$$\xi = -\frac{d\phi}{dt} = B\omega\left(\frac{\pi a^2}{2}\right)\sin(\omega t)$$

The induced current I, is then given by

$$I = \frac{\xi}{R} = \frac{\pi B\omega a^2}{2R}\sin(\omega t)$$

If P is the thermal power generated, then

$$P = \xi I = \frac{1}{R}\left(\frac{\pi B\omega a^2}{2}\right)^2 \sin^2(\omega t)$$

So, the average amount of thermal power being generated is

$$P_{av} = \langle P\rangle = \frac{1}{R}\left(\frac{\pi B\omega a^2}{2}\right)^2 \langle \sin^2(\omega t)\rangle$$

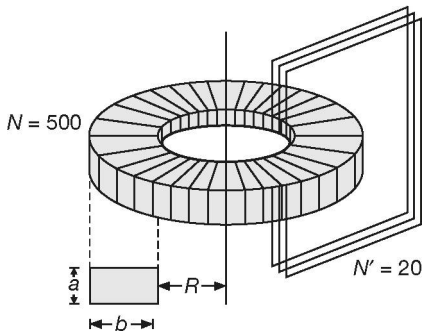
Since we know that $\langle \sin^2(\omega t)\rangle = \frac{1}{2}$

$$\Rightarrow \quad P_{av} = \langle P\rangle = \frac{1}{8R}(\pi B\omega a^2)^2$$

$$\Rightarrow \quad P_{av} = \frac{\pi^2 B^2\omega^2 a^4}{8R}$$

ILLUSTRATION 47

A toroid having a rectangular cross section $a \times b$, where $a = 2$ cm, $b = 3$ cm and inner radius $R = 4$ cm consists of 500 turns of wire that carries a sinusoidal current $I = I_0\sin(\omega t)$, with $I_0 = 50$ A and a frequency $f = 60$ Hz. A coil that consists of 20 turns of wire links with the toroid, as shown in Figure. Determine the emf induced in the coil as a function of time.



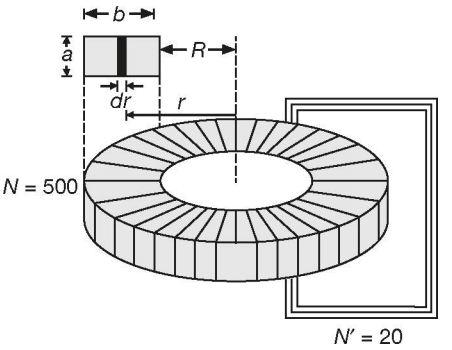
SOLUTION

In a toroid, all the field and hence the flux is confined to the inside of the toroid. Field inside the toroid is given by,

$$B = \frac{\mu_0 NI}{2\pi r}$$

$$B = \frac{500\mu_0 I}{2\pi r}, \text{ where } I = I_0\sin(\omega t)$$

Consider an infinitesimal area element of length a, width dr having area dA, then $dA = a\,dr$



Magnetic flux through this element is

$$d\phi_B = BdA\cos(0°) = BdA$$

$$\Rightarrow \quad d\phi_B = \left(\frac{\mu_0 NI}{2\pi r}\right)a\,dr$$

$$\Rightarrow \quad d\phi_B = \left(\frac{\mu_0 NI_0\sin(\omega t)}{2\pi r}\right)a\,dr$$

$$\Rightarrow \quad \phi_B = \int BdA = \frac{500\mu_0 I_0}{2\pi}\sin(\omega t)\int\frac{a\,dr}{r}$$

Please note that I is varying with t and not with r and hence can be taken out of the integral.

$$\Rightarrow \quad \phi_B = \frac{500\mu_0 I_{\max}}{2\pi}a\sin(\omega t)\log_e\left(\frac{b+R}{R}\right)$$

Since, $|\xi| = N'\left(\frac{d\phi_B}{dt}\right)$

$$\Rightarrow \quad \xi = N'\frac{d\phi_B}{dt}$$

$$\Rightarrow \quad \xi = 20\left(\frac{500\mu_0 I_{\max}}{2\pi}\right)\omega a\log_e\left(\frac{b+R}{R}\right)\cos(\omega t)$$

$$\Rightarrow \quad \xi = \frac{10^4}{2\pi}(4\pi\times10^{-7})(50)(377)(0.02)\times \log_e\left(\frac{3+4}{4}\right)\cos(\omega t)$$

$$\Rightarrow \quad \xi = (0.422)\cos(\omega t)\text{ V} = 422\cos(\omega t)\text{ mV}$$

Test Your Concepts-III

Based on Faraday's Laws: AC Generator

(Solutions on page H.136)

1. A steel guitar string vibrates. The component of magnetic field, in millitesla, perpendicular to the area of a pickup coil nearby is given by $B = 50 + 4\sin(1000t)$. The circular pickup coil has 50 turns and radius 7 mm. Find the emf induced in the coil as a function of time and the peak value.

2. A rectangular coil of 60 turns, dimensions 10 cm by 20 cm and total resistance 10 Ω, rotates with angular speed 30 $rads^{-1}$ about the *y*-axis in a region where a 1 T magnetic field is directed along the *x*-axis. The rotation is initiated so that the plane of the coil is perpendicular to the direction of $\vec{B}$ at $t = 0$. Calculate

(a) the maximum induced emf in the coil,

(b) the maximum rate of change of magnetic flux through the coil,

(c) the induced emf at $t = \frac{\pi}{120}$ s and

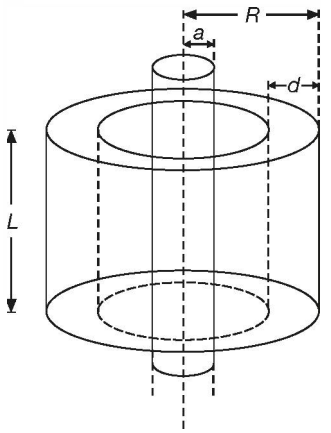
(d) the torque exerted by the magnetic field on the coil at the instant when the emf is a maximum.

3. A solenoid wound with 2000 $turnsm^{-1}$ is supplied with current that varies in time according to $I = 5\sin(100\pi t)A$, where t is in seconds. A small coaxial circular coil of 160 turns and radius $r = 5$ cm is located inside the solenoid near its center.

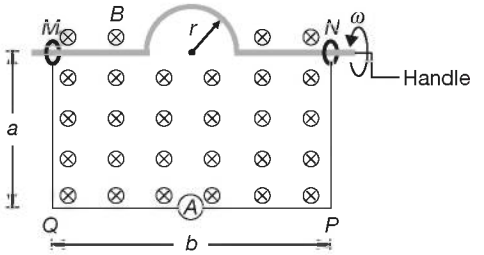
(a) Derive an expression that describes the manner in which the emf in the small coil varies in time.

(b) At what average rate is energy delivered to the small coil if the windings have a total resistance of 8 Ω?

4. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R thickness $d(d \ll R)$ and length L. A variable current $I = I_0\sin(\omega t)$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.



5. Consider the circuit depicted in Figure. The circuit is embedded in a uniform magnetic field *B* directed perpendicularly into the page. The upper branch of the circuit, *MN*, is rotated at a constant angular velocity *w*, by turning the handle shown in the Figure. The electric resistance of the circuit is *R*.

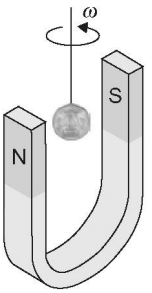


(a) Compute the time dependent magnetic flux through the circuit.

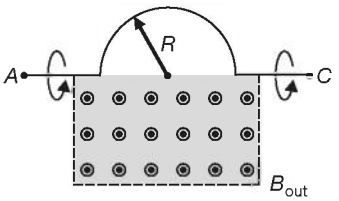
(b) Compute the EMF induced along the circuit.

(c) Compute the induced current in the circuit.

6. A coin is suspended from a thread and hung between the poles of a strong horseshoe magnet as shown in Figure. The coin rotates at constant angular speed *w* about a vertical axis. If θ represents the angle between the direction of $\vec{B}$ and the normal to the face of the coin, sketch a graph of the torque due to induced currents as a function of θ, for $0 < \theta < 2\pi$.



7. A semi-circular conductor of radius R = 0.25 m is rotated about the axis AC at a constant rate of 120 revmin^{-1} (shown in Figure). A uniform magnetic field in all of the lower half of the Figure is directed out of the plane of rotation and has a magnitude of 1.3 T.



(a) Calculate the maximum value of the emf induced in the conductor.
(b) What is the value of the average induced emf for each complete rotation?
(c) How would the answers to (a) and (b) change if $\vec{B}$ were allowed to extend a distance R above the axis of rotation?
(d) Sketch the emf versus time when the field is as shown in Figure and
(e) Sketch the emf versus time when the field is extended as described in (c).

8. The rotating loop in a AC generator is a square 10 cm on a side. It is rotated at 60 Hz in a uniform field of 800 mT. Calculate
(a) the flux through the loop as a function of time,
(b) the emf induced in the loop,
(c) the current induced in the loop for a loop resistance of 1 Ω,
(d) the power delivered to the loop, and
(e) the torque that must be exerted to rotate the loop.

9. A motor in normal operation carries a direct current of 5 A when connected to a 220 V power supply. The resistance of the motor windings is 24 Ω.
(a) What is the back emf generated by the motor during normal operation?
(b) At what rate is internal energy produced in the windings?
(c) Suppose that a malfunction stops the motor shaft from rotating. At what rate will internal energy be produced in the windings in this case?

INDUCED ELECTRIC FIELD

Since we know that the electric potential difference between two points A and B in an electric field $\vec{E}$ can be written as

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$$

When the electric field is conservative, as is the case of electrostatics, the line integral of $\vec{E}\cdot d\vec{l}$ is path-independent, which implies $\oint \vec{E}\cdot d\vec{l} = 0$.

Faraday's Law shows that when magnetic flux changes with time, an induced current begins to flow in the circuits or closed loops. **What causes the charges to move?**

It is the induced emf, which is the work done per unit charge that makes the charge to move. Since magnetic field does no work in moving the charge, so the work done on the mobile charges must be electric, and the electric field in this situation cannot be conservative because the line integral of a conservative electric field in a closed loop is always zero. **So, we can say that electric field associated with an induced emf/current (developed in a closed loop) has to be non-conservation in nature.** Hence

$$\xi = \oint \vec{E}_{nc}\cdot d\vec{l} \qquad \ldots(1)$$

From Faraday's Law, we have $\xi = -\frac{d\phi_B}{dt}$ …(2)

From (1) and (2), we get

$$\oint \vec{E}_{nc}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

The above expression implies that a changing magnetic flux will induce a non-conservative electric field which can vary with time. It is important to distinguish between the induced, non-conservative electric field and the conservative electric field which arises from electric charges.

So, $\oint \vec{E}_{nc} \cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A\left(\frac{dB}{dt}\right)$

where A is the area of the region in which magnetic flux (or magnetic field) is changing and the negative sign takes into account the opposing nature of the induced electric field. However, in magnitude we have

$$\oint \vec{E}_{nc} \cdot d\vec{l} = \left|\frac{d\phi_B}{dt}\right| = A\left(\frac{dB}{dt}\right)$$

Conceptual Note(s)

The induced electric field $\vec{E}_{nc}$ has following important properties.

(a) It is non-conservative in nature.

(b) The line integral of $\vec{E}_{nc}$ around a closed path/loop is non-zero.

(c) When a charge q goes once around the loop, the total work done on the charge by this induced electric field is equal to q times the induced emf. Hence

$$W = q\oint \vec{E}_{nc} \cdot d\vec{\ell} = q\xi$$

where, ξ is the induced emf given by

$$\xi = -\frac{d\phi_B}{dt} = -A\left(\frac{dB}{dt}\right)$$

and A is the area of the region in which magnetic field is changing.

(d) Always keep in mind that $\vec{E}_{nc}$ will exist both inside and outside the region of changing magnetic flux.

(e) The equation $\oint \vec{E}_{nc} \cdot d\vec{\ell} = \xi = -\frac{d\phi_B}{dt}$ is valid only if the path around which we integrate is stationary.

(f) Because of symmetry, the electric field $\vec{E}_{nc}$ has the same magnitude at every point on the circular imaginary contour (or loop) and is tangential to it at each point.

(g) Also note that the direction of $\vec{E}_{nc}$ is same as the direction of induced current.

(h) $\vec{E}_{nc}$ being a non-conservative field, the concept of potential has no meaning for such a field.

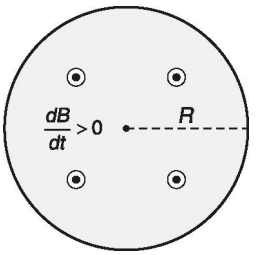
(i) This induced field is different from the electrostatic field produced by stationary charges (which is conservative in nature).

(j) The relation $\vec{F} = q\vec{E}_{nc}$ is still valid for this field.

(k) This field can vary with time. So, we observe that a changing magnetic field acts as a source of an induced electric field that cannot be produced from any static charge distribution.

INDUCED ELECTRIC FIELD DUE A TIME VARYING MAGNETIC FIELD CONFINED TO A CYLINDRICAL REGION

Let's consider a uniform magnetic field which points out of the page and is confined to a cylindrical region of radius R as shown in Figure.



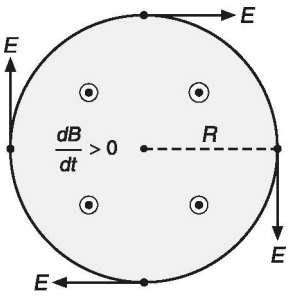
Let us assume that the magnitude of $\vec{B}$ increases with time, i.e., $\frac{dB}{dt} > 0$. To calculate the induced electric field everywhere due to the changing magnetic field we use the following thought process.

(a) Since the magnetic field is confined to a circular region, from symmetry arguments we select the integration path to be a circle of radius r.

(b) The magnitude of the induced field $\vec{E}_{nc}$ at all points on a circle is the same.

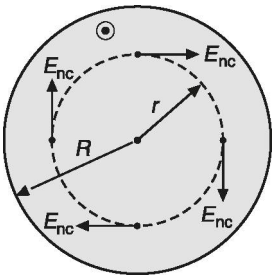
(c) According to Lenz's Law, the direction of induced field $\vec{E}_{nc}$ must be such that it would drive the induced current to produce a magnetic field that opposes the change in magnetic flux.

(d) For $\frac{dB}{dt} > 0$, the outward magnetic flux is increasing. Therefore, to counteract this change, the induced current (or the induced field) must flow clockwise so that it sets up an inward field which opposes the outward growth of magnetic flux. The direction of $\vec{E}_{nc}$ is shown in Figure.



Magnitude of $\vec{E}_{nc}$

Consider the region $r < R$ i.e. inside the cylindrical region as shown in Figure.



Inside the cylindrical region

The rate of change of magnetic flux is

$$\frac{d\phi_B}{dt} = \frac{d}{dt}(\vec{B}\cdot\vec{A}) = A\frac{dB}{dt} = \pi r^2\left(\frac{dB}{dt}\right)$$

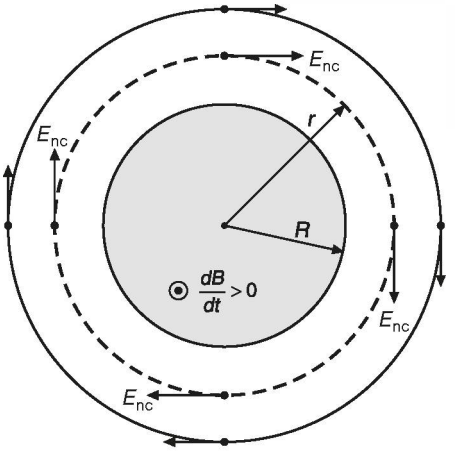
Since we know that

$$\oint \vec{E}_{nc}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E_{nc}(2\pi r) = -\pi r^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad |E_{nc}| = \frac{r}{2}\frac{dB}{dt}$$

Now, consider the region for $r > R$ i.e. outside the cylindrical region as shown in Figure.



Outside the cylindrical region

Outside the cylindrical region, we have

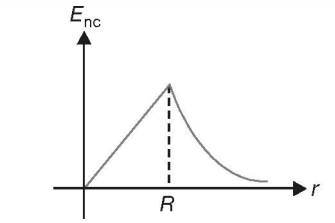
$$E_{nc}(2\pi r) = -A\left(\frac{dB}{dt}\right) = -\pi R^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad |E_{nc}| = \frac{R^2}{2r}\frac{dB}{dt}$$

So, we observe that

$$|E_{nc}| = \begin{cases} \frac{r}{2}\left(\frac{dB}{dt}\right) & \text{for} \quad r < R \\ \frac{R^2}{2r}\left(\frac{dB}{dt}\right) & \text{for} \quad r > R \end{cases}$$

A plot of E_{nc} as a function of r is shown in Figure.



Induced electric field as a function of r

FINDING DIRECTION OF INDUCED ELECTRIC FIELD

An easy way to find the direction of induced electric field is by making use of Right-Hand Thumb Rule. Curl the fingers of right hand in such a way that thumb points opposite to the direction of changing field i.e., $\frac{dB}{dt}$, then curl of the fingers gives the direction of induced electric field.

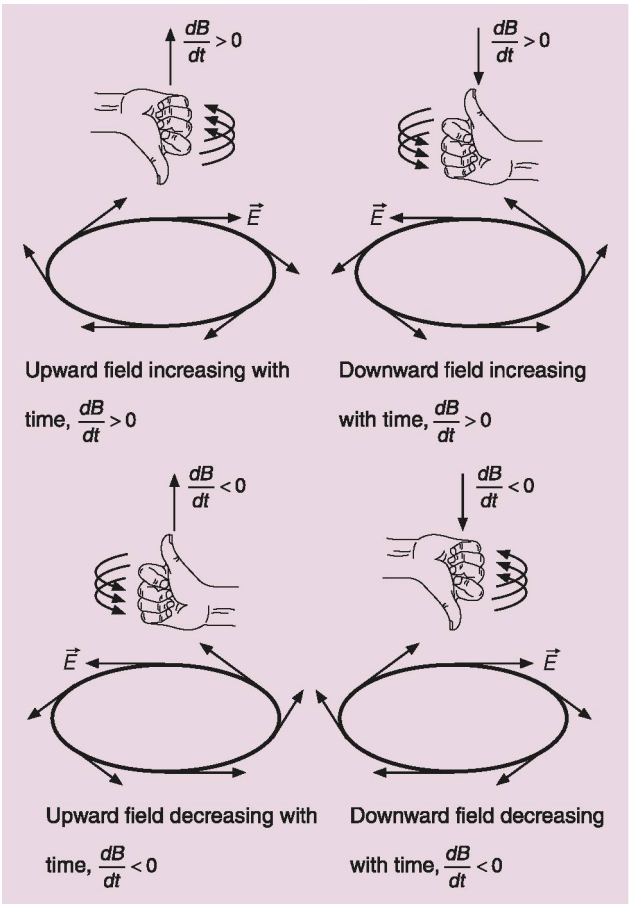
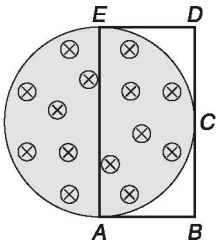

dB/dt > 0
E
Upward field increasing with time, dB/dt > 0
dB/dt > 0
E
Downward field increasing with time, dB/dt > 0
dB/dt < 0
E
Upward field decreasing with time, dB/dt < 0
dB/dt < 0
E
Downward field decreasing with time, dB/dt < 0

ILLUSTRATION 48

A time varying magnetic field $B = a + bt$ exists in a cylindrical region of radius R. A rectangular conducting loop $ABDE$ of dimension $R \times 2R$ is kept as shown in Figure. Calculate the value of emf induced across AE, ED and DB.



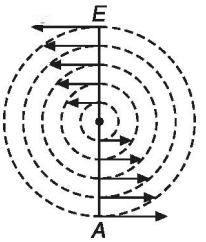
SOLUTION

Inside the circular region containing the field, the induced electric field is directly proportional to the radial distance r. So, we have

$$E_{\text{induced}} \propto r$$

EMF induced across AE

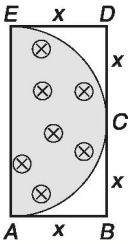
Induced electric field being concentric will be perpendicular to any section of AE as shown in Figure.



$$\text{So, } V_{AE} = -\int_{A}^{E} \vec{E} \cdot d\vec{l} = 0 \qquad \{\because \theta = 90^\circ\}$$

Due to symmetry, the emf induced across each of the sections ED, DC, CB and BA will be same.

Let the emf induced across each one of them be x.



Rate of change of flux through $ABCDE$ is

$$\frac{d\phi}{dt} = A\frac{dB}{dt} = \frac{\pi R^2}{2}\frac{d}{dt}(a+bt) = \frac{\pi R^2 b}{2}$$

$$\Rightarrow \quad |\xi_{\text{in}}| = \frac{d\phi}{dt} = \frac{\pi R^2 b}{2} = \oint \vec{E} \cdot d\vec{l} = 4x + 0 = 4x$$

$$\Rightarrow \quad 4x = \frac{\pi R^2 b}{2}$$

$$\Rightarrow \quad x = \frac{\pi R^2 b}{8}$$

EMF induced across ED is

$$V_{ED} = x = \frac{\pi R^2 b}{8}$$

EMF induced across BD is

$$V_{BD} = x = 2\left(\frac{\pi R^2 b}{8}\right) = \frac{\pi R^2 b}{4}$$

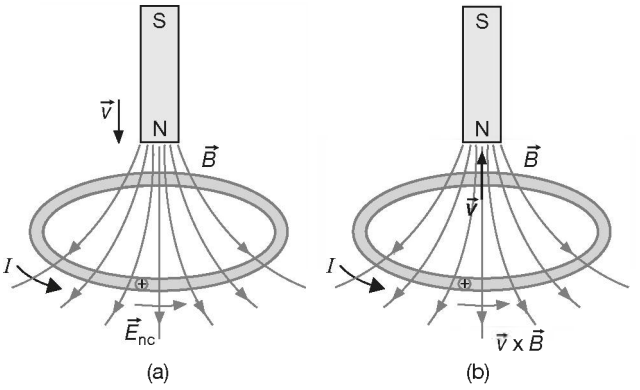
INDUCED EMF AND REFERENCE FRAME

Since we know that the induced emf ξ, for a moving conductor is also given by $\xi = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$. In addition, we also know that the induced emf associated

with a stationary conductor may be written as the line integral of the non-conservative electric field. So, we have

$$\xi = \oint \vec{E}_{nc} \cdot d\vec{l}$$

However, whether an object is moving or stationary actually depends on the reference frame. As an example, let's examine the situation where a bar magnet is approaching a conducting loop. An observer O in the rest frame of the loop sees the bar magnet moving toward the loop. An electric field $\vec{E}_{nc}$ is induced to drive the current around the loop, and a charge on the loop experiences an electric force $\vec{F}_e = q\vec{E}_{nc}$. Since the charge is at rest according to observer O, no magnetic force is present. On the other hand, an observer O' in the rest frame of the bar magnet sees the loop moving toward the magnet. Since the conducting loop is moving with a velocity $\vec{v}$, a motional emf is induced. In this frame, O' sees the charge q moving with a velocity $\vec{v}$, and concludes that the charge experiences a magnetic force $\vec{F}_B = q(\vec{v} \times \vec{B})$.



Induction observed in different reference frames. In (a) the bar magnet is moving, while in (b) the conducting loop is moving.

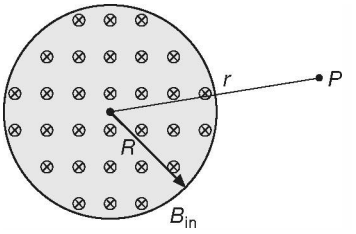
Since the event seen by the two observer is the same except the choice of reference frames, the force acting on the charge must be the same, $\vec{F}_e = \vec{F}_B$, which implies

$$\vec{E}_{nc} = \vec{v} \times \vec{B}$$

In general, as a consequence of relativity, an electric phenomenon observed in a reference frame O may appear to be a magnetic phenomenon in a frame O' that moves at a speed v relative to O.

ILLUSTRATION 49

For the situation shown in Figure, the magnetic field changes with time according to the expression $B = (2t^3 - 4t^2 + 0.8)$ T and $r = 2R = 5$ cm.



(a) Calculate the magnitude and direction of the force exerted on an electron located at point P_2 when $t = 2$ s.

(b) At what time is this force equal to zero?

SOLUTION

Since we are asked to calculate the force on an electron, so we should first calculate the induced electric field due to the variation in B with time. For that we know,

$$\oint \vec{E} \cdot d\vec{l} = \left| \frac{d\phi_B}{dt} \right|$$

$$\Rightarrow \quad E(2\pi r) = A\left(\frac{dB}{dt}\right)$$

Please note that A is the area to which the magnetic field is confined. So, $A = \pi R^2$

$$\Rightarrow \quad E(4\pi R) = \pi R^2 \left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{R}{4}\left(\frac{dB}{dt}\right), \text{ where } \frac{dB}{dt} = 6t^2 - 8t$$

$$\Rightarrow \quad E|_{t=2} = \frac{R}{4}(6t^2 - 8t)\Big|_{t=2} = \frac{R}{4}(6(4) - 8(2)) = 2R$$

$$\Rightarrow \quad E|_{t=2} = \left(\frac{5}{100}\right) \text{NC}^{-1} \qquad \{\because R = 2.5 \text{ cm}\}$$

So, $F|_{t=2} = qE|_{t=2} = 8 \times 10^{-21}$ N

$$\Rightarrow \quad F = 8 \times 10^{-21} \text{ N, downwards perpendicular to } r.$$

This force will become zero, when $E = 0$, which happens when $\frac{dB}{dt} = 0$

$$\Rightarrow \quad 6t^2 - 8t = 0$$

$$\Rightarrow \quad 6t-8=0$$

$$\Rightarrow \quad t=\frac{4}{3}\ \text{s}$$

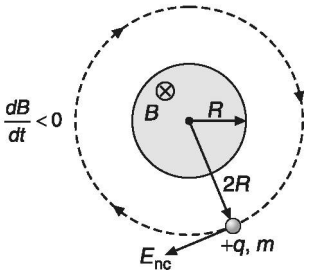
ILLUSTRATION 50

A long solenoid has n turns per unit length, radius R and carries a current I is kept in gravity free region. From its axis, at a distance twice that of its radius, a charge $+q$ and mass m is placed. If the current in solenoid is suddenly switched off, find the velocity attained by the charge.

SOLUTION

The magnetic field inside the solenoid is along the axis of the solenoid and is given by

$$B_{\text{inside}}=\mu_0 nI$$



Let the duration in which field drops to zero after switching off the current in the solenoid be Δt. Due to the switching off the current to the solenoid, an electric field E_{nc} is induced at a distance $x=2R$ i.e. outside the solenoid in this time interval Δt. So, we have

$$E_{nc}(2\pi x)=-A\left(\frac{\Delta B}{\Delta t}\right)=-\pi R^2\left(\frac{0-\mu_0 nI}{\Delta t}\right)$$

$$\Rightarrow \quad E_{nc}(2\pi x)=\pi R^2\left(\frac{\mu_0 nI}{\Delta t}\right)$$

$$\Rightarrow \quad E_{nc}=\frac{R^2}{2x}\left(\frac{\mu_0 nI}{\Delta t}\right)=\frac{R^2}{2(2R)}\left(\frac{\mu_0 nI}{\Delta t}\right)$$

$$\Rightarrow \quad E_{nc}=\frac{R}{4}\left(\frac{\mu_0 nI}{\Delta t}\right) \qquad \ldots(1)$$

Due to this induced electric field, the charge $+q$ experiences an impulse in this duration which is given by

$$F\Delta t=mv-0$$

$$\Rightarrow \quad (qE_{nc})\Delta t=mv$$

Using equation (1), we get

$$q\left[\frac{R}{4}\left(\frac{\mu_0 nI}{\Delta t}\right)\right]\Delta t=mv$$

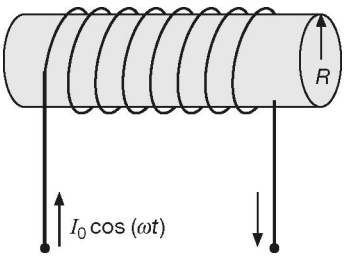
$$\Rightarrow \quad v=\frac{\mu_0 nIqR}{4m}$$

ILLUSTRATION 51

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I=I_0\cos(\omega t)$, where I_0 is the maximum current and ω is the angular frequency of the alternating current source.

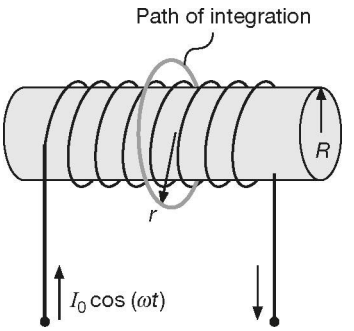
(a) Determine the magnitude of the induced electric field outside the solenoid at a distance $r>R$ from its long central axis.

(b) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?



SOLUTION

(a) First let us consider an external point and take the path (contour) for our line integral to be a circle of radius r centered on the solenoid, as illustrated in Figure.



By symmetry we see that the magnitude of $\vec{E}$ is constant on this path and that $\vec{E}$ is tangent to it. The magnetic flux through the area enclosed by this path is $BA=B\pi R^2$, so

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = E(2\pi r) = -\pi R^2 \frac{dB}{dt} \qquad \ldots(1)$$

The magnetic field inside a long solenoid is given by $B = \mu_0 nI$, where $I = I_0 \cos(\omega t)$. So, (1) becomes

$$E(2\pi r) = -\pi R^2 \mu_0 n I_0 \frac{d}{dt}(\cos \omega t)$$

$$\Rightarrow E(2\pi r) = \pi R^2 \mu_0 n I_0 \omega \sin(\omega t)$$

$$\Rightarrow E = \left(\frac{\mu_0 n I_0 \omega R^2}{2r}\right)\sin(\omega t) \quad (\text{for } r > R) \qquad \ldots(2)$$

Hence, the amplitude of the electric field outside the solenoid falls off as $\frac{1}{r}$ and varies sinusoidally with time.

(b) For an interior point $(r < R)$, the flux through an integration loop is given by $B(\pi r^2)$. Using the same procedure as in part (a), we find that

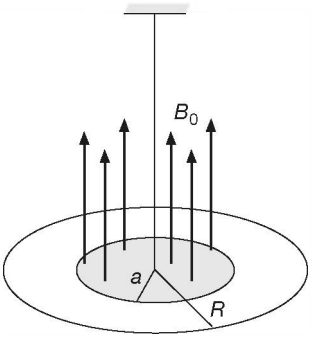
$$E(2\pi r) = -\pi r^2 \frac{dB}{dt} = \pi r^2 \mu_0 n I_0 \omega \sin(\omega t)$$

$$\Rightarrow E = \frac{\mu_0 n I_0 \omega}{2} r \sin(\omega t) \qquad (\text{for } r < R) \qquad \ldots(3)$$

This shows that the amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with r and varies sinusoidally with time.

ILLUSTRATION 52

A line charge λ is wound around an insulating disc of mass M and radius R, which is then suspended horizontally as shown in Figure, so that it is free to rotate.



In the central region of radius a, there is a uniform magnetic field B_0, pointing up. Now the magnetic field is turned off, which causes the disc to rotate. Find the angular speed ω with which the disc starts rotating. Can you explain the rotation of the disc.

SOLUTION

When the field is switched off, the flux linked with the central portion changes, due to which an induced circular electric field is set up. The line charge element will experience a torque/couple which makes the disc to rotate.

If $\vec{E}$ be the induced electric field, then

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt} = -\pi a^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow E(2\pi R) = -\pi a^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow E = -\frac{a^2}{2R}\left(\frac{dB}{dt}\right) \qquad \ldots(1)$$

If τ is the torque on the disc due to this field, then

$$\tau = FR = qER = \lambda(2\pi R)ER = -\pi\lambda a^2 R\left(\frac{dB}{dt}\right)$$

If ω be the angular speed of the disc, then

$$\tau = \frac{dL}{dt}$$

$$\Rightarrow \int \tau dt = \int dL$$

$$\Rightarrow -\pi\lambda a^2 R\int dB = \int dL$$

$$\Rightarrow -\pi\lambda a^2 R\Delta B = \Delta L$$

$$\Rightarrow -\pi\lambda a^2 R(B_f - B_i) = L_f - L_i$$

$$\Rightarrow -\pi\lambda a^2 R(0 - B_0) = I(\omega - 0), \text{ where } I = \frac{1}{2}MR^2$$

$$\Rightarrow \omega = \frac{2\pi\lambda a^2 R B_0}{MR}$$

ILLUSTRATION 53

A long solenoid of cross-sectional radius a has a thin insulated wire ring tightly put on its winding; one-half of the ring has the resistance $\eta(>1)$ times that of the other half. The magnetic induction produced by the solenoid varies with time as $B = bt$, where b is a constant. Find the magnitude of the electric field strength in the ring.

SOLUTION

The emf induced in the ring is

$$\xi = \left|\frac{d\phi}{dt}\right| = \pi a^2 b$$

The changing magnetic field produces an induced emf in the ring which is equal for both halves of the ring. So, for each half of the ring, the induced emf is $\frac{\xi}{2}$. Please note that the induced emf is independent of the resistance of the ring. The induced current flowing in the ring is

$$i = \frac{\xi}{R_{net}} = \frac{\xi}{r+\eta r}$$

Conceptual Note(s)

We are given that the two halves of the ring have different resistances, so our intuition says that the half ring with lesser resistance should have a higher current and the half ring with higher resistance should have a lower current which cannot be possible because

"How will a single loop carry two different currents?"

The answer to this puzzle is simple and logical. The current in the loop will have one single value if we use the concept of induced electric field. The induced electric field is set up such that according to Lenz's Law, in the half ring with smaller current it will be along the current and in the other half ring with larger current it will oppose the current.

If E is the induced electric field in the ring, then for the half ring with low resistance r carrying a current i, we have

$$ir = \frac{\xi}{2} - (\pi a)E \qquad \ldots(1)$$

and for the other half of the ring with high resistance ηr, also carrying a current i, we have

$$i(\eta r) = \frac{\xi}{2} + (\pi a)E \qquad \ldots(2)$$

Subtracting equation (1) from (2), we get

$$(\eta - 1)ir = 2\pi aE \qquad \ldots(3)$$

Adding equations (1) and (2), we get

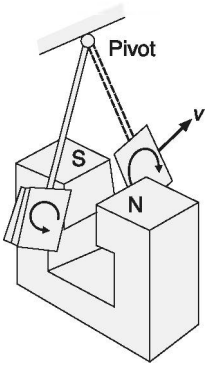
$$(\eta + 1)ir = \xi = \pi a^2 b \qquad \ldots(4)$$

Dividing equations (3) by (4), we get

$$\Rightarrow \quad E = \frac{ab}{2}\left(\frac{\eta - 1}{\eta + 1}\right)$$

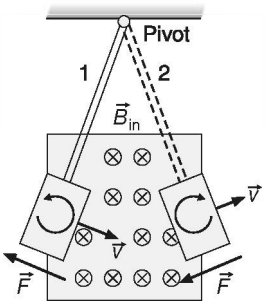
EDDY CURRENTS

Since we know that an emf and a current are induced in a circuit by a changing magnetic flux. In the same manner, circulating currents called **eddy currents** are induced in bulk pieces of metal (or metal sheets or metal plates) moving through a magnetic field. This can easily be demonstrated by allowing a flat copper or aluminum plate attached at the end of a rigid bar to swing back and forth through a magnetic field.



Formation of eddy currents in a conducting plate moving through a magnetic field. As the plate enters or leaves the field, the changing magnetic flux induces an emf, which causes eddy currents in the plate.

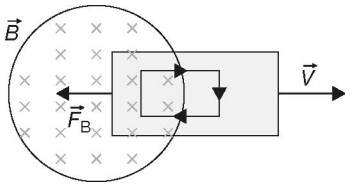
As the plate enters the field, the changing magnetic flux induces an emf in the plate, which in turn causes the free electrons in the plate to move, producing the **swirling eddy current**. According to Lenz's Law, the direction of the eddy currents is such that they create magnetic fields that oppose the change that causes the currents. For this reason, the eddy currents must produce effective magnetic poles on the plate, which are repelled by the poles of the magnet; this gives rise to a repulsive force that opposes the motion of the plate. (If the opposite were true, the plate would accelerate and its energy would increase after each swing, in violation of the Law of Conservation of Energy). As indicated in figure with $\vec{B}$ directed into the page, the induced eddy current is counterclockwise as the swinging plate enters the field at position 1.



As the conducting plate enters the field (position 1), the eddy currents are counterclockwise. As the plate leaves the field (position 2), the currents are clockwise. In either case, the force on the plate is opposite the velocity, and eventually the plate comes to rest.

This is because the flux due to the external magnetic field into the page through the plate is increasing, and hence by Lenz's Law the induced current must provide its own magnetic field out of the page. The opposite is true as the plate leaves the field at position 2, where the current is clockwise. Because the induced eddy current always produces a magnetic retarding force $\vec{F}$ when the plate enters or leaves the field, the swinging plate eventually comes to rest.

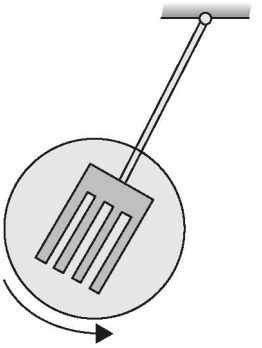
In simplified words, the induced eddy currents generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field.



Magnetic force arising from the eddy current that opposes the motion of the conducting slab.

Minimising Eddy Currents

Since the conductor has non-zero resistance R, Joule heating causes a loss of power by an amount $P=\frac{\xi^2}{R}$. Therefore, by increasing the value of R, power loss can be reduced. One way to increase R is to laminate the conducting slab, or construct the slab by gluing together thin strips that are insulated from one another. Another way is to make cuts in the slab, thereby disrupting the conducting path and preventing the formation of large current loops, as shown.



When slots are cut in the conducting plate, the eddy currents are reduces and the plate swings more freely through the magnetic field.

Eddy currents are often undesirable because they represent a transformation of mechanical energy to internal energy. To reduce this energy loss, conducting parts are often laminated that is, they are built up in thin layers separated by a non-conducting material such as lacquer or a metal oxide. This layered structure increases the resistance of eddy current paths and effectively confines the currents to individual layers. Such a laminated structure is used in transformer cores and motors to minimize eddy currents and thereby increase the efficiency of these devices.

Induction Brakes

The braking systems on many subway and rapid-transit cars make use of electromagnetic induction and eddy currents. An electromagnet attached to the train is positioned near the steel rails. (An electromagnet is essentially a solenoid with an iron core). The braking action occurs when a large current is passed through the electromagnet. The relative motion of the magnet and rails induces eddy currents in the rails, and the direction of these currents produces a drag force on the moving train. Because the eddy currents decrease steadily in magnitude as the train slows down, the braking effect is quite smooth. As a safety measure, some power tools use eddy currents to stop rapidly spinning blades once the device is turned off.

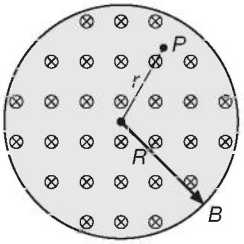
Test Your Concepts-IV

Based on Faraday's Laws: Induced Electric Field

(Solutions on page H.138)

1. A long solenoid with n turns per meter and radius R carries an oscillating current given by $I = I_0 \sin(\omega t)$. What is the electric field induced at a radius $r (< R)$ from the axis of the solenoid? What is the direction of this electric field when the current is increasing counter clockwise in the coil?

2. A magnetic field directed into the page changes with time according to $B = (0.03t^2 + 1.4)$ T, where t is in seconds. The field has a circular cross section of radius $R = 2.5$ cm. What are the magnitude and direction of the electric field at point P when $t = 3$ s and $r = 0.02$ m?

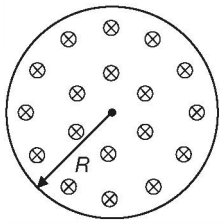


3. A betatron accelerates electrons to energies in the *MeV* range by means of electromagnetic induction. Electrons in a vacuum chamber are held in a circular orbit by a magnetic field perpendicular to the orbital plane. The magnetic field is gradually increased to induce an electric field around the orbit.

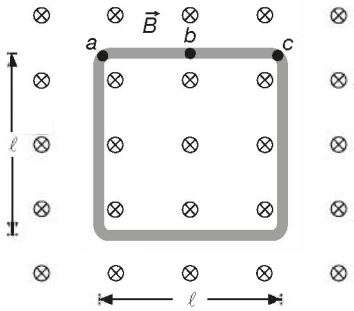
(a) Show that the electric field is set up in a direction so as to make the electrons speed up.

(b) Assume that the radius of the orbit remains constant. Show that the average magnetic field over the area enclosed by the orbit must be twice as large as the magnetic field at the circumference of the circle.

4. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the accompanying figure start increasing at a constant rate α Ts^{-1}. Find the magnitude of electric field as a function of r, the distance from the geometric centre of the region.



5. A square conducting loop, of side ℓ, is placed in a uniformly decreasing magnetic field shown. Assuming the center of the square loop is at the center of the magnetic field region.



(a) Draw vectors to show the directions and relative magnitudes of the induced electric field $\vec{E}$ at points a, b and c.

(b) Prove that the component of $\vec{E}$ along the loop has the same value at every point of the loop and is equal to that of the ring passing through that point.

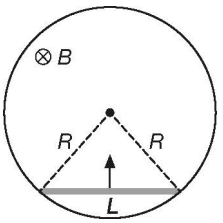
6. A thin non-conducting ring of mass m radius R carrying a charge q can rotate freely about its own axis which is vertical. At the initial moment, the ring was at rest and no magnetic field was present. At instant $t = 0$, a uniform magnetic field, inwards normal to the loop is switched on which increases with time according to the law $B = B_0 t$. Neglecting magnetism induced due to rotational motion of the ring, calculate

(a) angular acceleration and angular velocity of the ring at time t and

(b) power developed by the force acting on the ring as a function of time.

7. In a long straight solenoid with cross-sectional radius a and number of turns per unit length n, a current varies at a constant rate of I ampere per second. Calculate the magnitude of the induced electric field strength as a function of the distance r from the solenoid axis. Also plot the graph of magnitude of induced field vs the distance r from the axis of the solenoid.

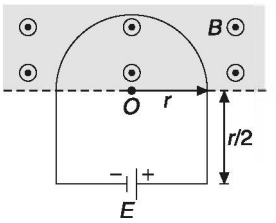
8. A cylindrical volume of radius R has a uniform axial magnetic field B, which is changing at the rate $\frac{dB}{dt}$. A metal rod of length L is placed in a plane normal to the axis of the cylinder as shown in Figure.



Show that the emf between the two ends of the rod is

$$e = \left(\frac{L}{2}\sqrt{R^2 - \frac{L^2}{4}}\right)\frac{dB}{dt}$$

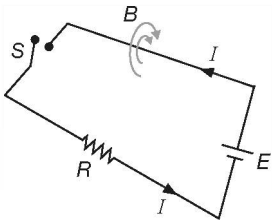
9. A conducting loop has its semi-circular part lying in a magnetic field B which varies with time as $B = (2t^3 + 3t^2 + 4)$ tesla . The wire is having a resistance $R\ \Omega m^{-1}$. Calculate the current in the loop at time $t = 2s$.



SELF AND MUTUAL INDUCTION

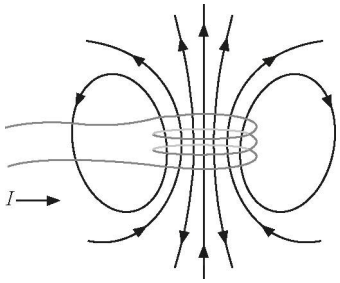
PHENOMENON OF SELF INDUCTION: AN INTRODUCTION

Consider a circuit having a source of emf E, a resistor R and a switch S as shown in Figure.



It is observed that, when the switch is thrown to its closed position, the current does not immediately jump from zero to its maximum value $\frac{E}{R}$. Explanation to this effect is provided by the Faraday's Law of Electromagnetic Induction. As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit. The direction of the induced emf is such that it produces an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field that opposes the change in the original magnetic field. Thus, the direction of the induced emf is opposite to the direction of the emf of the battery which results in a gradual (not instantaneous) increase in the current to its final equilibrium value. Because of the direction of the induced emf, it is also called a **back emf**, similar to that in a motor. This phenomenon is called the phenomenon of self-induction because the changing flux through the circuit and the resultant induced emf arise from the circuit is itself. The emf, ξ_L set up in this case in the circuit called the **self-induced emf**.

As a second example, consider a coil consisting of N turns carrying a current I in the counterclockwise direction, as shown in Figure.



If the current is steady, then the magnetic flux through the coil remains constant, so that no induced emf exists in the coil. However, suppose the current I changes with time, then according to Faraday's Laws of Electromagnetic Induction, an induced emf is set up in the coil so that it opposes the change producing it. The induced current will flow clockwise in the coil, if $\frac{dI}{dt} > 0$ i.e., I increases with time and it will flow counterclockwise in the coil, if $\frac{dI}{dt} < 0$ i.e., I decreases with time.

So, finally we observe that whenever the electric current flowing through a circuit (or loop) changes, the magnetic flux linked with the circuit (or loop) also changes so as to produce an induced emf current in the circuit (or loop). This phenomenon is called the **phenomenon of self-induction** and the emf so induced is called the **back emf**.

INDUCTOR AND SELF INDUCTANCE: BASIC INTRODUCTION AND SIGNIFICANCE

If a circuit contains a coil, such as a solenoid, the self-inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously. A circuit element that has a large self-inductance is called an inductor and has the circuit symbol •—ᔓᔓᔓ—•.

We always assume that the self-inductance of the remainder of a circuit is negligible compared with that of the inductor. However, keep in mind, that even a circuit without a coil has some self-inductance that can affect the behavior of the circuit.

Since the inductance of an inductor results in a back emf, an inductor in a circuit opposes changes in the current in that circuit. The inductor attempts to keep the current the same as it was before the change occurred. If the battery voltage in the circuit is increased so that the current rises, the inductor opposes this change, and the rise is not instantaneous. If the battery voltage is decreased, the presence of the inductor results in a slow drop in the current rather than an immediate drop. Thus, the inductor causes the circuit to be "sluggish" as it reacts to changes in the voltage. An inductor has no role to play in a circuit as long as the current in the circuit stays constant. It becomes active whenever the current linked with

the circuit changes. The self-inductance of an inductor depends on the shape, size, the number of turns and the magnetic properties of the material enclosed by the circuit.

SELF INDUCTANCE: DEFINITION

The phenomenon of the production of an induced e.m.f. in the coil when the flux linked with the coil changes is called **phenomenon of self-induction**.

If I is the current flowing in the circuit, then flux linked with the circuit is observed to be proportional to I.

$\Rightarrow \quad \phi_B \propto I$

$\Rightarrow \quad \phi_B = LI$

where L is called the self-inductance or coefficient of self-inductance or simply inductance of the coil and its SI unit is henry (H)

$\Rightarrow \quad 1\text{ H} = 1\text{ WbA}^{-1}$

If $I = 1$ A, then $L = \phi$ (numerically)

So, inductance of a coil is numerically equal to the flux linked with the coil when the current in the coil is 1 A.

Since, $\xi = -\dfrac{d\phi_B}{dt} = -\dfrac{d}{dt}(LI) = -L\left(\dfrac{dI}{dt}\right)$

$\Rightarrow \quad \xi = L\dfrac{dI}{dt}$ {in magnitude}

If $\dfrac{dI}{dt} = 1\text{ As}^{-1}$, then $L = \xi$ (numerically)

So, inductance of a coil is numerically equal to the e.m.f. induced in the coil when the current in the coil changes at the rate of 1 As^{-1}.

Physically, the inductance L is a measure of an **"Inductor's Opposition"** to the change of current. The larger the value of L, the lower the change in the value of current. So, L is also called **"Electrical Inertia"**.

Conceptual Note(s)

(a) Here we must note that if we are asked to calculate the induced e.m.f. in an inductor, then we have

$$\xi = -L\frac{dI}{dt}$$

But when we are asked to calculate the voltage (V) across the inductor then

$$V = |\xi| = L\frac{dI}{dt}$$

(b) Consider a coil that is wound on a cylindrical core and the current in the coil either increases or decreases. Then an induced emf is set up in the coil, which is in accordance with the Lenz's Law. The polarity of this induced emf must be such that it opposes the change in magnetic field due to the change in current. Due to this we can **think of replacing the coil with an equivalent self-induced emf with the polarity shown in Figure.**

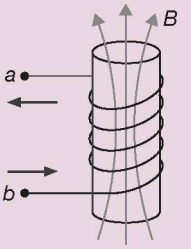


Figure 1(a)

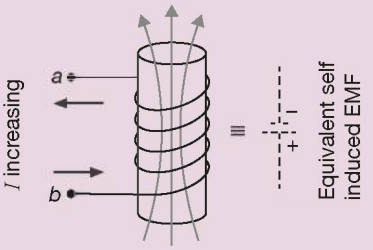


Figure 1(b)

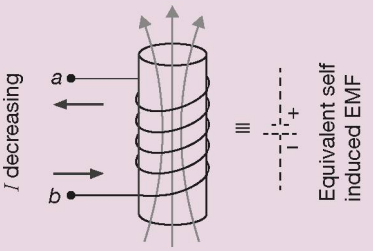


Figure 1(c)

Please note that the 'dotted battery' represents the equivalent self-induced emf produced by the coil, hence it is shown here as a replacement to the coil and is not the emf across the free ends a and b of the coil.

Finally, we have

$$|\text{Equivalent Self Induced EMF}| = L\left(\frac{dI}{dt}\right)$$

Also note that in giving the polarity of the equivalent self-induced emf, we must first see whether I is increasing or decreasing. Let us take the case of Figure 1(b), where I is increasing i.e., $\frac{dI}{dt} > 0$ and B is also increasing with t. Due to this $\xi < 0$. Now this means that the emf induced must produce an induced current that must not allow B to increase. So, this induced current, $I_{induced}$ must have a direction opposite to I (as in Figure 2(a)). Hence the equivalent self-induced emf must have a polarity as shown. A similar argument also holds good for explaining the polarity of the equivalent self-induced emf in Figure 1(c).

So, we can easily summarise the above discussions in the form of diagrams shown below.

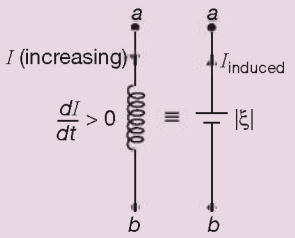


Figure 2(a) The polarity of $|\xi|$ will not allow I to increase, because it shall be producing an induced current, $I_{induced}$, that opposes I.

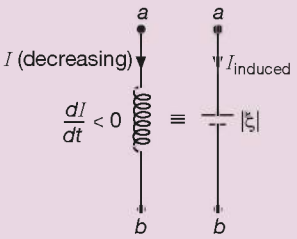


Figure 2(b) The polarity of $|\xi|$ will not allow I to decrease, because it shall be producing an induced current, $I_{induced}$, that favours I.

TECHNIQUE TO FIND THE SELF INDUCTANCE

To find the self-inductance of a given circuit, a good approach consists of the following steps.

STEP-1: Assume that a current I flows through the circuit (we can call the circuit an inductor).

STEP-2: Determine the magnetic field $\vec{B}$ produced due to the flow of this current.

STEP-3: Obtain the magnetic flux $\phi_B = \vec{B} \cdot \vec{A}$.

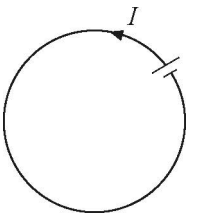
STEP-4: With the flux known, the self-inductance is given by

$$L = \frac{N\phi_B}{I}$$

The above steps have been implemented to find the self-inductance of the circuits discussed further.

Self-inductance for a Circular Coil

Consider a circular coil of radius r and number of turns N, as shown in Figure.



If current I passes in the coil, then magnetic field at centre of coil

$$B = \frac{\mu_0 NI}{2r}$$

The effective magnetic flux linked with this coil

$$\phi_B = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A$$

Since, by definition

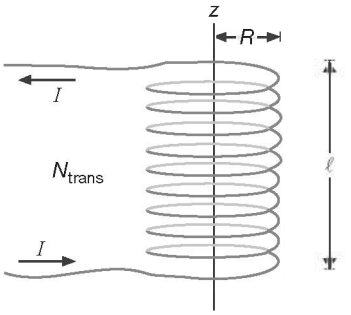
$$L = \frac{\phi_B}{I}$$

$$\Rightarrow \quad L = \frac{\mu_0 N^2 A}{2r} = \frac{\mu_0 N^2 \pi r^2}{2r}$$

$$\Rightarrow \quad L = \frac{\mu_0 N^2 \pi r}{2}$$

Self-inductance for a Solenoid

Consider a solenoid with n number of turns per metre as shown in Figure.



Let current I flow in the windings of solenoid, then magnetic field inside solenoid is given by

$$B = \mu_0 nI$$

The magnetic flux linked with its length l is $\phi_B = NBA$, where N is total number of turns in length l of solenoid.

$$\Rightarrow \quad \phi_B = (nl)\,BA = nl(\mu_0 nI)\,\mathrm{A} = (\mu_0 n^2 Al)I$$

Since, $L = \dfrac{\phi_B}{I}$

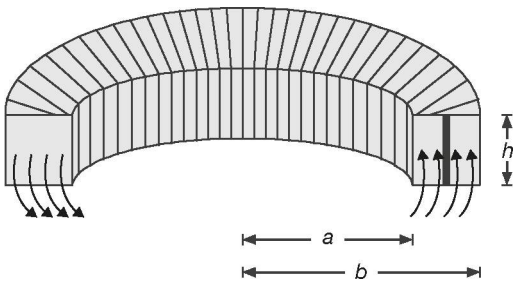
$$L = \mu_0 n^2 Al$$

Since, $n = \dfrac{N}{l}$

$$\Rightarrow \quad \text{Self-inductance, } L = \frac{\mu_0 N^2 A}{l}$$

Self-inductance of a Toroid

Let us calculate the self-inductance of a toroid having N turns and a rectangular cross section, with inner radius a, outer radius b and height h, as shown in Figure.



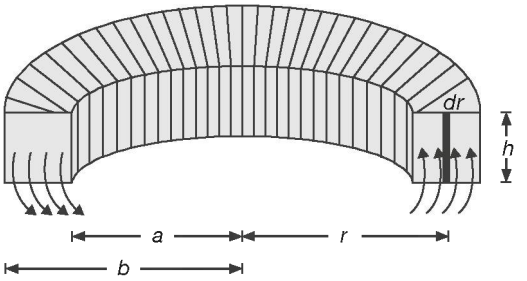
According to Ampere's Law, the magnetic field is given by

$$\oint \vec{B}\cdot d\vec{l} = \oint Bdl = B\oint dl = B(2\pi r) = \mu_0 NI$$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi r} \qquad \ldots(1)$$

where r is the distance from the central axis of the toroid.

The magnetic flux through one turn of the toroid is obtained by integrating over the rectangular cross section, with $dA = hdr$ as the differential area element shown in Figure.



$$\phi_B = \int \vec{B}\cdot d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right) hdr$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 NIh}{2\pi}\log_e\left(\frac{b}{a}\right) \qquad \ldots(2)$$

The total flux is $N\phi_B$, hence the self-inductance is

$$L = \frac{N\phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi}\log_e\left(\frac{b}{a}\right) \qquad \ldots(3)$$

Again, the self-inductance L depends only on the geometrical factors.

SPECIAL CASE

Let's consider the situation where $a \gg b-a$.
In this limit, the logarithmic term in the equation above will be expanded as

$$\log_e\left(\frac{b}{a}\right) = \log_e\left(1+\frac{b-a}{a}\right) \approx \frac{b-a}{a}$$

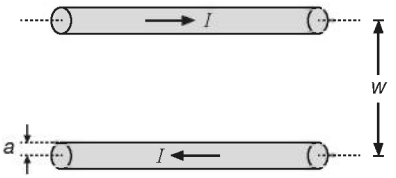
and the self-inductance now becomes

$$L \approx \frac{\mu_0 N^2 h}{2\pi}\left(\frac{b-a}{a}\right) = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{l}$$

where $A = h(b-a)$ is the cross-sectional area, and $l = 2\pi a$
We see that the self-inductance of the toroid in this limit has the same form as that of a solenoid.

ILLUSTRATION 54

Consider two wires 1 and 2 carrying equal currents I but in opposite direction as shown in Figure.



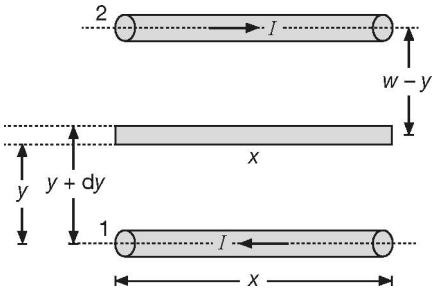
If a be the radius of the wires, and w be the centre to centre distance between them, then find the inductance L of a length x of this arrangement. In doing so, neglect magnetic flux inside the wires.

SOLUTION

Consider a section of arrangement of length x. Since we have to find L, so we must first find the flux ϕ, associated with a wire and then we get

$$L = \frac{\phi}{I} = \frac{1}{I}\int BdA$$

Consider a rectangular areal strip in between the wires at a distance y from centre of the lower wire as shown in Figure.



If dy be the width of the strip then $dA = xdy$ field at the strip due to the two wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{(w-y)}\right) \text{ inwards}$$

$$\Rightarrow \quad |d\phi_B| = BdA = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{w-y}\right)xdy$$

$$\Rightarrow \quad d\phi_B = \frac{\mu_0 Ix}{2\pi}\left(\frac{dy}{y} + \frac{dy}{w-y}\right)$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{2\pi}\left[\int_a^{w-a}\frac{dy}{y} + \int_a^{w-a}\frac{dy}{w-y}\right]$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{2\pi}\left(\log_e y\Big|_a^{w-a} - \log_e (w-y)\Big|_a^{w-a}\right)$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{2\pi}\left[\log_e\left(\frac{w-a}{a}\right) - \log_e\left(\frac{w-w+a}{w-a}\right)\right]$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{2\pi}\left[\log_e\left(\frac{w-a}{a}\right) - \log_e\left(\frac{a}{w-a}\right)\right]$$

Since, $\log_e\left(\frac{1}{m}\right) = -\log_e m$

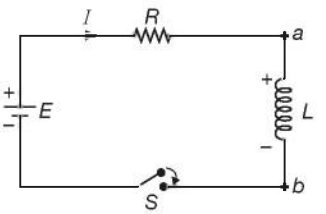
$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{2\pi}\left[2\log_e\left(\frac{w-a}{a}\right)\right]$$

$$\Rightarrow \quad \phi_B = \frac{\mu_0 Ix}{\pi}\log_e\left(\frac{w-a}{a}\right)$$

$$\Rightarrow \quad L = \frac{\phi_B}{I} = \frac{\mu_0 x}{\pi}\log_e\left(\frac{w-a}{a}\right)$$

MODIFIED KIRCHHOFF'S RULE FOR INDUCTORS

Consider the circuit which contains a battery of negligible internal resistance as shown in Figure.



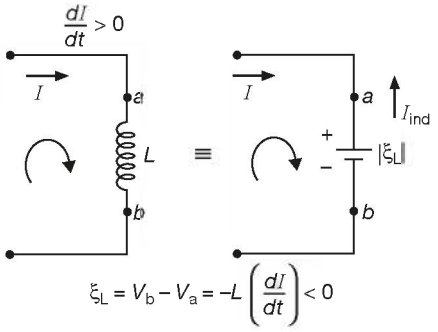
This is an RL circuit because the elements connected to the battery are a resistor and an inductor. Suppose that the switch S is open for $t < 0$ and then closed at $t = 0$. The current in the circuit begins to increase, and a back emf that opposes the increasing current is induced in the inductor. Because the current is increasing, $\frac{dI}{dt}$ is positive; thus, ξ_L is negative. This negative value reflects the decrease in electric potential that occurs in going from a to b across the inductor, as indicated by the positive and negative signs.

With this in mind, we can apply Kirchhoff's Loop Rule to this circuit, traversing the circuit in the clockwise direction

$$E - IR - L\frac{dI}{dt} = 0$$

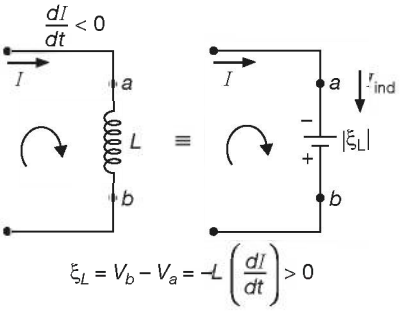
where IR is the voltage drop across the resistor. (We developed Kirchhoff's rules for circuits with steady currents, but they can also be applied to a circuit in which the current is changing if we imagine them to represent the circuit at one instant of time.

The expression $\Delta V = E - IR - L\frac{dI}{dt} = 0$ has been cast in a form that resembles Kirchhoff's Loop Rule, according to which, "the sum of the potential drops around a circuit is zero". To preserve the loop rule, we must specify the "potential drop" across an inductor.



The modified rule for inductors may be obtained as follows: The polarity of the self-induced emf is such as to oppose the change in current, in accord with Lenz's Law. If the rate of change of current is positive, as shown in Figure, the self-induced emf ξ_L sets up an induced current I_{ind} moving in the opposite direction of the current I to oppose such an increase. The inductor could be replaced by an emf $|\xi_L| = L\left|\frac{dI}{dt}\right| = +L\left(\frac{dI}{dt}\right)$ with the polarity shown in Figure. On the other hand, if $\frac{dI}{dt} < 0$, as shown in Figure, the induced current I_{ind} set up by the self-induced emf ξ_L flows in the same direction as I to oppose such a decrease.

We see that whether the rate of change of current in increasing $\left(\frac{dI}{dt} > 0\right)$ or decreasing $\left(\frac{dI}{dt} < 0\right)$, in both cases, the change in potential when moving from a and b along the direction of the current I is $V_b - V_a = -L\left(\frac{dI}{dt}\right)$.



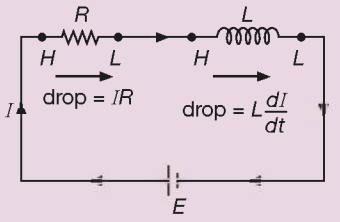
KIRCHHOFF'S LOOP RULE MODIFIED FOR INDUCTORS

(a) If an inductor is traversed in the direction of the current, the "potential change" is $-L\left(\frac{dI}{dt}\right)$.

(b) On the other hand, if the inductor is traversed in the direction opposite of the current, the "potential change" is $+L\left(\frac{dI}{dt}\right)$.

(c) So, for the circuit shown, when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop of $L\frac{dI}{dt}$ where $\frac{dI}{dt}$ is to be substituted with sign. For example, in the loop shown in Figure, Kirchhoff's Second Law gives the equation.

$$E - IR - L\frac{dI}{dt} = 0$$



Conceptual Note(s)

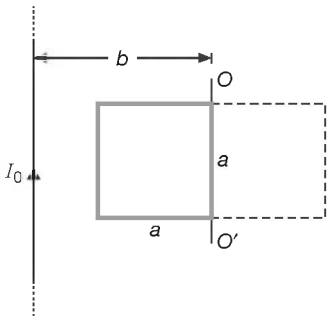
Use of this modified Kirchhoff's Rule will give the correct equations for circuit problems that contain inductors. However, keep in mind that it is misleading at best, and at some level wrong in terms of the physics. Again, *I* emphasize that Kirchhoff's Loop Rule

was originally based on the fact that the line integral of $\vec{E}$ around a closed loop was zero.

However, with time changing magnetic fields, this is no longer so, and thus the sum of the "potential drops" around the circuit, if we take that to mean the negative of the closed loop integral of $\vec{E}$, is no longer zero in fact it is $+L\left(\frac{dI}{dt}\right)$.

ILLUSTRATION 55

A square wire frame with side a and a straight conductor carrying a constant current I_0 are located in the same plane. The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current carrying conductor by a distance b. Find the electric charge having flown through the frame.



SOLUTION

Since we know that due to the change in flux a charge flows through the circuit. Since this circuit is a combination of resistor and an inductor, so we proceed as follows.

Let us first calculate the induced emf in the circuit without the inductor

$$\xi_R = -\frac{d\phi}{dt}$$

Now let us calculate the emf induced across the circuit without the resistor. We get

$$\xi_L = -L\frac{dI}{dt}$$

If ξ be the total emf induced, then

$$\xi = -\left(\frac{d\phi}{dt} + L\frac{dI}{dt}\right)$$

So, the induced current I, is

$$I = \frac{\xi}{R} = -\frac{1}{R}\left(\frac{d\phi}{dt} + L\frac{dI}{dt}\right)$$

$$\Rightarrow \quad Idt = -\frac{1}{R}(d\phi + LdI)$$

$$\Rightarrow \quad \int Idt = -\frac{1}{R}\int d\phi - \frac{L}{R}\int dI$$

$$\Rightarrow \quad \Delta q = -\frac{1}{R}\Delta\phi - \frac{L}{R}\Delta I$$

$$\Rightarrow \quad |\Delta q| = \frac{1}{R}(\Delta\phi + L\Delta I) \qquad \ldots(1)$$

The coil is stationary before and after rotation takes place. So,

$$\Delta I = 0$$

$$\Rightarrow \quad |\Delta q| = \frac{1}{R}(\phi_f - \phi_i) \qquad \ldots(2)$$

To calculate the ϕ_i, we consider a strip of length a, thickness dx at a distance x from the wire. Then

$$d\phi_i = BdA = \frac{\mu_0 I_0}{2\pi x}(adx)$$

$$\phi_i = \int d\phi_i = \frac{\mu_0 I_0 a}{2\pi}\int_{b-a}^{b}\frac{dx}{x} = \frac{\mu_0 I_0 a}{2\pi}\log_e\left(\frac{b}{b-a}\right) \qquad \ldots(3)$$

To calculate ϕ_f, we again repeat the same procedure, then

$$d\phi_f = BA\cos(180°) = -\frac{\mu_0 I_0}{2\pi y}(ady)$$

$$\Rightarrow \quad \phi_f = \int d\phi_f = -\frac{\mu_0 I_0 a}{2\pi}\int_{b}^{b+a}\frac{dy}{y}$$

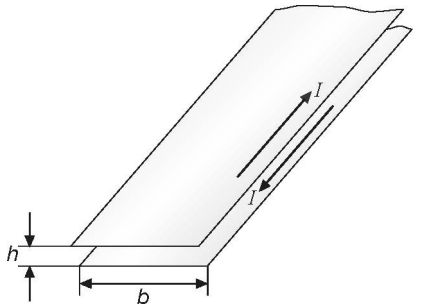
$$\Rightarrow \quad \phi_f = -\frac{\mu_0 I_0 a}{2\pi}\log_e\left(\frac{b+a}{b}\right) \qquad \ldots(4)$$

Put (3) and (4) in (2), we get

$$|\Delta q| = \frac{\mu_0 I_0 a}{2\pi R}\log_e\left(\frac{b+a}{b-a}\right)$$

ILLUSTRATION 56

Calculate the inductance per unit length of a double tape line if the tapes are separated by a distance h which is considerably less than their width b. The current flows in the two sheets of the tape line in opposite direction as shown in Figure.



SOLUTION

The current per unit length in the width of the sheet of tape line is $\lambda = \frac{I}{b}$.

Due to this current per unit length λ in each sheet, a magnetic field B_0 is produced by each sheet, so that

$$B_0 = \frac{\mu_0 \lambda}{2} = \frac{\mu_0}{2}\left(\frac{I}{b}\right)$$

The net field between the sheets due to both of the sheets is

$$B = 2B_0 = \frac{\mu_0 I}{b}$$

The magnetic flux associated with the region between the two sheets of tape line of length l is

$$\phi = BA = \left(\frac{\mu_0 I}{b}\right)(lh)$$

The self-inductance of the tape line is

$$L = \frac{\phi}{I} = \left(\frac{\mu_0}{b}\right)(lh)$$

The self-inductance per unit length of the line is

$$\frac{L}{l} = \frac{\mu_0 h}{b}$$

ILLUSTRATION 57

Calculate the self-inductance per unit length of a cable consisting of two thin walled coaxial metallic cylinders if the radius of the outside cylinder is $\eta(\eta > 1)$ times that of the inside one. The permeability of the medium between the cylinders is assumed to be equal to unity.

SOLUTION

A coaxial cable carries equal and opposite current in the two cylinders. the magnetic field in the region between the two cylinders is

$$B = \frac{\mu_0 I}{2\pi r} \text{ for } a \le r \le b$$

where, a and b are the radii of inner and outer cylindrical shells of the cable.

The magnetic flux in an elemental section of length l, width dr at a distance r from the axis of cable is

$$d\phi = B(ldr)$$

Total magnetic flux associated with a cross section in the region between the shells is

$$\phi = \int d\phi = \int B dr$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\ln\eta$$

Self-inductance of the cable

$$L = \frac{\phi}{I}$$

$$\Rightarrow \quad L = \frac{\mu_0 l}{2\pi}\ln\eta$$

Self-inductance per unit length of the cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\eta$$

Test Your Concepts-V

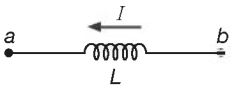
Based on Faraday's Laws: Self Induction

(Solutions on page H.141)

1. The current in a 90 mH inductor changes with time as $I = t^2 - 6t$ (in SI units). Find the magnitude of the induced emf at
(a) $t = 1$ s and
(b) $t = 4$ s
(c) At what time is the emf zero?

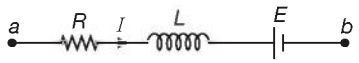
2. A self-induced emf in a solenoid of inductance L changes in time as $\xi = \xi_0 e^{-kt}$. Find the total charge that passes through the solenoid, assuming the charge is finite.

3. The inductor shown in Figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate $\frac{dI}{dt} = -0.03\ \text{As}^{-1}$.



(a) Find the self-induced emf,
(b) Which end of the inductor a or b is at a higher potential?

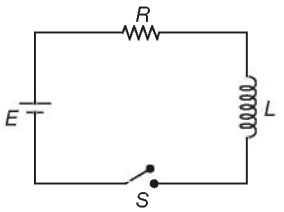
4. In the circuit diagram shown in Figure, $R = 10\ \Omega$, $L = 5$ H, $E = 20$ V, $I = 2$ A. If $\frac{dI}{dt} = -1\ \text{As}^{-1}$, find V_{ab} at this instant.



5. **(a)** Calculate the inductance of an air core solenoid containing 300 turns if the length of the solenoid is 25 cm and its cross-sectional area is 4 cm^2.
(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50 As^{-1}.

6. Calculate the self-inductance of a toroid whose inside radius is equal to b and cross-section has the form of a small square of side a. The solenoid winding consists of N turns. The space inside the solenoid is filled with uniform paramagnetic material having relative permeability μ_r.

7. For the RL circuit shown in Figure, let the inductance be 3 H, the resistance 8 Ω, and the battery emf 36 V.



(a) Calculate the potential difference across the resistor when the current is 2 A.
(b) Calculate the potential difference across the inductor when the current is 2 A.
(c) Calculate the voltage across the inductor when the current is 4.5 A.

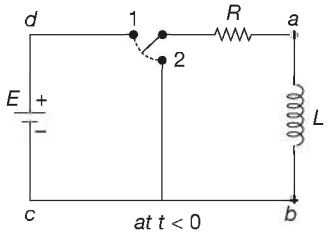
8. A long solenoid of diameter 0.1 m has 2×10^4 turns per meter. At the centre of the solenoid, a 100 turn coil of radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid is decreased at a constant rate from $+2A$ to $-2A$ in 0.05 s. Find the EMF induced in the coil (in mV). Also find the total charge flowing through the coil (in μC) during the same time interval if the resistance of the coil is $10\pi^2\ \Omega$.

9. Calculate the inductance per unit length of a double wire line if the radius of each wire is η times less than the distance between the axes of the wires. The field inside the wires is to be neglected, the permeability is assumed to be equal to unity throughout and $\eta \gg 1$.

SERIES *LR* CIRCUIT: CURRENT GROWTH AND DECAY

Growth of Current

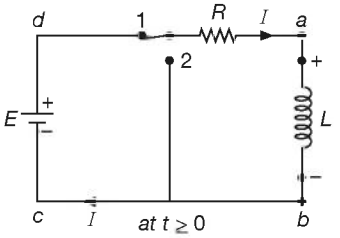
Consider a series LR circuit which contains a battery E of negligible internal resistance, a resistor R, an inductor L and a switch S, connected in series as shown in Figure.



Suppose that the switch S had been open for $t<0$ and then closed at $t=0$, by throwing it to position 1. The current in the circuit begins to increase, and an induced emf which opposes the increasing current is induced in the inductor. Because the current is increasing i.e., $\frac{dI}{dt}$ is positive so, we have

$$\xi = -L\frac{dI}{dt} < 0$$

This negative value reflects the decrease in electric potential that occurs in going from a to b across the inductor, as indicated by the positive and negative signs in Figure.



Apply Kirchhoff's loop Rule (as discussed before) to the loop $abcda$, we get

$$-L\frac{dI}{dt} + E - IR = 0 \qquad \ldots(1)$$

where IR is the voltage drop across the resistor. (We developed Kirchhoff's Rules for circuits with steady currents, but they can also be applied to a circuit in which the current is changing if we imagine them to represent the circuit at one instant of time). We must now look for a solution to this differential equation, which is similar to that for the RC circuit. Since, from (1), we get

$$E = IR + L\frac{dI}{dt}$$

$$\Rightarrow \quad E - IR = L\frac{dI}{dt}$$

$$\Rightarrow \quad \frac{dI}{E-IR} = \frac{1}{L}dt$$

$$\Rightarrow \quad \int_0^I \frac{dI}{E-IR} = \frac{1}{L}\int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{R}\log_e(E-IR)\Big|_0^I = \left(\frac{1}{L}\right)t\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{E-IR}{E}\right) = -\left(\frac{R}{L}\right)t$$

$$\Rightarrow \quad \frac{E-IR}{E} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad E - IR = Ee^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad I = \frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right) = I_0\left(1-e^{-\frac{Rt}{L}}\right)$$

where $I_0 = \frac{E}{R}$, is the peak value of current in the circuit i.e., current in the circuit when $t \to \infty$.

This expression shows how the inductor effects the current. The current does not increase instantly to its final equilibrium value when the switch is closed but instead increases according to an exponential function.

If we remove the inductance in the circuit, i.e., $L \to 0$, the exponential term becomes zero and we see that there is no time dependence of the current for $L \to 0$ i.e., the current increases instantaneously to its final equilibrium value in the absence of the inductance.

The above expression can also be written as

$$I = \frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right) = I_0\left(1-e^{-\frac{t}{\tau}}\right)$$

where τ is the time constant of the LR circuit, also called as Inductive Time Constant of LR circuit.

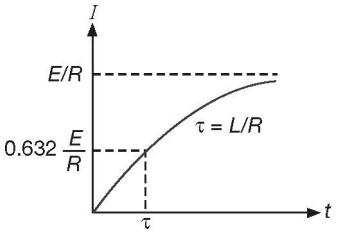
$$\tau = \frac{L}{R}$$

and I_0 is the maximum constant value given by

$$I_0 = \frac{E}{R}$$

Physically, τ is the time interval required for the current in the circuit to reach $(1-e^{-1}) = 0.632 = 63.2\%$ of its final value/maximum value $\frac{E}{R}$.

The time constant is a useful parameter for comparing the time responses of various circuits. Figure shows a graph of the current versus time in the RL circuit.

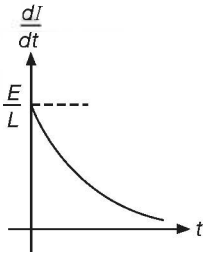


Plot of the current I versus time t for series LR circuit.

Note that the equilibrium value of the current, which occurs at t approaches infinity, is $\frac{E}{R}$. Thus, we see that the current initially increases very rapidly and then gradually approaches the equilibrium value $\frac{E}{R}$ as $t \to \infty$.

$$\frac{dI}{dt} = \frac{E}{L} e^{-\frac{t}{\tau}}$$

From this result, we see that the time rate of change of the current is a maximum $\left(\text{equal to } \frac{E}{L}\right)$ at $t = 0$ and falls off exponentially to zero as t approaches infinity as shown in Figure.

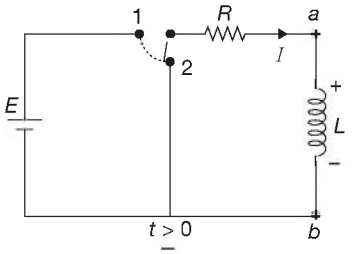


Plot of dI/dt versus time t for series LR circuit.

DECAY OF CURRENT

Again let us consider the LR circuit taken previously, when the switch had been kept at position 1 for long enough $(t \to \infty)$, such that the current reaches a maximum constant value $I_0 = \frac{E}{R}$.

Now let the switch S be thrown from position 1 to position 2, so that the circuit is now just described by the right loop of the diagram shown.



Please note that here too we have taken $t \geq 0$, because for $t < 0$, a constant maximum current I_0 flows through the circuit, which shall now decay to attain some new value $I(< I_0)$. So, for the decay circuit, we have

$$I = \begin{cases} I_0 & \text{at} \quad t = 0 \quad \text{and} \\ I(< I_0) & \text{at} \quad \text{time } t(> 0) \end{cases}$$

Since no battery is connected to the loop $ab2a$, so by applying Kirchhoff's Loop Rule to loop $ab2a$, we get

$$0 = -L\frac{dI}{dt} - IR$$

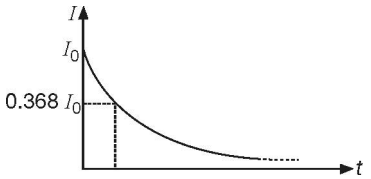
$$\Rightarrow \quad L\frac{dI}{dt} = -IR$$

$$\Rightarrow \quad \int_{I_0}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow \quad \log_e\left(\frac{I}{I_0}\right) = -\left(\frac{R}{L}\right)t$$

$$\Rightarrow \quad I = I_0 e^{-\frac{Rt}{L}} = I_0 e^{-\frac{t}{\tau}}$$

where $\tau = \frac{L}{R}$ and $I_0 = \frac{E}{R}$ $(\text{at } t < 0)$



Current I vs time t for series LR decay circuit.

The variation of I vs t is shown here. Please observe that if the circuit has got no inductor L (take $L \to 0$), then the current will immediately decay to zero as soon as the battery is removed (or the switch is thrown to position 2). However, the presence of an inductor causes the current to decrease exponentially.

Also note that here $\frac{dI}{dt} = -\left(\frac{I_0 R}{L}\right) e^{-\frac{Rt}{L}} < 0$

So, $\frac{dI}{dt}$ is always negative and hence $\xi > 0$

Also, note that the inductive time constant may also be defined as the time in which the current in the circuit decays from I_0 to $0.368\, I_0$.

Problem Solving Technique(s)

(a) If the growth current is denoted by I_g and the decay current by I_d, then at the same time instants from the start, we have

$$I_g + I_d = I_0 = \frac{E}{R}$$

(b) Theoretically the current grows from zero to I_0 (or decays from I_0 to zero) when $t \to \infty$, however practically, it is observed that the current grows from zero to I_0 (or decays from I_0 to zero) in five time constants i.e., as $t \to \frac{5L}{R}$.

So, $I\left(t = \frac{5L}{R}\right) \longrightarrow I_0$, for Growth Circuit and

$I\left(t = \frac{5L}{R}\right) \longrightarrow$ zero, for Decay Circuit.

(c) Also, we observe that

$$\frac{dI_g}{dt} = -\frac{dI_d}{dt} = \left(\frac{I_0 R}{L}\right) e^{-\frac{Rt}{L}}$$

(d) During the growth of current in the circuit, we have

$$\frac{dI}{dt} > 0 \text{ and hence } \xi < 0$$

(e) During the decay of current in the circuit, we have

$$\frac{dI}{dt} < 0 \text{ and hence } \xi > 0$$

(f) The magnitude of the induced emf (or the self-induced emf) is

$$|\xi| = \left|-L\frac{dI}{dt}\right| = I_0 R e^{-\frac{Rt}{L}}$$

(g) ξ is maximum when $t = 0$ i.e., initially and vanishes as t approaches infinity i.e., $t \to \infty$. This simply means, that a sufficiently long time after the switch is closed, self-induction disappears and the inductor simply acts as a conducting wire connecting two parts of the circuit.

(h) Since we have

$$E = IR + \frac{LdI}{dt}$$

Multiplying both sides by I, we get

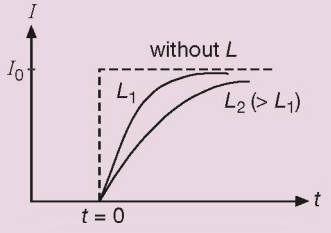
$$EI = I^2R + LI\frac{dI}{dt} \qquad \ldots(1)$$

where, EI is the rate at which battery delivers energy to the circuit, I^2R is the power dissipated by the resistor in the form of heat and $LI\frac{dI}{dt}$ is the rate at which energy is stored in the inductor.

So, equation (1) just represents the Law of Conservation of Energy for the LR circuit.

Please note that the energy dissipated by the resistor as heat is not recoverable, however the magnetic energy stored in the inductor can be released at some later time.

(i) In some growth graphs, we may have the situations shown in a single graph for a set of L values. Then from the graph we can conclude which graph corresponds to which value of L, so we observe $L_2 > L_1$.



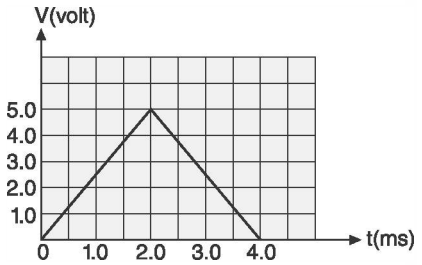
(j) So, while solving problems involving series inductor-resistor combination, we must keep in mind that at

$$t = 0,\ I = 0 \text{ and } t \to \infty,\ \frac{dI}{dt} = 0$$

This happens to be a helping tool to solve equations, as used in the ILLUSTRATIONS to come.

ILLUSTRATION 58

The potential difference across a 150 mH inductor as a function of time is shown in Figure.



Assume that the initial value of the current in the inductor is zero. Calculate the current at time 2 ms and 4 ms.

SOLUTION

The potential difference across an inductor is given by

$$V_L = L\frac{di}{dt}$$

$$\Rightarrow \quad di = \frac{1}{L}(V_L dt)$$

Integrating, we get

$$\int di = i = \frac{1}{L}\int V_L dt$$

$$\Rightarrow \quad i = \frac{1}{L}(\text{Area under } V_L \text{ versus } t \text{ graph})$$

At $t = 2$ ms, we have

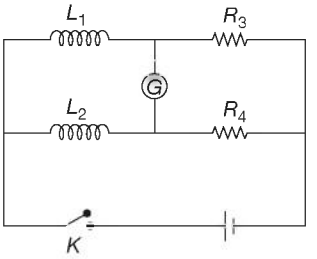
$$i = \frac{\frac{1}{2}(2\times10^{-3})(5)}{150\times10^{-3}} = 3.33\times10^{-2} \text{ A}$$

At $t = 4$ ms, area is just double and hence the current is also doubled.

$$\Rightarrow \quad i = \frac{2\left[\frac{1}{2}(2\times10^{-3})(5)\right]}{150\times10^{-3}} = 6.66\times10^{-2} \text{ A}$$

ILLUSTRATION 59

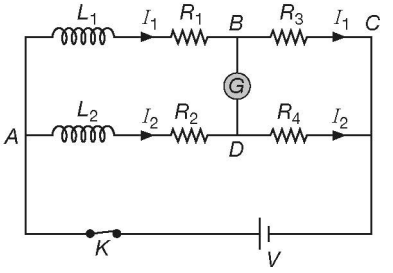
Two inductors of self-inductances L_1 and L_2 and of resistances R_1 and R_2 (not shown here) respectively, are connected in the circuit as shown in the Figure. At the instant $t = 0$, key K is closed, obtain an expression for which the galvanometer will show zero deflection at all times after the key is closed.



SOLUTION

Since no current flows through the branch BD, so

$$V_B = V_D$$



For loop $ABDA$, we have

$$\Rightarrow \quad V_A - V_B = V_A - V_D$$

$$\Rightarrow \quad L_1\frac{dI_1}{dt} + I_1R_1 = L_2\frac{dI_2}{dt} + I_2R_2 \qquad \ldots(1)$$

Similarly

$$V_B = V_D$$

$$\Rightarrow \quad V_C - V_B = V_C - V_D$$

$$\Rightarrow \quad I_1R_3 = I_2R_4 \qquad \ldots(2)$$

Take derivative of (2) w.r.t. t, we get

$$R_3\left(\frac{dI_1}{dt}\right) = R_4\left(\frac{dI_2}{dt}\right)$$

$$\Rightarrow \quad \frac{dI_1}{dt} = \frac{R_4}{R_3}\left(\frac{dI_2}{dt}\right) \qquad \ldots(3)$$

Substituting value of I_1 from (2) and $\frac{dI_1}{dt}$ from (3) in (1), we get

$$\left(\frac{R_4}{R_3}L_1 - L_2\right)\frac{dI_2}{dt} = I_2\left(R_2 - \frac{R_1R_4}{R_3}\right) \quad \ldots(4)$$

When $t = 0$, $I_2 = 0$

$$\Rightarrow \quad \frac{R_4}{R_3} = \frac{L_2}{L_1} \quad \ldots(5)$$

Now, when $t \to \infty$ i.e., I_2 grows to a maximum constant value, then

$$\frac{dI_2}{dt} = 0$$

$$\Rightarrow \quad R_2 = \frac{R_4R_1}{R_3}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \ldots(6)$$

From (5) and (6), we get

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

ILLUSTRATION 60

A straight solenoid of length l is having a single layer winding of copper wire whose total mass is m. The cross-sectional diameter of the solenoid is assumed to be considerably less than its length. Take resistivity of copper to be ρ and density is equal to d, calculate the time constant τ of the solenoid.

SOLUTION

Time constant of the solenoid i.e. an LR circuit is given by

$$\tau = \frac{L}{R}$$

Inductance of a solenoid is

$$L = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l} = \left(\frac{\mu_0 N^2}{l}\right)\pi r^2 \quad \ldots(1)$$

where, r is the radius of cross-section of the solenoid.

If l_0 is the total length of wire that makes the solenoid, then we have

$$l_0 = N(2\pi r)$$

$$\Rightarrow \quad r = \frac{l_0}{2\pi N}$$

So, equation (1) can be rewritten as

$$L = \left(\frac{\mu_0 N^2}{l}\right)\pi\left(\frac{l_0}{2\pi N}\right)^2 = \frac{\mu_0}{4\pi}\frac{l_0^2}{l} \quad \ldots(2)$$

The resistance of the wire is

$$R = \frac{\rho l_0}{A} = \frac{\rho l_0^2}{Al_0} = \frac{\rho l_0^2}{V} = \frac{\rho l_0^2}{m/d} = \frac{\rho l_0^2 d}{m}$$

$$\Rightarrow \quad R = \frac{\rho l_0^2 d}{m}$$

$$\Rightarrow \quad l_0^2 = \frac{mR}{\rho d}$$

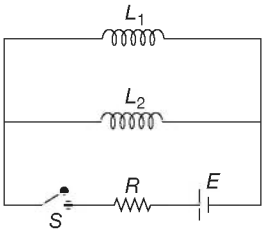
Substituting this value of l_0^2 in equation (2), we get

$$L = \frac{\mu_0}{4\pi l}\left(\frac{mR}{\rho d}\right)$$

$$\Rightarrow \quad \tau = \frac{L}{R} = \frac{\mu_0 m}{4\pi l \rho d}$$

ILLUSTRATION 61

In the circuit shown emf E, a resistance R and coil inductances L_1 and L_2 are known. The internal resistance of the source and the coil are negligible. Find the steady state currents in the coils after switch S was shorted.



SOLUTION

Since L_1 and L_2 are in parallel, so we have

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = E - (I_1 + I_2)R \quad \ldots(1)$$

At $t = 0$, $I_1 = I_2 = 0$, so we get from (1),

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$

$$\Rightarrow \quad L_1 dI_1 = L_2 dI_2$$

$$\Rightarrow \quad L_1 I_1 = L_2 I_2 \quad \ldots(2)$$

Also, at steady state, we have

$$\frac{dI_1}{dt}=\frac{dI_2}{dt}=0$$

$$\Rightarrow \quad E-(I_1+I_2)R=0$$

$$\Rightarrow \quad (I_1+I_2)=\frac{E}{R} \qquad \ldots(3)$$

From (2) and (3), we get

$$I_1=\frac{EL_2}{R(L_1+L_2)} \text{ and } I_2=\frac{EL_1}{R(L_1+L_2)}$$

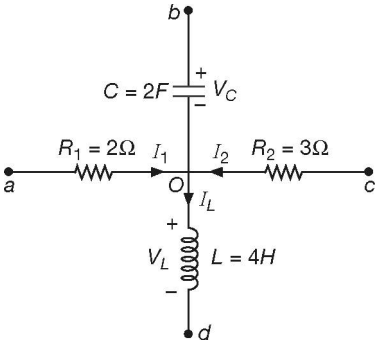
ILLUSTRATION 62

In the Figure shown if, $I_1=10e^{-2t}$ A, $I_2=4$ A and $V_C=3e^{-2t}$ V, then calculate

(a) the current and voltage, I_L and V_L, across L as a function of t.

(b) V_{ac}, V_{ab} and V_{cd} as function of t.

Show the variation of I_L, V_L, V_{ac}, V_{ab} and V_{cd} with time t.



SOLUTION

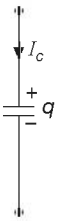
(a) Charge stored in the capacitor at time t,

$$q=CV_C$$

$$\Rightarrow \quad q=(2)(3e^{-2t})$$

$$\Rightarrow \quad q=6e^{-2t} \text{ C}$$

$$\Rightarrow \quad I_C=\frac{dq}{dt}=-12e^{-2t} \text{ A}$$



{direction of current is from b to O}

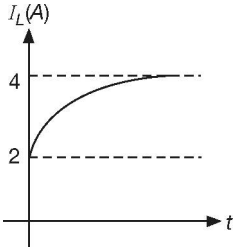
Applying junction rule at O,

$$I_L=I_1+I_2+I_C=10e^{-2t}+4-12e^{-2t}$$

$$I_L=(4-2e^{-2t}) \text{ A}$$

$$I_L=\left[2+2(1-e^{-2t})\right] \text{ A}$$

I_L versus time graph is as shown in Figure.



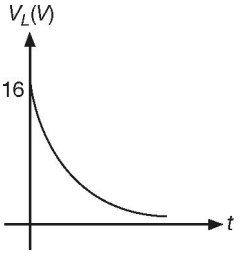
I_L increases from 2 A to 4 A exponentially.

$$V_L=V_{Od}=L\frac{dI_L}{dt}$$

$$\Rightarrow \quad V_L=(4)\frac{d}{dt}(4-2e^{-2t})$$

$$\Rightarrow \quad V_L=16e^{-2t} \text{ V}$$

V_L versus time graph is as shown in Figure.



V_L decreases exponentially from 16 V to 0.

(b) $V_{ac}=V_a-V_c$

$$V_a-I_1R_1+I_2R_2=V_c$$

$$\Rightarrow \quad V_a-V_c=V_{ac}=I_1R_1-I_2R_2$$

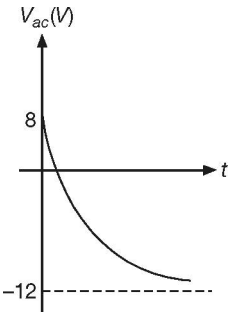
Substituting the values, we get,

$$V_{ac}=(10e^{-2t})(2)-(4)(3)$$

$$\Rightarrow \quad V_{ac}=(20e^{-2t}-12) \text{ V}$$

At $t=0$, $V_{ac}=8$ V. As $t\to\infty$, $V_{ac}=-12$ V

Therefore, V_{ac} decreases exponentially from 8 V to –12 V



Since, $V_{ab} = V_a - V_b$

$\Rightarrow V_a - I_1R_1 + V_C = V_b$

$\Rightarrow V_a - V_b = V_{ab} = I_1R_1 - V_C$

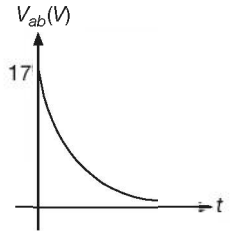
Substituting the values, we get,

$$V_{ab} = (10e^{-2t})(2) - 3e^{-2t}$$

$\Rightarrow V_{ab} = 17e^{-2t}$ V

Thus, V_{ab} decreases exponentially from 17 V to 0.

V_{ab} versus t graph is shown in Figure.



Since, $V_{cd} = V_c - V_d$

$$V_c - I_2R_2 - V_L = V_d$$

$\Rightarrow V_c - V_d = V_{cd} = I_2R_2 + V_L$

Substituting the values, we get,

$$V_{cd} = (4)(3) + 16e^{-2t}$$

$\Rightarrow V_{cd} = (12 + 16e^{-2t})$ V

At $t = 0$, $V_{cd} = 28$ V and as $t \to \infty$, $V_{cd} = 12$ V

i.e., V_{cd} decreases exponentially from 28 V to 12 V.

V_{cd} versus t graph is shown in Figure.

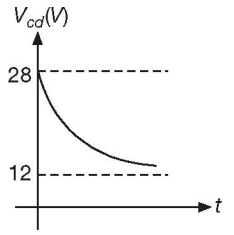
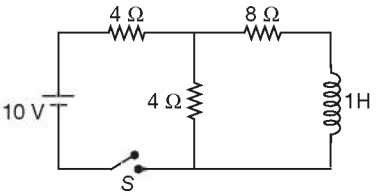


ILLUSTRATION 63

The switch in Figure is open for $t < 0$ and then closed at time $t = 0$. Find the current in the inductor and the current in the switch as functions of time thereafter.



SOLUTION

For loop $abefa$, we have

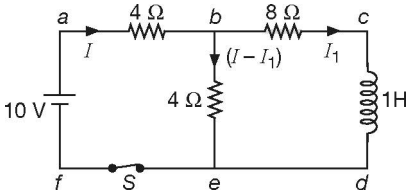
$$-4I - 4(I - I_1) + 10 = 0$$

$\Rightarrow -4I_1 + 8I = 10$ ...(1)

For loop $bcdeb$, we have

$$-8I_1 - (1)\frac{dI_1}{dt} + 4(I - I_1) = 0$$

$\Rightarrow -12I_1 + 4I = \frac{dI_1}{dt}$ ...(2)



Multiplying (2) by 2 and the subtracting from (1), we get

$$-4I_1 + 24I_1 = 10 - 2\frac{dI_1}{dt}$$

$\Rightarrow 20I_1 = 10 - 2\frac{dI_1}{dt}$

$\Rightarrow \frac{dI_1}{dt} = 5 - 10I_1$

$\Rightarrow \int_0^{I_1} \frac{dI_1}{5 - 10I_1} = \int_0^t dt$

$\Rightarrow -\frac{1}{10}\log_e(5 - 10I_1)\Big|_0^{I_1} = t$

$\Rightarrow \log_e\left(\frac{5 - 10I_1}{5}\right) = -10t$

$$\Rightarrow \quad 5-10I_1 = 5e^{-10t}$$

$$\Rightarrow \quad I_1 = 0.5\left(1-e^{-10t}\right) \text{ A}$$

{current through the inductor}

The current through the switch is I, calculated from either of the equations to be

$$I = \frac{1}{8}\left(10+4I_1\right)$$

$$\Rightarrow \quad I = 1.25+0.5I_1$$

$$\Rightarrow \quad I = 1.25+(0.5)(0.5)\left(1-e^{-10t}\right)$$

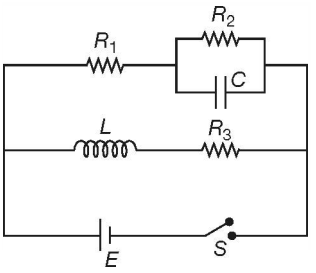
$$\Rightarrow \quad I = 1.25+0.25-0.25e^{-10t}$$

$$\Rightarrow \quad I = \left(1.5-0.25e^{-10t}\right) \text{ A}$$

{current through the switch}

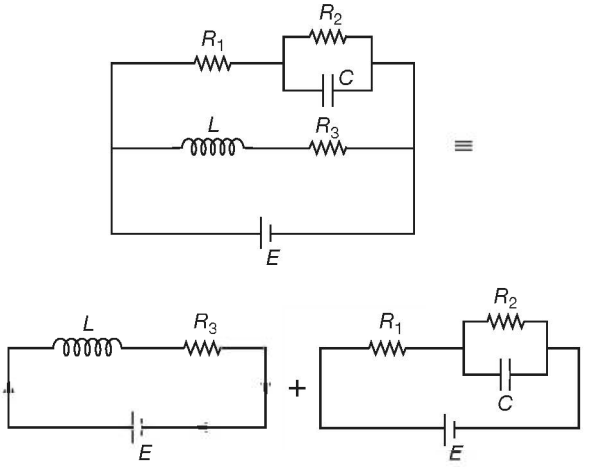
ILLUSTRATION 64

In the circuit shown, the switch S is closed at time $t=0$. Find the current through capacitor and inductor at any time t.



SOLUTION

From Superposition principle, the circuit may be assumed to be equivalent to two circuits in parallel as shown.

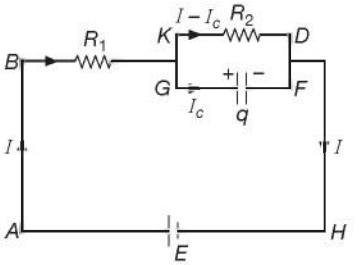


The first is a series LR circuit. Hence current I_L at any time t will be given by $I_L = I_0\left(1-e^{-\frac{t}{\tau_L}}\right)$

where $I_0 = \frac{E}{R_3}$ and $\tau_L = \frac{L}{R_3}$

$$\Rightarrow \quad I_L = \frac{E}{R_3}\left(1-e^{-\frac{R_3 t}{L}}\right) \qquad \ldots(1)$$

Let us now find the current through capacitor at time t.



Let the currents in different branches be I, I_C and $I-I_C$ at any time t and q be the charge stored in the capacitor.

Applying Kirchhoff's Loop Law to $ABKDHA$, we get

$$E-IR_1-\left(I-I_C\right)R_2 = 0 \qquad \ldots(2)$$

For loop $KDFGK$, we have

$$\frac{q}{C}-\left(I-I_c\right)R_2 = 0 \qquad \ldots(3)$$

Since, $I_C = \frac{dq}{dt}$ $\qquad \ldots(4)$

From equation (2)

$$I = \left(\frac{E+I_C R_2}{R_1+R_2}\right)$$

Substituting this in equation (2), we get

$$\frac{q}{C}-\left(\frac{E+I_C R_2}{R_1+R_2}-I_C\right)R_2 = 0$$

$$\Rightarrow \quad \frac{q}{C}-\frac{ER_2}{R_1+R_2} = I_C\left(\frac{R_2^2}{R_1+R_2}-R_2\right)$$

$$\Rightarrow \quad \left(\frac{dq}{dt}\right)\left(\frac{R_2^2}{R_1+R_2}-R_2\right) = \frac{q}{C}-\frac{ER_2}{R_1+R_2}$$

$$\Rightarrow \quad \int_0^q \frac{dq}{\left(\frac{ER_2}{R_1+R_2}\right)-\frac{q}{C}} = \frac{R_1+R_2}{R_1R_2}\int_0^t dt$$

Solving this we get

$$q=\frac{ECR_2}{R_1+R_2}\left(1-e^{-\left(\frac{R_1+R_2}{CR_1R_2}\right)t}\right)$$

$$\Rightarrow \quad I_C=\frac{dq}{dt}=\frac{E}{R_1}e^{-\left(\frac{R_1+R_2}{CR_1R_2}\right)t}$$

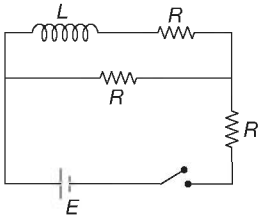
Hence currents through capacitor and inductor at any time t are

$$I_C=\frac{E}{R_1}e^{-\left(\frac{R_1+R_2}{CR_1R_2}\right)t} \text{ and}$$

$$I_L=\frac{E}{R_3}\left(1-e^{-\frac{R_3}{L}t}\right)$$

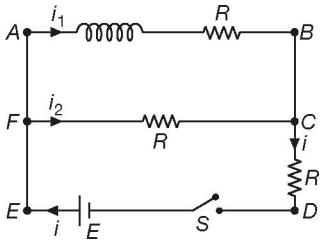
ILLUSTRATION 65

In the circuit shown, the switch is closed at $t=0$. Calculate the current drawn from the ideal battery as a function of time.



SOLUTION

At an instant t, the circuit is shown in Figure.



If i be the current through the battery, then we have

$$i=i_1+i_2 \qquad \ldots(1)$$

Applying KLL for the loop $ABCDEA$, we get

$$-L\frac{di_1}{dt}-i_1R-iR+E=0 \qquad \ldots(2)$$

Applying KLL for the loop $FCDEF$, we get

$$-i_2R-iR+E=0 \qquad \ldots(3)$$

From (1), we have

$$i_2=i-i_1$$

$$\Rightarrow \quad -(i-i_1)R-iR+E=0$$

$$\Rightarrow \quad -2iR+i_1R+E=0$$

$$\Rightarrow \quad i_1=-\frac{E+2iR}{R}=2i-\frac{E}{R} \qquad \ldots(4)$$

From (2) and (4), we get

$$-L\frac{d}{dt}\left(2i-\frac{E}{R}\right)-\left(2i-\frac{E}{R}\right)R-iR+E=0$$

$$\Rightarrow \quad -2L\frac{di}{dt}-2iR+E-iR+E=0$$

$$\Rightarrow \quad 2E-3iR=2L\frac{di}{dt}$$

$$\Rightarrow \quad E-\frac{3}{2}iR=L\frac{di}{dt}$$

$$\Rightarrow \quad \int_{\frac{E}{2R}}^{i}\frac{di}{E-\frac{3}{2}iR}=\int_0^t\frac{dt}{L}$$

$$\Rightarrow \quad \frac{\log_e\left(E-\frac{3}{2}iR\right)\Big|_{\frac{E}{2R}}^{i}}{-\frac{3}{2}R}=\frac{t}{L}$$

$$\Rightarrow \quad \log_e\left(\frac{E-\frac{3}{2}iR}{\frac{E}{4}}\right)=-\frac{3R}{2L}t$$

$$\Rightarrow \quad 4-\frac{6iR}{E}=e^{-\frac{3R}{2L}t}$$

$$\Rightarrow \quad \left(1-e^{-\frac{3R}{2L}t}\right)+3=\frac{6iR}{E}$$

$$\Rightarrow \quad i=\frac{E}{6R}\left[3+\left(1-e^{-\frac{3R}{2L}t}\right)\right]$$

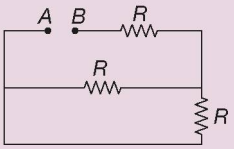
$$\Rightarrow \quad i=\frac{E}{2R}+\frac{E}{6R}\left(1-e^{-\frac{3R}{2L}t}\right)$$

Problem Solving Technique(s)

In these types of problems, we can first calculate the current flowing through the inductor using the following steps.

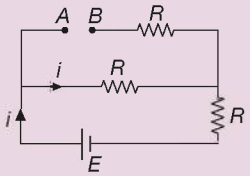
STEP-1

Remove the inductor, short the battery and then calculate the equivalent resistance across the open terminals (lets call then A and B) after removing the inductor as shown in Figure. Let that resistance be R_{AB} and we get $R_{AB}=\frac{3R}{2}$



STEP-2

Now calculate the current (i) in the battery circuit at the moment key is closed as shown in Figure.



$$\Rightarrow\ i=\frac{E}{2R}$$

STEP-3

Potential difference across the open ends A and B is given by

$$V_{AB}=iR=\left(\frac{E}{2R}\right)R=\frac{E}{2}$$

STEP-4

So, current through the inductor is given by

$$i_L=i_1=\frac{V_{AB}}{R_{AB}}\left(1-e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow\ i_L=i_1=\frac{E/2}{3R/2}\left(1-e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow\ i_L=i_1=\frac{E}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$$

STEP-5

Apply KLL on the loop $EFCDE$, we get

$$-(i-i_1)R-iR+E=0$$

$$\Rightarrow\ 2iR=E+i_1R$$

$$\Rightarrow\ i=\frac{E}{2R}+\frac{i_1}{2}$$

$$\Rightarrow\ i=\frac{E}{2R}+\frac{E}{6R}\left(1-e^{-\frac{3R}{2L}t}\right)$$

ILLUSTRATION 66

A closed circuit consists of a source of constant EMF E and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R. At the moment $t=0$, the choke coil inductance was decreased abruptly η times suddenly. Calculate the current in the circuit as a function of t.

SOLUTION

When the inductance was abruptly decreased η times i.e. made $\frac{L}{\eta}$, then at the same instant, current in the circuit becomes η times the original value so that the flux through the inductance remains the same.

Conceptual Note(s)

Dear Student, please understand that during the stepwise or sudden or abrupt change in the inductance, the total magnetic flux i.e. the flux linkage remains constant.

This statement can be understood based on the concept that when mass of a moving body is abruptly made half, then its speed becomes twice just because linear momentum $p=mv$ is conserved.

Similarly, inductance being analogous to electrical inertia and current being analogous to speed, then flux $\phi=LI$ becomes analogous to momentum and hence abrupt change in L leads to a corresponding change in current just to conserve the flux.

Just before changing the inductance, current in the circuit would have been

$$I_0=\frac{E}{R} \qquad \ldots(1)$$

At $t=0$, when the inductance is abruptly made $\frac{L}{\eta}$, then the current in the circuit becomes ηI_0.

At any instant, if I be the current in the circuit, then by using KLL for the given LR circuit, we get

$$IR+\frac{L}{\eta}\frac{dI}{dt}=E$$

$$\Rightarrow \quad \frac{dI}{E-IR}=\frac{\eta}{L}dt$$

Integrating within appropriate limits, we get

$$\Rightarrow \quad \int_{\eta I_0}^{I}\frac{dI}{E-IR}=\frac{\eta}{L}\int_0^t dt$$

Here while applying the limits we kept in mind that at $t=0$, the current in the circuit is ηI_0 and at time t, the current in the circuit is I.

$$\Rightarrow \quad -\frac{1}{R}\ln(E-IR)\Big|_{\eta I_0}^{I}=\frac{\eta}{L}t\Big|_0^t$$

$$\Rightarrow \quad \ln\left(\frac{E-IR}{E-\eta I_0 R}\right)=-\left(\frac{\eta R}{L}\right)t$$

From the Equation (1), we get

$$\ln\left(\frac{E-IR}{E-\eta E}\right)=-\left(\frac{\eta R}{L}\right)t$$

$$\Rightarrow \quad \frac{E-IR}{E-\eta E}=e^{-\left(\frac{\eta R}{L}\right)t}$$

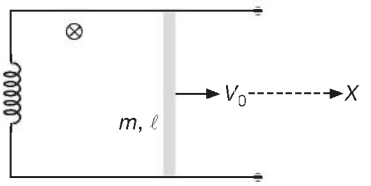
$$\Rightarrow \quad E-IR=E(1-\eta)e^{-\left(\frac{\eta R}{L}\right)t}$$

$$\Rightarrow \quad IR=E-E(1-\eta)e^{-\left(\frac{\eta R}{L}\right)t}$$

$$\Rightarrow \quad I=\frac{E}{R}\left[1-(1-\eta)e^{-\left(\frac{\eta R}{L}\right)t}\right]$$

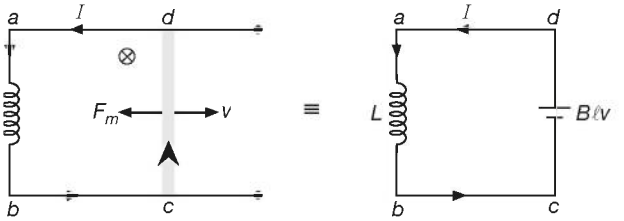
ILLUSTRATION 67

A loop is formed by two parallel conductors connected by a solenoid with inductance L and a conducting rod of mass m which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane in a uniform vertical magnetic field B. The distance between the conductors is l. At the moment $t=0$, the rod is imparted an initial velocity v_0 directed to the right. Find the law of its motion $x(t)$ if the electric resistance of the loop is negligible.



SOLUTION

Let at any instant of time, the velocity of the rod be v towards right.



Let the current in the circuit be I. For the loop $abcda$

$$Blv-L\frac{dI}{dt}=0$$

$$\Rightarrow \quad L\frac{dI}{dt}=Bl\frac{dx}{dt}$$

$$\Rightarrow \quad LdI=Bldx$$

Integrating, we get

$$LI=Blx$$

$$\Rightarrow \quad I=\frac{Bl}{L}x \qquad \ldots(1)$$

Magnetic force on the rod at this instant is,

$$F_m=BIl=\frac{B^2l^2}{L}x \qquad \ldots(2)$$

Since, this force is in opposite direction of $\vec{v}$, so from Newton's Second Law we can write,

$$m\left(\frac{d^2x}{dt^2}\right)=-\frac{B^2l^2}{L}x$$

$$\Rightarrow \quad \frac{d^2x}{dt^2}=-\frac{B^2l^2}{mL}x$$

$$\Rightarrow \quad \frac{d^2x}{dt^2}+\left(\frac{B^2l^2}{mL}\right)x=0$$

Comparing this with the standard equation of SHM $\frac{d^2x}{dt^2}+\omega^2x=0$ we get,

$$\omega = \frac{Bl}{\sqrt{mL}}$$

Therefore, the rod will oscillate simple harmonically with angular frequency $\omega = \frac{Bl}{\sqrt{mL}}$. At time $t = 0$, rod was at $x = 0$ and it was moving towards positive x-axis. Hence, x-t equation of the rod is,

$$x = A\sin(\omega t) \qquad \ldots(3)$$

METHOD I to find A

Since we have at $t = 0$, $v = \frac{dx}{dt} = v_0$

Since, $\frac{dx}{dt} = v = A\omega\cos(\omega t)$

$\Rightarrow \quad A\omega = v_0 \qquad \{\text{at } t = 0\}$

$\Rightarrow \quad A = \frac{v_0}{\omega}$

Substituting in equation (3), we get

$$x = \frac{v_0}{\omega}\sin(\omega t), \text{ where } \omega = \frac{Bl}{\sqrt{mL}}$$

METHOD II to find A

At $x = A$, $v = 0$, i.e., the entire kinetic energy is converted into magnetic energy. So,

$$\frac{1}{2}LI^2 = \frac{1}{2}mv_0^2$$

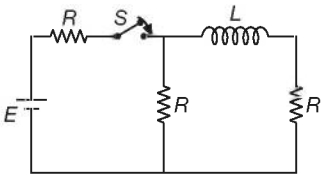
Substituting value of I from equation (1), with $x = A$, we get

$$L\left(\frac{Bl}{L}A\right)^2 = mv_0^2$$

$\Rightarrow \quad A = \frac{\sqrt{mL}}{Bl}v_0 = \frac{v_0}{\omega}$, where $\omega = \frac{Bl}{\sqrt{mL}}$

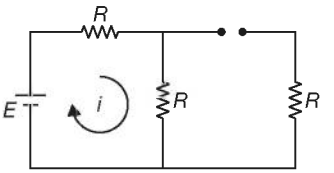
ILLUSTRATION 68

In circuit shown in Figure, find the current through battery just after closing the switch and after a long time in steady state.



SOLUTION

Just after closing the switch inductor behaves like open circuit so the circuit just after closing the switch is shown in Figure below.

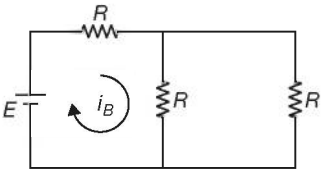


The current only flows in the left loop as shown which is given as

$$i_B = \frac{E}{2R}$$

In steady state after a long time the inductor behaves like a straight wire thus equivalent circuit in steady state is shown in Figure below. The two resistances on the right side can be considered in parallel and thus across the battery equivalent resistance will be

$$R + \frac{R}{2} = \frac{3R}{2}$$

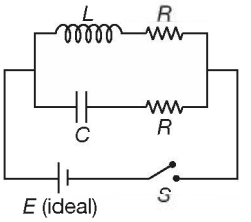


The current flowing through battery in steady state is given as

$$i_B = \frac{E}{\frac{3R}{2}} = \frac{2E}{3R}$$

ILLUSTRATION 69

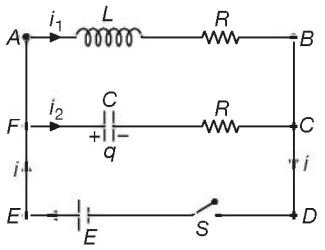
In the circuit shown in Figure, the switch is closed at $t = 0$.



Find the condition for which current drawn from the battery does not vary with time.

SOLUTION

The switch is closed at $t = 0$. At any instant, the current drawn from the battery is i. So, we have



$$i = i_1 + i_2$$

Applying KLL to the loop $ABCDEFA$, we get

$$-L\frac{di_1}{dt} - i_1R + E = 0$$

$$\Rightarrow \quad \frac{di_1}{E - i_1R} = \frac{dt}{L}$$

Integrating, we get

$$i_1 = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Applying KLL to the loop $FCDEF$, we get

$$\frac{-q}{C} - i_2R + E = 0$$

$$\Rightarrow \quad q = CE\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad i_2 = \frac{E}{R}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad i = i_1 + i_2$$

$$\Rightarrow \quad i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) + \frac{E}{R}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad i = \frac{E}{R} + \frac{E}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{Rt}{L}}\right)$$

For the current i not to vary with time, we observe that the terms containing t must cancel, so

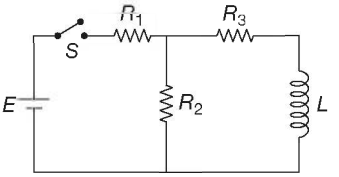
$$\Rightarrow \quad e^{-\frac{t}{RC}} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{t}{RC} = \frac{Rt}{L}$$

$$\Rightarrow \quad R = \sqrt{\frac{L}{C}}$$

ILLUSTRATION 70

For the circuit shown in Figure, we are given that $E = 50$ V, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$ and $L = 2.0$ mH. Find the current through R_1 and R_2,



(a) immediately after switch S is closed.
(b) long time after S is closed.
(c) immediately after S is reopened.
(d) long time after S is reopened.

SOLUTION

(a) Resistance offered by inductor immediately after switch is closed will be infinite as it behaves like open circuit. Therefore, current through R_3 will be zero and current through R_1 and R_2 is the same, given by

$$I_1 = I_2 = \frac{E}{R_1 + R_2} = \frac{50}{10 + 20} = \frac{5}{3}\ \text{A}$$

$$\Rightarrow \quad I_1 = I_2 = 1.67\ \text{A}$$

(b) Long time after the switch is closed, the resistance offered by inductor is zero and in steady state it behaves like a short circuit.
In that case R_2 and R_3 are in parallel and the resultant of these two is in series with R_1. So, equivalent resistance across the battery in steady state is

$$R_{\text{net}} = R_1 + \frac{R_2R_3}{R_2 + R_3} = 10 + \frac{(20)(30)}{20 + 30} = 22\ \Omega$$

Current through the battery is

$$I_2 = \frac{E}{R_{\text{net}}} = \frac{50}{22}\ \text{A} = \frac{25}{11}\ \text{A}$$

$$\Rightarrow \quad I_2 = \frac{50}{22}\text{A} = 2.3\ \text{A}$$

This current will distribute in R_2 and R_3 in inverse ratio of resistance, so current through R_2 is

$$I_{R_2} = \left(\frac{50}{22}\right)\left(\frac{R_3}{R_2 + R_3}\right)$$

$$\Rightarrow \quad I_{R_2} = \left(\frac{50}{22}\right)\left(\frac{30}{30 + 20}\right) = \frac{15}{11}\ \text{A}$$

(c) Immediately after switch is reopened, the current through R_1 will become zero. However, current through R_2 will be equal to the steady state current through R_3, which is

$$I_3 = \left(\frac{50}{22} - \frac{15}{11}\right) = 0.91 \text{ A}$$

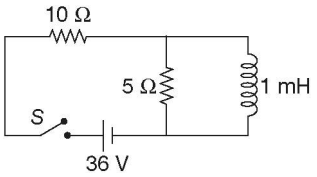
(d) Long time after S is reopened, current through all resistors will become zero.

Test Your Concepts-VI

Based on LR Circuits

(Solutions on page H.143)

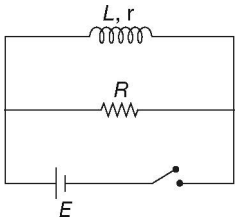
1. In the circuit arrangement shown in Figure, the switch S is closed at $t = 0$. Find the current in the inductance as a function of time? Does the current through 10 Ω resistor vary with time or remains constant.



2. A coil of resistance 20 Ω and inductance 0.5 H is switched to dc supply of 200 V. Calculate the
- **(a)** rate of increase of current at the instant of closing the switch and
- **(b)** rate of increase of current after one time constant.
- **(c)** steady state current in the circuit.

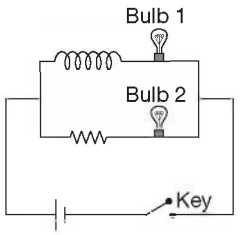
3. A coil of inductance 2 H and resistance 10 Ω are in closed series circuit with an open key and a cell of constant 100 V with negligible resistance. At time $t = 0$, the key is closed. Find
- **(a)** the time constant of the circuit.
- **(b)** the maximum steady current in the circuit.
- **(c)** the current in the circuit at $t = 1$ second.
- **(d)** the energy stored in the magnetic field linked with the coil at the steady state.

4. A solenoid of inductance L with resistance r is connected in parallel to a resistance R. A battery of emf E and of negligible internal resistance is connected across the parallel combination as shown in the Figure. At time $t = 0$, switch S is opened, calculate:



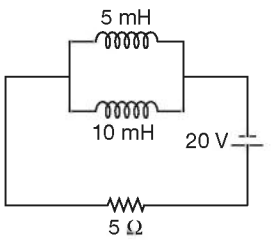
- **(a)** current through the solenoid after the switch is opened.
- **(b)** amount of heat generated in the solenoid.

5. Two identical bulbs 1 and 2 are connected in the circuit shown in Figure.

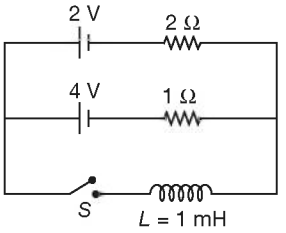


Assume that the key is closed at time $t = 0$ and when the bulbs start glowing with their full intensities the key is opened again. Will the bulbs die out suddenly or slowly? Discuss in detail.

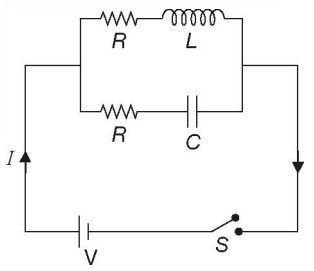
6. In the given circuit, find the current through the 5 mH inductor in steady state.



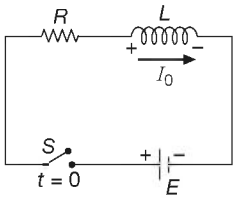
7. In the circuit shown, switch S is closed at time $t = 0$. Find the current through the inductor as a function of time t.



8. In the circuit diagram shown, initially there is no energy in the inductor and the capacitor. The switch is closed at $t = 0$. Find the current I as a function of time if $R = \sqrt{\frac{L}{C}}$.

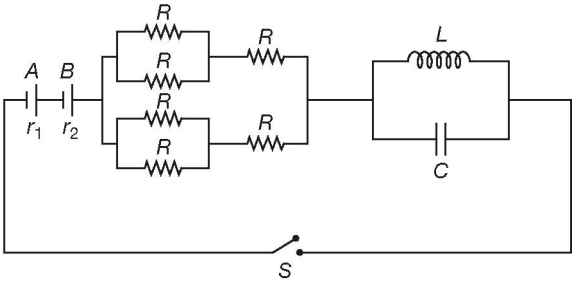


9. An active inductor with current I_0 is connected in series with a resistance and a battery as shown in Figure.

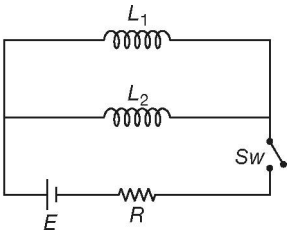


If at $t = 0$ the switch is closed, find the current in inductor as a function of time.

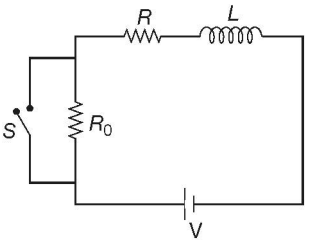
10. In the circuit shown A and B are two cells of same emf E but different internal resistances r_1 and $r_2 (r_1 > r_2)$ respectively. Find the value of R such that the potential difference across the terminals of cell A is zero a long time after the key K is closed.



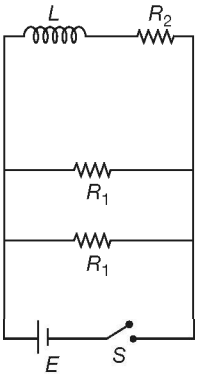
11. In the electric circuit shown in Figure, a battery of EMF E, a resistance R and coils of inductances L_1 and L_2 are used. The internal resistance of the battery and the coil resistances are negligible. Find the steady state currents in the coils long time after the switch Sw was shorted.



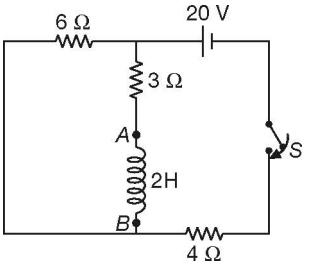
12. In the network shown, switch S is closed at $t = 0$ when a steady state current has been attained previously. Find the current in the circuit at time t.



13. An inductor of inductance $L = 400$ mH and three resistors of resistances $R_1 = 4\ \Omega$ and $R_2 = 4\ \Omega$ are connected to a battery of e.m.f. $E = 10$ V as shown in the Figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and magnitude of current through inductor as a function of time?



14. In the circuit shown in Figure, find the current in inductor as a function of time if switch is closed at $t = 0$.



15. A 5 H inductor is placed in series with a 10 Ω resistor. An emf of 5 V is suddenly applied to the combination. For the set up discussed, verify the principle of conservation of energy at time equal to the time constant of the circuit.

ENERGY STORED IN AN INDUCTOR/ ENERGY IN A MAGNETIC FIELD

As discussed, the emf induced in an inductor-resistor series circuit prevents a battery from establishing an instantaneous current in the circuit. Since we have

$$E = IR + L\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad EI = I^2R + LI\left(\frac{dI}{dt}\right)$$

From this equation, we observe that the battery must provide more energy to this LR circuit than it provides to a circuit without an inductor, because a part of the total energy supplied by the battery per second (EI) is the rate at which energy is dissipated as heat by the resistor (I^2R) and the remaining is the rate at which energy is stored in the magnetic field associated with the inductor $\left(L\frac{dI}{dt}\right)$.

If we denote the energy stored in the inductor at any instant of time t by U, then we get

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$\Rightarrow \quad dU = LIdI$$

$$\Rightarrow \quad U = \int dU = L\int_0^I IdI$$

$$\Rightarrow \quad U = \frac{1}{2}LI^2$$

We can also find the above expression by using,

$$dW = V_L Idt \text{, where } V_L = L\frac{dI}{dt} \quad \{\because dW = VIdt\}$$

$$\Rightarrow \quad dW = L\left(\frac{dI}{dt}\right)Idt = LIdI$$

$$\Rightarrow \quad W = \int dW = L\int_0^I IdI = \frac{1}{2}LI^2$$

This work done is stored in the form of energy of the magnetic field in the inductor, denoted by U. So,

$$U = \frac{1}{2}LI^2$$

$$\Rightarrow \quad L = \frac{2U}{I^2}$$

If $I = 1$ ampere, then $L = 2U$ (numerically)

So, the self-inductance of a circuit is numerically equal to twice the work done against the induced emf in establishing a current of 1 A in the coil.

ILLUSTRATION 71

A coil of inductance 1 H and resistance 10 Ω is connected to a resistance less battery of emf 50 V at time $t = 0$. Calculate the ratio of the rate at which magnetic energy is stored in the coil to the rate at which energy is supplied by the battery at $t = 0.1$ sec.

SOLUTION

Since, $E = L\frac{dI}{dt} + IR$

$\Rightarrow \quad EI = LI\frac{dI}{dt} + I^2R$

where, EI is the rate at which energy is supplied by the battery and $LI\frac{dI}{dt}$ is the rate at which magnetic energy is stored in the coil.
So, the desired ratio is

$$\text{Desired Ratio} = \frac{LI\left(\frac{dI}{dt}\right)}{EI} = \frac{L\left(\frac{dI}{dt}\right)}{E}$$

Since $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$

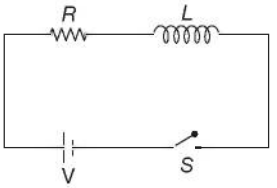
$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0R}{L}e^{-\frac{Rt}{L}} = \frac{E}{L}e^{-\frac{Rt}{L}} \qquad \{\because I_0R = E\}$$

$$\Rightarrow \quad \text{Ratio} = \frac{1}{L}e^{-\frac{Rt}{L}} = e^{-10(0.1)}$$

$$\Rightarrow \quad \text{Ratio} = \frac{1}{e} = 0.37$$

ILLUSTRATION 72

A resistor and an inductor are connected in series through a battery of potential difference V as shown in Figure.



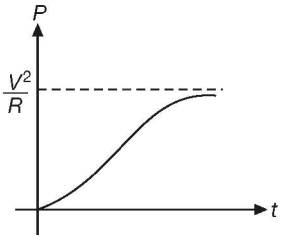
The switch S is closed at time $t = 0$. Plot the graph of rate of loss of heat (P) in the resistor versus time (t). Also calculate the magnitude of current flowing in the circuit when the rate of increase of magnetic energy in the inductor is maximum.

SOLUTION

The current i at any time $t = t$ is given by

$$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad P = i^2R = \frac{V^2}{R}\left(1 - e^{-\frac{Rt}{L}}\right)^2 R = \frac{V^2}{R}\left(1 - e^{-\frac{Rt}{L}}\right)^2$$



The magnetic energy stored in the inductor is

$$U_m = U = \frac{1}{2}Li^2$$

Rate of change of magnetic energy is

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) = Li\frac{di}{dt}$$

Let $Li\frac{di}{dt} = Z$

For the rate of increase of magnetic energy in the inductor to be maximum i.e. for Z to be maximum, we have

$$\frac{dZ}{dt} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\left(\frac{V}{L}e^{-\frac{Rt}{L}}\right)\right) = 0$$

$$\Rightarrow \quad \frac{V^2}{LR}\frac{d}{dt}\left(e^{-\frac{Rt}{L}} - e^{-\frac{2Rt}{L}}\right) = 0$$

$$\Rightarrow \quad -\frac{R}{L}e^{-\frac{Rt}{L}} - \frac{2R}{L}e^{-\frac{2Rt}{L}} = 0$$

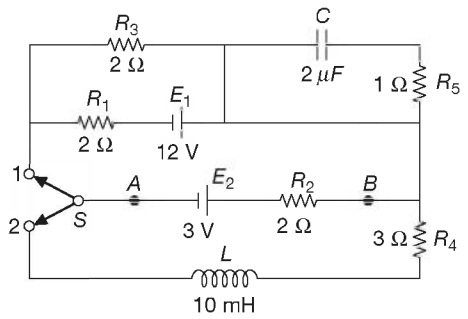
$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{1}{2}$$

So, current in the circuit at that instant is

$$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{V}{R}\left(1 - \frac{1}{2}\right) = \frac{V}{2R}$$

ILLUSTRATION 73

A circuit containing a two-position switch S is shown in Figure.



(a) The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(b) If now the switch S is put in position 2 at $t = 0$. Find:

(i) steady current in R_4 and

(ii) the time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time.

SOLUTION

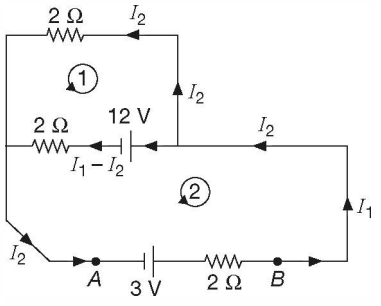
(a) In steady state no current will flow through capacitor, because a fully charged capacitor is a dc blocking element.

Also, no current flows through it initially. So, applying Kirchhoff's Second Law in Loop 1, we get

$$-2I_2 + 2(I_1 - I_2) + 12 = 0$$

$$\Rightarrow \quad 2I_1 - 4I_2 = -12$$

$$\Rightarrow \quad I_1 - 2I_2 = -6 \qquad \ldots(1)$$



Applying Kirchhoff's Second Law in Loop 2, we get

$$-12 - 2(I_1 - I_2) + 3 - 2I_1 = 0$$

$$\Rightarrow \quad 4I_1 - 2I_2 = -9 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

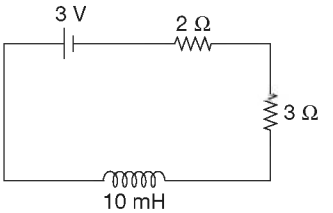
$$I_2 = 2.5 \text{ A} \text{ and } I_1 = -1 \text{ A}$$

Now, $V_A + 3 - 2I_1 = V_B$

$$\Rightarrow \quad V_A - V_B = 2I_1 - 3 = 2(-1) - 3 = -5 \text{ V}$$

$$\Rightarrow \quad P_{R_1} = (I_1 - I_2)^2 R_1 = (-1 - 2.5)^2 (2) = 24.5 \text{ W}$$

(b) In position 2, the circuit is shown in Figure



Steady current in R_4 is

$$I_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

Time when current in R_4 is half the steady value is

$$t_{1/2} = \frac{\log_e(2)}{1/\tau_L} = \tau_L(\log_e 2) = \frac{L}{R}\log_e(2)$$

$$t_{1/2} = \frac{(10 \times 10^{-3})}{5}\log_e(2) = 1.386 \times 10^{-3} \text{ s}$$

Energy stored in inductor at that time i.e. when $I = \frac{I_0}{2} = 0.3$ A is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(10 \times 10^{-3})(0.3)^2 = 4.5 \times 10^{-4} \text{ J}$$

ILLUSTRATION 74

An inductance L and a resistance R are connected in series with a battery of emf E. Find the maximum rate at which the energy is stored in the magnetic field.

SOLUTION

Energy stored in the magnetic field is

$$U = \frac{1}{2}LI^2$$

Since $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\Rightarrow \quad U=\frac{1}{2}LI_0^2\left(1-e^{-\frac{Rt}{L}}\right)^2$$

The rate at which the energy is stored is

$$P=\frac{dU}{dt}=LI_0^2\left(1-e^{-\frac{Rt}{L}}\right)\left(-e^{-\frac{Rt}{L}}\right)\left(-\frac{R}{L}\right)$$

$$\Rightarrow \quad P=I_0^2R\left(e^{-\frac{Rt}{L}}-e^{-\frac{2Rt}{L}}\right) \qquad \ldots(1)$$

For the maximum rate at which the energy is stored, we have

$$\frac{dP}{dt}=0$$

$$\Rightarrow \quad -\left(\frac{R}{L}\right)e^{-\frac{Rt}{L}}+\left(\frac{2R}{L}\right)e^{-\frac{2Rt}{L}}=0$$

$$\Rightarrow \quad -1+2e^{-\frac{Rt}{L}}=0$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}}=\frac{1}{2} \qquad \ldots(2)$$

So, put (2) in (1), we get $P_{\max}$ as

$$P_{\max}=I_0^2R\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{I_0^2R}{4}=\frac{E^2}{4R}$$

Magnetic Energy Density (u_m)

Let us now calculate the magnetic energy density or simply the energy density of a magnetic field. For the sake of convenience, consider a solenoid of length l, area A, having n number of turns per unit length and self-inductance L. Then, we have

$$L=\mu_0 n^2 Al \qquad \ldots(1)$$

If B is the magnetic field associated with the solenoid, then

$$B=\mu_0 nI \qquad \ldots(2)$$

Since, we know that the energy associated with the magnetic field is given by

$$U=\frac{1}{2}LI^2$$

$$\Rightarrow \quad U=\frac{1}{2}\left(\mu_0 n^2 Al\right)\left(\frac{B}{\mu_0 n}\right)^2 \qquad \{\because B=\mu_0 nI\}$$

$$\Rightarrow \quad U=\left(\frac{B^2}{2\mu_0}\right)Al$$

where Al = Volume of the solenoid.

Now, by definition, the magnetic energy density is defined as

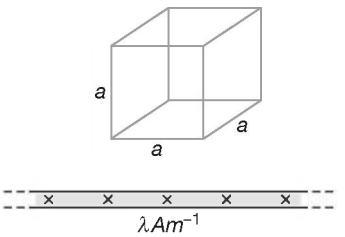
$$u_m=\frac{\text{Energy Associated with the Magnetic Field}}{\text{Volume of the Solenoid}}$$

$$\Rightarrow \quad u_m=\frac{U}{Al}=\frac{B^2}{2\mu_0}$$

Although this expression has been derived by considering the special case of a solenoid, however it is valid for any region of space where the magnetic field exists. Also, we observe that the above formula for magnetic energy density (u_m) matches with that of electric energy density given by $u_E=\frac{1}{2}\varepsilon_0 E^2$. In both the cases, the energy density is proportional to the square of the field.

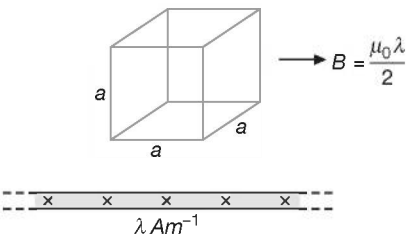
ILLUSTRATION 75

Calculate the magnetic field energy associated with a cubical region (of edge a) that lies above a large current carrying sheet which carries a uniform linear current density λ Am^{-1}.



SOLUTION

Magnetic field B due to a large current sheet in its surrounding is shown in Figure.



Since $B=\frac{1}{2}\mu_0\lambda$, so the magnetic field energy density due to the field associated with the sheet in the region of space is given by

$$u_m = \frac{B^2}{2\mu_0} = \frac{\left(\frac{1}{2}\mu_0\lambda\right)^2}{2\mu_0} = \frac{1}{8}\mu_0\lambda^2 \text{ Jm}^{-3}$$

So, magnetic field energy associated with this cubical region is

$$U = u_m V_{\text{cube}} = u_m a^3 = \frac{1}{8}\mu_0 a^3\lambda^2$$

ILLUSTRATION 76

A long cylinder of radius a carrying a uniform surface charge rotates about its axis with an angular velocity ω. Find the magnetic field energy per unit length of the cylinder inside of it if the linear charge density equals λ and consider permeability of the medium is equal to unity.

SOLUTION

Current due to rotation of the cylinder of length l is given as

$$I = \frac{q\omega}{2\pi} = \frac{(\lambda l)\omega}{2\pi}$$

From Ampere's Circuital law, the magnetic induction inside the cylinder is

$$B = \frac{\mu_0 I}{l}$$

The magnetic field energy density inside the cylinder is given as

$$u_m = \frac{B^2}{2\mu_0}$$

Energy per unit length of the cylinder is

$$\frac{U_m}{l} = \frac{u_m V_{\text{cylinder}}}{l} = \frac{1}{l}\left(\frac{B^2}{2\mu_0}\right)(\pi a^2 l)$$

$$\Rightarrow \quad \frac{U_m}{l} = \left(\frac{1}{2\mu_0 l}\right)\left(\frac{\mu_0\lambda l\omega}{2\pi l}\right)^2(\pi a^2 l)$$

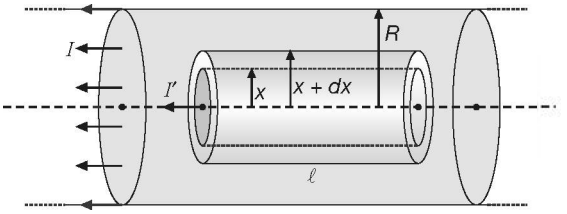
$$\Rightarrow \quad \frac{U_m}{l} = \frac{\mu_0\lambda^2\omega^2 a^2}{8\pi}$$

ILLUSTRATION 77

Calculate the total magnetic field energy stored per unit length inside a long cylindrical wire of radius R and carrying a current I.

SOLUTION

Let us consider an elemental shell of radius x and wall width dx inside the cylinder as shown in Figure.



The magnetic field inside the wire at a distance x from axis is calculated using Ampere's Circuital Law, according to which we get

$$B(2\pi x) = \mu_0\left(I_{\text{enc}}\right) = \mu_0 I'$$

$$\Rightarrow \quad B(2\pi x) = \mu_0\left(\frac{I}{\pi R^2}\right)(\pi x^2)$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 I}{2\pi R^2}\right)x$$

The volume of the elemental shell of length l is

$$dV = (2\pi x dx)l$$

magnetic field energy stored in elemental cylindrical sheet is

$$dU = \left(\frac{B^2}{2\mu_0}\right)dV = \frac{1}{2\mu_0}\left(\frac{\mu_0 I x}{2\pi R^2}\right)^2(2\pi l x dx)$$

$$\Rightarrow \quad dU = \frac{\mu_0 I^2 l}{4\pi R^4}x^3 dx$$

Total field energy inside the cylinder is

$$U = \int dU = \frac{\mu_0 I^2 l}{4\pi R^4}\int_0^R x^3 dx$$

$$\Rightarrow \quad \frac{U}{l} = \left(\frac{\mu_0 I^2}{4\pi R^4}\right)\frac{R^4}{4} = \frac{\mu_0 I^2}{16\pi}$$

Test Your Concepts-VII

Based on Magnetic Energy and Magnetic Energy Density

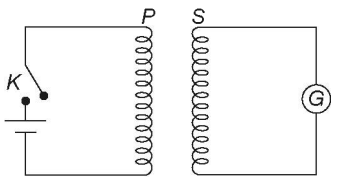
(Solutions on page H.148)

1. What inductance would be needed to store 1 kWh of energy in a coil carrying a current of 200 A?
2. Assume that the magnitude of the magnetic field outside a sphere of radius R is $B = B_0\left(\frac{R}{r}\right)^2$, where B_0 is a constant. Determine the total energy stored in the magnetic field outside the sphere.
3. A wire of non-magnetic material, with radius R, carries current uniformly distributed over its cross section. The total current carried by the wire is I. Find the magnetic energy per unit length from the surface of wire to twice its radius.
4. On a clear day at a certain location, a 100 Vm^{-1} vertical electric field exists near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of 50 μT. Compute the energy densities of the two fields. For what ratio of E to B can the energy density be equal. Can you recognise this ratio?
5. A long coaxial cable consists of two concentric cylinders of radii a and b. The central conductor of the cable carries a steady current i and the outer conductor provides the return path of the current. Calculate the energy stored in the magnetic field of length l of such a cable
6. The current (in ampere) in an inductor is given by $I = 5 + 16t$, where t is in seconds. The self-induced emf in it is 10 mV. Find
   (a) the self-inductance, and
   (b) the energy stored in the inductor and the power supplied to it at $t = 1$.
7. An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increases linearly in time as described by $I = kt$, where k is a constant. The capacitor is initially uncharged. Determine
   (a) the voltage across the inductor as a function of time,
   (b) the voltage across the capacitor as a function of time, and
   (c) the time when the energy stored in the capacitor first exceeds that in the inductor.
8. A solenoid has an inductance of 10 H and a resistance of 2 Ω. It is connected to a 10 V battery. How long will it take for the magnetic energy to reach one fourth of its maximum value?

MUTUAL INDUCTANCE

It is generally observed that the magnetic flux through the area enclosed by a circuit varies with time because of time varying currents in the neighbouring circuits. This phenomenon in which emf is produced due to the interaction of the two circuits is called the **phenomenon of Mutual Induction**.

Consider two coils, P (Primary) and S (Secondary) placed close to each other such that if a current passes in coil P, the coil S is in the magnetic field of coil P and vice-versa.



When the key K is closed then the current flowing through a coil (P) changes, the magnetic flux linked with the neighbouring coil (Q) also changes. As a result of this an induced e.m.f. and hence an induced current is set up in coil Q.

This phenomenon of production of e.m.f. in a coil when the current in neighbouring coil changes is called mutual induction. The circuit in which the current changes is called the **primary circuit**, while the neighbouring circuit in which e.m.f. is induced is called the **secondary circuit**.

Definition I

If I_1 is current flowing through primary coil then at any instant, the total flux linked with secondary coil is given by

$$\phi_{2,\text{ total}} \propto I_1$$

$$\Rightarrow \quad N_2\phi_2 \propto I_1$$

$$\Rightarrow \quad N_2\phi_2 = MI_1 \qquad \ldots(1)$$

$$\Rightarrow \quad M = \frac{N_2\phi_2}{I_1}$$

where M is called the coefficient of mutual induction or mutual inductance of the coils.

If $I_1 = 1$ A, then $M = N_2\phi_2$ (numerically)

So, M is numerically equal to the total flux associated with the secondary coil, when the current in the primary is 1 A.

Definition 2

Also induced e.m.f. in secondary coil,

$$\xi_2 = -\frac{d}{dt}(\phi_{2,\text{ total}}) = -N_2\frac{d\phi_2}{dt}$$

$$\Rightarrow \quad \xi_2 = -\frac{d}{dt}(MI_1) = -M\frac{dI_1}{dt} \qquad \ldots(2)$$

If $\frac{dI_1}{dt} = 1\ \text{As}^{-1}$, then $M = \xi_2$ (numerically)

So, M is numerically equal to the e.m.f. induced in the secondary coil, when the rate of change of current in the primary coil is $1\ \text{As}^{-1}$.

Conceptual Note(s)

(a) Like self-inductance, the unit of mutual inductance is henry (H). The direction of induced e.m.f. or induced current arising due to a change in magnetic flux in all cases is given by **Lenz's Law**.

(b) Mutual inductance depends on the geometry of both the circuits, their orientations with respect to each other, their closeness and the number of turns. As the circuit separation increases the flux linking the circuits decreases and hence the mutual inductance also decreases.

(c) To understand the concept of mutual inductance, we can understand that the induced emf will develop in coil 2, due to the change in its flux because of the coil 1. So, we get

$$\xi_2 = -N_2\left(\frac{d\phi_2}{dt}\right) = -N_2\frac{d}{dt}\left(\frac{M_{12}I_1}{N_2}\right)$$

Since $\phi_2 = \frac{M_{12}I_1}{N_2}$

$$\Rightarrow \quad \xi_2 = -M_{12}\left(\frac{dI_1}{dt}\right)$$

where M_{12} is the inductance of coil 2 due to change of current in coil 1.

The induced emf will develop in coil 1, due to the change in its flux because of the coil 2 is

$$\xi_1 = -N_1\left(\frac{d\phi_1}{dt}\right) = -N_1\frac{d}{dt}\left(\frac{M_{21}I_2}{N_1}\right)$$

Similarly, $\phi_1 = \frac{M_{21}I_2}{N_1}$

$$\Rightarrow \quad \xi_1 = -M_{21}\left(\frac{dI_2}{dt}\right)$$

where M_{21} is the inductance of the coil 1 due to the change of current in coil 2.

From **Reciprocity Theorem,** we have

$$M_{12} = M_{21} = M\,(\text{say})$$

So, (3) and (4), can be rewritten as

$$\xi_2 = -M\left(\frac{dI_1}{dt}\right) \text{ and } \xi_1 = -M\left(\frac{dI_2}{dt}\right)$$

CALCULATION OF MUTUAL INDUCTANCE

To calculate the mutual inductance of two circuits, simply follow the steps given.

STEP-1: Take any one circuit as the primary (the first one) and the other as secondary (the second one), for the sake of convenience.

STEP-2: Let a current I_1 flow in the primary circuit.

STEP-3: Calculate the strength of magnetic field $(\vec{B})$ produced by the current in the primary at the location of the secondary coil.

STEP-4: Now calculate $\phi_{2,\text{ total}} = N_2\phi_2 = N_2(\vec{B}\cdot\vec{A}_2)$

STEP-5: Get M by using

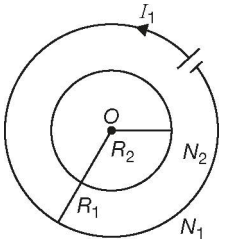
$$M = \frac{\phi_{2,\text{ total}}}{I_1} = \frac{N_2\phi_2}{I_1} = \frac{N_2(\vec{B}\cdot\vec{A}_2)}{I_1}$$

Let us apply these steps to calculate the mutual inductance for the following systems.

MUTUAL INDUCTANCE FOR A TWO COIL SYSTEM

Consider two co-axial coils having number of turns N_1 and N_2 and radii R_1 and R_2 respectively. If I_1 is current in outer coil, then magnetic field at its centre,

$$B_1 = \frac{\mu_0 N_1 I_1}{2R_1}$$



The flux linked with inner-coil

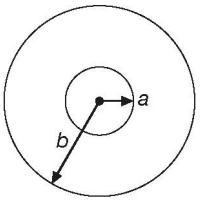
$$\phi_2 = N_2 B_1 A_2$$

$$\Rightarrow \quad \phi_2 = N_2 \left(\frac{\mu_0 N_1 I_1}{2R_1}\right) A_2$$

$$\Rightarrow \quad M = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A_2}{2R_1} = \frac{\mu_0 N_1 N_2 (\pi R_2^2)}{2R_1}$$

ILLUSTRATION 78

Two thin concentric wires shaped as circle with radii a and b lie in the same plane as shown in Figure.



Assuming that $a \ll b$, calculate their mutual inductance and the magnetic flux through the surface enclosed by the outside wire, when the inside wire carries a current I.

SOLUTION

If outer coil carries a current I, then the magnetic flux at the location of inner coil 1 due to current in outer coil 2 is

$$\phi_{12} = \phi = B_2 A_1 = \left(\frac{\mu_0 I}{2b}\right)(\pi a^2)$$

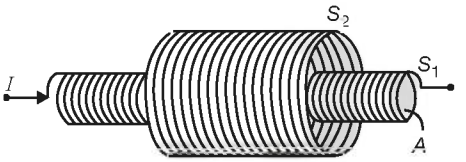
The mutual induction coefficient between the two coils is

$$M = \frac{\phi}{I} = \frac{\mu_0 \pi a^2}{2b}$$

MUTUAL INDUCTANCE FOR A SOLENOID-COIL SYSTEM

Let n_2 be number of turns per metre length of a long solenoid S_2. Let a coil S_1 of n_1 turns per unit length and of area A is placed within it. If I_1 is current in solenoid, then magnetic field within it,

$$B_1 = \mu_0 n_1 I_1$$



Since no magnetic field exists inside the annular region (region between S_1 and S_2), so magnetic flux linked with coil,

$$\phi_2 = N_2 B_1 A = (n_2 l) B_1 A = n_2 (\mu_0 n_1 I_1) A l$$

$$\Rightarrow \quad M = \frac{\phi_2}{I_1} = \mu_0 n_1 n_2 A l$$

Please note that M is independent of the area/radius of the surrounding coil S_2, because the field in the region between S_1 and S_2 (also called as the annular region) is zero.

Conceptual Note(s)

MUTUAL INDUCTANCE OF TWO CLOSE COILS AND COEFFICIENT OF COUPLING

If two coils of self-inductances L_1 and L_2 are placed near each other, then mutual inductance

$$M = K\sqrt{L_1 L_2}$$

where K is a constant, called **coefficient of coupling**, having values between zero and 1.

If flux linkage between coils is 100% then $K = 1$, so we have

$$M = \sqrt{L_1 L_2}$$

ILLUSTRATION 79

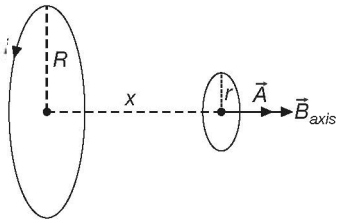
Two co-axial loops of radii R and $r(R \gg r)$ are placed such that separation between their centres is x. Calculate the mutual inductance of the arrangement.

SOLUTION

Let us imagine a current i to flow through the loop of radius R. The field produced at the centre of the smaller loop lying at the axis of the bigger loop is

$$B_{axis} = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}.$$

Since $R \gg r$, so we can assume that this field is acting uniformly on the entire area of the smaller loop.



Since $\phi = \vec{B} \cdot \vec{A} = BA\cos 0° = BA$

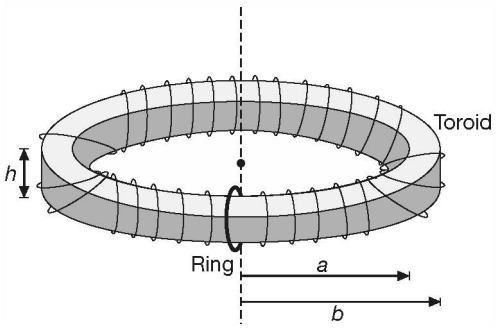
$$\Rightarrow \quad \phi = \left[\frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}\right]\pi r^2$$

Since $M = \frac{\phi}{i}$

$$\Rightarrow \quad M = \frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$$

ILLUSTRATION 80

A toroidal coil S_1 has a rectangular cross-section and contains N loops as shown in Figure.



Its inner and outer radii are a and b, respectively. The height of the toroid is h. A thin conducting ring S_2 of resistance R, encircles the toroid. The coil S_1 carries a time-dependent current $I = I_0 e^{-\alpha t}$ (where I_0 and α are constants). Calculate the mutual inductance between the ring and the coil and the induced time-dependent current i in the ring.

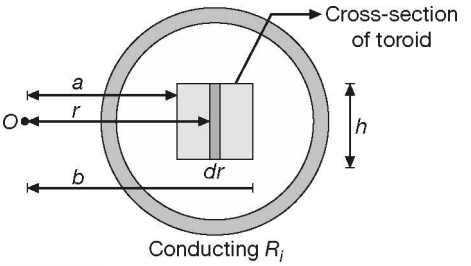
SOLUTION

For calculating the mutual inductance, let us assume that a current i is passed through the toroid. By Ampere's Circuital Law (ACL), the magnetic field at a radial distance r from the centre of toroid is

$$B = \frac{\mu_0 N i}{2\pi r}$$

Since magnetic field outside the toroid is zero, so the flux associated with the ring equals the flux associated with the toroid.

Now to calculate the flux associated with the ring, let us consider the cross-section of toroid. The flux associated with the shaded strip is



$$d\phi = BdA$$

$$\Rightarrow \quad d\phi = \frac{\mu_0 N i}{2\pi r}(h\,dr)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Since $M = \frac{\phi}{i}$

$$\Rightarrow \quad M = \frac{\mu_0 N h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Applying Faraday's Laws of EMI, we get

$$\xi = -\frac{M di}{dt}$$

$$\Rightarrow \quad \xi = -M\frac{d}{dt}\left(I_0 e^{-\alpha t}\right) = \frac{\mu_0 Nh}{2\pi}\ln\left(\frac{b}{a}\right)\left(I_0\alpha\right)e^{-\alpha t}$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{\mu_0 NhI_0\alpha e^{-\alpha t}}{2\pi R}\ln\left(\frac{b}{a}\right)$$

ILLUSTRATION 81

There are two stationary loops with mutual inductance M. The current in one of the loops starts varying as $I_1 = \alpha t$, where α is a positive constant and t is any time instant starting from $t = 0$. Calculate the time dependence $I_2(t)$ of the current in the other loop which has self-inductance L and resistance R.

SOLUTION

The induced EMF in the second loop when the current changes in the first loop is

$$\xi_2 = M\frac{dI_1}{dt} = M\frac{d}{dt}(\alpha t) = M\alpha$$

$$\Rightarrow \quad \xi_2 = M\alpha$$

For the second loop, on applying KLL we get

$$I_2 R + L\frac{dI_2}{dt} = M\alpha$$

$$\Rightarrow \quad L\frac{dI_2}{dt} = M\alpha - I_2 R$$

$$\Rightarrow \quad \frac{dI_2}{M\alpha - I_2 R} = \frac{dt}{L}$$

$$\Rightarrow \quad \int_0^{I_2}\frac{dI_2}{M\alpha - I_2 R} = \int_0^{t}\frac{dt}{L}$$

$$\Rightarrow \quad -\frac{1}{R}\ln\left(M\alpha - I_2 R\right)\Big|_0^{I_2} = \frac{t}{L}\Big|_0^{t}$$

$$\Rightarrow \quad \ln\left(\frac{M\alpha - I_2 R}{M\alpha}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \quad \frac{M\alpha - I_2 R}{M\alpha} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad M\alpha - I_2 R = M\alpha e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad I_2 R = M\alpha\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad I_2 = \frac{M\alpha}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

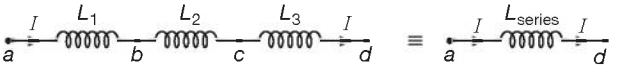
INDUCTANCES IN SERIES AND PARALLEL

Series

Consider a number of inductors L_1, L_2, L_3, ... connected in series. Assuming no interaction between them through mutual inductance, then we have

$$L_{\text{series}} = L_1 + L_2 + L_3 + ...$$

This can easily be shown by taking the following series inductor circuit and its equivalent circuit.



At any instant, assume that the current in the inductor circuit be I. Then

$$V_a - V_b = L_1\left(\frac{dI}{dt}\right) \qquad ...(1)$$

$$V_b - V_c = L_2\left(\frac{dI}{dt}\right) \qquad ...(2)$$

$$V_c - V_d = L_3\left(\frac{dI}{dt}\right) \qquad ...(3)$$

Now, for the equivalent circuit we have

$$V_a - V_d = L_{\text{series}}\left(\frac{dI}{dt}\right) \qquad ...(4)$$

From (1), (2), (3) and (4), we get

$$V_a - V_d = (V_a - V_b) + (V_b - V_c) + (V_c - V_d)$$

$$\Rightarrow \quad L_{\text{series}}\left(\frac{dI}{dt}\right) = L_1\left(\frac{dI}{dt}\right) + L_2\left(\frac{dI}{dt}\right) + L_3\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad L_{\text{series}} = L_S = L_1 + L_2 + L_3 + ...$$

SPECIAL CASE

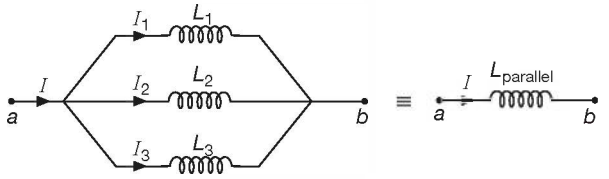
If two coils of self-inductances L_1 and L_2, having mutual inductance M are connected in series and placed close to each other, then the net inductance is given by

$$L_{\text{series}} = L_S = L_1 + L_2 \pm 2M$$

Parallel

Consider a number of inductors L_1, L_2, L_3, ... connected in parallel. Assuming no interaction between them through mutual inductance, then we have

$$\frac{1}{L_{\text{parallel}}}=\frac{1}{L_P}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\ldots\ldots$$



This can be shown by taking the following parallel inductor circuit and its equivalent circuit.

Since the inductors are connected in parallel, so we have

$$I=I_1+I_2+I_3$$

$$\Rightarrow \quad \frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}+\frac{dI_3}{dt}$$

Since, $V_a-V_b=L_P\left(\frac{dI}{dt}\right)$ and

$$V_a-V_b=L_1\left(\frac{dI_1}{dt}\right)=L_2\left(\frac{dI_2}{dt}\right)=L_3\left(\frac{dI_3}{dt}\right)$$

$$\Rightarrow \quad \frac{(V_a-V_b)}{L_{\text{parallel}}}=\frac{(V_a-V_b)}{L_1}+\frac{(V_a-V_b)}{L_2}+\frac{(V_a-V_b)}{L_3}$$

$$\Rightarrow \quad \frac{1}{L_{\text{parallel}}}=\frac{1}{L_P}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\ldots\ldots$$

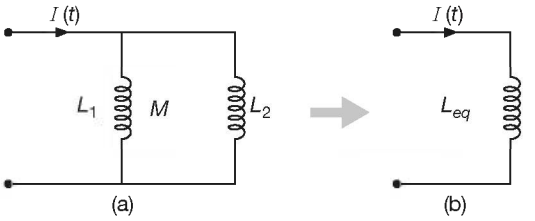
SPECIAL CASE

If two coils of self-inductances L_1 and L_2, having mutual inductance M are connected in parallel and placed close to each other, then the net inductance is given by

$$L_P=L_{\text{parallel}}=\frac{L_1L_2-M^2}{L_1+L_2\pm 2M}$$

ILLUSTRATION 82

Two inductors having self-inductances L_1 and L_2 are connected in parallel as shown in Figure (a). The mutual inductance between the two inductors is M. Determine the equivalent self-inductance L_{eq} for the system as shown in Figure (b).



SOLUTION

$$I=I_1+I_2 \qquad \ldots(1)$$

The voltage across the pair is

$$\Delta V=-L_1\frac{dI_1}{dt}-M\frac{dI_2}{dt_2} \qquad \ldots(2)$$

$$\Delta V=-L_2\frac{dI_2}{dt}-M\frac{dI_1}{dt} \qquad \ldots(3)$$

For the equivalent combination, we have

$$\Delta V=-L_{\text{eq}}\frac{dI}{dt} \qquad \ldots(4)$$

From (2), we get

$$-\frac{dI_1}{dt}=\frac{\Delta V}{L_1}+\frac{M}{L_1}\frac{dI_2}{dt}$$

Substituting in (3), we get

$$\Delta V=-L_2\frac{dI_2}{dt}+M\left(\frac{\Delta V}{L_1}+\frac{M}{L_1}\frac{dI_2}{dt}\right)$$

$$\Rightarrow \quad -L_2\frac{dI_2}{dt}+\frac{M(\Delta V)}{L_1}+\frac{M^2}{L_1}\frac{dI_2}{dt}=\Delta V$$

$$\Rightarrow \quad \left(M^2-L_1L_2\right)\frac{dI_2}{dt}=\Delta V\left(L_1-M\right) \qquad \ldots(5)$$

Now, from (3), we get

$$-\frac{dI_2}{dt}=\frac{\Delta V}{L_2}+\frac{M}{L_2}\frac{dI_1}{dt}$$

Substituting in (2), we get

$$\left(M^2-L_1L_2\right)\frac{dI_1}{dt}=\Delta V\left(L_2-M\right)$$

However, we have

$$I=I_1+I_2$$

$$\Rightarrow \quad \frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}$$

$$\Rightarrow \quad -\frac{\Delta V}{L_{eq}} = \frac{\Delta V(L_2 - M)}{M^2 - L_1 L_2} + \frac{\Delta V(L_1 - M)}{M^2 - L_1 L_2} \qquad \Rightarrow \quad -\frac{1}{L_{eq}} = \frac{L_2 - M + L_1 - M}{M^2 - L_1 L_2}$$

$$\left\{\because -L_{eq}\frac{dI}{dt} = \Delta V\right\} \qquad \Rightarrow \quad L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Test Your Concepts-VIII

Based on Faraday's Laws: Mutual Induction

(Solutions on page H.150)

1. A coil has 600 turns which produces 5×10^{-3} Wb per turn of flux when 3 A flows in the wire. This produced 6×10^{-3} Wb per turn in 1000 turns secondary coil. When a switch is opened the current drops to zero in 0.2 s. Find
- **(a)** mutual inductance
- **(b)** the induced emf in the secondary
- **(c)** the self-inductance of the primary coil

2. Calculate the mutual inductance between two coils when a current of 4 A changes to 12 A in 0.5 s and induces an emf of 50 mV in the secondary. Also calculate the induced emf in the secondary if current in the primary changes from 3 A to 9 A in 0.02 s.

3. A circular coil P of 100 turns and radius 2 cm is placed concentrically at the centre of another circular coil Q of 1000 turns and radius 20 cm. Calculate the mutual inductance of the coils. When the current in the coil Q decreases from 5 A to 3 A in 0.04 s, then calculate the emf induced in coil P, rate of change of flux through the coil P at this instant and the charge passing through coil P if its resistance is 8 Ω.

4. A large coil of radius R_1 and having N_1 turns is coaxial with a small coil of radius R_2 and having N_2 turns. The centres of the coils are separated by a distance x that is much larger than R_1 and R_2. What is the mutual inductance of the coils?

5. A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross section of the solenoid is 4 cm^2. Calculate the mutual inductance.

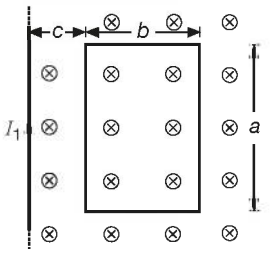
6. Find the mutual inductance of two thin coaxial loops of the same radius a if their centres are separated by a distance l, with $l \gg a$.

7. An air core solenoid 0.5 m in length contains 1000 turns and has a cross sectional area of 1 cm^2.
- **(a)** Ignoring end effects, find the self-inductance.
- **(b)** A secondary winding wrapped around the center of the solenoid has 100 turns. What is the mutual inductance?
- **(c)** The secondary winding carries a constant current of 1 A, and the solenoid is connected to a load of 1 kΩ. The constant current is suddenly stopped. How much charge flows through the load resistor?

8. A coil of 100 turns and 1 cm radius is kept coaxially within a long solenoid of 8 turns per cm and 5 cm radius. Find the mutual inductance.

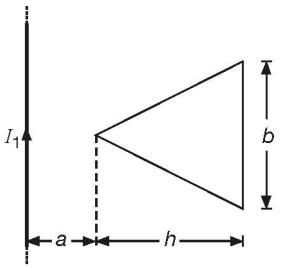
9. A very small circular loop of radius a is initially coplanar and concentric with a much larger circular loop of radius $b(\gg a)$. A constant current I is passed in the large loop which is kept fixed in space and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is R and the inductance is negligible.
- **(a)** Find the current in the small loop as a function of time.
- **(b)** Calculate how much torque must be exerted on the small loop to rotate it.
- **(c)** Calculate the induced emf in the large loop due to current [found in part (a)] in smaller loop as a function of time.

10. Figure shows a rectangular coil near a long wire. Calculate the mutual inductance of the combination.



11. An inductor of inductance L is cut in three equal parts and two of these parts are interconnected first in series and then in parallel. Assuming that the mutual inductance between the parts is negligible, calculate the inductance of the combination in both the cases.

12. Figure shows a long wire and a triangular coil. Calculate the mutual inductance of the combination.



OSCILLATIONS IN AN LC CIRCUIT

Introduction and Circuit

When a capacitor is connected to an inductor, as shown in Figure 1, then the combination is called an LC **circuit**.

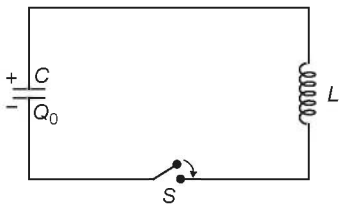


Figure 1 A simple LC circuit. The capacitor has an initial charge Q_0 and the switch is open for $t < 0$ and then closed at $t = 0$.

If the capacitor is initially charged and the switch is then closed, we find that both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values, if the resistance of the circuit is zero, i.e., no energy is dissipated as heat.

Assumptions

(a) In the following analysis, we neglect the resistance in the circuit.

(b) We also assume an idealized situation in which energy is not radiated away from the circuit.

(c) Assume that the capacitor has an initial charge Q_0 (the maximum charge) and that the switch is open for $t < 0$ and then closed at $t = 0$. Let us discuss what happens from an energy viewpoint.

With these idealizations of zero resistance and no radiation losses, the oscillations in the circuit persist indefinitely.

Explanation Based on Energy Viewpoint

When the capacitor is fully charged, the entire energy in the circuit is stored in the electric field of the capacitor and is $U = \frac{Q_0^2}{2C}$. At this time, the current in the circuit is zero, and therefore no energy is stored in the inductor. After the switch is closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. As the capacitor begins to discharge after the switch is closed, the energy stored in its electric field decreases. The discharge of the capacitor represents a current in the circuit, and hence some energy is now stored in the magnetic field of the inductor. Thus, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this moment, the current reaches its maximum value and now the entire energy is stored in the inductor. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite to the initial polarity. This is followed by another discharge until the circuit returns to its original state of maximum charge Q_0 and the plate polarity shown in Figure 1. The energy continues to oscillate between inductor and capacitor.

The total energy in the LC circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 = \text{constant} \qquad \ldots(1)$$

Since, U remains constant, so we get

$$\frac{dU}{dt} = 0$$

$$\Rightarrow \quad \frac{dU}{dt}=\frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2\right)=\frac{Q}{C}\frac{dQ}{dt}+LI\frac{dI}{dt}=0 \quad \ldots(2)$$

$$\Rightarrow \quad \frac{Q}{C}+L\frac{d^2Q}{dt^2}=0 \quad \ldots(3)$$

where $I=-\frac{dQ}{dt}$ and $\frac{dI}{dt}=-\frac{d^2Q}{dt^2}$

Notice the sign convention we have adopted here. **The negative sign implies that the current I is equal to the rate of decrease of charge in the capacitor plate immediately after the switch has been closed.** The same equation can be obtained by applying the modified Kirchhoff's Loop Rule, to the circuit. So, we get

$$\frac{Q}{C}-L\frac{dI}{dt}=0 \quad \ldots(4)$$

Since $I=-\frac{dQ}{dt}$, so we get

$$\frac{Q}{C}+L\frac{d^2Q}{dt^2}=0$$

The general solution to equation (3) is

$$Q(t)=Q_0\cos(\omega_0 t+\theta) \quad \ldots(5)$$

where Q_0 is the amplitude of the charge and θ is the phase. The angular frequency ω_0 is given by

$$\omega_0=\frac{1}{\sqrt{LC}} \quad \ldots(6)$$

The corresponding current in the inductor is

$$I(t)=-\frac{dQ}{dt}=\omega_0 Q_0\sin(\omega_0 t+\theta)$$

$$\Rightarrow \quad I(t)=I_0\sin(\omega_0 t+\theta) \quad \ldots(7)$$

where $I_0=\omega_0 Q_0$ $\ldots(8)$

From the initial conditions, we have

$$Q(t=0)=Q_0 \text{ and } I(t=0)=0.$$

So, the phase θ is given by $\theta=0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t)=Q_0\cos(\omega_0 t) \quad \ldots(9)$$

{Think smartly that at $t=0$, we have $Q=Q_0$, so Q must be a function of cosine with zero initial phase}

$$\Rightarrow \quad I(t)=\frac{dQ}{dt}=I_0\sin(\omega_0 t) \quad \ldots(10)$$

The time dependence of $Q(t)$ and $I(t)$ are depicted in Figure 2.

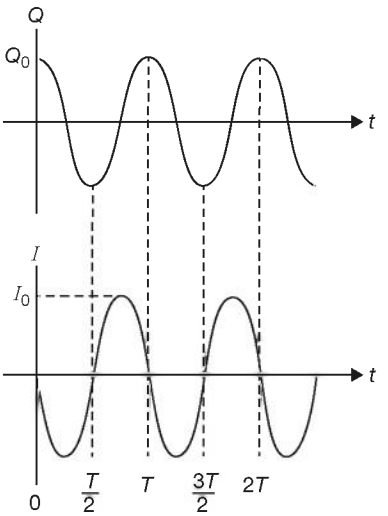


Figure 2 Charge and current in the LC circuit as a function of time.

From equations (9) and (10) (AND USING (8)), we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E=\frac{Q^2(t)}{2C}$$

$$\Rightarrow \quad U_E=\left(\frac{Q_0^2}{2C}\right)\cos^2(\omega_0 t) \quad \ldots(11)$$

and $U_B=\frac{1}{2}LI^2(t)$

$$\Rightarrow \quad U_B=\frac{1}{2}LI^2(t)=\frac{LI_0^2}{2}\sin^2(\omega t)$$

Since $I_0=\omega_0 Q_0$, so we have

$$\Rightarrow \quad U_B=\frac{L(\omega_0 Q_0)^2}{2}\sin^2(\omega_0 t) \quad \ldots(12)$$

Since we know that $\omega_0=\frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad U_B=\frac{1}{2}LI^2=\left(\frac{Q_0^2}{2C}\right)\sin^2(\omega_0 t)$$

The total energy is given by

$$\Rightarrow \quad U = U_E + U_B = \left(\frac{Q_0^2}{2C}\right)\cos^2(\omega_0 t) + \left(\frac{Q_0^2}{2C}\right)\sin^2(\omega_0 t)$$

$$\Rightarrow \quad U = \frac{Q_0^2}{2C} = \text{constant} \qquad \ldots(13)$$

The electric and magnetic energy oscillations are illustrated in Figure 3.

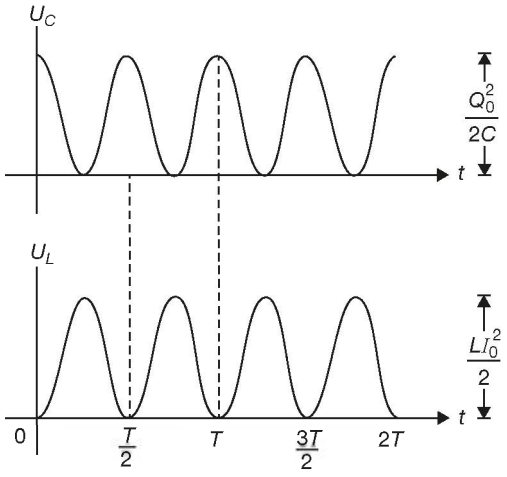


Figure 3 Plots of U_C versus t and U_L versus t for a resistanceless, non-radiating LC circuit. The sum of the two curves is a constant and equal to the total energy stored in the circuit.

Mechanical Analogue of LC Oscillations

The mechanical analogue of the LC oscillations is the mass-spring system, shown in Figure 4.

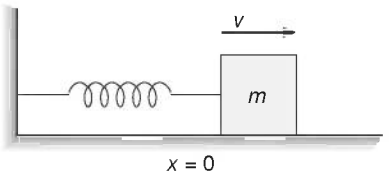


Figure 4 Mass spring oscillations

Consider a mass m moving with a speed v attached to a spring having a spring constant k. Let the mass m be displaced from its equilibrium position by x, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic, energy of the mass and the elastic potential energy of the spring, respectively. In the absence of friction, the total energy U is conserved and we get

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right)$$

$$\Rightarrow \quad \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right)$$

$$\Rightarrow \quad \frac{dU}{dt} = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = A\cos(\omega_0 t + \theta) \text{ where } \omega_0 = \sqrt{\frac{k}{m}}$$

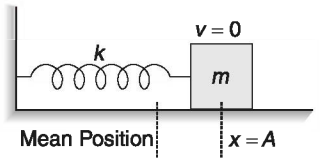
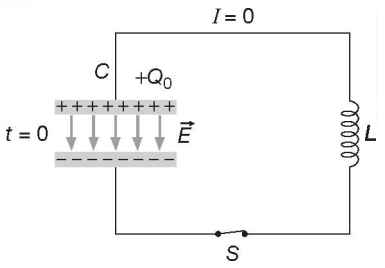
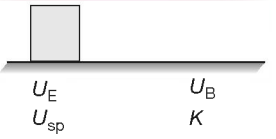
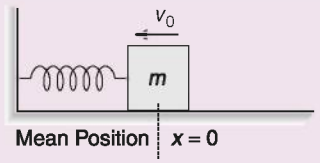
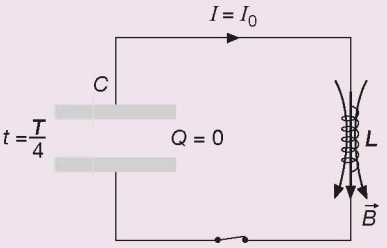
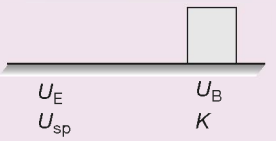
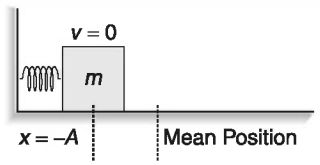
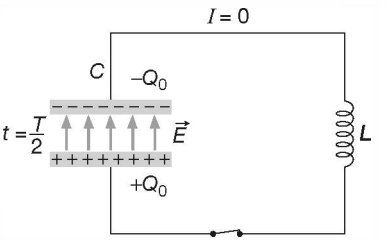
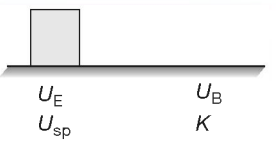
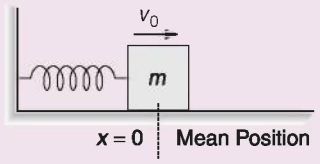
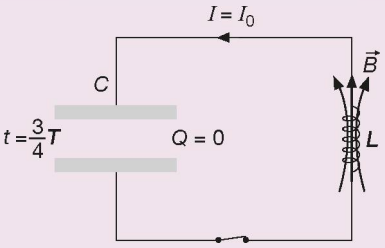
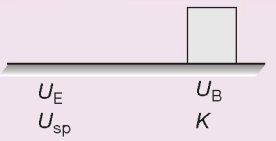
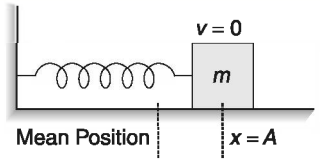
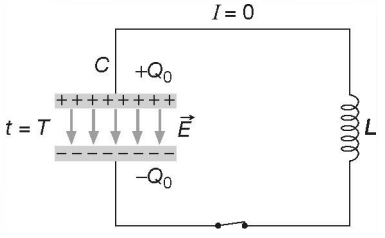
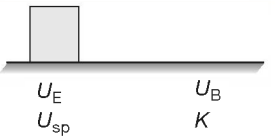
is the angular frequency and A is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system is

$$U = \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \theta) + \frac{1}{2}kA^2\cos^2(\omega_0 t + \theta)$$

$$\Rightarrow \quad U = \frac{1}{2}kA^2\left(\sin^2(\omega_0 t + \theta) + \cos^2(\omega_0 t + \theta)\right)$$

$$\Rightarrow \quad U = \frac{1}{2}kA^2 = \frac{1}{2}m\omega_0^2A^2 \qquad \left\{\because \omega_0 = \sqrt{\frac{k}{m}}\right\}$$

In this table, we have illustrated the energy oscillations in the LC Circuit and the mass-spring system (harmonic oscillator). Please note that U_E is the electrical energy stored in the capacitor, U_B is the magnetic energy stored in the inductor. Also, U_{sp} is the elastic potential energy of the spring and K is the kinetic energy of the mass attached to the spring.

Mass Spring System	LC Circuit	Energy
 Initially, spring stretched, has maximum elastic potential energy	 Initially capacitor full charged	
 Kinetic energy of the block is maximum, $U_{sp} = 0$	 Current grows to maximum in L	
 Kinetic energy of the block is zero, U_{sp} is maximum	 Capacitor becomes full charged, with polarity reversed	
 Kinetic energy of the block is maximum, $U_{sp} = 0$	 Current grows to maximum in L	
 Kinetic energy of the block is zero, U_{sp} is maximum	 Finally, capacitor returns to its original state	

Analogies between Electrical and Mechanical Systems.

One Dimensional Mechanical System	Electric Circuit	Analogy
Position	Charge	$Q \leftrightarrow x$
Velocity	Current	$I \leftrightarrow v_x$
Force	Potential difference	$\Delta V \leftrightarrow F_x$
Viscous damping coefficient	Resistance	$R \leftrightarrow b$
(k = spring constant)	Capacitance	$C \leftrightarrow \frac{1}{k}$
Mass	Inductance	$L \leftrightarrow m$
Velocity = time derivative of position	Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$
Acceleration = second time derivative of position	Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$
Kinetic energy of moving object	Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$
Potential energy stored in a spring	Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$
Rate of energy loss due to friction	Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$
Damped object on a spring	RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

ILLUSTRATION 83

A capacitor of capacitance 25 μF is charged to 300 V. It is then connected across a 10 mH inductor. The resistance in the circuit is negligible.

(a) Find the frequency of oscillation of the circuit.
(b) Find the potential difference across capacitor and magnitude of circuit current 1.2 ms after the inductor and capacitor are connected.
(c) Find the magnetic energy and electric energy at $t = 0$ and $t = 1.2$ ms.

SOLUTION

(a) The frequency of oscillation of the circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Substituting the given values, we get

$$f = \frac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}}$$

$$\Rightarrow \quad f = 318.3 \text{ Hz}$$

(b) Charge across the capacitor at time t is

$$q = q_0 \cos(\omega t)$$

$$\Rightarrow I = \frac{dq}{dt} = -q_0 \omega \sin(\omega t)$$

where $q_0 = CV_0 = (25\times10^{-6})(300) = 7.5\times10^{-3}$ C

Now, charge in the capacitor after $t = 1.2\times10^{-3}$ s is,

$$q = (7.5\times10^{-3})\cos((2\pi\times318.3)(1.2\times10^{-3}))\text{ C}$$

$$\Rightarrow q = -5.53\times10^{-3}\text{ C}$$

Potential Difference, across capacitor is,

$$V = \frac{|q|}{C} = \frac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2\text{ V}$$

The magnitude of current in the circuit at $t = 1.2\times10^{-3}$ s is,

$$|I| = q_0\omega\sin(\omega t)$$

$$\Rightarrow |I| = (7.5\times10^{-3})(2\pi)(318.3)\sin((2\pi\times318.3)(1.2\times10^{-3}))\text{ A}$$

$$\Rightarrow |I| = 10.13\text{ A}$$

(c) At $t = 0$ the current in the circuit is zero. Hence, $U_L = 0$. So, charge in the capacitor is maximum.

$$\Rightarrow U_C = \frac{1}{2}\frac{q_0^2}{C}$$

$$\Rightarrow U_C = \frac{1}{2}\times\frac{(7.5\times10^{-3})^2}{(25\times10^{-6})} = 1.125\text{ J}$$

$$\Rightarrow \text{Total energy } E = U_L + U_C = 1.125\text{ J}$$

At $t = 1.2$ ms, we have

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}(10\times10^{-3})(10.13)^2$$

$$\Rightarrow U_L = 0.513\text{ J}$$

$$\Rightarrow U_C = E - U_L = 1.125 - 0.513 = 0.612\text{ J}$$

Else, U_C can also be calculated as,

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\times\frac{(5.53\times10^{-3})^2}{(25\times10^{-6})} = 0.612\text{ J}$$

ILLUSTRATION 84

An inductor of inductance 2 mH is connected across a charged capacitor of capacitance 5 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is 200 μC.

(a) When $Q = 100\ \mu$C, what is the value of $\left|\frac{dI}{dt}\right|$?

(b) When $Q = 200\ \mu$C, what is the value of I?

(c) Find the maximum value of I

(d) When I is equal to one-half its maximum value, what is the value of $|Q|$?

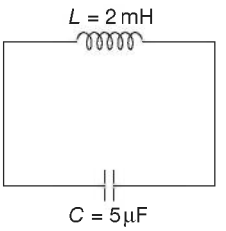
SOLUTION

This problem is dealing with LC oscillations. The charge stored in the capacitor oscillates simple harmonically as,

$$Q = Q_0\sin(\omega t \pm \phi)$$

where Q_0 is the maximum value of charge given by

$$Q_0 = 200\ \mu\text{C} = 2\times10^{-4}\text{ C}$$



$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3}\text{ H})(5\times10^{-6}\text{ F})}} = 10^4\text{ s}^{-1}$$

Let at $t = 0$, $Q = Q_0$ then,

$$Q(t) = Q_0\cos(\omega t) \quad \ldots(1)$$

$$\Rightarrow I(t) = \frac{dQ}{dt} = -Q_0\omega\sin(\omega t) \text{ and} \quad \ldots(2)$$

$$\Rightarrow \frac{dI(t)}{dt} = -Q_0\omega^2\cos(\omega t) \quad \ldots(3)$$

(a) $Q = 100\ \mu\text{C} = \frac{Q_0}{2}$

At $\cos(\omega t) = \frac{1}{2}$, from equation (3), we get

$$\left|\frac{dI}{dt}\right| = (2\times10^{-4}\ \text{C})(10^4\ \text{s}^{-1})^2\left(\frac{1}{2}\right)$$

$$\Rightarrow \left|\frac{dI}{dt}\right| = 10^4\ \text{As}^{-1}$$

(b) $Q = 200\ \mu\text{C} = Q_0$ when

$\cos(\omega t) = 1$, i.e., $\omega t = 0,\ 2\pi...$

At this time $I(t) = -Q_0\omega\sin(\omega t)$

$\Rightarrow I(t) = 0 \qquad \{\sin 0° = \sin 2\pi = 0\}$

(c) $I(t) = -Q_0\omega\sin(\omega t)$

The maximum value of I is $Q_0\omega$

$\Rightarrow I_{\max} = Q_0\omega = (2\times10^{-4}\ \text{C})(10^4\ \text{s}^{-1}) = 2\ \text{A}$

(d) From energy conservation,

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

$$\Rightarrow Q = \sqrt{LC\left(I_{\max}^2 - I^2\right)}$$

For, $I = \frac{I_{\max}}{2} = 1\ \text{A}$, we get

$$Q = \sqrt{(2\times10^{-3})(5\times10^{-6})(2^2 - 1^2)}$$

$$\Rightarrow Q = \sqrt{3}\times10^{-4}\ \text{C}$$

$$\Rightarrow Q = 1.732\times10^{-4}\ \text{C}$$

ILLUSTRATION 85

A 1.5 μF capacitor is charged to 20 V. The charging battery is then disconnected and a 15 mH coil is connected in series with the capacitance so that LC oscillations occur. Write the equation for variation of charge on capacitor and current in the inductor assuming that the inductor is connected to the capacitor at $t = 0$

SOLUTION

Given that $Q_{\max} = Q_0 = CV_0 = 30\ \mu\text{C}$

At $t = 0$, $Q = Q_0$

$$\Rightarrow \quad Q = Q_0\cos\omega t \text{ where } \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{1.5\times10^{-6}\times15\times10^{-3}}} = \frac{1}{15\times10^{-5}}\ \text{rads}^{-1}$$

Also, $I = -\frac{dQ}{dt} = Q_0\omega\sin\omega t = I_0\sin\omega t$

where, $I_0 = Q_0\omega = \frac{30\times10^{-6}}{15\times10^{-5}} = 0.2\ \text{A}$

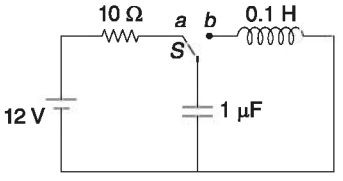
$$\Rightarrow \quad I = (0.2\ \text{A})\sin\omega t$$

Test Your Concepts-IX

Based on LC Oscillations

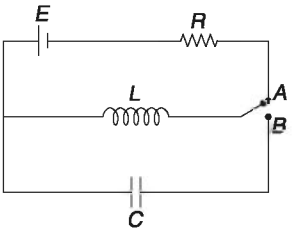
(Solutions on page H.153)

1. The switch in figure is connected to point *a* for a long time. After the switch is thrown to point *b*, what are
- **(a)** the frequency of oscillation of the *LC* circuit,
- **(b)** the maximum charge that appears on the capacitor,
- **(c)** the maximum current in the inductor, and
- **(d)** the total energy the circuit possesses at $t = 3$ s?

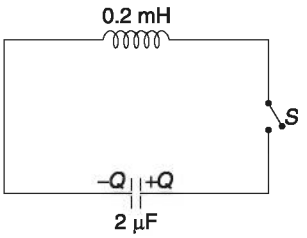


2. An *LC* circuit consists of a 20 mH inductor and a 0.5 μF capacitor. If the maximum instantaneous current is 0.1 A, what is the greatest potential difference across the capacitor?

3. In the arrangement shown, the switch is in position *A* for a long time. At time $t = 0$, it is switched to position *B*. Find the maximum charge which will accumulate on capacitor.



4. An LC circuit having a capacitor with initial charge of 200 μC is shown in Figure. If at $t = 0$, the switch is closed, calculate the first instant when energy stored in inductor becomes one third that of capacitor.



5. An *LC* circuit consists of an inductor with $L = 90$ mH and a capacitor of $C = 400$ μF. The initial charge on the capacitor is 5 μC, and the initial current in the inductor is zero.

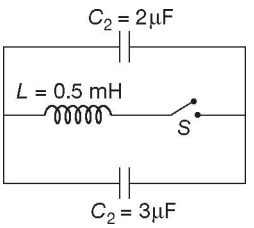
(a) What is the maximum voltage across the capacitor?

(b) What is the maximum current in the inductor?

(c) What is the maximum energy stored in the inductor?

(d) When the current in the inductor has half its maximum value, what is the charge on the capacitor and what is the energy stored in the inductor?

6. In an oscillating circuit shown in figure, the capacitors were charged to a voltage of 200 V and then the switch *S* was closed. Find

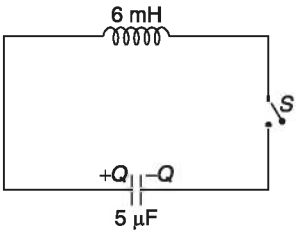


(a) the normal oscillation frequency.

(b) the peak value of the current flowing through the coil

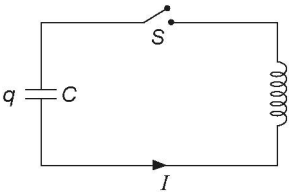
7. A capacitor in a series *LC* circuit has an initial charge *Q* and is being discharged. Find, in terms of *L* and *C*, the flux through each of the *N* turns in the coil, when the charge on the capacitor is $\frac{Q}{2}$.

8. In the circuit shown in figure charge on capacitor is $Q = 100$ μC. If switch is closed at $t = 0$ calculate the current in circuit when charge on capacitor reduces to 50 μC. Also find the maximum current in circuit.



9. Initially, the capacitor in a series *LC* circuit is charged. A switch is closed at $t = 0$, allowing the capacitor to discharge, and at time *t* the energy stored in the capacitor is one fourth of its initial value. Assuming *C* to be known, determine *L*.

10. In the *LC* circuit shown, $C = 1$ μF. With *C* charged to 100 V, switch *S* is suddenly closed at time $t = 0$. The circuit then oscillates at 10^3 cycles per second.



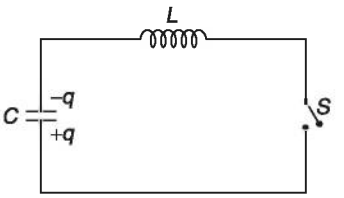
(a) Calculate ω and *T*

(b) Express *q* as a function of time

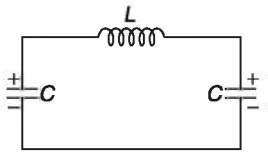
(c) Calculate *L*. Assume $\pi^2 = 10$

(d) Calculate the average current during the first quarter cycle.

11. In the circuit shown in Figure, if *S* is closed at $t = 0$, calculate the charge on capacitor as a function of time assuming that the inductor is active with initial current I_0 in it.



12. Two identical capacitors each having capacitance *C* are taken. The first capacitor is charged using a battery of potential difference *V* and the second capacitor is charged using the battery of potential difference 2*V*. Now the capacitors are connected to an inductor of inductance *L* as shown in Figure.

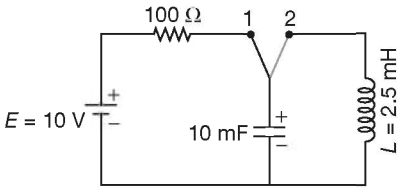


Calculate the time period of oscillation of the arrangement and the maximum current in the circuit

13. Initially the capacitor is charged to a potential of 5 V and then connected to position 1 with the shown polarity for 1 s. After 1 s it is connected across the inductor at position 2.

(a) Find the potential across the capacitor after 1 s of its connection to position 1.

(b) Find the maximum current flowing in the *LC* circuit when capacitor is connected across the inductor. Also find the frequency of *LC* oscillations.



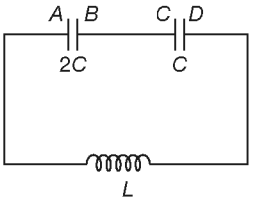
14. In an oscillating LC circuit in which $C = 4.00\ \mu F$, the maximum potential difference across the capacitor during the oscillations is 1.50 V and the maximum current through the inductor is 50.0 mA.

(a) What is the inductance *L*?

(b) What is the frequency of the oscillations?

(c) How much time does the charge on the capacitor take to rise from zero to its maximum value?

15. Two capacitors of capacitances 2*C* and *C* are connected in series through an inductor of inductance *L*. Initially capacitors have charge such that $V_B - V_A = 4V_0$, $V_C - V_D = V_0$ and the initial current in the circuit is zero. Calculate the



(a) maximum current that flows in the circuit.

(b) potential difference across each capacitor at that instant.

(c) equation of current flowing towards left in the inductor.

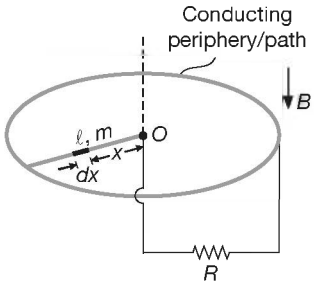
SOLVED PROBLEMS

PROBLEM 1

A conducting rod of length l, mass m is set in rotation with an angular velocity ω_0 such that it rotates on a rough horizontal plane of coefficient of friction μ. A uniform magnetic field of strength B is directed into the horizontal plane along the axis of rotation of the rod. The periphery of the conducting rod is in contact with a circular conducting path, having a resistance R. If the bar is free to rotate about the vertical axis passing through the centre of the circular conducting path, then find the variation of ω with t.

SOLUTION

Let us first visualise the arrangement discussed, in the form of a Figure.



Consider an infinitesimal element of the rod having length dx at a distance x from the centre O of the conducting path. The total force on this element must be the sum of the magnetic force and the frictional force. So,

$$dF = dF_m + df$$

where $df =$ frictional force $= \mu(dm)g = \mu\left(\frac{M}{L}dx\right)g$

and $dF_m =$ magnetic force $= i(dx)B$, where i is the induced current in the rod given by,

$$i = \frac{\xi}{R} = \frac{Bl^2\omega}{2R}$$

$$\Rightarrow \quad dF_m = \left(\frac{Bl^2\omega}{2R}\right)(dx)B$$

$$\Rightarrow \quad dF_m = \frac{B^2l^2\omega}{2R}dx$$

$$\Rightarrow \quad dF = dF_m + df$$

$$\Rightarrow \quad dF = \frac{B^2l^2\omega}{2R}dx + \frac{\mu Mg}{l}dx$$

$$\Rightarrow \quad dF = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)dx$$

The torque applied by the force dF about the axis of rotation is

$$d\tau = x\,dF$$

$$\Rightarrow \quad d\tau = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)x\,dx$$

So, the total torque is given by

$$\tau = \int d\tau = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)\int_0^t x\,dx$$

$$\Rightarrow \quad \tau = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)\frac{l^2}{2} \qquad \ldots(1)$$

If the angular retardation is α we have $\alpha = -\frac{d\omega}{dt}$.

If I be the moment of inertia of the rod, then equation (1), becomes

$$I\alpha = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)\frac{l^2}{2}$$

$$\Rightarrow \quad \frac{Ml^2}{3}\frac{d\omega}{dt} = \left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)\frac{l^2}{2} \qquad \left\{\because I = \frac{Ml^2}{3}\right\}$$

$$\Rightarrow \quad \int_{\omega_0}^{\omega}\frac{d\omega}{\left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)} = \frac{3}{2M}\int_0^t dt$$

$$\Rightarrow \quad \frac{2R}{B^2l^2}\log_e\left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)\Bigg|_{\omega_0}^{\omega} = -\frac{3}{2M}t$$

$$\Rightarrow \quad \log_e\left[\frac{\left(\frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l}\right)}{\left(\frac{B^2l^2\omega_0}{2R} + \frac{\mu Mg}{l}\right)}\right] = -3\left(\frac{B^2l^2}{4MR}\right)t$$

$$\Rightarrow \quad \frac{B^2l^2\omega}{2R} + \frac{\mu Mg}{l} = \left(\frac{B^2l^2\omega_0}{2R} + \frac{\mu Mg}{l}\right)e^{-3\left(\frac{B^2l^2}{4MR}\right)t}$$

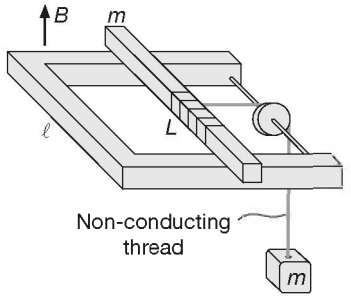
$$\Rightarrow \quad \omega = \frac{2R}{B^2l^2}\left[\left(\frac{B^2l^2\omega_0}{2R}\right)e^{-3\left(\frac{B^2l^2}{4MR}\right)t} - \frac{\mu Mg}{l}\left(1 - e^{-3\left(\frac{B^2l^2}{4MR}\right)t}\right)\right] \qquad \ldots(2)$$

CHECK POINT

At $t = 0$, we must get $\omega = \omega_0$, which is true when we put $t = 0$ in equation (2).

PROBLEM 2

A pair of parallel horizontal conducting rails of negligible resistance is shorted at one end of it and is fixed on the table as shown in Figure.



A constant magnetic field B exists perpendicular to the table. The distance between the rails is l. An inductor of negligible resistance and mass m can slide on the rails smoothly. The inductor is connected to the mass m by a light inextensible string passing over a light frictionless pulley as shown. Calculate velocity of the rod as a function of displacement form initial position. Also find the x_0 when terminal velocity is attained.

SOLUTION

For mass m and the rod

$$mg - T = ma$$

$$\frac{T - F_m = ma}{mg - F_m = 2ma} \quad \ldots(1)$$

Let at any instant inductor is moving with velocity v, then

$$E - L\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \frac{Blvdt}{L} = dI$$

$$\int_0^x \frac{Bl}{L}dx = \int_0^I dI$$

$$\Rightarrow \quad \frac{Bl}{L}x = I$$

Now $F_m = BIl = B^2l^2\frac{x}{L}$

Substituting in (1)

$$mg - \frac{B^2l^2x}{L} = 2ma = 2mv\frac{dv}{dx}$$

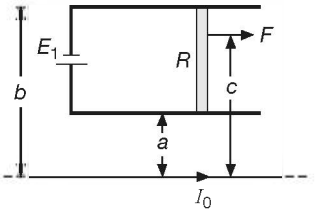
$$\Rightarrow \quad \int_0^x \left(mg - \frac{B^2l^2x}{L}\right)dx = 2m\int_0^v vdv$$

$$\Rightarrow \quad mgx - \frac{B^2\ell^2x^2}{2L} = 2m\frac{v^2}{2}$$

$$\Rightarrow \quad v = \sqrt{gx - \frac{B^2l^2x^2}{2mL}}$$

PROBLEM 3

A metallic rod of mass m and resistance R is sliding over two conducting frictionless rails as shown in Figure.



An infinitely long wire carries a current I_0. The distance of the rails from the wire are b and a respectively.

(a) Find the value of current in the circuit, if the rod slides with constant velocity v_0.

(b) Find the value of F, if the rod slides with constant velocity v_0.

(c) Find the value of c.

SOLUTION

(a) Magnetic flux linked with the infinitesimal area element shown in Figure is

$$d\phi = \frac{\mu_0 I_0}{2\pi y}(x\,dy)$$

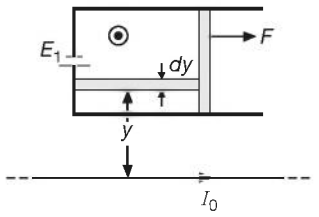
So, net flux is

$$\phi = \oint d\phi = \int_a^b \frac{\mu_0 I}{2\pi y}x\,dy = \frac{\mu_0 I_0}{2\pi}\log_e\left|\frac{b}{a}\right|$$

$$\xi = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 I_0}{2\pi}v_0\log_e\left|\frac{b}{a}\right| \qquad \left\{\because \frac{dx}{dt} = v_0\right\}$$

Induced emf developed due to motion of the conductor has an opposite sense to the emf of the cell. So, we get

$$I=\frac{E-\xi}{R}=\frac{1}{R}\left\{E-\frac{\mu_0 I_0 v_0}{2\pi}\log_e\left|\frac{b}{a}\right|\right\}$$



(b) $F=\left|\int d\vec{F}\right|=\left|\int Id\vec{l}\times\vec{B}\right|$

$$\Rightarrow F=\frac{1}{R}\int_a^b\left(E-\frac{\mu_0 I_0 v_0}{2\pi}\log_e\left|\frac{b}{a}\right|\right)(dy)\left(\frac{\mu_0 I_0}{2\pi y}\right)$$

$$\Rightarrow F=\frac{\mu_0 I_0}{2\pi R}\left(E-\frac{\mu_0 I_0 v_0}{2\pi}\log_e\left|\frac{b}{a}\right|\right)\log_e\left(\frac{b}{a}\right)$$

(c) $\tau=\left|\int d\vec{\tau}\right|=\int\vec{r}\times d\vec{F}$

$$\Rightarrow \tau=\int_a^b y\left(\frac{\mu_0 I_0}{2\pi R}\right)\left(E-\frac{\mu_0 I_0 v_0}{2\pi}\log_e\left(\frac{b}{a}\right)\right)\left(\frac{dy}{y}\right)$$

$$\Rightarrow \tau=\frac{\mu_0 I_0}{2\pi R}\left(E-\frac{\mu_0 v_0 I_0}{2\pi}\log_e\left|\frac{b}{a}\right|\right)(b-a)$$

Now $\vec{\tau}=\vec{F}_m\times\vec{c}$

$$\Rightarrow \tau=F_m c$$

$$\Rightarrow \frac{\mu_0 I_0}{2\pi R}\left[E-\frac{\mu_0 v_0 I_0}{2\pi}\log_e\left(\frac{b}{a}\right)\right](b-a)$$

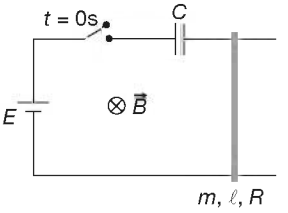
$$=\frac{\mu_0 I_0}{2\pi R}c\left[E-\frac{\mu_0 v_0 I_0}{2\pi}\log_e\left(\frac{b}{a}\right)\right]\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow c=\frac{b-a}{\log_e\left(\frac{b}{a}\right)}=\frac{b-a}{\log_e b-\log_e a}$$

PROBLEM 4

In the arrangement shown, the connector of mass m, length l and resistance R can smoothly slide along the two fixed parallel rails and the switch is closed at time $t=0$. The capacitor is initially uncharged and a uniform magnetic field B exists in the region directed into the plane of the paper. Find the

(a) charge on the capacitor after a long time.
(b) speed of the connector after a long time.



SOLUTION

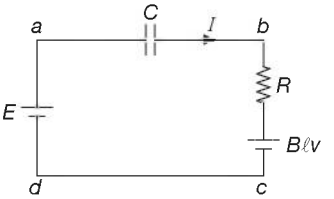
(a) Applying KVL to the loop $abcda$, we get

$$-\frac{q}{C}-IR-Blv+E=0 \qquad \ldots(1)$$

$$\Rightarrow \frac{q}{C}+R\frac{dq}{dt}=E-Blv$$

$$\Rightarrow \frac{dq}{dt}+\frac{q}{RC}=\frac{E}{R}-\frac{B^2l^2}{mR}q$$

$$\Rightarrow \int_0^q\frac{dq}{\frac{E}{R}-q\left(\frac{B^2\ell^2}{mR}+\frac{1}{RC}\right)}=\int_0^t dt$$



$$-\frac{1}{\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)}\log_e\left[\frac{\frac{E}{R}-q\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)}{\frac{E}{R}}\right]=t$$

$$\frac{E}{R}-q\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)=\frac{E}{R}e^{-\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)t}$$

$$\Rightarrow q=\frac{E}{R\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)}\left(1-e^{-\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)t}\right)$$

$$\Rightarrow q=\frac{CE}{\left(1+\frac{B^2l^2C}{m}\right)}\left(1-e^{-\left(\frac{B^2l^2}{mR}+\frac{1}{RC}\right)t}\right) \qquad \ldots(2)$$

When $t \to \infty$, we get $q = Q_0$

$$\Rightarrow \quad Q_0 = q_{t\to\infty} = \frac{CE}{\left(1+\frac{B^2l^2C}{m}\right)}$$

(b) Let at any moment the speed of the connector be v, current in the circuit is $I = \frac{dq}{dt}$ and charge on the capacitor is q. Then

$$F = m\frac{dv}{dt} = BIl$$

$$\Rightarrow \quad F = m\frac{dv}{dt} = B\left(\frac{dq}{dt}\right)l$$

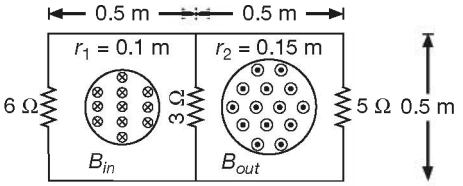
Integrating we get $v = \left(\frac{Bl}{m}\right)q$

From Equation (1), we get

$$v_{t\to\infty} = \frac{BlCE}{(m+CB^2l^2)}$$

PROBLEM 5

Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Figure.



The magnitude of $\vec{B}$ inside each is the same and is increasing at the rate of 100 Ts^{-1}. What is the current in each resistor?

SOLUTION

For the left loop, the emf induced is

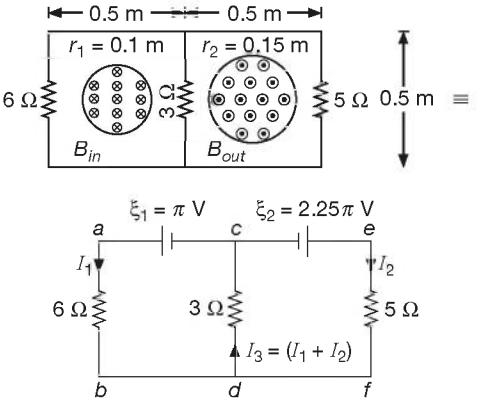
$$\xi_1 = \frac{d\phi_B}{dt} = A_1\left(\frac{dB}{dt}\right) = (\pi r_1^2)\frac{dB}{dt}$$

$$\Rightarrow \quad \xi_1 = \pi(0.1)^2(100) = \pi \text{ V}$$

Since the field is increasing inwards, so the induced current must set up a field which will not allow this increase. Hence induced current in the left loop is I_1 counter clockwise (to establish an outward field)
Similarly, for the right loop, we have

$$\xi_2 = \frac{d\phi_B}{dt} = A_2\left(\frac{dB}{dt}\right) = \pi(0.15)^2(100) = 2.25\pi \text{ V}$$

This induced emf produces an induced current, say I_2, clockwise to oppose the change in B here.



For loop $abdca$, we have

$$-6I_1 - 3(I_1 + I_2) + \pi = 0$$

$$\Rightarrow \quad 9I_1 + 3I_2 = \pi \qquad \ldots(1)$$

For loop $cdfec$, we have

$$3(I_1 + I_2) + 5I_2 - 2.25\pi = 0$$

$$3I_1 + 8I_2 = 2.25\pi \qquad \ldots(2)$$

Solving (1) and (2), we get

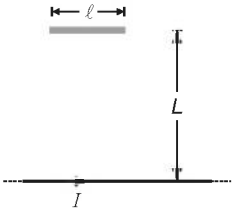
$$I_1 = 62 \text{ mA}$$

$$I_2 = 860 \text{ mA}$$

$$I_3 = 922 \text{ mA}$$

PROBLEM 6

A rod of length l is held parallel to and a distance L above a long wire carrying I and resting on the floor (shown in Figure).



The rod is released and it remains parallel to the current carrying wire as it falls. Assume that the rod falls under gravity g, derive an equation for the emf induced in it. Express your result as a function of the time t after the wire is dropped. What is the induced emf at half the time that the rod takes to hit the wire?

Find the instant and position when the induced emf is maximum. Also find the maximum induced emf. Neglect the interaction of induced current in the rod and current in the wire.

SOLUTION

Since, $\xi = Blv$

where $B = \frac{\mu_0 I}{2\pi y}$, $v = gt$ and $y = L - \frac{1}{2}gt^2$

$$\Rightarrow \quad \xi = \frac{\mu_0 Il}{2\pi}\left(\frac{gt}{L - \frac{1}{2}gt^2}\right)$$

The rod hits the wire in time T (say), then

$$L = \frac{1}{2}gT^2$$

$$\Rightarrow \quad T = \sqrt{\frac{2L}{g}}$$

at $t = \frac{T}{2} = \frac{1}{2}\sqrt{\frac{2L}{g}} = \sqrt{\frac{L}{2g}}$

$$\Rightarrow \quad \xi = \frac{\mu_0 Il}{2\pi}\left[\frac{g\sqrt{\frac{L}{2g}}}{L - \frac{1}{2}g\left(\frac{L}{2g}\right)}\right] = \frac{\mu_0 Il}{2\pi}\left[\frac{\sqrt{\frac{gL}{2}}}{\left(L - \frac{L}{4}\right)}\right]$$

$$\xi = \frac{\mu_0 Il}{2\pi}\frac{4}{3}\sqrt{\frac{g}{2L}} = \frac{2\mu_0 Il}{3\pi}\sqrt{\frac{g}{2L}}$$

ξ will be MAXIMUM, when

$$\frac{d\xi}{dt} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{t}{L - \frac{1}{2}gt^2}\right) = 0 \qquad \left\{\because \xi = \frac{\mu Ilg}{2\pi}\left(\frac{t}{L - \frac{1}{2}gt^2}\right)\right\}$$

$$\Rightarrow \quad \frac{\left(L - \frac{1}{2}gt^2\right) - t(gt)}{\left(L - \frac{1}{2}gt^2\right)^2} = 0$$

$$\Rightarrow \quad L - \frac{1}{2}gt^2 - gt^2 = 0$$

$$\Rightarrow \quad L = \frac{3}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2L}{3g}} = \frac{T}{\sqrt{3}}$$

$$\Rightarrow \quad y = L - \frac{1}{2}g\left(\frac{2L}{3g}\right) = \frac{2L}{3}, \text{ from the wire}$$

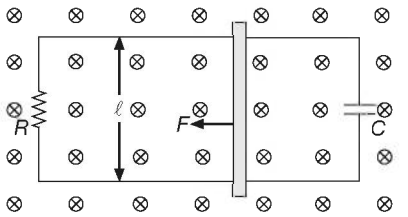
$$\xi_{MAX} = \frac{\mu_0 Il}{2\pi}\left(\frac{g\sqrt{\frac{2L}{3g}}}{L - \frac{1}{2}g\left(\frac{2L}{3g}\right)}\right)$$

$$\Rightarrow \quad \xi_{MAX} = \frac{\mu_0 Il}{2\pi}\left(\frac{\sqrt{\frac{2Lg}{3}}}{\frac{2L}{3}}\right)$$

$$\Rightarrow \quad \xi_{MAX} = \frac{\mu_0 Il}{2\pi}\sqrt{\frac{3g}{2L}}$$

PROBLEM 7

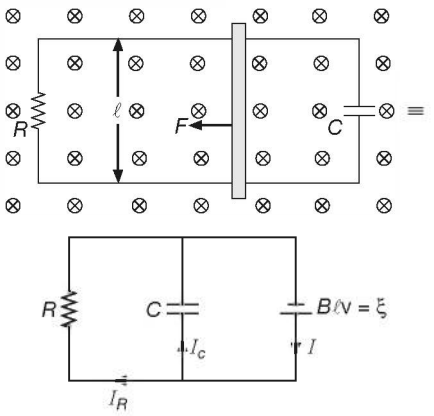
Two parallel long smooth conducting rails separated by a distance l are connected by a movable conducting connector of mass m. Terminals of the rails are connected by the resistor R and the capacitor C as shown in Figure.



A uniform magnetic field B perpendicular to the plane of the rails is switched on. The connector is dragged by a constant force F. Find the speed of the connector as a function of time if the force F is applied at $t = 0$. Also find the terminal velocity of the connector.

SOLUTION

Due to the external force F the connector begins to accelerate such that an emf is induced in the connector due to which an induced current flow through it. If the induced current is I (downwards), then due to this current a magnetic force BIl (acting opposite to F) acts on the connector. Let at some instant t, the connector has a velocity v. Then the current passing through the resistor is



$$I_R = \frac{\xi}{R} = \frac{Blv}{R} \qquad \ldots(1)$$

Since the source of emf (which is the moving connector) is connected in parallel to both R and C, hence the potential across both is Blv. So, at that instant if Q be the charge on the capacitor, then

$$Q = C\xi = C(Blv)$$

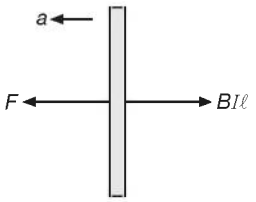
$$\Rightarrow \quad I_C = \frac{dQ}{dt} = BlC\left(\frac{dv}{dt}\right) \qquad \ldots(2)$$

Total induced current is then given by

$$I = I_R + I_C$$

$$\Rightarrow \quad I = \frac{Blv}{R} + BlC\left(\frac{dv}{dt}\right) \qquad \ldots(3)$$

Let us now draw a free body diagram of the connector.



Then, from Newton's Second Law, we get

$$F - BIl = ma$$

$$\Rightarrow \quad F - B\left(\frac{Blv}{R} + BlC\frac{dv}{dt}\right)l = m\left(\frac{dv}{dt}\right)$$

$$\Rightarrow \quad F - \frac{B^2l^2v}{R} = \left(B^2l^2C + m\right)\frac{dv}{dt} \qquad \ldots(4)$$

$$\Rightarrow \quad \frac{dv}{F - \left(\frac{B^2l^2}{R}\right)v} = \frac{dt}{\left(B^2l^2C + m\right)}$$

$$\Rightarrow \quad \int_0^v \frac{dv}{F - \left(\frac{B^2l^2}{R}\right)v} = \frac{1}{\left(B^2l^2C + m\right)}\int_0^t dt$$

$$\Rightarrow \quad -\frac{R}{B^2l^2}\log_e\left(F - \frac{B^2l^2v}{R}\right)\Bigg|_0^v = \frac{t}{\left(B^2l^2C + m\right)}$$

$$\Rightarrow \quad \log_e\left(\frac{F - \frac{B^2l^2v}{R}}{F}\right) = -\left[\frac{B^2l^2}{R\left(B^2l^2C + m\right)}\right]t$$

$$\Rightarrow \quad F - \frac{B^2l^2v}{R} = Fe^{-\left(\frac{B^2l^2}{R\left(B^2l^2C+m\right)}\right)t}$$

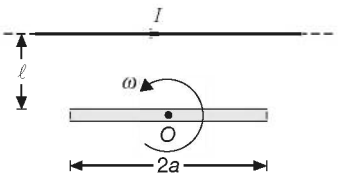
$$\Rightarrow \quad v = \frac{FR}{B^2l^2}\left(1 - e^{-\left(\frac{B^2l^2}{R\left(B^2l^2C+m\right)}\right)t}\right)$$

If v_T is the terminal velocity, then at $v = v_T$, $\frac{dv}{dt} = 0$. Then from (4), we get

$$v_T = \frac{FR}{B^2l^2}$$

PROBLEM 8

A rod of length $2a$ is free to rotate in a vertical plane, about a horizontal axis-O passing through its midpoint. A long straight, horizontal wire is in the same plane and is carrying a constant current I as shown in Figure.

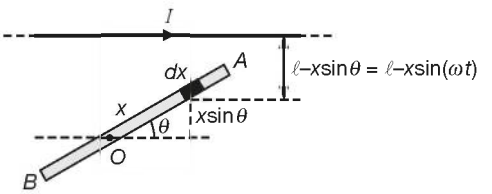


At initial moment of time, the rod is horizontal and starts to rotate with constant angular velocity ω, calculate emf induced in the rod as a function of time.

SOLUTION

At any instant of time, angular displacement of the rod is

$$\theta = \omega t$$



Consider an element of length dx at a distance x from O. If $d\xi$ be the emf induced due to motion of this element, then

$$d\xi = B(dx)v$$

$$\Rightarrow \quad d\xi = B(x\omega)dx$$

where, $B = \frac{\mu_0}{2\pi}\left(\frac{I}{l - x\sin(\omega t)}\right)$

$$\Rightarrow \quad d\xi = \frac{\mu_0 I\omega}{2\pi}\left(\frac{x}{l - x\sin(\omega t)}\right)dx$$

$$V_{OA} = V_0 - V_A = \int_0^a d\xi = \frac{\mu_0 I\omega}{2\pi}\int_0^a \frac{x}{l - x\sin(\omega t)}dx$$

$$V_{OA} = -\frac{\mu_0 I\omega}{2\pi\sin(\omega t)}\left[l\frac{\log_e\left(\frac{l - a\sin(\omega t)}{l}\right)}{\sin(\omega t)} + a\right]$$

Similarly, $V_{OB} = V_O - V_B = \frac{\mu_0 I\omega}{2\pi}\int_0^a \frac{x}{l + x\sin(\omega t)}dx$

$$\Rightarrow \quad V_{OB} = \frac{\mu_0 I\omega}{2\pi\sin(\omega t)}\left[a - l\frac{\log_e\left(\frac{l + a\sin(\omega t)}{l}\right)}{\sin(\omega t)}\right]$$

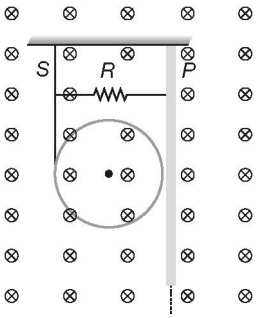
So, $V_{AB} = V_{OB} - V_{OA}$

$$\Rightarrow \quad V_{AB} = \frac{\mu_0 I\omega}{2\pi\sin(\omega t)}\left[2a + l\frac{\log_e\left(\frac{l - a\sin(\omega t)}{l + a\sin(\omega t)}\right)}{\sin(\omega t)}\right]$$

Please note that the function V_{AB} is discontinuous at $\omega t = n\pi$

PROBLEM 9

A conducting light string S is wound on the rim of a metal ring of radius r and mass m. The free end of the string is fixed to the ceiling. A vertical infinite smooth conducting plane P is always tangent to the ring as shown in Figure.



A uniform magnetic field B is applied perpendicular to the plane of the ring. The ring is always inside the magnetic field. The plane and the string are connected by a resistance R. When the ring is released, calculate the

(a) current in the resistance R as a function of time.
(b) terminal velocity of the ring.

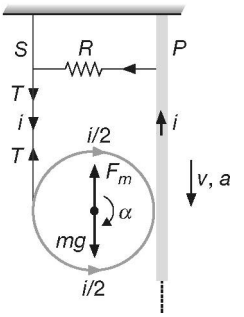
SOLUTION

Let at time t velocity of ring be v (downwards). Then

$$\xi = Bl_{\text{eff}}v = Bv(2r) = 2Bvr$$

The above result can also be obtained, if we observe the two batteries of emf $Bv(2r)$, connected in parallel. So, the induced current i is given by

$$i = \frac{\xi}{R} = \frac{2Bvr}{R}$$



Now, $mg - F_m - T = ma$, where

$$F_m = 2\left[\left(\frac{i}{2}\right)(2r)B\right] = 2irB = \frac{4B^2r^2v}{R}$$

$$\Rightarrow \quad a = g - \frac{4B^2r^2v}{mR} - \frac{T}{m} \qquad \ldots(1)$$

Also, if I be the moment of inertia of the ring, then

$$\alpha = \frac{\tau}{I} = \frac{Tr}{mr^2} = \frac{T}{mr} \qquad \ldots(2)$$

$$\Rightarrow \quad a = r\alpha = \frac{T}{m} \qquad \ldots(3)$$

From equations (1), (2) and (3), we get

$$a = \frac{g}{2} - \frac{2B^2r^2v}{mR} \qquad \ldots(4)$$

$$\Rightarrow \quad \int_0^v \frac{dv}{\frac{g}{2} - \frac{2B^2r^2v}{mR}} = \int_0^t dt$$

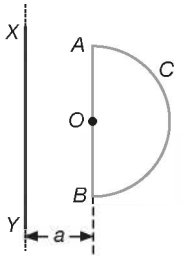
$$\Rightarrow \quad v = \frac{mgR}{4B^2r^2}\left(1 - e^{-\frac{2B^2r^2}{mR}t}\right)$$

At, $v = v_T$, we have $a = 0$. So, from equation (4), we get

$$v_T = \frac{mgR}{4B^2r^2}$$

PROBLEM 10

A wire $ACBOA$ shaped as a semi-circle ACB with the bounding diameter BOA $(OA = OB = a)$, has a total resistance R. It is placed in the same plane as infinitely long wire XY as shown in Figure.



A total charge Q flows through XY such that the current decreases linearly from an initial value I_0 to zero in time T, find the

(a) current in XY as a function of time.

(b) magnetic flux through $ACBOA$ when current in XY is I.

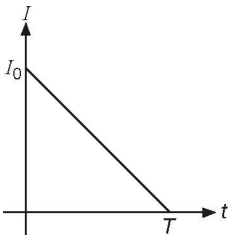
(c) induced current in $ACBOA$ as a function of time.

(d) heat generated in $ACBOA$ in time T.

(e) force acting on AOB as a function of time and its direction.

SOLUTION

(a) Variation of I with time is shown in the graph.



$$\begin{pmatrix}\text{Total}\\ \text{charge}\end{pmatrix} = \begin{pmatrix}\text{Area of the}\\ \text{triangle}\end{pmatrix} = \left(\frac{I_0 T}{2}\right) = (Q)$$

$$\Rightarrow \quad I_0 = \frac{2Q}{T}$$

Since, $I = I_0 - \frac{I_0}{T}t$

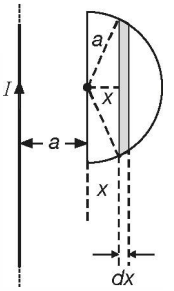
$$\Rightarrow \quad I = \frac{2Q}{T}\left(1 - \frac{t}{T}\right) \qquad \ldots(1)$$

(b) $\phi = \int \vec{B} \cdot d\vec{A}$, where

$$dA = 2\sqrt{a^2 - x^2}\,dx$$

$$\Rightarrow \quad \phi = \int_0^a \frac{\mu_0 I}{2\pi(a+x)}\left(2\sqrt{a^2 - x^2}\,dx\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Ia}{2\pi}(\pi - 2)$$



(c) Induced emf is given by

$$\xi = -\frac{d\phi}{dt} = -\frac{\mu_0 a(\pi - 2)}{2\pi}\frac{dI}{dt}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 a(\pi - 2)}{2\pi}\left(\frac{2Q}{T^2}\right) \quad \left\{\because \frac{dI}{dt} = \frac{2Q}{T^2}, \text{ from (1)}\right\}$$

So, the induced current is $I_1 = \frac{\mu_0 a(\pi - 2)}{\pi R}\frac{Q}{T^2}$

(d) $H = \int_0^T I_1^2 R\,dt$

$$\Rightarrow \quad H = I_1^2 RT$$

$$\Rightarrow \quad H = \frac{a^2\mu_0^2(\pi - 2)^2 Q^2}{\pi^2 RT^3}$$

(e) Force on the straight part is given by $F = BI_1(2a)$

$$\Rightarrow \quad F = \frac{\mu_0}{2\pi a} II_1(2a), \text{ where } I = \frac{2Q}{T}\left(1 - \frac{t}{T}\right)$$

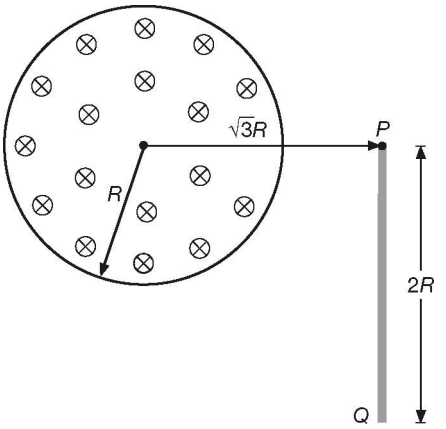
$$\Rightarrow \quad F = \frac{\mu_0}{2\pi a}\frac{2Q}{T}\left(1 - \frac{t}{T}\right)\frac{\mu_0 a(\pi - 2)Q}{\pi RT^2}2a$$

$$\Rightarrow \quad F = \frac{\mu_0^2 Q^2(\pi - 2)}{\pi^2 RT^3}\left(1 - \frac{t}{T}\right)2a$$

The force is directed towards the wire as the currents are parallel.

PROBLEM 11

A uniform but variable magnetic field of induction $B = \alpha + \beta t$ fills a cylindrical region of radius R. A conducting rod PQ of length $2R$ is pivoted at P situated at a horizontal distance $\sqrt{3}R$ from the centre of the magnetic field as shown in Figure.



At $t = 0$, the rod starts rotating clockwise with constant angular velocity ω. Find the emf induced in the rod at the instant when end Q of the rod enters the magnetic field.

SOLUTION

Before we start the problem, we must note that the emf is induced in the rod due to the

(a) motion of the rod in magnetic field (ξ_1) and
(b) changing magnetic field (ξ_2)

EMF produced due to motion of the rod in field

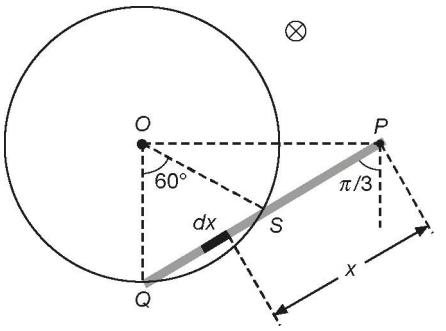
Just when the end Q enters the field, from simple geometry, we observe that ΔOSQ is equilateral. So,

$$QS = PS = R$$

$$\Rightarrow \quad \xi_1 = \int_R^{2R} B\omega x\, dx \qquad \{Q \text{ at higher potential}\}$$

$$\xi_1 = B\omega\left(\frac{3R^2}{2}\right) \qquad \ldots(1)$$

where, $B = \alpha + \beta\left(\frac{\pi}{3\omega}\right)$ $\qquad \ldots(2)$



EMF produced due to the changing magnetic field

Before calculating the induced emf, we must understand that a portion of rod is inside the field and a portion is outside it. At the same time we must not forget that the induced field has variable value from end Q of the rod till the end B.

Magnitude of electric field at the position of the small element in the magnetic field, E is given by

$$E(2\pi r) = \frac{dB}{dt}(\pi r^2)$$

$$\Rightarrow \quad E = \frac{r}{2}\frac{dB}{dt} = \frac{\beta r}{2}$$

If the element lies outside the magnetic field, then we have

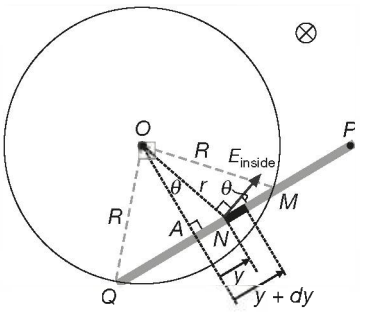
$$E'(2\pi r) = \frac{dB}{dt}(\pi R^2)$$

$$\Rightarrow \quad E' = \frac{\beta R^2}{2r}$$

Induced emf due to the inner portion in changing magnetic field

So, emf induced in the portion of wire inside the field is

$$\xi_{\text{inside}} = \int |d\xi_{\text{inside}}| = \left|\int_Q^M \vec{E}_{\text{inside}} \cdot d\vec{y}\right|$$



At the element, a distance r from O, E is $\perp$ to r and has a value $E_{\text{inside}} = \frac{r\beta}{2}$

$$\Rightarrow \quad \xi_{\text{inside}} = \left|\int_{-\frac{R}{2}}^{\frac{R}{2}} \frac{r\beta}{2} dy \cos\theta\right|$$

In triangle OAN $r\cos\theta = OA = \dfrac{\sqrt{3}R}{2}$

$$\Rightarrow \quad \xi_{\text{inside}} = \frac{\sqrt{3}\beta R}{4}\left|\int_{\frac{R}{2}}^{\frac{R}{2}} dy\right| = \frac{\sqrt{3}\beta R^2}{4} \qquad \ldots(3)$$

Induced emf due to outer portion in the changing magnetic field

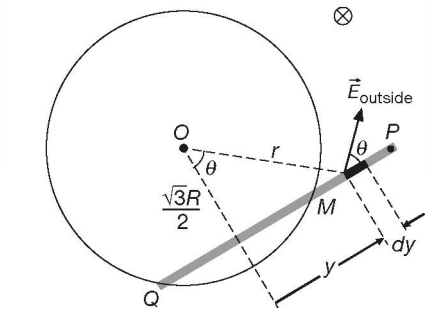
Again, emf induced in the portion of wire outside the field is

$$\xi_{\text{outside}} = \int |d\xi_{\text{outside}}| = \left|\int_M^P \vec{E}_{\text{outside}} \cdot d\vec{y}\right|$$

$$\Rightarrow \quad \xi_{\text{outside}} = \left|\int_M^P \frac{\beta R^2}{2r} dy \cos\theta\right|$$

$$\Rightarrow \quad \xi_{\text{outside}} = \left|\int_M^P \frac{\beta R^2}{2r}\left(\frac{\sqrt{3}R}{2r}\right)dy\right| \quad \left\{\because \cos\theta = \frac{\sqrt{3}R}{2r}\right\}$$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{\sqrt{3}bR^3}{4}\left|\int_{\frac{R}{2}}^{\frac{3R}{2}} \frac{dy}{r^2}\right|$$



But $r^2 = \dfrac{3R^2}{4} + y^2$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{\sqrt{3}bR^3}{4}\left|\int_{\frac{R}{2}}^{\frac{3R}{2}} \frac{dy}{\left(y^2 + \frac{3R^2}{4}\right)}\right|$$

Since $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{\sqrt{3}bR^3}{4\left(\frac{\sqrt{3}R}{2}\right)}\left[\tan^{-1}\left(\frac{y}{\frac{\sqrt{3}R}{2}}\right)\right]_{\frac{R}{2}}^{\frac{3R}{2}}$$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{bR^2}{2}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{bR^2}{2}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$\Rightarrow \quad \xi_{\text{outside}} = \frac{\pi bR^2}{12}$$

Hence total emf developed is

$$\xi = \xi_{\text{motional emf}} + \xi_{\text{changing } B}$$

$$\Rightarrow \quad \xi = \xi_{\text{motional emf}} = (\xi_{\text{inside portion}} + \xi_{\text{outside portion}})_{\text{changing } B}$$

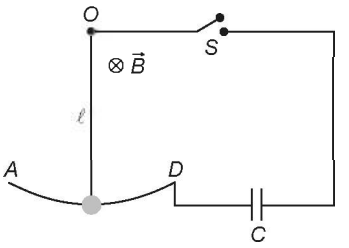
$$\Rightarrow \quad \xi = \xi_1 + (\xi_{\text{inside}} + \xi_{\text{outside}})$$

$$\Rightarrow \quad \xi = \frac{3B\omega R^2}{2} + \frac{\sqrt{3}\beta R^2}{4} + \frac{\pi\beta R^2}{12}$$

$$\Rightarrow \quad \xi = \frac{3B\omega R^2}{2} + \frac{\beta R^2}{4}\left(\sqrt{3} + \frac{\pi}{3}\right) \text{ where } B = \alpha + \frac{\pi\beta}{3\omega}$$

PROBLEM 12

A simple pendulum consists of a small conducting ball of mass m and a light conducting rod of length l. The pendulum oscillates with angular amplitude θ_0 in a vertical plane about a horizontal axis passing through O such that the ball remains always just in contact with a metallic strips AD bent into a circular arc of radius l as shown in Figure. A uniform magnetic field of induction B normal to plane of oscillation exists in the space. At time $t = 0$ when the ball is at its lowest position and moving towards right, the switch S is closed. Neglecting self-inductance of the circuit calculate external torque τ required to keep the pendulum oscillating as before. Assume that θ_0 is small.

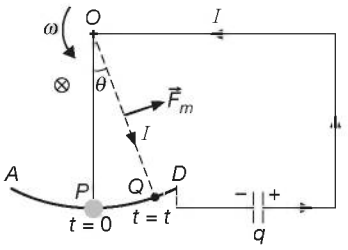


SOLUTION

At time t, the angular position of the rod is given by

$$\theta = \theta_0 \sin\left(\frac{2\pi}{T}\right)t \qquad \ldots(1)$$

where $T = 2\pi\sqrt{\frac{l}{g}}$



Since $\omega = \frac{d\theta}{dt}$, so differentiating equation (1) w.r.t., time, we get

$$\omega = \frac{d\theta}{dt} = \left(\frac{2\pi\theta_0}{T}\right)\cos\left(\frac{2\pi t}{T}\right)$$

The potential difference between the ends of the rod at this instant is,

$$V_O - V_Q = V = \frac{1}{2}B\omega l^2$$

$$\Rightarrow \quad V_O - V_Q = \left(\frac{\pi\theta_0 l^2 B}{T}\right)\cos\left(\frac{2\pi t}{T}\right)$$

From right hand rule we can see that $V_O > V_Q$. This is also the potential difference across the capacitor. If q is the charge stored in the capacitor at this instant, then

$$q = CV = \left(\frac{\pi\theta_0 l^2 CB}{T}\right)\cos\left(\frac{2\pi t}{T}\right)$$

As the rod move towards D, its ω decreases (due to Lenz's Law). Hence, q will decrease. Thus, current in the circuit is anticlockwise, which is given by

$$I = -\frac{dq}{dt} = \left(\frac{2B\pi^2\theta_0 l^2 C}{T^2}\right)\sin\left(\frac{2\pi t}{T}\right)$$

In the rod it points from O to Q.
Magnetic force on the rod due to this induced current I is,

$$F_m = BIl = \left(\frac{2\pi^2\theta_0 B^2 l^3 C}{T^2}\right)\sin\left(\frac{2\pi t}{T}\right)$$

This force acts on the rod in the direction shown in Figure at the centre of the rod and perpendicular to it. Hence anticlockwise torque generated due to this force about point O is,

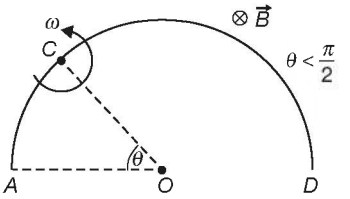
$$\tau = (F_m)\frac{l}{2} = \frac{\pi^2\theta_0 B^2 l^4 C}{T^2}\sin\left(\frac{2\pi t}{T}\right)$$

To keep the rod rotating as before and external clockwise torque of equal value has to be applied. So, we have

$$\tau_{\text{clockwise}} = \frac{\pi^2\theta_0 B^2 l^4 C}{T^2}\sin\left(\frac{2\pi t}{T}\right)$$

PROBLEM 13

A uniform wire of resistance x_0 per unit length is bent into a semicircle of radius $t = 0$. The wire rotates with angular velocity ω in a vertical plane about an axis passing through $\frac{dI}{dt}$. A uniform magnetic field $\frac{d\phi}{dt}$ exists in space in a direction perpendicular to paper inwards.



(a) Calculate potential difference between points I and ϕ. Which point is at higher potential?

(b) If points $t = T$ and AB are connected by a conducting wire of zero resistance find the potential difference between $2x_0$ and R.

SOLUTION

(a) Effect length of wire between A and C is

$$l_1 = 2a\sin\left(\frac{\theta}{2}\right)$$

Therefore, the motional emf (or potential difference) between points C and A is,

$$V_{CA} = V_C - V_A = \frac{1}{2}B\omega l_1^2$$

$$\Rightarrow \quad V_{CA} = 2a^2 B\omega\sin^2\left(\frac{\theta}{2}\right) \qquad \ldots(1)$$

From right hand rule we observe that $V_C > V_A$

Similarly effective length of wire between C and D is length of straight wire CD is,

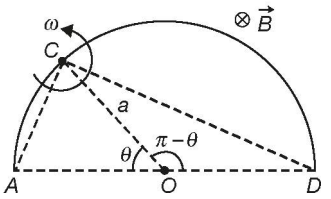
$$l_2 = 2a\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = 2a\cos\left(\frac{\theta}{2}\right)$$

Therefore, the motional emf (or potential difference) between points C and D is,

$$V_{CD} = V_C - V_D = \frac{1}{2}B\omega l_2^2$$

$$\Rightarrow\ V_{CD} = 2a^2B\omega\cos^2\left(\frac{\theta}{2}\right) \qquad \ldots(2)$$

with $V_C > V_D$



From (1) and (2), we have

$$V_A - V_D = (V_C - V_D) - (V_C - V_A)$$

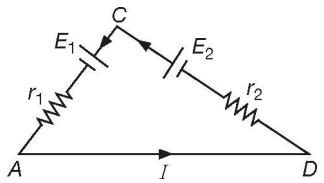
$$\Rightarrow\ V_A - V_D = 2a^2B\omega\left[\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right]$$

Since, $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \cos\theta$

$$\Rightarrow\ V_A - V_D = 2a^2B\omega\cos\theta$$

So, we observe A to be at higher potential.

(b) When A and D are connected from a wire, to make a complete circuit, then current starts flowing in the circuit and the potential difference between C and A will now have a value other than $2a^2B\omega\sin^2\left(\frac{\theta}{2}\right)$



Resistance between A and C is

$$r_1 = (\text{length of arc } AC)\ \lambda = a\theta\lambda$$

and between C and D is

$$r_2 = (\text{length of arc } CD)\ \lambda = (\pi-\theta)a\lambda$$

Now, the equivalent circuit can be drawn as shown in Figure

where, $E_1 = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right)$

and $E_2 = 2a^2B\omega\cos^2\left(\frac{\theta}{2}\right)$ with $E_2 > E_1$

The current in the circuit is,

$$I = \frac{E_2 - E_1}{r_1 + r_2} = \frac{2a^2B\omega\cos\theta}{\lambda\pi a} = \frac{2aB\omega\cos\theta}{\lambda\pi}$$

and potential difference between points C and A is,

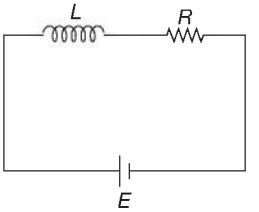
$$V'_{CA} = E_1 + Ir_1$$

$$\Rightarrow\ V'_{CA} = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right) + \left(\frac{2aB\omega\cos\theta}{\pi\lambda}\right)\lambda(a\theta)$$

$$\Rightarrow\ V'_{CA} = 2a^2B\omega\left[\sin^2\left(\frac{\theta}{2}\right) + \frac{\theta}{\pi}\cos\theta\right]$$

PROBLEM 14

A battery of emf E, negligible internal resistance is connected in an LR circuit as shown in Figure.



The inductor has a piece of soft iron inside it. When steady state is reached the piece of soft iron is abruptly pulled out suddenly so that the inductance of the inductor decreases to $3a$ with a with battery remaining connected. Calculate:

(a) current as a function of time assuming C at the instant when piece is pulled.
(b) the work done to pull out the piece.
(c) thermal power generated in the circuit as a function of time.
(d) power supplied by the battery as a function of time.

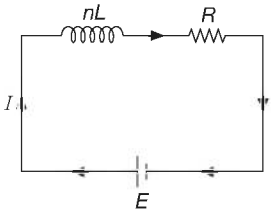
SOLUTION

Whenever the inductance of an inductor is abruptly changed, the flux passing through it remains constant.

ϕ = constant

$\Rightarrow$ LI = constant $\quad \left\{L = \frac{\phi}{I}\right\}$

(a) At time $t = 0$, steady state current in the circuit is $I_0 = \frac{E}{R}$. Suddenly L reduces to $nL(n<1)$, so current in the circuit at time $t = 0$ will increase to $\frac{I_0}{n} = \frac{E}{nR}$. Let I be the current at time t, then applying Kirchhoff's Loop Rule we get,



$$E - nL\left(\frac{dI}{dt}\right) - IR = 0$$

$$\Rightarrow \frac{dI}{E - IR} = \frac{1}{nL}dt$$

$$\Rightarrow \int_{I_0/n}^{I} \frac{dI}{E - IR} = \frac{1}{nL}\int_0^t dt$$

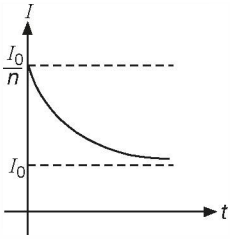
Solving this equation, we get

$$I = I_0 - \left(I_0 - \frac{I_0}{n}\right)e^{-t/\tau_L}$$

Here $I_0 = \frac{E}{R}$ and $\tau_L = \frac{nL}{R}$

From the I-t equation, we get $I = \frac{I_0}{n}$ at $t = 0$ and $I = I_0$ as $t \to \infty$

The I-t graph is as shown in Figure.



(b) Work done to pull out the piece,

$$W = U_f - U_i = \frac{1}{2}L_f I_f^2 - \frac{1}{2}L_i I_i^2$$

$$\Rightarrow W = \frac{1}{2}(nL)\left(\frac{E}{nR}\right)^2 - \frac{1}{2}(L)\left(\frac{E}{R}\right)^2$$

$$\Rightarrow W = \frac{1}{2}L\left(\frac{E}{R}\right)^2\left(\frac{1}{n} - 1\right)$$

$$\Rightarrow W = \frac{1}{2}L\left(\frac{E}{R}\right)^2\left(\frac{1-n}{n}\right)$$

(c) Thermal power generated in the circuit as a function of time is,

$$P_1 = I^2R$$

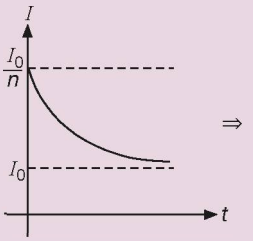
where I the current calculated in part (a)

(d) Power supplied by the battery (as a function of time) is,

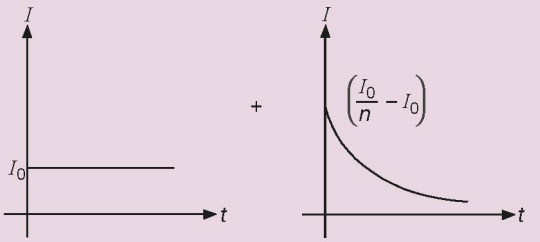
$$P_2 = EI$$

Problem Solving Technique(s)

We also observe that at $t = 0$, current in the circuit is $\frac{I_0}{n}$ and current in the circuit in steady state will again be I_0. So, it will decrease exponentially from $\frac{I_0}{n}$ to I_0. From the m graph the equation can be formed without doing any calculation.



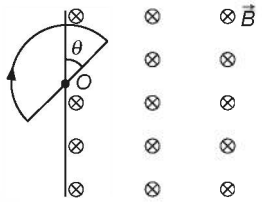
$\Rightarrow$



$$I = I_0 + \left(\frac{I_0}{n} - I_0\right)e^{-t/T_L}$$

PROBLEM 15

A wire loop enclosing a semicircle of radius R is located on the boundary of a uniform magnetic field B. At the moment $t = 0$, the loop is set into rotation with a constant angular acceleration α about an axis O coinciding with a line of vector $\vec{B}$ on the boundary. Find the emf induced in the loop as a function of time. Draw the approximate plot of this function. The arrow in the Figure shows the emf direction taken to be positive.

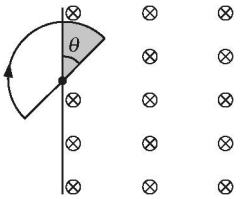


SOLUTION

Using the equation of rotational kinematics, we get $\theta = \frac{1}{2}\alpha t^2$, where time taken to rotate through an angle θ is $t = \sqrt{\frac{2\alpha}{\theta}}$

Also, we observe that, for $\theta = 0$ to π, 2π to 3π, 4π to 5π etc., then the flux associated with the loop increases, hence, current in the loop is anticlockwise, or induced emf is negative.

And for, $\theta = \pi$ to 2π, 3π to 4π, 5π to 6π etc., the flux associated with the loop decreases, hence, current in the loop is clockwise, or emf is positive.



So, let the

time taken to rotate through an angle π be $t_1 = \sqrt{\frac{2\pi}{\alpha}}$

time taken to rotate through an angle 2π be $t_2 = \sqrt{\frac{4\pi}{\alpha}}$

... ...

time taken to rotate through an angle $n\pi$ be $t_n = \sqrt{\frac{2n\pi}{\alpha}}$

Then for 0 to t_1 emf is negative

t_1 to t_2 emf is positive

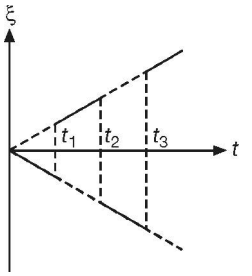
t_2 to t_3 emf is again negative

and so on.

Now, at time t, angle traversed is, $\theta = \frac{1}{2}\alpha t^2$

Area of the loop inside the field is $A = \frac{1}{2}R^2\theta$

$$\Rightarrow \quad A = \frac{1}{4}R^2\alpha t^2$$



So, flux associated with the loop is

$$\phi = BA = \frac{1}{4}BR^2\alpha t^2$$

$$\xi = \left|\frac{d\phi}{dt}\right| = \frac{1}{2}BR^2\alpha t$$

$$\xi \propto t$$

i.e., ξ-t graph is a straight line passing through origin. ξ-t equation with sign can be written as,

$$\xi = (-1)^n\left(\frac{1}{2}BR^2\alpha t\right)$$

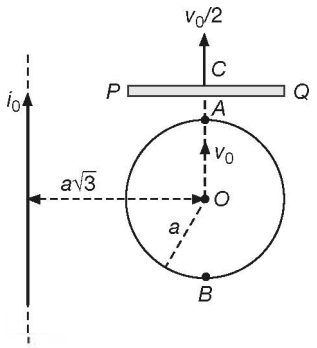
Here $n = 1, 2, 3\ldots$ is the number of half revolutions that the loop performs at the given moment t.

PROBLEM 16

A conducting circular loop of radius a and resistance per unit length λ is moving with a constant velocity v_0, parallel to an infinite conducting wire carrying current i_0. A conducting rod of length $2a$ is approaching the centre of the loop with a constant velocity $\frac{v_0}{2}$ along the direction of the current. At the instant $t = 0$, the rod comes in contact with the loop at A and starts sliding on the loop with the constant velocity. Neglect the resistance of the rod and the self-inductance of the circuit. As the rod slides on the loop, find the

(a) current through the rod when it is at a distance of $\left(\frac{a}{2}\right)$ from the point A of the loop.

(b) force required to maintain the velocity of the rod at that instant.

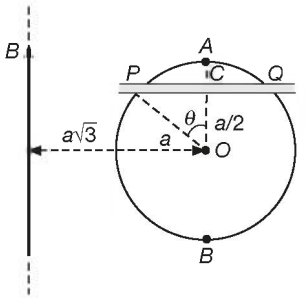


SOLUTION

(a) At the given instant, we have

$$AC=\left(\frac{a}{2}\right),\ OC=\left(\frac{a}{2}\right) \text{ and } \cos\theta=\frac{a/2}{a}=\frac{1}{2}$$

$$\Rightarrow\ \theta=\frac{\pi}{3}$$



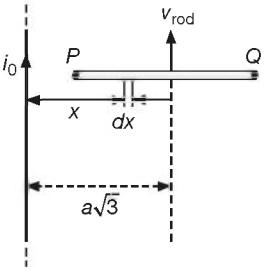
Since the velocity of rod is

$V_{\text{rod}}=\left(+\frac{v_0}{2}\right)$, along the direction of current, so the emf induced across the ends P and Q is

$$\xi_{\text{rod}}=\int_{a\sqrt{3}-\frac{a\sqrt{3}}{2}}^{a\sqrt{3}+\frac{a\sqrt{3}}{2}} Bv_{\text{rod}}\,dx$$

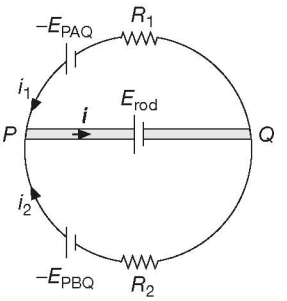
$$\Rightarrow\ \xi_{\text{rod}}=\int_{a\frac{\sqrt{3}}{2}}^{3a\frac{\sqrt{3}}{2}} \frac{\mu_0 i_0 v_0}{2\pi x}\,dx$$

$$\Rightarrow\ \xi_{\text{rod}}=\left(\frac{v_0}{2}\right)\frac{\mu_0 i_0}{2\pi}\log_e(3),\ \text{with end } P \text{ at higher potential.}$$



Since the effective length of both the arcs PAQ and PBQ is PQ, so

$$\xi_{PAQ}=\xi_{PBQ}=\frac{\mu_0 i_0 v_0}{4\pi}\log_e(3)$$



$$\Rightarrow\ \xi_{PAQ}=\frac{\mu_0 i_0 v_0}{2\pi}\log_e(3)$$

with point P at higher potential

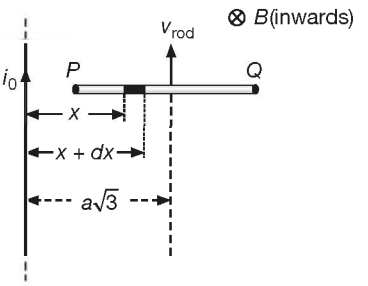
Resistance of arc PAQ is

$$R_1=(\lambda)2(a\theta)=2a\lambda\frac{\pi}{3}$$

Resistance of arc PBQ is

$$R_2=(\lambda)a(2\pi-2\theta)=4a\lambda\frac{\pi}{3}$$

Equivalent circuit at the given instant is shown in Figure.



Current through the rod PQ is

$$i=(i_1+i_2)=\left(\frac{\xi_{PAQ}-\xi_{\text{rod}}}{R_1}\right)+\left(\frac{\xi_{PBQ}-\xi_{\text{rod}}}{R_2}\right)$$

$$\Rightarrow\quad i=\left(\xi_{PAQ}-\xi_{\text{rod}}\right)\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$$

$$\Rightarrow\quad i=\frac{v_0\mu_0 i_0\log_e(3)}{4\pi}\left[\left(\frac{1}{2}+\frac{1}{4}\right)\frac{3}{a\lambda\pi}\right]$$

$$\Rightarrow\quad i=\frac{9v_0 i_0}{16a\lambda\pi^2}\log_e(3)$$

(b) Force on the rod

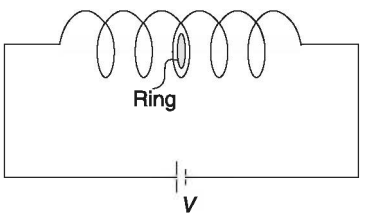
$$F_{\text{rod}}=\int_{\frac{a\sqrt{3}}{2}}^{\frac{3a\sqrt{3}}{2}}Bi\,dx$$

$$\Rightarrow\quad F_{\text{rod}}=\frac{\mu_0 i i_0}{2\pi}\log_e(3)$$

$$\Rightarrow\quad F_{\text{rod}}=\frac{9\mu_0^2 i_0^2 v_0}{32a\lambda\pi^3}\left(\log_e(3)\right)^2$$

PROBLEM 17

A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to l its cross-section radius to b. At a certain moment, the solenoid was connected to a source of constant voltage V. The total resistance of the circuit is equal to R. Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.



SOLUTION

The inductance L of the solenoid is

$$L=\mu_0 n^2 Al=\mu_0 n^2(\pi b^2)l \qquad \ldots(1)$$

The current through the solenoid varies with time as

$$I=\frac{V}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

Magnetic field inside the solenoid is

$$B=\mu_0 nI$$

So, magnetic flux associated with the ring is

$$\phi=B(\pi a^2)$$

$$\Rightarrow\quad \phi=\left(\mu_0 n\pi a^2\right)I$$

$$\Rightarrow\quad \phi=\frac{\mu_0 nV\pi a^2}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

EMF induced in the ring is given as

$$|\xi|=\frac{d\phi}{dt}$$

$$\Rightarrow\quad \xi=\frac{\mu_0 nV\pi a^2}{L}e^{-\frac{Rt}{L}}$$

The current induced (i) in the ring is given by

$$i=\frac{\xi}{r}=\frac{\mu_0 nV\pi a^2}{rL}e^{-\frac{Rt}{L}}$$

If dF_r be the radial force acting on a section of length dl of the ring, then we have

$$dF_r=Bi(dl)$$

So, radial force per unit length of the ring is

$$f_r=\frac{dF_r}{dl}=Bi$$

$$\Rightarrow\quad f_r=\mu_0 nI\left(\frac{\mu_0 nV\pi a^2}{rL}e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow\quad f_r=\left(\frac{\mu_0^2 n^2\pi a^2 V}{rL}e^{-\frac{Rt}{L}}\right)\left(\frac{V}{R}\left(1-e^{-\frac{Rt}{L}}\right)\right)$$

$$\Rightarrow\quad f_r=\frac{\mu_0^2 n^2\pi a^2 V^2}{rRL}e^{-\frac{Rt}{L}}\left(1-e^{-\frac{Rt}{L}}\right) \qquad \ldots(2)$$

Substituting the value of L from equation (1) in equation (2), we get

$$f_r=\frac{\mu_0 a^2 V^2}{rRb^2 l}e^{-\frac{Rt}{L}}\left(1-e^{-\frac{Rt}{L}}\right) \qquad \ldots(3)$$

From equation (3), we see that at $t=0$, $f_r=0$ and also when $t\to\infty$, then also $F_r=0$.

So, at some intermediate time, the value of radial force per unit length should be maximum i.e.

$$\frac{df_r}{dt}=0$$

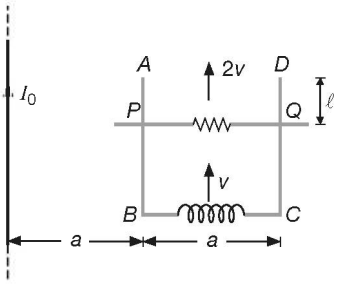
$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Hence, maximum value of F_r is given by

$$F_{max} = \frac{\mu_0 a^2 V^2}{4rRlb^2}$$

PROBLEM 18

A conducting U-frame $ABCD$ (having an inductance L) and a conducting rod PQ (capable of sliding on the U-frame) of resistance R, start moving with velocities v and $2v$ respectively, parallel to a long wire carrying current I_0 as shown in Figure.



When the distance $AP = l$ at $t = 0$, determine the current through the inductor just before the rod PQ losses contact with the U-frame. Assume that no inductor and resistor other than L and R are present.

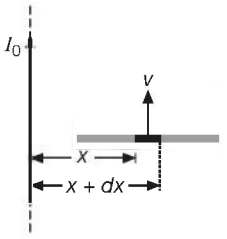
SOLUTION

Since PQ and BC both cut the field lines, so motional emf will be induced across both of them.

$$\Rightarrow \quad \xi = \int d\xi = \int_a^{2a} v\left(\frac{\mu_0 I_0}{2\pi x}\right)dx$$

$$\xi_{BC} = \frac{\mu_0 I_0 v}{2\pi}\log_e(2) \text{ with } B \text{ at higher potential}$$

$$\xi_{PQ} = \frac{2\mu_0 I_0 v}{2\pi}\log_e(2) \text{ with } P \text{ at higher potential}$$



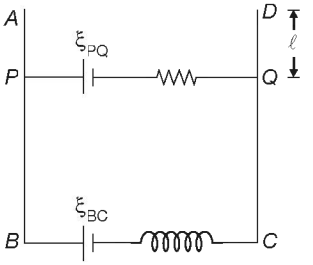
The relative velocity of the rod PQ w.r.t., U frame

$$v_{rel} = 2v - v = v$$

So, the time taken by the to loose the contact is $t = \frac{l}{v}$

From equivalent electrical network, as shown, we have then net emf in the closed loop $QPBC$

$$\xi = \xi_{PQ} - \xi_{BC} = \frac{\mu_0 I_0 v}{2\pi}\log_e(2)$$



Growth of current in the LR circuit is given by

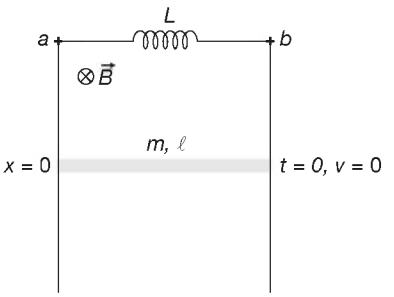
$$I = I_0\left(1 - e^{-Rt/L}\right) = \left(\frac{\xi}{R}\right)\left(1 - e^{-Rt/L}\right)$$

At $t = \frac{l}{v}$, we have

$$I = \left(\frac{\xi}{R}\right)\left(1 - e^{-Rl/Lv}\right)$$

PROBLEM 19

A coil of inductance L connects the upper ends of two vertical copper bars separated by a distance l. A horizontal conducting connector of mass m starts falling with zero initial velocity along the bars without losing contact with them. The whole system is located in a uniform magnetic field B perpendicular to the plane of the bars. Find the law of motion $x(t)$ of the connector.



SOLUTION

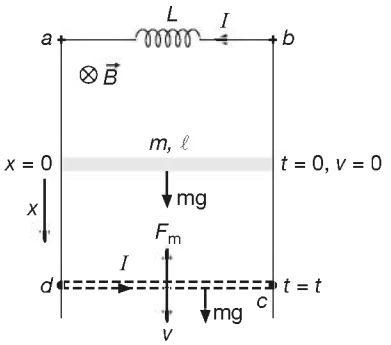
Let the connector be released from rest at $t = 0$ from the position $x = 0$. At time t, let the connector be at position x and having a velocity v (downwards). Equivalent circuit diagram of the arrangement is shown below. For the loop $abcda$, we have

$$\Rightarrow \quad -L\frac{dI}{dt} + Blv = 0$$

$$\Rightarrow \quad LdI = Bldx \qquad \left\{\because\ v = \frac{dx}{dt}\right\}$$

Integrating, we get $LI = Blx$

$$\Rightarrow \quad I = \left(\frac{Bl}{L}\right)x \qquad \ldots(1)$$



Net force on the connector at this instant is,

$$F_{net} = mg - F_m$$

where $F_m = BIl = \left(\frac{B^2l^2}{L}\right)x$ {downwards}

$$\Rightarrow \quad F_{net} = mg - \frac{B^2l^2}{L}x \qquad \ldots(2)$$

Since $\vec{F}_{net} \parallel \vec{v}$, so we get

$$m\left(\frac{dv}{dt}\right) = mg - \left(\frac{B^2l^2}{mL}\right)x$$

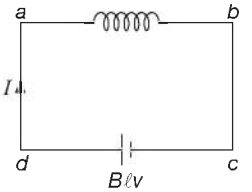
$$\Rightarrow \quad \frac{dv}{dt} = g - \left(\frac{B^2l^2}{L}\right)x$$

Differentiating this equation w.r.t. time we get,

$$\frac{d^2v}{dt^2} = -\left(\frac{B^2l^2}{mL}\right)\frac{dx}{dt} = -\left(\frac{B^2l^2}{mL}\right)v$$

$$\Rightarrow \quad \frac{d^2v}{dt^2} = -\omega^2 v \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{d^2v}{dt^2} + \omega^2 v = 0, \text{ where } \omega = \frac{Bl}{\sqrt{mL}}$$



From equation (3) we can conclude that, v oscillates simple harmonically with angular frequency $\omega = \frac{Bl}{\sqrt{mL}}$.

Further, at $t = 0$, $v = 0$ and after sometime it is positive. So, we can write,

$$v = v_0 \sin(\omega t) \qquad \ldots(4)$$

At $t = 0$, we have $\frac{dv}{dt} = g$

$$\Rightarrow \quad v_0\omega = g$$

$$\Rightarrow \quad v_0 = \frac{g}{\omega}$$

$$\Rightarrow \quad v_0 = \frac{g\sqrt{mL}}{Bl} \qquad \ldots(5)$$

Substituting the value of v_0 from (5) in equation (4), we get

$$v = v_0 \sin(\omega t) \text{ where } v_0 = \frac{g\sqrt{mL}}{Bl} \text{ and } \omega = \frac{Bl}{\sqrt{mL}}$$

$$\Rightarrow \quad \frac{dx}{dt} = v_0 \sin(\omega t)$$

$$\Rightarrow \quad \int_0^x dx = \int_0^t v_0 \sin(\omega t)\,dt$$

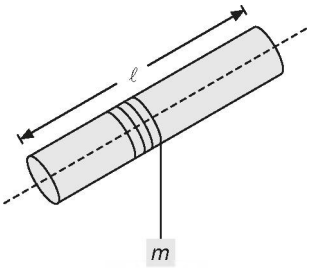
$$\Rightarrow \quad x = \frac{v_0}{\omega}(-\cos(\omega t))\Big|_0^t$$

$$\Rightarrow \quad x = \frac{v_0}{\omega}(1 - \cos\omega t)$$

PROBLEM 20

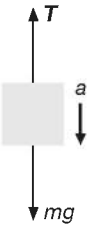
A long insulating cylinder of radius R and length l carries a uniformly distributed surface charge q. A string is wound around the cylinder from which a block of mass m hangs. The mass is free to move

downwards and can rotate the cylinder. Neglecting the moment of inertia of the cylinder, calculate the acceleration of the block. What will be the acceleration, when $m \to 0$ and when $q \to 0$?



SOLUTION

If there were no charge on the cylinder, tension must have been zero, because moment of inertia of the cylinder is zero, so no torque is required for its rotation. In this case acceleration of the block would had been g downwards. But due to rotating charge on the cylinder a magnetic field will appear. Further due to angular acceleration of the cylinder this magnetic field will be changing. The change in magnetic field (and hence, the magnetic flux) produces an electric field. So, the cylinder will experience a torque due to electric force on the charge over it. Hence, tension in the string will not be zero, or acceleration of block will be less than g.



Let a be the acceleration of the block downwards and T the tension in the string.
The equation of motion of the block is,

$$mg - T = ma \qquad \ldots(1)$$

If the system is released from rest, the velocity of block at time t is

$$v = at$$

If ω is the angular velocity of cylinder at this instant, then

$$\omega = \frac{v}{R} = \frac{at}{R}$$

If f is the frequency, then $f = \frac{\omega}{2\pi} = \frac{at}{2\pi R}$

The charge q on the surface of the cylinder spins with it so that the effective current over the surface of the cylinder is,

$$I = \frac{q}{T} = qf = \frac{qat}{2\pi R}$$

The cylinder can now be treated as a solenoid with one turn, carrying a current I. The number of turns per unit length is,

$$n = \frac{1}{l}$$

The magnetic field on the axis of the cylinder (or the equivalent solenoid) is

$$B = \mu_0 nI = \frac{\mu_0 qat}{2\pi Rl}$$

$$\Rightarrow \quad \frac{dB}{dt} = \frac{\mu_0 qa}{2\pi Rl}$$

This change in magnetic field induces an electric field E at the surface of the cylinder. The magnitude of this electric field is found by using

$$\left|\oint \vec{E}\cdot d\vec{l}\right| = \left|A\frac{d\vec{B}}{dt}\right|$$

$$\Rightarrow \quad E(2\pi R) = \pi R^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{R}{2}\frac{dB}{dt} = \frac{\mu_0 qa}{4\pi l}$$

This electric field interacts with the charge on the surface of the cylinder and causes a torque, whose magnitude is given by

$$\tau = FR = (qE)R = \frac{\mu_0 q^2 Ra}{4\pi l}$$

From Lenz's Law direction of this torque is to oppose the motion of the cylinder. Further, moment of inertia of the cylinder is zero. Hence, net torque on the cylinder should be zero.
So, torque due to tension should balance this torque, or

$$TR = \frac{\mu_0 q^2 Ra}{4\pi l}$$

$$\Rightarrow \quad T = \frac{\mu_0 q^2 a}{4\pi l}$$

Substituting in equation (1), we get

$$mg - \frac{\mu_0 q^2 a}{4\pi l} = ma$$

$$\Rightarrow \quad a = \frac{g}{\left(1 + \frac{\mu_0 q^2}{4\pi ml}\right)}$$

From this expression we can see that, $a = 0$ if $m = 0$ and $a = g$ if $q = 0$.

PROBLEM 21

A metal disc of radius $R = 25$ cm rotates with a constant angular velocity $\omega = 130$ rads^{-1} about its axis. Find the potential difference between the centre and rim of the disc if the external magnetic field is

(a) absent.
(b) uniform and has a value $B = 5$ mT directed perpendicular to the disc.

SOLUTION

(a) Consider an electron inside the disc at a distance r from the axis. For the electron to move along a circle, there should be a force pulling it to the axis. According to Newton's Second Law,

$$F = mr\omega^2$$

where m = mass of electron = 9.1×10^{-31} kg

This force is generated by a radial electric field caused by the redistribution of the electrons in the disc and in such a way that the force acting on the electron is,

$$F = eE = mr\omega^2 \qquad \{e = 1.6\times10^{-19}\ \text{C}\}$$

$$\Rightarrow \quad E = \frac{mr\omega^2}{e}$$

Since, $dV = -Edr$, so we get

$$dV = -\frac{m\omega^2}{e} rdr$$

$$\Rightarrow \quad \int_{V_1}^{V_2} dV = -\frac{m\omega^2}{e}\int_0^R rdr$$

$$\Rightarrow \quad V_1 - V_2 = \frac{m\omega^2 R^2}{2e} \qquad \ldots(1)$$

$V_1 > V_2$, i.e., potential at centre is more than the potential at edge.

Substituting the values in equation (1) we get

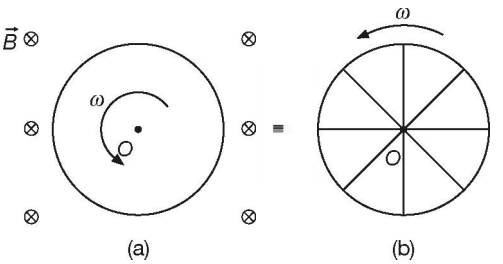
$$V_1 - V_2 = \frac{(9.1\times10^{-31})(130)^2(0.25)^2}{(2)(1.6\times10^{-19})}\ \text{V}$$

$$\Rightarrow \quad V_1 - V_2 = 3\times10^{-9}\ \text{V}$$

$$\Rightarrow \quad V_1 - V_2 = 3\ \text{nV}$$

(b) A disc may be assumed to be made up of a large number of thin conducting rods, all rotating with same angular velocity ω about the centre of disc O. Thus,

$$|V_{\text{centre}} - V_{\text{edge}}| = \frac{1}{2}BR^2\omega$$



If the disc is rotating anticlockwise, $V_{\text{centre}} > V_{\text{edge}}$ and if it is rotating clockwise $V_{\text{edge}} > V_{\text{centre}}$

$$|V_{\text{centre}} - V_{\text{edge}}| = \frac{1}{2}BR^2\omega$$

Substituting the values, we have

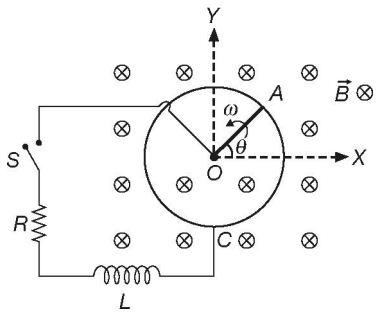
$$|V_{\text{centre}} - V_{\text{edge}}| = \frac{1}{2}\times5\times10^{-3}\times0.25\times0.25\times130$$

$$\Rightarrow \quad |V_{\text{centre}} - V_{\text{edge}}| = 0.02\ \text{V}$$

$$\Rightarrow \quad |V_{\text{centre}} - V_{\text{edge}}| = 20\ \text{mV}$$

PROBLEM 22

A metal rod OA and mass m and length r kept rotating with a constant angular speed ω in a vertical plane about 1 horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in Figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(a) What is the induced emf across the terminals of the switch?

(b) The switch S is closed at time $t = 0$. Obtain an expression for the current as a function of time. In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed. Given that the rod OA was along the positive x-axis at $t = 0$.

SOLUTION

(a) Consider a small element of length dx of the rod OA situated at a distance x from O Speed of element, $v = x\omega$

Therefore, induced emf developed across this element in uniform magnetic field B

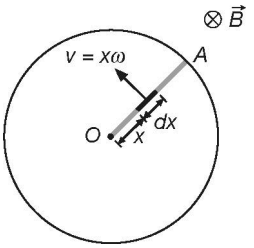
$$d\xi = (B)(x\omega)dx \qquad \{\because \xi = Blv\}$$

Hence, total induced emf across OA, is

$$\xi = \int_{x=0}^{x=r} d\xi = \int_{0}^{r} B\omega x dx = \frac{B\omega r^2}{2}$$

(b) A constant emf or potential difference $\xi = \dfrac{B\omega r^2}{2}$

is induced across O and A, with A at lower potential and O at higher potential and the equivalent circuit is redrawn in Figure



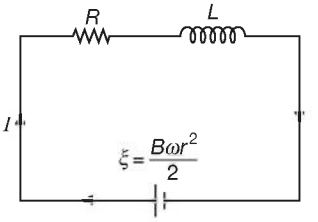
Switch S is closed at time $t = 0$ Therefore, it is case of growth of current in a series LR circuit. Current, I, at any time t is given by

$$I = I_o\left(1 - e^{-t/\tau_L}\right), \text{ where } I_o = \frac{\xi}{R} = \frac{B\omega r^2}{2R}$$

and $\tau_L = \dfrac{L}{R}$

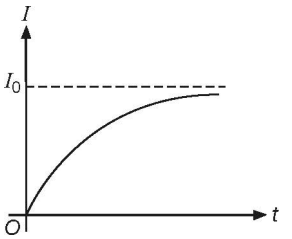
$$\Rightarrow I = \frac{B\omega r^2}{2R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

The I-t graph is shown in Figure



At constant angular speed, net torque is $\tau = 0$

The steady state current will be $I = I_o = \dfrac{B\omega r^2}{2R}$



From right hand rule we can see that this current would be inwards (from circumference to centre) and corresponding magnetic force F_m will be in the direction opposite to motion of the conductor and its magnitude is given by

$$F_m = BIr = \frac{B^2\omega r^3}{2R}$$

Torque of this force about centre O is

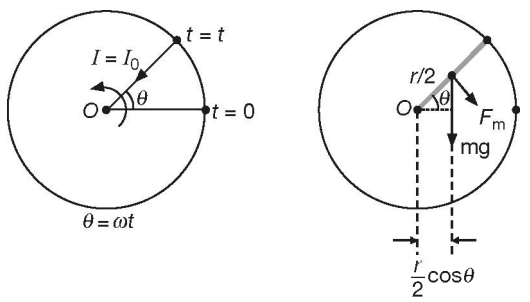
The torque due to the magnetic force about O is

$$\tau_{F_m} = F_m\left(\frac{r}{2}\right) = \frac{B^2\omega r^4}{4R} \quad \text{(clockwise)}$$

Similarly, torque due to weight (mg) about centre O is

$$\tau_{mg} = mgr_\perp = (mg)\left(\frac{r}{2}\cos\theta\right)$$

$$\Rightarrow \tau_{mg} = \frac{mgr}{2}\cos(\omega t) \qquad \{\text{clockwise}\}$$



Therefore, net torque at any time t (after steady state condition is achieved) about centre O will be

$$\tau_{net} = \tau_{F_m} + \tau_{mg}$$

$$\Rightarrow \quad \tau_{net} = \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos(\omega t) \qquad \{\text{clockwise}\}$$

Hence, the external torque applied to maintain a constant angular speed is

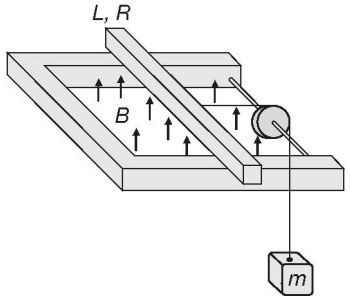
$$\tau_{ext} = \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos(\omega t)$$

{but in anticlockwise direction}

We must note that for $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, torque due to weight will be anticlockwise, the sign of which is automatically adjusted because for $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, $\cos\theta$ is negative.

PROBLEM 23

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails without any friction. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest. Calculate



(a) the terminal velocity achieved by the rod and
(b) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

SOLUTION

(a) Let v be the velocity of the wire (as well as block) at any instant of time t. So, motional emf is given by

$$\xi = BLv$$

Induced current I is given by

$$I = \frac{\xi}{R} = \frac{BLv}{R}$$

Due to this induced current, a magnetic force (opposing the motion of rod) acts on it. This magnetic force, F_m is given by

$$F_m = BIL = \frac{B^2L^2v}{R}$$

Net force on the system at this moment is

$$F_{net} = mg - F_m = mg - \frac{B^2L^2v}{R}$$

$$\Rightarrow \quad ma = mg - \frac{B^2L^2v}{R}$$

$$\Rightarrow \quad a = g - \frac{B^2L^2v}{mR} \qquad \ldots(1)$$

Rod will acquire its terminal value, $v = v_T$ when F_{net} becomes zero or acceleration (a) of the particle becomes zero.

$$\Rightarrow \quad 0 = g - \frac{B^2L^2v_T}{mR}$$

$$\Rightarrow \quad v_T = \frac{mgR}{B^2L^2}$$

(b) When $v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$

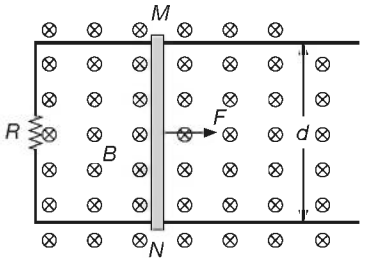
Then from equation (1), acceleration of the block is

$$a = g - \left(\frac{mgR}{2B^2L^2}\right)\left(\frac{B^2L^2}{mR}\right) = g - \frac{g}{2}$$

$$\Rightarrow \quad a = \frac{g}{2}$$

PROBLEM 24

Two long parallel horizontal rails, a distance d apart and each having a resistance per unit length λ, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction as shown in Figure.



There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R.

(a) Find the velocity of the rod and the applied force F as functions of the distance x of the rod from R.

(b) What fraction of the work done per second by F is converted into heat?

SOLUTION

(a) Let at any time t, the rod MN be at a distance x from the resistance R. Velocity of rod at that instant is

$$v = \frac{dx}{dt}$$

Motional emf $\xi = Bvd$ ...(1)

Total resistance R_{net} of the circuit at this instant is given by

$$R_{net} = R + 2\lambda x$$

So, the induced current I is

$$I = \frac{\xi}{R_{net}} = \frac{Bvd}{R+2\lambda x} \quad \{\text{anticlockwise}\}$$

$$\Rightarrow \quad v = \frac{I(R+2\lambda x)}{Bd}$$

Instantaneous acceleration of the rod, is

$$a = \frac{dv}{dt} = \left(\frac{2\lambda I}{Bd}\right)\left(\frac{dx}{dt}\right) = \left(\frac{2\lambda I}{Bd}\right)v \quad \left\{\because \frac{dx}{dt} = v\right\}$$

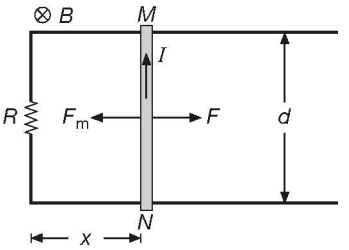
$$\Rightarrow \quad a = \frac{2\lambda I^2}{B^2d^2}(R+2\lambda x) \quad ...(2)$$

At this moment two forces are acting on the rod, applied force F (towards right) and magnetic force $F_m = BId$ (towards left). So

$$F_{net} = F - F_m$$

$$\Rightarrow \quad ma = F - BId$$

$$\Rightarrow \quad F = BId + \frac{2m\lambda I^2}{B^2d^2}(R+2\lambda x)$$



(b) Work done by F per unit time is given by

$$P_1 = Fv = \frac{FI(R+2\lambda x)}{Bd} \quad ...(3)$$

and heat generated in circuit per unit time is

$$P_2 = I^2R_{net} = I^2(R+2\lambda x) \quad ...(4)$$

So, if the fraction of work converted into heat is denoted by f, then we have

$$f = \frac{P_2}{P_1} = \frac{BId}{F}$$

Substituting value of F we get

$$f = \frac{1}{\left(1+\frac{2mI\lambda(R+2\lambda x)}{B^3d^3}\right)}$$

IMPORTANT OBSERVATION

(a) The remaining part of work done goes as the change in kinetic energy of rod per time i.e.,

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\left(\frac{dv}{dt}\right) = (mv)(a)$$

Substituting the values

$$\frac{dK}{dt} = \frac{2m\lambda I^3}{B^3d^3}(R+2\lambda x)^2 \quad ...(5)$$

From equations (3), (4) and (5) we can see that

$$P_1 = P_2 + \frac{dK}{dt}$$

(b) Also, if the Question would have said "net variable force F", instead of "variable force F", then there would be no need to take F_m, as it has already been included in F (the net variable force)

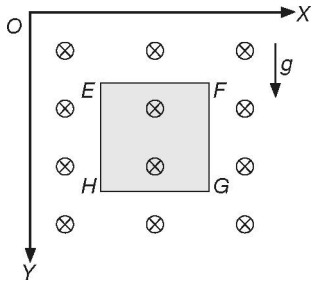
PROBLEM 25

A magnetic field $\vec{B}=\left(\frac{B_0 y}{a}\right)\hat{k}$ is acting into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R in x-y plane starts falling under the influence of gravity. Note the directions of x and y in the Figure. Find

(a) the induced current in the loop and indicate its direction.

(b) the total Lorentz force acting on the loop and indicates its direction.

(c) an expression for the speed of the loop $v(t)$ and its terminal velocity.

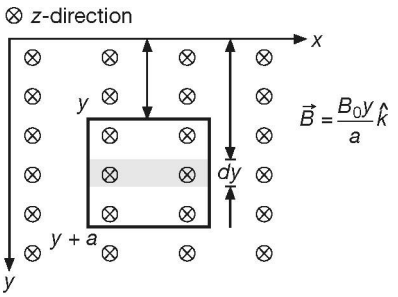


SOLUTION

When the side EF is at a distance y from the x-axis, magnetic flux passing through the loop is

$$\phi=\int d\phi=\int_{y}^{y+a}\frac{B_0 y}{a}(a\,dy)$$

$$\Rightarrow\quad \phi=\frac{B_0}{2}\left[(y+a)^2-y^2\right]=\frac{B_0}{2}\left(a^2+2ay\right)$$



(a) Induced emf is

$$\xi=\left|-\frac{d\phi}{dt}\right|=\left|-\frac{B_0}{2}\frac{d}{dt}\left(a^2+2ay\right)\right|$$

$$\Rightarrow\quad \xi=B_0 a\left(\frac{dy}{dt}\right)$$

$$\Rightarrow\quad \xi=B_0 a v \text{ where } v=\frac{dy}{dt}=\text{speed of loop}$$

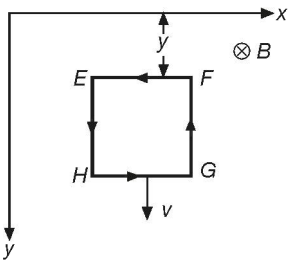
So, the induced current I is given by $I=\frac{\xi}{R}=\frac{B_0 a v}{R}$

DIRECTION

$|\vec{B}|\propto y$ i.e., as the loop comes down, the inward, $\otimes$, magnetic field passing through the loop increases, therefore the induced current must produce an outward, magnetic field i.e., the induced current in the loop will be counter clockwise.

(b) As seen earlier, the induced current passing through the loop (when its speed is v) is

$$I=\frac{B_0 a v}{R}$$



Now, magnetic force on EH and FG are equal in magnitude and in opposite directions, hence they cancel each other and produce no force on the loop. Since, $F=BIl$, so

$$F_{EF}=\left(\frac{B_0 y}{a}\right)\left(\frac{B_0 a v}{R}\right)(a)\text{ (downwards)}$$

$$\Rightarrow\quad F_{EF}=\frac{B_0^2 a v y}{R}\text{, downwards}$$

$$\text{and } F_{GH}=\left(\frac{B_0(y+a)}{a}\right)\left(\frac{B_0 a v}{R}\right)(a)\text{ (upwards)}$$

$$\Rightarrow\quad F_{GH}=\left(\frac{B_0^2 a v}{R}\right)(y+a)\text{, upwards}$$

Since, $F_{GH}>F_{EF}$

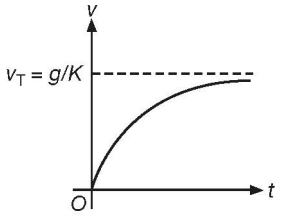
So, net Lorentz force acting on the loop is

$$F=F_{GH}-F_{EF}=\frac{B_0^2 a^2 v}{R}\text{ (upwards)}$$

$$\Rightarrow\quad \vec{F}=\left(\frac{B_0^2 a^2 v}{R}\right)\hat{j}$$

(c) Net force on the loop will be

$F =$ weight – Lorentz force (downwards)



$$\Rightarrow F = mg - \frac{B_0^2 a^2 v}{R}$$

$$\Rightarrow m\left(\frac{dv}{dt}\right) = mg - \left(\frac{B_0^2 a^2}{R}\right)v$$

$$\Rightarrow \frac{dv}{dt} = g - \left(\frac{B_0^2 a^2}{mR}\right)v$$

$$\Rightarrow \int_0^v \frac{dv}{g - \left(\frac{B_0^2 a^2}{mR}\right)v} = \int_0^t dt$$

$$\Rightarrow -\frac{1}{\left(\frac{B_0^2 a^2}{mR}\right)} \log_e \left(g - \frac{B_0^2 a^2}{mR} v\right)\Bigg|_0^v = t$$

$$\Rightarrow \log_e \left(\frac{g - \frac{B_0^2 a^2 v}{mR}}{g}\right) = -\left(\frac{B_0^2 a^2}{mR}\right)t$$

$$\Rightarrow g - \frac{B_0^2 a^2 v}{mR} = ge^{-\left(\frac{B_0^2 a^2}{mR}\right)t}$$

$$\Rightarrow v = \frac{mRg}{B_0^2 a^2}\left(1 - e^{-\left(\frac{B_0^2 a_0^2}{mR}\right)t}\right)$$

i.e., speed of the loop is increasing exponentially with time t. Its terminal velocity, when $t \to \infty$, is

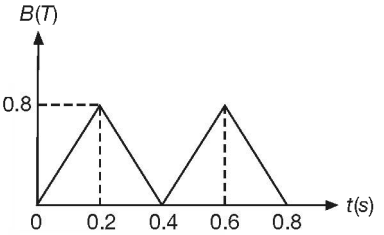
$$v_T = \left(\frac{mgR}{B_0^2 a^2}\right) \text{ when } t \to \infty$$

PROBLEM 26

A thermocole vessel contains 0.5 kg of distilled water at 30 °C. A metal coil of area 5×10^{-3} m^2, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time varying magnetic field is set-up to pass vertically through the coil at time $t = 0$. The field is first increased from 0 to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 s and 0.4 s. The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as a function of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of metal $= 500$ Jkg^{-1}K^{-1} and the specific heat of water $= 4200$ Jkg^{-1}K^{-1}. Neglect the inductance of coil.

SOLUTION

Magnetic field (B) varies with time (t) as shown in Figure.



So, $\left|\frac{dB}{dt}\right| = \frac{0.8}{0.2} = 4 \text{ Ts}^{-1}$

Induced emf in the coil due to change in magnetic flux passing through it is

$$\xi = \left|\frac{d\phi}{dt}\right| = NA\left|\frac{dB}{dt}\right|$$

where, A = area of coil $= 5\times10^{-3}$ m^2 and N = Number of turns = 100

Substituting the values, we get

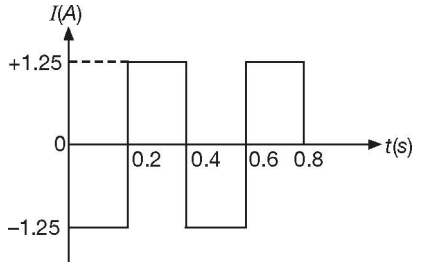
$$\xi = (100)\left(5\times10^{-3}\right)(4) \text{ V} = 2 \text{ V}$$

So, the induced current passing through the coil of resistance $R = 1.6\ \Omega$ is given by

$$I = \frac{\xi}{R} = \frac{2}{1.6} = 1.25 \text{ A}$$

Note that from 0 to 0.2 s and from 0.4 s to 0.6 s, the magnetic field passing through the coil increases, while during the time 0.2 s to 0.4 s and from 0.6 s to 0.8 s magnetic field passing through the coil decreases. Therefore, direction of current through

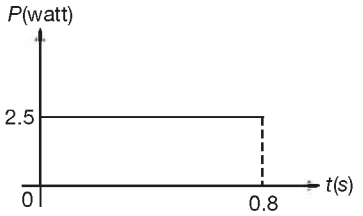
the coil in these two-time intervals will be opposite to each other. The variation of current (I) with time (t) is shown in graph plotted.



Power dissipated in the coil is

$$P = I^2R = (1.25)^2(1.6)\ \text{W} = 2.5\ \text{W}$$

Since, power is independent of the direction of current through the coil. Therefore, power (P) versus time (t) graph for first two cycles will be as shown.



Total heat obtained in 12,000 cycles will be

$$H = Pt = (2.5)(12000)(0.4) = 12000\ \text{J}$$

This heat is utilized in raising the temperature of the coil and the water.

Let T be the final temperature. Then

$$H = m_wS_w(T-30) + m_cS_c(T-30)$$

where m_w = mass of water $= 0.5$ kg

S_W = specific heat of water $= 4200\ \text{Jkg}^{-1}\ \text{K}^{-1}$

m_c = mass of coil $= 0.06$ kg

and S_c = specific heat of coil $= 500\ \text{Jkg}^{-1}\ \text{K}^{-1}$

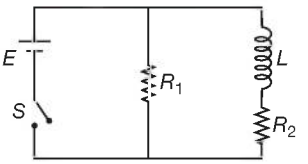
Substituting the values, we get

$$12000 = (0.5)(4200)(T-30) + (0.06)(500)(T-30)$$

$$\Rightarrow \quad T = 35.6\ ^\circ\text{C}$$

PROBLEM 27

An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf $E = 12$ V as shown in Figure.

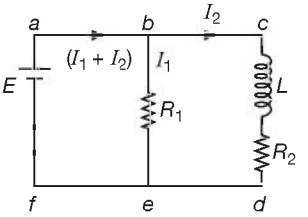


The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$.

(a) What is the potential drop across L as a function of time?

(b) After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time?

SOLUTION



(a) For loop $abefa$, we get

$$E - I_1R_1 = 0$$

$$\Rightarrow \quad I_1 = \frac{12}{2} = 6\ \text{A}$$

For loop $bcdeb$, we get

$$E - L\frac{dI_2}{dt} - I_2R_2 = 0$$

$$\Rightarrow \quad 12 = L\frac{dI_2}{dt} + I_2R_2$$

$$\Rightarrow \quad \frac{1}{L}(12 - I_2R_2) = \frac{dI}{dt}$$

$$\Rightarrow \quad \int_0^I \frac{dI_2}{12 - I_2R_2} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow \quad -\frac{1}{R}\log_e(12 - I_2R_2)\Big|_0^{I_2} = \frac{t}{L}$$

$$\Rightarrow \quad \log_e\left(\frac{12 - I_2R_2}{12}\right) = -\frac{R_2t}{L}$$

$$\Rightarrow \quad 12 - I_2R_2 = 12e^{-\frac{R_2t}{L}}$$

$$\Rightarrow \quad I_2 = \frac{12}{R_2}\left(1 - e^{-\frac{R_2t}{L}}\right)$$

$$\Rightarrow \quad I_2 = 6\left(1-e^{-\frac{2t}{400\times10^{-3}}}\right)$$

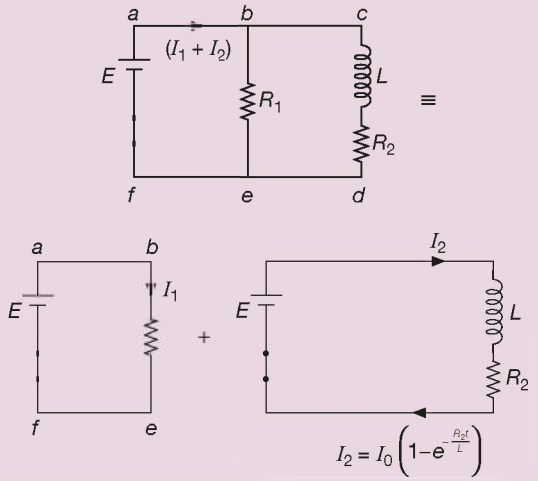
$$\Rightarrow \quad I_2 = 6\left(1-e^{-5t}\right)$$

So, potential difference across L at time t is

$$V_L = \left|L\frac{dI_2}{dt}\right| = L\left(30e^{-5t}\right) = (30)(0.4)e^{-5t}$$

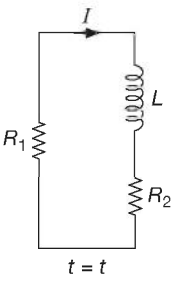
$$\Rightarrow \quad V_L = 12e^{-5t} \text{ V}$$

We could have obtained the result for I_2, by using the superposition principle,



(b) The steady state current in L or R_2 is

$$I = 6 \text{ A}$$



Now, as soon as the switch is opened, current in R_1 is reduced to zero immediately. But in L and R_2 it decreases exponentially, the new time constant of the circuit being

$$\tau'_L = \frac{L}{R_1+R_2} = \frac{0.4}{(2+2)} = 0.1 \text{ s}$$

So, current through R_1 at any time t is

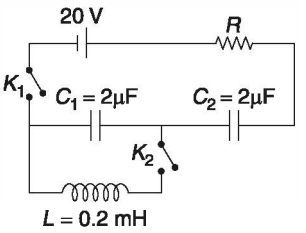
$$I' = I_0 e^{-t/\tau'_L} = 6e^{-t/0.1}$$

$$\Rightarrow \quad I' = 6e^{-10t} \text{ A}$$

Direction of current in R_1 is as shown in Figure i.e., clockwise.

PROBLEM 28

A circuit containing capacitors C_1 and C_2, is in the steady state with key K_1 closed and K_2 opened as shown in Figure.



At the instant $t = 0$, K_1 is opened and K_2 is closed. Calculate the angular frequency of oscillations of LC circuit. Find the first instant t, when energy in the inductor becomes one third that in the capacitor. Also calculate the charge on plates of the capacitor at that instant.

SOLUTION

With key K_1 closed, C_1 and C_2 are in series with the battery. In steady state, we have charges on the capacitors given by

$$q_0 = C_{eq}V = 1\times20 = 20 \ \mu\text{C}$$

With K_1 opened and K_2 closed, charge on C_2 will remain the same, whereas charge on C_1 will oscillate in the LC_1 circuit with frequency given by

$$\omega = \frac{1}{\sqrt{LC_1}}$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{0.2\times10^{-3}\times2\times10^{-6}}}$$

$$\Rightarrow \quad \omega = 5\times10^4 \text{ rads}^{-1}$$

Since at $t = 0$, charge is maximum i.e. q_0. Therefore, current will be zero at $t = 0$.

When energy in the inductor is one third that of capacitor, then

$$\frac{1}{2}Li^2 = \frac{1}{3}\left(\frac{1}{2}\frac{q^2}{C}\right)$$

$$\Rightarrow \quad i = \frac{q}{\sqrt{3LC}} = \frac{q\omega}{\sqrt{3}}$$

In LC oscillations, current in circuit at any instant is

$$i = \omega\sqrt{q_0^2 - q^2}$$

$$\Rightarrow \quad \frac{q\omega}{\sqrt{3}} = \omega\sqrt{q_0^2 - q^2}$$

$$\Rightarrow \quad q = \frac{\sqrt{3}}{2}q_0$$

Since at $t = 0$, charge is maximum i.e. q_0, so we have

$$q = q_0 \cos \omega t$$

$$\Rightarrow \quad \frac{\sqrt{3}q_0}{2} = q_0 \cos \omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6\times 5\times 10^4}$$

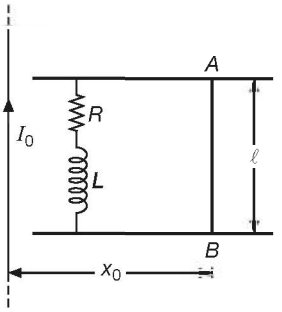
$$\Rightarrow \quad t = 1.05\times 10^{-5} \text{ s}$$

At this instant, charge on capacitor plates is

$$q = \frac{\sqrt{3}}{2}q_0 = \frac{\sqrt{3}}{2}\times 20 = 10\sqrt{3}\ \mu\text{C}$$

PROBLEM 29

A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in Figure.



A long straight wire, carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it made to slide on the rails away from the wire. Answer the following questions.

(a) Find a relation among I, $\frac{dI}{dt}$ and $\frac{d\phi}{dt}$, where I is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current I_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{I_1}{4}$ at time $2T$. Find the value of $\frac{L}{R}$ in terms of the other given quantities.

SOLUTION

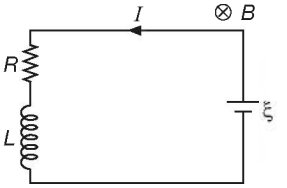
(a) Applying Kirchhoff's Second Law, we get

$$\xi - IR - L\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \frac{d\phi}{dt} - IR - L\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \frac{d\phi}{dt} = IR + L\frac{dI}{dt} \qquad \ldots(1)$$

This is the desired relation between I, $\frac{dI}{dt}$ and $\frac{d\phi}{dt}$



Equivalent Circuit

(b) Equation (1) can be written as

$$d\phi = IRdt + LdI$$

Integrating we get,

$$\Delta\phi = R\Delta q + LI_1$$

$$\Rightarrow \quad \Delta q = \frac{\Delta\phi}{R} - \frac{LI_1}{R} \qquad \ldots(2)$$

Here, $\Delta\phi = \phi_f - \phi_i = \int_{2x_0}^{x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x} l dx = \frac{\mu_0 I_0 l}{2\pi}\log_e(2)$

So, from equation (2), the charge that flows through the resistance up to time $t = T$, when current is I_1, is

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\log_e(2) - LI_1\right]$$

(c) This is the case of current decay in an LR circuit.

$\Rightarrow I = I_0 e^{-\Delta t/\tau_L}$ ...(3)

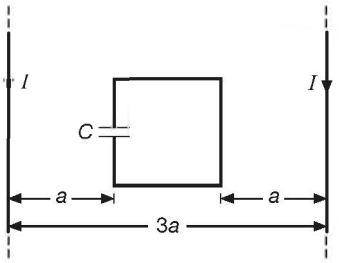
where, $I = \frac{I_1}{4}$, $I_0 = I_1$, $\Delta t = (2T - T) = T$ and $\tau_L = \frac{L}{R}$

Substituting these values in equation (3), we get

$$\tau_L = \frac{L}{R} = \frac{T}{\log_e(4)}$$

PROBLEM 30

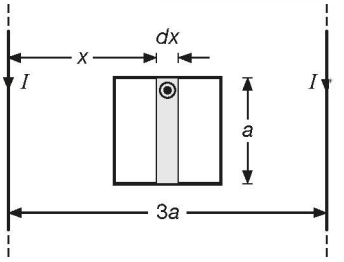
Two infinitely long parallel wires carrying currents $I = I_0 \sin(\omega t)$ in opposite directions are placed a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive.



SOLUTION

(a) For an infinitesimal strip of thickness dx at a distance x from left wire, net magnetic field (due to both wires) is

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a-x} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right)$$



Magnetic flux in this strip,

$$d\phi = BdA = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right)a dx$$

So, total flux $\phi = \int_a^{2a} d\phi = \frac{\mu_0 I a}{2\pi}\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a-x}\right)dx$

$$\Rightarrow \phi = \frac{\mu_0 I a}{\pi}\log_e(2)$$

$$\Rightarrow \phi = \frac{\mu_0 a \log_e(2)}{\pi}(I_0 \sin(\omega t)) \quad ...(1)$$

Magnitude of induced emf,

$$\xi = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \log_e(2)}{\pi}\cos(\omega t)$$

$$\Rightarrow \xi = \xi_0 \cos(\omega t)$$

where, $\xi_0 = \frac{\mu_0 a I_0 \omega \log_e(2)}{\pi}$

Charge stored in the capacitor,

$$Q = C\xi = C\xi_0 \cos(\omega t) \quad ...(2)$$

And current in the loop,

$$I = \frac{dQ}{dt} = C\omega\xi_0 \sin(\omega t) \quad ...(3)$$

$$\Rightarrow I_{max} = C\omega\xi_0 = \frac{\mu_0 a I_0 \omega^2 C \log_e(2)}{\pi}$$

(b) Since, magnetic flux passing through the square loop

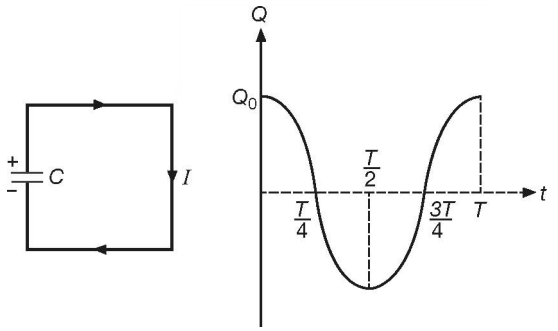
$\phi \propto \sin\omega t$ $\{\because$ of (1)$\}$

i.e., magnetic field passing through the loop is increasing at $t = 0$. Hence, the induced current will produce $\otimes$ magnetic field (from Lenz's Law). Or the current in the circuit at $t = 0$ will be clockwise (or negative as per the given convention). Therefore, charge on upper plate could be written as,

$$Q = +Q_0 \cos(\omega t) \qquad \{\because \text{ of } (2)\}$$

Where, $Q_0 = C\xi_0 = \dfrac{\mu_0 aCI_0\omega \log_e(2)}{\pi}$

The corresponding Q-t graph is shown in Figure.



PROBLEM 31

A non-conducting ring of mass m and radius R has a charge Q uniformly distributed over its circumference. The ring is placed on a rough horizontal surface such that plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0 t^2$ tesla is switched on. After 2 second from switching on the magnetic field the ring is just about to rotate about vertical axis through its centre.

(a) Find friction coefficient μ between the ring and the surface.

(b) If magnetic field is switched off after 4 second, then find the angle rotated by the ring before coming to stop after switching off the magnetic field.

SOLUTION

(a) Magnitude of induced electric field due to change in magnetic flux is given by

$$\oint \vec{E}\cdot d\vec{l} = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

$$\Rightarrow El = \pi R^2 (2B_0 t) \qquad \left\{\because \frac{dB}{dt} = 2B_0 t\right\}$$

$$\Rightarrow E(2\pi R) = 2\pi R^2 B_0 t$$

$$\Rightarrow E = B_0 Rt$$

Hence $F = QE = B_0 QRt$

This force is tangential to ring. The ring starts rotating when torque of this force is greater than the torque due to maximum friction $(f_{\max} = \mu mg)$ So, $\tau_F \geq \tau_{f_{\max}}$

Taking the limiting case. $\tau_F = \tau_{f_{\max}}$

$$\Rightarrow FR = (\mu mg)R$$

$$\Rightarrow F = \mu mg$$

$$\Rightarrow B_0 QRt = \mu mg$$

It is given that ring starts rotating after 2 second.

So, putting $t = 2$ second, we get $\mu = \dfrac{2B_0 RQ}{mg}$

(b) After 2 second, we have

$$\tau_F > \tau_{f_{\max}}$$

Therefore, net torque is

$$\tau = \tau_F - \tau_{f_{\max}} = B_0 QR^2 t - \mu mgR$$

Substituting $\mu = \dfrac{2B_0 QR}{mg}$, we get

$$\tau = B_0 QR^2 (t-2)$$

$$\Rightarrow I\alpha = I\left(\frac{d\omega}{dt}\right) = B_0 QR^2 (t-2)$$

$$\Rightarrow mR^2\left(\frac{d\omega}{dt}\right) = B_0 QR^2 (t-2)$$

$$\Rightarrow \int_0^{\omega} d\omega = \frac{B_0 Q}{m}\int_2^4 (t-2)\,dt$$

$$\Rightarrow \omega = \frac{2B_0 Q}{m} \qquad \ldots(1)$$

Now magnetic field is switched off i.e., only retarding torque is present due to the friction. So, angular retardation will be

$$\alpha = \frac{\tau_{f_{\max}}}{I} = \frac{\mu mgR}{mR^2} = \frac{\mu g}{R}$$

Since $\omega^2 = \omega_0^2 - 2\alpha\theta$

$$\Rightarrow 0 = \left(\frac{2B_0 Q}{m}\right)^2 - 2\left(\frac{\mu g}{R}\right)\theta$$

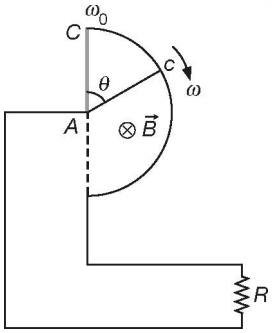
$$\Rightarrow \theta = \frac{2B_0^2 Q^2 R}{\mu m^2 g}$$

Substituting $\mu = \dfrac{2B_0 RQ}{mg}$, we get

$$\theta = \frac{B_0 Q}{m}$$

PROBLEM 32

A conducting rod AC of mass m, resistance r free to rotate in a horizontal plane about one end A over a semi-circular conducting ring of radius a is joined with an external resistance R as shown in Figure.



The rod is given an initial angular velocity ω_0. A uniform magnetic field of magnitude B exists perpendicular to the plane of semi-circular loop. Find the current in the circuit at angle θ. Neglect effect of gravity on the rod.

SOLUTION

Let at a time t, angular velocity of rod be ω. Then induced emf in the circuit is

$$\xi = \frac{B\omega a^2}{2}$$

and therefore, the induced current in the circuit is

$$i = \frac{\xi}{R+r} = \frac{B\omega a^2}{2(R+r)} \qquad \{\text{radially outwards}\}$$

Magnetic force on element dx, at a distance x from A is

$$F = (Bidx)$$

Torque on this element about A is given by

$$d\tau = Fx = Bixdx$$

So, total torque on the rod is

$$\tau = \int_{x=0}^{x=a} d\tau$$

$$\Rightarrow \quad \tau = \frac{Bia^2}{2} \qquad \{\text{anticlockwise}\}$$

Now angular retardation is

$$\alpha = \frac{\tau}{I}$$

where, $I = I_A = \frac{ma^2}{3}$

$$\Rightarrow \quad -\omega\frac{d\omega}{d\theta} = \frac{iBa^2}{2\left(\frac{ma^2}{3}\right)}$$

$$\Rightarrow \quad -\omega\frac{d\omega}{d\theta} = \frac{3}{2}\frac{Bi}{m} = \frac{3}{2}\frac{B}{m}\frac{B\omega a^2}{2(R+r)}$$

$$\Rightarrow \quad d\omega = -\frac{3}{4}\frac{B^2a^2}{m(R+r)}d\theta$$

$$\Rightarrow \quad \int_{\omega_0}^{\omega} d\omega = -\frac{3}{4}\frac{B^2a^2}{m(R+r)}\int_0^{\theta} d\theta$$

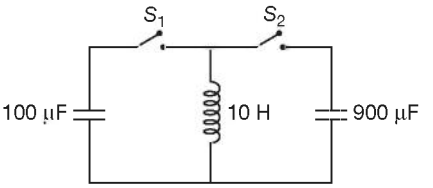
$$\Rightarrow \quad \omega = \omega_0 - \frac{3}{4}\frac{B^2a^2}{m(R+r)}\theta$$

Since, $i = \frac{B\omega a^2}{2(R+r)}$

$$\Rightarrow \quad i = \frac{Ba^2}{2(R+r)}\left(\omega_0 - \frac{3}{4}\frac{B^2a^2\theta}{m(R+r)}\right)$$

PROBLEM 33

Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged shown in Figure.



Then the switch S_2 is closed for a time t_1, after which it is opened and at the same instant switch S_1 is closed for a time t_2 and then opened. It is now found that the 100 μF capacitor is charged to 300 V. Find the minimum possible values of the time intervals t_1 and t_2.

SOLUTION

Let the energy stored in the 900 μF and the 100 μF capacitor be U_1 and U_2 respectively. Then

$$U_1 = \frac{1}{2}(900)(10^{-6})(100)^2 = 4.5 \text{ J}$$

$$U_2 = \frac{1}{2}(100)(10^{-6})(300)^2 = 4.5 \text{ J}$$

That is the entire energy of 900 μF capacitor has been transferred to the 100 μF capacitor.

First electrical energy of the 900 μF capacitor is converted into magnetic energy in the inductor and then this energy is converted into electrical energy once again using S_2 and S_1 appropriately. In a LC circuit the transfer of electrical energy into magnetic energy and vice-versa takes place in a time $\frac{T}{4}$ where $T = 2\pi\sqrt{LC}$ is the time period of the electrical oscillations. Thus

$$T_1 = 2\pi\sqrt{10\times900\times10^{-6}} = 0.6 \text{ s and}$$

$$T_2 = 2\pi\sqrt{10\times100\times10^{-6}} = 0.2 \text{ s}$$

Therefore, switch S_2 is first closed for time $t_1 = \frac{0.6}{4} =$ 0.15 s, during which time the 900 μF capacitor gets fully discharged and the current in the inductor is fully established. Next, the switch S_2 is opened and simultaneously switch S_1 is closed for time $t_2 = \frac{0.2}{4} = 0.05$ s during which the current in the inductor disappears and the 100 μF capacitor gets fully charged. After this time, the switch S_1 is also opened. The 100 μF capacitor is now charged to 300 V. So, we have

$$t_1 = 0.15 \text{ s and } t_2 = 0.05 \text{ s}$$

$$\Rightarrow \quad t_1 = 150 \text{ ms and } t_2 = 50 \text{ ms}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A vertically oriented, square loop of copper wire falls from a region where the field $\vec{B}$ is horizontal, uniform and perpendicular to the plane of the loop, into a region where the field is zero. The loop is released from rest and initially is entirely within the magnetic field region. Let the side length of the loop be l, diameter of the wire be d, the conductivity of copper be σ and the density of copper be ρ. If the loop reaches its terminal speed v_T while its upper segment is still in the magnetic field region, then

(A) $v_T = \dfrac{4\rho g}{\sigma B^2}$ (B) $v_T = \dfrac{8\sigma g}{\rho B^2}$

(C) $v_T = \dfrac{16\rho g}{\sigma B^2}$ (D) $v_T = \dfrac{8\rho g}{\sigma B^2}$

2. Magnetic flux (in weber) in a closed circuit of resistance $10\ \Omega$ varies with time t (in seconds) according to the equation $\phi = 6t^2 - 5t + 1$. What is the magnitude of the induced current at $t = 0.25$ seconds

(A) 1.2 A (B) 0.8 A

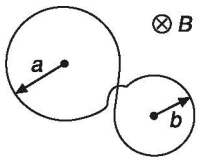
(C) 0.6 A (D) 0.2 A

3. The dimensions of self-inductance are

(A) $[MLT^{-2}A^{-2}]$ (B) $[ML^2T^{-1}A^{-2}]$

(C) $[ML^2T^{-2}A^{-2}]$ (D) $[ML^2T^{-2}A^{-1}]$

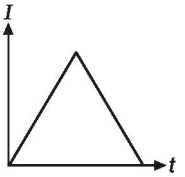
4. Figure shows a pattern of loops of radii a and b $(a > b)$ placed in a time varying magnetic field $\dfrac{dB}{dt} = k$, which is perpendicular to the plane of the loops. If the resistance per unit length of the wire is λ, then induced current is



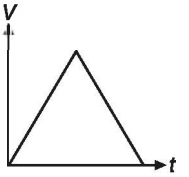
(A) $\dfrac{k(a^2+b^2)}{2\lambda(a+b)}$ (B) $\dfrac{k(a+b)}{2\lambda}$

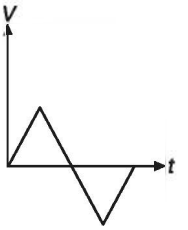
(C) $\dfrac{k(a-b)}{2\lambda}$ (D) $\dfrac{k(a^2-b^2)}{2\lambda}$

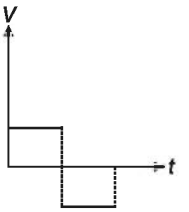
5. An alternating current I in an inductance coil varies with time t according to the graph as shown.

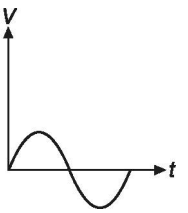


Which one of the following graphs gives the variation of voltage across inductance coil with time?

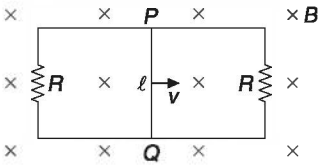
(A)


(B)


(C)


(D)


6. A rectangular loop circuit has a sliding wire PQ as shown. The loop is placed in a magnetic field B perpendicular to its plane. The resistance of the wire PQ is R. If the wire PQ moves with constant velocity v, then what is the current through the wire PQ?



(A) $\dfrac{Blv}{3R}$ (B) $\dfrac{Blv}{2R}$

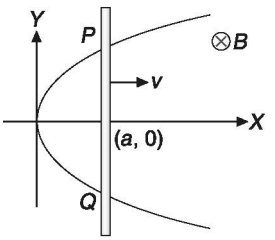
(C) $\dfrac{3Blv}{2R}$ (D) $\dfrac{2Blv}{3R}$

7. The dimensional formula for the ratio of electric flux to magnetic flux is same as that of

(A) $\frac{1}{\mu_0\varepsilon_0}$ (B) $\mu_0\varepsilon_0$

(C) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (D) $\sqrt{\mu_0\varepsilon_0}$

8. A rod PQ slides with constant velocity v on a wire that is shaped like a parabola $y^2 = 4ax$. At the instant the rod crosses the point $(a, 0)$, the induced emf in the rod is



(A) Bva (B) $2Bav$

(C) $3Bav$ (D) $4Bav$

9. A constant voltage is applied to a series R-L circuit by closing the switch. The voltage across inductor $(L = 2\text{ H})$ is 20 V at $t = 0$ and drops to 5 V at 20 ms. The value of R in Ω is

(A) $100\ln 2\ \Omega$ (B) $100(1-\ln 2)\ \Omega$

(C) $100\ln 4\ \Omega$ (D) $100(1-\ln 4)\ \Omega$

10. A coil of cross-sectional area 400 cm^2 having 30 turns is making 1800 revmin^{-1} in a magnetic field of 1 T. The peak value of the induced emf is

(A) 113 V (B) 226 V

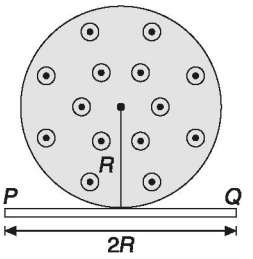
(C) 339 V (D) 452 V

11. In a gravity free space, a smooth insulating ring of radius R, with a bead having charge q is placed horizontally in a uniform magnetic field of induction B_0 and perpendicular to the plane of ring. Starting from $t = 0$, the magnetic field is varying with time as $B(t) = B_0 + \alpha t$, where α is a positive constant. The contact force between the ring and bead as a function of time is

(A) $\frac{\alpha q^2 Rt}{m}(2B_0 + \alpha t)$ (B) $\frac{\alpha q^2 Rt}{4m}(2B_0 + \alpha t)$

(C) $\frac{\alpha q^2 Rt}{4m}(B_0 + \alpha t)$ (D) $\frac{4\alpha q^2 Rt}{m}(2B_0 + \alpha t)$

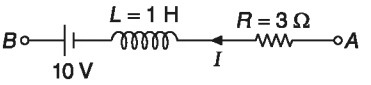
12. Magnetic field in a cylindrical region is given by $B = \alpha t$. Find emf across rod PQ as shown in Figure.



(A) αR^3 (B) $\pi\alpha R^2$

(C) $\frac{\pi}{2}\alpha R^2$ (D) $\frac{\pi}{4}\alpha R^2$

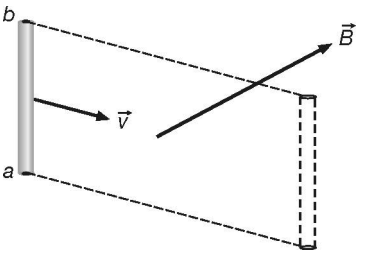
13. In the given branch AB of a circuit a current, $I = (10t + 5)$ A is flowing, where t is time in second. At $t = 0$, the potential difference between points A and $B(V_A - V_B)$ is



(A) 15 V (B) –5 V

(C) –15 V (D) 5 V

14. A straight wire of length $\vec{L}$ moves with constant velocity $\vec{v}$ (no rotation) through a uniform magnetic field $\vec{B}$ as shown in Figure.



$\vec{L}$ is a vector directed from a to b. ξ_i is the induced emf in the process.

(A) $\xi_i = 0$ (B) $\xi_i = \vec{B}\cdot(\vec{v}\times\vec{L})$

(C) $\xi_i = \vec{L}\cdot(\vec{B}\times\vec{v})$ (D) $\xi_i = \vec{B}\cdot(\vec{L}\times\vec{v})$

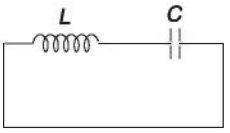
15. Two conducting rings of radius R and r $(r \ll R)$ have a common axis. The distance between them is x. If the smaller ring carries current I, then the mutual inductance of bigger ring due to smaller ring is

(A) $\dfrac{\mu_0\pi R^2r^2}{(R^2+x^2)^{\frac{3}{2}}}$ (B) $\dfrac{\mu_0\pi R^2r^2}{4(R^2+x^2)^{\frac{3}{2}}}$

(C) $\dfrac{\mu_0\pi R^2r^2}{16(R^2+x^2)^{\frac{3}{2}}}$ (D) $\dfrac{\mu_0\pi R^2r^2}{2(R^2+x^2)^{\frac{3}{2}}}$

16. Identify the incorrect statement. Induced electric field
(A) is produced by varying magnetic field
(B) is non-conservative in nature
(C) cannot exist in a region not occupied by magnetic field
(D) None of these

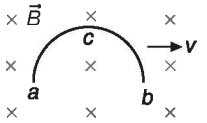
17. In an LC circuit, the capacitor has maximum charge q_0. The maximum value of rate of change of current is



(A) $\dfrac{q_0}{LC}$ (B) $\dfrac{q_0}{\sqrt{LC}}$

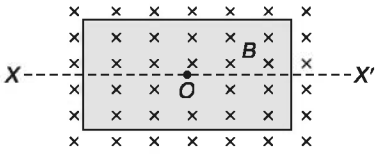
(C) $\dfrac{q_0}{LC}-1$ (D) $\dfrac{q_0}{LC}+1$

18. A semi-circular conducting ring acb of radius R moves with constant speed v in a plane perpendicular to uniform magnetic field B as shown in Figure. Identify the correct statement.



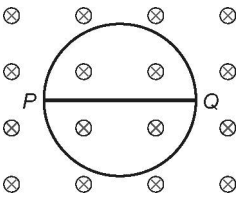
(A) $V_a - V_c = BRv$ (B) $V_b - V_c = BRv$
(C) $V_a - V_b = 0$ (D) None of these

19. A rectangular coil is placed in a region having a uniform magnetic field B perpendicular to the plane of the coil. An emf will not be induced in the coil if the



(A) magnetic field is increased uniformly
(B) magnetic field is switched off
(C) coil is rotated about the axis XX'
(D) coil is rotated about an axis perpendicular to the plane of the coil and passing through its centre O

20. A circular conducting loop of radius R and resistance per unit length λ is placed in a magnetic field decreasing at a constant rate α. Two points lying at the diametric ends are connected through a wire PQ. If I be the current in PQ then



(A) $I=\dfrac{R\alpha}{2\lambda}$ from P to Q

(B) $I=\dfrac{R\alpha}{2\lambda}$ from Q to P

(C) $I=\dfrac{2R\alpha}{\lambda}$ from P to Q

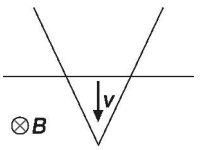
(D) $I=0$

21. A metal disc of radius R rotates with an axis perpendicular to its plane passing through its centre in a magnetic field of induction B acting perpendicular to the plane of the disc. The induced emf between the rim and the axis of the disc is

(A) $-B\pi R^2$ (B) $-\dfrac{2B\pi^2R^2}{\omega}$

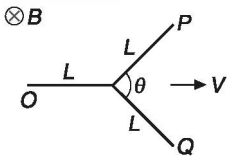
(C) $-B\pi R^2\omega$ (D) $-\dfrac{BR^2\omega}{2}$

22. A wire is bent in the form of a V shape and placed in a horizontal plane. There exists a uniform magnetic field B perpendicular to the plane of the wire. A uniform conducting rod starts sliding over the V shaped wire with a constant speed v as shown in Figure.



If the wire has no resistance, the current in rod will
(A) increase with time (B) decrease with time
(C) remain constant (D) always be zero

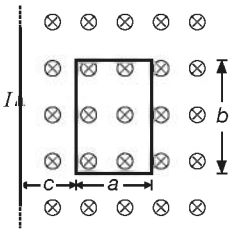
23. A conducting wire in the shape of Y with each side of length L is moving in a uniform field B with uniform speed V. The induced emf between P and Q of the wire will be



(A) ZERO (B) $2BLV$

(C) $2BLV\sin\left(\frac{\theta}{2}\right)$ (D) $2BLV\sin\theta$

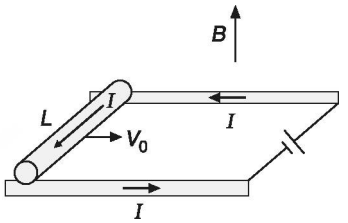
24. A rectangular loop of width a, length b is located at a distance c from a long straight wire carrying a current I. The wire is parallel to the long side of the loop. The magnetic flux through the loop is ϕ.



(A) $\phi = 0$ (B) $\phi = \frac{\mu_0 Ib}{2\pi}\ln\left(\frac{a}{c}\right)$

(C) $\phi = \frac{\mu_0 Ib}{2\pi}\ln\left(\frac{a+c}{c}\right)$ (D) $\phi = \frac{\mu_0 Ic}{2\pi}\ln\left(\frac{a+b}{b}\right)$

25. A constant current I flows through a metal rod of length L and mass m that slides on frictionless rails as shown in Figure.

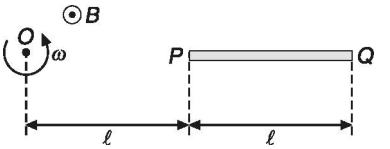


If the initial speed of the rod is V_0 and a magnetic field B acts vertically up, find the total distance moved by the rod before coming to a stop

(A) $\frac{mV_0^2}{BIL}$ (B) $\frac{mV_0^2}{2BIL}$

(C) $\frac{BLV_0}{m}$ (D) $\frac{BILV_0 m}{2}$

26. A rod of length l is rotating about an axis at distance l from one end as shown in Figure.

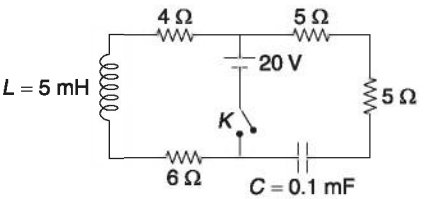


If angular speed of rod is ω and magnetic field is B_0 out of the page, find emf developed between end points of rod.

(A) $\frac{1}{2}B\omega l^2$ (B) $B\omega l^2$

(C) $\frac{3}{2}B\omega l^2$ (D) ZERO

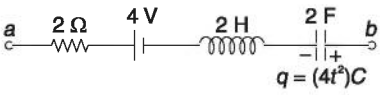
27. In the circuit shown, key (K) is closed at $t = 0$. The current through the key at the instant $t = 10^{-3}\ln 2$ s is



(A) 2 A (B) 3.5 A

(C) 2.5 A (D) 0

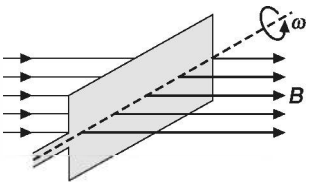
28. In the Figure shown, V_{ab} at $t = 1$ s is



(A) 30 V (B) –30 V

(C) 20 V (D) –20 V

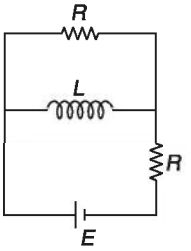
29. A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B with an angular velocity ω about an axis perpendicular to the field. If initially the plane of the coil is perpendicular to the field, then the average induced emf when its has rotated through 90° is



(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$

(C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

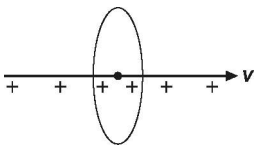
30. The Figure shows an L-R circuit, the time constant for the circuit is



(A) $\frac{L}{2R}$ (B) $\frac{2L}{R}$

(C) $\frac{2R}{L}$ (D) $\frac{R}{2L}$

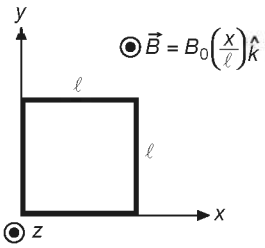
31. A very long uniformly charged non-conducting rod (linear charge density $= \lambda$) is moving with a constant velocity v through the centre of a circular conducting loop as shown. If radius of the loop is r, then induced emf in loop is



(A) $\frac{\mu_0 \lambda v r}{4}$ (B) $\frac{\mu_0 \lambda v r^2}{4}$

(C) $\frac{\mu_0 \lambda v r}{2}$ (D) ZERO

32. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{l}\right)\hat{k}$. A square loop of side l is placed with its edges along the x and y axis. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is



(A) $B_0 l v_0$ (B) $2B_0 l v_0$

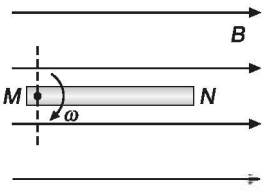
(C) $\frac{1}{2}(B_0 l v_0)$ (D) $\frac{1}{4}(B_0 l v_0)$

33. In a uniform magnetic field of induction B, a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(A) $\frac{B\pi r^2\omega}{2R}$ (B) $\frac{(B\pi r^2\omega)^2}{8R}$

(C) $\frac{(B\pi r\omega)^2}{2R}$ (D) $\frac{(B\pi r\omega^2)^2}{8R}$

34. A metallic rod of length l is hinged at the point M and is rotating about an axis perpendicular to the plane of paper with a constant angular velocity ω. A uniform magnetic field of intensity B is acting in the region (as shown in Figure) parallel to the plane of paper. The potential difference between the points M and N



(A) is always zero

(B) varies between $\frac{1}{2}B\omega l^2$ to 0

(C) is always $\frac{1}{2}B\omega l^2$

(D) is always $B\omega l^2$

35. In a region of space, magnetic field exists in a cylindrical region of radius a centred at origin with magnetic field along negative z-direction. The field is given by $\vec{B} = -(B_0 t)\hat{k}$. The force experienced by a stationary charge q placed at $(r, 0, 0)$, where $r > a$, is

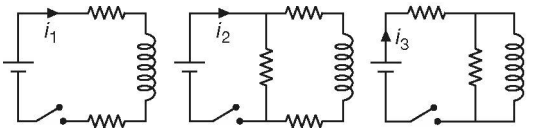
(A) qB_0 (B) $\frac{qB_0a^2}{2r}$

(C) $\frac{qB_0r}{2}$ (D) ZERO

36. A magnet is taken towards a conducting ring in such a way that a constant current of 10 mA is induced in it. The total resistance of the ring is $0.5\ \Omega$. In 5 s, the magnetic flux through the ring changes by

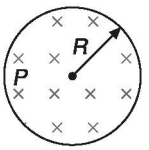
(A) 0.25 mWb (B) 25 mWb
(C) 50 mWb (D) 15 mWb

37. Figure shows three circuits with identical batteries, inductors and resistors. Rank the circuits according to the currents thought the battery just after the switch is closed, greatest first.



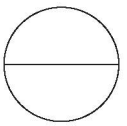
(A) $i_2 > i_3 > i_1$ (B) $i_2 > i_1 > i_3$
(C) $i_1 > i_2 > i_3$ (D) $i_1 > i_3 > i_2$

38. The Figure shows a circular region of radius R occupied by a time varying magnetic field $\vec{B}(t)$ such that $\frac{dB}{dt} < 0$. The magnitude of induced electric field at the point P at a distance $r < R$ is

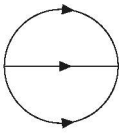
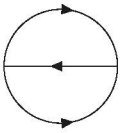
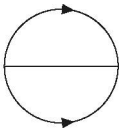
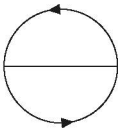


(A) decreasing with r (B) increasing with r
(C) not varying with r (D) varying as r^{-2}

39. A wire loop consists of two parts as shown in Figure.



It is placed in a magnetic field directed into the plane of the paper and increases with time. Which of the following shows the induced current correctly?

(A) 
(B) 
(C) 
(D) 

40. A non-conducting ring of radius r has charge q. A magnetic field perpendicular to the plane of the ring changes at the rate $\frac{dB}{dt}$. The torque experienced by the ring is

(A) ZERO (B) $qr^2\frac{dB}{dt}$
(C) $\frac{1}{2}qr^2\frac{dB}{dt}$ (D) $\pi qr^2\frac{dB}{dt}$

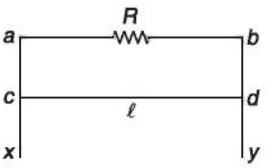
41. A ring of radius r is spun about its axis with an angular velocity ω in a horizontal plane. A uniform magnetic field of magnitude B exists in the region in vertical direction. the emf induced in the ring is

(A) ZERO (B) $\pi r^2\omega B$
(C) $\left(\frac{1}{2}\right)Br^2\omega$ (D) $Br^2\omega$

42. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\phi = at(\tau - t)$. Find the amount of heat generated in the loop during that time

(A) $\frac{a\tau^2}{2R}$ (B) $\frac{a^2\tau^3}{3R}$
(C) $\frac{2a^2\tau^3}{3R}$ (D) $\frac{a\tau}{3R}$

43. A wire cd of length l and mass m is sliding without friction on conducting rails ax and by as shown in Figure.



The vertical rails are connected to each other with a resistance R between a and b. A uniform magnetic field B is applied perpendicular to the plane $abcd$ such that cd moves with a constant velocity of

(A) $\frac{mgR}{Bl}$ (B) $\frac{mgR}{B^2l^2}$
(C) $\frac{mgR}{B^2l^3}$ (D) $\frac{mgR}{B^2l}$

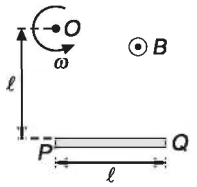
44. Two long parallel wires have radii equal to a and are at distance d measured between their centres as shown in Figure.

Their far ends are connected together so that the wires form two sides of the same loop. For a section far from the ends, if the flux within the wires is neglected, then the self-inductance per meter is

(A) $L=\frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$ (B) $L=\frac{\mu_0}{\pi}\ln\left(\frac{a}{d-a}\right)$

(C) $L=\frac{\mu_0}{\pi}\ln\left(\frac{d+a}{a}\right)$ (D) $L=\frac{\mu_0}{\pi}\ln\left(\frac{a}{d+a}\right)$

45. A rod of length l is rotating about an axis at P distance l from one end as shown in Figure.



If angular speed of rod is ω and magnetic field is B_0 out of the page, find emf developed between end points of rod.

(A) $\frac{1}{2}B\omega l^2$ (B) $B\omega l^2$

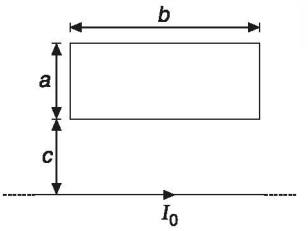
(C) $\frac{3}{2}B\omega l^2$ (D) ZERO

46. In the steady state condition, the rate of heat produced in a choke coil is P. The time constant of the choke coil is τ. If now the choke coil is short-circuited, then the total heat dissipated in the coil is

(A) $P\tau$ (B) $\frac{1}{2}P\tau$

(C) $\frac{P\tau}{\ln 2}$ (D) $P\tau\ln 2$

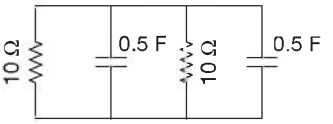
47. A rectangular loop of side a and b has a resistance R and lies at a distance c from an infinite straight wire carrying current I_0. The current decreases to zero in time t_0. $I(t)=I_0\left[\frac{t_0-t}{t_0}\right]$, $0<t<t_0$. The charge flowing through the rectangular loop is



(A) $\mu_0 I_0 t_0$ (B) $\mu_0 I_0 \frac{ab}{c^2}t_0$

(C) $\frac{\mu_0 bI_0}{2\pi R}\ln\left(\frac{a+c}{c}\right)$ (D) $\frac{\mu_0 It_0}{R}\ln\left(\frac{ab}{c^2}\right)$

48. The time constant of the circuit shown in Figure is



(A) 5 s (B) 10 s

(C) $\frac{1}{5}$ s (D) 0.1 s

49. A copper ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is

(A) equal to that due to gravity
(B) less than that due to gravity
(C) more than that due to gravity
(D) depends on the diameter of the ring and the length of the magnet

50. A non-conducting ring of mass m, radius R, having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying magnetic field $B=8t^2$ is switched on at time $t=0$. After 3 s, the ring starts rotating. The coefficient of friction between the ring and the table is

(A) $\frac{1}{24}\left(\frac{qR}{mg}\right)$ (B) $\frac{qR}{16mg}$

(C) $\frac{16qR}{mg}$ (D) $\frac{24qR}{mg}$

51. When a wheel with metal spokes, 1 m long, is rotated in a magnetic field of flux density 5×10^{-4} Wbm^{-2} normal to the plane of the wheel, an emf of 0.01 V is induced between the rim and the axle. The frequency of rotation of the wheel is

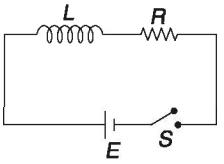
(A) 10π revolutions per second

(B) $\frac{20}{\pi}$ revolutions per second

(C) $\frac{10}{\pi}$ revolutions per second

(D) 20π revolutions per second

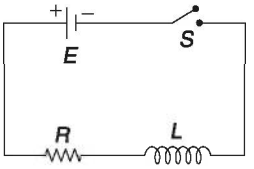
52. In the circuit shown in Figure, $L = 10$ H, $R = 5\ \Omega$, $E = 15$ V. The switch S is closed at $t = 0$. At $t = 2$ s, the current in the circuit is

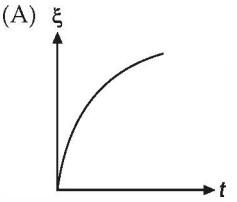


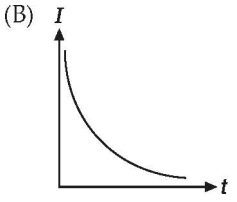
(A) $3\left(1-\frac{1}{e}\right)$ A (B) $3\left(1-\frac{1}{e^2}\right)$ A

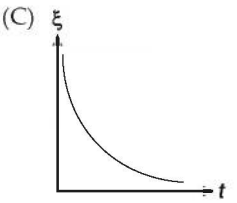
(C) $3\left(\frac{1}{e}\right)$ A (D) $3\left(\frac{1}{e^2}\right)$ A

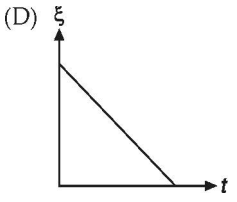
53. Switch S of the circuit shown in Figure is closed at $t = 0$. If ξ denotes the induced emf in L and I the current flowing through the circuit at time t, then which of the following graphs is correct?



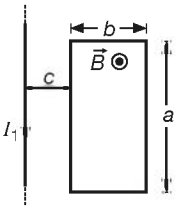
(A) 

(B) 

(C) 

(D) 

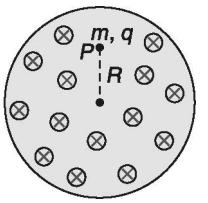
54. The mutual inductance between the rectangular loop and the long straight wire as shown in Figure is M.



(A) $M = \text{ZERO}$ (B) $M = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{c}{b}\right)$

(C) $M = \frac{\mu_0 b}{2\pi}\ln\left(\frac{a+c}{b}\right)$ (D) $M = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)$

55. A charge q, mass m is lying in a cylindrical region of magnetic field at point P. If magnetic field in the region is given by B_0 and P is at distance R from centre of region, find speed of charge if the magnetic field is suddenly switched off



(A) $\frac{qRB_0}{m}$ (B) $\frac{2qRB_0}{3m}$

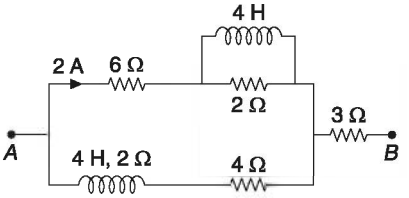
(C) $\frac{qRB_0}{2m}$ (D) $\frac{qRB_0}{3m}$

56. A flat circular coil of n turns, area A and resistance R is placed in a uniform magnetic field B. The plane of coil is initially perpendicular to B. When the coil is rotated through an angle of $180°$ about one of its diameter, a charge Q_1 flows through the coil. When the same coil after being brought to its initial position, is rotated through an angle of $360°$ about the same axis a charge Q_2 flows through it. Then, $\frac{Q_2}{Q_1}$ is

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) 0

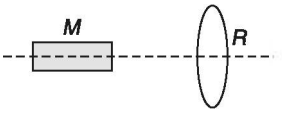
57. Find $V_A - V_B$ in steady state



(A) 8 V (B) 16 V

(C) 24 V (D) Data is insufficient

58. A conducting ring R is placed on the axis of a bar magnet M. The plane of R is perpendicular to this axis. M can move along this axis. Then,



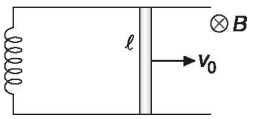
(A) M will repel R when it is moving towards R

(B) M will attract R when it is moving towards R

(C) M will repel R when moving towards as well as away from R

(D) M will attract R when moving towards as well as away from R

59. Two ends of an inductor of inductance L are connected to two parallel conducting wires. A rod of length l and mass m is given velocity v_0 as shown in Figure.

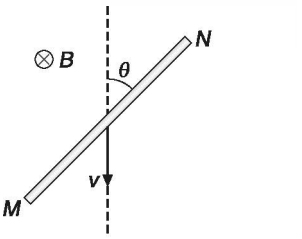


The whole system is placed in perpendicular magnetic field B. The maximum current in the inductor is (Neglect gravity and friction)

(A) $\frac{mv_0}{L}$ (B) $\sqrt{\frac{m}{L}}v_0$

(C) $\frac{mv_0^2}{L}$ (D) $2\sqrt{\frac{m}{L}}v_0$

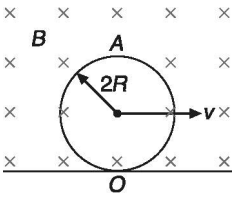
60. A conducting rod MN of length $2l$ moves through a uniform magnetic field of induction B, acting into the plane of the paper. The rod makes an angle θ with the vertical while it falls with a uniform linear velocity v. The potential difference between the two ends of the rod equals



(A) ZERO (B) $2Bvl\sin\theta$

(C) $2Bvl$ (D) Bvl

61. A conducting ring of radius $2R$ rolls on a smooth horizontal conducting surface as shown in Figure.

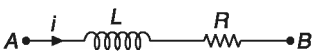


A uniform horizontal magnetic field B is perpendicular to the plane of the ring. The potential of A with respect to O is

(A) $2BvR$ (B) $\frac{1}{2}BvR$

(C) $8BvR$ (D) $4BvR$

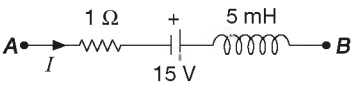
62. In a part of circuit shown in Figure, $R = 0.2\ \Omega$. At the instant shown, $V_A - V_B = 0.5$ V, $i = 0.5$ A, $\frac{di}{dt} = 8\ \text{As}^{-1}$. Then inductance L is



(A) 0.01 H (B) 0.02 H

(C) 0.05 H (D) 0.5 H

63. The network shown in Figure is part of a complete circuit. If at a certain instant, the current I is 5 A and it is decreasing at a rate of $10^3\ \text{As}^{-1}$ then $V_B - V_A$ equals



(A) 20 V (B) 15 V

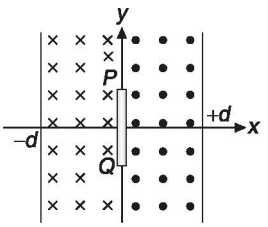
(C) 10 V (D) 5 V

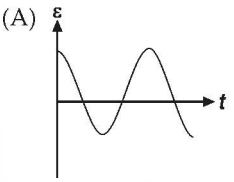
64. In PROBLEM 63, if I is reversed in direction, then $V_B - V_A$ equals

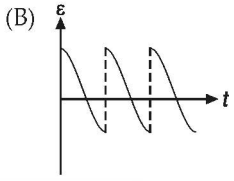
(A) 5 V (B) 10 V

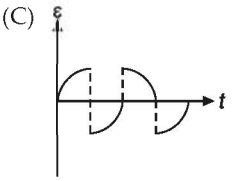
(C) 15 V (D) 20 V

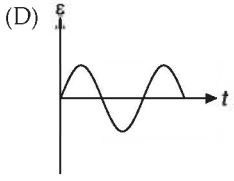
65. A straight conducting rod PQ is executing SHM in xy plane from $x=-d$ to $x=+d$. Its mean position is $x=0$ and its length is along y-axis. There exists a uniform magnetic field B from $x=-d$ to $x=0$ pointing inward normal to the paper and from $x=0$ to $x=+d$ there exists another uniform magnetic field of same magnitude B but pointing outward normal to the plane of the paper. At the instant $t=0$, the rod is at $x=0$ and moving to the right. The variation of induced EMF across the rod PQ with time is best represented as



(A) 

(B) 

(C) 

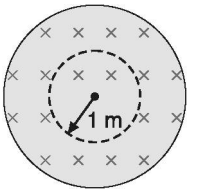
(D) 

66. An LR circuit is connected to a battery at time $t=0$. The energy stored in the inductor reaches half its maximum value at time

(A) $\frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$ (B) $\frac{L}{R}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

(C) $\frac{R}{L}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$ (D) $\frac{R}{L}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

67. A uniform but time varying magnetic field exists in a cylindrical region as shown in Figure. The direction of magnetic field is into the plane of the paper and its magnitude is decreasing at a constant rate of $2\times10^{-3}\ \text{Ts}^{-1}$. A particle of charge $1\ \mu\text{C}$ is moved slowly along a circle of radius 1 m by an external force as shown in Figure.

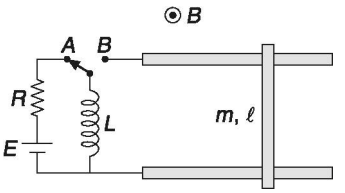


The plane of the circle lies in the plane of the paper and it is concentric with the cylindrical region. The work done by the external force in moving this charge along the circle will be

(A) ZERO (B) $2\pi\times10^{-9}$ J

(C) $\pi\times10^{-9}$ J (D) $4\pi\times10^{-6}$ J

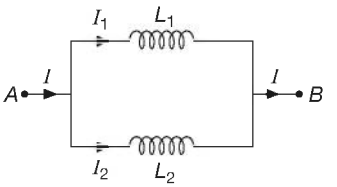
68. Switch is in position A for a long time. It is suddenly switched to the side B. Mass of rod is m and length is l. Maximum speed of rod will be



(A) $\left(\frac{E}{R}\right)\sqrt{\frac{L}{m}}$ (B) $\left(\frac{E}{R}\right)\sqrt{\frac{m}{L}}$

(C) $2\left(\frac{E}{R}\right)\sqrt{\frac{L}{m}}$ (D) $2\left(\frac{E}{R}\right)\sqrt{\frac{m}{L}}$

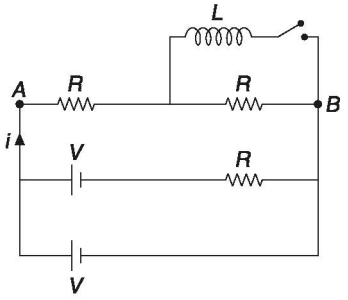
69. The parallel combination of the inductors shown is connected across a battery that delivers a current I to the combination. If I_1 and I_2 be the currents in the branches containing L_1 and L_2, then



(A) $\frac{I_1}{I_2}=\frac{L_2}{L_1}$ (B) $\frac{I_1}{I_2}=\sqrt{\frac{L_2}{L_1}}$

(C) $\frac{I_1}{I_2}=\frac{L_1}{L_2}$ (D) $\frac{I_1}{I_2}=\frac{L_1}{L_1+L_2}$

70. Key is closed at time $t = 0$. If $i = i_1$ at $t = 0$ and $i = i_2$ at time $t = \infty$, then $\frac{i_1}{i_2}$ equals



(A) 3 (B) $\frac{1}{3}$

(C) 1 (D) $\frac{1}{2}$

71. In a decaying L-R circuit, the time after which energy stored in the inductor reduces to one-fourth of its initial value is

(A) $(\ln 2)\frac{L}{R}$ (B) $0.5\frac{L}{R}$

(C) $\sqrt{2}\frac{L}{R}$ (D) $\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)\frac{L}{R}$

72. The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in time Δt. Then the total quantity of electric charge Q that during this time passes any point of the circuit is given by

(A) $Q = \frac{\Delta\phi}{\Delta t}$ (B) $Q = R\frac{\Delta\phi}{\Delta t}$

(C) $Q = \frac{\Delta\phi}{\Delta t} + R$ (D) $Q = \frac{\Delta\phi}{R}$

73. The magnetic flux density B is changing in magnitude at a constant rate $\frac{dB}{dt}$. A given mass m of copper, drawn into a wire of radius a and formed into a circular loop of radius r is placed perpendicular to the field B. The induced current in the loop is i. The resistivity of copper is ρ and density is d.

(A) $i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$ (B) $i = \frac{m}{4\pi a^2 r}\frac{dB}{dt}$

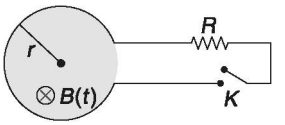
(C) $i = \frac{m}{4\pi a d}\frac{dB}{dt}$ (D) $i = \frac{m}{2\pi\rho d}\frac{dB}{dt}$

74. A conducting square loop is placed in a magnetic field B with its plane perpendicular to the field. Somehow the sides of the loop start shrinking at a constant rate α. The induced emf in the loop at an instant when its side has length a is

(A) $2a\alpha B$ (B) $a^2\alpha B$

(C) $2a^2\alpha B$ (D) $a\alpha B$

75. A circular loop of radius r has resistance R. A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil as shown in Figure.



If the key K is closed, the power developed in the coil just after closing the key is

(A) $\frac{B_0^2\pi r^2}{R}$ (B) $\frac{B_0 10 r^3}{R}$

(C) $\frac{B_0^2\pi^2 r^4 R}{5}$ (D) $\frac{B_0^2\pi^2 r^4}{R}$

76. An emf of 15 volt is applied in a circuit containing 5 henry inductance and 10 Ω resistance. The ratio of the current at time $t \to \infty$ and $t = 1$ second is

(A) $\frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}}-1}$ (B) $\frac{e^2}{e^2-1}$

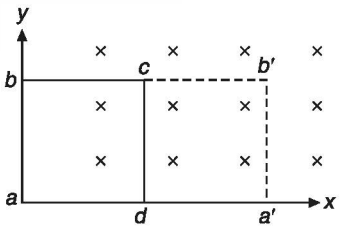
(C) $1 - e^{-1}$ (D) e^{-1}

77. Two coils have a mutual inductance of 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin\omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf (in volt) in the second coil is

(A) 2π (B) 5π

(C) π (D) 4π

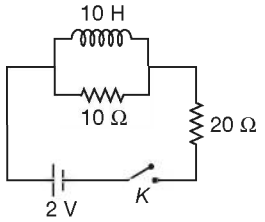
78. A square loop $abcd$ of side l is placed as shown in Figure with point a lying at origin. A magnetic field $B = B_0 x(-\hat{k})$ exists in the space. The change in flux $|\Delta\phi|$ when the loop is rotated through 180° about cd (as shown in the dotted lines in Figure) is



(A) ZERO (B) $2B_0l^3$

(C) $3B_0l^3$ (D) B_0l^3

79. Two resistors of 10 Ω and 20 Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown in Figure.



The key K is inserted at time $t = 0$. The initial $(t = 0)$ and final $(t \to \infty)$ currents through battery are

(A) $\frac{1}{15}$ A, $\frac{1}{10}$ A (B) $\frac{1}{10}$ A, $\frac{1}{15}$ A

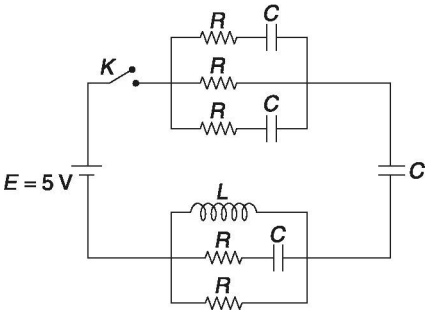
(C) $\frac{2}{15}$ A, $\frac{1}{10}$ A (D) $\frac{1}{15}$ A, $\frac{2}{25}$ A

80. An inductor L and a resistor R are connected in series with a direct current source of emf E. The maximum rate at which energy is stored in the magnetic field is

(A) $\frac{E^2}{4R}$ (B) $\frac{E^2}{R}$

(C) $\frac{4E^2}{R}$ (D) $\frac{2E^2}{R}$

81. In the circuit shown, initially all the capacitors are uncharged. Given that $R = 6\ \Omega$ and $C = 4\ \mu F$, the current passing through battery immediately after key (K) is closed is



(A) 1 A (B) 5 A

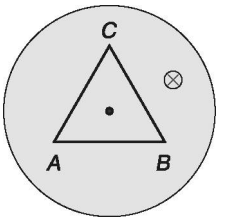
(C) 3 A (D) 2 A

82. A thin circular ring of area A is held perpendicular to a uniform magnetic field B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(A) $\frac{B^2A}{R^2}$ (B) ABR

(C) $\frac{AB}{R}$ (D) $\frac{A^2B^2}{R}$

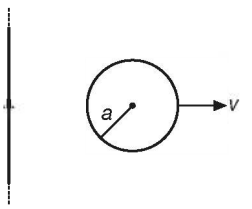
83. A triangular wire frame having each side equal to 2 m is placed in a region of time varying magnetic field having $\frac{dB}{dt} = \sqrt{3}\ \text{Ts}^{-1}$. The magnetic field is perpendicular to the plane of the triangle as shown in Figure. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be



(A) 0.4 V (B) 0.6 V

(C) 1.2 V (D) None of these

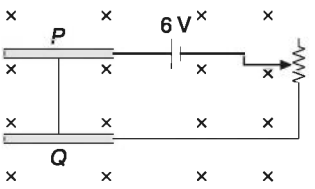
84. A circular loop of radius a is placed in the same plane as a long straight wire carrying a current I. The centre of the loop is at a distance r from the wire where $r \gg a$. The loop is moved away from the wire with a constant velocity v. The induced emf in the loop is



(A) $\xi = \frac{\mu_0 Iv}{2\pi}$ (B) $\xi = \frac{\mu_0 Ia^2v}{2\pi r^2}$

(C) $\xi = \frac{\mu_0 Ia^2v}{2r^2}$ (D) $\xi = \frac{\mu_0 Ia^3v}{2r^3}$

85. A metal wire PQ having a mass of 10 g and length 4.9 cm lies at rest on the metal rails as shown in Figure.



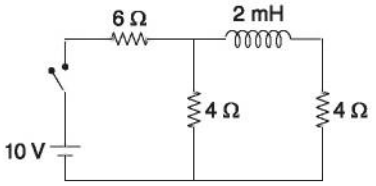
A vertically downward magnetic field of magnitude 0.8 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below $20\ \Omega$, the wire starts sliding on the rails. The coefficient of friction is

(A) 0.25 (B) 0.31
(C) 0.43 (D) 0.12

86. When a loop moves towards a stationary magnet with speed v, the induced emf in the loop is E. If the magnet also moves away from the loop with the same speed, then the emf induced in the loop is

(A) E (B) $2E$
(C) $\frac{E}{2}$ (D) ZERO

87. In the given circuit, find the ratio of i_1 and i_2, where i_1 is the initial (at $t = 0$) current and i_2 is steady state (at $t = \infty$) current through the battery is

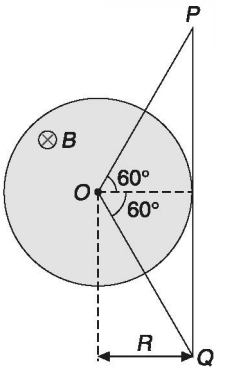


(A) 1.0 (B) 0.8
(C) 1.2 (D) 1.5

88. The time constant of an inductance coil is 2×10^{-3} s. When a 90 Ω resistance is joined in series, the same constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are

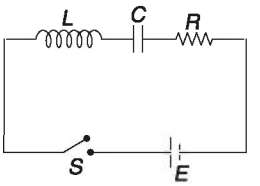
(A) 30 mH; 30 Ω (B) 30 mH; 60 Ω
(C) 60 mH; 30 Ω (D) 60 mH; 60 Ω

89. In a cylindrical region having radius R, magnetic field varies with time as $B = a + bt$. OPQ is a triangular loop made of wire having resistance per unit length λ. Current induced in the loop is



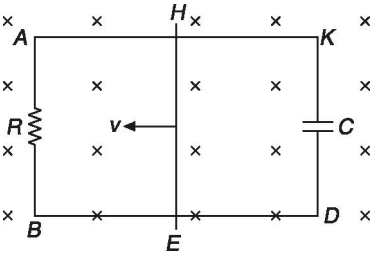
(A) $\frac{\pi Rb(2-\sqrt{3})}{6\lambda}$ (B) $\frac{\sqrt{3}Rb}{2\lambda(2+\sqrt{3})}$
(C) $\frac{\pi Rb\sqrt{3}}{6\lambda(2+\sqrt{3})}$ (D) $\frac{\pi Rb(2+\sqrt{3})}{6\lambda}$

90. When the switch S is closed at $t = 0$, identify the correct statement just after closing the switch as shown in Figure.



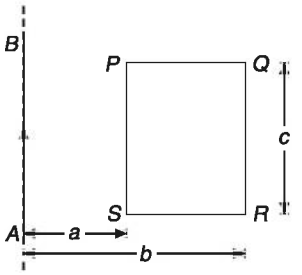
(A) The current in the circuit is maximum
(B) Equal and opposite voltages are dropped across inductor and resistor
(C) The entire voltage is dropped across inductor
(D) All of these

91. In the circuit shown in Figure, a conducting wire HE is moved with a constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of circuit inwards. The current in $HKDE$ after a long time is



(A) clockwise (B) anticlockwise
(C) alternating (D) zero

92. AB is an infinitely long wire placed in the plane of rectangular coil of dimensions as shown in Figure.

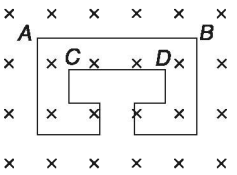


Calculate the mutual inductance of wire AB and coil $PQRS$

(A) $\frac{\mu_0 b}{2\pi}\ln\frac{a}{b}$ (B) $\frac{\mu_0 c}{2\pi}\ln\frac{b}{a}$

(C) $\frac{\mu_0 abc}{2\pi(b-a)^2}$ (D) None of these

93. A wire is bent to form a double loop shown in Figure. There is a uniform magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, current will flow from.



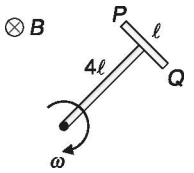
(A) A to B and C to D

(B) B to A and D to C

(C) A to B and D to C

(D) B to A and C to D

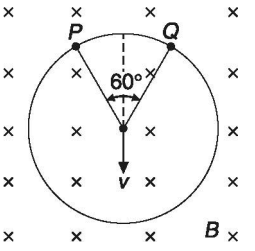
94. In the Figure shown, a T shaped conductor moves with constant angular velocity ω in a plane perpendicular to uniform magnetic field $\vec{B}$. The potential difference $V_P - V_Q$ is



(A) ZERO (B) $\frac{1}{2}B\omega l^2$

(C) $2B\omega l^2$ (D) $B\omega l^2$

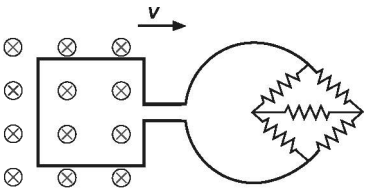
95. A conducting ring of radius a falls vertically downward with a velocity v in a magnetic field B. The potential difference between two points P and Q located symmetrically on both sides of the vertical has the value



(A) ZERO (B) $\frac{Bva}{2}$

(C) $\frac{\sqrt{3}}{2}Bva$ (D) Bva

96. A square metal loop of side 10 cm and resistance 1 Ω is moved with a constant velocity partly inside a magnetic field of 2 Wbm^{-2}, directed into the paper, as shown in Figure.

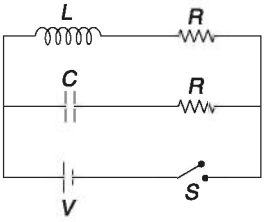


This loop is connected to a network of five resistors each of value 3 Ω. If a steady current of 1 mA flows in the loop, then the speed of the loop is

(A) 0.5 cms^{-1} (B) 1 cms^{-1}

(C) 2 cms^{-1} (D) 4 cms^{-1}

97. In the circuit shown in Figure, $R=\sqrt{\frac{L}{C}}$. Switch S is closed at time $t=0$. The current through C and L would be equal after a time t equal to



(A) CR (B) $CR\ln(2)$

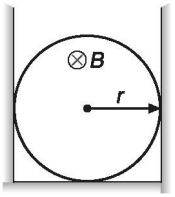
(C) $\frac{L}{R\ln(2)}$ (D) LR

98. An inductance of 2 H carries a current of 2 A. To prevent sparking when the circuit is broken a capacitor of $4\ \mu F$ is connected across the inductance. The voltage rating of the capacitor is of the order of

(A) 10^3 V (B) 10 V

(C) 10^5 V (D) 10^6 V

99. A uniformly charged, insulating ring of charge Q, mass m and radius r is placed as shown in Figure.



Vertical surfaces are smooth and the coefficient of friction between horizontal surface and ring is μ. Uniform magnetic field is present, in the perpendicular direction of the plane of ring. If magnetic field changes at the rate $\frac{dB}{dt} = kt$, $(k = 1\ \text{Tsec}^{-2})$, the time after which ring starts motion is

(A) $\frac{\mu mg}{3rQ}$ (B) $\frac{2\mu mg}{rQ}$

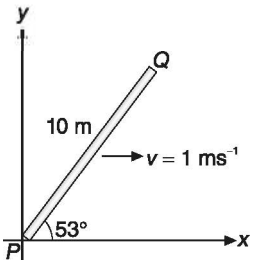
(C) $\frac{\mu mg}{rQ}$ (D) $\frac{3\mu mg}{rQ}$

100. A coil has an inductance of 50 mH and a resistance of $0.3\ \Omega$. If a 12 V emf is applied across the coil, the energy stored in the magnetic field after the current has built up to its steady state value is

(A) 40 J (B) 40 mJ

(C) 20 J (D) 20 mJ

101. A conducting rod PQ of length 10 m oriented as shown in Figure is moving with velocity $1\hat{i}\ \text{ms}^{-1}$ without any rotation in a uniform magnetic induction of $(3\hat{j} + 4\hat{k})$ T. The emf induced across the rod is



(A) 32 V (B) 40 V

(C) 50 V (D) 64 V

102. A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B with an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced emf when it has rotated through 90° is

(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$

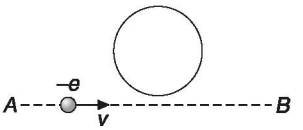
(C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

103. A circuit contains an inductance L, a resistance R and a battery of emf E. The circuit is switched on at $t = 0$. The charge flown through the battery in one time constant (τ) is

(A) $\frac{2E\tau}{\text{Re}}$ (B) $\frac{E\tau}{2\text{Re}}$

(C) $\frac{E\tau}{Re}$ (D) ZERO

104. An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in Figure.



What will be the direction of the current induced in the loop?

(A) No current will be induced

(B) The current will be clockwise

(C) The current will anticlockwise

(D) The current will change direction as the electron passes by

105. When the number of turns in the two circular coils closely wound are doubled (in both), their mutual inductance becomes

(A) four times (B) two times

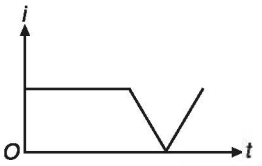
(C) remains same (D) sixteen times

106. In a DC motor, if E is the applied emf and e is the back emf, then the efficiency is

(A) $\frac{(E-e)}{E}$ (B) $\frac{e}{E}$

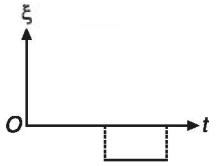
(C) $\left\{\frac{(E-e)}{E}\right\}^2$ (D) $\left(\frac{e}{E}\right)^2$

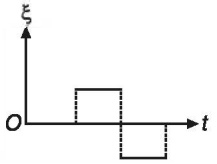
107. The current i in an induction coil varies with time t according to the graph shown in Figure.

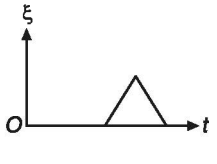


Which of the following graphs shows the induced emf (ξ) in the coil with time?

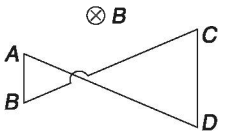
(A)

(B) 

(C) 

(D) 

108. A conducting wire frame is placed in a magnetic field, which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



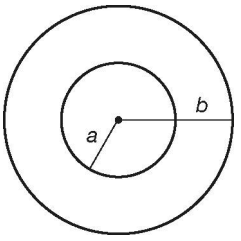
(A) A to B and C to D

(B) B to A and C to D

(C) A to B and D to C

(D) B to A and D to C

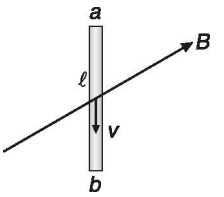
109. Two concentric and coplanar circular coils have radii a and $b(\gg a)$ as shown in Figure. Resistance of the inner coil is R. Current in the outer coil is increased from zero to I. The total charge circulating the inner coil is Q. Then Q equals



(A) $\dfrac{\mu_0 Iab}{2R}$ (B) $\dfrac{\mu_0 Ia^2}{2Rb}$

(C) $\dfrac{\mu_0 \pi Ib^2}{2aR}$ (D) $\dfrac{\mu_0 \pi Ia^2}{2bR}$

110. A conducting rod of length l falls vertically under gravity in a region of uniform magnetic field $\vec{B}$. The field vectors are inclined at an angle θ with the horizontal as shown in Figure.

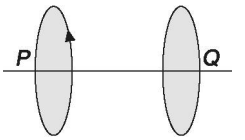


If the instantaneous velocity of the rod is v, the induced emf in the rod ab is

(A) Blv (B) $Blv\cos\theta$

(C) $Blv\sin\theta$ (D) ZERO

111. Two circular loops P and Q are placed with their planes parallel to each other. A current is flowing through P. If this current is increased, then



(A) the loops will attract each other
(B) the loops will repel each other
(C) the loops will neither attract nor repel each other
(D) loop Q will start rotating

112. A capacitor of capacitance $2\ \mu F$ is first charged and then discharged through a resistance of $5\times10^4\ \Omega$. The time in which the charge on the capacitor will fall to 36.8% of its initial value is

(A) 0.0001 second (B) 0.001 second

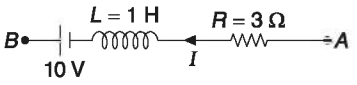
(C) 0.01 second (D) 0.1 second

113. A short magnet is allowed to fall from rest along the axis of a horizontal conducting ring. The distance fallen by the magnet in one second may be

(A) 5 m (B) 6 m

(C) 4 m (D) 7 m

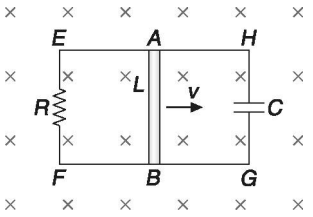
114. In the given branch AB of a circuit, a current $I=(10t+5)$ A is flowing, where t is time in second. At $t=0$, the potential difference between points A and B i.e. (V_A-V_B) is



(A) 15 V (B) –5 V

(C) –15 V (D) 5 V

115. A conducting rod is moving with a constant velocity v over the parallel conducting rails which are connected at the ends through a resistor R and capacitor C as shown in Figure.



Magnetic field B is into the plane. Consider the following statements.

(i) Current in loop $AEFBA$ is anti-clockwise.

(ii) Current in loop $AEFBA$ is clockwise.

(iii) Current through the capacitor is zero.

(iv) Energy stored in the capacitor is $\frac{1}{2}CB^2L^2v^2$.

Which of the following options is correct?

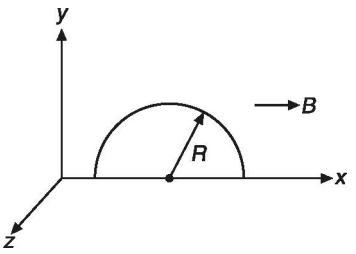
(A) Statements (i) and (iii) are correct

(B) Statements (ii) and (iv) are correct

(C) Statements (i), (iii) and (iv) are correct

(D) None of these

116. A semi-circular conducting loop of radius R is placed in the xy plane, as shown in Figure.



A uniform magnetic field is set up along the x-axis. No emf will be induced in the ring, if

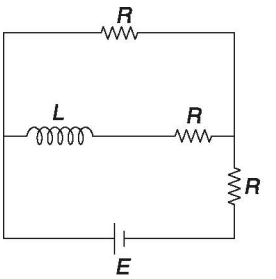
(A) it moves along the x-axis/y-axis

(B) it moves along the y-axis/z-axis

(C) it remains stationary

(D) All of these

117. The Figure shows a specific LR circuit, the time constant for this circuit is



(A) $\frac{L}{2R}$ (B) $\frac{2L}{R}$

(C) $\frac{3L}{2R}$ (D) $\frac{2L}{3R}$

118. A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Ignoring the field variation across the cross-section of the toroid, the self-inductance of the toroid is

(A) 4.6 mH (B) 6.9 mH

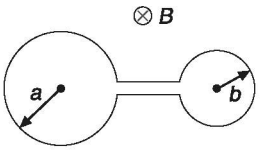
(C) 2.3 mH (D) 9.2 mH

119. A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. A second coil of 300 turns is wound closely on this toroid. If the current in the toroid is increased from zero to 2.0 A in 0.05 s, the emf induced in the second coil is

(A) 0.046 V (B) 0.069 V

(C) 0.023 V (D) 0.092 V

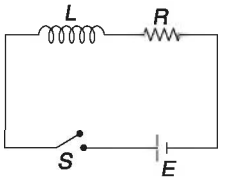
120. Figure shows two circular rings of radii a and $b(a>b)$ joined together by wire of negligible resistance. If the arrangement is placed in a time varying magnetic field, $\frac{dB}{dt}=k$ and if the resistance per unit length of wire is λ, then induced current is



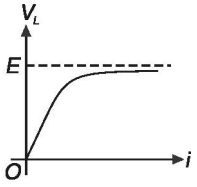
(A) $\frac{k(a+b)}{2\lambda}$ (B) $\frac{k(a-b)}{2\lambda}$

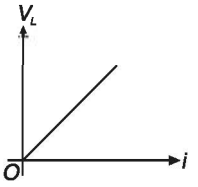
(C) $\frac{k(a^2+b^2)}{2\lambda(a-b)}$ (D) $\frac{k(a^2+b^2)}{2\lambda(a+b)}$

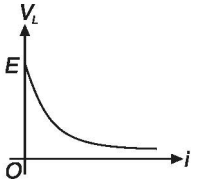
121. The switch S is closed at $t = 0$ in the circuit shown in Figure.

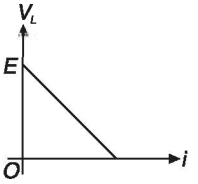


If V_L is the voltage induced across the inductor and i is the instantaneous current, the correct variation of V_L versus i is given by

(A) 

(B) 

(C) 

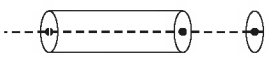
(D) 

122. Two coils P and S have 2000 turns and 5000 turns respectively. A current of 1 A in coil P causes a flux per turn of 0.8×10^{-3} weber to link with P and 0.4×10^{-3} weber through S. The ratio of coefficient of self-inductance of P and the coefficient of mutual inductance of P and S is

(A) $\left(\frac{2}{3}\right)$ (B) $\left(\frac{3}{4}\right)$

(C) $\left(\frac{4}{5}\right)$ (D) $\left(\frac{5}{4}\right)$

123. A solenoid carries a time varying current given by. $I = I_0 t$. At the axis of this solenoid, a ring is placed as shown in Figure.

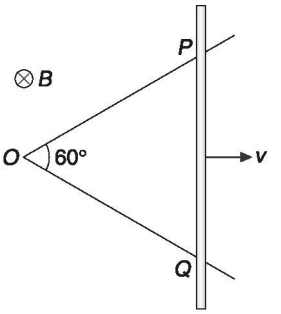


The mutual inductance of the ring and the solenoid is M and the self-inductance of the ring is L. If the resistance of the ring is R then maximum current which can flow through the ring is

(A) $\frac{(2M+L)I_0}{R}$ (B) $\frac{MI_0}{R}$

(C) $\frac{(2M-L)I_0}{R}$ (D) $\frac{(M+L)I_0}{R}$

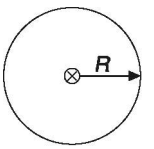
124. A rod PQ slides on a V-shaped wire with speed v as shown such that at any time $OP = OQ = l$. Magnetic field in the region is perpendicular into the paper and has strength B. Induced emf in the rod is



(A) Bvl (B) $\frac{Bvl}{2}$

(C) $\frac{\sqrt{3}Bvl}{2}$ (D) ZERO

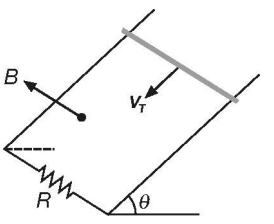
125. A circuit consists of a circular loop of radius R kept in the plane of paper and an infinitely long current carrying wire kept perpendicular to the plane of paper and passing through the centre of loop. The mutual inductance of wire and loop will be



(A) $\frac{\mu_0 \pi R}{2}$ (B) 0

(C) $\mu_0 \pi R^2$ (D) $\frac{\mu_0 R^2}{2}$

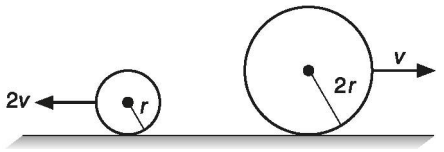
126. A copper rod of mass m slides under gravity on two smooth parallel rails l distance apart and set at an angle $30°$ to the horizontal. At the bottom, the rails are joined by a resistance R. There is a uniform magnetic field perpendicular to the plane of the rails. The terminal velocity of the rod is v_T. Then



(A) $v_T = \frac{\sqrt{3}mgR}{2B^2l^2}$ (B) $v_T = \frac{\sqrt{3}mgR}{B^2l^2}$

(C) $v_T = \frac{mgR}{2B^2l^2}$ (D) $v_T = \frac{mgR}{\sqrt{3}B^2l^2}$

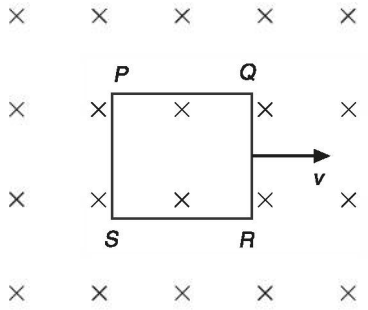
127. Two conducting rings of radii r and $2r$ rotate uniformly in opposite directions with centre of mass velocities $2v$ and v respectively on a conducting horizontal surface. There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is



(A) zero (B) $4Bvr$

(C) $8Bvr$ (D) $16Bvr$

128. A metallic square loop $PQRS$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in Figure.



If V_P, V_Q, V_R and V_S are the potential of points P, Q, R and S, then incorrect statement is

(A) $V_P = V_Q$ (B) $V_P > V_S$

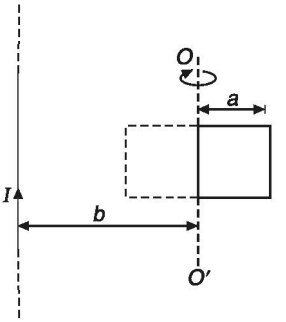
(C) $V_P > V_R$ (D) $V_S > V_R$

129. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in

(A) 0.15 s (B) 0.3 s

(C) 0.05 s (D) 0.1 s

130. A square loop of side a and a straight long wire are placed in the same plane as shown in Figure.

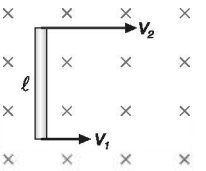


The loop has a resistance R and inductance L. The frame is turned through 180° about the axis OO'. The charge that flows through the loop in this process is

(A) $\frac{\mu_0 Ia}{2\pi R}\ln\left(\frac{2a+b}{b}\right)$ (B) $\frac{\mu_0 Ia}{2\pi R}\ln\left(\frac{b}{b^2-a^2}\right)$

(C) $\frac{\mu_0 Ia}{2\pi R}\ln\left(\frac{a+2b}{b}\right)$ (D) $\frac{\mu_0 ia}{2\pi R}\ln\left(\frac{b+a}{b-a}\right)$

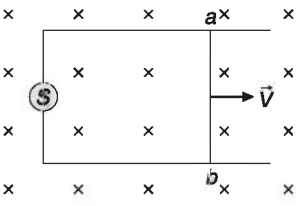
131. In the Figure, magnetic field points into the plane of paper and the conducting rod of length l is moving in this field such that the lowest point has a velocity v_1 and the topmost point has the velocity $v_2 (v_2 > v_1)$. The emf induced is given by



(A) Bv_1l (B) Bv_2l

(C) $\frac{1}{2}B(v_2+v_1)l$ (D) $\frac{1}{2}B(v_2-v_1)l$

132. The following diagram shows a wire ab of length l and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant current I in the circuit. Then the potential difference between a and b is



(A) Bvl (B) IR

(C) $Bvl - IR$ (D) $Bvl + IR$

133. A uniformly wound long solenoid of inductance L and resistance R is cut into two equal parts. The two parts are then joined in parallel. Further this combination is joined to a cell of emf E. The time constant of the circuit is

(A) $\frac{L}{4R}$ (B) $\frac{L}{2R}$

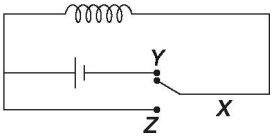
(C) $\frac{2L}{R}$ (D) $\frac{L}{R}$

134. A uniformly wound long solenoid of inductance L and resistance R is cut into two equal parts. The two parts are then joined in parallel. Further this combination is joined to a cell of emf E. The steady-state current is

(A) $\frac{2E}{R}$ (B) $\frac{4E}{R}$

(C) $\frac{8E}{R}$ (D) $\frac{E}{R}$

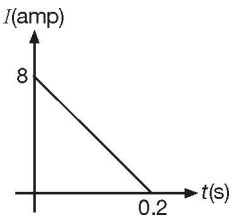
135. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z. Then the total heat produced in the coil is



(A) τP (B) $\frac{1}{2}\tau P$

(C) $2\tau P$ (D) $\frac{1}{4}\tau P$

136. Some magnetic flux is changed from a coil of resistance $10\ \Omega$. As a result, an induced current is developed in it, which varies with time as shown in Figure. The magnitude of change in flux through the coil in weber is



(A) 2 (B) 4

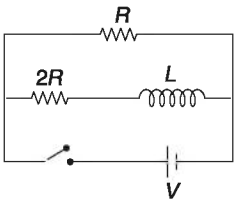
(C) 6 (D) 8

137. A loop of area $1\ m^2$ is placed in a magnetic field $B = 2T$, such that plane of the loop is parallel to the magnetic field. If the loop is rotated by $180°$, the amount of net charge passing through any point of loop, if its resistance is $10\ \Omega$, is

(A) 0.4 C (B) 0.2 C

(C) 0.8 C (D) 0 C

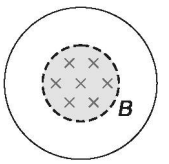
138. The ratio of time constants during current growth and current decay of the circuit shown in Figure is



(A) 1:1 (B) 3:2

(C) 2:3 (D) 1:3

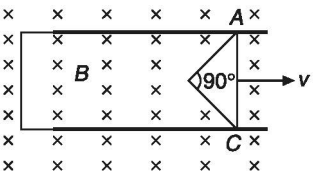
139. The Figure shows a conducting ring of radius R. A uniform steady magnetic field B lies perpendicular to the plane of the ring in a circular region of radius $r(< R)$. If the resistance per unit length of the ring is λ, then the current induced in the ring when its radius gets doubled is



(A) $\frac{BR}{\lambda}$ (B) $\frac{2BR}{\lambda}$

(C) ZERO (D) $\frac{Br^2}{4R\lambda}$

140. Two parallel straight rails of negligible resistance are L apart. At one end, they are connected to each other by a wire of negligible resistance. An isosceles right-angled triangle ABC made of a uniform wire of resistance per unit length λ slides along the rail with a constant velocity v. Force required to pull it is



(A) $\frac{B^2Lv(\sqrt{2}+1)}{\sqrt{2}\lambda}$ (B) $\frac{B^2Lv}{\lambda(\sqrt{2}+1)}$

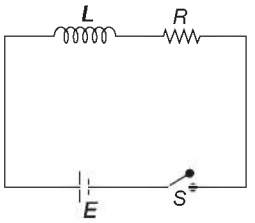
(C) $\frac{B^2Lv\sqrt{2}}{\lambda(\sqrt{2}+1)}$ (D) $\frac{B^2Lv}{\lambda}$

141. When a choke coil carrying a steady current is short-circuited, the current in it decreases to $\beta(>1)$ times its initial value in a time T. The time constant of the choke coil is

(A) $\frac{T}{\beta}$ (B) $\frac{T}{\ln\left(\frac{1}{\beta}\right)}$

(C) $\frac{T}{\ln\beta}$ (D) $T\ln\beta$

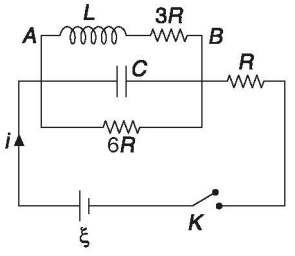
142. In the circuit shown in Figure switch S is closed at time $t=0$. The charge which passes through the battery in one time constant is Q. Then



(A) $Q=\frac{EL}{R^2}\left(1-\frac{1}{e}\right)$ (B) $Q=\frac{EL}{R^2}$

(C) $Q=\frac{EL}{eR^2}$ (D) $Q=\frac{EL}{R^2}\left(1+\frac{1}{e}\right)$

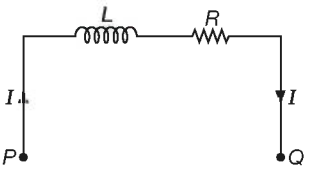
143. In the given circuit diagram, the key K is switched on at $t=0$. The ratio of the current i through the cell at $t=0$ and $t=\infty$ will be



(A) 3:1 (B) 1:3

(C) 1:2 (D) 2:1

144. When the current in the portion of the circuit shown in Figure is 4 A and increasing at the rate of 4 As^{-1}, the measured potential difference $V_{PQ}=16$ V. However when the current is 2 A and decreasing at the rate of 1 As^{-1}, the measured potential difference $V_{PQ}=5$ V. Then



(A) $L=1$ H, $R=3\ \Omega$ (B) $L=2$ H, $R=4\ \Omega$

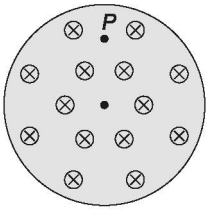
(C) $L=3$ H, $R=1\ \Omega$ (D) $L=4$ H, $R=2\ \Omega$

145. A magnet is moving towards the coil along the axis and the emf induced in the coil is ξ. If the coil also starts moving towards the magnet with the same speed, the induced emf will be

(A) $\frac{\xi}{2}$ (B) ξ

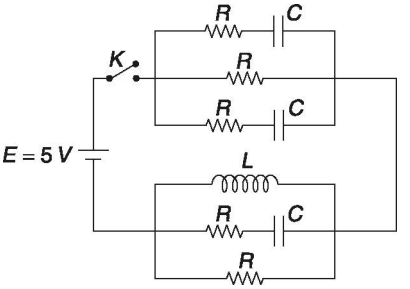
(C) 2ξ (D) 4ξ

146. A uniform but increasing with time magnetic field exists in a cylindrical region. The direction of force on an electron at P is



(A) towards right
(B) towards left
(C) into the plane of paper
(D) out of the plane of paper

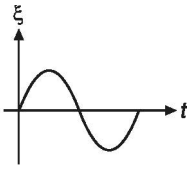
147. In the circuit shown, initially all the capacitors are uncharged, $R=6\ \Omega$ and $C=4\ \mu$F. The current passing through the battery immediately after the key K is closed is i_1 and long time after the key K is closed is i_2, then $\frac{i_1}{i_2}$ equals

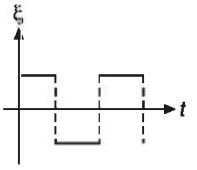


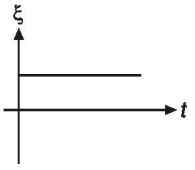
(A) 1 (B) 5

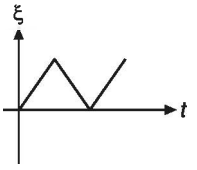
(C) 3 (D) 1.2

148. A conducting rod is rotated about one end in a plane perpendicular to a uniform magnetic field with constant angular velocity. The correct graph between the induced emf (ξ) across the rod and time (t) is

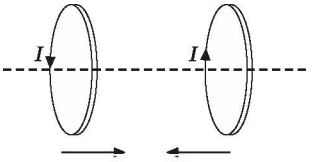
(A) 

(B) 

(C) 

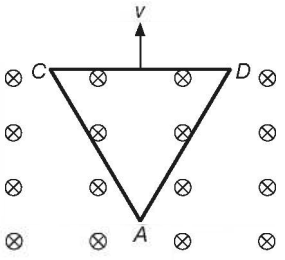
(D) 

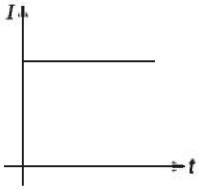
149. Two coils carrying current in opposite direction are placed co-axially with centres at some finite separation. If they are brought close to each other then, current flowing in them should

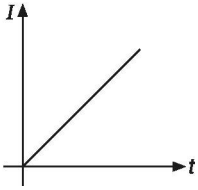


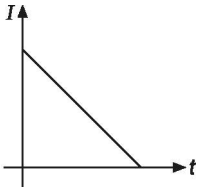
(A) decrease (B) increase
(C) remain same (D) become zero

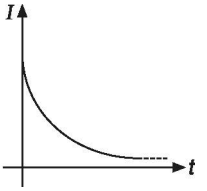
150. An equilateral triangular loop ADC of side length $2a$, having resistance R is pulled with a constant velocity v out of a uniform magnetic field directed into the paper. At time $t = 0$, side DC of the loop is at edge of the magnetic field. The induced current (I) versus time (t) graph will be as



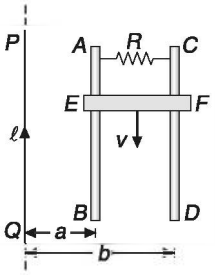
(A) 

(B) 

(C) 

(D) 

151. PQ is an infinite current carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity v as shown in Figure.



The force required to maintain constant speed of EF is

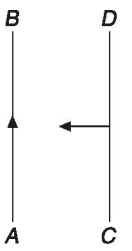
(A) $\frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\log_e\left(\frac{b}{a}\right)\right]^2$

(B) $\frac{v}{R}\left[\frac{v_0 Iv}{2\pi}\log_e\left(\frac{a}{b}\right)\right]^2$

(C) $\frac{v}{R}\left[\frac{v_0 Iv}{2\pi}\log_e\left(\frac{b}{a}\right)\right]^2$

(D) $\frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\log_e\left(\frac{a}{b}\right)\right]^2$

152. Two infinitely long straight wires AB and CD are placed side by side as shown in Figure. AB carries a current i. If CD is moved towards AB, then



(A) C will be at a higher potential
(B) D will be at a higher potential
(C) No emf is induced in CD
(D) Force on CD will be away from AB

153. Two coils of inductances L_1 and L_2 are linked such that their mutual inductance is M.

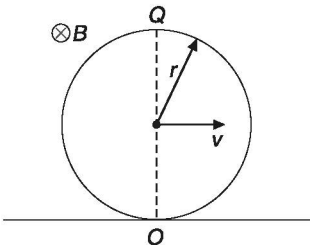
(A) $M = L_1 + L_2$

(B) $M = \frac{1}{2}(L_1 + L_2)$

(C) The maximum value of M is $\sqrt{L_1 L_2}$

(D) The maximum value of M is $(L_1 + L_2)$

154. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform normal to the plane of the loop.



(A) The induced emf between O and Q is $2Bvr$.

(B) An induced current $I = \frac{2Bvr}{R}$ flows in the clockwise direction.

(C) An induced current $I = \frac{2Bvr}{R}$ flows in the anticlockwise direction.

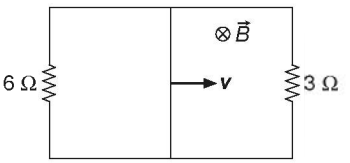
(D) None of these

155. The armature of a DC motor has $20\ \Omega$ resistance. It draws a current of 1.5 A when run by 200 V DC supply. The value of back emf induced in it will be

(A) 150 V (B) 170 V

(C) 180 V (D) 190 V

156. A rectangular loop with a sliding connector of length $l = 2$ m is situated in a uniform magnetic field $B = 2$ T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistances of $6\ \Omega$ and $3\ \Omega$ are connected as shown in Figure.

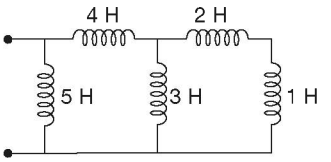


The external force required to keep the connector moving with a constant velocity $v = 4\ \text{ms}^{-1}$ is

(A) 4 N (B) 8 N

(C) 16 N (D) 32 N

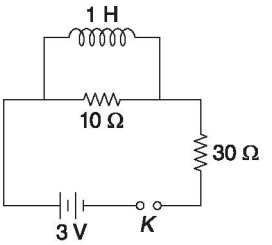
157. The equivalent inductance of the network shown here is



(A) 2.619 H (B) 9.126 H

(C) 6.219 H (D) 1.296 H

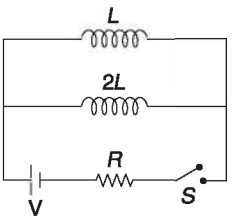
158. In Figure, final value of current in $10\ \Omega$ resistor, when plug of key K is inserted is



(A) $\frac{3}{10}$ A (B) $\frac{3}{20}$ A

(C) $\frac{3}{11}$ A (D) ZERO

159. In the circuit shown in Figure, steady state current in the resistor R after the switch S is closed will be equal to



(A) V/R (B) $2V/3R$

(C) $V/3R$ (D) $V/2R$

160. The current through an inductor of 1 H is given by $I = 3t\sin t$. The voltage across the inductor of 1 H is

(A) $3\sin t + 3\cos t$ (B) $3\sin t + 3t\cos t$

(C) $3\cos t + t\sin t$ (D) $3t\cos t + \sin t$

161. A Player with 3 m long iron rod runs towards east with a speed of 30 kmhr^{-1}. Horizontal component of earth's magnetic field is $4 \times 10^{-5}\ \text{Wbm}^{-2}$. If he is running with rod in horizontal and vertical positions,

then the potential difference induced between the two ends of the rod in two cases will be

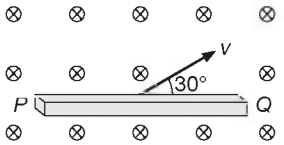
(A) zero in vertical position and 10^{-3} V in horizontal position

(B) 10^{-3} V in vertical position and zero in horizontal position

(C) zero in both cases

(D) 10^{-3} V in both cases

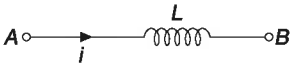
162. A conducting rod PQ of length $l = 2$ m is moving at a velocity $v = 8$ ms^{-1} making an angle $30°$ with its length. A uniform magnetic field $B = 3$ T exists in a direction perpendicular to the plane of motion. Then



(A) $V_P - V_Q = 41.6$ V (B) $V_P - V_Q = 24$ V

(C) $V_Q - V_P = 41.6$ V (D) $V_Q - V_P = 24$ V

163. In Figure, if the current i decreases at a rate α, then $V_A - V_B$ is



(A) zero (B) $-\alpha L$

(C) αL (D) No relation exists

164. A rectangular loop of sides a and b is placed in xy plane. A uniform but time varying magnetic field of strength $\vec{B} = 20t\hat{i} + 10t^2\hat{j} + 50\hat{k}$ is present in the region. The magnitude of induced emf in the loop at time t is

(A) $20 + 20t$ (B) 20

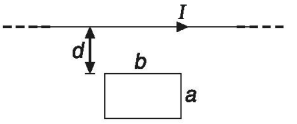
(C) $20t$ (D) zero

165. The annular metallic disk of inner and outer radii a and b respectively is rotating with angular velocity ω in the uniform magnetic field of induction B normal to the plane. The induced emf between inner and outer edge of disk is

(A) $\frac{B\omega}{2}(a^2 + b^2)$ (B) $\frac{B\omega}{2}(b^2 - a^2)$

(C) $B\omega(b-a)^2$ (D) $B\omega(b+a)^2$

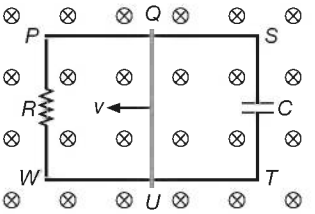
166. A long straight wire is parallel to one edge of a rectangular loop as shown in Figure. If the current in the long wire varies with time as $I = I_0 e^{-\frac{t}{\tau}}$, then the emf induced in the loop at $t = \tau$ is



(A) $\frac{\mu_0 bI}{\pi\tau}\ln\left(\frac{d+a}{d}\right)$ (B) $\frac{\mu_0 bI}{2\pi e\tau}\ln\left(\frac{d+a}{d}\right)$

(C) $\frac{2\mu_0 bI}{\pi\tau}\ln\left(\frac{d+a}{d}\right)$ (D) $\frac{\mu_0 bI}{\pi e\tau}\ln\left(\frac{d}{d+a}\right)$

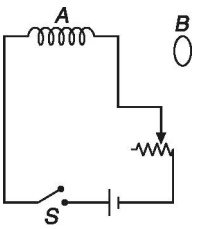
167. In the loop $PQSTUWP$ shown in Figure, a conducting wire QU of mass m, length l is moved with a constant velocity v towards left. The complete circuit is placed in a uniform inward magnetic field $\vec{B}$ perpendicular to the plane of loop. The current in the branch $QSTU$ is



(A) $\frac{Blv}{R}$, clockwise (B) $\frac{BlC}{R}$, anticlockwise

(C) $\frac{BlCv}{R}$, clockwise (D) zero

168. The ring B is coaxial with a solenoid A as shown in Figure.



As the switch S is closed at $t = 0$, the ring B

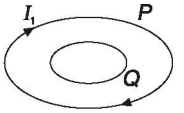
(A) is attracted towards A

(B) is repelled by A

(C) is initially repelled and then attracted

(D) is initially attracted and then repelled

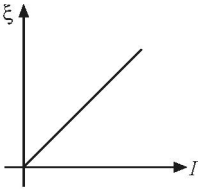
169. Two circular loops P and Q are concentric and coplanar as shown in Figure.

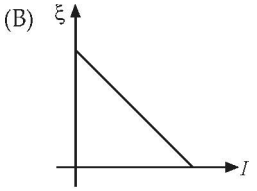


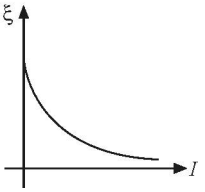
The loop Q is smaller than P. If the current I_1 flowing in loop P is decreasing with time, then the current I_2 in the loop Q

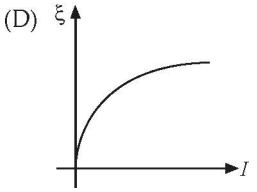
(A) flows in the same direction as that of P
(B) flows in the opposite direction as that of Q
(C) is zero
(D) None of these

170. Consider a series LR combination connected to a battery of constant emf E and switch S in series. The switch S is closed at time $t = 0$. If ξ denotes the induced emf across inductor and I the current in the circuit at any time t. The graph showing the variation of ξ with I is

(A) 

(B) 

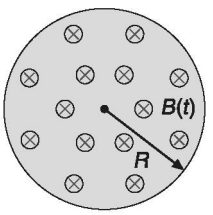
(C) 

(D) 

171. If the instantaneous magnetic flux and induced emf produced in a coil is ϕ and E respectively, then according to Faraday's law of electromagnetic induction

(A) E must be zero if $\phi = 0$
(B) $E \neq 0$ if $\phi = 0$
(C) $E \neq 0$ but ϕ may or may not be zero
(D) $E = 0$ then ϕ must be zero

172. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius R and is directed into the plane of the paper as shown. For the magnitude of induced electric field $\vec{E}$ at a point at a distance r from centre, it can be said that

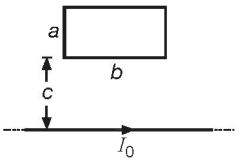


(A) $\vec{E} = \vec{0}$ for $r > R$
(B) $\vec{E} = \vec{0}$ for $r < R$
(C) $|\vec{E}| \propto \frac{1}{r^2}$ for $r < R$
(D) $|\vec{E}| \propto \frac{1}{r}$ for $r > R$

173. A rectangular loop of sides a and b, has a resistance R and lies at a distance c from an infinite straight wire carrying current I_0. The current decreases to zero in time τ

$$I(t) = I_0 \frac{(\tau - t)}{\tau}, \quad 0 < t < \tau$$

The charge flowing through the rectangular loop is

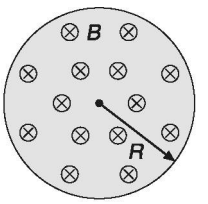


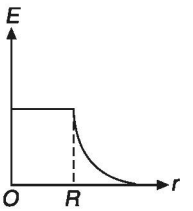
(A) $\mu_0 I_0 \tau$
(B) $\mu_0 I_0 \frac{ab}{c^2} \tau$
(C) $\frac{1}{2} \frac{\mu_0 b I_0}{\pi R} \ln\left(\frac{c+a}{c}\right)$
(D) $\frac{\mu_0 I \tau}{R} \frac{ba}{c^2}$

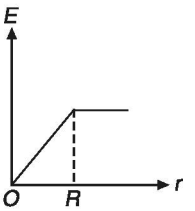
174. A conducting square loop is placed in a magnetic field B with its plane perpendicular to the field. Now the sides of the loop start shrinking at a constant rate α. The induced emf in the loop at an instant when its side is a, is

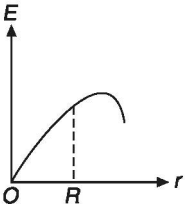
(A) $2a\alpha B$
(B) $a^2\alpha B$
(C) $2a^2\alpha B$
(D) $a\alpha B$

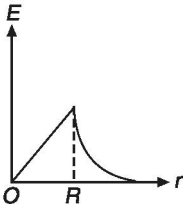
175. A uniform magnetic field is confined in a cylindrical region of radius R. Induction of magnetic field is increasing at a constant rate $\frac{dB}{dt} = \alpha$. Strength of induced electric field (E) varies with distance r from the axis of cylindrical region as



(A) 

(B) 

(C) 

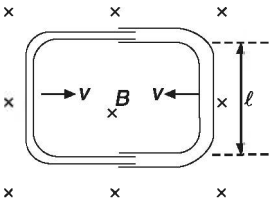
(D) 

176. A small coil of radius r is placed at the centre of a large coil of radius R, where $R \gg r$. The coils are coplanar. The mutual induction between the coils is

(A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R}$

(C) $\frac{\mu_0 \pi r^2}{2R^2}$ (D) $\frac{\mu_0 \pi r}{2R^2}$

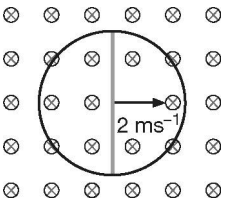
177. One conducting U tube can slide inside another as shown in Figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the Figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v (l is the width of each tube) will be



(A) ZERO (B) $2Blv$

(C) Blv (D) $-Blv$

178. A circular loop of radius 2 m is kept in a magnetic field of strength $2T$ (plane of loop is perpendicular to the direction of magnetic field). Resistance of the loop wire is $\frac{2}{\pi}\ \Omega m^{-1}$. A conductor of length 4 m is sliding with a speed 2 ms^{-1} as shown in Figure.

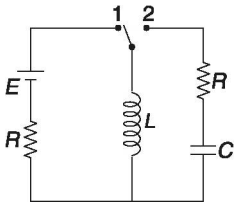


Find the instantaneous force acting on the rod: Assuming the rod to have a negligible resistance, the instantaneous force acting on the rod is

(A) 8 N (B) 16 N

(C) 32 N (D) 64 N

179. Initially, the switch is in position 1 for a long time and then shifted to position 2 at $t = 0$ as shown in Figure.

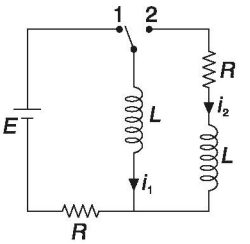


Just after closing the switch, the magnitude of current through the capacitor is

(A) ZERO (B) $\frac{E}{2R}$

(C) $\frac{E}{R}$ (D) None of these

180. In Figure, the switch is in the position 1 for a long time, then the switch is shifted to position 2 at $t = 0$. At this instant the value of i_1 and i_2 are



(A) $\frac{E}{R}, 0$ (B) $\frac{E}{R}, -\frac{E}{R}$

(C) $\frac{E}{2R}, -\frac{E}{2R}$ (D) None of these

181. A coil is joined to a cell such that the current through the cell grows with a time constant τ. The current reaches 10% of its steady-state value in time t. Then

(A) $t = \tau \ln(1.1)$ (B) $t = \tau \ln(0.1)$

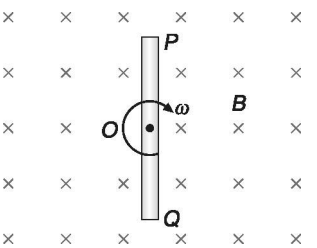
(C) $t = \tau \ln(0.9)$ (D) $t = 0.1\tau$

182. A coil is joined to a cell such that the current through the cell grows with a time constant τ. The current will be 10% less than its steady-state value after a time

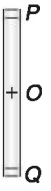
(A) $\tau \ln(9)$ (B) $\tau \ln(10)$

(C) 0.9τ (D) $\tau \ln(0.9)$

183. A rod is rotating with a constant angular velocity ω about point O (its centre) in a magnetic field B as shown. Which of the following Figure correctly shows the distribution of charge inside the rod?



(A)

(B) 

(C)

(D)

184. An inductor coil stores energy U when a current I is passed through it and dissipates energy at the rate of P. The time constant of the circuit when this coil is connected across a battery of zero internal resistance is

(A) $\frac{U}{2P}$ (B) $\frac{U}{P}$

(C) $\frac{2U}{P}$ (D) $\frac{4U}{P}$

185. Two identical coaxial circular loops carry a current i each circulating in the same direction. If the loops approach each other the current in

(A) each decreases (B) each increases

(C) remains same (D) None of these

186. A current carrying ring is placed in a horizontal plane. A charged particle is dropped along the axis of the ring to fall under the influence of gravity

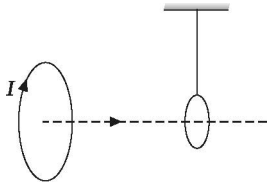
(A) the current in the ring may increase

(B) the current in the ring may decrease

(C) the velocity of the particle will increase till it reaches the centre of the ring

(D) the acceleration of the particle will decrease continuously till it reaches the centre of the ring

187. A small circular loop is suspended from an insulating thread. Another coaxial circular loop carrying a current I and having radius much larger than the first loop starts moving towards the smaller loop. The smaller loop will



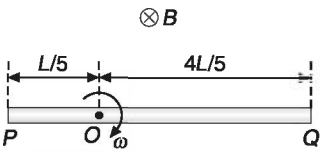
(A) be attracted towards the bigger loop

(B) be repelled by the bigger loop

(C) experience no force

(D) All of these

188. A straight rod of length L is rotating about an axis passing through O and perpendicular to the plane. In the space, a uniform magnetic field B exists normal to the plane of rotation. Potential difference between P and Q is equal to



(A) $\frac{8}{25}B\omega L^2$ (B) $\frac{7}{25}B\omega L^2$

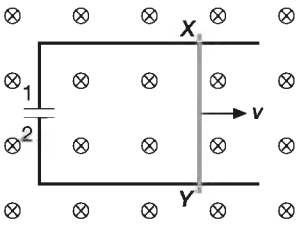
(C) $\frac{3}{10}B\omega L^2$ (D) ZERO

189. The time constant of a circuit containing resistance and inductance, when the current rises to 63.2% of its steady value in one second, is

(A) 0.15 second (B) 0.5 second

(C) 1 second (D) 2 second

190. A conducting rod XY of length $L = 0.5$ m is moving with a uniform speed $v = 4\ \text{ms}^{-1}$ in a uniform magnetic field $B = 2T$ directed into the paper. A capacitor of capacity $C = 5\ \mu\text{F}$ is connected as shown in Figure. Then



(A) charge stored in the capacitor increases exponentially with time

(B) $q_1 = q_2 = 0$

(C) $q_1 = -20\ \mu\text{C}$ and $q_2 = +20\ \mu\text{C}$

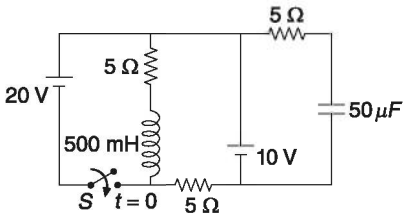
(D) $q_1 = +20\ \mu\text{C}$ and $q_2 = -20\ \mu\text{C}$

191. The magnetic field in a region is given by $\vec{B} = B_0\left(1 + \frac{x}{a}\right)\hat{k}$. A square loop of edge-length d is placed with its edges along the X and Y-axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is

(A) ZERO (B) v_0B_0d

(C) $\frac{v_0B_0d^3}{a^2}$ (D) $\frac{v_0B_0d^2}{a}$

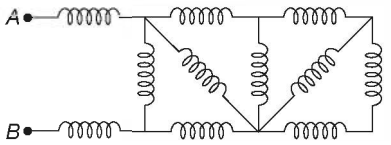
192. Switch S is closed at $t = 0$, in the circuit shown in Figure.



The change in flux in the inductor $(L = 500\ \text{mH})$ from $t = 0$ to an instant when it reaches steady state is

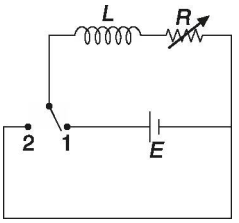
(A) 2 Wb (B) 1.5 Wb
(C) 0 Wb (D) None of these

193. A network of inductances, each of value 1 H, is shown in Figure. The equivalent inductance of the circuit between points A and B is



(A) 6.218 H (B) 0.268 H
(C) 8.162 H (D) 2.618 H

194. In the circuit shown in Figure initially the switch is in position 1 for a long time, then suddenly at $t = 0$, the switch is shifted to position 2. It is required that a constant current should flow in the circuit, the value of resistance R in the circuit

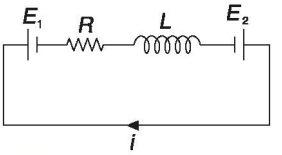


(A) should be decreased at a constant rate
(B) should be increased at a constant rate
(C) should be maintained constant
(D) Not possible

195. A capacitor of capacitance 2 μF is first charged and then discharged through a resistance of 1 MΩ. The time in which the charge on the capacitor will fall to 50% of its initial value is

(A) 1.38 second (B) 1.04 second
(C) 0.35 second (D) 0.693 second

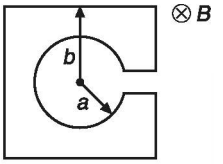
196. In the following loop, which one of the following is correct expression for Kirchhoff voltage law?



(A) $E_1 - E_2 = iR + L\frac{di}{dt}$ (B) $E_2 - E_1 = iR + L\frac{di}{dt}$

(C) $L\frac{di}{dt} = iR + E_1 - E_2$ (D) $L\frac{di}{dt} = iR + E_2 - E_1$

197. A loop shown in Figure is immersed in the varying magnetic field $B = B_0 t$, directed into the page. If the total resistance of the loop is R, then the direction and magnitude of induced current in the inner circle is



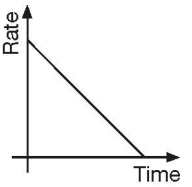
(A) clockwise $\frac{B_0(\pi a^2 - b^2)}{R}$

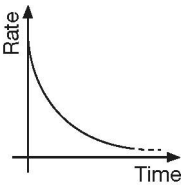
(B) anti-clockwise $\frac{B_0\pi(a^2 + b^2)}{R}$

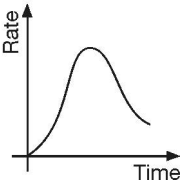
(C) clockwise $\frac{B_0(\pi a^2 + 4b^2)}{R}$

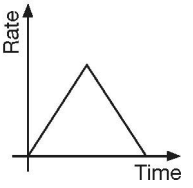
(D) clockwise $\frac{B_0(4b^2 - \pi a^2)}{R}$

198. In a series LR circuit connected to a battery the rate at which energy is stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?

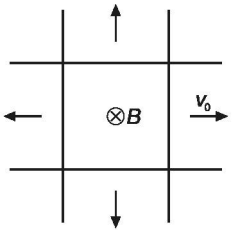
(A)


(B)


(C)


(D)


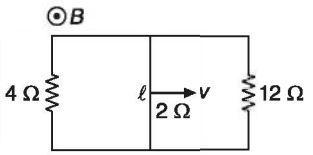
199. Two parallel long straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now, conductors start moving outward with a constant velocity v_0 at $t=0$. Then, induced current in the loop at any time t is (λ is resistance per unit length of the conductors)



(A) $\dfrac{aBv_0}{\lambda(a+v_0t)}$ (B) $\dfrac{aBv_0}{2\lambda}$

(C) $\dfrac{Bv_0}{\lambda}$ (D) $\dfrac{Bv_0}{2\lambda}$

200. A uniform magnetic field $|\vec{B}|=0.5$ T is perpendicular to the plane of circuit as shown in Figure.



The sliding rod of length $l=0.25$ m moves uniformly with constant speed $v=4$ ms^{-1}. If the resistance of the slides is 2 Ω, then the current flowing through the sliding rod is

(A) 0.1 A (B) 0.17 A

(C) 0.08 A (D) 0.03 A

201. Initially an inductor of zero resistance is joined to a cell of emf E through a resistance. The current increases with a time constant τ. The potential difference across the coil after time t is

(A) $E\left(1-e^{-\frac{t}{\tau}}\right)$ (B) $\dfrac{Et}{\tau}$

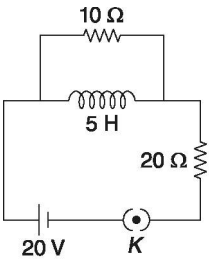
(C) $Ee^{-\frac{2t}{\tau}}$ (D) $Ee^{-\frac{t}{\tau}}$

202. Initially an inductor of zero resistance is joined to a cell of emf E through a resistance. The current increases with a time constant τ. The time after which the potential difference across the coil be equal to that across the resistance is

(A) τ (B) $\tau(1-\ln 2)$

(C) $\tau\ln 2$ (D) $\dfrac{\tau}{\ln 2}$

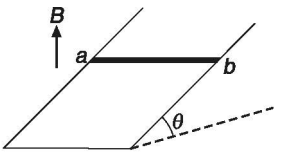
203. Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a 20 V battery through a key K, as shown in Figure. The key is closed at $t=0$. What is the final value of current in the 10 Ω resistor?



(A) $\dfrac{2}{3}$ A (B) $\dfrac{1}{3}$ A

(C) $\dfrac{1}{6}$ A (D) ZERO

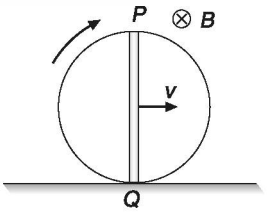
204. A wire ab of length l, mass m and resistance R slides on a smooth thick pair of metallic rails joined at the bottom as shown in Figure. The plane of the rails makes an angle θ with the horizontal. A vertical magnetic field B exist in the region. If the wire slides on the rails at a constant speed v, then the value of B is



(A) $\sqrt{\dfrac{mgR}{vl^2\cos^2\theta}}$ (B) $\sqrt{\dfrac{mgR\cos\theta}{vl^2\sin^2\theta}}$

(C) $\sqrt{\dfrac{mgR}{v^2l^2\sin^2\theta}}$ (D) $\sqrt{\dfrac{mgR\sin\theta}{vl^2\cos^2\theta}}$

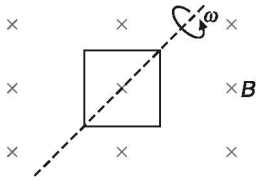
205. A conducting straight rod PQ of length l is fixed along a diameter of a non-conducting ring as shown in Figure. The ring is given a pure rolling motion on a horizontal surface such that its centre of mass has a velocity v. There exists a uniform horizontal magnetic field B in horizontal direction perpendicular to the plane of ring. The magnitude of induced emf in the rod PQ at the position shown in Figure, will be



(A) Bvl (B) $2Bvl$

(C) $3Bvl/2$ (D) ZERO

206. A square loop of side b is rotated in a constant magnetic field B at angular frequency ω as shown in Figure.



What is the emf induced in it?

(A) $b^2B\omega\sin\omega t$ (B) $bB\omega\sin^2\omega t$

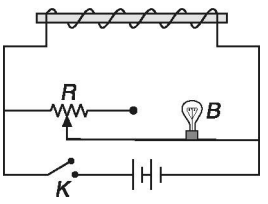
(C) $bB^2\omega\cos\omega t$ (D) $b^2B\omega$

207. A uniformly wound long solenoid of inductance L and resistance R is cut into two parts in the ratio $\eta:1$, which are then connected in parallel. The combination is then connected to a cell of emf E. The time constant of the circuit is

(A) $\dfrac{L}{R}$ (B) $\dfrac{L}{(\eta+1)R}$

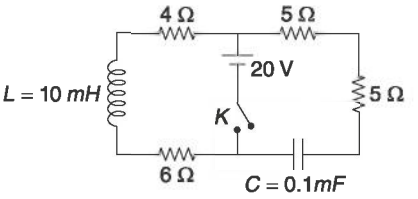
(C) $\left(\dfrac{\eta}{\eta+1}\right)\dfrac{L}{R}$ (D) $\left(\dfrac{\eta+1}{\eta}\right)\dfrac{L}{R}$

208. Figure shows a solenoid connected to a resistance R, a bulb B and a source of emf. The resistance R is adjusted so that when K is closed, the bulb just glows. Then the switch K is suddenly opened. The bulb B will



(A) glow same as before
(B) fuse
(C) glow very brightly for a moment
(D) None of these

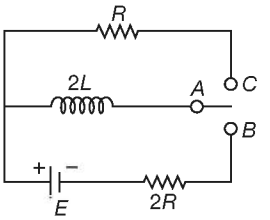
209. In the circuit shown, the key (K) is closed at $t=0$, the current through the key at the instant $t=10^{-3}\ln 2$, is



(A) 2 A (B) 8 A

(C) 4 A (D) ZERO

210. In the circuit shown, A is joined to B for a long time, and then A is joined to C. The total heat produced in R is



(A) $\dfrac{LE^2}{R^2}$ (B) $\dfrac{LE^2}{2R^2}$

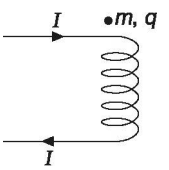
(C) $\dfrac{LE^2}{4R^2}$ (D) $\dfrac{LE^2}{8R^2}$

211. A coil carrying a steady current is short-circuited. The current in it decreases α times in time t_0. The time constant of the circuit is τ.

(A) $\tau=t_0\ln\alpha$ (B) $\tau=\dfrac{t_0}{\alpha}$

(C) $\tau=\dfrac{t_0}{\ln\alpha}$ (D) $\tau=\dfrac{t_0}{\alpha-1}$

212. A long solenoid carrying a current I is placed with its axis vertical as shown in Figure. A particle of mass m and charge q is released from the top of the solenoid. Its acceleration is (g being acceleration due to gravity).



(A) greater than g (B) less than g

(C) equal to g (D) None of these

213. A coil of inductance 2 H and negligible resistance is connected to a source of supply whose voltage, in volt, is given by $V = 4t$. If the voltage is applied when $t = 0$, then the energy stored in the coil in 4 s is

(A) 144 J (B) 256 J
(C) 512 J (D) 1024 J

214. A solenoid having 2000 turns is wound over a length of 0.3 m. Its cross-sectional area is 1.2×10^{-3} m^2. Around its central cross-section a coil of 300 turns is wound. If an initial current of 2 A flowing in the solenoid is reversed in 0.25 s, the emf induced in the coil is

(A) 0.6 mV (B) 60 mV
(C) 0.48 mV (D) 48 mV

215. Two coils, X and Y, are linked such that emf E is induced in Y when the current in X is changing at the rate $\dot{I}\left(=\frac{dI}{dt}\right)$. If a current I_0 is now made to flow through Y, the flux linked with X will be

(A) $EI_0\dot{I}$ (B) $\left(\frac{E}{\dot{I}}\right)I_0$
(C) $\left(E\dot{I}\right)I_0$ (D) $\frac{I_0\dot{I}}{E}$

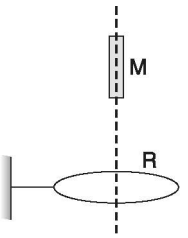
216. A metal disc of radius a rotates with a constant angular velocity ω about its axis. Assuming, m and e to be the mass and charge of electron, the potential difference between the centre and the rim of the disc is

(A) $\frac{1}{2}\left(\frac{m\omega^2a^2}{e}\right)$ (B) $\frac{m\omega^2a^2}{e}$
(C) $\frac{2m\omega^2a^2}{e}$ (D) $\frac{4m\omega^2a^2}{e}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

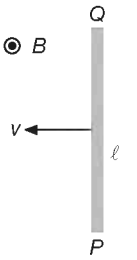
This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be



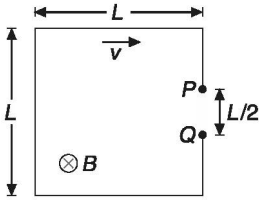
(A) less than g when it is above R and moving towards R
(B) greater than g when it is above R and moving towards R
(C) less than g when it is below R and moving away R
(D) greater than g when it is below R and moving away R

2. The conductor PQ moves to the left in a uniform magnetic field directed out of the plane the paper.



(A) The free electrons in PQ will move towards P
(B) Q will acquire a positive potential with respect to P
(C) If Q and P are joined by a conductor externally, a current will flow from P to Q in PQ
(D) The current in PQ flows from lower to higher potential

3. A loop is moving with a velocity v in a uniform magnetic field of magnitude B, directed into the paper as shown in Figure.



The potential difference between points P and Q is V. Select the correct statement(s)

(A) $V = \frac{1}{2}BLv$

(B) $V = BLv$

(C) P is positive with respect to Q

(D) Q is positive with respect to P

4. A planar loop of wire of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{B}$ varies in time according to the expression $B = B_0 e^{-at}$, where a is a constant and B_0 is the value of field at $t = 0$. The induced emf

(A) increases exponentially with time with minimum value aB_0A at $t = 0$.

(B) decreases exponentially with time with maximum value aB_0A at $t = 0$

(C) increases exponentially with time to a value $\frac{aB_0A}{e}$ at $t = \frac{1}{a}$

(D) decreases exponentially with time to a value $\frac{aB_0A}{e}$ at $t = \frac{1}{a}$

5. A rectangular coil C having N turns, length $2l$, width l and resistance per unit length λ is placed in a uniform magnetic field of magnitude B. The plane of the coil is initially perpendicular to B. When the coil is rotated by an angle θ about the axis XY, a charge Q flows through it

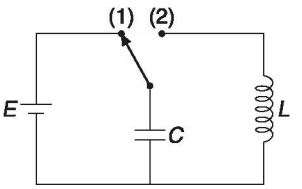
(A) $Q = \frac{NBl}{3\lambda}$, for $\theta = \frac{\pi}{2}$

(B) $Q = \frac{2NBl}{3\lambda}$, for $\theta = \pi$

(C) $Q = \frac{2NBl}{\lambda}$, for $\theta = \frac{3\pi}{2}$

(D) $Q = \frac{NBl}{6\lambda}$, for $\theta = 2\pi$

6. In the circuit shown, the key is placed at position-1 till the capacitor gets fully charged. Now the key is shifted to position-2 at $t = 0$. The time(s) when equal energies are stored as electric and magnetic field is/are



(A) $\frac{\pi\sqrt{LC}}{4}$ (B) $\frac{\pi\sqrt{LC}}{2}$

(C) $\frac{5\pi\sqrt{LC}}{4}$ (D) $\frac{5\pi\sqrt{LC}}{2}$

7. A magnetic flux ϕ linked with a closed coil produces an emf ξ.

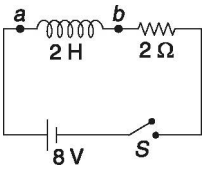
(A) ξ must be zero for $\phi = 0$

(B) ξ may or may not be zero for $\phi \neq 0$

(C) ξ is non-zero for $\phi \neq 0$

(D) ξ is non-zero constant for ϕ varying linearly with time.

8. In the circuit shown in Figure the switch S is closed at time $t = 0$. At time $t = \ln(2)$ second, the



(A) rate of energy supplied by the battery is 16 Js^{-1}

(B) rate of heat dissipated across resistance is 8 Js^{-1}

(C) rate of heat dissipated across resistance is 16 Js^{-1}

(D) $V_a - V_b = 4$ V

9. A conducting ring of radius r, charge Q, resistance R is placed in a magnetic field normal to its plane that changes at a rate B.

(A) The emf induced in the ring is $\pi r^2 \frac{dB}{dt}$

(B) The emf induced in the ring is $\pi r^2 B$

(C) The potential difference between diametrically opposite points on the ring is half of the induced emf

(D) All points on the ring are at the same potential

10. The magnetic flux ϕ_B linked with a closed coil depends on time t as $\phi = at^{n^2-1}$, where a is a positive constant and n is any real number. The emf induced in the coil is e.

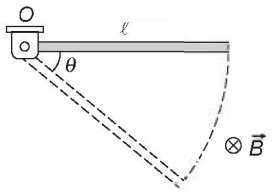
(A) $|e| = 0$ for $n = \pm 1$

(B) $|e| = a$ for $n = \pm\sqrt{2}$

(C) $|e|$ decreases with time for $-1 < n < 1$

(D) $|e|$ increases with time for all values of n expect those lying between -1 and 1

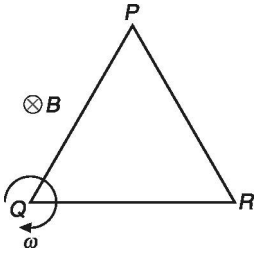
11. A conducting rod of mass m, length l, free to rotate in a vertical plane, is hinged at point O. There exists a uniform magnetic field $\vec{B}$ in horizontal direction. The rod is released from the position shown. The potential difference between the two ends of the rod, when it reaches a position at an angle θ below the initial position, is proportional to



(A) $\sqrt{\sin\theta}$ (B) l^2

(C) B (D) $l^{3/2}$

12. An equilateral triangular conducting frame of side l is rotating with uniform angular velocity ω in a uniform magnetic field B directed inwards as shown in Figure.



Select the correct option(s).

(A) $V_P - V_R = 0$ (B) $V_P - V_R = \frac{B\omega l^2}{2}$

(C) $V_P - V_Q = \frac{B\omega l^2}{2}$ (D) $V_R - V_Q = -\frac{B\omega l^2}{2}$

13. In an LR circuit, if an iron core is inserted inside the coil, then select the incorrect statements

(A) steady state current will increase

(B) steady state current will decrease

(C) time constant will increase

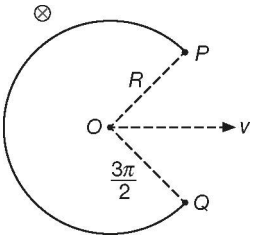
(D) time constant will decrease

14. A bar magnet is falling towards a fixed closed conducting loop of radius r. At $t = 2$ s, the possible distance(s) moved by the magnet towards the loop is

(A) 18 m (B) $21r$ m

(C) 19 m (D) $20r$ m

15. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between P and Q is ξ.



(A) $\xi = BRv$ (B) $\xi = \sqrt{2}BRv$

(C) $V_P > V_Q$ (D) $V_Q > V_P$

16. A coil of area $2\ m^2$ and resistance $4\ \Omega$ is placed perpendicular to a uniform magnetic field of 4 T. The loop is rotated by $90°$ in 0.1 second. Select the correct options

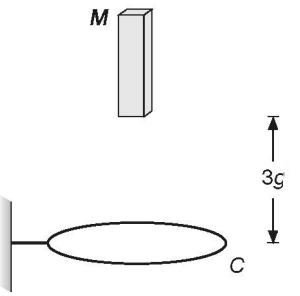
(A) Average induced emf in the coil is 8 V.

(B) Average induced current in the circuit is 20 A.

(C) $2C$ charge will flow in the coil in above period.

(D) Heat produced in the coil in the above period cannot be determined from the given data.

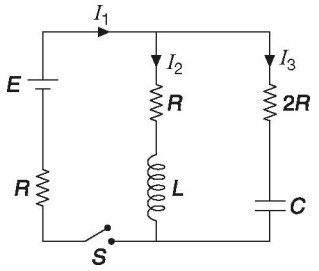
17. A bar magnet M is allowed to fall towards a fixed conducting ring C. If g is the acceleration due to gravity, v is the velocity of the magnet at $t = 2$ s and s is the distance travelled by it in the same time then,



(A) $v > 2g$ (B) $v < 2g$

(C) $s > 2g$ (D) $s < 2g$

18. Consider the circuit shown. The switch is closed at $t = 0$. Currents in various branches are marked. Which of the following is correct?



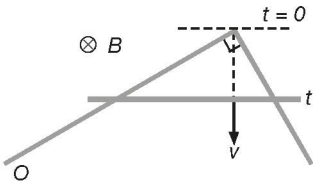
(A) At $t = 0$, $I_1 = \frac{E}{3R}$, $I_2 = \frac{E}{6R}$, $I_3 = \frac{E}{3R}$

(B) At $t = 0$, $I_1 = \frac{E}{3R}$, $I_3 = \frac{E}{3R}$

(C) As $t \to \infty$, $I_1 = \frac{E}{2R}$, $I_2 = \frac{E}{4R}$, $I_3 = \frac{E}{4R}$

(D) As $t \to \infty$, $I_1 = \frac{E}{2R}$, $I_2 = \frac{E}{2R}$

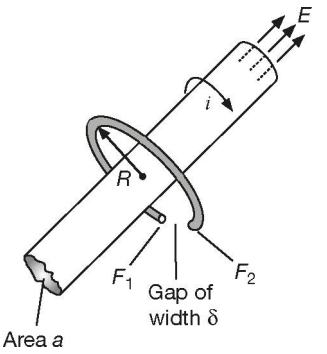
19. Two straight conducting rails form a right angle where their ends are joined. A long conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with constant velocity v along them as shown in Figure. A magnetic field $\vec{B}$ is directed into the page. The induced emf in the circuit at any time t is proportional to



(A) t^2 (B) t

(C) v^2 (D) v

20. A highly conducting ring of radius R is perpendicular to and concentric with the axis of a long solenoid, as shown. The ring has a narrow gap of width δ in its circumference. The cross-sectional area of the solenoid is a. The solenoid has a uniform internal field of magnitude $B(t) = B_0 + \beta t$, where $\beta > 0$. Assuming that no charge can flow across the gap, the face(s) accumulating an excess of positive charge is/are



(A) F_1

(B) F_2

(C) F_1 and F_2 both

(D) difficult to conclude as data given is insufficient

21. In PROBLEM 20, the accumulation of the charge on the gap faces will cease when the total electric field within the ring becomes zero. For this to happen, the electric field in the gap is E_0.

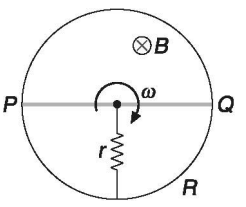
(A) $E_0 = \frac{a\beta}{\delta}$

(B) $E_0 = \frac{2a\beta}{\delta}$

(C) E_0 is dependent on R for $R > \sqrt{\frac{a}{\pi}}$

(D) E_0 is independent of R for $R > \sqrt{\frac{a}{\pi}}$

22. A fixed conducting ring R of negligible resistance and radius a has a uniform rod PQ of resistance r hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There exists a uniform magnetic field of strength B pointing inwards in this region. A fixed resistor of resistance r is connected between centre of ring and its circumference which does not hinder the path of rotating rod as shown in Figure. Select the correct statement(s).



(A) The current through r is zero

(B) The current through r is $\frac{2B\omega a^2}{5r}$

(C) The direction of current in external r is from centre to circumference

(D) The direction of current in external r is from circumference to centre

23. The magnetic field perpendicular to the plane of a conducting ring of radius a changes at the rate of α. Then,

(A) all the points on the ring are at the same potential

(B) the emf induced in the ring is $\pi a^2\alpha$

(C) electric field intensity E at any point on the ring is zero

(D) $E=\frac{a\alpha}{2}$

24. A magnetic field B exists perpendicular to the plane of the paper in a cylindrical volume of radius R. If it is increasing at the rate $\frac{dB}{dt}$, then electric field at a distance r from its centre is

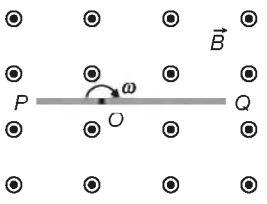
(A) $E=-\frac{r}{2}\left(\frac{dB}{dt}\right)$ for $r<R$

(B) $E=-\frac{r}{4}\left(\frac{dB}{dt}\right)$ for $r<R$

(C) $E=-\frac{R^2}{2r}\frac{dB}{dt}$ for $r>R$

(D) $E=-\frac{r^2}{2R}\frac{dB}{dt}$ for $r>R$

25. A conducting rod PQ of length $8l$ is rotated about a point O at distance $3l$ from P, in a uniform magnetic field $\vec{B}$ directed into the paper. Then



(A) $V_P-V_O=\frac{B\omega l^2}{2}$ (B) $V_O-V_Q=\frac{25}{2}B\omega l^2$

(C) $V_P-V_Q=8B\omega l^2$ (D) $V_O-V_P=\frac{9}{2}B\omega l^2$

26. Select the correct option(s).

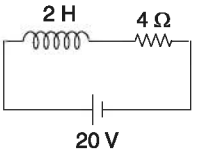
(A) SI unit of magnetic flux is henry-ampere

(B) SI unit of coefficient of self-inductance is JA^{-1}

(C) SI unit of coefficient of self-inductance is $\frac{\text{volt-second}}{\text{ampere}}$

(D) SI unit of magnetic induction is weber

27. In the LR circuit shown in Figure, potential difference across the resistance at some instant is 4 V. Then,



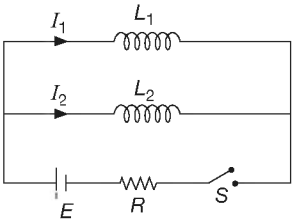
(A) current is increasing at a rate of $8\ As^{-1}$ at this instant

(B) power supplied by the battery at this instant is 20 W

(C) power stored in the magnetic field at this instant is 16 W

(D) current in the circuit at this instant is 1 A

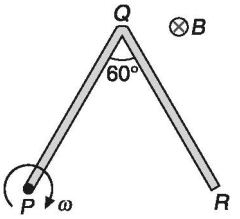
28. In the circuit shown in Figure. The steady state currents I_1 and I_2 in the coils after the switch S is closed are given by



(A) $I_1=\frac{EL_2}{R(L_1+L_2)}$ (B) $I_1=\frac{EL_1}{R(L_1+L_2)}$

(C) $I_2=\frac{EL_2}{R\sqrt{L_1L_2}}$ (D) $I_2=\frac{EL_1}{R(L_1+L_2)}$

29. A bent rod PQR with $PQ=QR=l$ shown in Figure is rotating about its end P with a uniform angular velocity ω in a region of transverse magnetic field of induction B as shown in Figure. Select the correct statement(s).



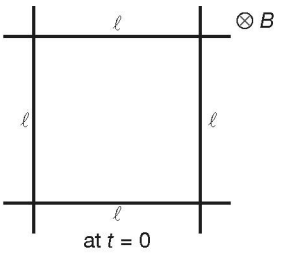
(A) No emf is induced across the rod PQ.

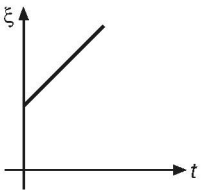
(B) Induced emf across the rod PQ is $\frac{B\omega l^2}{2}$

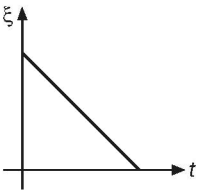
(C) Point P is at a lower potential with respect to point R.

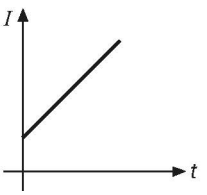
(D) Points Q and R are at the same potential.

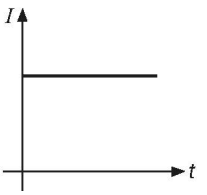
30. Four long conductors having resistance per unit length λ, forming a square, capable of sliding smoothly on each other, are placed in a magnetic field B, as shown. At $t = 0$, the square formed by them has side length l. Now all the four conductors start moving outwards with a constant velocity v. The induced emf ξ and induced current I will vary with time t as



(A) 

(B) 

(C) 

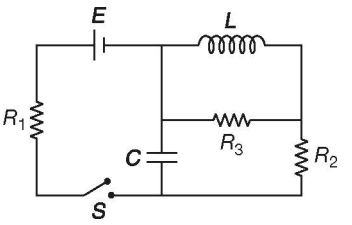
(D) 

31. A conducting circular loop of radius a, resistance R, having resistance per unit length λ is kept on a horizontal plane. A vertical time varying magnetic field $B = 2t$ is switched on at time $t = 0$. If P is the power generated in the coil, $\frac{dq}{dt}$ is the rate of flow of charge from any section of the coil, Q is the total charge passing through coil from $t = 0$ to $t = 3$ s and ξ be the emf induced in the coil. Then

(A) $\xi = 2\pi a^2$ (B) $\frac{dq}{dt} = \frac{a}{\lambda}$

(C) $Q = \frac{6\pi a^2}{R}$ (D) $P = \frac{4\pi^2 a^4}{R}$

32. In the circuit shown, current in R_3



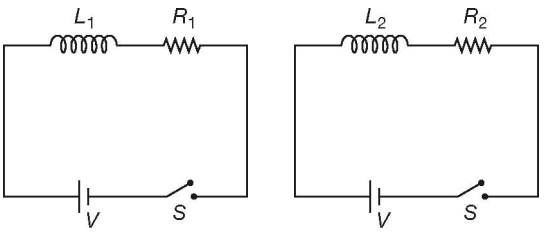
(A) just after closing the switch is zero

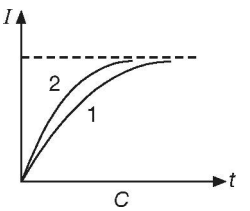
(B) long after closing the switch is zero

(C) just after closing the switch is $\frac{E}{R_3}$

(D) long after closing the switch is $\frac{E}{R_3}$

33. The growth of current in two series LR circuits A and B is shown in C. Let L_1, R_1 and L_2, R_2 be the corresponding inductor, resistor values in the two circuits. Then

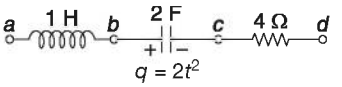




(A) $R_1 > R_2$ (B) $R_1 = R_2$

(C) $L_1 > L_2$ (D) $L_1 < L_2$

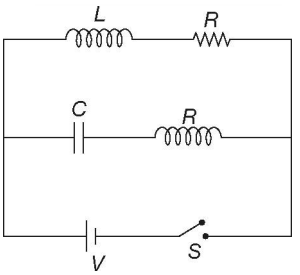
34. In the setup shown in Figure, q is in coulomb and t in second. At time $t = 1$ s, we have



(A) $V_a - V_b = 4$ V (B) $V_b - V_c = 1$ V

(C) $V_c - V_d = 16$ V (D) $V_a - V_d = 20$ V

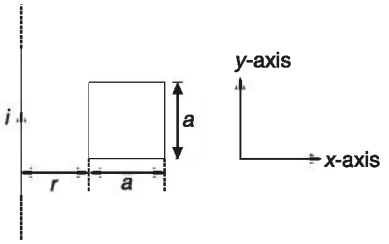
35. In the circuit shown, $R=\sqrt{\frac{L}{C}}$, the switch S is closed at time $t=0$. Equal currents will flow through L and C at time t_0. Then



(A) $t_0 = RC$ (B) $t_0 = RC\ln(2)$

(C) $t_0 = \frac{L\ln(2)}{R}$ (D) $t_0 = LR$

36. Figure shows a long straight conductor carrying current i. A square loop of side a is kept at a distance r from it. Which of the following is correct?



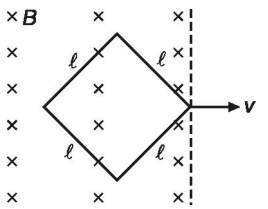
(A) Mutual inductance $M=\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{r}\right)$

(B) If loop is pulled with constant speed v along x-axis, emf induced at the instant is $\frac{\mu_0 i a^2 v}{2\pi r(r+a)}$

(C) If loop is pulled with constant speed v, no emf is induced

(D) If i increases with time, the loop is repelled away from the straight conductor

37. Figure shows a square loop being pulled out with a constant speed out of a region of uniform magnetic field. The induced emf in the loop



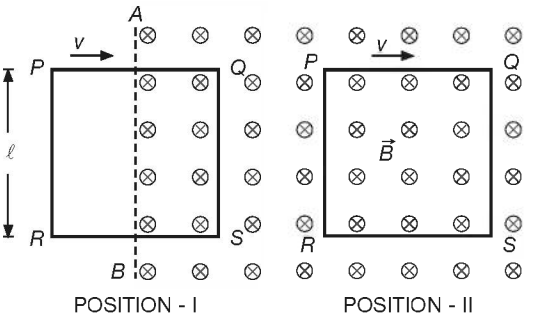
(A) first increases, then decreases

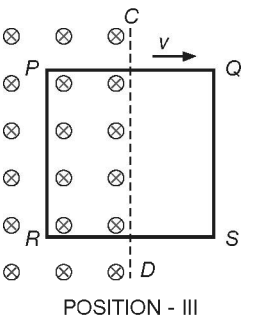
(B) first decreases, then increases

(C) has a maximum value $Bvl\sqrt{2}$

(D) has a maximum value $2Bvl$

38. A square loop $PQRS$ of edge l moves to the right with a velocity v, parallel to PQ. There is a uniform magnetic field of magnitude B, directed into the paper, in the region between AB and CD only. I, II and III are three positions of the loop.





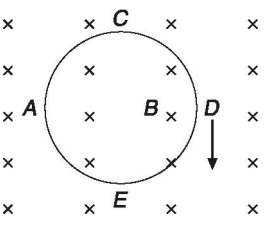
(A) In all the three positions, the emf induced in the loop has a magnitude Blv

(B) In position I, the induced current is anticlockwise

(C) In position II, the induced emf is zero

(D) In position III, the induced current is clockwise

39. A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B. The direction of B is perpendicular to the plane of the ring. Then,



(A) no current flows in the ring

(B) A and D are at the same potential

(C) C and E are at the same potential

(D) the potential difference between A and D is $2BRv$, with D at a higher potential

40. An ideal inductor with initial current zero, a resistor and an ideal battery are connected in series at time $t=0$. At any time t, the battery supplies energy at the rate P_B, the resistor dissipates energy at the rate P_R and the inductor stores energy at the rate P_L then

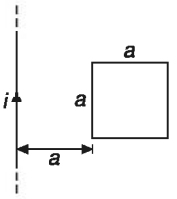
(A) at all instants of time, $P_B = P_R + P_L$

(B) at all instants of time, $P_R < P_L$

(C) in steady state, $P_L < P_R$

(D) at the instant when the circuit is just closed, $P_R > P_L$

41. An infinitely long wire is placed near a square loop as shown in Figure.



Choose the correct statement(s)

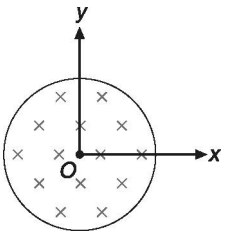
(A) The mutual inductance between the two is $\frac{\mu_0 a}{2\pi}\ln(2)$.

(B) The mutual inductance between the two is $\frac{\mu_0 a^2}{2\pi}\ln(2)$.

(C) If a constant current is passed in the straight wire in upward direction and loop is brought close to the wire, then induced current in the loop is counter clockwise.

(D) In the above condition, induced current in the loop is anti-clockwise.

42. Magnetic field in a cylindrical region of radius R, an inward magnetic field exists as shown in Figure.



Select the correct statement(s).

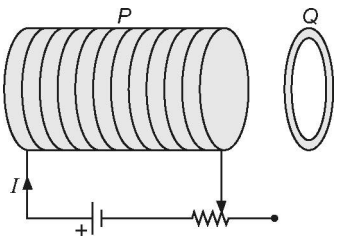
(A) An electron kept at $(2R, 0, 0)$ will experience no force if magnetic field increases with time.

(B) An electron kept at $(2R, 0, 0)$ will experience a force along negative y direction if the magnetic field increases with time.

(C) A proton kept at $\left(0, \frac{R}{2}, 0\right)$ will experience the force along positive x-direction if magnetic field is decreasing with time.

(D) A proton kept at $(-R, 0, 0)$ will experience force along negative y-direction if magnetic field is increasing with time.

43. In the arrangement shown, an aluminium ring Q faces an electromagnet P. The current I through P can be altered using a variable resistor. Then



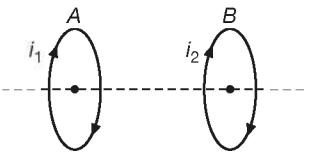
(A) P repels Q, when I increases

(B) P attracts Q, when I increases

(C) P repels Q, when I decreases

(D) P attracts Q, when I decreases

44. Two circular coils are placed adjacent to each other. Their planes are parallel and currents through them i_1 and i_2 are in same direction. Select the correct statement(s).



(A) When A is brought near B, current i_2 will decrease

(B) In the above process, current i_2 will increase

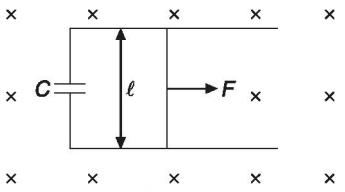
(C) When current i_1 is increased, current i_2 will decrease

(D) In the above process, current i_2 will increase

45. Which of the following statement(s) is/are correct regarding the nature of induced electric field produced by a changing magnetic field?

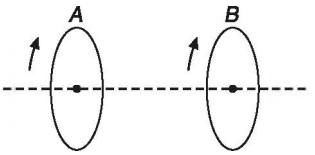
(A) The lines of this field are closed curves
(B) It is non-electrostatic in nature
(C) A corresponding potential can be associated with this induced electric field
(D) It is non-conservative in nature

46. A wire of mass m and length l can freely slide on a pair of smooth, horizontal rails placed in a vertical magnetic field B. The rails are connected by a capacitor of capacitance C. The electric resistance of the rails and wire is zero. If a constant force F acts on the wire and the resulting acceleration is a, then,



(A) current in the circuit is $BlCa$
(B) charge on the capacitor is $CBla\sqrt{\frac{2l}{a}}$
(C) acceleration $a=\frac{F}{m+B^2l^2C}$
(D) acceleration $a=\frac{F}{m}$

47. Two circular coils A and B are arranged co-axially and the currents flowing in the direction of arrow are taken positive. Then,

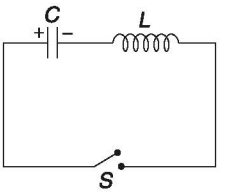


(A) if A carries a steady positive current and A is moved towards B, a negative current is induced in B
(B) if A carries a steady positive current and B is moved towards A, a negative current is induced in B
(C) if both coils carry positive current, they attract each other
(D) None of these

48. In LC oscillations, the quantities that are analogue to each other are

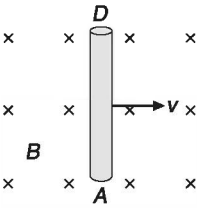
(A) L and m (B) C and K
(C) I and v (D) $\dot{I}$ and $\dot{v}$

49. A capacitor is charged to a potential of V_0. It is connected with an inductor through a switch S. The switch is closed at time $t=0$. Which of the following statement(s) is/are correct?



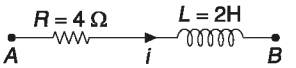
(A) The maximum current in the circuit is $V_0\sqrt{\frac{C}{L}}$
(B) Potential across capacitor becomes zero for the first time at $t=\pi\sqrt{LC}$
(C) Energy stored in the inductor at time $t=\frac{\pi}{2}\sqrt{LC}$ is $\frac{1}{2}CV_0^2$
(D) Maximum energy stored in the inductor is $\frac{1}{2}CV_0^2$

50. The conductor AD moves to the right in a uniform magnetic field directed into the paper. Then,



(A) the free electrons in AD will move towards A
(B) D will acquire a positive potential with respect to A
(C) if D and A are joined by a conductor externally, a current will flow from A to D in AD
(D) the current in AD flows from lower to higher potential

51. In Figure shown the current in circuit is $i=10e^{-4t}$ A. If V_L be the potential difference across the inductor and V_{AB} be the potential difference across the points A and B, then we have



(A) $V_{AB}=40e^{40t}$
(B) $V_{AB}=-40e^{-40t}$
(C) $V_L=-80e^{40t}$
(D) $V_L=-80e^{-40t}$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** The induced e.m.f. in a conducting loop of wire will be non-zero when it rotates in a uniform magnetic field.
Statement-2: The e.m.f. may be induced due to change in magnetic field.

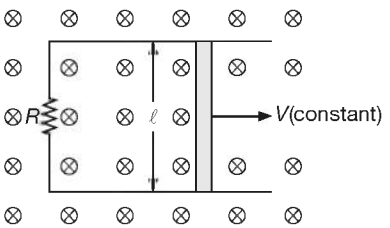
2. **Statement-1:** If electric current, changes through a circuit, eddy currents are induced in nearby iron piece.
Statement-2: Due to change of electric current, the magnetic flux through iron piece changes, so eddy currents are induced in the iron piece.

3. **Statement-1:** The work done by a charge in a closed (induced) current carrying loop is non-zero.
Statement-2: Induced electric field is non-conservative in nature.

4. **Statement-1:** Lenz's Law violates the principle of conservation of energy.
Statement-2: Induced e.m.f. always opposes the change in magnetic flux responsible for its production.

5. **Statement-1:** The self-inductance (L) is given by the relation $\phi_B = LI$.
Statement-2: When current I is increased, self-inductance increases.

6. **Statement-1:** In the situation shown, a conductor can be moved with constant velocity by an external agent, that applies a force on it so that it moves with constant velocity.



Statement-2: As conductor is moved, a current is induced in the circuit as a result of which magnetic force acts on conductor opposite to its velocity.

7. **Statement-1:** An emf is induced in a long solenoid by a bar magnet that moves while totally inside it along the solenoid axis.
Statement-2: As the magnet moves inside the solenoid the flux through individual turns of the solenoid changes.

8. **Statement-1:** Time dependent magnetic field generates an induced electric field.
Statement-2: Direction of electric field generated from time variable magnetic field does not obey Lenz's Law.

9. **Statement-1:** If a coil is rotated in uniform magnetic field about an axis perpendicular to the field, e.m.f. induced in coil is maximum for orientation of coil in which magnetic flux through the coil is zero.
Statement-2: Work done to rotate the coil will get converted into electrical energy.

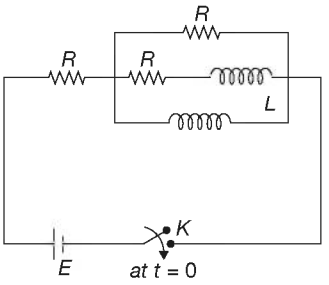
10. **Statement-1:** The direction of the induced electric field is always perpendicular to the direction of the (changing) magnetic field.
Statement-2: The induced electric field is a non-conservative field.

11. **Statement-1:** If a charged particle is released from rest in a time varying magnetic field, it moves in a circle.
Statement-2: In time varying magnetic field electric field is induced.

12. **Statement-1:** Varying magnetic field produces an electric field, which is non-conservative.
Statement-2: Charged particle(s) in motion produces only magnetic field.

13. **Statement-1:** When two coils are wound on each other, the mutual induction between the coils is maximum.
Statement-2: Mutual induction does not depend on the orientation of the coils.

14. **Statement-1:** At $t = 0$, current through e.m.f. source $I = \frac{E}{2R}$ and at $t \to \infty$ the I (through e.m.f. source will be) is $\frac{E}{R}$.
Statement-2: At $t = 0$, inductor will behave like open circuit and at $t \to \infty$, inductor will behave like short circuit.



15. **Statement-1:** The growth of current in LR circuit is uniform.
Statement-2: Inductor (L) opposes the growth of current.

16. **Statement-1:** The nature of induced e.m.f. is always such, so that it always opposes the change that causes it.
Statement-2: The direction of induced e.m.f. is given by Lenz's Law.

17. **Statement-1:** A system cannot have mutual inductance without having self-inductance.
Statement-2: If mutual inductance of system is zero, its self-inductance must be zero.

18. **Statement-1:** Two identical co-axial circular loops carry equal currents in same direction. When both loops start approaching each other, the current in both coils will decreases.
Statement-2: Current in a circuit is independent of any other circuit.

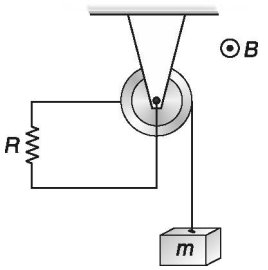
LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension 1

A metal ring having three metallic spokes of length $r = 0.2$ m is in a vertical plane and can spin around a fixed horizontal axis in a homogeneous magnetic field of a magnetic induction $B = 0.5$ T. The lines of magnetic field are perpendicular to the plane of metal ring. Between the axis of the metal ring and its perimeter we connect a consumer of resistance of $0.15\ \Omega$ with the help of two sliding contacts. We fix a thread of negligible mass to the rim of the ring and wind it several times around the ring and to its end we fix a body of a mass of 20 g. At a given moment we release the body of mass m. The friction is negligible everywhere, the resistance of the ring, the spokes and the connected wiring is also negligible. Based on the above facts, answer the following questions.

1. The torque exerted on the ring with spokes by the magnetic forces when the body of mass m is moving with a constant velocity is



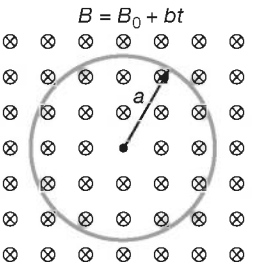
(A) 0.02 Nm (B) 0.04 Nm
(C) 0.06 Nm (D) 0.08 Nm

2. The current is flowing through the resistor when the velocity of the body of mass m is 3 ms^{-1}
(A) 1 A (B) 3 A
(C) 2 A (D) 4 A

3. The highest velocity of the body of mass m
(A) 6 ms^{-1} (B) 7 ms^{-1}
(C) 8 ms^{-1} (D) 12 ms^{-1}

Comprehension 2

A circular loop of wire of radius a, resistance per unit length λ is placed in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field, as shown in Figure.



The magnetic field varies with time according to $B(t) = B_0 + bt$, where B_0 and b are positive constants.

4. The magnetic flux through the loop at $t = 0$ is
(A) πba^2 (B) $\pi B_0 a^2$
(C) $\frac{\pi B_0 a^2}{2}$ (D) ZERO

5. The induced emf in the loop is

(A) πba^2 (B) $\pi B_0 a^2$

(C) $\frac{\pi ba^2}{2}$ (D) $\frac{\pi B_0 a^2}{2}$

6. The induced current and its direction of flow is

(A) $\frac{ba}{\lambda}$, CW (B) $\frac{ba}{\lambda}$, CCW

(C) $\frac{ba}{2\lambda}$, CW (D) $\frac{ba}{2\lambda}$, CCW

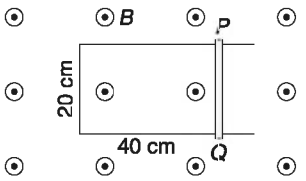
7. The power dissipated due to the resistance of the loop is

(A) $\frac{\pi b^2 a^3}{\lambda}$ (B) $\frac{\pi b^3 a^2}{2\lambda}$

(C) $\frac{\pi b^2 a^3}{2\lambda}$ (D) $\frac{\pi b^3 a^2}{\lambda}$

Comprehension 3

Figure shows a conducting rod PQ of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance $1\ \Omega m^{-1}$. Position of the conducting rod at $t = 0$ is shown. A time dependent magnetic field $B = 2t$ tesla is switched on at $t = 0$. Based on the above facts, answer the following questions.



8. The current in the loop at $t = 0$ due to induced emf is

(A) 0.16 A, clockwise (B) 0.08 A, clockwise

(C) 0.16 A, anti-clockwise (D) ZERO

9. At $t = 0$, when the magnetic field is switched on, the conducting rod is moved to the left at constant speed $5\ cms^{-1}$ by some external means. At $t = 2\ s$, net induced emf has magnitude

(A) 0.12 V (B) 0.08 V

(C) 0.04 V (D) 0.02 V

10. The magnitude of the force required at $t = 2\ s$ to move the conducting rod at a constant speed of $5\ cms^{-1}$ is

(A) 0.096 N (B) 0.12 N

(C) 0.08 N (D) 0.064 N

Comprehension 4

An induction furnace uses electromagnetic induction to produce eddy currents in a conductor, thereby raising the conductor's temperature. Commercial units operate at frequencies ranging from 60 Hz to about 1 MHz and deliver powers from a few watts to several megawatts. Induction heating can be used for welding in a vacuum enclosure, to avoid oxidation and contamination of the metal. At high frequencies, induced currents occur only near the surface of the conductor this is the "skin effect". By creating an induced current for a short time at an appropriately high frequency, one can heat a sample down to a controlled depth. For example, the surface of a farm tiller can be tempered to make it hard and brittle for effective cutting while keeping the interior metal soft and ductile to resist breakage.

To explore induction heating, consider a flat conducting disk of radius R, thickness b and resistivity ρ. A sinusoidal magnetic field $B_0 \cos(\omega t)$ is applied perpendicular to the disk. Assume that the frequency is so low that the skin effect is not important. Assume the eddy currents occur in circles concentric with the disk. Based on the above facts, answer the following questions.

11. The average power delivered to disc is

(A) $\frac{1}{8}\left(\frac{\pi bB_0^2 R^4 \omega^2}{\rho}\right)$ (B) $\frac{1}{4}\left(\frac{\pi RB_0^3 b^4 \omega}{\rho}\right)$

(C) $\frac{1}{8}\left(\frac{\pi b^2 B_0^2 R^3 \omega^4}{\rho}\right)$ (D) $\frac{1}{16}\left(\frac{\pi bB_0^2 R^4 \omega^2}{\rho}\right)$

12. The amplitude of the field is doubled (keeping others same), the factor by which the power changes is

(A) 2 (B) 4

(C) 16 (D) 8

13. The frequency of the field is doubled (keeping others same), then the factor by which the power changes is

(A) 2 (B) 4

(C) 8 (D) 16

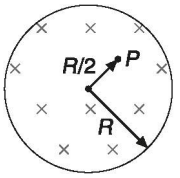
14. The radius of the disc is doubled (keeping others same), then the factor by which the power changes is

(A) 2 (B) 4

(C) 8 (D) 16

Comprehension 5

A uniform but time varying magnetic field $B = (2t^3 + 24t)$ T is present in a cylindrical region of radius $R = 2.5$ cm as shown in Figure.



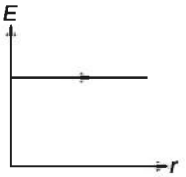
Based on the above facts, answer the following questions.

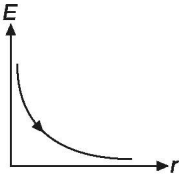
15. The force on an electron at P at $t = 2$ s is

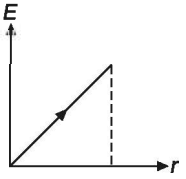
(A) 96×10^{-21} N (B) 48×10^{-21} N

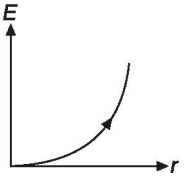
(C) 24×10^{-21} N (D) ZERO

16. At any instant, the variation of induced electric field as a function of distance measured from the centre of cylindrical region is

(A) 

(B) 

(C) 

(D) 

17. At $t = 1$ s, the induced electric field is

(A) clockwise (B) counter-clockwise

(C) zero (D) vertically upwards

Comprehension 6

A circular wire loop of radius a and resistance R initially has a magnetic flux through it due to an external magnetic field. The external field then decreases to zero at a constant rate. A current is induced in the loop while the external field is changing. However, this current does not stop at the instant that the external field stops changing. The reason is that the current itself generates a magnetic field, which gives rise to a flux through the loop. If the current changes, the flux through the loop changes as well and an induced emf appears in the loop to oppose the change. Based on the information provided, answer the following questions.

18. Assuming that the field generated by the induced current I (say) is constant over the loop at all the points, the flux due to this field through the loop is

(A) $\dfrac{\mu_0 \pi a I}{2}$ (B) $2\mu_0 \pi a I$

(C) $\dfrac{\mu_0 \pi a I}{4}$ (D) ZERO

19. After the external field has stopped changing, the rate of fall of induced current is given by

(A) $\dfrac{2R}{\mu_0 \pi a}$ (B) $\dfrac{R}{2\mu_0 \pi a}$

(C) $\left(\dfrac{2R}{\mu_0 \pi a}\right) I$ (D) $\left(\dfrac{R}{2\mu_0 \pi a}\right) I$

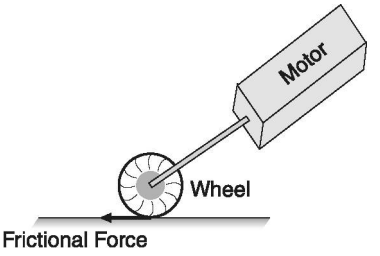
20. Designating the instant when the external field stops changing as $t = 0$ and assuming $I = I_0$ at $t = 0$, then

(A) $I = I_0 + \left(\dfrac{2R}{\mu_0 \pi a}\right) t$ (B) $I = I_0 + \left(\dfrac{R}{2\mu_0 \pi a}\right) t$

(C) $I = I_0 e^{-\left(\frac{R}{2\mu_0 \pi a}\right) t}$ (D) $I = I_0 e^{-\left(\frac{2R}{\mu_0 \pi a}\right) t}$

Comprehension 7

A small electric car overcomes a 250 N friction force between the road and its wheel when travelling at 30 kmh^{-1}. The electric motor is powered by ten 12 V batteries connected in series and is directly coupled to the wheels whose diameters are 50 cm. The 10 cm × 15 cm rectangular motor coils have 300 turns and rotate in a 0.60 T magnetic field. You may assume that the geometry of the motor is such that the magnetic field is always parallel to the plane of the coils. Based on the above facts, answer the following questions.



21. The current that the motor draws to produce the required torque is

(A) 2 A (B) 23.15 A

(C) 13 A (D) 53 A

22. The back emf or induced emf generated in the coil is

(A) 30 V (B) 20 V

(C) 90 V (D) 50 V

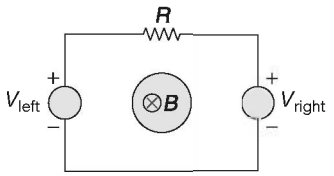
23. The percentage of the input power used to drive the car is

(A) 75% (B) 50%

(C) 95% (D) None of these

Comprehension 8

Two voltmeters V_{right} and V_{left} each with an internal resistance of $10^4\ \Omega$ are in series with a resistor R of $5\times10^3\ \Omega$. They are connected through wires of negligible resistances. The positive side of voltmeters is up as shown in Figure. A changing magnetic field is present in the region shown in Figure. At a particular moment in time V_{left} reads +0.1 volt . Based on the above facts, answer the following questions.



24. At that moment when reading of V_{left} is +0.1 Volt , the reading of V_{right} is

(A) –0.1 Volt (B) –0.2 Volt

(C) –0.3 Volt (D) –0.4 Volt

25. At that moment current in ampere (magnitude and direction)

(A) 10^{-4} A , clockwise

(B) 10^{-4} A , anticlockwise

(C) 10^{-5} A , clockwise

(D) 10^{-5} A , anticlockwise

26. At that moment induced emf is

(A) 0.5 V (B) 0.25 V

(C) 0.75 V (D) 1 V

Comprehension 9

A thin non-conducting ring of mass m, radius a, carrying a charge q can rotate freely about its own axis which is vertical. Initially, the ring was at rest in horizontal position and no magnetic field was present. At instant $t=0$, a uniform magnetic field is switched on which is vertically downward and increases with time as $B=B_0t$, where B_0 is a positive constant. Neglecting magnetism induced due to rotational motion of ring, answer the following questions.

27. The magnitude of emf induced on the closed surface of ring is

(A) πa^2B_0 (B) $2a^2B_0$

(C) ZERO (D) $\frac{1}{2}\pi a^2B_0$

28. The magnitude of electric field at the circumference of the ring is

(A) aB_0 (B) $2aB_0$

(C) $\frac{1}{2}aB_0$ (D) ZERO

29. Angular acceleration of ring is

(A) $\frac{qB_0}{2m}$ (B) $\frac{qB_0}{4m}$

(C) $\frac{qB_0}{m}$ (D) $\frac{2qB_0}{m}$

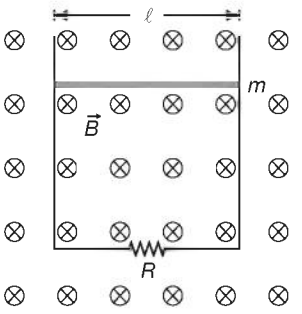
30. The instantaneous power developed by electric force acting on the ring at $t=1$ s is

(A) $\frac{2q^2B_0^2a^2}{14m}$ (B) $\frac{q^2B_0^2a^2}{8m}$

(C) $\frac{3q^2B_0^2a^2}{m}$ (D) $\frac{q^2B_0^2a^2}{4m}$

Comprehension 10

A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length l. Resistance of the circuit is R. A uniform magnetic field B is directed perpendicular to the frame. Based on the facts provided, answer the following questions.



31. The terminal speed of the wire as it falls under the force of gravity is v_t. Then v_t equals

(A) $\frac{mgR}{Bl}$ (B) $\frac{mgl}{BR}$

(C) $\frac{mgR}{B^2l^2}$ (D) $\frac{B^2l^2}{mgR}$

32. Assuming $m = 1$ kg, $g = 10$ ms^{-2} and terminal velocity attained by it to be 4 ms^{-1} after falling through 1 m, the energy dissipated as heat till then is

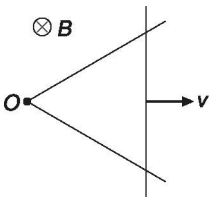
(A) 2 J (B) 8 J
(C) 10 J (D) 12 J

33. The rate at which energy is dissipated in resister, once the terminal speed is attained is

(A) $\frac{B^2l^2v_T^2}{2R}$ (B) $\frac{B^2l^2v_T^2}{R}$
(C) mgv_T (D) $\frac{1}{2}mgv_T$

Comprehension 11

The mobile side of the triangular conducting frame shown in Figure is sliding at uniform speed of $v = 0.1$ ms^{-1} along the other two side. This horizontal frame is in a vertical homogeneous magnetic field with an induction of $B = 0.4$ T. Resistance per unit length is $1\ \Omega$. Initially at $t = 0$ sliding rod was at O. Based on the above facts, answer the following questions.



34. The induced emf
(A) increases with time
(B) decreases with time
(C) remains constant
(D) initially increases, then decreases

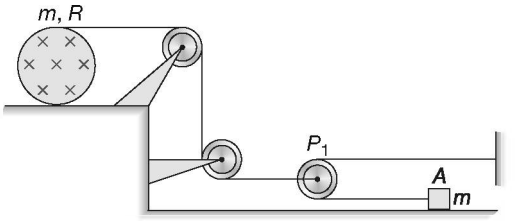
35. Induced current in the frame is
(A) increasing with time
(B) decreasing with time
(C) remain constant
(D) initially increase then decrease

36. Work done in sliding the rod in 10 seconds is

(A) 3×10^{-3} J (B) 9×10^{-3} J
(C) 12×10^{-3} J (D) 14×10^{-3} J

Comprehension 12

A solid non-conducting disc of mass m and radius R has total charge Q is uniformly distributed on its surface. A light thin string is wound around the disc is connected to the light pulley P_1 as shown in Figure. A block of same mass rest on smooth surface. Now uniform magnetic field given by $B = B_0t$ perpendicular into the plane of paper is switched on. The surface on which disc lies is sufficiently rough to ensure pure rolling. Then



37. Tension in string wound around the disc is

(A) $\frac{4QB_0R}{35}$ (B) $\frac{QB_0R}{35}$
(C) $\frac{4QB_0R}{70}$ (D) $\frac{QB_0R}{20}$

38. Angular acceleration of disc is

(A) $\frac{QB_0}{m}$ (B) $\frac{4QB_0}{35m}$
(C) $\frac{QB_0}{70m}$ (D) $\frac{2QB_0}{35m}$

39. Acceleration of block

(A) $\frac{2QB_0R}{35m}$ (B) $\frac{20QB_0R}{7m}$
(C) $\frac{4QB_0R}{41m}$ (D) $\frac{4QB_0R}{35m}$

Comprehension 13

Figure below shows a circular wire loop of radius $r = 0.1$ m with $N = 100$ turns and resistance $R = 2\ \Omega$. The plane of the loop coincides with the plane of paper. There is a uniform magnetic field $B = 4$ T perpendicular to the plane of the paper and directed into page. Since the loop is flexible copper wire, you can grab it on opposite sides and pull it flat. Figure B shows the loop when you are in the middle of pulling it flat. You can assume that with no magnetic field, it would take almost no work to pull the loop flat. Figure C shows how the area enclosed by the loop changes with time. It starts at its initial area and drops linearly to zero in $\Delta t = 3$ sec.

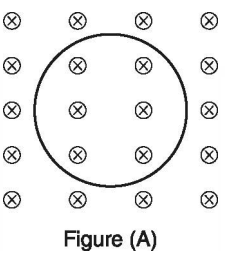


Figure (A)

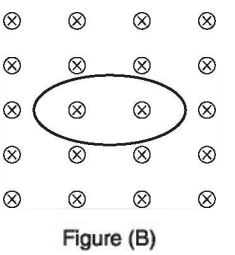


Figure (B)

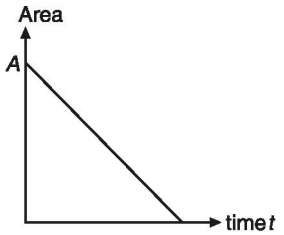


Figure (C)

40. While you are flattening the loop, there is an induced current in the loop. The
(A) induced current is anticlockwise
(B) induced current will increase during flattening the loop
(C) induced current will decrease during flattening the loop
(D) induced current will remain same during flattening the loop

41. At $t = 2$ sec the magnitude of current is

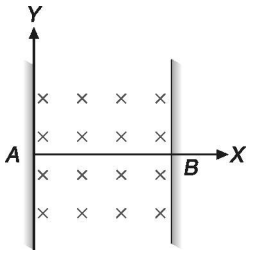
(A) 2.09 A (B) 6.08 A
(C) 1.09 A (D) 0.09 A

42. Average force required to flatten it

(A) 111 N (B) 231 N
(C) 350 N (D) 160 N

Comprehension 14

A standing wave $y = 2A\sin(kx)\cos(\omega t)$ is set up in the wire AB fixed at both ends by two vertical walls. The region between the walls contains a constant magnetic field B. Now answer the following questions.



43. The wire is found to vibrate in the third harmonic. The maximum emf induced is

(A) $\frac{4(AB)\omega}{k}$ (B) $\frac{3(AB)\omega}{k}$
(C) $\frac{2(AB)\omega}{k}$ (D) $\frac{(AB)\omega}{k}$

44. In the above problem, the instant when the emf becomes maximum for the first time is

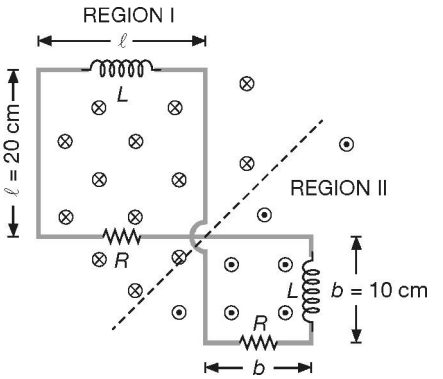
(A) $\frac{2\pi}{\omega}$ (B) $\frac{\pi}{\omega}$
(C) $\frac{\pi}{2\omega}$ (D) $\frac{\pi}{4\omega}$

45. In which of the following modes the emf induced in AB is always zero?

(A) Fundamental mode (B) Second harmonic
(C) Second overtone (D) Forth overtone

Comprehension 15

Consider a frame consisting of two square loops having resistors and inductors as shown. This frame is placed in uniform but time varying magnetic field is such a way that the bigger loop is placed in Region I (above dotted line) having an inward, $\otimes$, field and other is placed in Region II (below the dotted line) having an outward, $\odot$, field. Both magnetic fields are perpendicular to the planes of loops. If the magnetic field is given by $B = (20 + 10t)\ \text{Wbm}^{-2}$ in both regions, $l = 20$ cm, $b = 10$ cm, $R = 10\ \Omega$ and $L = 10$ H, then answer the following questions



46. The direction of induced current in bigger loop will be
(A) clockwise
(B) counter clockwise
(C) first clockwise for some time then counter clockwise and so on
(D) first counter clockwise for some time then clockwise and so on

47. The induced emf in the frame is

(A) 0.1 V (B) 0.3 V
(C) 0.4 V (D) 0.5 V

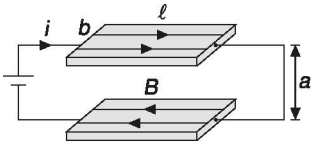
48. The current, in ampere, in the frame as a function of time will be

(A) $\frac{1}{20}(1-e^{-t})$ (B) $\frac{1}{40}(1-e^{-t})$

(C) $\frac{1}{20}e^{-t}$ (D) $\frac{1}{10}e^{-t}$

Comprehension 16

The system shown in Figure consists of two flat conducting strips of length l, width b (perpendicular to plane of diagram) separated by a small gap a (where $a \ll b$ and $a \ll l$). The right ends of the strips are shorted and a battery of voltage V_0 is connected across the left ends. The current is assumed to flow only parallel to the l-dimension of the strips. Neglect all resistances. Based on the above facts, answer the following questions.



49. The self-inductance of the circuit is

(A) $\frac{\mu_0 la}{b}$ (B) $\frac{\mu_0 bl}{a}$

(C) $\frac{\mu_0 al^2}{b^2}$ (D) $\frac{\mu_0 a^2 l}{b^2}$

50. The voltage across the strips as a function of the distance x from the shorted end (if L is the self-inductance) is

(A) $\frac{\mu_0 xaV_0}{bL}$ (B) $\frac{\mu_0 aV_0}{xbL}$

(C) $\frac{\mu_0 ax}{V_0 bL}$ (D) $\frac{\mu_0 aV_0 L}{bx}$

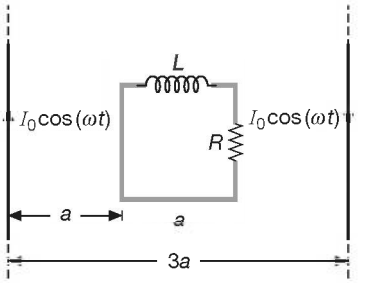
51. The rate of flow of energy down the system as a function of distance x from shorted end and the time t (if L is self-inductance) is

(A) $\frac{\mu_0 xa}{b}\frac{V_0 t}{L}$ (B) $\frac{\mu_0 xa}{b}\frac{V_0^2 t}{L^2}$

(C) $\frac{\mu_0 xaV_0^2 t}{bL}$ (D) ZERO

Comprehension 17

A square loop, consisting of an inductor L and a resistor R is placed between two long parallel wires each carrying time varying current of magnitude $I = I_0\cos(\omega t)$ in opposite directions, as shown. Based on the above facts, answer the following questions.



52. Total magnetic flux associated with the loop is

(A) $\frac{\mu_0 Ia}{\pi}\log_e(2)$ (B) $\frac{2\mu_0 Ia}{\pi}\log_e(2)$

(C) $\frac{4\mu_0 Ia}{\pi}\log_e(2)$ (D) $\frac{\mu_0 Ia}{2\pi}\log_e(2)$

53. Magnitude of emf in this circuit only due to flux change associated with two long straight current carrying wires will be

(A) $\frac{\mu_0 I_0 \omega a\log_e(2)}{\pi}\sin(\omega t)$

(B) $\frac{2\mu_0 I_0 \omega a\log_e(2)}{\pi}\sin(\omega t)$

(C) $\frac{\mu_0 I_0 \omega a\log_e(2)}{2\pi}\cos(\omega t)$

(D) $\frac{\mu_0 I_0 \omega a\log_e(2)}{\pi}\cos(\omega t)$

54. The instantaneous current in the circuit will be

(A) $\frac{2\mu_0 I_0 \omega a\log_e(2)}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\phi)$

(B) $\frac{2\mu_0 I_0 \omega a\log_e(2)}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi)$

(C) $\frac{\mu_0 I_0 \omega a\log_e(2)}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t)$

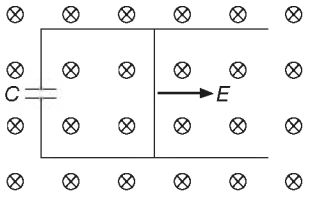
(D) $\frac{\mu_0 I_0 \omega a\log_e(2)}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\phi)$

$\left(\text{where } \tan\phi = \frac{\omega L}{R}\right)$

Comprehension 18

A uniform wire of mass m and length l can slide freely on a pair of parallel, frictionless, horizontal rails placed in a vertical constant magnetic field B (as shown in figure).

The rails are connected by a capacitor of capacitance C. The electric resistance of the circuit is zero. At $t = 0$, a constant horizontal force F acts on the middle point of wire and the wire was at rest initially. Based on the above facts, answer the following questions.



55. At $t = 0$, electric current in the circuit is

(A) zero (B) $\dfrac{BlCF}{(m + B^2l^2C)}$

(C) $\dfrac{BlCF}{m}$ (D) $\dfrac{Blv}{R}$

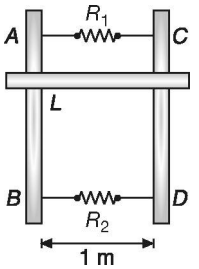
56. At $t = 0$, acceleration of wire is

(A) $\dfrac{F}{m}$ (B) $\dfrac{F}{(m + B^2l^2C)}$

(C) $\dfrac{F}{B^2l^2C}$ (D) $\dfrac{F}{2m}$

Comprehension 19

Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in Figure.



A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively $(g = 9.8\ \text{ms}^{-2})$. Based on the above facts, answer the following questions.

57. The terminal velocity of the bar L will be

(A) $2\ \text{ms}^{-1}$ (B) $3\ \text{ms}^{-1}$

(C) $1\ \text{ms}^{-1}$ (D) None of these

58. The value of R_1 is

(A) 0.47 Ω (B) 0.82 Ω

(C) 0.12 Ω (D) None of these

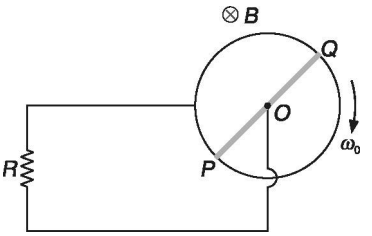
59. The value of R_2 is

(A) 0.6 Ω (B) 0.5 Ω

(C) 0.4 Ω (D) 0.3 Ω

Comprehension 20

A conducting rod PQ of mass M rotates without friction on a horizontal plane about O on circular rails of diameter l. The centre O and the periphery are connected by resistance R. The system is located in a uniform magnetic field perpendicular to the plane of the loop. At $t = 0$, PQ starts rotating clockwise with angular velocity ω_0. Neglect the resistance of the rails and rod, as well as self-inductance.



Based on the above facts, answer the following questions.

60. Magnitude of current as a function of time

(A) $\dfrac{B\omega_0 l^2}{2R}e^{-\alpha t}$ (B) $\dfrac{B\omega_0 l^2}{16R}e^{-2\alpha t}$

(C) $\dfrac{B\omega_0 l^2}{8R}e^{-\alpha t}$ (D) $\dfrac{B\omega_0 l^2}{8R}e^{-2\alpha t}$

where $\alpha = \dfrac{3B^2l^2}{8RM}$

61. Total charge flow through the resistance till rod PQ stops rotating is

(A) $\dfrac{\omega_0 M}{8B}$ (B) $\dfrac{\omega_0 M}{3B}$

(C) $\dfrac{\omega_0 M}{6B}$ (D) $\dfrac{\omega_0 M}{9B}$

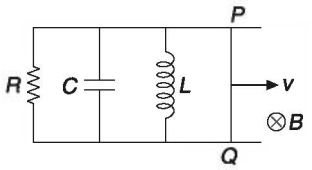
62. Heat generated in the circuit by $t = \infty$

(A) $\frac{Ml^2\omega_0^2}{24}$ (B) $\frac{Ml^2\omega_0^2}{8}$

(C) $\frac{Ml^2\omega_0^2}{3}$ (D) $\frac{Ml^2\omega_0^2}{32}$

Comprehension 21

A conducting wire PQ of length $l = 1$ m, is moved in a uniform magnetic field $B = 4$ T with constant velocity $v = 2$ ms^{-1} towards right as shown in Figure.



If $R = 2\,\Omega$, $C = 1$ F and $L = 4$ H then, answer the following questions.

63. If V_P and V_Q are potentials of the point P and Q respectively, then

(A) $V_P - V_Q = 0$ (B) $V_P - V_Q = -8$ V

(C) $V_Q - V_P = 4$ V (D) $V_Q - V_P = -8$ V

64. The potential difference across the inductor is

(A) 0 V (B) 2 V

(C) 4 V (D) 8 V

65. The potential difference across the capacitor is

(A) 0 V (B) 2 V

(C) 4 V (D) 8 V

66. The potential difference across the resistor is

(A) 0 V (B) 2 V

(C) 4 V (D) 8 V

67. The rate of change of current in the inductor is

(A) 8 As^{-1} (B) 4 As^{-1}

(C) 2 As^{-1} (D) 1 As^{-1}

68. The current through capacitor at $t = 2$ s is

(A) ZERO (B) 2 A

(C) 4 A (D) 6 A

69. The current through resistor at $t = 2$ s is

(A) ZERO (B) 2 A

(C) 4 A (D) 6 A

70. The current through inductor at $t = 2$ s is

(A) ZERO (B) 2 A

(C) 4 A (D) 6 A

71. The current through wire PQ at $t = 2$ s is

(A) 2 A (B) 4 A

(C) 6 A (D) 8 A

72. The force required to move the wire with the given constant velocity of 2 ms^{-1} at $t = 2$ s is

(A) 8 N (B) 16 N

(C) 24 N (D) 32 N

73. The energy supplied per second by the source is

(A) 16 W (B) 32 W

(C) 64 W (D) 128 W

74. The total power generated by the applied external force is

(A) 16 W (B) 32 W

(C) 64 W (D) 128 W

75. The magnetic energy stored per second in inductor at $t = 2$ s is

(A) 16 W (B) 32 W

(C) 64 W (D) 128 W

76. The energy dissipated per second in the resistor at $t = 2$ s is

(A) 16 W (B) 32 W

(C) 64 W (D) 128 W

77. The electrostatic stored per second in capacitor at $t = 2$ s is

(A) 0 W (B) 16 W

(C) 32 W (D) 64 W

Comprehension 22

A fan operates at 200 volt (DC) consuming 1000 W when running at full speed. It's internal wiring has resistance $1\,\Omega$. When the fan runs at full speed, it's speed becomes constant. This is because the torque due to magnetic field inside the fan is balanced by the torque due to air resistance on the blades of the fan and torque due to friction between the fixed part and the shaft of the fan. The electrical power going into the fan is spent (i) in the internal resistance as heat, call it P_1 (ii) in doing work against internal friction and air resistance producing heat, sound etc. call if P_2.

When the coil of fan rotates, an emf is also induced in the coil. This opposes the external emf applied to send the current into the fan. This emf is called bake emf, call it ξ. Answer the following questions when the fan is running at full speed.

78. The current flowing into the fan and the value of back emf ξ is

(A) 200 A, 5 volt (B) 5 A, 200 volt

(C) 5 A, 195 volt (D) 1 A, 0 volt

79. The value of power P_1 is

(A) 1000 W (B) 975 W

(C) 25 W (D) 200 W

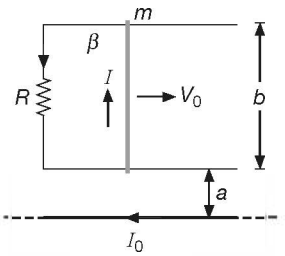
80. The value of power P_2 is

(A) 1000 W (B) 975 W

(C) 25 W (D) 200 W

Comprehension 23

A long straight wire carries a current I_0. At distances a and b from it, there are two other wires, parallel to the former one, which are interconnected by a resistance R. A connector of mass m slides without friction along the wires with an initial velocity v_0 at time $t=0$. Assume the resistances of the wires, the connector, the sliding contacts and the self-inductances of the frame to be negligible. Based on the above facts, answer the following questions.



81. The current I, induced in the circuit is

(A) ZERO

(B) $\frac{\mu_0 I_0 bv}{2\pi R}$, anticlockwise

(C) $\frac{\mu_0 I_0 v_0}{2\pi R}\log_e\left(\frac{b}{a}\right)$, anticlockwise

(D) $\frac{\mu_0 I_0 v_0}{2\pi R}\log_e\left(\frac{a}{b}\right)$, clockwise

82. The time after which velocity becomes half is

(A) $\frac{4\pi^2 mR\ln 2}{\mu_0^2 I_0^2\left(\ln\left(\frac{b}{a}\right)\right)^2}$ (B) $\frac{2\pi^2 mR}{\mu_0^2 I_0^2\left(\ln\left(\frac{b}{a}\right)\right)^2}$

(C) $\frac{\pi^2 mR\log_e(2)}{\mu_0^2 I_0^2\left(\ln\left(\frac{b}{a}\right)\right)^2}$ (D) infinity

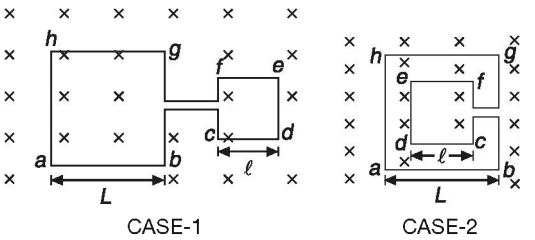
83. The distance covered by the rod until it comes to rest is proportional to

(A) v_0 (B) $\sqrt{v_0}$

(C) $\frac{1}{\sqrt{v_0}}$ (D) $v_0^{-\frac{2}{3}}$

Comprehension 24

Two different arrangements in which two square wire frames are placed in a uniform constantly decreasing magnetic field B are shown in Figure.



In both cases, length of larger square is L and that of the smaller square is l. Based on the above facts, answer the following questions.

84. The value of magnetic flux in CASE-1 and CASE-2, respectively are

(A) $\phi=\pi(L^2+l^2)B$, $\phi=\pi(L^2-l^2)B$

(B) $\phi=\pi(L^2+l^2)B$, $\phi=\pi(L^2+l^2)B$

(C) $\phi=(L^2+l^2)B$, $\phi=(L^2+l^2)B$

(D) $\phi=(L^2+l^2)B$, $\phi=(L^2-l^2)B$

85. The direction of induced current in the CASE-1 is

(A) from a to b and from c to d

(B) from a to b and from f to e

(C) from b to a and from d to c

(D) from b to a and from e to f

86. The direction of induced current in the CASE-2 is
(A) from a to b and from c to d
(B) from b to a and from f to e
(C) from b to a and from c to d
(D) from a to b and from d to c

87. If I_1 and I_2 are the magnitudes of induced current in the CASES-1 and 2 respectively, then,
(A) $I_1 = I_2$ (B) $I_1 > I_2$
(C) $I_1 < I_2$ (D) Nothing can be said

Comprehension 25

An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC . The resistance of the circuit is negligible. Let the instant when the circuit is closed, be at $t = 0$. Based on the above facts, answer the following questions.

88. The total energy stored initially is and is during the LC oscillations. Then
(A) 0.01 J, conserved (B) 0.1 J, not conserved
(C) 1 J, conserved (D) 2 J, not conserved

89. The natural frequency of the circuit is
(A) 50 Hz (B) 159 Hz
(C) 259 Hz (D) 300 Hz

90. The times at which the energy stored is completely electrical i.e., stored in the capacitor are
(A) $0, \frac{T}{2}, T, \frac{3T}{2}$ (B) $0, \frac{T}{4}, \frac{3T}{4}$
(C) $\frac{T}{6}, \frac{T}{12}$ (D) $\frac{T}{4}, \frac{7T}{4}, \frac{9T}{4}$

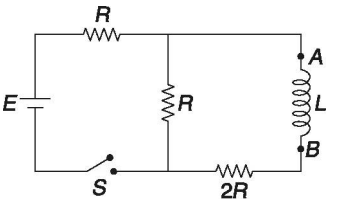
91. The times at which the energy stored is completely magnetic i.e., stored in the inductor are
(A) $0, \frac{T}{2}, T, \frac{3T}{2}$ (B) $0, \frac{T}{4}, \frac{7T}{4}, \frac{9T}{4}$
(C) $\frac{T}{6}$ (D) $\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$

92. The times at which the total energy is equally shared between the inductor and the capacitor are
(A) $0, \frac{T}{2}, T, \frac{3T}{2}$ (B) $\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$
(C) $\frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$ (D) $\frac{T}{12}, \frac{3T}{12}, \frac{5T}{12}$

93. If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?
(A) 0.01 J (B) 0.1 J
(C) 1 J (D) 1.4 J

Comprehension 26

An inductor is connected to three resistors as shown in Figure. Assuming that the switch S is closed at $t = 0$, then answer the following questions.



94. The current flowing through the resistor $2R$ just when the switch is closed is
(A) $\frac{E}{R}$ (B) $\frac{2E}{R}$
(C) $\frac{E}{2R}$ (D) $\frac{E}{4R}$

95. The time constant of the circuit is
(A) $\frac{5R}{2L}$ (B) $\frac{2R}{5L}$
(C) $\frac{5L}{2R}$ (D) $\frac{2L}{5R}$

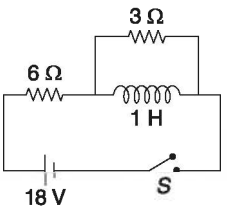
96. The open circuit voltage across the terminals of the inductor just when the switch is closed is
(A) E (B) $\frac{E}{2}$
(C) $\frac{E}{3}$ (D) $\frac{E}{4}$

97. The current through the inductor as a function of time is
(A) $\frac{E}{5R}\left(1-e^{-\frac{2Rt}{5L}}\right)$ (B) $\frac{E}{5R}\left(1-e^{-\frac{5Rt}{2L}}\right)$
(C) $\frac{5E}{R}\left(1-e^{-\frac{5Rt}{2L}}\right)$ (D) $\frac{5E}{R}\left(1-e^{-\frac{2Rt}{5L}}\right)$

98. The current supplied by the battery as a function of time is
(A) $\frac{E}{2R}-\frac{E}{10R}\left(1-e^{-\frac{5Rt}{2L}}\right)$ (B) $\frac{E}{R}+\frac{E}{10R}\left(1-e^{-\frac{5Rt}{2L}}\right)$
(C) $\frac{E}{2R}+\frac{E}{10R}\left(1-e^{-\frac{5Rt}{2L}}\right)$ (D) $\frac{E}{R}-\frac{E}{10R}\left(1-e^{-\frac{5Rt}{2L}}\right)$

Comprehension 27

In LR circuits, the time constant of the circuit can be obtained by the expression $\tau = \frac{L}{R_{net}}$, where R_{net} is the net resistance across inductor after short circuiting the battery. In the circuit shown in Figure, switch S is closed at time $t = 0$. Based on the above facts, answer the following questions.



99. The potential difference across $3\ \Omega$ resistance at time t is given by

(A) $9e^{-2t}$ (B) $6e^{-2t}$

(C) $3e^{-2t}$ (D) $18\left(1-e^{-\frac{t}{9}}\right)$

100. Current I from the battery at time t is given by

(A) $3\left(1-e^{-2t}\right)$ (B) $3+e^{-2t}$

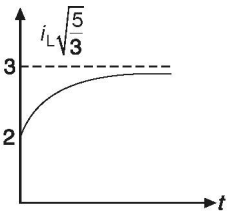
(C) $3\left(1-e^{-\frac{t}{9}}\right)$ (D) $3-e^{-2t}$

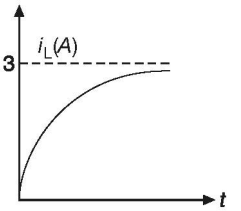
101. The time when the currents through $3\ \Omega$ resistance and 1 H inductor are equal is

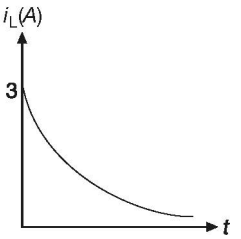
(A) $\ln\sqrt{\frac{5}{3}}$ (B) $\ln\sqrt{\frac{3}{5}}$

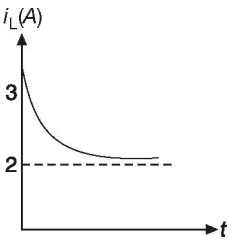
(C) $\ln\left(\frac{5}{3}\right)$ (D) $\ln\left(\frac{3}{5}\right)$

102. Taking left to right current through the inductor as positive current, current through inductor varies with time t as

(A)


(B)


(C)


(D)


MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

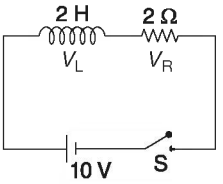
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. A uniform circular disc of mass M, radius R, having uniformly distributed charge Q is placed on a horizontal rough floor of coefficient of friction μ. A uniform, vertically downward magnetic field, varying with time t as $B = B_0 t^2$, is confined in a cylindrical region of radius $r(> R_0)$ coaxial with the disc. Assuming B_0 to be a positive constant, match quantities in **COLUMN-I** (given as a multiple of k) with their respective k values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Rotation of disc begins at time $t_0 = k\left(\frac{\mu Mg}{QB_0R}\right)$.	(p) $\frac{2}{3}$
(B) Torque due to friction at $t = 2$ s is $\tau_f = k(\mu MgR)$.	(q) $\frac{8}{9}$
(C) Torque due to magnetic field at time values $t = 0$, $t = t_0$ and $t = 3t_0$ are $k(\mu MgR)$, with multiple respective values of k to be selected from **COLUMN-II**.	(r) ZERO
(D) Angular velocity of disc at $t = 2t_0$ is $\omega = k\left(\frac{M\mu^2 g^2}{QB_0R^2}\right)$.	(s) $\frac{4}{3}$
	(t) 2

2. In the circuit shown in Figure, switch is closed at time $t = 0$. Match the contents of **COLUMN-I** with their values in **COLUMN-II**.



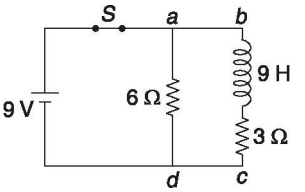
COLUMN-I	COLUMN-II
(A) V_L at $t = 0$	(p) zero
(B) V_R at $t = 0$	(q) 10 V
(C) V_L at $t = 1$ s	(r) $\frac{10}{e}$ V
(D) V_R at $t = 1$ s	(s) $10\left(1 - \frac{1}{e}\right)$ V

(Continued)

3. Match the dimensional formulae with the corresponding units.

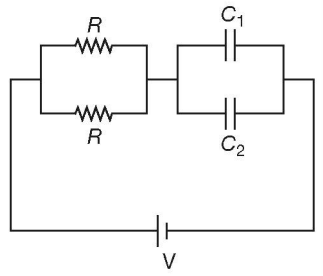
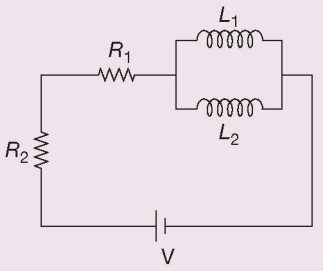
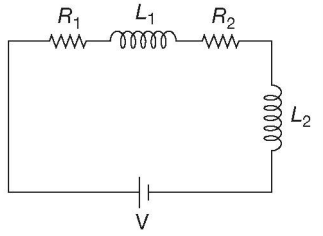
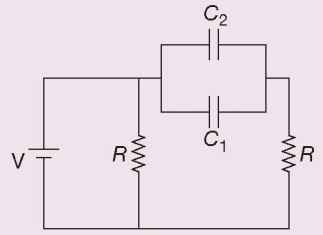
COLUMN-I	COLUMN-II
(A) $ML^2T^{-2}A^{-1}$	(p) farad
(B) $M^{-1}L^{-2}T^4A^2$	(q) weber
(C) $ML^2T^{-2}A^{-2}$	(r) ohm
(D) $ML^2T^{-3}A^{-2}$	(s) henry

4. In the circuit shown in Figure, switch remains closed for long time. It is opened at time $t = 0$. Match the following two columns at $t = (\ln 2)$ second. Match the potential difference specified in **COLUMN-I** to the respective values given in **COLUMN-II**.

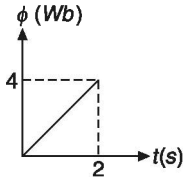


COLUMN-I	COLUMN-II
(A) Across inductor	(p) 9 V
(B) Across 3 Ω resistance	(q) 4.5 V
(C) Across 6 Ω resistance	(r) 6 V
(D) Between points b and c	(s) 13.5 V

5. Match the circuit with their respective time constants.

COLUMN-I	COLUMN-II
(A) 	(p) $\frac{(L_1+L_2)}{(R_1+R_2)}$
(B) 	(q) $\frac{R}{2}(C_1+C_2)$
(C) 	(r) $R(C_1+C_2)$
(D) 	(s) $\frac{\left(\frac{L_1L_2}{L_1+L_2}\right)}{R_1+R_2}$

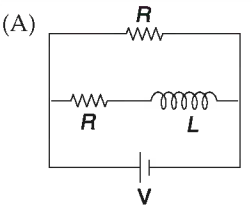
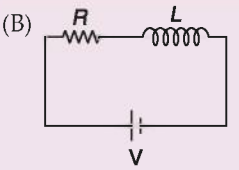
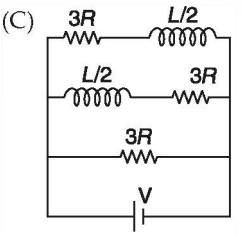
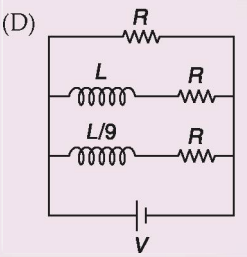
6. The variation of magnetic flux associated with a coil of resistance 2 Ω varies with time t as shown in Figure.



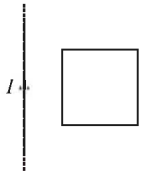
Match the quantities in **COLUMN-I** with their respective values (in SI units) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Induced emf produced	(p) 4
(B) Induced current	(q) 1
(C) Charge flow in 2 s	(r) 8
(D) Heat generation in 2 s	(s) 2

7. Different LR circuits connected to a battery of potential difference V are shown in **COLUMN-I. COLUMN-II** shows the amount of current passing through inductor and magnetic energy stored in each inductor separately if there are two inductors in steady state. Match the situations in **COLUMN-I** with appropriate value in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) $\frac{V}{R}$
(B) 	(q) $\frac{V}{3R}$
(C) 	(r) $\frac{LV^2}{2R^2}$ Joule
(D) 	(s) $\frac{LV^2}{36R^2}$ Joule
	(t) $\frac{LV^2}{18R^2}$ Joule

8. A square loop of perimeter $4l$, is placed near a long straight wire carrying a current I as shown. Match phenomenon in **COLUMN-I** with the consequences in **COLUMN-II**.

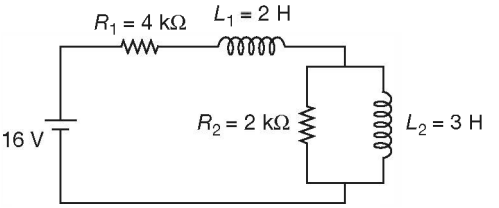


COLUMN-I	COLUMN-II
(A) I is increased	(p) Induced current in loop is clockwise
(B) I is decreased	(q) Induced current in loop is anticlockwise
(C) If loop is moved away from the wire	(r) wire will attract the loop
(D) If loop is moved towards the wire	(s) wire will repel the loop

9. **COLUMN-I** gives certain situations in which a straight metallic wire of resistance R is used and **COLUMN-II** gives some resulting effects. Match the statements in **COLUMN-I** with the statements in **COLUMN-II**.

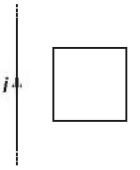
COLUMN-I	COLUMN-II
(A) A charged capacitor is connected to the ends of the wire.	(p) A constant current flow through the wire.
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion.	(q) Thermal energy is generated in the wire.
(C) The wire is placed in a constant electric field that has a direction along the length of the wire.	(r) A constant potential difference develops between the ends of the wire.
(D) A battery of constant emf is connected to the ends of the wire.	(s) Charges of constant magnitude appear at the ends of the wire.

10. In the circuit shown, $L_1 = 2\ \text{H}$, $L_2 = 3\ \text{H}$, $R_1 = 4\ \text{k}\Omega$ and $R_2 = 2\ \text{k}\Omega$. In steady state, match the items of **COLUMN-I** to the values in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) The current through inductor L_1 in mA.	(p) 16
(B) The voltage across inductor L_2 in volt.	(q) 10
(C) The energy stored in inductor L_1 in μJ.	(r) 4
(D) The energy stored in inductor L_2 in μJ.	(s) 0
	(t) 24

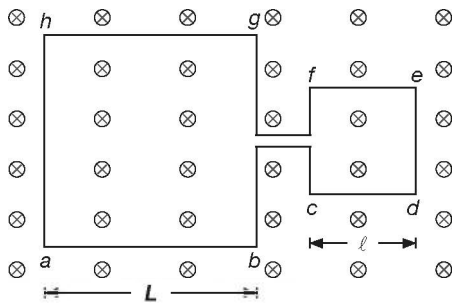
11. A square loop is placed near a long straight current carrying wire as shown in Figure.



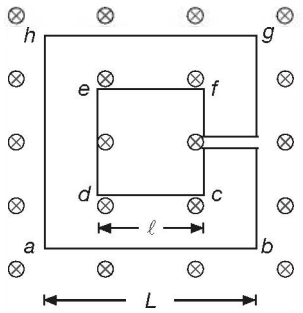
Match the following two **COLUMNS**.

COLUMN-I	COLUMN-II
(A) If current is increased	(p) induced current in loop is clockwise
(B) If current is decreased	(q) induced current in loop is anti-clockwise
(C) If loop is moved away from the wire	(r) wire will attract the loop
(D) If loop is moved towards the wire	(s) wire will repel the loop

12. The figure shows two different arrangements in which two square wire frames are placed in a uniform constantly decreasing magnetic field B.



FRAME I



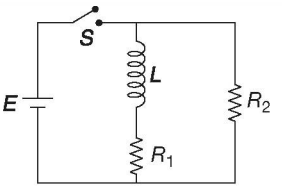
FRAME II

COLUMN-I	COLUMN-II
(A) Magnetic flux in wire frame I	(p) $(L^2 - l^2)B$
(B) Magnetic flux in wire frame II	(q) $(L^2 + l^2)B$
(C) Direction of induced current in wire frame I	(r) from b to a and from f to e
(D) Direction of induced current in wire frame II	(s) from b to a and from c to d
	(t) from b to a and from d to c

13. Consider an LC oscillating circuit and oscillations of a spring block system. Let Q and V be the initial charge and potential on the capacitor, I be the current in the circuit at time t, $\frac{dI}{dt}$ be the rate of change of current, U_C and U_L be the maximum energy across the capacitor and the inductor. Further if mass of block is m, spring constant of spring is k, displacement of block or extension in the spring is x, acceleration of block at any instant is a, F be the restoring force, K be the kinetic energy of block and U be the potential energy of the spring, then match the pairs in **COLUMN-I** to those in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) L, C	(p) x, F
(B) I, $\frac{dI}{dt}$	(q) m, $\frac{1}{k}$
(C) Q, V	(r) U, K
(D) U_C, U_L	(s) v, a
	(t) K, U

14. In the circuit shown in Figure, $E = 18$ V, $L = 2$ H, $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$. Switch S is closed at $t = 0$. Match the following currents through a particular element (at an instant) in **COLUMN-I** with their respective values in **COLUMN-II.**

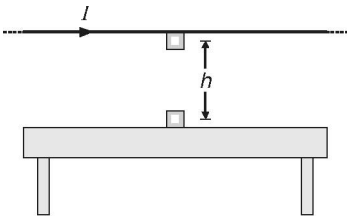


COLUMN-I	COLUMN-II
(A) R_1 at $t = 0$	(p) 6 A
(B) R_1 at $t \to \infty$	(q) 3 A
(C) R_2 at $t = 0$	(r) zero
(D) R_2 at $t \to \infty$	(s) Infinite

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

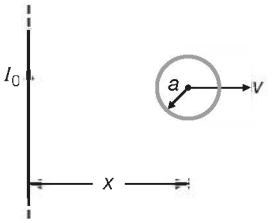
1. A small square washer of side 0.5 cm is held directly below a long, straight wire carrying a current of 10 A. The washer is located 0.5 m above the top of a table.



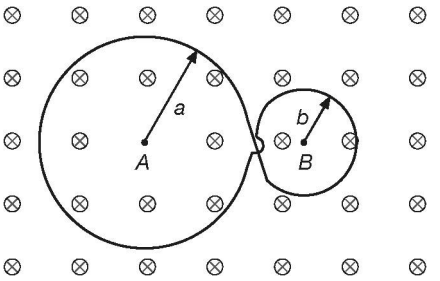
(a) If the washer is dropped from rest, what is the magnitude of the average induced emf, in nano volt, in the washer from the time it is released to the moment it hits the table-top? Assume that the magnetic field is nearly constant over the area of the washer, and equal to the magnetic field at the center of the washer.

(b) What is the direction of the induced current in the washer?

2. A small conducting loop of radius a and resistance per unit length λ, is pulled with velocity v perpendicular to a long straight conductor carrying a current I_0. If a constant power P is dissipated in the loop, the variation of velocity of the loop as a function of x is $\frac{x^2}{\mu_0 I_0 a^2}\sqrt{ka\pi\lambda P}$, where k is an integer. {Given that $x \gg a$}.



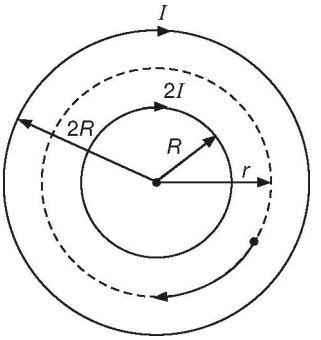
3. A plane loop is shaped in the form as shown in Figure with radii $a = 20$ cm and $b = 10$ cm and is placed in a uniform time varying magnetic field $B = B_0 \sin(\omega t)$ where $B_0 = 10$ mT and $\omega = 10$ rads^{-1}.



Find the amplitude of the induced current, in ampere, in the loop if its resistance per unit length $\lambda = 50 \times 10^{-3}$ Ωm^{-1}. The inductance of the loop is negligible.

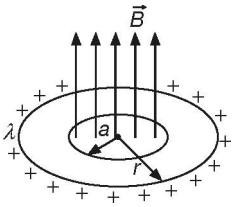
4. Calculate the length of a thin wire in metre required to manufacture a solenoid of length 100 cm and inductance 1 mH. Assume that the solenoid's cross-sectional diameter is considerably less than its length.

5. A long solenoid of radius $2R$ contains another coaxial solenoid of half the radius. The coils have the same number of turns per unit length and initially both carry no current. At a same instant, the currents in both solenoids start increasing linearly with time. At any moment the current flowing in the inner coil is twice as large as that in the outer one and their directions are the same. Due the increasing currents, a charged particle, initially at rest between the solenoids, starts moving along a circular trajectory (see Figure) of radius $r = \sqrt{k}R$. Then find k.



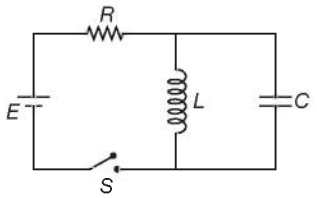
6. A ring of mass 4 kg is uniformly charged with $\lambda = 4$ cm^{-1} and kept on rough horizontal surface with friction coefficient $\mu = \frac{\pi}{4}$. A time varying magnetic

field $B = B_0 t^2$ is applied in a circular region of radius $a (a < r)$ perpendicular to the plane of ring as shown in Figure.

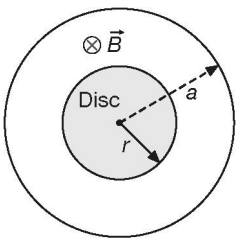


Find out the time when ring just starts to rotate on surface. (Take $a = 5$ cm and $g = 10\ \text{ms}^{-2}$) $\beta_0 = 125$ SI unit.

7. In the circuit of Figure, the battery emf is 50 V, the resistance is 250 Ω and the capacitance is 0.5 μF. The switch S is closed for a long time and no voltage is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. Calculate the inductance, in mH, to the nearest three-digit integer.



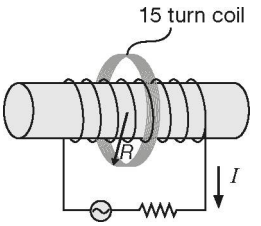
8. A uniform disc of radius r and mass m is charged uniformly with the charge q. This disc is placed flat on a rough horizontal surface having coefficient of friction μ. A uniform magnetic field is present in a circular region $(a > r)$ but varying as kt^3 as shown in Figure.



Find the time, in second, after which the disc begins to rotate. (Given $r = 1$ m, $m = 18$ kg, $q = 1$ C, $\mu = 0.1$, $K = 4$, $g = 10\ \text{ms}^{-2}$)

9. A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.4 s, an emf of magnitude 80 mV is induced in the coil. Calculate the total length of the wire, in metre.

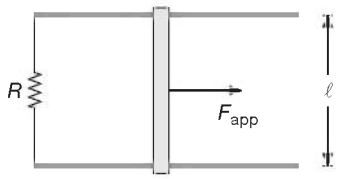
10. A coil of 15 turns and radius 10 cm surrounds a long solenoid of radius 2 cm and 1000 turns per metre as shown. The current in the solenoid changes as $I(\text{in ampere}) = 4\cos(250t)$. Find the induced peak emf, in millivolt, in the 15 turn coil as a function of time. (Take $\pi^2 = 10$)



11. A thin charged ring of radius $a = 9$ cm rotates about its axis with an angular velocity $\omega = 100\ \text{rads}^{-1}$. The ratio of volume energy densities of magnetic and electric field at the axis of the ring at a point lying on the axis of the ring at a distance equal to the radius of the ring is $(*) \times 10^{-16}$, where $*$ is not readable. Find $*$.

12. It has been observed that very large magnetic fields can be produced by adopting a method called **flux compression**. In this method a metallic cylindrical tube of radius R is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder of radius $r < R$. If the collapse happens so rapidly that the induced current in tube maintains a nearly constant magnetic flux inside the tube. If the initial magnetic field in the solenoid is 2.5 T, and $\frac{R}{r} = 12$, what maximum value of magnetic field, in tesla, can be achieved?

13. Figure shows a top view of a bar of length $l = 1.2$ m that can slide without friction. The resistor is 6 Ω and a 2.5 T magnetic field is directed perpendicularly downward, into the paper.

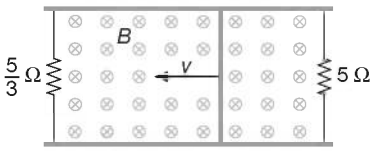


(a) Calculate the applied force, in newton, required to move the bar to the right at a constant speed of $2\ \text{ms}^{-1}$.

(b) At what rate, in SI units, is energy delivered to the resistor?

14. An iron core is inserted into a solenoid 0.5 m long with 4 turns per unit centimetre. The area of cross-section of the solenoid is $0.001\ \text{m}^2$. If the magnetic flux associated with the cross-section of the solenoid is 1.6×10^{-3} Wb, then calculate the relative permeability of the core when a current of 5 A flows through the solenoid and the inductance of the solenoid in mH.

15. A conducting rod of length $l = 50$ cm is free to slide on two parallel conducting bars as shown in Figure. Two resistors $R_1 = \frac{5}{3}\ \Omega$ and $R_2 = 5\ \Omega$ are connected across the ends of the bars to form a loop. A constant magnetic field $B = 2.5$ T is directed perpendicularly into the page. An external agent pulls the rod to the left with a constant speed of $v = 8\ \text{ms}^{-1}$. Find (in SI units)



(a) the current in both resistors and the rod.

(b) the total power delivered to the resistance of the circuit, and

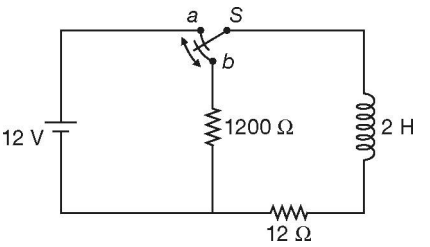
(c) the magnitude of the applied force required to move the rod with this constant velocity.

16. The magnetic flux through a metal ring varies with time t according to $\phi_B = 3(at^3 - bt^2)\ \text{Tm}^2$, with $a = 2\ \text{s}^{-3}$ and $b = 6\ \text{s}^{-2}$. The resistance of the ring is $3\ \Omega$. Determine the maximum current induced in the ring during the interval from $t = 0$ to $t = 2$ s.

17. A 42 H inductor carries a current of 20 A. Calculate the amount of ice in gram at 0 °C that could be melted by the energy stored as magnetic field in the inductor if the latent heat of ice is taken to be $80\ \text{calg}^{-1}$.

18. A $1\ \mu$F capacitor is charged by a 40 V power supply. The fully charged capacitor is then discharged through a 10 mH inductor. Find the maximum current in, mA, in the resulting oscillations.

19. For the circuit shown, answer the following questions.

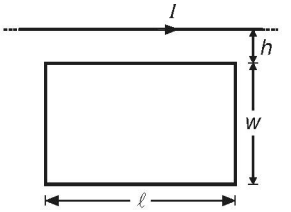


(a) What is the current, in ampere, in the circuit a long time after the switch has been in position a?

(b) Now the switch is thrown quickly from a to b. Compute the initial voltage, in volt, across each resistor and across the inductor.

(c) The time elapsed before the voltage across the inductor drops to 12 V is $x\times10^{-4}$ s. Then find x. Take $\log_e(101) = 4.6$.

20. A relatively long straight conductor and a conducting rectangular loop lie in the same plane, as shown in Figure.



Taking $h = 0.4$ mm, $w = 1.3$ mm and $l = 2.7$ mm, find their mutual inductance, in pH (picohenry).

21. A series RL circuit with $L = 3$ H and a series RC circuit with $C = 3\ \mu$F have equal time constants. The two circuits contain the same resistance R. Find the value of R, in kΩ and the time constant, in millisecond.

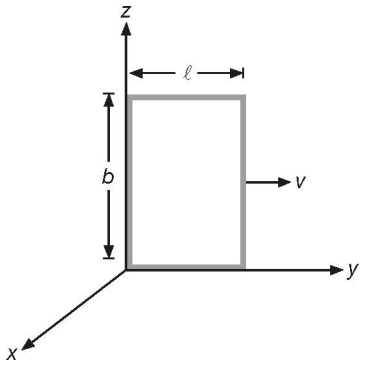
22. Assuming the magnetic field in the space surrounding the earth to be varying according to the expression $B = \frac{B_0R^2}{r^2}$ where B_0 is the field close to the surface of earth, R is the radius of the earth and r is the radial distance from its centre. Take $B_0 = 50\ \mu$T and $R = 6\times10^6$ m, find the energy associated with this magnetic field surrounding the earth. Assume all other fields to be absent or negligible in comparison to B_0. Give your answer in peta joule.

23. A solenoid has an inductance of 20 H and a resistance of $2\ \Omega$. It is connected to a 10 V battery through a switch S. Calculate the time taken (in second) for the magnetic energy to reach one fourth of its maximum value. Take $\ln(2) \approx 0.7$

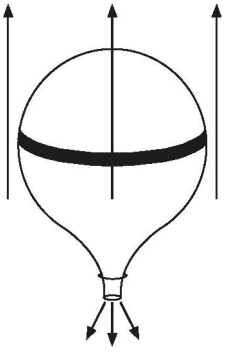
24. A rectangular loop $l \times b$, where $l = 2$ m and $b = 5$ m is moved through a region in which the magnetic field is given by $B_y = B_z = 0$, $B_x = (6 - y)T$. Find the induced emf, in volt, in the loop, at $t = 1$ s, if at $t = 0$, the loop starts moving from rest, from the position (shown in Figure)

(a) at constant velocity $v = 3$ ms^{-1}

(b) at constant acceleration of 2 ms^{-2}.

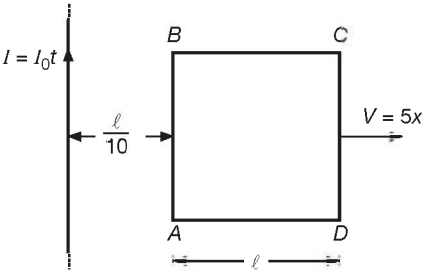


25. An elasticized conducting band is wrapped around a spherical balloon. Its plane passes through the centre of the balloon. A uniform magnetic field of magnitude 0.04 T is directed perpendicular to the plane of the band. Air is let out of the balloon at 100 cm^{3}s^{-1} at an instant when the radius of the balloon is 6 cm. What is the emf induced, in μV, in the band?



26. As shown in Figure a square shaped conducting loop having side length l and resistance $R = \frac{\mu_0 I_0 l}{2\pi}$ is moving away with a velocity $v = 5x$, where x is the separation between conductor and side AB of the loop at any instant, from an infinitely long current carrying conductor carrying a time dependent current given by $I = I_0 t$. At $t = 0$ the side AB is at a distance $\frac{l}{10}$ from the conductor. Find the current, in milliampere, to the nearest three digit integer, in the loop when the side AB is at a distance $10l$ from the conductor.

(Given: $\log_e(100) = 4.605$ and $\log_e(1.1) = 0.095$)



27. A 1 mH inductor and a 1 μF capacitor is connected in series. The current in the circuit is described by $I = 20t$, where t is in second and I is in ampere. The capacitor initially has no charge. Determine,

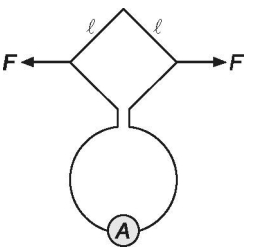
(a) the voltage across the inductor at $t = 1$ s, in millivolt.

(b) the voltage across the capacitor as a function of time at $t = 1$ ms, in volt.

(c) the time, closest to two-digit integer, in microsecond, when the energy stored in the capacitor first exceeds that in the inductor.

28. A total charge of 5×10^{-4} C passes through a 100 turn coil of resistance $200\ \Omega$, cross-sectional area 40 cm^2, when it is rotated in a uniform field from a position where the plane of the coil is perpendicular to the field to a position where the coil's plane is parallel to the field. Calculate B, in millitesla.

29. The plane of a square loop of wire with edge length $l = 0.2$ m is perpendicular to the Earth's magnetic field at a point where $B = 25$ μT, as shown in Figure.



The total resistance of the loop and the wires connecting it to a sensitive ammeter is $0.5\ \Omega$. If the loop is suddenly collapsed by horizontal forces as shown, what is the total charge, in μC, that passes through the ammeter?

30. A small coil is introduced between the poles of an electromagnet so that its axis coincides with the magnetic field direction. The cross-sectional area of the coil is $A = 3\ \text{mm}^2$, the number of turns is $N = 30$. When the coil turns through $180°$ about its diameter, a galvanometer connected to the coil indicates a charge $q = 4.5\ \mu\text{C}$ flowing through it. Find the magnetic induction magnitude between the poles provided the total resistance of the electric circuit equals $R = 40\ \Omega$.

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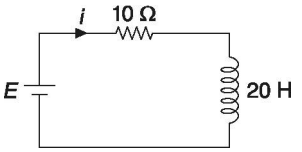
1. **[Online April 2019]**
A 20 H inductor coil is connected to a $10\ \Omega$ resistance in series as shown in Figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(A) $\frac{2}{\ln 2}$ (B) $2\ln 2$

(C) $\ln 2$ (D) $\frac{1}{2}\ln 2$

2. **[Online April 2019]**
The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to

(A) $\frac{1}{L}$ (B) L

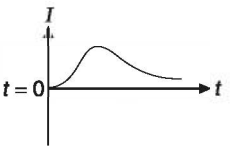
(C) $\frac{1}{L^2}$ (D) L^2

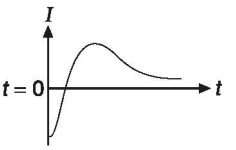
3. **[Online April 2019]**
Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is

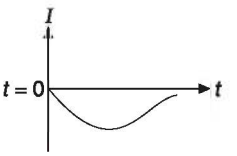
(A) 6.67×10^{-3} Wb (B) 6.67×10^{-4} Wb

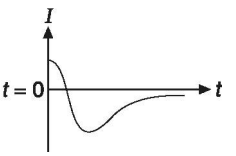
(C) 3.67×10^{-3} Wb (D) 3.67×10^{-4} Wb

4. **[Online April 2019]**
A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k>0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by

(A) 

(B) 

(C) 

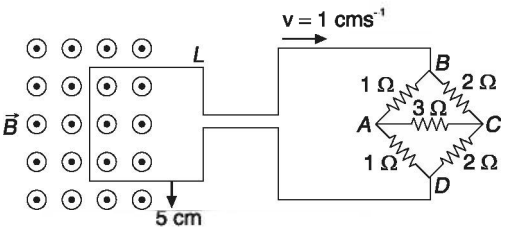
(D) 

5. **[Online April 2019]**
A coil of self-inductance 10 mH and resistance $0.1\ \Omega$ is connected through a switch to a battery of internal resistance $0.9\ \Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [take $\ln 5 = 1.6$]

(A) 0.002 s (B) 0.324 s

(C) 0.103 s (D) 0.016 s

6. **[Online April 2019]**
The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cms^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is $1.7\ \Omega$, the current in the loop at that instant will be close to

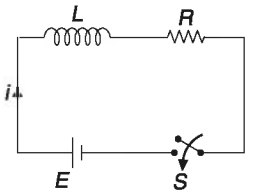


(A) 170 μA (B) 60 μA

(C) 150 μA (D) 115 μA

7. **[Online April 2019]**

Consider the LR circuit shown in Figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is



(A) $\frac{2.7EL}{R^2}$ (B) $\frac{EL}{2.7R^2}$

(C) $\frac{7.3EL}{R^2}$ (D) $\frac{EL}{7.3R^2}$

8. **[Online January 2019]**

A conducting circular loop made of a thin wire, has area 3.5×10^{-3} m^2 and resistance $10\ \Omega$. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to

(A) 7 mC (B) 21 mC

(C) 6 mC (D) 14 mC

9. **[Online January 2019]**

A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is

(A) 520 Am^{-1} (B) 2600 Am^{-1}

(C) 1200 Am^{-1} (D) 285 Am^{-1}

10. **[Online January 2019]**

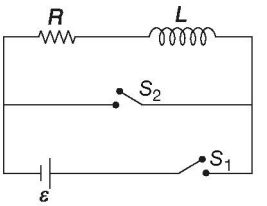
The self-induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is

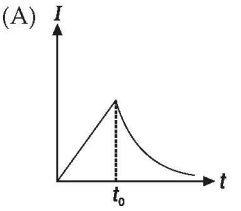
(A) 437.5 J (B) 740 J

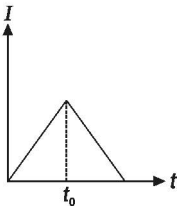
(C) 637.5 J (D) 540 J

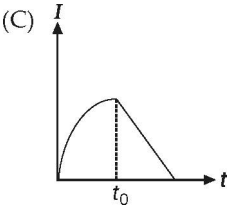
11. **[Online January 2019]**

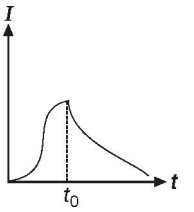
In the circuit shown, the switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time t is given by



(A)


(B)


(C)


(D)


12. **[Online January 2019]**

There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner-coil is

(A) $\frac{n_2}{n_1}\frac{r_2^2}{r_1^2}$ (B) $\frac{n_2}{n_1}$

(C) $\frac{n_2}{n_1}\frac{r_1}{r_2}$ (D) $\frac{n_1}{n_2}$

13. **[Online January 2019]**

A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil

(A) Increases by a factor of 3

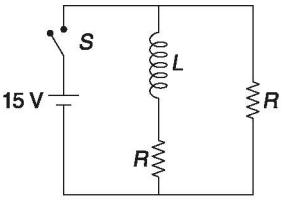
(B) Decreases by a factor of $9\sqrt{3}$

(C) Decreases by a factor of 9

(D) Increases by a factor of 27

14. **[Online January 2019]**

In the Figure shown, a circuit contains two identical resistors with resistance $R = 5\ \Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed?



(A) 7.5 A (B) 3 A

(C) 6 A (D) 5.5 A

15. [Online January 2019]

A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, of 0.3×10^{-4} Wbm^{-2}. The value of the induced emf in wire is

(A) 1.1×10^{-3} V (B) 0.3×10^{-3} V

(C) 2.5×10^{-3} V (D) 1.5×10^{-3} V

16. [Online 2018]

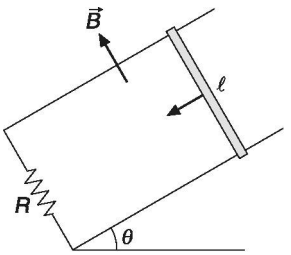
At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

(A) $\frac{\mu_0 I}{4R}\omega\pi r^2\sin\omega t$ (B) $\frac{\mu_0 I}{4R}\omega r^2\sin\omega t$

(C) $\frac{\mu_0 I}{2R}\omega r^2\sin\omega t$ (D) $\frac{\mu_0 I}{2R}\omega\pi r^2\sin\omega t$

17. [Online 2018]

A copper rod of mass m slides under gravity on two smooth parallel rails, with separation l and set an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in Figure. The terminal speed of the copper rod is



(A) $\frac{mgR\sin\theta}{B^2l^2}$ (B) $\frac{mgR\cot\theta}{B^2l^2}$

(C) $\frac{mgR\tan\theta}{B^2l^2}$ (D) $\frac{mgR\cos\theta}{B^2l^2}$

18. [Online 2018]

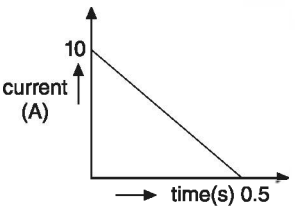
A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω, the maximum e.m.f. induced in the coil will be

(A) $\frac{3}{2}nBA\omega$ (B) $nBA\omega$

(C) $3nBA\omega$ (D) $\frac{1}{2}nBA\omega$

19. [2017]

In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in Figure. The magnitude of change in flux through the coil is



(A) 200 Wb (B) 225 Wb

(C) 250 Wb (D) 275 Wb

20. [Online 2017]

A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_0\cos(\omega t)$. The emf induced in the smaller inner loop is nearly

(A) $\frac{\pi\mu_0 I_0}{2}\frac{a^2}{b}\omega\cos(\omega t)$ (B) $\frac{\pi\mu_0 I_0 b^2}{a}\omega\cos(\omega t)$

(C) $\frac{\pi\mu_0 I_0}{2}\frac{a^2}{b}\omega\sin(\omega t)$ (D) $\pi\mu_0 I_0\frac{a^2}{b}\omega\sin(\omega t)$

21. [Online 2016]

A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where B_0 and τ are constants, at time $t = 0$. If the resistance of the loop is R then the heat generated in the loop after a long time $(t\to\infty)$ is

(A) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (B) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$

(C) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (D) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

22. [Online 2016]

A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of 15 m and height 5 m is flying towards east over Delhi. Its speed is 240 ms^{-1}. The earth's magnetic field over Delhi is 5×10^{-5} T with the declination angle ~0° and dip of θ such that $\sin\theta=\frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to

(A) $V_B = 40$ mV ; $V_W = 135$ mV with left side of pilot at higher voltage

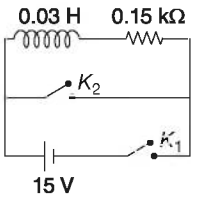
(B) $V_B = 45$ mV ; $V_W = 120$ mV with right side of pilot at higher voltage

(C) $V_B = 40$ mV ; $V_W = 135$ mV with right side of pilot at higher voltage

(D) $V_B = 45$ mV ; $V_W = 120$ mV with left side of pilot at higher voltage.

23. [2015]

An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15 V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be ($e^5 \cong 150$)

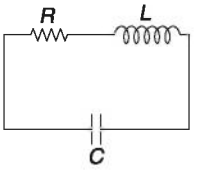


(A) 6.7 mA (B) 0.67 mA

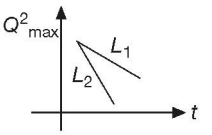
(C) 100 mA (D) 67 mA

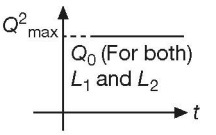
24. [2015]

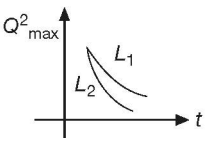
An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown here.

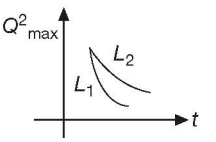


If a student plots graphs of the square of maximum charge (Q^2_{max}) on capacitor with time (t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(A) 

(B) 

(C) 

(D) 

25. [Online 2015]

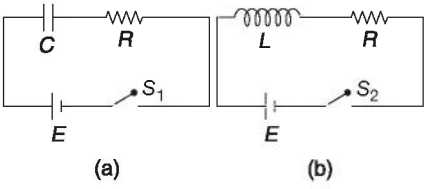
When current in a coil changes from 5 A to 2 A in 0.1 s, an average voltage of 50 V is produced. The self-inductance of the coil is

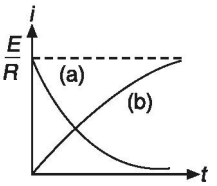
(A) 0.67 H (B) 1.67 H

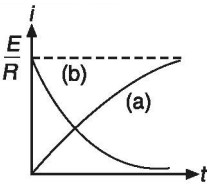
(C) 3 H (D) 6 H

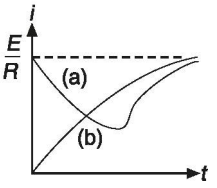
26. [Online 2015]

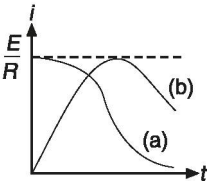
In the circuits (a) and (b) switches S_1 and S_2 are closed at $t = 0$ and are kept closed for a long time. The variation of currents in the two circuits for $t \geq 0$ are roughly shown by (figures are schematic and not drawn to scale)



(A) 

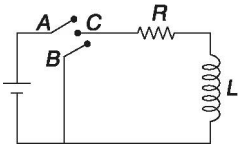
(B) 

(C) 

(D) 

27. [2014]

In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly, point C is disconnected from point A and connected to point B at time $t=0$. Ratio of the voltage across resistance and the inductor at $t=\frac{L}{R}$ will be equal to

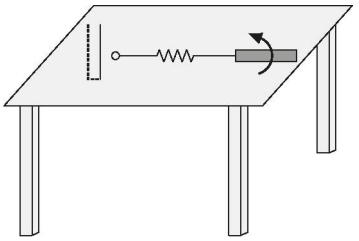


(A) $\frac{1-e}{e}$ (B) $\frac{e}{1-e}$

(C) 1 (D) -1

28. [2013]

A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the e.m.f. induced across the ends of the rod is

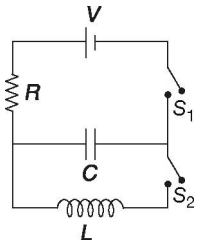


(A) $\frac{5B\omega l^2}{2}$ (B) $\frac{2B\omega l^2}{2}$

(C) $\frac{3B\omega l^2}{2}$ (D) $\frac{4B\omega l^2}{2}$

29. [2013]

In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau=RC$ is capacitive time constant). Which of the following statement is correct?



(A) At $t=\frac{\tau}{2}$, $q=CV\left(1-e^{-1}\right)$

(B) Work done by the battery is half of the energy dissipated in the resistor

(C) At $t=\tau$, $q=\frac{CV}{2}$

(D) At $t=2\tau$, $q=CV\left(1-e^{-2}\right)$

30. [2013]

A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(A) 6.6×10^{-9} weber (B) 9.1×10^{-11} weber

(C) 6×10^{-11} weber (D) 3.3×10^{-11} weber

31. [2011]

A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(A) 1 mV (B) 0.75 mV

(C) 0.50 mV (D) 0.15 mV

32. [2011]

A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. $\left(\log_{10}2.5=0.4\right)$

(A) 1.3×10^{4} Ω (B) 1.7×10^{5} Ω

(C) 2.7×10^{6} Ω (D) 3.3×10^{7} Ω

33. [2011]

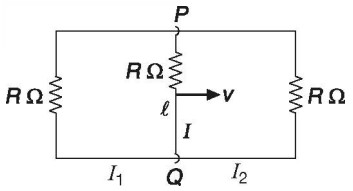
A fully charged capacitor C with initial charge q_0 is connected to a coil of self-inductance L at $t=0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(A) $\pi\sqrt{LC}$ (B) $\frac{\pi}{4}\sqrt{LC}$

(C) $2\pi\sqrt{LC}$ (D) $\sqrt{LC}$

34. [2010]

A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(A) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$

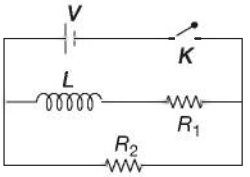
(B) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$

(C) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

(D) $I_1 = I_2 = I = \frac{Blv}{R}$

35. [2010]

In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(A) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

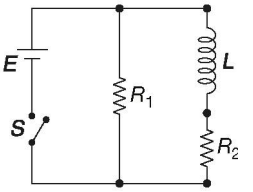
(B) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(C) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(D) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

36. [2009]

An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in Figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(A) $6e^{-5t}$ V

(B) $\frac{12}{t}e^{-3t}$ V

(C) $6\left(1 - e^{\frac{t}{0.2}}\right)$ V

(D) $12e^{-5t}$ V

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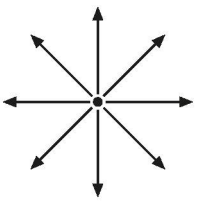
Single Correct Choice Type Problems

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

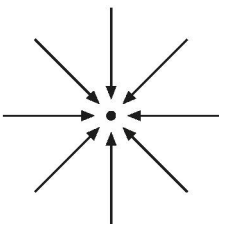
1. [IIT-JEE 2011]

Which of the field patterns given in Figure is valid for electric field as well as for magnetic field?

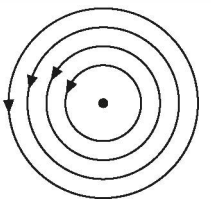
(A)



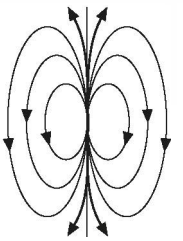
(B)



(C)

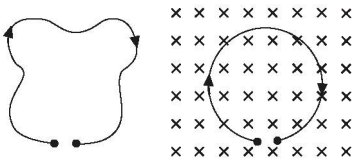


(D)



2. [IIT-JEE 2010]

A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in Figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is

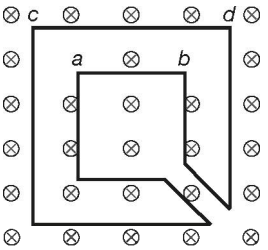


(A) IBL (B) $\frac{IBL}{\pi}$

(C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

3. [IIT-JEE 2009]

The Figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the Figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then,



(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction ba and I_2 is in the direction cd

(D) I_1 is in the direction ab and I_2 is in the direction dc

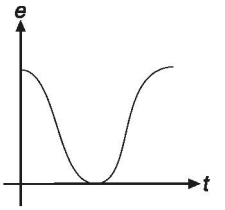
4. [IIT-JEE 2005]

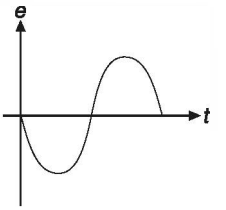
An infinitely long cylinder is kept parallel to a uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be

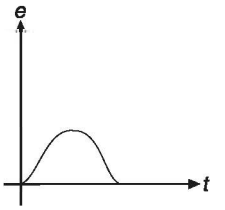
(A) clockwise of the +ve z-axis
(B) anticlockwise of the +ve z-axis
(C) ZERO
(D) along the magnetic field

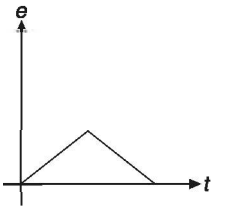
5. [IIT-JEE 2004]

The variation of induced emf (e) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as

(A)


(B)


(C)


(D)


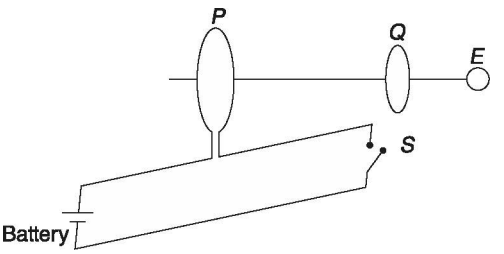
6. [IIT-JEE 2002]

A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(A) halved (B) the same
(C) doubled (D) quadrupled

7. [IIT-JEE 2002]

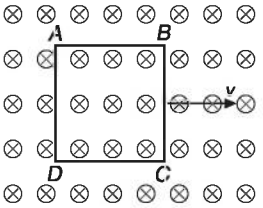
As shown in Figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are



(A) respectively clockwise and anti-clockwise.
(B) both clockwise.
(C) both anti-clockwise.
(D) respectively anti-clockwise and clockwise.

8. [IIT-JEE 2001]

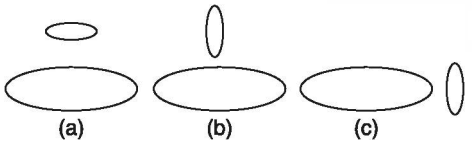
A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in Figure. Electric field is induced



(A) in AD, but not in BC
(B) in BC, but not in AD
(C) neither in AD nor in BC
(D) in both AD and BC

9. [IIT-JEE 2001]

Two circular coils can be arranged in any of the three situations shown in Figure. Their mutual inductance will be



(A) maximum in situation (a)
(B) maximum in situation (b)
(C) maximum in situation (c)
(D) the same in all situations

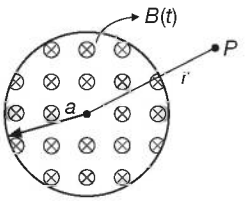
10. [IIT-JEE 2000]

A coil of wire having finite inductance and resistance has a conducting ring placed co-axially within it. The coil is connected to a battery at time $t=0$, so that a time dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time $(t>0)$, the product $I_2(t)B(t)$

(A) increases with time
(B) decreases with time
(C) does not vary with time
(D) passes through a maximum

11. [IIT-JEE 2000]

A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



(A) is ZERO
(B) decreases as $\frac{1}{r}$
(C) increases as r
(D) decreases as $\frac{1}{r^2}$

12. [IIT-JEE 1999]

A circular loop of radius R, carrying current I, lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is

(A) directly proportional to I
(B) directly proportional to R
(C) inversely proportional to R
(D) ZERO

13. [IIT-JEE 1999]

A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1 A at approximately the time

(A) 500 s
(B) 20 ms
(C) 35 ms
(D) 1 ms

14. [IIT-JEE 1999]

Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increase with time. In response, the loop B

(A) remains stationary
(B) is attracted by the loop A
(C) is repelled by the loop A
(D) rotates about its CM, with CM fixed

15. [IIT-JEE 1998]

A small square loop of wire of side l is placed inside a large square loop of side $L(L>>l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

(A) $\frac{l}{L}$
(B) $\frac{l^2}{L}$
(C) $\frac{L}{l}$
(D) $\frac{L^2}{l}$

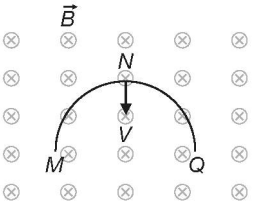
16. [IIT-JEE 1998]

A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following

(A) The entire rod is at same electric potential
(B) There is an electric field in the rod
(C) The electric potential is highest at the centre of the rod and decreases towards its ends
(D) The electric potential is lowest at the centre of the rod and increases towards its ends

17. [IIT-JEE 1996]

A thin semi-circular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is V, and the potential difference developed across the ring is



(A) ZERO

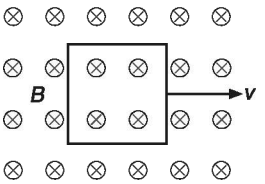
(B) $\frac{BV\pi R^2}{2}$ and M is at a higher potential

(C) πRBV and Q is at a higher potential

(D) $2RBV$ and Q is at a higher potential

18. [IIT-JEE 1989]

A conducting square loop of side L is moving with uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into plane of the loop exists everywhere. Current induced in the loop is



(A) $\frac{BLv}{R}$, clockwise (B) $\frac{BLv}{R}$, anticlockwise

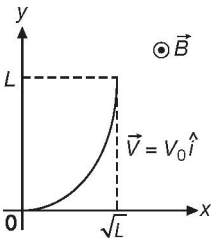
(C) $\frac{2BLv}{R}$, anticlockwise (D) ZERO

Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2019]

A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in Figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are



(A) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

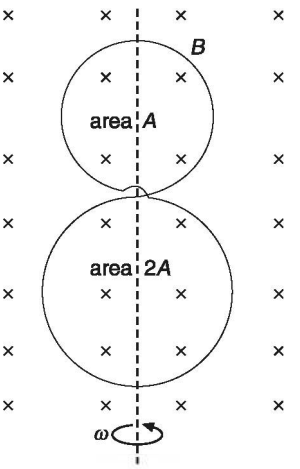
(B) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

(C) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis

(D) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

2. [JEE (Advanced) 2017]

A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in Figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(A) The emf induced in the loop is proportional to the sum of the areas of the two loops

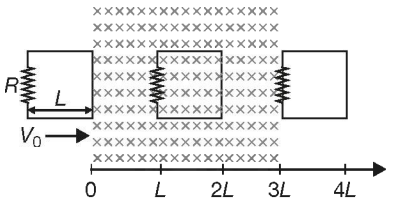
(B) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(C) The net emf induced due to both the loops is proportional to $\cos\omega t$

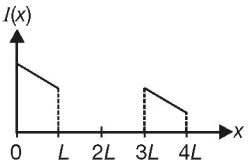
(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

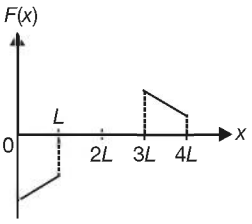
3. [JEE (Advanced) 2016]

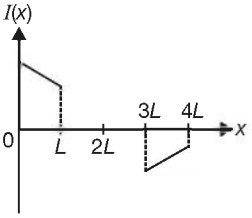
A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in Figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.

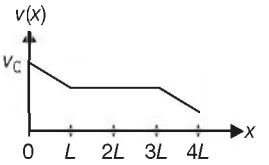


Which of the following schematic plot(s) is(are) correct? (Ignore gravity)

(A)


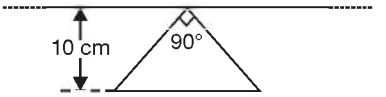
(B)


(C)


(D)


4. [JEE (Advanced) 2016]

A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the $90°$ vertex is very close to an infinitely long conducting wire (see the Figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counter clockwise direction and increased at a constant rate of $10\ \text{As}^{-1}$. Which of the following statement(s) is(are) true?



(A) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(C) The induced current in the wire is in opposite direction to the current along the hypotenuse

(D) There is a repulsive force between the wire and the loop

5. [IIT-JEE 2012]

A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are)

(A) The emf induced in the loop is zero if the current is constant

(B) The emf induced in the loop is finite if the current is constant

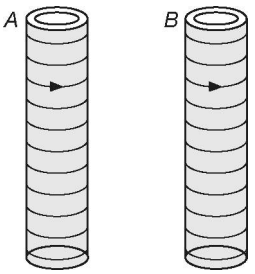
(C) The emf induced in the loop is zero if the current decreases at a steady rate

(D) The emf induced in the loop is finite if the current decreases at a steady rate

6. [IIT-JEE 2009]

Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in Figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$.

The possible relation(s) between their resistivities and their masses m_A and m_B is (are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$

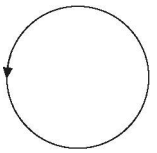
(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

(D) $\rho_A < \rho_B$ and $m_A < m_B$

7. [IIT-JEE 2006]

A field line is shown in Figure. This field cannot represent



(A) magnetic field
(B) electrostatic field
(C) induced electric field
(D) gravitational field

8. [IIT-JEE 1994]

Two different coils have self-inductances, $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, v_1 and w_1 respectively. Corresponding values for the second coil at the same instant are i_2, v_2 and w_2 respectively. Then,

(A) $\frac{i_1}{i_2} = \frac{1}{4}$ (B) $\frac{i_1}{i_2} = 4$

(C) $\frac{w_2}{w_1} = 4$ (D) $\frac{v_2}{v_1} = \frac{1}{4}$

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

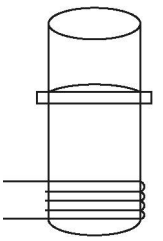
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2007]

Statement-1: A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flow in the coil. The rod goes through a conducting ring as shown in Figure.



The ring can float at a certain height above the coil.

Statement-2: In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit, the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T \gg RC$). In the process, some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_c.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T \gg RC$. Then, the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T \gg RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in process 1. These two processes are depicted in Figure 2.

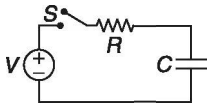


Figure 1

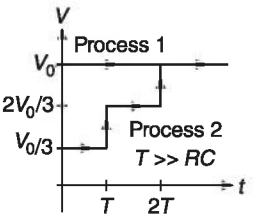


Figure 2

1. [JEE (Advanced) 2017]

In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

(A) $E_C = E_D \ln 2$ (B) $E_C = E_D$

(C) $E_C = 2E_D$ (D) $E_C = \frac{1}{2}E_D$

2. [JEE (Advanced) 2017]

In process 2, total energy dissipated across the resistance E_D is

(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(C) $E_D = 3CV_0^2$ (D) $E_D = \frac{1}{2}CV_0^2$

Comprehension 2

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The applications of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

3. [JEE (Advanced) 2013]

The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(A) $\frac{BR}{4}$ (B) $-\frac{BR}{2}$

(C) BR (D) $2BR$

4. [JEE (Advanced) 2013]

The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(A) γBQR^2 (B) $-\gamma\frac{BQR^2}{2}$

(C) $\gamma\frac{BQR^2}{2}$ (D) γBQR^2

Comprehension 3

Modern trains are based on Maglev technology in which trains are magnetically leviated, which runs its EDS Maglev system. There are coils on both sides of wheels. Due to motion of train, current induces in the coil of track which levitate it. This is in accordance with Lenz's law. If trains lower down then due to Lenz's law a repulsive force increases due to which train gets uplifted and if it goes much high then there is a net downward force disc to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

Disadvantage of Maglev train is that as it slows down the electromagnetic forces decreases and it becomes difficult to keep it leviated and as it moves forward according to Lenz law there is an electromagnetic drag force. Based on the above facts, answer the following questions.

5. [IIT-JEE 2006]

What is the advantage of this system?

(A) No friction hence no power consumption
(B) No electric power is used
(C) Gravitation force is zero
(D) Electrostatic force draws the train

6. [IIT-JEE 2006]

What is the disadvantage of this system?

(A) Train experiences upward force according to Lenz's law
(B) Friction force create a drag on the train
(C) Retardation
(D) By Lenz's law train experience a drag

7. [IIT-JEE 2006]

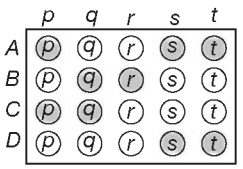
Which force causes the train to elevate up?

(A) Electrostatic force
(B) Time varying electric field
(C) Magnetic force
(D) Induced electric field

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



1. [IIT-JEE 2006]

Match the following

COLUMN-I	COLUMN-II
(A) Dielectric ring uniformly charged	(p) Time independent electrostatic field out of system
(B) Dielectric ring uniformly charged rotating with angular velocity ω	(q) Magnetic field
(C) Constant current in ring i_0	(r) Induced electric field
(D) $i = i_o \cos \omega t$	(s) Magnetic moment

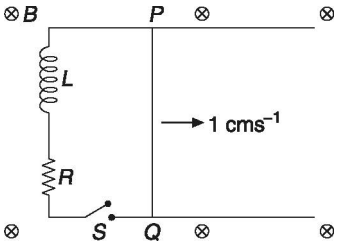
Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2019]

A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cms^{-1} on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor $L = 1$ mH and a resistance $R = 1\ \Omega$ as shown in Figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field $B = 1$ T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is

(Assume the velocity of wire PQ remains constant $(1\ cms^{-1})$ after key S is closed. Given: $e^{-1} = 0.37$, where e is base of the natural logarithm)

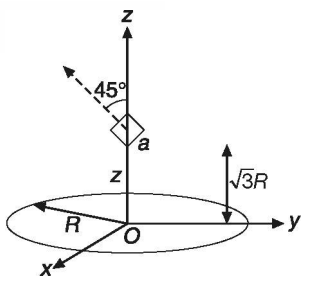


2. [IIT-JEE 2016]

Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current $\left(\frac{I_{max}}{I_{min}}\right)$ drawn from the battery is

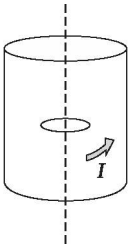
3. [IIT-JEE 2012]

A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side $a\,(a \ll R)$ having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in Figure. The plane of the square loop makes an angle of 45° with respect to the Z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{\frac{p}{2}} R}$, then the value of p is



4. [IIT-JEE 2011]

A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube.



The current varies as $I = I_0 \cos 300t$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then N is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Magnetic Flux, Faraday's Laws and Induced EMF)

1. Negative
2. No current is induced in the loop
3. Clockwise
4. (b) Zero
8. (a) 2.54×10^{5} ms^{-1}
   (b) 215 V
10. (a) πar^2
    (b) $-\pi br^2$
    (c) $\dfrac{br}{2\lambda}$
    (d) $\dfrac{\pi b^2 r^3}{2\lambda}$
11. 1.6 mC
12. 6.75 V
13. $2a^2B_0v(2vt+a)$
14. 62 mV
15. 4.1 V
16. aAB_0e^{-at}
17. 13.3 mA
18. $(68)e^{-1.6t}$ mV, counterclockwise
19. $27\pi\times10^{-3}$ rads^{-1}
20. $\dfrac{\mu_0 Ia^2v}{2\pi x(x+a)}$
21. 0.5 A
22. (a) πa^2CK
    (b) Upper
23. (a) 3.7 A
    (b) 54 μA
    (c) Counterclockwise

Test Your Concepts-II (Based on Faraday's Laws: Motional EMF)

1. (a) 0.5 A
   (b) 2 W
   (c) 2 W
2. $\dfrac{MgR}{B^2w^2}$
3. $\dfrac{\mu_0 Iv\ell}{2\pi rR}$
4. $\dfrac{B\ell v}{\left(R+\dfrac{R_1R_2}{R_1+R_2}\right)}$
5. $\dfrac{mgR\sin\alpha}{B^2\ell^2}$
6. 1 mV
7. 0.02 ms^{-1}, clockwise
8. (a) a to b
   (b) $\dfrac{mgR\sin\phi}{B^2\ell^2\cos^2\phi}$
   (c) $\dfrac{mg\tan\phi}{B\ell}$
   (d) $\dfrac{m^2g^2R\tan^2\phi}{B^2\ell^2}$
   (e) $\dfrac{m^2g^2R\tan^2\phi}{B^2\ell^2}$
10. $\dfrac{Bv}{\lambda(1+\sqrt{2})}$
11. (a) $4Bv(\ell-2vt)$
    (b) $\dfrac{Bv}{\lambda}$
    (c) $\dfrac{B^2v}{\lambda}(\ell-2vt)$
    (d) $\dfrac{4B^2v^2}{\lambda}(\ell-2vt)$
    (e) $\dfrac{4B^2v^2(\ell-2vt)}{\lambda}$
12. (a) $\dfrac{N^2B^2w^2v}{R}$, to the left
    (b) 0
    (c) $\dfrac{N^2B^2w^2v}{R}$, to the left again
13. 2.83 mV
14. $\dfrac{7ak}{22\lambda}$, from P to S
    $\dfrac{ak}{22\lambda}$, from R to Q
    $\dfrac{3ak}{11\lambda}$, from U to T

15. $\frac{\mu_0 Iv}{2\pi}\log_e\left(\frac{d+2r\cos\theta}{d-2r\cos\theta}\right)$

16. $\frac{MgR}{B^2\ell^2}\left(1-e^{-\frac{B^2\ell^2 t}{R(M+m)}}\right)$, $\frac{MgR}{B^2\ell^2}$

Test Your Concepts-III (Based on Faraday's Laws: AC Generator)

1. $30.8\cos(1000t)$ mV, 30.8 mV
2. (a) 36 V
   (b) 0.6 Wbs^{-1}
   (c) $18\sqrt{2}$ V
   (d) 4.32 Nm
3. (a) $5\cos(100\pi t)$
   (b) 1.57 W
4. $\frac{\mu_o Lnda^2 I_o \omega\cos(\omega t)}{2\rho R}$
5. (a) $B\left(\frac{\pi r^2}{2}\cos(\omega t)+ab\right)$
   (b) $\frac{\pi Br^2\omega}{2}\sin(\omega t)$
   (c) $\frac{\pi Br^2\omega}{2R}\sin(\omega t)$
7. (a) 1.6 V
   (b) 0
   (c) Both remain unchanged
8. (a) $(8\times10^{-3})\cos(377t)\ Tm^2$
   (b) $3\sin(377t)$ V
   (c) $3\sin(377t)$ A
   (d) $9\sin^2(377t)$ W
   (e) $24\times10^{-3}\sin^2(377t)$ Nm
9. (a) 100 V
   (b) 600 W
   (c) 2 kW

Test Your Concepts-IV (Based on Faraday's Laws: Induced Electric Field)

1. $\left(\frac{\mu_0 nr\omega I_0}{2}\right)\cos(\omega t)$, clockwise
2. 1.8×10^{-3} NC^{-1}
4. $E=\begin{cases}\frac{r\alpha}{2} & \text{for } r\le R\\ \frac{\alpha R^2}{2r} & \text{for } r\ge R\end{cases}$
6. (a) $\frac{qB_0}{2m}$, $\left(\frac{qB_0}{2m}\right)t$
   (b) $\frac{q^2B_0^2R^2t}{4m}$
7. $\left(\frac{\mu_0 nI}{2}\right)r$, $\left(\frac{\mu_0 nIa^2}{2}\right)\frac{1}{r}$
9. $\frac{E-18\pi r^2}{rR(3+\pi)}$

Test Your Concepts-V (Based on Faraday's Laws: Self Induction)

1. (a) 360 mV
   (b) 180 mV
   (c) 3 s
2. $\frac{\xi_0}{k^2L}$
3. (a) 1.62×10^{-2} V
   (b) **a** is at higher potential.
4. 35 V
5. (a) 1.81×10^{-4} H
   (b) 9.05 mV
6. $\frac{\mu_r\mu_0}{2\pi}N^2a\ln\left(\frac{b+a}{a}\right)$
7. (a) 16 V
   (b) 20 V
   (c) 0 V
8. 64 mV, 32 μC
9. $\frac{\mu_0}{\pi}\ln(\eta)$

Test Your Concepts-VI (Based on Growth and Decay of Current in LR Circuits)

1. $3.6\left(1-e^{-\frac{t}{3\times10^{-4}}}\right)$
2. (a) 400 As^{-1}
   (b) 148 As^{-1}
   (c) 10 A

3. (a) 0.2 s
   (b) 10 A
   (c) 9.93 A
   (d) 100 J
4. (a) $\frac{E}{r}e^{-\left(\frac{R+r}{L}\right)t}$
   (b) $\frac{E^2L}{2r(R+r)}$
5. Dies out slowly
6. $\frac{8}{3}$ A
7. $5\left(1-e^{-\frac{2000t}{3}}\right)$ A
8. $\frac{V}{R}$
9. $\frac{1}{R}\left[E-(E-I_0R)e^{-\frac{Rt}{L}}\right]$
10. $\frac{4}{3}(r_1-r_2)$
11. $\frac{EL_2}{R(L_1+L_2)}, \frac{EL_1}{R(L_1+L_2)}$
12. $\frac{V}{R}-\left(\frac{VR_0}{R+R_0}\right)e^{-\frac{Rt}{L}}$
13. $10e^{-5t}$ V, 5 e^{-10t} A (direction of current is from c to d)
14. $2.22\left(1-e^{-2.7t}\right)$ A

Test Your Concepts-VII (Based on Magnetic Energy and Magnetic Energy Density)

1. 180 H
2. $\frac{2\pi B_0^2R^3}{\mu_0}$
3. $\frac{\mu_0I^2}{4\pi}\ln 2$
4. $u_e = 44.25\ \text{nJm}^{-3}$, $u_m = 995\ \mu\text{Jm}^{-3}$
   $3\times10^8\ \text{ms}^{-1}$, velocity of light
5. $\frac{\mu_0i^2l}{4\pi}\ln\left(\frac{b}{a}\right)$
6. (a) 6.25×10^{-4} H
   (b) 138 mJ, 210 mW
7. (a) $-Lk$
   (b) $\frac{kt^2}{2C}$
   (c) $2\sqrt{LC}$
8. 3.465 s

Test Your Concepts-VIII (Based on Faraday's Laws : Mutual Induction)

1. (a) 2 H
   (b) 30 V
   (c) 1 H
2. 3.125 mH, 937.5 mV
3. $M\approx4\times10^{-4}$ H, $0.02\ \text{V}=20\ \text{mV}$, $20\times10^{-3}\ \text{Wbs}^{-1}$, $10^{-4}\ \text{C}=100\ \mu\text{C}$
4. $\frac{\mu_0\pi(N_1N_2)(R_1R_2)^2}{2(x^2+R_1^2)^{3/2}}$
5. 5×10^{-4} H
6. $\frac{\phi}{i}=\frac{\mu_0\pi a^4}{2l^3}$
7. (a) 251 μH
   (b) 25.1 μH
   (c) 25.1 nC
8. 3.2×10^{-5} H
9. (a) $\frac{\mu_0\omega\pi a^2I}{2bR}\sin(\omega t)$
   (b) $\frac{\mu_0^2\omega\pi^2a^4I^2}{4b^2R}\sin^2(\omega t)$
   (c) $\frac{\mu_0^2\pi^2a^4\omega^2l}{4b^2}\cos(\omega t)$
10. $\frac{\mu_0a}{2\pi}\log_e\left(1+\frac{b}{c}\right)$
11. $L_s=\frac{L}{3}+\frac{L}{3}=\frac{2L}{3}$, $L_p=\frac{L}{6}$
12. $\frac{\mu_0b}{2\pi h}\left[h-a\log_e\left(\frac{a+h}{a}\right)\right]$

Test Your Concepts-IX (Based on LC Oscillations)

1. (a) 503 Hz
   (b) 12 μC

(c) 38 mA

(d) 72 μJ

2. 20 V

3. $\frac{E\sqrt{LC}}{R}$

4. 10.5 μs

5. (a) 12.5 mV

(b) 8.33×10^{-4} A

(c) 3.125×10^{-8} J

(d) 4.33×10^{-6} C, 7.8×10^{-7} J

6. (a) $\frac{10^4}{\pi}$ Hz

(b) 20 A

7. $\frac{Q}{2N}\sqrt{\frac{3L}{C}}$

8. 0.5 A, $\frac{1}{\sqrt{3}}$ A

9. $\frac{9t^2}{\pi^2 C}$

10. (a) 6.28×10^3 rads^{-1}, 10^{-3} s

(b) $10^{-4}\cos(6.28\times10^3 t)$

(c) 25 mH

(d) 0.4 A

11. $I_0\sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right)$

12. $2\pi\sqrt{\frac{LC}{2}}$, $V\sqrt{\frac{C}{2L}}$

13. (a) 8.15 V

(b) 16.3 A, $\frac{20}{\pi}$ Hz

14. (a) 3.6×10^{-3} H

(b) 1.33 kHz

(c) 0.188 ms

15. (a) $q_0\omega$, when $\omega t=\frac{\pi}{2}$

(b) $3V_0$, $3V_0$

(c) $q_0\omega\sin(\omega t)$

Single Correct Choice Type Questions

1. C	**2.** D	**3.** C	**4.** C	**5.** C	**6.** D	**7.** C	**8.** D	**9.** C	**10.** B
11. B	**12.** D	**13.** A	**14.** D	**15.** D	**16.** C	**17.** A	**18.** C	**19.** D	**20.** D
21. D	**22.** C	**23.** C	**24.** C	**25.** B	**26.** C	**27.** C	**28.** B	**29.** D	**30.** B
31. D	**32.** A	**33.** B	**34.** A	**35.** B	**36.** B	**37.** A	**38.** B	**39.** D	**40.** C
41. A	**42.** B	**43.** B	**44.** A	**45.** A	**46.** B	**47.** C	**48.** A	**49.** B	**50.** D
51. B	**52.** A	**53.** C	**54.** D	**55.** C	**56.** D	**57.** C	**58.** A	**59.** B	**60.** B
61. D	**62.** C	**63.** B	**64.** C	**65.** B	**66.** A	**67.** B	**68.** A	**69.** A	**70.** D
71. A	**72.** D	**73.** A	**74.** A	**75.** D	**76.** B	**77.** B	**78.** B	**79.** A	**80.** A
81. A	**82.** C	**83.** A	**84.** C	**85.** D	**86.** D	**87.** B	**88.** C	**89.** A	**90.** C
91. D	**92.** B	**93.** C	**94.** A	**95.** D	**96.** C	**97.** B	**98.** A	**99.** B	**100.** A
101. A	**102.** D	**103.** C	**104.** D	**105.** A	**106.** B	**107.** C	**108.** D	**109.** D	**110.** D
111. B	**112.** D	**113.** C	**114.** A	**115.** C	**116.** D	**117.** D	**118.** C	**119.** C	**120.** D
121. D	**122.** C	**123.** B	**124.** A	**125.** B	**126.** C	**127.** D	**128.** D	**129.** D	**130.** D
131. C	**132.** C	**133.** D	**134.** B	**135.** B	**136.** D	**137.** A	**138.** B	**139.** C	**140.** A
141. C	**142.** C	**143.** A	**144.** A	**145.** C	**146.** A	**147.** D	**148.** C	**149.** B	**150.** C
151. A	**152.** A	**153.** C	**154.** A	**155.** B	**156.** D	**157.** A	**158.** D	**159.** A	**160.** B
161. B	**162.** B	**163.** B	**164.** D	**165.** B	**166.** B	**167.** D	**168.** B	**169.** A	**170.** B

171. C	172. D	173. C	174. A	175. D	176. B	177. B	178. D	179. C	180. B
181. A	182. B	183. A	184. C	185. A	186. C	187. B	188. C	189. C	190. D
191. D	192. B	193. D	194. D	195. A	196. B	197. D	198. C	199. C	200. A
201. D	202. C	203. D	204. D	205. A	206. A	207. A	208. C	209. A	210. C
211. C	212. C	213. B	214. D	215. B	216. A				

Multiple Correct Choice Type Questions

1. A, C	2. A, B, C, D	3. A, C	4. B, D	5. A, B
6. A, C	7. A, B, D	8. A, B, D	9. B, D	10. A, B
11. A, C, D	12. A, C	13. A, B, D	14. A, C	15. B, C
16. B, C, D	17. B, D	18. B, D	19. B, C	20. A
21. A, D	22. B, D	23. A, B, D	24. A, C	25. B, C, D
26. A, C	27. A, B, C, D	28. A, D	29. B, C, D	30. A, D
31. A, B, C, D	32. A, B	33. B, C	34. A, B, C	35. B, C
36. A, B, D	37. A, C	38. B, C, D	39. A, C, D	40. A, C
41. A, C	42. B, C, D	43. A, D	44. A, C	45. A, B, D
46. A, C	47. A, B, C	48. A, C, D	49. A, C, D	50. A, B, C, D
51. B, D				

Reasoning Based Questions

1. D	2. A	3. A	4. D	5. C	6. A	7. D	8. C	9. B	10. B
11. D	12. C	13. C	14. A	15. D	16. B	17. C	18. C		

Linked Comprehension Type Questions

1. B	2. A	3. D	4. B	5. A	6. D	7. C	8. A	9. B	10. C
11. D	12. B	13. B	14. D	15. B	16. C	17. B	18. A	19. C	20. D
21. B	22. C	23. A	24. A	25. D	26. B	27. A	28. C	29. A	30. D
31. C	32. A	33. B, C	34. A	35. C	36. A	37. A	38. C	39. A	40. D
41. A	42. B	43. A	44. C	45. B	46. B	47. D	48. B	49. A	50. A
51. B	52. A	53. A	54. D	55. B	56. B	57. C	58. A	59. D	60. C
61. B	62. A	63. D	64. D	65. D	66. D	67. C	68. A	69. C	70. C
71. D	72. D	73. C	74. C	75. B	76. B	77. A	78. C	79. C	80. B
81. C	82. A	83. A	84. D	85. C	86. B	87. B	88. C	89. B	90. A
91. D	92. C	93. C	94. D	95. C	96. B	97. B	98. C	99. B	100. D
101. A	102. B								

Matrix Match/Column Match Type Questions

1. A → (s)	B → (p)	C → (p, r, t)	D → (q)
2. A → (q)	B → (p)	C → (r)	D → (s)
3. A → (q)	B → (p)	C → (s)	D → (r)

4.	A → (s)	B → (q)	C → (p)	D → (p)
5.	A → (q)	B → (s)	C → (p)	D → (r)
6.	A → (s)	B → (q)	C → (s)	D → (p)
7.	A → (p, r)	B → (p, r)	C → (q, s)	D → (p, r, t)
8.	A → (q, s)	B → (p, r)	C → (p, r)	D → (q, s)
9.	A → (q)	B → (r, s)	C → (s)	D → (p, q, r)
10.	A → (r)	B → (s)	C → (p)	D → (t)
11.	A → (q, s)	B → (p, r)	C → (p, r)	D → (q, s)
12.	A → (q)	B → (p)	C → (t)	D → (r)
13.	A → (q)	B → (s)	C → (p)	D → (r)
14.	A → (r)	B → (p)	C → (q)	D → (q)

Integer/Numerical Answer Type Questions

1. (a) 31, (b) clockwise		**2.** 8	**3.** 1	**4.** 100
5. 2	**6.** 4	**7.** 281	**8.** 2	**9.** 272
10. 24	**11.** 9	**12.** 360	**13.** (a) 3, (b) 6	**14.** 637, 64
15. (a) 2, (b) 80, (c) 10		**16.** 6	**17.** 25	**18.** 400
19. (a) 1 (b) Voltage across 12 Ω, 12 Ω and L (in volt) is 12, 1200, 1212 (c) $x = 76$				**20.** 783
21. 3	**22.** 2700	**23.** 7	**24.** (a) 30, (b) 20	**25.** 333
26. 324	**27.** (a) 20, (b) 10, (c) 63		**28.** 250	**29.** 2
30. 1				

ARCHIVE: JEE MAIN

1. B	**2.** A	**3.** B	**4.** B	**5.** D	**6.** A	**7.** B	**8.** *	**9.** B	**10.** A
11. *	**12.** B	**13.** A	**14.** C	**15.** A	**16.** D	**17.** A	**18.** B	**19.** C	**20.** C
21. B	**22.** D	**23.** B	**24.** C	**25.** B	**26.** A	**27.** D	**28.** A	**29.** D	**30.** B
31. D	**32.** C	**33.** B	**34.** C	**35.** C	**36.** D				

* No given option is correct

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Single Correct Choice Type Problems

1. C	**2.** C	**3.** D	**4.** C	**5.** B	**6.** B	**7.** D	**8.** D	**9.** A	**10.** D
11. B	**12.** D	**13.** D	**14.** C	**15.** B	**16.** B	**17.** D	**18.** D		

Multiple Correct Choice Type Problems

1. B, C, D	**2.** B, D	**3.** C, D	**4.** A, D	**5.** A, C
6. B, D	**7.** B, D	**8.** A, C, D		

Reasoning Based Questions

1. A

Linked Comprehension Type Questions

1. B	**2.** A	**3.** B	**4.** B	**5.** A	**6.** D	**7.** C

Matrix Match/Column Match Type Questions

1. A → (p)	B → (p, q, s)	C → (q, s)	D → (q, r, s)

Integer/Numerical Answer Type Questions

1. 0.63	**2.** 8	**3.** 7	**4.** 6

CHAPTER

4 Alternating Currents

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Introduction to AC, Average Value and RMS Value of AC
(b) AC Through Resistor, Inductor, Capacitor, Their Combinations and Applications
(c) Resonance in LCR Circuit
(d) Average Power Consumed in an AC Circuit
(e) Half Power Frequencies, Band Width and Sharpness of Resonance
(f) Parallel LCR Circuit
(g) Transformer and Applications

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

ALTERNATING CURRENT: AN INTRODUCTION

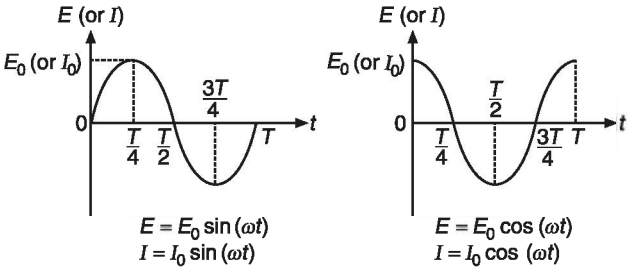
As studied earlier, we know that a changing magnetic flux can induce an emf and hence a current in a closed circuit. Also, we have seen that when a coil rotates in the presence of a magnetic field the induced emf varies sinusoidally with time leading to an alternating current (AC) and provides a source of AC power. The symbol for an AC voltage is

An alternating current change in magnitude and direction periodically and is abbreviated as AC (alternating current).

The alternating e.m.f. E at any instant may be expressed as

$$E = E_0 \sin(\omega t) \qquad \textbf{OR} \qquad E = E_0 \cos(\omega t)$$

where ω is angular frequency of alternating e.m.f. and E_0 is the peak value or amplitude of alternating e.m.f.



The frequency of alternating e.m.f.,

$$f = \frac{\omega}{2\pi} = \frac{1}{T}$$

where T is the time period.

Similarly, the alternating current in the circuit is given by

$$I = I_0 \sin(\omega t) \qquad \textbf{OR} \qquad I = I_0 \cos(\omega t)$$

where I_0 is the peak value of current and other symbols have the same notations.

Alternating current in circuits fed by an alternating source of emf may be controlled by inductance L, resistance R and capacitance C. Due to presence of

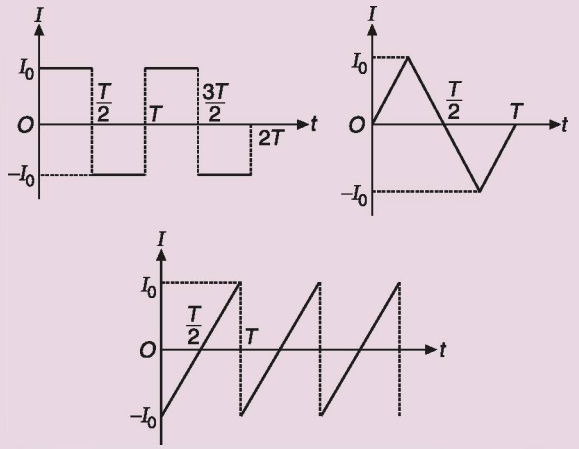
elements L and C, the current is not necessarily in phase with the applied emf. Hence alternating current, in general, may be expressed as

$$I = I_0 \sin(\omega t + \phi)$$

where ϕ is the phase angle (in radian) which may be positive, zero or negative depending on the values of reactive components R, L and C.

Conceptual Note(s)

An alternating current change its direction of flow periodically. For a half cycle, it flows in one direction and for next half cycle, it flows in opposite direction. The following graphs can also be a good representation of alternating currents



MEAN VALUE OF AC

Since current represents the rate of flow of charge, so average current is defined as charge flown per unit time interval and can be written as

$$I_{\text{mean}} = I_{\text{av}} = \langle I \rangle = \frac{\Delta q}{\Delta t}$$

where, Δq represents the charge flown in a time interval Δt. If the current is varying, then we have

$$I = \frac{dq}{dt}$$

$$\Rightarrow \quad dq = Idt$$

So, the charge flown through the circuit in a time T is obtained by integrating the above expression.

$$\Rightarrow \quad \Delta q = \int_0^T I dt$$

$$\Rightarrow \quad I_{\text{mean}} = I_{\text{av}} = \langle I \rangle = \frac{\Delta q}{T} = \frac{\int_0^T I dt}{T}$$

For an alternating current, mean value during one complete cycle is zero as there is a reversal in the direction of current after every half cycle. Therefore, we shall be finding the mean value of ac for the half cycle.

AVERAGE VALUE OR MEAN VALUE OF AN A.C. [I_{av} OR E_{av} OR $\langle I \rangle$ OR $\langle E \rangle$]

Physically, the mean value of an ac for one half cycle is defined as the steady current which when passes through the circuit makes the same amount of charge to pass through it as is done by an ac for the same time. The average value of AC over full cycle is zero since there are equal positive and negative half cycles. Mathematically, if $I = I_0 \sin(\omega t)$ or $I = I_0 \cos(\omega t)$, then for both the sine and cosine functions we have

$$\langle I \rangle = (I_{\text{av}})_{\text{full cycle}} = \frac{\int_0^T I\, dt}{\int_0^T dt} = 0$$

The average value of AC over half cycle $\left(t = 0 \text{ to } \frac{T}{2}\right)$ of sine function is given by

$$\langle I \rangle = (I_{\text{av}})_{\text{half cycle}} = \frac{\int_0^{T/2} I dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi}$$

Similarly, the average values of alternating voltage are given by

$$\langle E \rangle = (E_{\text{av}})_{\text{full cycle}} = 0$$

$$\langle E \rangle = (E_{\text{av}})_{\text{half cycle}} = \frac{2E_0}{\pi}$$

ROOT MEAN SQUARE OR VIRTUAL VALUE OF A.C. (R.M.S VALUE)

Physically, the r.m.s. value of alternating current is defined as the direct current which produces the same heating effect in a given resistor as is produced by an alternating current flowing through the resistor for the same time.

Due to this reason the r.m.s. value of current is also known as **effective or apparent or virtual value of current,**

$$\Rightarrow \quad I_{\text{effective}} = I_{\text{virtual}} = \frac{I_0}{\sqrt{2}}$$

Similarly, the r.m.s. value of alternating voltage is called the **effective or virtual value of alternating voltage (or e.m.f.).** So,

$$E_{\text{virtual}} = E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

Mathematically, the root mean square value of AC is defined by the expression given by

$$I_V = I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$

For $I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$, we have

$$\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = \frac{1}{2}$$

$$\Rightarrow \quad I_V = I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \frac{I_0}{\sqrt{2}}$$

Similarly, $E_V = E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$

RMS represents the squared mean of currents for a given time interval.

Remark(s)

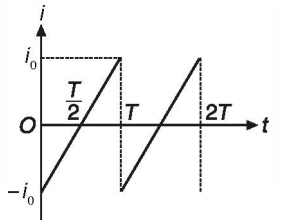
(a) Alternating currents show only heating effect, hence meters used for AC are based on heating effect and are called **hot wire meters**. The deflection in hot wire meters is proportional to mean square value of current but the scale is so calibrated that its reading is proportional to the square root of deflection. Hence these meters read directly the r.m.s values. As direct current also shows the same heating effect as produced by r.m.s value, therefore hot wire (i.e., AC) meters can read both AC and DC while dc meters based on magnetic effects of current can only read d.c.

(b) We must remember that all labelled values or the designated values of voltages and currents are the virtual values.

EXAMPLE: If we are given a 50 Hz, 220 V ac then 220 V is the virtual voltage and to get the peak value we must multiply it by $\sqrt{2}$.

ILLUSTRATION 1

Calculate the mean and rms values (for half cycle) for the current for which the graph here shows its variation with time?



SOLUTION

From the graph, current as function of time is

$$i = \left(\frac{2i_0}{T}\right)t - i_0$$

$$\Rightarrow \quad I_{\text{mean}} = \frac{\int_0^{\frac{T}{2}} i\,dt}{\frac{T}{2}} = \frac{\int_0^{\frac{T}{2}} \left[\left(\frac{2i_0}{T}\right)t - i_0\right] dt}{\frac{T}{2}} = -\frac{i_0}{2}$$

Negative sign appears as we have computed the mean for first half cycle. If we had calculated the mean for the second half cycle, then it would have been $\frac{i_0}{2}$. Since we know that

$$i_{rms}^2 = \langle i^2 \rangle = \frac{\int_0^{\frac{T}{2}} i^2 dt}{\frac{T}{2}}$$

$$\Rightarrow \quad i_{rms}^2 = \frac{\int_0^{\frac{T}{2}} i^2 dt}{\frac{T}{2}} = \frac{\int_0^{\frac{T}{2}} \left[\left(\frac{2i_0}{T}\right)t - i_0\right]^2 dt}{\frac{T}{2}} = \frac{i_0^2}{3}$$

$$\Rightarrow \quad i_{rms} = \frac{i_0}{\sqrt{3}}$$

ILLUSTRATION 2

An AC voltage is given as $e = e_1 \sin \omega t + e_2 \cos \omega t$. Calculate the RMS value of this voltage.

SOLUTION

The given AC voltage can be written as

$$e = e_1 \sin \omega t + e_2 \cos \omega t \qquad \ldots(1)$$

Let $e_1 = e_0 \cos \theta \qquad \ldots(2)$

$$e_2 = e_0 \sin \theta \qquad \ldots(3)$$

Substituting these values in equation (1), we get

$$e = e_0 (\sin \omega t \cos \theta + \cos \omega t \sin \theta)$$

$$\Rightarrow \quad e = e_0 \sin (\omega t + \theta)$$

Squaring and adding equations (2) and (3), we get

$$e_1^2 + e_2^2 = e_0^2$$

$$\Rightarrow \quad e_0 = \sqrt{e_1^2 + e_2^2}$$

$$\Rightarrow \quad e_{rms} = \frac{e_0}{\sqrt{2}} = \sqrt{\frac{e_1^2 + e_2^2}{2}}$$

We can also calculate the RMS value of the given time function of voltage by using the formula for RMS value given by

$$e_{rms} = \sqrt{\frac{1}{T}\int_0^T e^2 dt}, \text{ where } T = \frac{2\pi}{\omega}$$

ILLUSTRATION 3

In a wire, direct current i_1 and an AC current $i_2 = i_0 \sin \omega t$ are superposed. Find the RMS value of current in wire.

SOLUTION

Total current in wire is given by

$$i = i_1 + i_0 \sin \omega t$$

Since $i_{rms} = \sqrt{\langle i^2 \rangle}$

where, $i^2 = (i_1 + i_0 \sin \omega t)^2$

$$\Rightarrow \quad i^2 = i_1^2 + i_0^2 \sin^2 \omega t + 2 i_1 i_0 \sin \omega t$$

$$\Rightarrow \quad \langle i^2 \rangle = \langle i_1^2 + i_0^2 \sin^2 \omega t + 2 i_1 i_0 \sin \omega t \rangle$$

$$\Rightarrow \quad \langle i^2 \rangle = i_1^2 + i_0^2 \langle \sin^2 \omega t \rangle + 2 i_1 i_0 \langle \sin \omega t \rangle$$

Since we know that $\langle \sin^2 \omega t \rangle = \frac{1}{2}$ and $\langle \sin \omega t \rangle = 0$

$$\Rightarrow \quad \langle i^2 \rangle = i_1^2 + i_0^2 \left(\frac{1}{2}\right)$$

$$\Rightarrow \quad i_{rms} = \sqrt{\langle i^2 \rangle} = \sqrt{i_1^2 + \frac{i_0^2}{2}}$$

ILLUSTRATION 4

For a given AC flowing through a specific branch of a circuit, the variation of current as a function of time is given by

$$i = \begin{cases} i_0 \sin^2 \omega t & \text{for} \quad 0 \le \omega t < \pi \\ i_0 \sin \omega t & \text{for} \quad \pi \le \omega t < 2\pi \end{cases}$$

Calculate the average current per cycle for this AC.

SOLUTION

Average value of the given AC current is

$$\Rightarrow \quad i_{av} = \frac{1}{T}\left[\int_0^{\frac{T}{2}} (i_0 \sin^2 \omega t) dt + \int_{\frac{T}{2}}^{T} (i_0 \sin \omega t) dt\right]$$

$$\Rightarrow \quad i_{av} = \frac{i_0}{T}\left(\int_0^{\frac{T}{2}} \left(\frac{1 - \cos 2\omega t}{2}\right) dt + \int_{\frac{T}{2}}^{T} \sin \omega t dt\right)$$

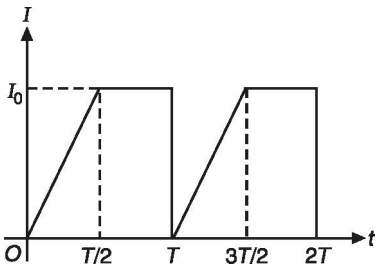
$$\Rightarrow \quad i_{av} = \frac{i_0}{T}\left[\frac{1}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right)\Big|_0^{\frac{T}{2}} + \left(-\frac{\cos \omega t}{\omega}\right)\Big|_{\frac{T}{2}}^{T}\right]$$

$$\Rightarrow \quad i_{av} = \frac{i_0}{T}\left[\frac{1}{2}\left(\frac{T}{2}\right) - \frac{2}{\omega}\right] = \frac{i_0}{T}\left[\frac{T}{4} - \frac{2}{2\pi/T}\right]$$

$$\Rightarrow \quad i_{av} = i_0\left(\frac{1}{4} - \frac{1}{\pi}\right)$$

ILLUSTRATION 5

Calculate the average value of the AC per cycle for which the time variation of current is shown in Figure.



SOLUTION

Since $I_{av} = \dfrac{\text{Charge Flown}}{\text{Time Duration}} = \dfrac{\Delta q}{\Delta t}$

The charge flown Δq in one cycle i.e. from $t = 0$ to $t = T$ is the area under the current-time graph. So, we have

$$\Rightarrow \quad \Delta q = \frac{1}{2} I_0\left(\frac{T}{2}\right) + I_0\left(T - \frac{T}{2}\right)$$

$$\Rightarrow \quad \Delta q = \frac{3}{4} I_0 T$$

$$\Rightarrow \quad I_{av} = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{T} = \frac{3}{4} I_0$$

ILLUSTRATION 6

If the current in an ac circuit is represented by the equation, $I = 5\sin\left(300t - \dfrac{\pi}{4}\right)$, where, t is in second and I in ampere. Calculate the

(a) peak and r.m.s. value of current.
(b) frequency of ac.
(c) average current for the half cycle.

SOLUTION

(a) Since, for an ac input we have

$$I = I_0 \sin(\omega t \pm \phi)$$

So, the peak value $I_0 = 5$ A

and $I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{5}{\sqrt{2}} = 3.535$ A

(b) Angular frequency $\omega = 300$ rads^{-1}

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{300}{2\pi} \simeq 47.75 \text{ Hz}$$

(c) $I_{av} = \left(\dfrac{2}{\pi}\right) I_0 = \left(\dfrac{2}{\pi}\right)(5) = 3.18$ A

IMPEDANCE, REACTANCE AND ADMITTANCE

Impedance (Z)

In alternating current circuit, the ratio of e.m.f. applied and consequent current produced is called the **impedance** and is denoted by Z i.e.,

$$Z = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}}$$

Physically, impedance of ac circuit is the opposition by the circuit to the flow of ac through it.

Reactance (X)

The opposition offered by pure inductance or capacitance to the flow of ac in an ac circuit is called **reactance** and is denoted by X. In other words, when there is no ohmic resistance in the circuit, the reactance is equal to impedance.

The reactance due to inductance alone is called **inductive reactance** and is denoted by X_L while the reactance due to capacitance alone is called the **capacitive reactance** and is denoted by X_C.

Z, X_L and X_C all have same SI unit i.e. ohm (Ω)

Admittance (Y)

The reciprocal of impedance is called the **admittance** and is denoted by Y i.e.,

$$Y = \frac{1}{Z}$$

Its SI unit is ohm^{-1} (i.e. Ω^{-1}), sometimes written as mho.

PHASOR DIAGRAMS AND PHASORS

The concept of phasor diagrams is introduced when resultant of scalar quantities having a constant phase difference (such as voltage, current) is to be found. Actually, the concept of phasors fits well at those places where the quantities are scalars and have a constant phase difference between them, because on superposition they behave like vectors e.g., ac currents, ac voltages, amplitudes of superimposing waves etc. So, phasor quantities are those quantities which possess magnitude along with a phase angle. In simple words we can say that a **phasor** is a rotating vector having the following properties.

(a) **Length**, which corresponds to the amplitude.
(b) **Angular speed**, with which the vector rotates counter clockwise.
(c) **Projection**, of the vector along the vertical axis corresponds to the value of alternating current or voltage at that time.

EXAMPLE

Alternating Current and Voltages. If an alternating voltage is given by $V = 100\sin\left(\omega t + \frac{\pi}{6}\right)$ and the current given by $I = 10\cos(\omega t)$, then before concluding anything, we must first convert both to sine or to cosine. Using the property $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, we conclude that voltage **lags behind the current** by a phase angle of

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

Alternatively, it can be said that the **current leads the voltage** by a phase angle of

$$\phi = \frac{\pi}{3}$$

Problem Solving Technique(s)

While drawing phasor diagram for a pure element (e.g. a resistance, a capacitance or an inductance) either of the current or voltage can be plotted along x-axis

But when phasor diagrams for a combination of elements is drawn then the quantity which remains constant for the combination must be plotted along x-axis So, we observe that

(a) in series circuits current has to be plotted along x-axis.

(b) in parallel circuits voltage has to be plotted along x-axis and the phasor diagram is then completed by plotting the variation of other quantity for a particular element.

IMPEDANCES AND PHASES OF AC CIRCUITS CONTAINING DIFFERENT ELEMENTS

As already pointed out that in an ac circuit the current and applied emfs are not necessarily in same phase. The applied e.m.f. (E) and current produced (I) may be expressed as

$$E = E_0 \sin \omega t$$

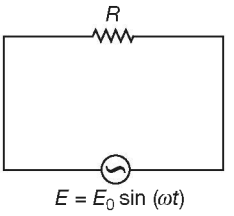
and $I = I_0 \sin(\omega t + \phi)$ with $I_0 = \frac{E_0}{Z}$

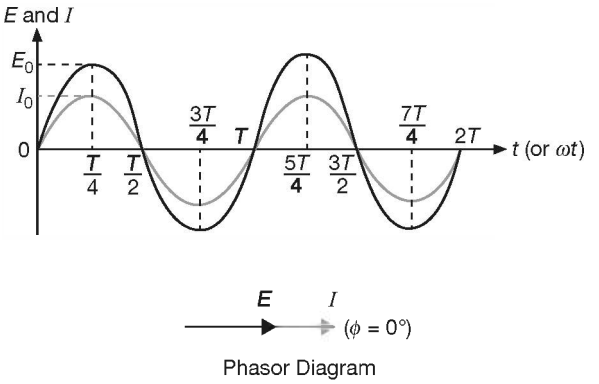
where E_0 and I_0 are peak values of alternating e.m.f. and current.

A.C. THROUGH A PURE RESISTOR

Consider a circuit, fed by an alternating e.m.f. $E = E_0 \sin(\omega t)$, containing pure resistance R, then current is given by

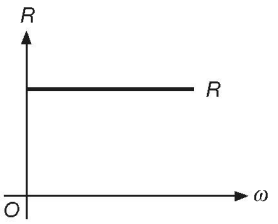
$$I = \frac{E}{R} = \frac{E_0 \sin(\omega t)}{R} = I_0 \sin(\omega t), \text{ where } I_0 = \frac{E_0}{R}$$





Comparing this with standard equation $E = E_0 \sin(\omega t)$, we observe that Impedance of circuit, $Z = R$ and phase lead of current over e.m.f., $\phi = 0$ i.e. current and e.m.f. both are in the same phase, the phasor diagram for which is also shown.

Graph of R vs ω is also shown here.

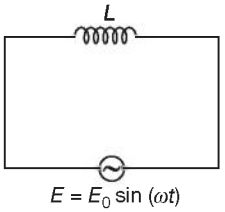


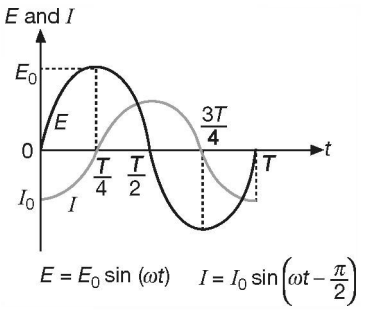
AC THROUGH A PURE INDUCTOR

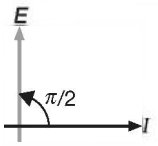
Let an alternating e.m.f. $E = E_0 \sin(\omega t)$ be applied across a pure inductance L, then at any instant we have

$$L\frac{dI}{dt} = E_0 \sin(\omega t)$$

or $$dI = \frac{E_0}{L}\sin(\omega t)\,dt$$







Phasor Diagram

Integrating, we get

$$I = \int \frac{E_0}{L}\sin(\omega t)\,dt = -\frac{E_0}{\omega L}\cos(\omega t)$$

$$\Rightarrow \quad I = -\frac{E_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\Rightarrow \quad I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{E_0}{L\omega} = \dfrac{E_0}{X_L}$

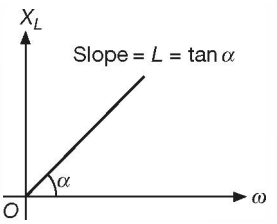
where X_L = Inductive Reactance = $L\omega$

In a purely inductive circuit the current lags behind the applied voltage by $\dfrac{\pi}{2}$ or the voltage leads the current by a phase angle of $\dfrac{\pi}{2}$.

Also, we note that since

$$X_L = L\omega = 2\pi fL$$

Graph of X_L vs ω is shown in figure.



Problem Solving Technique(s)

(a) For a dc $f = 0$
$\Rightarrow \quad X_L = 0$
i.e. an inductor offers zero resistance path to the flow of dc.

(b) If an oscillating voltage of a given amplitude V_0 is applied across an inductor the resulting current will have a smaller amplitude I_0 for larger value of ω. Since, X_L is proportional to frequency, a high frequency voltage applied to the inductor gives only a small current while a lower frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio interference filters to block high frequencies while permitting lower frequencies to pass through. A circuit device that uses an inductor for this purpose is called a **Low Pass Filter**.

AC THROUGH A PURE CAPACITOR

Let a circuit contain pure capacitance and the applied alternating e.m.f. be $E = E_0 \sin(\omega t)$.

The current in circuit is given by

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE) = \frac{d}{dt}(CE_0 \sin \omega t)$$

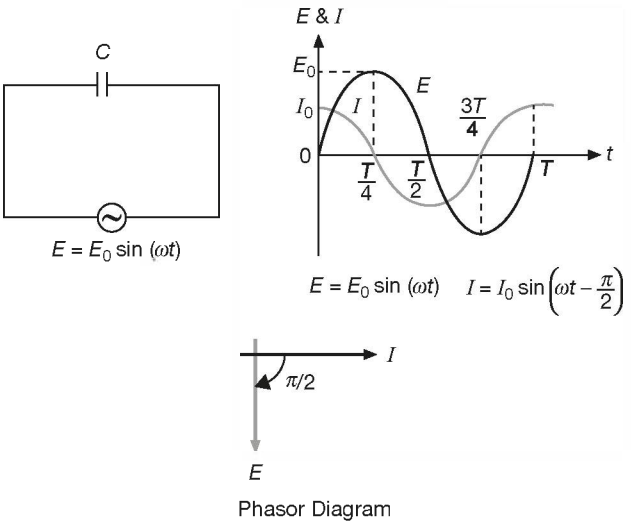
$$\Rightarrow \quad I = \omega C E_0 \cos \omega t$$

$$\Rightarrow \quad I = \frac{E_0}{1/\omega C} \sin\left(\frac{\pi}{2} + \omega t\right) = \frac{E_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{E_0}{1/\omega C} = \dfrac{E_0}{X_C}$

where $X_C = \dfrac{1}{C\omega}$ = Capacitance Reactance

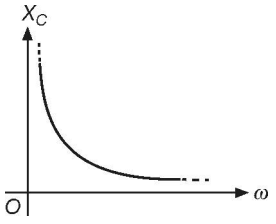


In a purely capacitive circuit the current leads the applied e.m.f. by a phase angle $\dfrac{\pi}{2}$ or the voltage lags behind the current by a phase angle of $\dfrac{\pi}{2}$.

Also, we note that since

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C}$$

Graph of X_C vs ω is shown in figure.



Problem Solving Technique(s)

(a) For a dc circuit $f = 0$

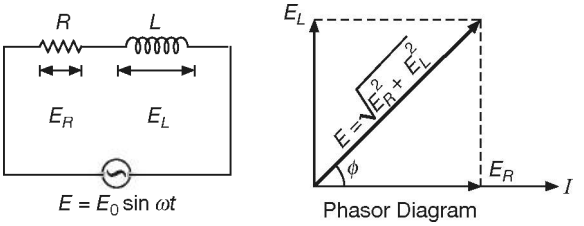
$\Rightarrow \quad X_C \to \infty$

i.e. a capacitor offers an infinite resistance path to the flow of dc i.e. it bypasses all the dc components of supply and hence is called a **dc blocking element.**

(b) The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to angular frequency ω. The greater the capacitance and the higher the frequency, the smaller is the capacitive reactance X_C. Capacitors tend to pass high frequency current and to block low frequency current, just the opposite of inductors. A device that passes signals of high frequency is called a **High Pass Filter**.

AC THROUGH A NON-IDEAL INDUCTOR OR SERIES *LR* CIRCUIT

Consider a circuit containing resistance R and inductance L in series.



Let I be the current flowing in the circuit and E_R $(= IR)$ the potential difference across resistance and E_L $(= IX_L)$ the potential difference across inductance.

The current I and the potential difference E_R are always in phase, but the potential difference E_L across inductance leads the current I by an angle $\dfrac{\pi}{2}$.

Therefore, resultant voltage is given by

$$E = \sqrt{\left(E_R^2 + E_L^2\right)} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$\Rightarrow \quad \frac{E}{I} = \sqrt{R^2 + L^2\omega^2}$$

So, impedance of RL circuit,

$$Z = \frac{E}{I} = \frac{E_v}{I_v} = \sqrt{R^2 + L^2\omega^2}$$

From phasor diagram we observe that net voltage E leads the current by ϕ. Further,

$$\tan\phi = \frac{E_L}{E_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

ILLUSTRATION 7

An 220 V AC voltage at a frequency of 40 cycles/s is applied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 Ω in series. Calculate

(a) The current supplied by source
(b) The potential difference across the resistance
(c) The potential difference across the inductance
(d) The time lag between maxima of current and voltage in circuit

SOLUTION

The impedance of RL series circuit is given as

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$$

$$\Rightarrow \quad Z = \sqrt{(6)^2 + 4(3.14)^2 (40)^2 (0.01)^2}$$

$$\Rightarrow \quad Z \approx 6.5\ \Omega$$

(a) Current supplied by ac source is

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{220}{6.5} = 33.84\ \text{mA}$$

(b) The potential difference across the resistance is

$$E_R = I_{rms} R = 33.83 \times 6 = 202.98\ \text{V}$$

(c) Potential difference across inductance is

$$E_L = I_{rms} X_L = I_{rms} (L\omega)$$

$$\Rightarrow \quad E_L = 33.83 \times (2 \times 3.14 \times 40 \times 0.01)$$

$$\Rightarrow \quad E_L = 96.83\ \text{V}$$

(d) Phase angle between current and voltage is

$$\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{2 \times 3.14 \times 40 \times 0.01}{6}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.419) = 22.73^\circ$$

$$\Rightarrow \quad \phi = 22.73 \times \frac{\pi}{180}\ \text{radian} \approx 0.4\ \text{radian}$$

Since $\phi = \omega t$, so the time lag corresponding to the above phase angle is given by

$$t = \frac{\phi}{\omega} = \frac{0.4}{80\pi} \approx 1.6\ \text{ms}$$

ILLUSTRATION 8

A resistance and inductance are connected in series across a voltage, $V = 283\sin(314t)$. The current is found to be $I = 4\sin\left(314t - \frac{\pi}{4}\right)$. Find the values of the inductance and resistance.

SOLUTION

In series LR circuit, the current lags the voltage by an angle, ϕ given by

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

Since $\phi = \frac{\pi}{4}$

$$\Rightarrow \quad X_L = L\omega = R \qquad \{\because\ \omega = 314\ \text{rads}^{-1}\}$$

$$\Rightarrow \quad 314L = R \qquad \ldots(1)$$

Further, since $V_0 = I_0 |Z|$

$$\Rightarrow \quad 283 = 4\sqrt{R^2 + X_L^2}$$

$$\Rightarrow \quad R^2 + (\omega L)^2 = \left(\frac{283}{4}\right)^2 = 5005.56$$

$$\Rightarrow \quad 2R^2 = 5005.56 \qquad \{\because\ \omega L = R\}$$

$$\Rightarrow \quad R \approx 50\ \Omega$$

and from equation (1), we get $L = 0.16$ H

ILLUSTRATION 9

An electric lamp which runs at 40 V and consumes 10 A current is connected to 100 V, 50 Hz AC main supply. Calculate the inductance of the required choke for lamp to glow at full brightness.

SOLUTION

Resistance of the lamp is given as

$$R = \frac{V}{I} = \frac{40}{10} = 4\ \Omega$$

Current in the circuit is

$$I = \frac{E}{\sqrt{R^2 + (\omega L)^2}}$$

$$\Rightarrow \quad 10 = \frac{100}{\sqrt{(4)^2 + (2 \times 3.14 \times 50 \times L)^2}}$$

Solving, we get

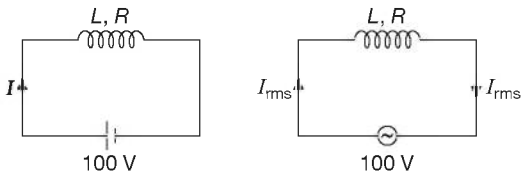
$$L \approx 0.029 \text{ H}$$

ILLUSTRATION 10

When 100 V dc is applied across a solenoid, a steady current of 1 A flows in it. When 100 V ac is applied across the same solenoid, current drops to 0.5 A. If the frequency of ac source is $\frac{150\sqrt{3}}{\pi}$ Hz, find the resistance and inductance of the solenoid.

SOLUTION

Steady current through solenoid with dc source is given by



$$I = \frac{E}{R} = \frac{100}{R} = 1$$

$$\Rightarrow \quad R = 100 \ \Omega$$

When ac is applied, the current through the solenoid is given by

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{100}{\sqrt{R^2 + X_L^2}} = 0.5$$

$$\Rightarrow \quad \sqrt{R^2 + X_L^2} = 200$$

$$\Rightarrow \quad X_L^2 = 40000 - 10000 = 30000$$

$$\Rightarrow \quad X_L = 100\sqrt{3} \ \Omega$$

Inductive reactance is given by

$$X_L = L\omega$$

$$\Rightarrow \quad L = \frac{X_L}{\omega} = \frac{100\sqrt{3}}{2\pi} \frac{\pi}{150\sqrt{3}} = \frac{1}{3} \text{ H}$$

ILLUSTRATION 11

A $130\sqrt{2}$ V, 50 Hz ac source is applied across a series LR circuit having $L = \frac{175}{11}$ mH and $R = 12 \ \Omega$. Calculate the impedance of the circuit and the phase difference between voltage and current in the circuit.

SOLUTION

For a series LR circuit, impedance is given by

$$Z = \sqrt{R^2 + X_L^2}$$

Inductive reactance X_L is given by

$$X_L = L\omega = 2\pi fL$$

$$\Rightarrow \quad X_L = 2\left(\frac{22}{7}\right)(50)\left(\frac{175}{11} \times 10^{-3}\right) \Omega$$

$$\Rightarrow \quad X_L = 5000 \times 10^{-3} \ \Omega = 5 \ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13 \ \Omega$$

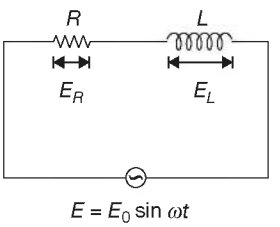
Since we know that in a series LR circuit, the voltage leads the current by a phase angle of

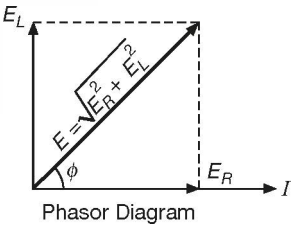
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{5}{12}\right)$$

AC THROUGH A NON-IDEAL CAPACITOR OR SERIES *CR* CIRCUIT

Let a circuit contain resistance R and capacitance C in series.





Let I be the current flowing in the circuit, E_R $(= I_R)$ the potential difference across resistance and E_C $(= IX_C)$ the potential difference across capacitance.

The potential difference E_R and current I are in same phase and the potential difference E_C lags behind the current I (and hence E_R) by phase angle $\frac{\pi}{2}$.

The resultant e.m.f is given by

$$E = \sqrt{E_R^2 + E_C^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\Rightarrow \quad \text{Impedance, } Z = \frac{E}{I} = \frac{E_V}{I_V} = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

The current leads the applied e.m.f by phase angle ϕ given by

$$\tan\phi = \frac{E_C}{E_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

or $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{RC\omega}\right)$$

ILLUSTRATION 12

A 100 μF capacitor in series with a 40 Ω resistor is connected to a 110 V, 60 Hz supply. Calculate the maximum current in the circuit. Also find the time lag between current maxima and voltage maxima.

SOLUTION

The peak current in a series RC circuit is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

Given that $E_{rms} = 110$ V, so peak voltage is given by

$$E_0 = \sqrt{2}E_{rms} = 1.414 \times 110 = 155.5 \text{ V}$$

$$\Rightarrow \quad I_0 = \frac{155.5}{\sqrt{(40)^2 + \left(\frac{1}{376.8 \times 10^{-4}}\right)^2}}$$

$$\Rightarrow \quad I_0 = 3.24 \text{ A}$$

In RC circuit, the voltage lags behind the current by phase angle ϕ, where

$$\tan\phi = \frac{X_C}{R} = \frac{1}{RC\omega} = \frac{1}{(376.8 \times 10^{-4})(40)}$$

$$\Rightarrow \quad \tan\phi = 0.6635$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.6635) = 33.56^\circ$$

The time lag is given by $\phi = \omega t$

$$\Rightarrow \quad t = \frac{\phi}{\omega}, \text{ where } \phi = 33.56 \times \frac{\pi}{180} \text{ radian and}$$

$$\omega = 2\pi f = 376.8 \text{ rads}^{-1}$$

$$\Rightarrow \quad t = 1.55 \times 10^{-3} \text{ s}$$

ILLUSTRATION 13

In a circuit, a resistance of 40 Ω and capacitor of capacitance $\frac{1250}{9\pi}$ μF are connected in series across a 500 V, 120 Hz ac source. Find the effective current in circuit and phase difference between current and voltage source.

SOLUTION

Impedance Z of the series RC circuit is given by

$$Z = \sqrt{R^2 + X_C^2}$$

where $X_C = \frac{1}{C\omega}$ is the capacitive reactance.

$$\Rightarrow \quad X_C = \frac{1}{C\omega} = \frac{1}{2\pi fC}$$

$$\Rightarrow \quad X_C = \frac{9\pi \times 10^6}{2\pi(120)(1250)} \ \Omega$$

$$\Rightarrow \quad X_C = \frac{3 \times 10^6}{10^5} = 30 \ \Omega$$

Circuit impedance is given by

$$Z = \sqrt{(40)^2 + (30)^2} = 50 \ \Omega$$

Current in the circuit is

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{500}{50} = 10 \text{ A}$$

Phase difference between current and voltage source is given by

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{30}{40}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

Since in RC circuit, the current leads voltage, so in this case, current leads voltage by a phase angle of 37°.

ILLUSTRATION 14

An ac source of angular frequency ω is fed across resistor R and a capacitor C in series. The current registered is i. If now the frequency of the source is changed to $\frac{\omega}{3}$ but maintaining the same voltage, the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency.

SOLUTION

At angular frequency ω, the current in RC circuit is given as

$$I_{rms} = \frac{E_{rms}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \ldots(1)$$

When frequency is changed to $\frac{\omega}{3}$, the current is halved so we have

$$\frac{I_{rms}}{2} = \frac{E_{rms}}{\sqrt{R^2 + \frac{1}{(\omega/3)^2 C^2}}}$$

$$\Rightarrow \quad \frac{I_{rms}}{2} = \frac{E_{rms}}{\sqrt{R^2 + \frac{9}{C^2\omega^2}}} \quad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{2}{\sqrt{R^2 + \frac{9}{C^2\omega^2}}}$$

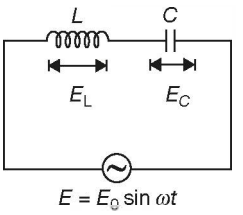
$$\Rightarrow \quad 3R^2 = \frac{5}{C^2\omega^2}$$

$$\Rightarrow \quad 3R^2 = 5X_C^2$$

$$\Rightarrow \quad \frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

AC THROUGH A SERIES *LC* CIRCUIT

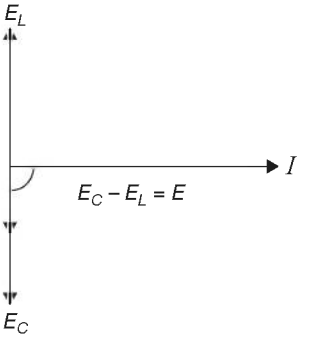
Let a circuit contain inductance L and capacitance C in series.



Let I be the current flowing in circuit, $L = IX_L$ the potential difference across inductance L and $E_C = IX_C$ the potential difference across capacitance C.

The potential difference E_C lags behind the current by angle $\frac{\pi}{2}$ and the potential difference E_C leads the current by angle $\frac{\pi}{2}$.

So, resultant applied e.m.f, $E = E_C \sim E_L = IX_C \sim IX_L$



So, impedance of circuit is

$$Z = \frac{E}{I} = X_C - X_L = \left(\frac{1}{\omega C} - \omega L\right)$$

The leading of current over applied e.m.f is $\pm\frac{\pi}{2}$

In case $X_C = X_L$ then $Z = 0$

$$\Rightarrow \quad \frac{1}{\omega_0 C} = \omega_0 L$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

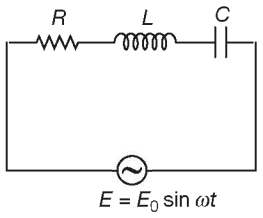
where ω_0 is the resonant angular frequency.

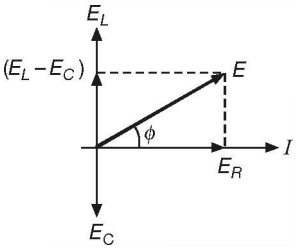
$$\Rightarrow \quad \text{Frequency } f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

This frequency is called the **resonant frequency**.

AC THROUGH A SERIES *LCR* CIRCUIT

Let a circuit contain a resistance R, inductance L and capacitance C in series.





Let I be the current flowing in circuit E_R, E_L and E_C the respective potential differences across resistance R, inductance L and capacitance C. The potential difference E_R is in phase with current I. The potential difference E_C lags behind the current by angle $\frac{\pi}{2}$. The potential difference V_L leads the current by angle $\frac{\pi}{2}$ as shown in the phasor diagram.

So, Resultant e.m.f is

$$E = \sqrt{E_R^2 + (E_L - E_C)^2}$$

$$\Rightarrow \quad E = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\Rightarrow \quad Z = \frac{E}{I} = \frac{E_V}{i_V} = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase lead of e.m.f over current is

$$\tan\phi = \frac{E_L - E_C}{E_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(a) If $X_C > X_L$ i.e. $\omega_0 > \omega$ the value of ϕ is negative i.e., current leads the applied e.m.f by a phase angle ϕ.

(b) If $X_C < X_L$ i.e. $\omega_0 < \omega$ the value of ϕ is positive i.e., current lags behind the applied e.m.f by a phase angle ϕ.

(c) If $X_C = X_L$ i.e. $\omega_0 = \omega$ the value of ϕ is zero i.e., current and e.m.f are in same phase. This is called the case of **resonance** and **resonant frequency** is given by condition $X_C = X_L$

$$\Rightarrow \quad \frac{1}{\omega_0 C} = \omega_0 L$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Problem Solving Technique(s)

At resonance

(a) $X_L = X_C$

(b) $\omega_0 = \frac{1}{\sqrt{LC}}$

(c) E and I both are in the same phase.

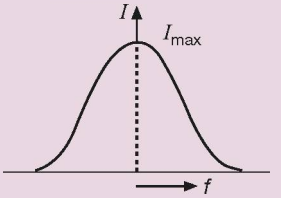
(d) Impedance is minimum and $Z_{min} = R$

(e) Current in the circuit is maximum and $I_{max} = \frac{E}{R}$

(f) Resistance is the only active element in the *LCR* series circuit.

(g) Power factor $(\cos\phi)$ is maximum i.e. Power factor $= 1$

(h) The variation of current with the frequency of the applied voltage is shown in the figure.



If the applied voltage consists of a number of frequency components, the current will be large for the component having frequency f_0. This resonant behaviour of the *LCR* circuit is used in radio tuning.

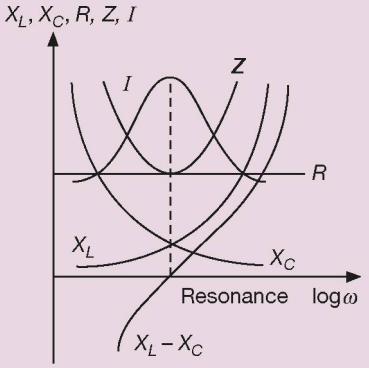
The tuning circuit of a radio receiver contains an *LCR* circuit, usually having a variable capacitor (also called **Variac**). It is varied till the resonant frequency of the circuit is equal to the particular frequency from some radio station. Then the current corresponding to this signal is maximum and the receiver responds to it.

(i) In *L*-*C*-*R*, circuit whenever voltage across various elements is asked, find r.m.s. values unless stated in the question for the peak or instantaneous value.

The r.m.s. values are, $V_R = I_{rms}R$, $V_L = I_{rms}X_L$ and $V_C = I_{rms}X_C$

The peak values can be obtained by multiplying the r.m.s. values by $\sqrt{2}$. The instantaneous values across different elements is rarely asked.

(j) Response curves of series circuit: The impedence of an *LCR* circuit depends on the frequency. The dependence is shown in figure. The frequency is taken on logarithmic scale because of its wide range. From the figure we can see that at resonance.



(i) $X_L = X_C$ or $\omega = \dfrac{1}{\sqrt{LC}}$

(ii) $Z = Z_{min} = R$ and

(iii) I is maximum.

Here by Z we mean the modulus of Z and I means I_{rms}.

(k) Acceptor Circuit

If the frequency of the ac supply can be varied (e.g., in radio or television signal) then in series *LCR* circuit, at a frequency $f = \dfrac{1}{2\pi\sqrt{LC}}$ maximum current flows in the circuit and have a maximum P.D. across its inductance (or capacitance). This is the method by which a radio or television set is tuned at a particular frequency. The circuit is known as **acceptor circuit**.

ILLUSTRATION 15

A series *LCR* circuit containing a resistance of 120 Ω has angular frequency 4×10^5 rads^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of *L* and *C*. At what angular frequency the current in the circuit lags the voltage by $\dfrac{\pi}{4}$.

SOLUTION

At resonance, $X_L - X_C = 0$

$$\Rightarrow \quad Z = R = 120\ \Omega \qquad \left\{\because\ Z^2 = (X_L - X_C)^2 + R^2\right\}$$

$$\Rightarrow \quad I_{rms} = \frac{(V_R)_{rms}}{R} = \frac{60}{120} = \frac{1}{2}\ \text{A}$$

$$(V_L)_{rms} = I_{rms}X_L = I_{rms}(L\omega)$$

$$\Rightarrow \quad L = \frac{(V_L)_{rms}}{I_{rms}\omega} = \frac{40}{(4\times10^5)\left(\frac{1}{2}\right)} = 2\times10^{-4}\ \text{H}$$

$$\Rightarrow \quad L = 0.2\ \text{mH}$$

The resonant frequency is given by,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad C = \frac{1}{\omega_0^2 L}$$

Substituting the values, we have

$$C = \frac{1}{(4\times10^5)^2(2\times10^{-4})} = \frac{1}{32\times10^6}$$

$$\Rightarrow \quad C = \frac{1}{32}\times10^{-6} = 3.125\times10^{-8}\ \text{F} = 31.25\ \text{nF}$$

For the current to lag behind the voltage by $\dfrac{\pi}{4}$, we have

$$\tan\left(\frac{\pi}{4}\right) = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Substituting the values of *L*, *C*, *R* and $\tan\left(\dfrac{\pi}{4}\right)$, we get

$$(1)(120) = 0.2\omega - \frac{1}{\omega\left(\frac{10^{-6}}{32}\right)}$$

$$\Rightarrow \quad \omega^2 - 6\times10^5\omega - 16\times10^{10} = 0$$

$$\Rightarrow \quad \omega = \frac{6\times10^5 \pm \sqrt{36\times10^{10} + 64\times10^{10}}}{2}$$

$$\Rightarrow \quad \omega = \frac{6\times10^5 + 10\times10^5}{2} = 8\times10^5 \text{ rads}^{-1}$$

$$\Rightarrow \quad \omega = 8\times10^5 \text{ rads}^{-1}$$

ILLUSTRATION 16

A 200 km long telegraph line wire has a capacitance of 0.014 μFkm^{-1}. If it carries an alternating current of 50 kHz, what should be the value of an inductance required to be connected in series so that impedance is minimum.

SOLUTION

The total capacitance of telegraph line wire is

$$C = 0.014\times200 = 2.8\ \mu\text{F} = 2.8\times10^{-6}\ \text{F}$$

For minimum impedance, we have $X_L = X_C$

$$\Rightarrow \quad L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \quad L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C} = \frac{1}{4\pi^2 f^2 C}$$

$$\Rightarrow \quad L = \frac{1}{4\times(3.14)^2\times(50\times10^3)^2\times(2.8\times10^{-6})}\ \text{H}$$

$$\Rightarrow \quad L = 3.6\times10^{-6} = 3.6\ \mu\text{H}$$

ILLUSTRATION 17

An AC source with $V_{max} = 205$ V and $f = 50$ Hz is connected between points a and d having a resistance of $R = 40\ \Omega$, inductance of $L = 185$ mH and a capacitance of $C = 65\ \mu$F as shown in figure. Calculate the maximum voltages between points (a) a and b, (b) b and c, (c) c and d, and (d) b and d. A power supply with $\Delta V_{rms} = 120$ V is connected between points a and d in figure. At what frequency will it deliver a power of 250 W ?

SOLUTION

Let us first calculate X_L and X_C. Since

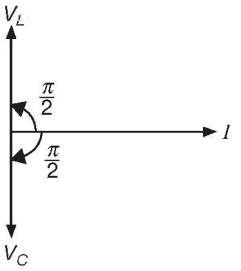
$$X_L = L\omega = (2\pi)(50)(185\times10^{-3}) = 58\ \Omega$$

$$X_C = \frac{1}{C\omega} = \frac{1}{(2\pi)(50)(65\times10^{-6})} = 49\ \Omega$$

Since R, L and Care connected in series, so we have

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{1600 + 81} = 41\ \Omega$$



Now $I_{max} = \dfrac{\Delta V_{max}}{Z} = \dfrac{205}{41} = 5$ A

(a) $(V_{ab})_{max} = I_{max}R = (5)(40) = 200\ \text{V} = (V_R)_{max}$

(b) $(V_{bc})_{max} = I_{max}X_L = (5)(58) = 290\ \text{V} = (V_L)_{max}$

(c) $(V_{cd})_{max} = I_{max}X_C = (5)(49) = 245\ \text{V} = (V_C)_{max}$

(d) $(V_{bd})_{max} = (V_L)_{max} - (V_C)_{max} = (V_{bc})_{max} - (V_{cd})_{max}$

$\Rightarrow \quad (V_{bd})_{max} = 45$ V

Also note that, for this arrangement we have

$$V_{max} = \sqrt{(V_L - V_C)^2_{max} + (V_R)^2_{max}} = 205\ \text{V}$$

ILLUSTRATION 18

A resistance R and inductance L and a capacitor C are all connected in series with an AC supply. The resistance is of 16 Ω and for a given frequency, the inductive reactance is 24 Ω and the capacitive reactance is 12 Ω. If the current in the circuit is 5 A, calculate the

(a) potential difference across R, L and C.
(b) impedance of the circuit.
(c) ac supply voltage.
(d) phase angle between current and voltage.

SOLUTION

(a) Potential difference across resistance is

$$E_R = IR = 5\times 16 = 80 \text{ V}$$

Potential difference across inductor is

$$E_L = IX_L = 5\times 24 = 120 \text{ V}$$

Potential difference across capacitor is

$$E_C = IX_C = 5\times 12 = 60 \text{ V}$$

(b) The impedance of circuit is

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\Rightarrow \quad Z = \sqrt{(16)^2 + (24-12)^2} = 20\ \Omega$$

(c) The AC supply voltage is

$$E = IZ = 5\times 20 = 100 \text{ V}$$

(d) Phase angle between current and voltage is given as

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{24-12}{16}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.75) = 37^\circ$$

ILLUSTRATION 19

A series circuit consists of a resistance of $15\ \Omega$, an inductance of 0.08 H and a capacitor of capacitance $30\ \mu$F. The applied voltage has a frequency of 500 rads^{-1}. Find whether the current leads or lags the applied voltage and also the angle of lead or lag.

SOLUTION

The inductive reactance of the circuit is

$$X_L = L\omega = 500\times 0.08 = 40\ \Omega$$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{C\omega} = \frac{1}{(30\times 10^{-6})(500)} = 66.7\ \Omega$$

Since $X_C > X_L$, so the circuit is capacitive in nature and hence the current leads the voltage by a phase angle ϕ given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{66.7-40}{15} = 1.78$$

$$\Rightarrow \quad \phi = 60.65^\circ$$

Hence current leads voltage by a phase angle of 60.65°.

ILLUSTRATION 20

An inductor coil, a capacitor and an AC source of 24 V are connected in series. When the frequency of the source is varied, a maximum current of 6 A flows through the circuit. If the inductor coil is connected to a battery of emf 12 V and internal resistance $4\ \Omega$, what will be steady state current through battery.

SOLUTION

Let R be the resistance of the inductor coil, then at resonance, current in the circuit is maximum.

$$\Rightarrow \quad I_{\max} = \frac{E_V}{Z_{\min}} = \frac{E_V}{R} = \frac{24}{R} = 6 \text{ A}$$

$$\Rightarrow \quad R = 4\ \Omega$$

When connected across a DC source, the steady current in circuit is

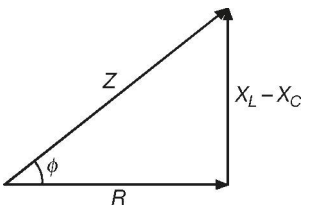
$$I = \frac{E}{R+r} = \frac{12}{4+R} = \frac{12}{8} = 1.5 \text{ A}$$

IMPEDANCE: REVISITED

We have already seen that the inductive reactance $X_L\,(=\omega L)$ and capacitance reactance $X_C\left(=\frac{1}{C\omega}\right)$ play the role of an effective resistance in the purely inductive and capacitive circuits, respectively. In the series RLC circuit, the effective resistance is the impedance Z, given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The relationship between Z, X_L and X_C is best represented by the diagram shown.



The SI unit of impedance is ohm (Ω). In terms of Z, the current may be rewritten as

$$I(t) = \frac{V_0}{Z}\sin(\omega t - \phi)$$

Notice that the impedance Z also depends on the angular frequency ω, as do X_L and X_C. Using $\tan\phi = \left(\frac{X_L - X_C}{R}\right)$ and $Z^2 = R^2 + (X_L - X_C)^2$, we can easily get the limits for simple circuit (with only one element) as provided in the table.

Table 1 Simple-circuit limits of the series *RLC* circuit

Simple Circuit	R	L	C	$X_L = \omega L$	$X_C = \frac{1}{C\omega}$	$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Pure resistance	R	0	∞	0	0	0	R
Pure inductance	0	L	∞	X_L	0	$\frac{\pi}{2}$	X_L
Pure capacitance	0	0	C	0	X_C	$-\frac{\pi}{2}$	X_C

Problem Solving Technique(s)

(a) Please note that

$$I_0 = \frac{V_0}{|Z|},\ I_{rms} = \frac{V_{rms}}{|Z|}.$$

(b) The complex impedance for an inductor is

$$X_L = j(L\omega), \text{ where } j = \sqrt{-1}$$

(c) The complex impedance of a capacitor is

$$X_C = \frac{1}{jC\omega} = -\frac{j}{C\omega}, \text{ where } j = \sqrt{-1}$$

Please note that here we are using the complex number symbol as $j = \sqrt{-1}$, because i can be misled to be current. Also note that $\frac{1}{j} = \frac{j}{j^2} = -j$.

(d) If we have any complex number written as

$$z = x + jy, \text{ where } j = \sqrt{-1}$$

Then the real part of this number is x and the imaginary part is y. The magnitude of the number is

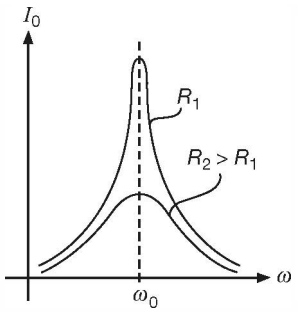
$$|z| = \sqrt{x^2 + y^2}$$

(e) When any complex number is multiplied by $j = \sqrt{-1}$, then that number is rotated counter clockwise through an angle of 90°.

QUALITY FACTOR OR Q-FACTOR

The Q-factor of an LCR series circuit is defined as the ratio of the voltage across inductor or capacitor at resonance to the voltage across the resistor. So,

$$Q = \frac{E_L}{E_R} = \frac{E_C}{E_R}$$



The amplitude of the current as a function of ω in the driven RLC circuit.

$$\Rightarrow \quad Q = \frac{IX_L}{IR} = \frac{IX_C}{IR}$$

$$\Rightarrow \quad Q = \frac{\omega_0 L}{R} = \frac{1}{RC\omega_0}$$

Since $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Note that, we get the same result from both equalities. Quality factor is an indicator of the sharpness of the current peak. Higher the value of the quality factor Q, sharper is the current peak.

ILLUSTRATION 21

Prove that in a series LCR circuit, the frequencies at which the current amplitude falls to $\frac{1}{\sqrt{2}}$ of the current at resonance are separated by an interval equal to $\frac{R}{2\pi L}$.

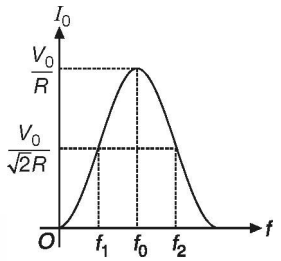
SOLUTION

At resonance, we have

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad I_0 = \frac{V_0}{R} \qquad \{\text{as } Z = R\}$$

According to the problem, we have $I = \frac{I_0}{\sqrt{2}}$



$$\Rightarrow \quad \frac{1}{\sqrt{2}}\frac{V_0}{R} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\Rightarrow \quad 2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} = \pm R$$

If f_1 and f_2 are two corresponding frequencies, then

$$2\pi f_1 L - \frac{1}{2\pi f_1 C} = R \qquad \ldots(1)$$

$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = -R \qquad \ldots(2)$$

Dividing (1) by f_2, (2) by f_1 and solving, we get

$$(f_1 - f_2) = \frac{R}{2\pi L}$$

AVERAGE POWER CONSUMED IN AN AC CIRCUIT

The power is defined as the rate at which work is being done in the circuit.
In an ac circuit, the current and e.m.f are not necessarily in the same phase. Consider

$$E = E_0 \sin(\omega t + \phi) \text{ and } I = I_0 \sin(\omega t)$$

If dW is the work done in time dt, then

$$dW = EIdt$$

$$\Rightarrow \quad dW = E_0 I_0 \sin(\omega t + \phi)\sin(\omega t)dt$$

$$\Rightarrow \quad dW = \frac{E_0 I_0}{2}\left[2\sin(\omega t + \phi)\sin(\omega t)\right]dt$$

From Trigonometry

$$2\sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\Rightarrow \quad dW = \frac{E_0 I_0}{2}\left[\cos(\omega t + \phi - \omega t) - \cos(2\omega t + \phi)\right]dt$$

So, work done in one complete cycle is

$$W = \int dW = \frac{E_0 I_0}{2}\left[\int_0^T \cos\phi dt - \int_0^T \cos(2\omega t + \phi)dt\right]$$

Since, $\int_0^T \cos(2\omega t + \phi)dt = 0$

$$\Rightarrow \quad W = \left(\frac{E_0 I_0}{2}\cos\phi\right)T - 0$$

$$\Rightarrow \quad P_{av} = \frac{W}{T} = \frac{E_0 I_0}{2}\cos\phi$$

$$\Rightarrow \quad P_{av} = \frac{E_0 I_0}{2}\cos\phi = \left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\phi \qquad \ldots(1)$$

Since, $E_V = \frac{E_0}{\sqrt{2}}$, $I_V = \frac{I_0}{\sqrt{2}}$ and from phasor diagrams of various cases already discussed we know that

$$\cos\phi = \frac{\text{Resistance } (R)}{\text{Impedance } (Z)}$$

$\cos\phi$ is called the **Power Factor (PF) of an AC circuit**. So, we must keep in mind that power factor

$$PF = \cos\phi = \frac{R}{Z}$$

So, equation (1) can be re-written as

$$P_{av} = E_V I_V \cos\phi \text{, where } I_V = \frac{E_V}{Z}$$

$$\Rightarrow \quad P_{av} = (I_V Z) I_V \left(\frac{R}{Z}\right) = I_V^2 R$$

This result is completely justified as we are aware that there is no power consumption across an ideal inductor and an ideal capacitor because in both, the power factor is zero. Hence the power is consumed only across the resistor in the circuit.

Also $P_{av} = \frac{E_V^2}{Z}\cos\phi = I_V^2 Z\cos\phi \qquad \left\{\because Z = \frac{E_V}{I_V}\right\}$

CASE-1:

If $R = 0$, $\cos\phi = 0$ and $P_{av} = 0$ i.e., in resistance less circuit the power consumed is zero. Such a circuit is called the **wattless circuit** and the current flowing is called the **wattless current**.

CASE-2:

If $R = Z$ (in purely resistive circuit and *LCR* circuit at resonance), the power factor $\cos\phi = R/Z = 1$ and the power loss is maximum. Therefore, the use of resistance is avoided in AC circuits.

HALF POWER FREQUENCIES (HPF), BAND WIDTH AND SHARPNESS OF RESONANCE

Since the average power is given by

$$P_{av} = \langle P\rangle = E_{rms} I_{rms}\cos\phi = E_V I_V \cos\phi \qquad \ldots(1)$$

This average power consumed in the circuit varies with the frequency, because the impedance Z depends upon the frequency.

We want to find the frequencies where the power is half the maximum power. These frequencies are called the Half Power Frequencies.

Equation (1) can be re-written as

$$\langle P\rangle = \left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\phi = \frac{E_0 I_0}{2}\cos\phi \qquad \ldots(2)$$

Further we know that $I_0 = \frac{E_0}{Z}$ and $\cos\phi = \frac{R}{Z}$

So, from equation (2), we get

$$\langle P\rangle = \frac{E_0}{2}\left(\frac{E_0}{Z}\right)\left(\frac{R}{Z}\right) = \left(\frac{E_0^2 R}{2}\right)\frac{1}{Z^2}$$

where, $Z^2 = R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2$

$$\Rightarrow \quad \langle P\rangle = \frac{E_0^2 R}{2\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]} \qquad \ldots(3)$$

We can clearly see from equation (3), that the average power is dependent on the frequency fed across the circuit.

From equation (3), we also conclude that

(a) $\langle P\rangle = 0$, when $\omega \to 0$ or when $\omega \to \infty$ i.e. $\langle P\rangle$ is zero for extremely small and extremely large frequencies.

(b) $\langle P\rangle$ is maximum at resonance i.e. when

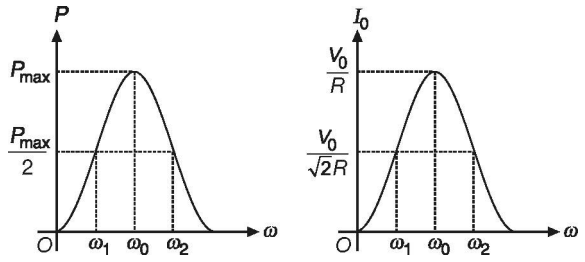
$$L\omega_0 = \frac{1}{C\omega_0}$$

where, $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonant frequency.

This maximum value of average power i.e. $\langle P\rangle_{max}$ is given by

$$\langle P\rangle_{max} = \frac{E_0^2}{2R} \qquad \ldots(4)$$

The variation of average power with frequency is shown in Figure.



The frequencies at which the average power consumed in the circuit is half of the maximum average power (i.e. the power at resonance) are called Half Power Frequencies (HPF). So, at half power frequencies we have

$$\langle P\rangle = \frac{\langle P\rangle_{max}}{2}$$

Using equations (3) and (4), we get

$$\frac{E_0^2 R}{2\left[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2\right]} = \frac{1}{2}\left(\frac{E_0^2}{2R}\right)$$

$$\Rightarrow \quad R^2+\left(L\omega-\frac{1}{C\omega}\right)^2 = 2R^2$$

$$\Rightarrow \quad \left(L\omega-\frac{1}{C\omega}\right)^2 = R^2$$

$$\Rightarrow \quad L\omega-\frac{1}{C\omega} = \pm R$$

From this equation, we conclude that

EITHER, $L\omega-\frac{1}{C\omega} = +R$ ...(5)

OR, $L\omega-\frac{1}{C\omega} = -R$ ...(6)

To calculate these frequencies, we proceed using equations (5) and (6). From equation (5), we have

$$L\omega-\frac{1}{C\omega} = +R$$

Multiplying both sides by $\frac{\omega}{L}$, we get

$$\omega^2-\frac{1}{LC} = \frac{R\omega}{L}$$

Rearranging this equation, we get

$$\omega^2-\frac{R\omega}{L}-\frac{1}{LC} = 0$$

Solving this quadratic equation, we get

$$\omega = \frac{1}{2}\left(\frac{R}{L}\pm\sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}\right)$$

Rejecting the negative value of frequency ω, we get

$$\omega = \frac{1}{2}\left(\frac{R}{L}+\sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}\right)$$

$$\Rightarrow \quad \omega = \frac{R}{2L}+\sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}} \qquad ...(7)$$

Now from equation (6), we have

$$L\omega-\frac{1}{C\omega} = -R$$

$$\Rightarrow \quad \omega^2+\frac{R\omega}{L}-\frac{1}{LC} = 0$$

Solving this quadratic equation, we get

$$\omega = \frac{1}{2}\left(-\frac{R}{L}+\sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}\right)$$

$$\Rightarrow \quad \omega = -\frac{R}{2L}+\sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}} \qquad ...(8)$$

From equations (7) and (8), we observe that one frequency is higher and the other frequency is lower that the resonant frequency. So, they are denoted by

$$\omega_{Lower} = \omega_L = \omega_1 = -\frac{R}{2L}+\sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}} \text{ and}$$

$$\omega_{Higher} = \omega_H = \omega_2 = \frac{R}{2L}+\sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}$$

The circuit is capacitive in nature for $\omega < \omega_0$ *and inductive in nature for* $\omega > \omega_0$. *For* ω_L, *the circuit is capacitive in nature and for* ω_H, *the circuit is inductive in nature.*
Also, we see that

$$\omega_L\omega_H = \frac{1}{LC} = \omega_0^2$$

which simply makes us conclude that

$$\omega_L < \omega_0 < \omega_H$$

and ω_0 is the geometric mean of ω_L and ω_H.
The difference of these two frequencies is called the band-width given by

$$\Delta\omega = \omega_H - \omega_L = \frac{R}{L}$$

The quality factor Q is defined as the ratio of the resonant frequency ω_0 to the band width $\Delta\omega$, so we have

$$Q = \frac{\text{Resonant Frequency}}{\text{Band Width}} = \frac{\omega_0}{\Delta\omega}$$

$$\Rightarrow \quad Q = \frac{1/\sqrt{LC}}{L/R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The larger the value of Q, the smaller is the value of bandwidth and sharper is the resonance. If sharpness of the circuit is small, then its selectivity is also less. If quality factor is large, then R is small, C is small and L is large, so the circuit is more selective.

ILLUSTRATION 22

A DC of 2 A and an AC of peak value 2 A flow through a resistance of 2 Ω and 1 Ω respectively. Calculate the ratio of heat produced in the two resistances in the same time interval.

SOLUTION

In time t heat produced by a DC current i_{dc} is given by

$$H_1 = i_{dc}^2 Rt$$

$$\Rightarrow \quad H_1 = (2)^2(2)t = 8t$$

RMS value of AC is given by

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ \text{A}$$

Heat produced by the rms current in time t is given by

$$H_2 = i_{\text{rms}}^2 Rt$$

$$\Rightarrow \quad H_2 = \left(\sqrt{2}\right)^2(1)t = 2t$$

$$\Rightarrow \quad \frac{H_1}{H_2} = \frac{8t}{2t} = 4$$

ILLUSTRATION 23

A $\frac{2.2}{\pi}$ H, 220 Ω coil is applied across a 220 V, 50 Hz ac. Calculate wattless current in the circuit.

SOLUTION

For a series LR circuit, impedance is given by

$$Z = \sqrt{R^2 + X_L^2}$$

Inductive reactance is given by

$$X_L = \omega L$$

$$\Rightarrow \quad X_L = 2\pi(50) \times \frac{2.2}{\pi}$$

$$\Rightarrow \quad X_L = 220\ \Omega$$

Since, $X_L = R = 220\ \Omega$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(1) = 45^\circ$$

Circuit in the circuit is

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{220}{\sqrt{(220)^2 + (220)^2}} = \frac{1}{\sqrt{2}}\ \text{A}$$

Wattless current in an ac circuit is given by

$$I_{WL} = I_V \sin\phi = I_{\text{rms}} \cos\phi$$

$$\Rightarrow \quad I_{WL} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2} = 0.5\ \text{A}$$

ILLUSTRATION 24

A series LCR circuit has $L = 10$ mH, $R = 3\ \Omega$ and $C = 1\ \mu$F connected in series to an ac source of $E = 15\cos(\omega t)$ volt. Calculate the current amplitude and the average power dissipated per cycle at a frequency which is 10% lower than the resonant frequency.

SOLUTION

At resonance, the frequency of circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \omega_0 = \sqrt{\frac{1}{(10\times10^{-3})(1\times10^{-6})}} = 10^4\ \text{rads}^{-1}$$

A frequency which is 10% less than the resonant frequency will be

$$\omega = 10^4 - 10^4 \times \frac{10}{100} = 9\times10^3\ \text{rads}^{-1}$$

At this frequency, the inductive reactance is

$$X_L = L\omega = 9\times10^3 \times (10\times10^{-3}) = 90\ \Omega$$

and the capacitive reactance is

$$X_C = \frac{1}{C\omega} = \frac{1}{(1\times10^{-6})(9\times10^3)} = 111.1\ \Omega$$

Since, $Z=\sqrt{R^2+(X_L-X_C)^2}$

$\Rightarrow \quad Z=\sqrt{3^2+(90-111.1)^2}=21.32\ \Omega$

Current amplitude is

$$I_0=\frac{E_0}{Z}=\frac{15}{21.32}=0.704\ \text{A}$$

Average power consumed in the circuit is

$$P_{av}=E_V I_V\cos\phi=I_V^2R=\frac{I_0^2R}{2}$$

$$\Rightarrow \quad P=\frac{(0.704)^2\,3}{2}=0.744\ \text{W}$$

ILLUSTRATION 25

In an AC circuit, the voltage applied is $E=5\sin(\omega t)$ and a current $I=3\cos(\omega t)$ flows in circuit. Calculate the average power dissipated in the circuit.

SOLUTION

Given that

$$E=5\sin(\omega t)\ \text{and}$$

$$I=3\cos(\omega t)=3\sin\left(\omega t+\frac{\pi}{2}\right)$$

From above we conclude that, the phase difference between voltage and current is $\frac{\pi}{2}$, the current leads voltage by $\phi=\frac{\pi}{2}$ and hence the circuit is purely capacitive in nature. Since, $P_{av}=E_V I_V\cos\phi$

$$\Rightarrow \quad P_{av}=E_V I_V\cos\left(\frac{\pi}{2}\right)=0$$

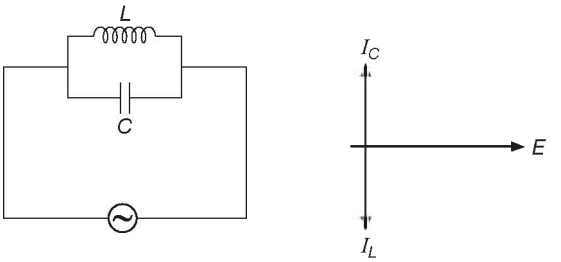
So, the average power dissipated in circuit will be zero.

CHOKE COIL

Choke coil is a coil having high inductance and negligible resistance. It is used to control current in AC circuit and is used in fluorescent tubes. The power loss in a circuit containing choke coil is least.

PARALLEL RESONANT CIRCUIT: REJECTOR CIRCUIT

A parallel resonant circuit consists of an inductance L and a capacitance C in parallel as shown in figure.



The condition of resonance is again that the current and applied e.m.f must be in same phase.

i.e. $\quad I_C=I_L$

$$\Rightarrow \quad \frac{V}{X_C}=\frac{V}{X_L}$$

$$\Rightarrow \quad X_L=X_C$$

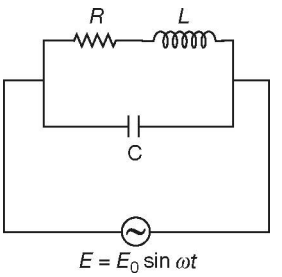
$$\Rightarrow \quad \omega_0=\frac{1}{\sqrt{LC}}$$

The frequency of parallel resonant circuit at resonance is

angular frequency $\omega_0=\frac{1}{\sqrt{LC}}$

and linear frequency $f_0=\frac{1}{2\pi\sqrt{LC}}$

At resonance the current in the circuit is minimum and impedance is infinite.



If inductance has also a resistance R as shown. The condition of resonant angular frequency is,

$$\omega_0=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

So, resonant frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

The impedance at resonance,

$$Z = \frac{R^2 + \omega_0^2 L^2}{R} = \frac{L}{RC}$$

In parallel resonant circuit, the impedance is maximum (or admittance is minimum) and the current is minimum, when $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

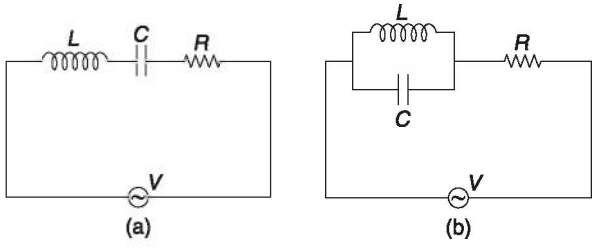
If $R \to 0$, then $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and $Z \to \infty$

REJECTOR CIRCUIT

We observe that at $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$, the admittance in the circuit is the minimum or the impedance is maximum or the current is the minimum. Thus, the parallel circuit does not allow this particular frequency from the source to pass in the circuit and due to this reason, the circuit with such frequency (f_0 or ω_0) is called a REJECTOR CIRCUIT.

ILLUSTRATION 26

An AC source is connected to two circuits as shown in figure. Obtain the current through resistance R at resonance in both the circuits.



SOLUTION

The first circuit is a series LCR circuit. The impedance for this circuit is

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

At resonance, we have

$$L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \quad Z = R$$

The current in the circuit at resonance is given as

$$I = \frac{V}{Z} = \frac{V}{R}$$

In the second circuit, the inductor and the capacitor are connected in parallel and hence the potential difference across each will be the same.
At resonance, we have

$$X_L = X_C$$

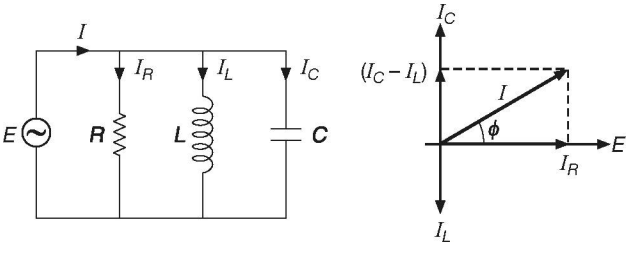
$$\Rightarrow \quad L\omega = \frac{1}{C\omega}$$

So, the current in both inductor and capacitor will be equal in magnitude. Also, we know that the phase difference between currents through the inductor L and the capacitor C will be $180°$ or the currents in them will be out of phase. So, we conclude that the two currents will be equal in magnitude but $180°$ out of phase and hence the current through the resistor R will be zero in this circuit.

THE *LCR* PARALLEL CIRCUIT

In this case the voltage E across each element is the same. The currents are related as

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$



$$\Rightarrow \quad I = \sqrt{\left(\frac{E}{R}\right)^2 + \left(C\omega E - \frac{E}{L\omega}\right)^2}$$

$$I = E\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}$$

$$\Rightarrow \quad Z = \frac{E}{I} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}}$$

So, we observe that at resonance, i.e. when $L\omega = \frac{1}{C\omega}$ then the current in circuit is minimum.

Problem Solving Technique(s)

In this chapter, we have seen how phasors provide a powerful tool for analysing the AC circuits. Below are some important techniques that help in problem solving.

(a) Keep in mind the phase relationships for simple circuits.
- **(i)** For a resistor, the voltage and the phase are always in phase.
- **(ii)** For an inductor, the current lags the voltage by $90°$.
- **(iii)** For a capacitor, the current leads to voltage by $90°$.

(b) When circuit elements are connected in series, the instantaneous current is the same for all elements, and the instantaneous voltages across the elements are out of phase.
On the other hand, when circuit elements are connected in parallel, the instantaneous voltage is the same for all elements, and the instantaneous currents across the elements are out of phase.

(c) For series connection, draw a phasor diagram for the voltages. The amplitudes of the voltage drop across all the circuit elements involved should be represented with phasors. In figure the phasor diagram for a series RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$.
From figure (a), we see that $V_L > V_C$ in the inductive case and $\vec{V}$ leads $\vec{I}$ by a phase ϕ. On the other hand, in the capacitive case shown in figure (b), $V_C > V_L$ and $\vec{I}$ leads $\vec{V}$ by a phase ϕ.

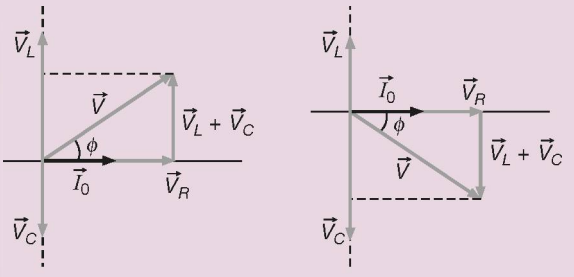


FIGURE (a) FIGURE (b)

Phasor diagram for the series RLC circuit for (a) $X_L > X_C$ and (b) $X_L < X_C$.

(d) When $V_L = V_C$, or $\phi = 0$, the circuit is at resonance. The corresponding resonant frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$, and the power delivered to the resistor is a maximum.

(e) For parallel connection, draw a phasor diagram for the currents. The amplitudes of the currents across all the circuit elements involved should be represented with phasors. In figure the phasor diagram for a parallel RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$.

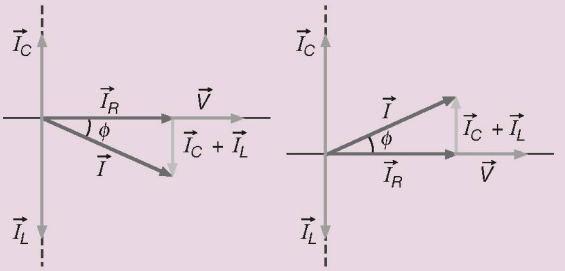


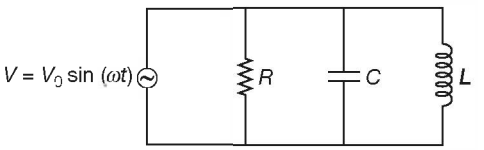
FIGURE (a) FIGURE (b)

Phasor diagram for the paralled RLC circuit for (a) $X_L > X_C$ and (b) $X_L < X_C$.

From figure (a), we see that $I_L > I_C$ in the inductive case and $\vec{V}$ leads $\vec{I}$ by a phase ϕ. On the other hand, in the capacitive case shown in figure (b), $I_C > I_L$ and $\vec{I}$ leads $\vec{V}$ by a phase ϕ.

ILLUSTRATION 27

For the circuit shown in figure, find the instantaneous current through each element. Also find the total instantaneous current through the source, and find expressions for phase angle of this current and the impedance of the circuit.



SOLUTION

The three current equations are,

$$V = I_R R,\ V = L\frac{dI_L}{dt} \text{ and } \frac{dV}{dt} = \frac{1}{C} I_C \qquad \ldots(1)$$

The steady state solutions of equation (1) are,

$$I_R = \frac{V_0}{R}\sin(\omega t) \equiv (I_0)_R \sin(\omega t)$$

$$I_L = -\frac{V_0}{\omega L}\cos(\omega t) = -\frac{V_0}{X_L}\cos(\omega t)$$

$$\Rightarrow \quad I_L = -(I_0)_L \cos(\omega t)$$

and $$I_C = V_0 \omega C \cos(\omega t) = \frac{V_0}{X_C}\cos(\omega t)$$

$$\Rightarrow \quad I_C = (I_0)_C \cos(\omega t)$$

where, $X_L = L\omega$ and $X_C = \frac{1}{C\omega}$

For the total current we have,

$$I = I_R + I_L + I_C$$

$$I = V_0\left[\frac{1}{R}\sin(\omega t) + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)\cos(\omega t)\right] \quad \ldots(2)$$

Let $\frac{V_0}{R} = I_0 \cos\phi$ and $V_0\left(\frac{1}{X_L} - \frac{1}{X_C}\right) = I_0 \sin\phi$

then equation (2), becomes

$$I = I_0 \sin(\omega t + \phi)$$

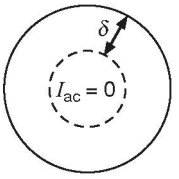
where $I_0 = \frac{V_0}{Z} = V_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

and $\tan\phi = \frac{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)}{\left(\frac{1}{R}\right)} = R\left(C\omega - \frac{1}{L\omega}\right)$

SKIN EFFECT

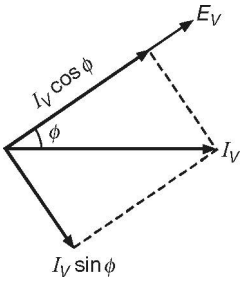
A direct current flows uniformly throughout the cross-section of the conductor. An alternating current, on the other hand, flows mainly along the surface of the conductor. This effect is known as **skin effect**.



The reason is that when alternating current flows through a conductor, the flux changes in the inner part of the conductor are higher. Therefore, the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect. The depth upto which ac current flows through a wire is called **skin depth** (δ).

WATTLESS CURRENT ($I_V \sin\phi$)

Since $P_{av} = E_V (I_V \cos\phi)$



i.e. the component $I_V \cos\phi$ contributes to the power whereas $I_V \sin\phi$ does not contribute to the power and hence is called the **Wattless current**.

ILLUSTRATION 28

An *LCR* series circuit with 100 Ω resistance is connected to an ac source of 200 V and angular frequency 300 rads^{-1}. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the *LCR* circuit.

SOLUTION

When capacitance is removed,

$$\tan\phi = \frac{X_L}{R}$$

$$\Rightarrow \quad \tan 60° = \frac{X_L}{R}$$

$$\Rightarrow \quad X_L = \sqrt{3}R \quad \ldots(1)$$

When inductance is removed, $\tan\phi = \frac{X_C}{R}$

$$\Rightarrow \quad \tan 60° = \frac{X_C}{R}$$

$$\Rightarrow \quad X_C = \sqrt{3}R \quad \ldots(2)$$

From equations (1) and (2) we get

$$X_L = X_C$$

So, the LCR circuit is in resonance and hence

$$Z = R$$

$$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100} = 2 \text{ A}$$

$$\Rightarrow \quad \langle P \rangle = V_{rms} I_{rms} \cos\phi$$

At resonance current and voltage are in phase i.e., $\phi = 0°$

$$\Rightarrow \quad \langle P \rangle = (200)(2)(1) = 400 \text{ W}$$

ILLUSTRATION 29

A 750 Hz, 20 V source is connected to a resistance of 100 Ω, an inductance of 180 mH and a capacitance of 10 μF all in series. Calculate the time in which the resistance having a thermal capacity 2 J °C^{-1}, will get heated by 10 °C.

SOLUTION

The impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

Now, $X_L = 2\pi fL = 2 \times 3.14 \times 750 \times 0.18 \approx 848 \ \Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 750 \times 10^{-5}} \approx 21 \ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(100)^2 + (848 - 21)^2} = \sqrt{(100)^2 + (827)^2}$$

$$\Rightarrow \quad Z = 833 \ \Omega$$

In case of an ac,

$$\langle P \rangle = V_{rms} I_{rms} \cos\phi$$

$$\Rightarrow \quad \langle P \rangle = (V_{rms})\left(\frac{V_{rms}}{Z}\right)\left(\frac{R}{Z}\right) = \left(\frac{V_{rms}}{Z}\right)^2 R$$

$$\Rightarrow \quad \langle P \rangle = \left(\frac{20}{833}\right)^2 \times 100 = 0.058 \text{ Js}^{-1}$$

Now, $\langle P \rangle \times t = (mc)\Delta T$

where, mc = heat capacity of wire = 2 J(°C)$^{-1}$.

$$\Rightarrow \quad t = \frac{(mc)\Delta T}{\langle P \rangle} = \frac{2 \times 10}{0.058} = 345 \text{ s}$$

ILLUSTRATION 30

A resistor R, inductor L, and capacitor C are connected in series to an AC source of rms voltage V and variable frequency. Find the energy that is delivered to the circuit during one period if the operating frequency is twice the resonance frequency.

SOLUTION

Since, at resonance, the angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Now, when $\omega = 2\omega_0 = \frac{2}{\sqrt{LC}}$, then

$$X_L = L\omega = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} \text{ and}$$

$$X_C = \frac{1}{C\omega} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Since by definition, we have

$$Z^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow \quad Z^2 = R^2 + 2.25\left(\frac{L}{C}\right)$$

Further, we know that

$$P = \left(\frac{V^2}{Z}\right)\cos\phi = \left(\frac{V^2}{Z}\right)\left(\frac{R}{Z}\right) = \frac{V^2 R}{Z^2}$$

$$\Rightarrow \quad P = \frac{V^2 R}{R^2 + 2.25\left(\frac{L}{C}\right)}$$

So, energy delivered to the circuit in one cycle is

$$E = PT = P\left(\frac{2\pi}{\omega}\right) = \left(\frac{2\pi RCV^2}{R^2C + 2.25L}\right)\frac{\sqrt{LC}}{2}$$

$$\Rightarrow \quad E = \frac{4\pi RCV^2\sqrt{LC}}{4R^2C + 9L}$$

Test Your Concepts-I

Based on AC

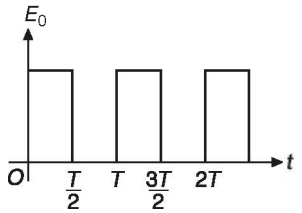
(Solutions on page H.227)

1. Show that average heat produced during a cycle of ac is same as produced by dc with $I = I_{rms}$.

2. Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by $V = 200\sin(\omega t) + 100\sin(3\omega t) + 50\sin(5\omega t)$.

3. The current flowing through a resistor R varies as $i = kt$, where k is constant. Calculate the rms current for first 2 seconds.

4. Calculate the RMS value of the voltage (for a cycle) whose time variation is shown in Figure.



5. An AC voltage is given by $V = V_0 + V_1\cos\omega t$. Calculate the rms value for one cycle.

6. A 100 Ω resistance is connected in series with a 4 H inductor. The voltage across the resistor is, $V_R = (2\ V)\sin\left((10^3\ \text{rads}^{-1})t\right)$

(a) Find the expression of circuit current

(b) Find the inductive reactance

(c) Derive an expression for the voltage across the inductor.

7. A 200 V, 50 Hz AC is connected to a circuit of resistance 1 Ω and inductance 0.01 H. What is the phase difference between the current and the emf in the circuit? Also find the virtual current in the circuit.

8. A sinusoidal voltage of frequency 60 Hz and peak value 150 V is applied to a series LR circuit, where $R = 20\ \Omega$ and $L = 40$ mH. Calculate the values of

(a) T, ω, X_L, Z and ϕ

(b) current amplitude, $(V_R)_{max}$ and $(V_L)_{max}$.

9. A choke coil is needed to operate an arc lamp at 160 V (r.m.s.) and 50 Hz. The lamp has an effective resistance of 5 Ω when running at 10 A (r.m.s.). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power loses in both cases.

10. A 300 Ω resistor is connected in series with a 0.8 H inductor. The voltage across the resistor as a function of time is $V_R = (2.5\ \text{V})\cos\left[(950\ \text{rads}^{-1})t\right]$.

(a) Derive an expression for the circuit current.

(b) Determine the inductive reactance of the inductor.

(c) Derive an expression for the voltage V_L across the inductor.

11. A 300 Ω resistor, a 250 mH inductor, and a 8 μF capacitor are in series with an ac source with voltage amplitude 120 V and angular frequency 400 rads^{-1}.

(a) What is the current amplitude?

(b) What is the phase angle of the source voltage with respect to the current? Does the source voltage lag or lead the current?

(c) What are the voltage amplitudes across the resistor, inductor, and capacitor?

12. (a) What is the reactance of a 2 H inductor at a frequency of 50 Hz ?

(b) What is the inductance of an inductor whose reactance is 2 Ω at 50 Hz?

(c) What is the reactance of a 2 μF capacitor at a frequency of 50 Hz?

(d) What is the capacitance of a capacitor whose reactance is 2 Ω at 50 Hz ?

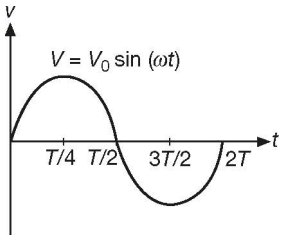
13. An ac circuit consists of a 220 Ω resistance and a 0.7 H choke. Find the power absorbed from 220 V and 50 Hz source connected in this circuit if the resistance and choke are joined in

(a) series

(b) parallel

14. In an LR series circuit, a sinusoidal voltage $V = V_0\sin(\omega t)$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = \frac{22}{7}$. Find the amplitude of current in the steady state and obtain the phase difference between the

current and the voltage. Also plot the variation of current for one cycle on the given graph.



15. In an *RLC* series circuit that includes a source of alternating current operating at fixed frequency and voltage, the resistance R is equal to the inductive reactance. If the plate separation of the capacitor is reduced to half of its original value, the current in the circuit doubles. Find the initial capacitive reactance in terms of R.

16. A voltage $V = 100\sin(\omega t)$ (in SI units) is applied across a series combination of a 2 H inductor, a 10 μF capacitor, and a 10 Ω resistor.
- **(a)** Determine the angular frequency ω_0 at which the power delivered to the resistor is a maximum.
- **(b)** Calculate the power delivered at that frequency.
- **(c)** Determine the two angular frequencies ω_1 and ω_2 at which the power is half the maximum value.

17. Voltage and current for a circuit with two elements in series are expressed as follows:

$$V(t) = 170\sin\left(6280t + \frac{\pi}{3}\right) \text{ volt}$$

$$I(t) = 8.5\sin\left(6280t + \frac{\pi}{2}\right) \text{ ampere}$$

- **(a)** Determine the frequency in Hz.
- **(b)** Determine the power factor stating its nature.
- **(c)** What are the values of the elements?

18. A capacitor of capacitance 250 pF is connected in parallel with a choke coil having inductance of 1.6×10^{-2} H and resistance 20 Ω. Calculate the
- **(a)** resonance frequency and
- **(b)** circuit impedance at resonance

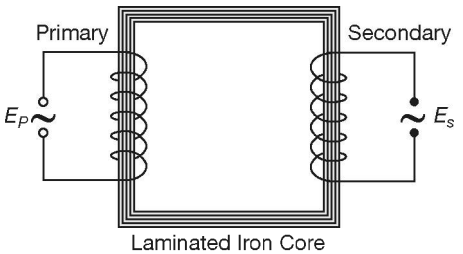
19. A radar transmitter contains an LC circuit oscillating at 1000 MHz.
- **(a)** What capacitance will resonate with a one-turn loop of inductance 400 pH at this frequency?
- **(b)** If the capacitor has square parallel plates separated by 1 mm of air, what should the edge length of the plates be?
- **(c)** What is the common reactance of the loop and capacitor at resonance? Take $\pi^2 = 10$.

20. Two LC circuits have same resonant frequency f_0. When connected in series (assume all the elements to be in series), find the new resonant frequency.

TRANSFORMER

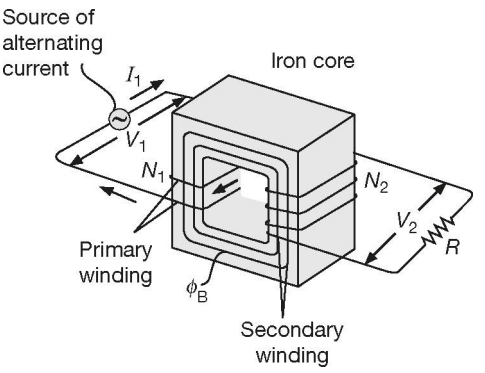
A transformer is a device for converting high voltage at low current to low voltage at high current and vice-versa.

It consists of two coils wound on a soft iron core. The primary coil is connected to an a.c. source. The secondary coil is connected to the load which may be a resistor or any other electrical device.



Construction

Figure shows an idealized transformer. The key components of the transformer are two coils or windings, electrically insulated from each other but wound on the same core.



The core is typically made of a material, such as iron, with a very large relative permeability K_m. This keeps the magnetic field lines due to a current in one winding almost completely within the core. Hence almost all of these field lines pass through the other winding, maximizing the mutual inductance of the two windings. The winding to which power is supplied is called the primary; the winding from which power is delivered is called the secondary. The circuit symbol for a transformer with an iron core, such as those used in power distribution systems, is shown here

Types

There are two types of transformers.

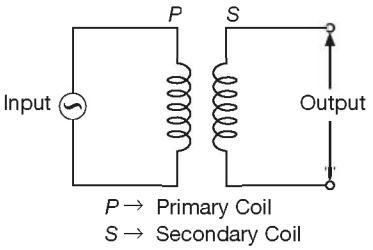
Step Up Transformer

It converts low voltage at high current into high voltage at low current.

Step Down Transformer

It converts high voltage at low current into low voltage at high current.

The principle of transformer is based on mutual induction and a transformer always works on AC. The input is applied across primary terminals and output across secondary terminals. If the resistance of primary coil is zero then voltage across the primary is equal to the applied voltage.



The ratio of number of turns in secondary and primary is called the **turn ratio.** So,

$$\text{Turn Ratio } N = \frac{N_S}{N_P}$$

If E_P and E_S are alternating voltages, I_P and I_S the alternating currents, ϕ_P and ϕ_S be the magnetic flux across primary and secondary terminals respectively, then

For an Ideal Transformer (Efficiency = 1) we have

$$\eta = 1 = \frac{\text{Output Power}}{\text{Input Power}} = \frac{\xi_S I_S}{\xi_P I_P}$$

$$\Rightarrow \quad \frac{\xi_S}{\xi_P} = \frac{I_P}{I_S}$$

Also, we observe that

$$\frac{\xi_S}{\xi_P} = \frac{\phi_S}{\phi_P} = \frac{N_S}{N_P}$$

So, for an Ideal Transformer

$$\frac{\xi_S}{\xi_P} = \frac{\phi_S}{\phi_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} \qquad \ldots(1)$$

Since $\xi_S I_S = \xi_P I_P$

The value of the load resistance (across which output is to be taken), determines the value of current in the secondary coil. So,

$$I_S = \frac{\xi_S}{R_L} \qquad \ldots(2)$$

Also, the current in the primary is

$$I_P = \frac{\xi_P}{R_{\text{eq}}} \qquad \ldots(3)$$

From (2) and (3), we get

$$\frac{I_P}{I_S} = \frac{\xi_P}{\xi_S}\frac{R_L}{R_{\text{eq}}}$$

From (1), we get

$$\frac{N_S}{N_P} = \frac{N_P}{N_S}\left(\frac{R_L}{R_{\text{eq}}}\right)$$

$$\Rightarrow \quad R_{\text{eq}} = \left(\frac{N_P}{N_S}\right)^2 R_L$$

From this analysis, we observe that a transformer may be used to match resistances between the primary circuit and the load. In this way, maximum power transfer can be achieved between a given power source and the load resistance. In other words, we can say that a transformer 'transforms' not only voltages and currents, but resistances as well.

For a non-ideal transformer (with efficiency $\eta < 1$) we have $\eta = \dfrac{\xi_S I_S}{\xi_P I_P}$

$$\frac{\xi_S}{\xi_P} = \eta\frac{I_P}{I_S}$$

So, we get

$$\frac{\xi_S}{\xi_P} = \frac{N_S}{N_P} = \frac{\phi_S}{\phi_P} = \frac{\eta I_P}{I_S}$$

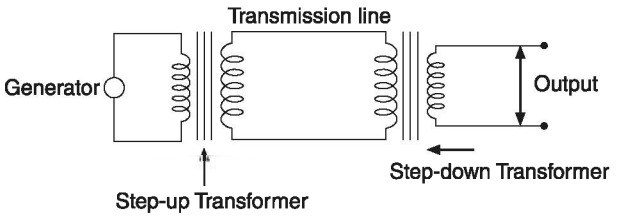
In actual transformers, there is some power loss. The main sources of power loss are:

(a) I^2R loss due to Joule heat in copper windings.
(b) Heating produced due to Eddy currents in the iron core. This is reduced by using laminated core.
(c) Hysteresis loss due to repeated magnetisation of the iron core.
(d) Loss due to flux leakage.

When all the losses are minimized, the efficiency of the transformer becomes very high (90-99%).

LONG DISTANCE TRANSMISSION OF ELECTRIC POWER

One of the great advantages of AC over DC for electric power distribution is that it is much easier to step voltage levels up and down with AC than with DC. For long-distance power transmission it is desirable to use as high a voltage and as small a current as possible; this reduces I^2R losses in the transmission lines, and smaller wires can be used, saving on material costs. To minimize the I^2R loss during long distance transmission of electric power, the voltage is first stepped up to a high value using a transformer.



Since $P = VI$ is constant, as V is increased I decreases and hence I^2R loss becomes small. By increasing V to a very high value, this power loss can be made negligible.

INDUCTION COIL

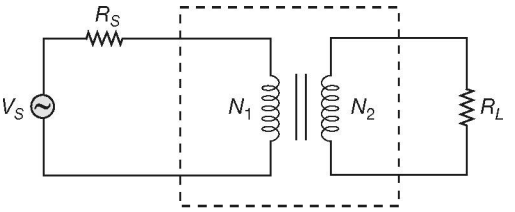
Induction coil is based on mutual induction and is used to produce a larger e.m.f. from a source of low e.m.f.

It has two coils, Primary and Secondary. When current in primary coil changes, the magnetic flux linked with the secondary also changes thus setting up an induced e.m.f. in the secondary. During self-induction of primary the rate of growth of current is slow but when the circuit breaks rate of change of current $\left|\frac{dI}{dt}\right|$ in the secondary is very large and so induced e.m.f. in the secondary is also very large.

Induction coil can be used to produce an e.m.f. of the order of 50,000 V from a 12 V battery. Induction coil is often used to operate a discharge tube.

ILLUSTRATION 31

In the transformer shown in figure, the load resistor is 50 Ω. The turns ratio $N_1 : N_2$ is 10 : 2 and the source voltage is 200 V (r.m.s.). If a voltmeter across the load measures 25 V (r.m.s.), what is the source resistance R_S?



SOLUTION

The r.m.s. voltage across the primary of the transformer is given by

$$V_P = V_1 = \left(\frac{N_1}{N_2}\right)V_2$$

So, the source voltage, if current in the primary is I_1 is

$$V_s = I_1R_s + V_P$$

$$\Rightarrow \quad V_s = I_1R_s + \left(\frac{N_1}{N_2}\right)V_2$$

The current in the secondary coil is I_2, then

$$I_1 = \left(\frac{N_2}{N_1}\right)I_2 = \left(\frac{N_2}{N_1}\right)\left(\frac{V_2}{R_L}\right)$$

$$\Rightarrow \quad V_s = \left(\frac{N_2}{N_1}\right)\left(\frac{V_2}{R_L}\right)R_s + \left(\frac{N_1}{N_2}\right)V_2$$

$$\Rightarrow \quad V_s = V_2\left[\left(\frac{N_2}{N_1}\right)\left(\frac{R_s}{R_L}\right) + \frac{N_1}{N_2}\right]$$

$$R_s = \frac{N_1 R_L}{N_2 V_2}\left(V_s - V_2\left(\frac{N_1}{N_2}\right)\right)$$

Since $\frac{N_1}{N_2} = \frac{10}{2}$, $V_s = 200\ \text{V}$, $R_L = 50\ \Omega$, $V_2 = 25\ \text{V}$, so

$$R_s = \left(\frac{10}{2}\right)\left(\frac{50}{25}\right)\left(200 - (25)\left(\frac{10}{2}\right)\right)$$

$\Rightarrow \quad R_s = (10)(200 - 125)$

$\Rightarrow \quad R_s = 750\ \Omega$

ILLUSTRATION 32

An ideal transformer has 50 turns in its primary winding and 25 turns in its secondary winding. If the current in the secondary winding in 4 A, calculate the current in primary winding if a 200 V AC is applied across it.

SOLUTION

Since the ratio of turns in primary to secondary winding is 2:1, so it is a step-down transformer for which, we have

$$\frac{\xi_s}{\xi_p} = \frac{N_s}{N_p}$$

$\Rightarrow \quad \xi_s = \left(\frac{N_s}{N_p}\right)\xi_p$

$\Rightarrow \quad \xi_s = \frac{25}{50} \times 200 = 100\ \text{V}$

For an ideal transformer, the power equation is

$$\xi_s i_s = \xi_p i_p$$

$\Rightarrow \quad i_p = \frac{\xi_s i_s}{\xi_p} = \frac{100 \times 4}{200} = 2\ \text{A}$

ILLUSTRATION 33

An ideal power transformer is used to step up an alternating voltage of 220 V to 4.4 kV for transmitting 0.6 kW of power. If the primary coil has 1000 turns, calculate the number of turns in the secondary coil and the current rating of the secondary coil.

SOLUTION

Since, we know that

$$N_s = \left(\frac{\xi_s}{\xi_p}\right)N_p$$

$\Rightarrow \quad N_s = \left(\frac{4.4 \times 1000}{220}\right) \times 1000 = 20000\ \text{turns}$

Power supplied at the primary coil is

$$P_p = i_p \xi_p = 6.6 \times 10^3\ \text{W}$$

$\Rightarrow \quad i_p = \frac{6.6 \times 10^3}{220} = 30\ \text{A}$

For an ideal transformer we use

$$\frac{i_s}{i_p} = \frac{N_p}{N_s} = \left(\frac{1000}{20{,}000}\right) = \frac{1}{20}$$

$\Rightarrow \quad i_s = \frac{i_p}{20} = 1.5\ \text{A}$

ILLUSTRATION 34

A transformer has secondary turns to primary turns ratio of 4. If a 200 V AC is applied across its primary and it carries a current of 1 A, then calculate the current in circuit connected to secondary coil if transformer is 80% efficient.

SOLUTION

Power supplied at primary coil is

$$P_1 = \xi_1 i_1 = (200)(1) = 200\ \text{W}$$

Voltage across secondary coil is

$$\xi_2 = \left(\frac{N_2}{N_1}\right)\xi_1 = 200 \times 4 = 800\ \text{V}$$

Power available at the secondary coil is

$$P_2 = \left(\frac{80}{100}\right)200\ \text{W} = 160\ \text{W}$$

$\Rightarrow \quad P_2 = \xi_2 i_2 = 160\ \text{W}$

$\Rightarrow \quad i_2 = \frac{160}{800} = 0.2\ \text{A}$

ILLUSTRATION 35

In a step-down transformer having primary to secondary turn ratio $20:1$, the input voltage applied is $250\ \text{V}$ and output current is $8\ \text{A}$. Assuming 100% efficiency, calculate the voltage across secondary coil, current in primary coil and power output.

SOLUTION

Since we know that

$$\frac{\xi_S}{\xi_P}=\frac{N_S}{N_P}$$

$$\Rightarrow \quad \xi_S=\left(\frac{N_S}{N_P}\right)\xi_P$$

where, $\frac{N_S}{N_P}=\frac{1}{20}$, $\xi_P=250\ \text{V}$ and $i_S=8\ \text{A}$

$$\Rightarrow \quad \xi_S=\frac{1}{20}(250)=12.5\ \text{V}$$

For 100% efficiency, we have

$$\frac{i_P}{i_S}=\frac{N_S}{N_P}$$

$$\Rightarrow \quad i_P=\left(\frac{N_S}{N_P}\right)i_S$$

$$\Rightarrow \quad i_P=\frac{1}{20}\times 8=0.4\ \text{A}$$

Power output of transformer is

$$P_{\text{output}}=\xi_S i_S$$

$$\Rightarrow \quad P_{\text{output}}=12.5\times 8=100\ \text{W}$$

ILLUSTRATION 36

A $2000\ \text{V}-200\ \text{V}$, $20\ \text{kVA}$ transformer has 66 turns in the secondary. Calculate the primary and secondary full-load current when power losses in the transformer are ignored.

SOLUTION

Primary and secondary coil voltage are

$$\xi_P=2000\ \text{V} \text{ and } \xi_S=200\ \text{V}$$

Power of the transformer at primary coil is 20 kVA, so we have

$$P_P=\xi_P i_P=20\times 10^3\ \text{W}$$

$$\Rightarrow \quad i_P=\frac{20\times 10^3}{2000}=10\ \text{A}$$

Since power losses in the transformer are ignored, so power of the transformer at secondary coil is also $20\ \text{kVA}$ and hence we have

$$P_S=\xi_S i_S=20\times 10^3\ \text{W}$$

$$\Rightarrow \quad i_S=\frac{20\times 10^3}{200}=100\ \text{A}$$

ILLUSTRATION 37

A transformer is used to light a $140\ \text{W}$, $24\ \text{V}$ lamp from $240\ \text{V}$ ac mains. If the current in the mains cable is $0.7\ \text{A}$, calculate the efficiency of transformer.

SOLUTION

After reading the problem, we can understand that the input voltage is $240\ \text{V}$ and the output voltage is, hence we have

$$\xi_P=240\ \text{V} \text{ and } \xi_S=24\ \text{V}$$

Resistance of lamp is

$$R=\frac{V^2}{P}=\frac{(24)^2}{140}=\frac{144}{35}\ \Omega$$

So, current in the secondary coil is

$$i_S=\frac{\xi_S}{R}=\frac{24}{144}\times 35=\frac{35}{6}\ \text{A}$$

Power supplied at the input coil i.e. the primary coil is

$$P_P=\xi_P i_P=240\times 0.7=168\ \text{W}$$

Power output at the secondary coil is

$$P_S=\xi_S i_S=24\times\frac{35}{6}=140\ \text{W}$$

So, efficiency of the transformer is

$$\eta=\frac{\xi_S i_S}{\xi_P i_P}\times 100\%=\frac{148}{168}\times 100\%$$

$$\Rightarrow \quad \eta=83.33\%$$

Test Your Concepts-II

Based on Transformer

(Solutions on page H.231)

1. A transformer has 200 turns in primary coil and 600 turns in secondary coil. If a 220 V DC is applied across primary coil, what will be the voltage across secondary coil.
2. The secondary voltage of an ignition transformer in a furnace is 10 kV. When the primary operates at rms voltage of 120 V, the primary impedance is 24 Ω and the transformer is 90% efficient. Calculate the
   (a) turns ratio.
   (b) current in the secondary.
   (c) impedance in the secondary.
3. In a transformer there are 10000 turns in primary coil and 25000 turns in secondary coil. An AC of voltage $E = 50\sin(100\pi t)$ is applied across the primary coil, calculate the peak voltage output across secondary coil in ideal conditions.
4. A transmission line that has a resistance per unit length of $5\times10^{-4}\ \Omega m^{-1}$ is to be used to transmit 5 MW over 600 km. The output voltage of the generator is 4.5 kV.
   (a) What is the line loss if a transformer is used to step up the voltage to 500 kV ?
   (b) What fraction of the input power is lost to the line under these circumstances?
   (c) What difficulties will be encountered in attempting to transmit the 5 MW at the generator voltage of 4.5 kV ?

SOLVED PROBLEMS

PROBLEM 1

A series LCR circuit with inductance $0.12\ \text{H}$, capacitance $480\ \text{nF}$ and resistance $23\ \Omega$ is connected to a $230\ \text{V}$ variable frequency ac supply.

(a) Calculate the ac source frequency for which current in the circuit is maximum. Calculate the maximum current amplitude.

(b) Calculate the source frequency for which average power consumed by the circuit is maximum. Calculate the value of maximum power.

(c) For which angular frequencies of the source, the power transferred to the circuit is half of the power at resonant frequency?

(d) Calculate the Q factor of the circuit.

SOLUTION

(a) The current in the circuit is I and is given by

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

Current in circuit is maximum at resonance i.e. when $X_L = X_C$. If ω_0 be the resonant angular frequency, then we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.12)(480\times10^{-9})}}$$

$$\Rightarrow \quad \omega_0 = 4167\ \text{rads}^{-1}$$

The resonant frequency f_0 is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{4167}{2\times3.14} = 663.5\ \text{Hz}$$

The maximum current amplitude at resonance is given

$$I_0 = \frac{E_0}{R} = \frac{\sqrt{2}E_{\text{rms}}}{R} = \frac{\sqrt{2}\times230}{23} = 14.14\ \text{A}$$

(b) The average power consumed by the circuit is

$$P = E_{\text{rms}} I_{\text{rms}} \cos\phi$$

At resonance, power factor is maximum, so

$$\cos\phi = 1$$

$$\Rightarrow \quad P_{\max} = E_{\text{rms}} I_{\text{rms}} = \frac{E_{\text{rms}}^2}{R}$$

$$\Rightarrow \quad P_{\max} = \frac{(230)^2}{23} = 2300\ \text{W}$$

(c) The half power frequencies are the frequencies at which the power in the circuit is half the maximum power. These are given by

$$\omega_{hp} = \omega_0 \pm \frac{R}{2L} = 4167 \pm \left(\frac{23}{2\times0.12}\right)$$

$$\Rightarrow \quad \omega_{hp} = 4263\ \text{rads}^{-1} \text{ and } 4071\ \text{rads}^{-1}$$

(d) The quality factor of the circuit is given as

$$Q = \frac{L\omega_0}{R} = \frac{(0.12)(4167)}{23} = 21.74$$

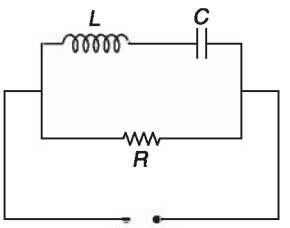
PROBLEM 2

A box contains an inductor, capacitor and a resistor. When $250\ \text{V}$ DC voltage is applied to the terminals of the box, a current of $1\ \text{A}$ flows in the circuit. When a $250\ \text{V}$, $2250\ \text{rads}^{-1}$ AC source is applied across the box, a current of $1.25\ \text{A}$ flows through it. It is observed that the current rises with frequency and becomes maximum at $4500\ \text{rads}^{-1}$. Calculate the values of inductance, capacitance and the resistance in the box. Also draw the circuit diagram.

SOLUTION

Let L be the inductance, C be the capacitance and R be the resistance in the circuit. When a dc voltage is applied to the circuit containing L, C and R, then the capacitor acts as a dc blocking element and if all L, C and R are in series then no current should flow in the circuit.

However, we observe a current to flow through the circuit which makes us conclude that the resistor R must have been in parallel to the series combination of L and C as shown in figure.



So, for the dc source, we have

$$R = \frac{E_{\text{dc}}}{I_{\text{dc}}} = \frac{250}{1} = 250\ \Omega$$

The impedance Z of the circuit at 2250 rads^{-1} is

$$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\frac{1}{\left(L\omega-\frac{1}{C\omega}\right)^2}}$$

Circuit impedance can also be directly given as

$$Z=\frac{V}{i}=\frac{250}{1.25}=200\ \Omega$$

$$\Rightarrow \quad \frac{1}{(200)^2}=\frac{1}{R^2}+\frac{1}{\left(L\omega-\frac{1}{C\omega}\right)^2}$$

$$\Rightarrow \quad \frac{1}{\left(L\omega-\frac{1}{C\omega}\right)^2}=\frac{1}{(200)^2}-\frac{1}{(250)^2}=\frac{9}{(10)^6}$$

$$\Rightarrow \quad L\omega-\frac{1}{C\omega}=\frac{1000}{3}$$

$$\Rightarrow \quad 2250L-\frac{1}{2250C}=\frac{1000}{3} \qquad \ldots(1)$$

At resonance, the frequency is $\omega_0 = 4500$ rads^{-1}

$$\Rightarrow \quad \omega_0^2=\frac{1}{LC}$$

$$\Rightarrow \quad 4500\times 4500=\frac{1}{LC}$$

$$\Rightarrow \quad 2250L=\frac{1}{9000C} \qquad \ldots(2)$$

Substituting equation (2) in equation (1), we get

$$\frac{1}{9000C}-\frac{1}{2250C}=\frac{1000}{3}$$

$$\Rightarrow \quad C=10^{-6}\ \text{F}=1\ \mu\text{F}$$

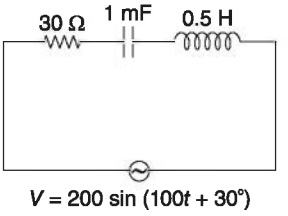
From equation (2), we get

$$2250L=\frac{1}{9000\times 10^{-6}}$$

$$\Rightarrow \quad L=\frac{1}{2250\times 9000\times 10^{-6}}\approx 0.05\ \text{H}$$

PROBLEM 3

An *LCR* circuit is fed with an ac voltage given by $V = 200\sin(100t+30°)$ volt as shown in Figure.



Find the current in the circuit, voltage across resistor, capacitor and inductor as a function of time. Also, calculate the average power consumed in the circuit.

SOLUTION

From the voltage equation given in the problem, we conclude that $\omega = 100$ rads^{-1} and $V_0 = 200$ V.

The inductive reactance is

$$X_L=\omega L=50\ \Omega$$

The capacitive reactance is

$$X_C=\frac{1}{\omega C}=\frac{1}{100\times 10^{-3}}=10\ \Omega$$

The impedance of the circuit is

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$$

$$\Rightarrow \quad Z=\sqrt{(30)^2+(50-10)^2}=50\ \Omega$$

Maximum current flowing through the circuit is

$$I_0=\frac{V_0}{Z}=\frac{200}{50}=4\ \text{A}$$

From the calculations done, we see that $X_L > X_C$, therefore the circuit is inductive in nature and hence voltage leads the current by a phase difference ϕ where,

$$\cos\phi=\frac{R}{Z}=\frac{30}{50}=\frac{3}{5}$$

$$\Rightarrow \quad \phi=53°$$

So, we conclude that current in the circuit lags behind the voltage by a phase angle of $\phi = 53°$.

The current function is given by

$$I=4\sin(100t+30°-53°)$$

$$\Rightarrow \quad I=4\sin(100t-23°)$$

For Resistor

Maximum value of potential drop across the resistor i.e. $(V_R)_{\max}$ is given by

$$(V_R)_{max} = I_0 R = 4 \times 30 = 120 \text{ volt}$$

Since, V_R and I are in same phase, so we have

$$V_R = 120 \sin(100t - 23°)$$

For Capacitor

Maximum value of potential drop across the capacitor i.e. $(V_C)_{max}$ is given by

$$(V_C)_{max} = I_0 X_C = 4 \times 10 = 40 \text{ volt}$$

Since voltage across the capacitor lags behind the current by a phase angle of $90°$, so we have

$$V_C = 40 \sin(100t - 23° - 90°)$$

$$\Rightarrow \quad V_C = 40 \sin(100t - 113°)$$

For Inductor

Maximum value of potential drop across the inductor i.e. $(V_L)_{max}$ is given by

$$(V_L)_{max} = I_0 X_L = 4 \times 50 = 200 \text{ volt},$$

Since voltage across the inductor leads the current by a phase angle of $90°$, so we have

Therefore,

$$V_L = 200 \sin(100t - 23° + 90°)$$

$$\Rightarrow \quad V_C = 200 \sin(100t - 67°)$$

Please note that at any instant in the circuit, we have

$$V = V_R + V_L + V_C$$

Average power is consumed in an AC circuit only across a resistance and this power is given by

$$\langle P \rangle = V_{rms} I_{rms} \cos\phi = I_{rms}^2 R = \frac{I_0^2 R}{2}$$

$$\Rightarrow \quad \langle P \rangle = \frac{I_0^2 R}{2} = \frac{(4)^2 (30)}{2} = 240 \text{ W}$$

PROBLEM 4

An AC circuit draws a power of 550 W when connected across a 220 V, 50 Hz ac source. If the power factor of the circuit is 0.8 and the current lags the voltage. For making the power factor of the circuit to be the maximum, calculate the capacitance required to be connected in series with it.

SOLUTION

Since the current lags behind the voltage, so the circuit must be having a resistance and an inductance. Since the average power consumed by the circuit is

$$P_{av} = E_{rms} I_{rms} \cos\phi$$

$$\Rightarrow \quad P = \left(\frac{E_{rms}^2}{Z}\right) \cos\phi$$

$$\Rightarrow \quad Z = \frac{E_{rms}^2 \cos\phi}{P}$$

$$\Rightarrow \quad Z = \frac{(220)^2 \times 0.8}{550} = 70.4 \ \Omega$$

Power factor of circuit is given to be 0.8, so we have

$$\cos\phi = \frac{R}{Z}$$

$$\Rightarrow \quad R = Z \cos\phi$$

$$\Rightarrow \quad R = 70.4 \times 0.8 = 56.32 \ \Omega$$

Also, the initial circuit impedance is

$$Z^2 = R^2 + (\omega L)^2$$

$$\Rightarrow \quad L\omega = \sqrt{Z^2 - R^2}$$

$$\Rightarrow \quad L\omega = \sqrt{(70.4)^2 - (56.32)^2} = 42.24 \ \Omega$$

Now, when the capacitor is connected in the circuit for making the power factor to be maximum i.e. $\cos\phi = 1$, then the circuit obeys the condition of resonance i.e. $X_L = X_C$

$$\Rightarrow \quad L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \quad C = \frac{1}{\omega(L\omega)} = \frac{1}{2\pi f (L\omega)}$$

$$\Rightarrow \quad C = \frac{1}{(2 \times 3.14 \times 50)(42.24)}$$

$$\Rightarrow \quad C \approx 75 \times 10^{-6} \text{ F} = 75 \ \mu\text{F}$$

PROBLEM 5

A series circuit consists of a resistance, inductance and capacitance. The applied voltage and the current at any instant are given by

$$E = 141.4 \cos(3000t - 10°)$$

and $\quad I = 5 \cos(3000t - 55°)$

The inductance is 0.01 H. Calculate the values of the resistance and capacitance.

SOLUTION

The phase difference between current and voltage is given as

$$\phi = -10° - (-55°) = 45°$$

This makes us conclude that the voltage is leading the current by a phase angle of $\phi = 45°$

For a series LCR circuit, we have

$$\tan\phi = \frac{L\omega - \frac{1}{C\omega}}{R}$$

$$\Rightarrow \quad \tan 45° = \frac{L\omega - \frac{1}{C\omega}}{R} = 1$$

$$\Rightarrow \quad L\omega - \frac{1}{C\omega} = R \qquad \ldots(1)$$

The impedance Z of the circuit is given by

$$Z = \frac{E_0}{I_0} = \frac{141.4}{5} = 28.28\ \Omega$$

Also, we know that

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + R^2} = 1.414R$$

$$\Rightarrow \quad 1.414R = 28.28\ \Omega$$

$$\Rightarrow \quad R = 20\ \Omega \qquad \ldots(2)$$

From equation (1), we get

$$L\omega - \frac{1}{C\omega} = 20$$

$$\Rightarrow \quad (0.01)(3000) - \frac{1}{3000C} = 20$$

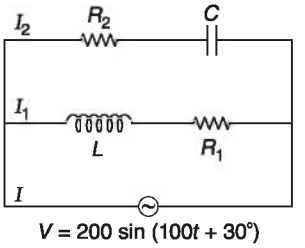
$$\Rightarrow \quad \frac{1}{3000C} = 30 - 20 = 10$$

$$\Rightarrow \quad C = \frac{1}{(3000)(10)} = 33.33 \times 10^{-6}\ \text{F}$$

$$\Rightarrow \quad C = 33.33\ \mu\text{F}$$

PROBLEM 6

Two resistors, a capacitor and an inductor are joined to form a circuit shown in Figure.



An ac source voltage given by $V = 200\sin(100t + 30°)$ is applied to the circuit. If $R_1 = 30\ \Omega$, $R_2 = 40\ \Omega$, $L = 0.4$ H and $C = \frac{1}{3}$ mF, then find I, I_1, I_2, voltage across resistor R_1, voltage across resistor R_2, voltage across inductor and voltage across capacitor as function of time. Also, calculate the average power consumed in the circuit.

SOLUTION

In this problem, let us consider the circuit containing inductor and resistor R_1 (thinking that R_2 and C being absent) and then reconsider the same circuit but now containing R_2 and capacitor (thinking L and R_1 to be absent) as shown in Figures 4.1 and 4.2.

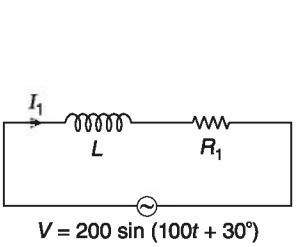


Figure 4.1

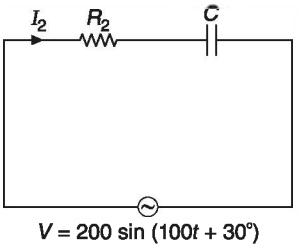


Figure 4.2

Then by Principle of Superposition as already discussed in Current Electricity, we know that

$$I = I_1 + I_2$$

Let us now calculate I_1 (i.e. current in the circuit when R_2 and C are absent).

From the voltage equation given in the problem, we conclude that $\omega = 100\ \text{rads}^{-1}$ and $V_0 = 200$ V.

FOR CIRCUIT IN FIGURE 4.1

Inductive reactance of this circuit is

$$X_L = \omega L = 100 \times 0.4 = 40\ \Omega$$

Since $R_1 = 30\ \Omega$, so the impedance for this circuit is

$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(30)^2 + (40)^2} = 50\ \Omega$$

Maximum value of current in this first circuit is

$$(I_1)_{max} = \frac{V_0}{Z_1} = \frac{200}{50} = 4 \text{ A}$$

Since, this first circuit only contains L and R_1, so in this inductive natured circuit, the current will lag behind the voltage by an angle ϕ_1 given by

$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{30}{50} = \frac{3}{5}$$

$$\Rightarrow \quad \phi_1 = 53^\circ$$

So, for the applied voltage

$$V = 200\sin(100t + 30^\circ) \text{ volt}$$

the current in this circuit should lag behind the applied voltage by 53° and hence is given by

$$\Rightarrow \quad I_1 = 4\sin(100t + 30^\circ - 53^\circ)$$

$$\Rightarrow \quad I_1 = 4\sin(100t - 23^\circ)$$

Maximum value of potential drop across the resistor i.e. $(V_{R_1})_{max}$ is given by

$$(V_{R_1})_{max} = (I_1)_{max} R_1 = (4)(30) = 120 \text{ V}$$

However, current in the resistor is in phase with the voltage, so we have

$$V_{R_1} = 120\sin(100t - 23^\circ)$$

Maximum value of potential drop across the inductor i.e. $(V_L)_{max}$ is given by

$$(V_L)_{max} = (I_1)_{max} X_L = (4)(40) = 160 \text{ V}$$

However, voltage in an inductor leads the current by 90°, so we have

$$\Rightarrow \quad V_L = 160\sin(100t - 23^\circ + 90^\circ)$$

$$\Rightarrow \quad V_L = 160\sin(100t + 67^\circ)$$

In this circuit, power will be consumed only across R_1. If $\langle P_1 \rangle$ is the average power consumed in this circuit, then we have

$$\langle P \rangle = V_{rms} I_{rms} \cos\phi = I_{rms}^2 R = \frac{I_0^2 R}{2}$$

$$\Rightarrow \quad \langle P_1 \rangle = \frac{I_0^2 R_1}{2} = \frac{(4)^2 (30)}{2} = 240 \text{ W}$$

FOR CIRCUIT IN FIGURE 4.2

Now, let us calculate I_2 (i.e. current in the circuit when R_1 and L are absent).

Capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times \frac{1}{3} \times 10^{-3}} = 30 \ \Omega$$

Since $R_2 = 40 \ \Omega$, so the impedance of this second circuit is

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{(40)^2 + (30)^2} = 50 \ \Omega$$

Maximum value of current in this second circuit is

$$(I_2)_{max} = \frac{V_0}{Z_2} = \frac{200}{50} = 4 \text{ A}$$

Since, this second circuit only contains R_2 and C, so in this capacitive natured circuit, current leads the voltage by a phase angle ϕ_2 given by

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{40}{50} = \frac{4}{5}$$

$$\Rightarrow \quad \phi_2 = 37^\circ$$

So, for the applied voltage

$$V = 200\sin(100t + 30^\circ) \text{ volt}$$

the current in this circuit should lead the applied voltage by 37° and hence is given by

$$\Rightarrow \quad I_2 = 4\sin(100t + 30^\circ + 37^\circ)$$

$$\Rightarrow \quad I_2 = 4\sin(100t + 67^\circ)$$

Maximum value of potential drop across the resistor R_2 is $(V_{R_2})_{max}$, where

$$(V_{R_2})_{max} = (I_2)_{max} R_2 = (4)(40) = 160 \text{ V}$$

Since, current in the resistor is in phase with the voltage, so we get

$$V_{R_2} = 160\sin(100t + 67^\circ)$$

Maximum value of potential drop across the capacitor i.e. $(V_C)_{max}$ is given by

$$(V_C)_{max} = (I_2)_{max} X_C = (4)(30) = 120 \text{ V}$$

Since, voltage in a capacitor lags behind the current by 90°, so we have

$$V_C = 120\sin(100t + 67° - 90°)$$

$$\Rightarrow \quad V_C = 120\sin(100t - 23°)$$

In this circuit, power will be consumed only across R_2. If $\langle P_2 \rangle$ is the average power consumed in this circuit, then we have

$$\langle P \rangle = V_{rms} I_{rms} \cos\phi = I_{rms}^2 R = \frac{I_0^2 R}{2}$$

$$\Rightarrow \quad \langle P_2 \rangle = \frac{I_0^2 R_2}{2} = \frac{(4)^2 (40)}{2} = 320 \text{ W}$$

Total average power consumed in the circuit is

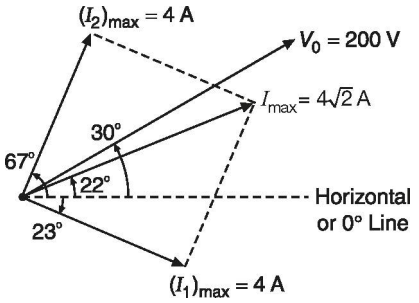
$$\langle P \rangle = \langle P_1 \rangle + \langle P_2 \rangle = 240 + 320 = 560 \text{ W}$$

The current I is given by

$$I = I_1 + I_2$$

$$\Rightarrow \quad I = 4\sin(100t - 23°) + 4\sin(100t + 67°)$$

The current amplitude can be found by drawing the phasor shown in Figure.



Resultant of 4 A and 4 A at 90° is $4\sqrt{2}$ A at 45° from both currents or at 22° with the horizontal i.e. the $100t$ line.

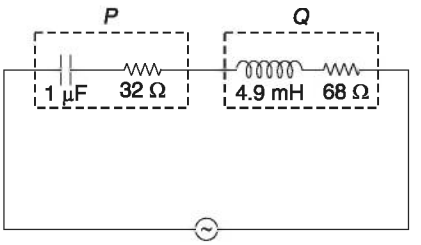
$$I = 4\sqrt{2}\sin(100t + 22°)$$

PROBLEM 7

A box P and a coil Q are connected in series with a 100 V ac source of variable frequency. The box P contains a 32 Ω, 1 μF capacitor and the coil Q is a 68 Ω, 4.9 mH inductor. The frequency is adjusted so that the maximum current flows both through P and Q. Calculate the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.

SOLUTION

The situation described in the problem is shown in Figure.



The current flowing in a series LCR circuit is

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

where $R = R_P + R_Q = 100\ \Omega$

The maximum current flows through P and Q when the circuit is in resonance i.e. when $X_L = X_C$

$$\Rightarrow \quad L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \quad \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{(4.9\times10^{-3})(10^{-6})}} = \frac{10^5}{7} \text{ rads}^{-1}$$

The current flowing in the circuit is

$$I_{max} = \frac{E_{rms}}{R_P + R_Q} = \frac{100}{32+68} = \frac{100}{100} = 1 \text{ A}$$

The impedance of P is

$$Z_P = \sqrt{R_P^2 + X_C^2} = \sqrt{R_1^2 + \left(\frac{1}{C\omega}\right)^2}$$

$$\Rightarrow \quad Z_P = \sqrt{(32)^2 + \left(\frac{7}{10^5}\times\frac{1}{10^{-6}}\right)^2}$$

$$\Rightarrow \quad Z_p = \sqrt{(32)^2 + (70)^2}$$

$$\Rightarrow \quad Z_p = \sqrt{1024 + 4900} = \sqrt{5924} \approx 77\ \Omega$$

The impedance of Q is

$$Z_Q = \sqrt{R_Q^2 + L^2\omega^2}$$

$$\Rightarrow \quad Z_Q = \sqrt{(68)^2 + \left(4.9\times10^{-3}\times\frac{10^5}{7}\right)^2}$$

$$\Rightarrow \quad Z_Q = \sqrt{(68)^2 + (70)^2} = \sqrt{9524} \approx 98\ \Omega$$

Voltage across P is

$$V_P = I_{\text{rms}} Z_P = 1\times76 = 76\ \text{V}$$

Voltage across Q is given as

$$V_Q = I_{\text{rms}} Z_Q = 1\times98 = 98\ \text{V}$$

PROBLEM 8

A 10 Ω resistor is joined in series with a 0.5 H inductor. What capacitance should be connected in series with the combination to obtain the maximum current? Find the maximum current. Assuming that the current is being supplied by a 200 V, 50 Hz ac source, calculate the potential difference across the resistor, inductor and capacitor.

SOLUTION

The current in the circuit is maximum at resonance i.e. when

$$L\omega = \frac{1}{C\omega}$$

$$\Rightarrow \quad C = \frac{1}{\omega^2 L}$$

$$\Rightarrow \quad C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\times3.14\times50)^2\times0.5}$$

$$\Rightarrow \quad C = 20.24\times10^{-6}\ \text{F}$$

At resonance, impedance is minimum, circuit is resistive in nature and current is maximum, so

$$Z_{\min} = R = 10\ \Omega$$

$$\Rightarrow \quad I_{\max} = \frac{E}{R} = \frac{200}{10} = 20\ \text{A}$$

Potential difference across resistance

$$E_R = IR = 20\times10 = 200\ \text{V}$$

Potential difference across inductor

$$E_L = I(L\omega) = I(2\pi fL)$$

$$\Rightarrow \quad E_L = 20[2\pi(50)(0.5)] = 3142\ \text{V}$$

Potential difference across capacitor

$$E_C = IX_C = \frac{I}{C\omega} = IL\omega = 3142\ \text{V}$$

PROBLEM 9

A 20 V, 5 W lamp is applied across a 200 V, 50 Hz AC source.

(a) Calculate the capacitance to be connected in series with the lamp to run the lamp at its peak brightness.

(b) Calculate the inductance to be connected in series with the lamp to run the lamp at its peak brightness.

(c) How much additional resistance can be connected in series with the lamp (as a replacement to inductor or a capacitor) so that the lamp can run on its peak brightness?

(d) Which of the above arrangements will be more economical and why?

SOLUTION

The current required by the lamp (a resistor) to run at its peak brightness is

$$I_{\text{rms}} = \frac{P}{E_{\text{rms}}} = \frac{5}{20} = 0.25\ \text{A}$$

The resistance of the lamp is

$$R = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{20}{0.25} = 80\ \Omega$$

(a) When a capacitor C is connected in series with the lamp, then its impedance is

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$$

The current through the circuit is

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{200}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}} = 0.25\ \text{A}$$

$$\Rightarrow \quad \frac{200}{\sqrt{(80)^2 + \frac{1}{4\pi^2(50)^2C^2}}} = 0.25$$

$$\Rightarrow \quad C = 4.0\times10^{-6}\ \text{F} = 4.0\ \mu\text{F}$$

(b) When an inductance L is connected in series with the lamp, then the impedance is

$$Z = \sqrt{R^2 + L^2\omega^2}$$

The current through the circuit is

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{200}{\sqrt{R^2 + L^2\omega^2}} = 0.25 \text{ A}$$

$$\Rightarrow \quad \frac{200}{\sqrt{(80)^2 + 4\pi^2 (50)^2 L^2}} = 0.25$$

$$\Rightarrow \quad L = 2.53 \text{ H}$$

(c) When an additional resistance r replaces the inductor or capacitor and it is connected in series with the lamp of resistance, then the current through the lamp resistor circuit is

$$I_{rms} = \frac{200}{R + r} = 0.25 \text{ A}$$

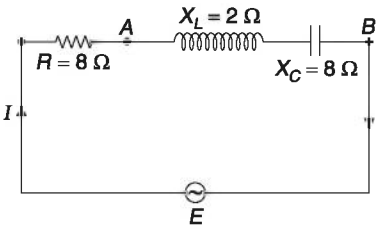
$$\Rightarrow \quad \frac{200}{80 + r} = 0.25$$

$$\Rightarrow \quad r = 720 \ \Omega$$

(d) It will be more economical to use inductor or capacitor in series with the lamp to run it, because the reactive components of circuit (i.e. inductor or capacitor) do not consume any power, whereas there would be dissipation of power when resistance is inserted in series with the lamp.

PROBLEM 10

An ac source $E = 10\cos(100\pi t)$ is applied across the circuit shown in Figure.



Calculate the circuit impedance. When the potential difference across the points A and B i.e. V_{AB} is half of the voltage of source, then calculate V_{AB}.

SOLUTION

Circuit impedance is given as

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(8)^2 + (6)^2}$$

$$\Rightarrow \quad Z = 10 \ \Omega$$

Since $X_L < X_C$, so current will lead the voltage by a phase angle

$$\phi = \tan^{-1}\left(\frac{6}{8}\right) = 37^\circ$$

So, at any instant, current in the circuit is given by

$$I = I_0 \cos(100\pi t + 37^\circ)$$

where, $I_0 = \frac{E_0}{Z} = 1 \text{ A}$

$$\Rightarrow \quad I = \cos(100\pi t + 37^\circ) \qquad \ldots(1)$$

Since between the points A and B, only capacitor and inductor are connected and $X_C > X_L$, so V_{AB} must lag behind the current by 90° which is possible only when we have potential difference across points is given as

$$V_{AB} = 6\cos(100\pi t - 53^\circ) \qquad \ldots(2)$$

Please note that from equations (1) and (2), we see that I is leading V_{AB} by a phase angle of 90°.
Now, according to the problem, we have

$$V_{AB} = \frac{1}{2}E$$

$$\Rightarrow \quad 6\cos(100\pi t - 53^\circ) = 5\cos 100\pi t \qquad \ldots(3)$$

$$\Rightarrow \quad 6\left[\frac{3}{5}\cos(100\pi t) + \frac{4}{5}\sin(100\pi t)\right] = 5\cos(100\pi t)$$

$$\Rightarrow \quad \frac{24}{5}\sin(100\pi t) = \frac{7}{5}\cos(100\pi t)$$

$$\Rightarrow \quad \tan(100\pi t) = \frac{7}{24}$$

Since we know that, $1 + \tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta}$

$$\Rightarrow \quad \cos(100\pi t) = \frac{24}{25}$$

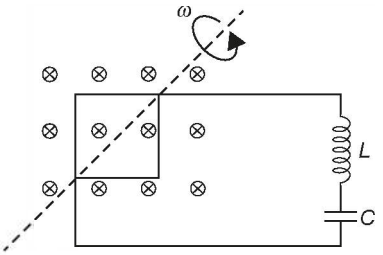
From equation (3), we get

$$V_{AB} = 5\cos(100\pi t) = 5\left(\frac{24}{25}\right) \text{ V}$$

$$\Rightarrow \quad V_{AB} = \frac{24}{5} \text{ V}$$

PROBLEM 11

In the given arrangement the square loop of area $10\ \text{cm}^2$ rotates with an angular velocity ω about its diagonal. The loop is connected to a inductance of $L = 100\ \text{mH}$. I and a capacitance of 10 mF in series. The lead wires have a net resistance of $10\ \Omega$. Given that $B = 0.1\ \text{T}$ and $\omega = 63\ \text{rads}^{-1}$, find the



(a) rms current in the circuit.
(b) energy dissipated in 50 sec.
(c) if the current is in phase with voltage, what should be the frequency of rotation of the coil.

SOLUTION

Since, $\phi = BA\cos(\omega t)$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = BA\omega\sin(\omega t)$$

(a) The current in the circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}, \text{ where } V_{\text{rms}} = \frac{\xi_0}{\sqrt{2}} = \frac{BA\omega}{\sqrt{2}}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{\frac{BA\omega}{\sqrt{2}}}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{(0.1)(10^{-3})(63)/\sqrt{2}}{\sqrt{\left(63\times 0.1 - \frac{1}{63\times 0.01}\right)^2 + (10)^2}}$$

$$\Rightarrow \quad I_{\text{rms}} = 4\times 10^{-4}\ \text{A}$$

(b) The energy dissipated in time interval t is

$$E = P_{av}t = V_{\text{rms}}I_{\text{rms}}t\cos\phi$$

$$\Rightarrow \quad E = \left(\frac{B^2A^2\omega^2}{2Z}\right)\left(\frac{R}{Z}\right)(50)$$

$$\Rightarrow \quad E = \frac{(0.1)^2(10^{-3})^2(63)^2(10)(50)}{2\left[\left(63\times 0.1 - \frac{1}{63\times 0.01}\right)^2 + (10)^2\right]}$$

$$\Rightarrow \quad E = 8.12\times 10^{-5}\ \text{J}$$

(c) Since the current is in phase with the voltage which happens only at resonance, so we have

$$X_L = X_C$$

$$\Rightarrow \quad L\omega_0 = \frac{1}{C\omega_0}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

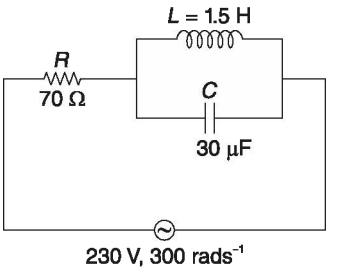
$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{(0.1)(0.01)}} = 31.6\ \text{rads}^{-1}$$

PROBLEM 12

A circuit with $R = 70\ \Omega$ in series with a parallel combination of $L = 1.5\ \text{H}$ and $C = 30\ \mu\text{F}$ is driven by a $230\ \text{V}$ ac supply of angular frequency $300\ \text{rads}^{-1}$. Calculate the impedance of the circuit. Find the RMS value of current. Calculate the current amplitude in the L and C arms of the circuit. How will the circuit behave when $\omega = \frac{1}{\sqrt{LC}}$?

SOLUTION

The situation described in the problem is shown in Figure.



In the given circuit, the inductor and capacitor are connected in parallel. Let Z' be their complex ac impedance. Then

$$\frac{1}{Z'} = \frac{1}{Z_L} + \frac{1}{Z_C}, \text{ where}$$

$$Z_L = j(L\omega) \text{ and } Z_C = -\frac{j}{C\omega} = \frac{1}{jC\omega}$$

$$\Rightarrow \quad \frac{1}{Z'} = \frac{1}{jL\omega} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{j\omega L} + j\omega C$$

$$\Rightarrow \quad \frac{1}{Z'} = \frac{1-\omega^2 LC}{jL\omega} \qquad \{\because j^2 = -1\}$$

$$\Rightarrow \quad Z' = \frac{jL\omega}{1-\omega^2 LC}$$

The total complex ac impedance of the circuit is given by

$$Z_{net} = Z = R + Z'$$

$$\Rightarrow \quad Z = R + \frac{j\omega L}{1-\omega^2 LC}$$

$$\Rightarrow \quad |Z| = \sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}$$

Substituting these values and solving, we get

$$|Z| = 163.3\ \Omega$$

$$\Rightarrow \quad I_{rms} = \frac{E_{rms}}{Z} = \frac{230\ \text{V}}{163.3\ \Omega} = 1.41\ \text{A}$$

Let I_L and I_C be the rms values of current in L and C respectively.

$$I_L = \left(\frac{Z_C}{Z_L + Z_C}\right) I_{rms} \text{ and } I_C = \left(\frac{Z_L}{Z_L + Z_C}\right) I_{rms}$$

where, $Z_L = jL\omega$ and $Z_C = \frac{1}{jC\omega} = -jC\omega$

$$\Rightarrow \quad I_L = \left(\frac{1}{1-\omega^2 LC}\right) I_{rms},\ I_C = \left(\frac{\omega^2 LC}{\omega^2 LC - 1}\right) I_{rms}$$

Substituting the values and solving, we get

$$I_L = 0.462\ \text{A} \text{ and } I_C = 1.87\ \text{A}$$

So, the corresponding current amplitudes are

$$(I_L)_0 = \sqrt{2} \times 0.462 = 1.414 \times 0.462 = 0.653\ \text{A}$$

and $(I_C)_0 = \sqrt{2} \times 1.87 = 1.414 \times 1.87 = 2.64\ \text{A}$

When $\omega = \frac{1}{\sqrt{LC}}$, then $\omega^2 LC = 1$ and hence

$$Z = R + \frac{j\omega L}{1-\omega^2 LC} \to \infty$$

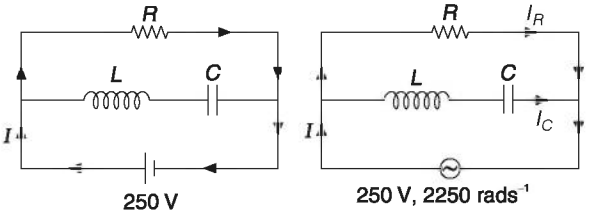
So, the current flowing in the circuit is zero.

PROBLEM 13

A box contains an inductor of inductance L, a capacitor of capacitance C and a resistor of resistance R. When a 250 V dc is applied to the terminals of the box, a current of 1 A flows in the circuit. When a 250 V, 2250 rads^{-1} ac source is connected, a current of 1.25 A flows. It is observed that the current rises with frequency and becomes maximum at 4500 rads^{-1}. Find the values of L, C and R. Also draw the circuit diagram.

SOLUTION

Since, the capacitor is a dc blocking element, but still the current in the circuit for a dc source is not zero so, all the elements must not be series. Further in case of ac, current rises with frequency and has a maximum value at 4500 rads^{-1}, L and C should be in series. The circuit diagram should therefore, be as shown in Figure.



CASE I: When dc is applied CASE II: When ac is applied

CASE-1:

When dc is applied, we have

$$R = \frac{V}{I} = \frac{250}{1} = 250\ \Omega$$

CASE-2:

When AC is applied, then for $\omega = 2250$ rads^{-1}, let the applied voltage be $V = V_0 \sin(\omega t)$. Then,

$$I_R = \frac{V_0}{R}\sin(\omega t)$$

Since $V_{rms} = 250$ volt, so $V_0 = 250\sqrt{2}$ volt

$$I_R = \frac{250\sqrt{2}}{250}\sin(\omega t) = \sqrt{2}\sin(\omega t)$$

and $I_L = I_C = \dfrac{V_0}{X_C \sim X_L}\sin\left(\omega t \pm \dfrac{\pi}{2}\right)$

$\Rightarrow \quad I_L = \left(\dfrac{V_0}{X_C \sim X_L}\right)\cos(\omega t)$

where, the sign '$\sim$' indicates positive difference between X_L and X_C

So, the total current through the circuit at any instant is

$$I = I_R + I_C = \frac{V_0}{R}\sin(\omega t) + \frac{V_0}{X_C \sim X_L}\cos(\omega t)$$

$\Rightarrow \quad I = I_0 \sin(\omega t + \phi)$

where $I_0 = V_0\sqrt{\dfrac{1}{R^2} + \dfrac{1}{(X_C \sim X_L)^2}}$

$\Rightarrow \quad I_{\text{rms}} = V_{\text{rms}}\sqrt{\dfrac{1}{R^2} + \dfrac{1}{(X_C \sim X_L)^2}}$

Given that, $V_{\text{rms}} = 250$ volt, $I_{\text{rms}} = 1.25$ A

$\Rightarrow \quad \dfrac{1}{R^2} + \dfrac{1}{(X_C \sim X_L)^2} = \left(\dfrac{1.25}{250}\right)^2$

$\Rightarrow \quad \dfrac{1}{(X_C \sim X_L)^2} = \left(\dfrac{1.25}{250}\right)^2 - \left(\dfrac{1}{250}\right)^2$

$\Rightarrow \quad X_C \sim X_L = \dfrac{1000}{3}\Omega$

$\Rightarrow \quad \dfrac{1}{\omega C} \sim \omega L = \dfrac{1000}{3}\Omega \qquad \ldots(1)$

Current in the circuit will be maximum at

$$X_L = X_C$$

$\Rightarrow \quad L\omega_0 = \dfrac{1}{C\omega_0}$

$\Rightarrow \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow \quad LC = \dfrac{1}{\omega_0^2} = \dfrac{1}{(4500)^2} \qquad \ldots(2)$

Solving equations (1) and (2) with $\omega = 2250$ rads^{-1}, we get $C = 10^{-6}$ F $= 1\,\mu$F and $L = 0.049$ H

$\Rightarrow \quad C = 1\,\mu$F, $L = 49$ mH and $R = 250\,\Omega$

PROBLEM 14

A solenoid with inductance $L = 7$ mH and active resistance $R = 44\,\Omega$ is first connected to a source of direct voltage V_0 and then to a source of sinusoidal voltage with effective value $V = V_0$. At what frequency of the oscillator will the power consumed by the solenoid be $\eta = 5.0$ times less than in the former case?

SOLUTION

For the case of direct current flowing through the circuit, we have

$$P_{\text{dc}} = \frac{V_0^2}{R}$$

When connected across an ac source of effective voltage V_0 (i.e. $V_{\text{rms}} = V_0$), the impedance of this series LR circuit is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$\Rightarrow \quad I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$\Rightarrow \quad P_{\text{ac}} = I_{\text{rms}}^2 R = \dfrac{V_0^2 R}{R^2 + \omega^2 L^2}$

According to problem, the power consumed by the solenoid is $\eta = 5.0$ times less than the power consumed when solenoid is connected across dc power. So, we have

$$P_{\text{dc}} = \eta P_{\text{ac}}, \text{ where } \eta = 5$$

$\Rightarrow \quad \dfrac{V_0^2}{R} = \eta\dfrac{V_0^2 R}{R^2\left(1 + \left(\dfrac{\omega L}{R}\right)^2\right)}$

$\Rightarrow \quad 1 + \left(\dfrac{\omega L}{R}\right)^2 = \eta$

$\Rightarrow \quad \dfrac{\omega L}{R} = \sqrt{\eta^2 - 1}$

$\Rightarrow \quad \omega = \dfrac{R}{L}\sqrt{\eta^2 - 1}$

$\Rightarrow \quad 2\pi f = \dfrac{R}{L}\sqrt{\eta^2 - 1}$

$\Rightarrow \quad f = \dfrac{R}{2\pi L}\sqrt{\eta^2 - 1} = 2$ kHz

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. An alternating voltage $V = 200\sqrt{2}\sin(100t)$, where V is in volt and t in seconds, is connected to a series combination of $1\ \mu F$ capacitor and $10\ k\Omega$ resistor through an ac ammeter. The reading of the ammeter will be

(A) $\sqrt{2}$ mA (B) $10\sqrt{2}$ mA

(C) 2 mA (D) 20 mA

2. An LCR series circuit containing a resistance of $120\ \Omega$ has angular resonance frequency 4×10^5 rads^{-1}. At resonance the voltage across resistance and inductance are 60 V and 40 V respectively. The values of L and C are

(A) 0.2 mH, $\frac{1}{32}\ \mu F$ (B) 0.4 mH, $\frac{1}{16}\ \mu F$

(C) 0.2 mH, $\frac{1}{16}\ \mu F$ (D) 0.4 mH, $\frac{1}{32}\ \mu F$

3. A transformer is used to light a 140 W, 24 V bulb from a 240 V a.c. mains. The current in the main cable is 0.7 A. The efficiency of the transformer is

(A) 63.8% (B) 83.3%

(C) 16.7% (D) 36.2%

4. In an AC circuit, twice the impedance is $\sqrt{3}$ times the reactance, then the phase angle is

(A) 60° (B) 30°

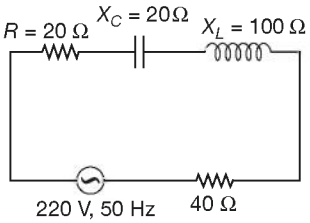
(C) ZERO (D) $\phi = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

5. An alternating voltage, given by $V = 300\sqrt{2}\sin(50t)$ is connected across a $1\ \mu F$ capacitor through an AC ammeter. The reading of the ammeter is

(A) 10 mA (B) 15 mA

(C) 40 mA (D) 100 mA

6. The power factor of the circuit shown in the figure is



(A) 0.2 (B) 0.4

(C) 0.6 (D) 0.8

7. If L and R be the inductance and resistance respectively for an ideal choke coil, then identify the correct statement.

(A) L is very high compared to R

(B) R is very high compared to L

(C) Both L and R are high

(D) Both L and R are low

8. The tuning circuit of a radio receiver has a resistance of $50\ \Omega$, an inductor of 10 mH and a variable capacitor. A 1 MHz radio wave produces a potential difference of 0.1 mV. The value of the capacitor to produce resonance is $(\text{Take } \pi^2 = 10)$

(A) 2.5 pF (B) 5.0 pF

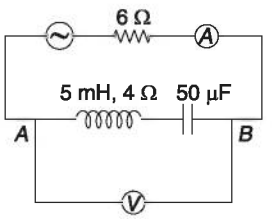
(C) 25 pF (D) 50 pF

9. In PROBLEM 8, the current at resonance is

(A) $1\ \mu A$ (B) $1.5\ \mu A$

(C) $2\ \mu A$ (D) $4\ \mu A$

10. An AC source of voltage $V = 20\cos(2000t)$ having negligible resistance is applied across the circuit shown in Figure. The voltmeter and ammeter readings are



(A) 0 V, 2 A (B) 0 V, 1.4 A

(C) 5.6 V, 1.4 A (D) 8 V, 2 A

11. When 100 V dc is applied across a coil, a current of 1 A flows through it. When 100 V ac of 50 Hz is applied across the same coil, only 0.5 A flows. The resistance and inductance of the coil are $(\text{Take } \pi^2 = 10)$

(A) $50\ \Omega$, 0.3 H (B) $50\ \Omega$, $\sqrt{0.3}$ H

(C) $100\ \Omega$, 0.3 H (D) $100\ \Omega$, $\sqrt{0.3}$ H

12. A generator produces a time varying voltage given by $V = 240\sin 120t$, where t is in second. The rms voltage and frequency are
(A) 170 V and 19 Hz (B) 240 V and 60 Hz
(C) 170 V and 60 Hz (D) 120 V and 19 Hz

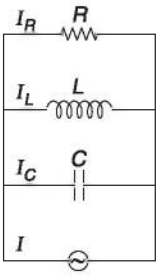
13. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.02 H. The power consumed is 3 kW and the power factor is 0.75. The impedance of the circuit is
(A) 0.144 Ω (B) 1.44 Ω
(C) 14.4 Ω (D) 144 Ω

14. In PROBLEM 13, the resistance of the coil B is
(A) 0.58 Ω (B) 5.8 Ω
(C) 1.16 Ω (D) 11.6 Ω

15. In PROBLEM 13, the inductance of coil A is
(A) 0.01 H (B) 0.02 H
(C) 0.03 H (D) 0.04 H

16. Two coils have mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin\omega t$ where $I_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf in the second coil (in volt) is
(A) 2π (B) 5π
(C) π (D) 4π

17. In a parallel LCR circuit as shown in Figure.



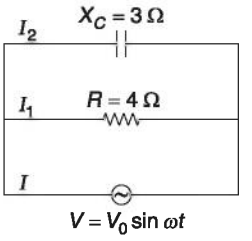
If I_R, I_L, I_C and I represent the rms values of current flowing through resistor, inductor, capacitor and the source respectively, then select the correct option.
(A) $I = I_R + I_L + I_C$
(B) $I = I_R + I_L - I_C$
(C) $I = \sqrt{I_R^2 + I_L^2 + I_C^2}$
(D) I_L or I_C may be greater than I

18. An LCR series circuit contains $L = 8$ H, $C = 0.5\ \mu$F and $R = 100\ \Omega$. The resonant frequency of the circuit is
(A) $\frac{1000}{\pi}$ Hz (B) $\frac{500}{\pi}$ Hz
(C) $\frac{250}{\pi}$ Hz (D) $\frac{125}{\pi}$ Hz

19. In a series LCR circuit connected across an ac input, the current in the circuit will
(A) decrease if R is increased
(B) decrease if L is increased
(C) increase if C is increased
(D) decrease if C is increased

20. An inductive coil has a resistance of 100 Ω. When an ac signal of frequency 1000 Hz is applied to the coil, the voltage leads the current by 45°. The inductance of the coil is
(A) $\frac{1}{10\pi}$ (B) $\frac{1}{20\pi}$
(C) $\frac{1}{40\pi}$ (D) $\frac{1}{60\pi}$

21. A capacitor and resistor are connected with an AC source as shown in Figure.



The capacitive reactance is 3 Ω and the resistance of resistor is 4 Ω. The phase difference between currents I and I_1 is

Given that $\sin^{-1}\left(\frac{3}{5}\right) = 37°$

(A) 90° (B) ZERO
(C) 53° (D) 37°

22. An ideal inductor takes a current of 10 A when connected to a 125 V, 50 Hz ac supply. A pure resistor across the same source takes 12.5 A. If the two are connected in series across a $100\sqrt{2}$ V, 40 Hz supply, the current through the circuit will be
(A) 10 A (B) 12.5 A
(C) 20 A (D) 25 A

23. Voltage applied to an AC circuit and the current flowing in it are respectively given by

$$V = 200\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and}$$

$$i = -\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

The power consumed in the circuit is

(A) 200 W (B) 400 W

(C) $200\sqrt{2}$ W (D) None of these

24. If $E = 8\sin(\omega t) + 6\sin(2\omega t)$, then the rms values is

(A) $5\sqrt{2}$ V (B) $7\sqrt{2}$ V

(C) 10 V (D) $10\sqrt{2}$ V

25. An AC voltage is applied across a series combination of L and R. If the voltage drop across the resistor and inductor are 20 V and 15 V respectively, then applied peak voltage is

(A) 25 V (B) 35 V

(C) $25\sqrt{2}$ V (D) $5\sqrt{7}$ V

26. The number of turns of primary and secondary coils of a transformer are 5 and 10 respectively and the mutual inductance of the transformer is 25 H. If the number of turns in the primary and secondary are made 10 and 5 respectively, then the mutual inductance of the transformer will be

(A) 6.25 H (B) 12.5 H

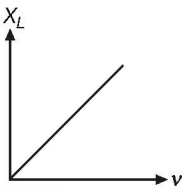
(C) 25 H (D) 50 H

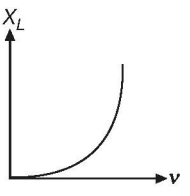
27. An ideal transformer is used to step up an alternating emf 220 V to 4.4 kV to transmit 6.6 kW power. The current rating the secondary is

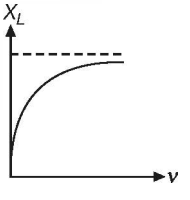
(A) 30 A (B) 3 A

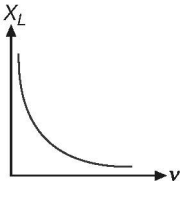
(C) 1.5 A (D) 1 A

28. Which one of the following represents the variation of inductive reactance (X_L) with the frequency of the voltage source?

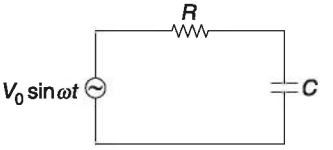
(A) 

(B) 

(C) 

(D) 

29. An AC voltage source $V = V_0 \sin \omega t$ is connected across resistance R and capacitance C as shown in Figure.



It is given that the resistance R equals the capacitive reactance and the peak current is I_0. If the angular frequency of the voltage source is changed to $\frac{\omega}{\sqrt{3}}$, then the new peak current in the circuit is

(A) $\frac{I_0}{2}$ (B) $\frac{I_0}{\sqrt{2}}$

(C) $\frac{I_0}{\sqrt{3}}$ (D) $\frac{I_0}{3}$

30. If resistance in an ac circuit is increased, then its power factor

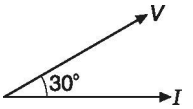
(A) decreases

(B) increases

(C) remains same

(D) decreases and becomes zero

31. Phasor diagram of a series AC circuit is shown in Figure. Then the circuit



(A) must be containing resistor and capacitor only.

(B) must be containing resistor and inductor only

(C) must be containing all three elements L, C and R

(D) cannot have only capacitor and inductor

32. Hot wire ammeters can be used for measuring

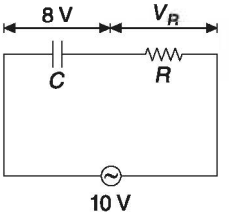
(A) alternating current only

(B) direct current only

(C) both alternating and direct current

(D) neither alternating nor direct current

33. In a series CR circuit shown in Figure, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. The voltage across R and the phase difference between current and the applied voltage will respectively be



(A) 6 V, $\tan^{-1}\left(\frac{4}{3}\right)$ (B) 3 V, $\tan^{-1}\left(\frac{3}{4}\right)$

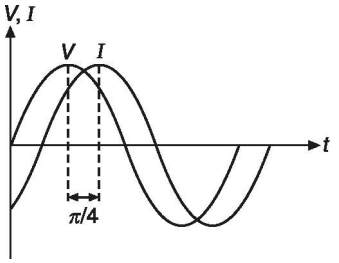
(C) 6 V, $\tan^{-1}\left(\frac{3}{4}\right)$ (D) None of these

34. An inductance, a capacitance and a resistance are connected in series across a source of alternating voltage. At resonance, the applied voltage and the current flowing through the circuit will have a phase difference of

(A) $\frac{\pi}{4}$ (B) ZERO

(C) π (D) $\frac{\pi}{2}$

35. An AC voltage $V = V_0 \sin 100t$ is applied to the circuit for which the phase difference between current and voltage is found to be $\frac{\pi}{4}$. The best suitable combination for satisfying the given condition is



(A) $R = 100\ \Omega$, $C = 1\ \mu F$

(B) $R = 1\ k\Omega$, $C = 10\ \mu F$

(C) $R = 10\ k\Omega$, $L = 1\ H$

(D) $R = 1\ k\Omega$, $L = 10\ H$

36. The average emf during the positive half cycle of an a.c. supply of peak value E_0 is

(A) $\frac{E_0}{\pi}$ (B) $\frac{E_0}{\sqrt{2}\pi}$

(C) $\frac{E_0}{2\pi}$ (D) $\frac{2E_0}{\pi}$

37. When an alternating voltage of 220 V is applied across a device P, a current of 0.25 A flows through the circuit and it leads the applied voltage by an angle $\frac{\pi}{2}$ radian. When the same voltage source is connected across another device Q, the same current is observed in the circuit but in phase with the applied voltage. What is the current when the same source is connected across a series combination of P and Q?

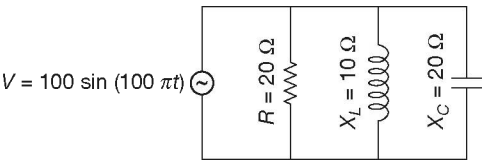
(A) $\frac{1}{4\sqrt{2}}$ A lagging in phase by $\frac{\pi}{4}$ with voltage

(B) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(C) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(D) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\frac{\pi}{2}$ with voltage

38. In the given circuit, the current drawn from the source is



(A) 20 A (B) 10 A

(C) 5 A (D) $5\sqrt{2}$ A

39. In a step-down transformer the input voltage is 22 kV and the output voltage 550 V. The ratio of the number of turns in the secondary to that in the primary

(A) 1 : 20 (B) 20 : 1

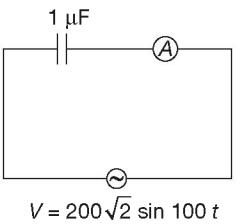
(C) 1 : 40 (D) 40 : 1

40. In an LCR series circuit, the capacitance is changed from C to $4C$. For the same resonant frequency, the inductance should be changed from L to

(A) $2L$ (B) $\frac{L}{2}$

(C) $\frac{L}{4}$ (D) $4L$

41. In the circuit shown in Figure, the reading of the AC ammeter is



(A) $20\sqrt{2}$ mA (B) $40\sqrt{2}$ mA

(C) 20 mA (D) 40 mA

42. An ideal transformer steps down 220 V to 22 V in order to operate a device with an impedance of 220 Ω. The current in the primary is

(A) 0.01 A (B) 0.1 A

(C) 0.5 A (D) 1.0 A

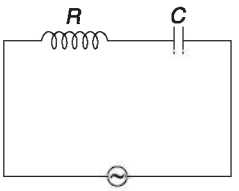
43. The frequency of an alternating current is 50 Hz. The minimum time taken by it in reaching from zero to peak value is

(A) 5 ms (B) 10 ms
(C) 20 ms (D) 50 ms

44. In a noiseless transformer an alternating current of 2 A is flowing in the primary coil. The number of turns in the primary and secondary coil are 100 and 20 respectively. The value of the current in the secondary coil is

(A) 0.08 A (B) 0.4 A
(C) 5 A (D) 10 A

45. A 50 Hz AC source of 20 V is connected across R and C as shown in Figure. If the voltage across R is 12 V, then the voltage across C is



(A) 8 V (B) 16 V
(C) 10 V (D) 14 V

46. If ϕ is phase difference between current and voltage, the wattless component of current is

(A) $I\cos\phi$ (B) $I\sin\phi$
(C) $I\tan\phi$ (D) $I\cos^2\phi$

47. In a series LCR circuit, current in the circuit is 11 A when the applied voltage is 220 V. Voltage across the capacitor is 200 V. If the value of resistor is 20 Ω, then the voltage across the unknown inductor is

(A) ZERO (B) 200 V
(C) 20 V (D) None of these

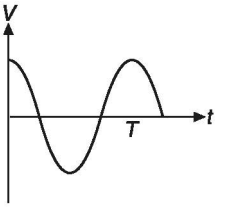
48. A steady potential difference of 10 V produces heat at a rate x in a resistor. The peak value of the alternating voltage which will produce heat at a rate $\frac{x}{2}$ in the same resistor is

(A) 5 V (B) $5\sqrt{2}$ V
(C) 10 V (D) $10\sqrt{2}$ V

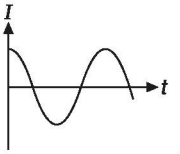
49. A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If coil is connected to a DC battery of emf 12 V and internal resistance 4 Ω, then current through it in steady state is

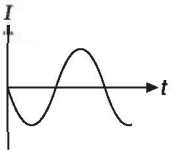
(A) 2.4 A (B) 1.8 A
(C) 1.5 A (D) 1.2 A

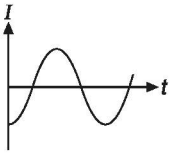
50. The voltage across a pure inductor is represented by the graph shown in Figure.

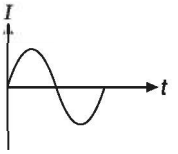


Which one of the following diagrams will represent the current?

(A) 

(B) 

(C) 

(D) 

51. An alternating voltage is applied across the series combination of resistor and an inductor. If the applied voltage is $V = 220\sin 120t$ and current flowing in the circuit is $I = 4\sin(120t - 60°)$. The power consumed in the circuit is

(A) ZERO (B) 100 W
(C) 220 W (D) 440 W

52. A resistance R, an inductance L and a capacitance C are connected in series across an ac source of angular frequency ω. If the resonant frequency is ω_0 then the current will lag behind the voltage if

(A) $\omega < \omega_0$ (B) $\omega > \omega_0$
(C) $\omega = \omega_0$ (D) $\omega = 0$

53. The impedance of a circuit consists of 3 Ω resistance and 4 Ω reactance. The power factor of the circuit is

(A) 0.4 (B) 0.6
(C) 0.8 (D) 1.0

54. In series LCR circuit, voltage drop across resistance is 8 V, across inductor is 6 V and across capacitor is 12 V. Then,

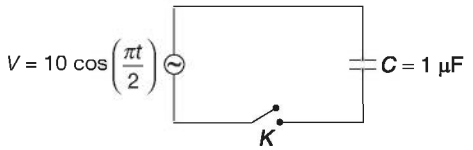
(A) voltage of the source will be leading in the circuit.
(B) voltage drop across each element will be less than the applied voltage.
(C) power factor of the circuit will be $\frac{3}{4}$.
(D) current leads the voltage.

55. An ideal inductor of $\left(\frac{1}{\pi}\right)$ H is connected in series with a $300\ \Omega$ resistor. If a 20 V, 200 Hz ac source is connected across the combination, the phase difference between the voltage and the current is

(A) $\tan^{-1}\left(\frac{5}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{5}\right)$

(C) $\tan^{-1}\left(\frac{3}{4}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

56. An AC voltage source described by $V = 10\cos\left(\frac{\pi t}{2}\right)$ is connected to a $1\ \mu F$ capacitor as shown in Figure. The key K is closed at $t = 0$. The time $(t > 0)$ after which the magnitude of current i reaches its maximum value for the first time is



(A) 1 s (B) 2 s

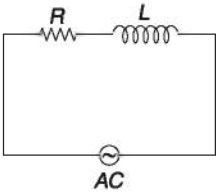
(C) 3 s (D) 4 s

57. A low-loss transformer has 230 V applied to the primary and gives 4.6 V in the secondary. The secondary is connected to a load which draws 5 A of current. The current (in ampere) in the primary is

(A) 0.1 (B) 1.0

(C) 10 (D) 250

58. A circuit contains resistance R and an inductance L in series. An alternating voltage $V = V_0 \sin\omega t$ is applied across it. The currents in R and L respectively will be



(A) $I_R = I_0\cos\omega t$, $I_L = I_0\cos\omega t$

(B) $I_R = -I_0\sin\omega t$, $I_L = I_0\cos\omega t$

(C) $I_R = I_0\sin\omega t$, $I_L = -I_0\cos\omega t$

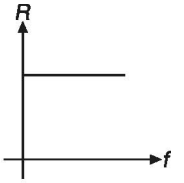
(D) $I_R = I_0\sin\omega t$, $I_L = I_0\cos\omega t$

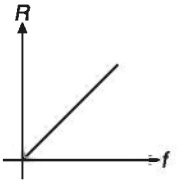
59. The rate of heat production in a resistor due to an alternating current of rms value 10 A is same as that due to a direct current of

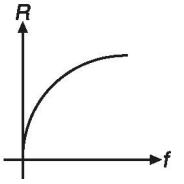
(A) 10 A (B) $\frac{10}{\sqrt{2}}$ A

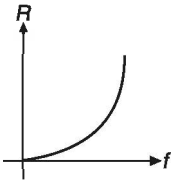
(C) $10\sqrt{2}$ A (D) 5 A

60. The correct variation of resistance R with frequency f is given by

(A) 

(B) 

(C) 

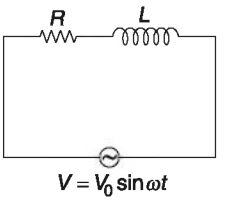
(D) 

61. A 220 V main supply is connected to a resistance of $100\ k\Omega$. The effective current is

(A) 2.2 mA (B) $2.2\sqrt{2}$ mA

(C) $\frac{2.2}{\sqrt{2}}$ mA (D) None of the above

62. The power consumed in the circuit shown in Figure, is



(A) zero (B) $\frac{V_0^2}{2R}$

(C) $\frac{V_0^2 R}{2(R^2+\omega^2L^2)}$ (D) $\frac{V_0^2 R}{2\sqrt{R^2+\omega^2L^2}}$

63. In a transformer, N_P and N_S are 1000 and 3000 respectively. If the primary is connected across 80 V A.C., the potential difference across each turn of the secondary will be

(A) 240 V (B) 0.24 V

(C) 0.8 V (D) 0.08 V

64. A. C. power is transmitted from a power house at a high voltage as

(A) the rate of transmission is faster at high voltages

(B) it is more economical due to less power loss

(C) power cannot be transmitted at low voltages
(D) a precaution against theft of transmission lines

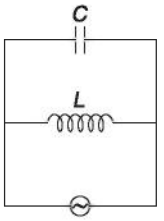
65. An LCR series circuit has a maximum current of 5 A. If $L = 0.5$ H and $C = 8\ \mu$F, then the angular frequency of AC voltage is

(A) 500 rads^{-1} (B) 5000 rads^{-1}
(C) 400 rads^{-1} (D) 250 rads^{-1}

66. The electric current in a circuit is given by $I = 4t$, where, t is in second and I in ampere. The rms current for the period $t = 0$ to $t = 2$ s is

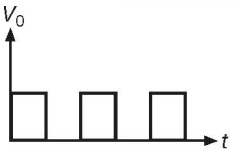
(A) 3 A (B) 4 A
(C) $\frac{8}{\sqrt{3}}$ A (D) $8\sqrt{3}$ A

67. In the AC network shown in Figure, the respective rms current flowing through the inductor and capacitor are 0.6 A and 0.8 A. The current supplied by the ac source is



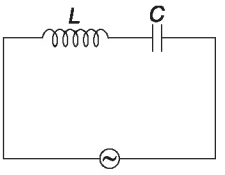
(A) 1.0 A (B) 1.4 A
(C) 0.2 A (D) 2.2 A

68. The rms value of the potential difference shown is



(A) V_0 (B) $\frac{V_0}{\sqrt{2}}$
(C) $\frac{V_0}{2}$ (D) $2V_0$

69. In a series LC circuit, the applied voltage is V_0. If ω is very low, then the voltage drop across the inductor V_L and capacitor V_C are

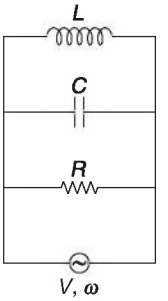


(A) $V_L = \frac{V_0}{2}$, $V_C = \frac{V_0}{2}$ (B) $V_L = 0$, $V_C = V_0$
(C) $V_L = V_0$, $V_C = 0$ (D) $V_L = -V_C = \frac{V_0}{2}$

70. In a step-up transformer, the turns ratio of primary and secondary is 1 : 2. A Laclanche cell of emf 1.5 V is connected across the primary. The voltage developed across the secondary would be

(A) ZERO (B) 3.0 V
(C) 1.5 V (D) 0.75 V

71. An AC source of peak value V_0 and angular frequency ω is applied across an LCR parallel circuit shown in Figure. The peak value of current through the AC source is



(A) $\frac{V_0}{\sqrt{\frac{1}{R^2} + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ (B) $V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

(C) $\frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ (D) $V_0\left(\frac{1}{R} + \omega C - \frac{1}{\omega L}\right)$

72. An inductor of 1 H is connected across a 220 V, 50 Hz supply. The peak value of the current is approximately

(A) 0.5 A (B) 0.7 A
(C) 1 A (D) 1.4 A

73. If a current $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ flows in a circuit across which an alternating potential $E = E_0 \sin \omega t$ has been applied, then the power consumed in the circuit is

(A) $\frac{E_0 I_0}{\sqrt{2}}$ (B) $\frac{E_0 I_0}{2}$
(C) $\frac{EI}{\sqrt{2}}$ (D) ZERO

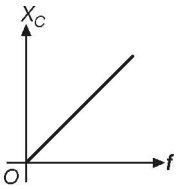
74. A steady current of magnitude I and an AC current of peak value I are allowed to pass through identical resistors for the same time. The ratio of heat produced in the two resistors will be

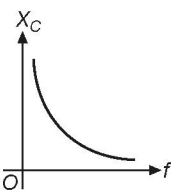
(A) 2 : 1 (B) 1 : 2
(C) 1 : 1 (D) None of these

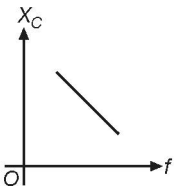
75. An AC source producing emf $V = V_0[\sin(\omega t) + \sin(2\omega t)]$ is connected in series with a capacitor and a resistor. The current found in the circuit is $I = I_1 \sin(\omega t + \phi_1) + I_2 \sin(2\omega t + \phi_2)$. Then, we have

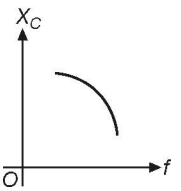
(A) $I_1 = I_2$ (B) $I_1 = \frac{I_2}{2}$
(C) $I_1 < I_2$ (D) $I_1 > I_2$

76. Identify the graph which correctly represents the variation of capacitive reactance X_C with frequency

(A) 

(B) 

(C) 

(D) 

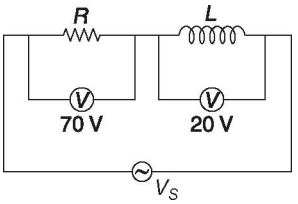
77. An LCR series circuit consists of a resistance of 10 Ω, a capacitance of reactance 60 Ω and an inductor coil. The circuit is found to resonate when put across a 300 V, 100 Hz supply. The inductance of the coil is (Take $\pi = 3$).

(A) 0.1 H (B) 0.01 H
(C) 0.2 H (D) 0.02 H

78. In PROBLEM 77, the current in the circuit at resonance is

(A) 10 A (B) 15 A
(C) 30 A (D) 60 A

79. The figure shows an AC circuit with resistance R, inductance L and source voltage V_s. Then,



(A) the source voltage $V_s = 72.8$ V
(B) the phase angle between current and source voltage is $\tan^{-1}\left(\frac{7}{2}\right)$
(C) Both (A) and (B) are correct
(D) Both (A) and (B) are wrong

80. A 60 μF capacitor, 0.3 H inductor and a 50 Ω resistor are connected in series with a 120 V, 60 Hz source. The current in the circuit approximately is

(A) 1.5 A (B) 2 A
(C) 3 A (D) 4 A

81. In PROBLEM 80, the power dissipated is

(A) 178 W (B) 89 W
(C) 56.25 W (D) 112.5 W

82. In an ac circuit, the rms value of the current, I_{rms}, is related to the peak current I_0 as

(A) $I_{rms} = \frac{I_0}{\pi}$ (B) $I_{rms} = \frac{I_0}{\sqrt{2}}$
(C) $I_{rms} = \sqrt{2} I_0$ (D) $I_{rms} = \pi I_0$

83. An alternating emf given by $V = V_0 \sin(\omega t)$ has peak value 10 V and frequency 50 Hz. The instantaneous emf at $t = \frac{1}{600}$ s is

(A) 10 V (B) $5\sqrt{3}$ V
(C) 5 V (D) 1 V

84. A step-up transformer connected to 220 V ac line is to supply 22 kV for a neon sign in secondary circuit. In primary circuit, a fuse wire is connected which is to blow when the current in the secondary circuit exceeds 10 mA. The turn ratio of the transformer is

(A) 50 (B) 100
(C) 150 (D) 200

85. A complex current wave function is represented by the equation is given by $i = (6 + 5\sin 100\omega t)$ A. Its average value over one time period is

(A) 12 A (B) 6 A
(C) $\sqrt{61}$ A (D) Zero

86. In an ac circuit $V(\text{in volt}) = 100\sin(100t)$ and $I(\text{in mA}) = 100\sin\left(100t + \frac{\pi}{3}\right)$. The power dissipated in the circuit is

(A) 10^4 W (B) 10 W
(C) 2.5 W (D) 5 W

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. In an LCR circuit, the resistance R, inductance L and capacitance C are connected in series. An ac voltage $V = V_0 \sin(\omega t)$ is fed to this circuit. The frequency ω of the source can be varied, theoretically from zero (i.e. extremely small values) to infinity (i.e. extremely large values). If $\langle P \rangle$ is the average power consumed in the circuit, then select the correct statement(s).

(A) $\langle P \rangle \approx 0$ for $\omega_0 \to \infty$

(B) $\langle P \rangle$ is independent of ω.

(C) $\langle P \rangle = \dfrac{V_0^2}{2R}$ for $\omega^2 LC = 1$

(D) $\langle P \rangle = \dfrac{V_0^2}{4R}$ for frequencies that differ in value by $\dfrac{R}{L}$

2. In an AC circuit, the power factor

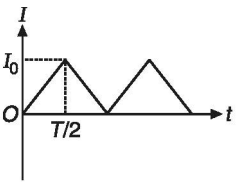
(A) is unity when the circuit is purely resistive.

(B) is unity when the circuit is purely inductive.

(C) is zero when the circuit is purely capacitive.

(D) is 0.5 when the difference of inductive reactance and capacitive reactance is 1.732 times the resistance.

3. The current in a certain circuit varies with time as shown. The peak value of current is I_0. If i_v and i_m represent the virtual (rms) and mean value of current for a complete cycle respectively, then



(A) $i_m = \dfrac{2i_0}{\pi}$ (B) $i_m = \dfrac{i_0}{2}$

(C) $i_{rms} = \dfrac{i_0}{\sqrt{3}}$ (D) $i_{rms} = \dfrac{i_0}{\sqrt{2}}$

4. A tube light of 60 V, 60 W rating is connected across an AC source of 100 V and 50 Hz frequency. Then

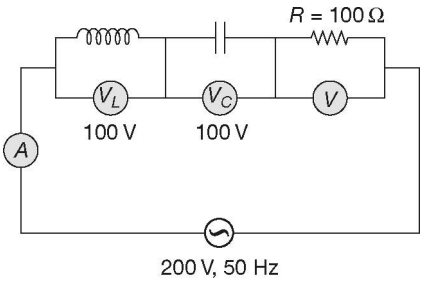
(A) an inductance of $\dfrac{4}{5\pi}$ H may be connected in series.

(B) a capacitor of $\dfrac{250}{\pi}$ μF may be connected in series to it.

(C) an inductor of $\dfrac{2}{5\pi}$ H may be connected in series

(D) a resistance of 40 Ω may be connected in series

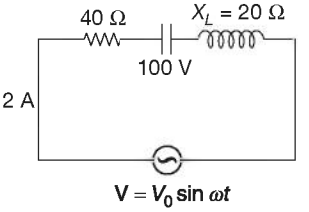
5. In the circuit shown if V_R and A be the readings of the voltmeter and ammeter, then



(A) $V_R = 300$ V (B) $A = 1$ A

(C) $V_R = 200$ V (D) $A = 2$ A

6. For the circuit shown in Figure, labelled with the specific values, select the correct statements.



(A) The voltage across the resistor is 80 V

(B) The capacitive reactance is 50 Ω

(C) The voltage across the inductor is 40 V

(D) The peak value of the applied voltage is 100 V

7. During resonance, in a series LCR circuit, the power

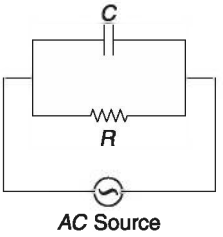
(A) factor is one

(B) factor is zero

(C) consumed across inductor is zero

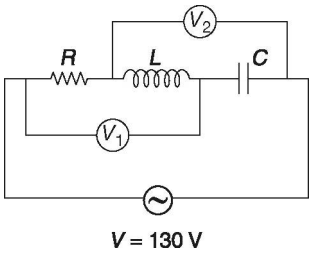
(D) consumed across resistor is $\dfrac{I_0^2 R}{2}$

8. A 200 V, 5kHz AC source is applied to the circuit having a capacitor $C = \dfrac{1}{\pi}$ μF and a resistor $R = 100$ Ω as shown in Figure. Then, select the correct statement(s).



(A) The current through the resistor is 2 A
(B) The current through the capacitor is 0.126 A
(C) Total current supplied by ac source is ≈ 0.283 A
(D) Current in both the branches is same

9. In a series LCR circuit shown in Figure, the readings of voltmeters V_1 and V_2 are 100 V and 120 V. Select the correct statement(s).



(A) Voltage across resistor, inductor and capacitor are 50 V, 86.6 V and 206.6 V respectively.
(B) Voltage across resistor, inductor and capacitor are 10 V, 90 V and 30 V respectively.
(C) Power factor of the circuit is $\frac{5}{13}$.
(D) Circuit is capacitive in nature.

10. In a series *LCR* circuit, the rms voltage supply is 170 V. If V_R, V_L and V_C represent the rms voltage drop across resistor, inductor and capacitor, then
(A) $V_R \le 170$ V (always)
(B) $V_L \le 170$ V (always)
(C) V_L or V_C may be greater than 170 V
(D) $|V_L - V_C| < 170$

11. In an AC series circuit, $R = 20\ \Omega$, $X_L = 20\ \Omega$ and $X_C = 40\ \Omega$. Then, select the correct option(s).
(A) The given values are at frequency less than the resonance frequency.
(B) The given values are at frequency more than the resonance frequency.
(C) If frequency is increased from the given value, impedance of the circuit will decrease.
(D) If frequency is increased from the given value, current in the circuit may increase or decrease.

12. Consider three *LC* circuits, first with capacitance *C*, inductance *L*, second with capacitance $\frac{C}{2}$, inductance $2L$ and third with capacitance $2C$, inductance $\frac{L}{2}$. All the three capacitors are charged to the same potential *V* and then made to oscillate. The respective frequencies of the circuits are f_1, f_2 and f_3. Also the currents in the circuits have respective maximum values I_1, I_2 and I_3. Then
(A) $f_1 = f_2 = f_3$ (B) $f_1 = f_2 = \frac{f_3}{\sqrt{2}}$
(C) $I_1 = I_2 < I_3$ (D) $I_2 < I_1 < I_3$

13. Current in an *AC* circuit is given by $i = 3\sin\omega t + 4\cos\omega t$, then
(A) rms value of current is $2.5\sqrt{2}$ A
(B) mean value of this current in positive one-half period will be 3.2 A
(C) if voltage applied is $V = V_0 \sin\omega t$, then the circuit may contain resistance and capacitance
(D) if voltage applied is $V = V_0 \cos\omega t$, then the circuit may contain resistance and inductance only

14. In an AC series circuit, $R = 10\ \Omega$, $X_L = 20\ \Omega$ and $X_C = 10\ \Omega$. Then, select the correct option(s).
(A) Voltage function will lead the current function.
(B) Total impedance of the circuit is $10\sqrt{2}\ \Omega$.
(C) Phase angle between voltage function and current function is $45°$.
(D) Power factor of circuit is $\frac{1}{\sqrt{2}}$.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** In series LCR circuit resonance can take place.
Statement-2: Resonance takes if inductive and capacitive reactance are equal.

2. **Statement-1:** AC source is connected across a circuit. Power dissipated in circuit is P. The power is dissipated only across resistance.
Statement-2: Inductor and capacitor will not consume any power in AC circuit.

3. **Statement-1:** In a LC circuit, the charge on the capacitor oscillates simple harmonically.
Statement-2: The total energy in a LC circuit is a constant.

4. **Statement-1:** Average value of a.c. over a complete cycle is always zero.
Statement-2: Average value of a.c. is always defined over half cycle.

5. **Statement-1:** Capacitor serves as a block for d.c. and offers an easy path to a.c.
Statement-2: Capacitive reactance is inversely proportional to frequency.

6. **Statement-1:** When frequency is greater than resonance frequency in a series LCR circuit, it will be an inductive circuit.
Statement-2: Resultant voltage will lead the current.

7. **Statement-1:** A capacitor blocks direct current in the steady state.
Statement-2: The capacitive reactance of the capacitor is inversely proportional to frequency f of the source of emf.

8. **Statement-1:** In the purely resistive element of a series LCR AC circuit, the maximum value of r.m.s. current increases with increase in the angular frequency of the applied e.m.f.
Statement-2: $I_{max}=\frac{E_{max}}{Z}$, $Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$, where I_{max} is the peak current in a cycle.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension I

A 25 μF capacitor, a 0.10 H inductor and a 25 Ω resistor are connected in series with an AC source whose emf is given by $E=310\sin(314t)$ volt. Based on the above facts, answer the following questions.

1. The frequency of the emf is
(A) 25 cps (B) 50 cps
(C) 75 cps (D) 100 cps

2. The reactance of the circuit is
(A) 95.9 Ω (B) 65.7 Ω
(C) 127.3 Ω (D) 31.4 Ω

3. The impedance of the circuit is
(A) 79.1 Ω (B) 89.1 Ω
(C) 99.1 Ω (D) 100.1 Ω

4. The current in the circuit is
(A) 1.21 A (B) 2.21 A
(C) 3.21 A (D) 4.21 A

5. The phase angle of the current by which it leads the applied emf is
(A) 75.4° (B) 30°
(C) 60° (D) 90°

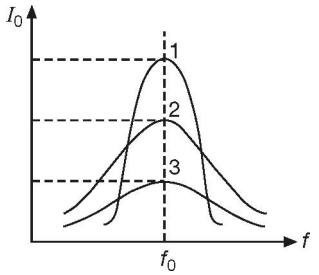
6. The expression for the instantaneous value of current in the circuit is
(A) $3.13\sin(314t+1.32)$ (B) 2 A
(C) $3.13\cos(314t+1.32)$ (D) $\sqrt{2}A$

7. The effective voltages across the capacitor, the inductor and the resistor are
(A) 281.3 V, 69.4 V, 55.3 V
(B) 0, 0, 0
(C) 3 V, 2 V, 6 V
(D) None of these

8. The value of inductance that will make the impedance of circuit to be minimum is
(A) 0.11 H (B) 0.21 H
(C) 0.31 H (D) 0.41 H

Comprehension 2

The figure represents variation of peak current I_0 with applied frequency f of the AC source of three different LCR circuits having different resistances. The value of inductance L and capacitance C are same for all the three circuits. Based on the above facts, answer the following questions.



9. If R_1, R_2 and R_3 be the resistance of circuit 1, 2 and 3 respectively, then

(A) $R_1 > R_2 > R_3$ (B) $R_1 < R_2 < R_3$

(C) $R_1 > R_2 = R_3$ (D) $R_1 = R_2 = R_3$

10. If $R_1 = 1\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 10\ \Omega$ and $L = \frac{900}{\pi}$ mH, $C = \frac{40}{\pi}\ \mu$F, then the value of f_0 is

(A) 250 Hz (B) 125 Hz

(C) $\frac{250}{6}$ Hz (D) $\frac{250}{3}$ Hz

11. A capacitor of capacitance C is charged to a potential V and then connected to an inductor of inductance L in a closed circuit. Choose the correct statement

(A) There is no current in the circuit, initially

(B) The current will flow through circuit until the capacitor gets fully discharged

(C) The current in the circuit will be very high since there is no resistance in the circuit

(D) There will always be flow of current in the circuit

12. In context to Question No. 37, the maximum current that flows through the inductor is

(A) $V\sqrt{\frac{C}{L}}$

(B) $V\sqrt{\frac{L}{C}}$

(C) $\frac{1}{2\pi}V\sqrt{\frac{C}{L}}$

(D) There is no flow of current

13. In context to Question No. 31, the frequency with which energy oscillates between Electric Field Energy and Magnetic Field Energy is

(A) $\frac{1}{4\pi\sqrt{LC}}$

(B) $\frac{1}{\pi\sqrt{LC}}$

(C) $\frac{1}{2\pi\sqrt{LC}}$

(D) the energy in the electric field does not oscillate

Comprehension 3

A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\ \Omega$, $L = 25.48$ mH, $C = 796\ \mu$F. Based on the above facts, answer the following questions.

14. The rms voltage is

(A) 50 V (B) 100 V

(C) 200 V (D) 250 V

15. X_L is

(A) 2 Ω (B) 4 Ω

(C) 6 Ω (D) 8 Ω

16. X_C is

(A) 2 Ω (B) 4 Ω

(C) 6 Ω (D) 8 Ω

17. Z is

(A) 2 Ω (B) 3 Ω

(C) 4 Ω (D) 5 Ω

18. The peak current is

(A) 56.6 A (B) 26.6 A

(C) 16.2 A (D) 7.2 A

19. The phase angle is

(A) 43.13° (B) 53.13°

(C) 60° (D) 90°

20. The rms current is

(A) 10 A (B) 20 A

(C) 30 A (D) 40 A

21. The rms voltage across capacitor is

(A) 120 V (B) 160 V

(C) 320 V (D) 0 V

22. The rms voltage across inductor is

(A) 120 V (B) 160 V

(C) 320 V (D) 0 V

23. The rms voltage across resistor is
(A) 120 V (B) 160 V
(C) 320 V (D) 0 V

24. The power dissipated in the circuit is
(A) 4800 W (B) 240 W
(C) 120 W (D) 12 W

25. The power factor is
(A) 0.1 (B) 0.4
(C) 0.5 (D) 0.6

26. The power input is
(A) 4800 W (B) 240 W
(C) 120 W (D) 12 W

27. The resonance frequency is
(A) 35.4 Hz (B) 45 Hz
(C) 50 Hz (D) 60 Hz

28. Value of Z at resonance is
(A) 1 Ω (B) 2 Ω
(C) 3 Ω (D) 4 Ω

29. Current at resonance is
(A) 66.67 A (B) 1 A
(C) 0 A (D) ∞

30. The power consumed at resonance is
(A) 13.33 kW (B) 23.33 kW
(C) 4 kW (D) 7 kW

Comprehension 4

In an AC series RC circuit, the voltage applied V and the current I are given by

$$V(t)=170\sin\left(6280t+\frac{\pi}{3}\right)\text{ V}$$

$$I(t)=8.5\sin\left(6280t+\frac{\pi}{2}\right)\text{ A}$$

31. The resistance of the circuit is
(A) 10.32 Ω (B) 17.32 Ω
(C) 6.32 Ω (D) 25.32 Ω

32. The capacitance of the circuit will be
(A) 15.92 μΩ (B) 10.92 μΩ
(C) 13 μΩ (D) 50.92 μΩ

33. The power factor will be given by
(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

Comprehension 5

A series LCR circuit with $L=0.12$ H, $C=480$ nF, $R=23$ Ω is connected to a 230 V variable frequency supply. Based on the above facts, answer the following questions.

34. The source frequency for which current amplitude is maximum is
(A) 263 Hz (B) 363 Hz
(C) 463 Hz (D) 663 Hz

35. The current amplitude is
(A) 7.1 A (B) 8.1 A
(C) 11.1 A (D) 14.1 A

36. What is the source frequency for which average power absorbed by the circuit is maximum?
(A) 263 Hz (B) 363 Hz
(C) 463 Hz (D) 663 Hz

37. The maximum average power is
(A) 186 W (B) 1186 W
(C) 2286 W (D) 3386 W

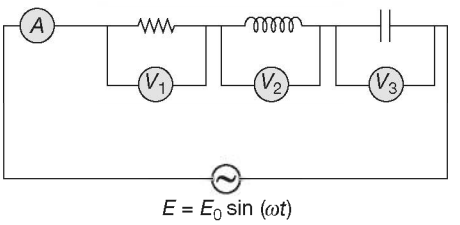
38. The frequencies of the source for which the power transferred to the circuit is half the power at resonant frequency are
(A) 648 Hz, 678 Hz (B) 630 Hz, 640 Hz
(C) 330 Hz, 440 Hz (D) 1 Hz, 2 Hz

39. The current amplitude at half the peak power points is
(A) 2 A (B) 6 A
(C) 8 A (D) 10 A

40. The Q-factor of the circuit is
(A) 21.7 (B) 11.7
(C) 9.7 (D) 7.7

Comprehension 6

For the circuit shown in figure a voltage of $E=E_0\sin(\omega t)$ is applied. The voltmeter readings are $V_1=100$ V, $V_2=125$ V, $V_3=150$ V and ammeter reading is 5 A.



41. The net impedance of circuit is
(A) $5\sqrt{37}$ Ω (B) $5\sqrt{26}$ Ω
(C) $5\sqrt{17}$ Ω (D) $5\sqrt{29}$ Ω

42. The power factor of circuit is

(A) $\frac{4}{\sqrt{17}}$ (B) $\frac{4}{\sqrt{29}}$

(C) $\frac{4}{\sqrt{26}}$ (D) $\frac{4}{\sqrt{37}}$

43. The value of E_0 is

(A) $25\sqrt{17}$ V (B) $20\sqrt{17}$ V

(C) $20\sqrt{34}$ V (D) $25\sqrt{34}$ V

Comprehension 7

A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 volt, 50 Hz supply. The resistance of the circuit is negligible. Based on the above facts, answer the following questions.

44. The current amplitude is

(A) 7.6 A (B) 8.6 A

(C) 9.6 A (D) 11.6 A

45. The rms value of current is

(A) 5.2 A (B) 6.2 A

(C) 7.2 A (D) 8.2 A

46. The rms value of potential drop across inductor is

(A) 106 V (B) 206 V

(C) 306 V (D) 406 V

47. The rms value of potential drop across capacitor is

(A) 135.3 V (B) 235.3 V

(C) 335.3 V (D) 435.3 V

48. The applied rms voltage is

(A) 229.3 V (B) 119.3 V

(C) 19.3 V (D) 9.3 V

49. The average power transferred to the inductor is

(A) 0 W (B) 1 W

(C) 4 W (D) 19.7 W

50. The average power transferred to the capacitor is

(A) 0 W (B) 1 W

(C) 4 W (D) 19.7 W

51. The total average power absorbed by the circuit is

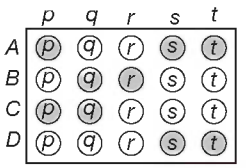
(A) 0 W (B) 1 W

(C) 4 W (D) 19.7 W

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

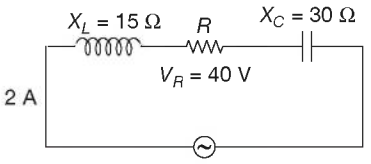


1. An LCR series circuit has a current which lag behind the applied voltage by ϕ. The voltage across the inductance has a maximum value equal to twice the maximum of the voltage across the capacitor. $E_L = 30\sin(100t)$. If $R = 20\ \Omega$, then match the items given in **COLUMN-I** with that in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Reactance of capacitor if $\phi = 45°$.	(p) 120 Ω
(B) Reactance of inductor if $\phi = 45°$.	(q) $20\sqrt{3}$ Ω
(C) Impedance of circuit if $\phi = 45°$.	(r) 20 Ω
(D) Reactance of circuit if $\phi = 60°$.	(s) 40 Ω
	(t) $20\sqrt{2}$ Ω

(Continued)

2. For the circuit shown, match descriptions in **COLUMN-I** with the respective values in SI units given in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) The resistance is	(p) 20
(B) Voltage across the capacitor	(q) 30
(C) Voltage across the inductor	(r) 50
(D) Applied voltage	(s) 60

3. Match the elements given in **COLUMN-I**, with their respective current phase relation given in **COLUMN-II** for an AC input applied across each element or the combination of elements.

COLUMN-I	COLUMN-II
(A) Ideal capacitor	(p) current leads voltage
(B) Non ideal inductor	(q) voltage leads current
(C) LCR circuit for the minimum impedance	(r) $\phi = 90°$
(D) Non ideal capacitor	(s) $\phi = 0°$

4. For a series LCR circuit connected to an AC source, match the variations in **COLUMN-I** to the effects in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) If resistance is increased	(p) current will increase
(B) If capacitance is increased	(q) current will decrease
(C) If inductance is increased	(r) current may increase or decrease
(D) If frequency is increased	(s) power may decrease or increase

5. An AC voltage $V = V_0 \sin \omega t$ is applied across a circuit. Corresponding to this applied voltage, match the currents in **COLUMN-I** to the respective circuit combinations in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $I = I_0 \sin \omega t$	(p) only R circuit
(B) $I = -I_0 \cos \omega t$	(q) only L circuit
(C) $I = I_0 \sin\left(\omega t + \frac{\pi}{6}\right)$	(r) may be CR circuit
(D) $I = I_0 \sin\left(\omega t - \frac{\pi}{3}\right)$	(s) may be LR circuit
	(t) may be LCR circuit

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

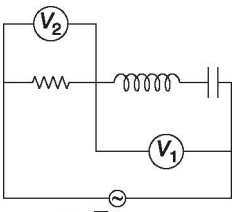
In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. In a series LCR circuit where $R = 300\ \Omega$, $L = 0.8$ H, and $C = 0.04\ \mu$F, the source has voltage amplitude $V = 300$ V and a frequency equal to the resonance frequency of the circuit.
   (a) What is the power factor?
   (b) What is the average power delivered, in watt, by the source?
   (c) The capacitor is replaced by one with $C = 0.08\ \mu$F, and the source frequency is adjusted to the new resonance value. What is then the average power delivered, in watt, by the source?

2. Consider a series LR circuit in which $L = \frac{1}{\pi} H$ and resistance $R = 100\ \Omega$. When the circuit is connected to a 220 V, 50 Hz a.c. source, calculate the current drawn in ampere from the source.

3. A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V, 50 rads^{-1} ac source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil in millihenry. Also find the power developed in the circuit, closest to two-digit integer in watt, if a 2500 μF capacitor is connected in series with the coil.

4. In the circuit shown, the reading of voltmeter V_1 is 40 V. Calculate the reading of voltmeter V_2 in volt.



5. A fixed inductance $L = 2\ \mu$H is used in series with a variable capacitor in the tuning section of a radiotelephone on a ship. What capacitance, in pF, tunes the circuit to the signal from a transmitter broadcasting at 5 MHz ? Take $\pi^2 \cong 10$.

6. A series LCR circuit having a resistance of $100\sqrt{5}\ \Omega$ is connected to a 200 V ac source. When the capacitor is removed from the circuit, the current lags behind voltage by $45°$. When the inductor is removed from the circuit keeping the capacitor and resistor in the circuit, the current leads the voltage by an angle of $\tan^{-1}\left(\frac{1}{2}\right)$. Calculate the current (in ampere) in the LCR circuit and the average power dissipated (in watt) in LCR circuit.

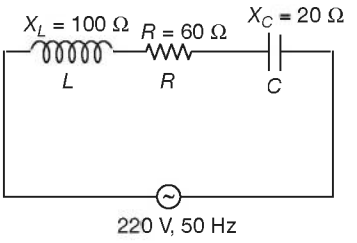
7. For a series LCR circuit with $L = 2$ H, $C = 1\ \mu$F and $R = 1\ \text{k}\Omega$, connected across an ac input given by $V = 100\sqrt{2}\sin(1000t)$ V, find the voltage across L, C and R, rounded off to the nearest integer, in volt.

8. A coil of inductance 0.4 mH is connected to a capacitor of capacitance 400 pF. The wave length for which this circuit is tuned is found to be $(*)\pi$, where $*$ is not readable. Calculate $*$.

9. A $10\ \Omega$ resistor, 10 mH inductor and $100\ \mu$F capacitor are connected in series to a 50 V (rms) source having variable frequency. Find the energy, in millijoule, that is delivered to the circuit during one period if the operating frequency is twice the resonance frequency.

10. When a coil is connected to a 100 V DC supply, the current in it is 2 A. When the same coil is connected to an AC source $V = 100\sqrt{2}\sin\omega t$, the current is 1 A. Find the inductive reactance (in ohm) used in the circuit.

11. For the circuit shown in Figure, calculate the power factor.



12. A series LCR circuit having resistor of $120\ \Omega$ has a resonant frequency of 4000 rads^{-1}. At resonance, the voltage across resistance and inductance is 60 V and 40 V respectively. Calculate the value of inductance (in millihenry) and capacitance (in microfarad) in the circuit.

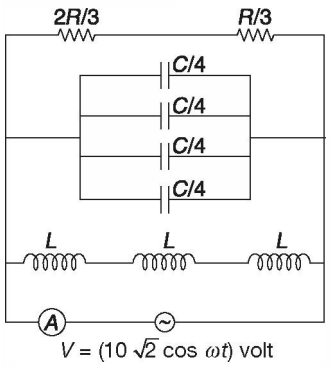
13. A $12\ \Omega$ resistor and an inductor of inductance $\frac{0.05}{\pi}$ H having negligible resistance are connected in series. Across the end of this circuit a 130 V, 50 Hz ac source is connected. Calculate the current (in ampere) in the circuit, the potential difference across resistance and inductance in volt.

14. A series LCR circuit is connected across an ac source $E = 10\sin\left(100\pi t - \frac{\pi}{6}\right)$ and the current in the circuit is observed to be $I = 2\sin\left(100\pi t + \frac{\pi}{12}\right)$. Calculate the average power (in watt) dissipated in the circuit.

15. Calculate the capacitance of the capacitor (in microfarad) required to be connected in series with a 30 V, 10 W bulb to run it when connected across a 220 V, 50 Hz ac.

16. A light bulb has the rating 200 W, 220 V. Find the resistance of the bulb (in ohm) and the rms value of current flowing through the bulb (in ampere).

17. In the circuit shown, reactance of each capacitor is $4R$ and that of each inductor is $\frac{R}{3}$. If $R = 5\ \Omega$ then calculate the reading of ammeter, in ampere.



18. A capacitor, a resistor and a 40 mH inductor is connected in series to an ac source of frequency 50 Hz. Calculate the capacitance of the capacitor in μF, if the current is in phase with the voltage.

ARCHIVE: JEE MAIN

1. **[Online April 2019]**
An alternating voltage $V(t) = 220\sin(100\pi t)$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is
(A) 2.2 ms (B) 7.2 ms
(C) 5 ms (D) 3.3 ms

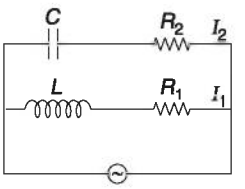
2. **[Online April 2019]**
A circuit connected to an ac source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?
(A) RL circuit with $R = 1$ kΩ and $L = 10$ mH
(B) RL circuit with $R = 1$ kΩ and $L = 1$ mH
(C) RC circuit with $R = 1$ kΩ and $C = 10$ μF
(D) RC circuit with $R = 1$ kΩ and $C = 1$ μF

3. **[Online April 2019]**
One kg of water, at 20 °C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to
[Specific heat of water $= 4200$ Jkg^{-1} °C^{-1}, Latent heat of water $= 2260$ kJkg^{-1}]
(A) 16 minute (B) 3 minute
(C) 22 minute (D) 10 minute

4. **[Online January 2019]**
A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of $\frac{24\text{ V}}{50\text{ Hz}}$. The energy dissipated in the circuit in 60 s is
(A) 5.65×10^2 J (B) 5.17×10^2 J
(C) 2.26×10^3 J (D) 3.39×10^3 J

5. **[Online January 2019]**
A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary winding of the transformer is 5 A and its efficiency is 90%, the output current would be
(A) 25 A (B) 50 A
(C) 45 A (D) 35 A

6. **[Online January 2019]**
In the circuit shown $C = \frac{\sqrt{3}}{2}$ μF, $R_2 = 20$ Ω, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10$ Ω. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volt. The phase difference between I_1 and I_2 is



(A) 0° (B) 60°
(C) 30° (D) 90°

7. **[2018]**
In an ac circuit, the instantaneous emf and current are given by $e = 100\sin 30t$, $i = 20\sin\left(30t - \frac{\pi}{4}\right)$. In one cycle of ac, the average power consumed by the circuit and the wattless current are, respectively

(A) $50, 10$ (B) $\frac{1000}{\sqrt{2}}, 10$

(C) $\frac{50}{\sqrt{2}}, 0$ (D) $50, 0$

8. [2018]

For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by

(A) $\frac{\omega_0 L}{R}$ (B) $\frac{\omega_0 R}{L}$

(C) $\frac{R}{\omega_0 C}$ (D) $\frac{CR}{\omega_0}$

9. [Online 2018]

An ideal capacitor of capacitance $0.2\ \mu F$ is charged to a potential difference of $10\ V$. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self-inductance $0.5\ mH$. The current at a time when the potential difference across the capacitor is $5\ V$, is

(A) $0.34\ A$ (B) $0.17\ A$

(B) $0.25\ A$ (C) $0.15\ A$

10. [Online 2018]

A power transmission line feeds input power at $2300\ V$ to a step down transformer with its primary windings having 4000 turns, giving the output power at $230\ V$. If the current in the primary of the transformer is $5\ A$ and its efficiency is 90%, the output current would be

(A) $50\ A$ (B) $25\ A$

(C) $45\ A$ (D) $20\ A$

11. [Online 2017]

A sinusoidal voltage of peak value $283\ V$ and angular frequency $320\ s^{-1}$ is applied to a series LCR circuit. Given that $R = 5\ \Omega$, $L = 25\ mH$ and $C = 1000\ \mu F$. The total impedance and phase difference between the voltage across the source and the current will respectively be

(A) $10\ \Omega$ and $\tan^{-1}\left(\frac{5}{3}\right)$ (B) $10\ \Omega$ and $\tan^{-1}\left(\frac{8}{3}\right)$

(C) $7\ \Omega$ and $\tan^{-1}\left(\frac{5}{3}\right)$ (D) $7\ \Omega$ and $45°$

12. [2016]

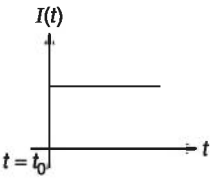
An arc lamp requires a direct current of $10\ A$ at $80\ V$ to function. If it is connected to a $220\ V$ (rms), $50\ Hz$ ac supply, the series inductor needed for it to work is close to

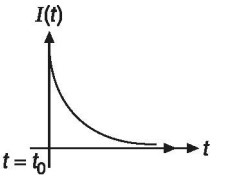
(A) $80\ H$ (B) $0.08\ H$

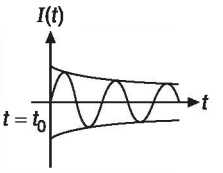
(C) $0.044\ H$ (D) $0.065\ H$

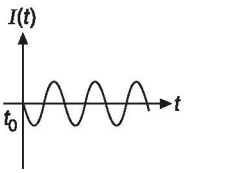
13. [Online 2016]

A series LR circuit is connected to a voltage source with $V(t) = V_0 \sin \omega t$. After very large time, current $I(t)$ behaves as $\left(t_0 \gg \frac{L}{R}\right)$

(A) 

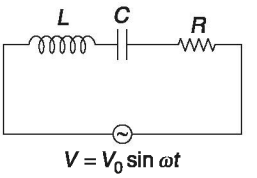
(B) 

(C) 

(D) 

14. [Online 2015]

For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in



(A) series with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(B) series with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

(C) parallel with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

(D) parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$.

15. [2010]

In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is $220\ V$ and $50\ Hz$ respectively. On taking out the capacitance from the circuit the current lags behind the voltage by $30°$. On taking out the inductor from the circuit the current leads the voltage by $30°$. The power dissipated in the LCR circuit is

(A) $242\ W$ (B) $305\ W$

(C) $210\ W$ (D) ZERO W

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

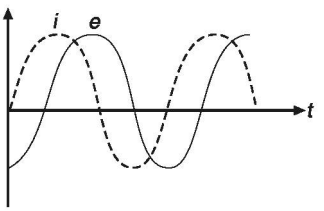
1. [IIT-JEE 2010]

An AC voltage source of variable angular frequency ω and fixed amplitude V connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedance of the circuit is unchanged
(D) total impedance of the circuit increases

2. [IIT-JEE 2003]

When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\frac{\pi}{4}$, as shown in Figure. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements



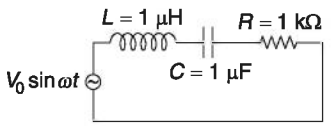
(A) $R = 1\ \text{k}\Omega,\ C = 10\ \mu\text{F}$ (B) $R = 1\ \text{k}\Omega,\ C = 1\ \mu\text{F}$
(C) $R = 1\ \text{k}\Omega,\ L = 10\ \text{H}$ (D) $R = 1\ \text{k}\Omega,\ L = 1\ \text{H}$

Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2017]

In the circuit shown, $L = 1\ \mu\text{H}$, $C = 1\ \mu\text{F}$ and $R = 1\ \text{k}\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(A) At $\omega \sim 0$, the current flowing through the circuit becomes nearly zero
(B) The frequency at which the current will be in phase with the voltage is independent of R
(C) The current will be in phase with the voltage if $\omega = 10^4\ \text{rads}^{-1}$
(D) At $\omega \gg 10^6\ \text{rads}^{-1}$, the circuit behaves like a capacitor

2. [JEE (Advanced) 2017]

The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin \omega t$,

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) $(V_{YZ})_{\text{rms}} = V_0\sqrt{\frac{1}{2}}$

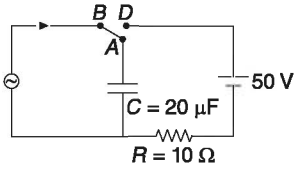
(B) $(V_{XY})_{\text{rms}} = V_0\sqrt{\frac{3}{2}}$

(C) independent of the choice of the two terminals

(D) $(V_{XY})_{\text{rms}} = V_0$

3. [JEE (Advanced) 2014]

At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500\ \text{rads}^{-1}$ starts flowing in it with the initial direction shown in the Figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu\text{F}$, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

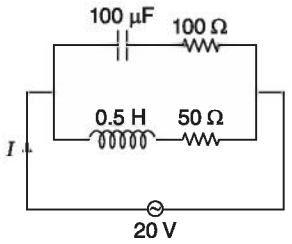
(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(C) Immediately after A is connected to D, the current in R is 10 A

(D) $Q = 2 \times 10^{-3}$ C

4. [IIT-JEE 2012]

In the given circuit, the AC source has $\omega = 100$ rads^{-1}. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)



(A) the current through the circuit, I is 0.3 A

(B) the current through the circuit, I is $0.3\sqrt{2}$ A

(C) the voltage across 100 Ω resistor $= 10\sqrt{2}$ V

(D) the voltage across 50 Ω resistor $= 10$ V

5. [IIT-JEE 2011]

A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the current and voltages mentioned are rms values.

1. [JEE (Advanced) 2013]

If the direct transmission method with a cable of resistance 0.4 Ω km^{-1} is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30

(C) 40 (D) 50

2. [JEE (Advanced) 2013]

In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1

(C) 100 : 1 (D) 50 : 1

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

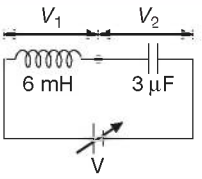
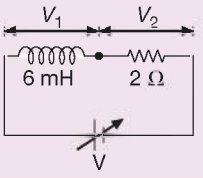
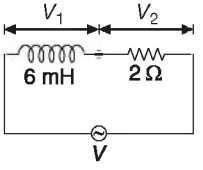
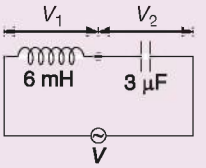
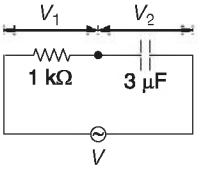
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

1. [IIT-JEE 2010]

You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source

of 50 Hz frequency (the next three circuits) in different ways as shown in **COLUMN II.** When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **COLUMN I.**

COLUMN-I	COLUMN-II
(A) $I \neq 0$, V_1 is proportional to I	(p) 
(B) $I \neq 0$, $V_2 > V_1$	(q) 
(C) $V_1 = 0$, $V_2 = V$	(r) 
(D) $I \neq 0$, V_2 is proportional to I	(s) 
	(t) 

(*Continued*)

Integer/Numerical Answer Type Questions

(In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s)).

1. [IIT-JEE 2011]

A series RC combination is connected to an ac voltage of angular frequency $\omega = 500$ rads^{-1}. If the impedance of the RC circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on AC)

2. 162 V

3. $I_{\text{rms}} = \frac{2k}{\sqrt{3}}$

4. $\frac{E_0}{\sqrt{2}}$

5. $V_{\text{rms}} = \sqrt{V_0^2 + \frac{V_1^2}{2}}$

6. (a) $I = (2\times10^{-2}\text{ A})\sin\left((10^3\text{ rads}^{-1})t\right)$

(b) $4\times10^3\ \Omega$

(c) $V_L = 80\sin\left\{(10^3\text{ rads}^{-1})t + \frac{\pi}{2}\right\}\text{ V}$

7. 72.3°, 60.67 A

8. (a) $\frac{1}{60}$ s, 377 rads^{-1}, 15.08 Ω, 25.05 Ω, 37°

(b) 6 A, 120 V, 90.5 V

9. 48.4 mH, 11 Ω

In case of ac, power loss in choke is zero.

In case of dc, the loss in additional resistance R' is 1100 W

10. (a) $I = 8.33\times10^{-3}\cos\left[(950\text{ rads}^{-1})t\right]$

(b) 760 Ω

(c) $V_L = -(6.33\text{ V})\sin\left((950\text{ rads}^{-1})t\right)$

11. (a) 326 mA

(b) 35.3°, voltage lagging behind current

(c) $V_R = 97.8$ V, $V_L = 32.6$ V, $V_C = 102$ V

12. (a) 628 Ω

(b) 6.37 mH

(c) 1.6 kΩ

(d) 1.6 mF

13. (a) 110.08 W

(b) 220 W

14. 20 A, $\frac{\pi}{4}$

15. $3R$

16. (a) 224 rads^{-1}

(b) 500 W

(c) 226 rads^{-1}, 221 rads^{-1}

17. (a) 1 kHz

(b) $\frac{\sqrt{3}}{2}$

(c) $R = 17.32\ \Omega$, $C = 16\ \mu\text{F}$

18. (a) 7.96×10^4 Hz

(b) $3.2\times10^6\ \Omega$

19. (a) 62.5 pF

(b) 84 mm

(c) 2.51 Ω

20. f_0

Test Your Concepts-II (Based on Transformer)

1. ZERO

2. (a) 83.3

(b) 54 mA

(c) 185 kΩ

3. 125 V

4. (a) 30 kW

(b) 6×10^{-3}

Single Correct Choice Type Questions

1. B	**2.** A	**3.** B	**4.** A	**5.** B	**6.** C	**7.** A	**8.** A	**9.** C	**10.** C
11. D	**12.** A	**13.** C	**14.** B	**15.** B	**16.** B	**17.** D	**18.** C	**19.** A	**20.** B
21. C	**22.** A	**23.** D	**24.** A	**25.** C	**26.** C	**27.** C	**28.** A	**29.** B	**30.** B
31. D	**32.** C	**33.** A	**34.** B	**35.** B	**36.** D	**37.** B	**38.** D	**39.** C	**40.** C
41. C	**42.** A	**43.** A	**44.** D	**45.** B	**46.** B	**47.** B	**48.** C	**49.** C	**50.** B

51. C	**52.** B	**53.** B	**54.** D	**55.** D	**56.** A	**57.** A	**58.** C	**59.** A	**60.** A
61. A	**62.** C	**63.** D	**64.** B	**65.** A	**66.** C	**67.** C	**68.** B	**69.** B	**70.** A
71. B	**72.** C	**73.** D	**74.** A	**75.** C	**76.** B	**77.** A	**78.** C	**79.** A	**80.** A
81. D	**82.** B	**83.** C	**84.** B	**85.** B	**86.** C				

Multiple Correct Choice Type Questions

1. A, C, D	**2.** A, C, D	**3.** B, C	**4.** A, D	**5.** C, D
6. A, B, C	**7.** A, C, D	**8.** A, C, D	**9.** A, C, D	**10.** A, C, D
11. A, C, D	**12.** A, D	**13.** A, B, C, D	**14.** A, B, C, D	

Reasoning Based Questions

1. A	**2.** A	**3.** B	**4.** B	**5.** A	**6.** A	**7.** A	**8.** D

Linked Comprehension Type Questions

1. B	**2.** A	**3.** C	**4.** B	**5.** A	**6.** A	**7.** A	**8.** D	**9.** B	**10.** D
11. D	**12.** A	**13.** C	**14.** C	**15.** D	**16.** B	**17.** D	**18.** A	**19.** B	**20.** D
21. B	**22.** C	**23.** A	**24.** A	**25.** D	**26.** A	**27.** A	**28.** C	**29.** A	**30.** A
31. B	**32.** A	**33.** C	**34.** D	**35.** D	**36.** D	**37.** C	**38.** A	**39.** D	**40.** A
41. C	**42.** A	**43.** D	**44.** D	**45.** D	**46.** B	**47.** D	**48.** A	**49.** A	**50.** A
51. A									

Matrix Match/Column Match Type Questions

1. A → (r)	B → (s)	C → (t)	D → (q)
2. A → (q)	B → (p)	C → (r)	D → (s)
3. A → (r)	B → (q)	C → (s)	D → (p)
4. A → (q, s)	B → (r, s)	C → (r, s)	D → (r, s)
5. A → (p, t)	B → (q)	C → (r, t)	D → (s, t)

Integer/Numerical Answer Type Questions

1. (a) 1, (b) 150, (c) 150	**2.** 1.56	**3.** 17	**4.** 30	
5. 500	**6.** 0.8, 143	**7.** Voltage across *L*, *C*, *R* (in volt) is 141, 71, 71	**8.** 240	
9. 242	**10.** 87	**11.** 0.6	**12.** 20, 3.125	**13.** 10, 120, 50
14. 7	**15.** 4.8	**16.** 242, 0.9	**17.** 2	**18.** 250

ARCHIVE: JEE MAIN

1. D	**2.** C	**3.** C	**4.** B	**5.** C	**6.** *	**7.** B	**8.** A	**9.** B	**10.** C
11. D	**12.** D	**13.** D	**14.** D	**15.** A					

* No given option is correct

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Single Correct Choice Type Problems

1. B **2.** A

Multiple Correct Choice Type Problems

1. A, B **2.** B, C **3.** C, D **4.** A, C **5.** B, C

Linked Comprehension Type Questions

1. B **2.** A

Matrix Match/Column Match Type Questions

1. A → (r, s, t) B → (q, r, s, t) C → (q, p) D → (q, r, s, t)

Integer/Numerical Answer Type Questions

1. 4

CHAPTER 5

Electromagnetic Waves

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Introduction and History of EM Waves
(b) Ampere Circuital Law and Concept of Displacement Current
(c) Maxwell's Equations
(d) Sources of Electromagnetic Waves
(e) Properties of EM Waves (Energy, Intensity, Momentum, Radiation Pressure, Poynting Vector)
(f) Electromagnetic Spectrum and its Properties

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION

In our daily routine, we come across a number of electromagnetic (EM) waves. The electromagnetic waves in the form of visible light enable us to view the world around us, the infrared waves warm our environment, the radio waves carry our favourite TV and radio programs and the list goes on and on.

As studied earlier that a time varying electric field produces a magnetic field and a time varying magnetic field produces an electric field. Maxwell first predicted that there exists a wave containing electric and magnetic fields, both varying with space and time and acting as sources of each other. Both electric and magnetic fields in this wave are mutually perpendicular and also perpendicular to the direction of wave-propagation.

In this chapter, we shall be studying the concept of displacement current along with various properties, uses and parts of electromagnetic waves.

HISTORICAL FACTS ABOUT ELECTROMAGNETIC WAVES

(a) In 1865 Maxwell predicted the presence of electromagnetic waves. He formulated the theory in terms of four Maxwell's equations which predict that the electromagnetic waves of all frequencies should propagate with the speed of light.

(b) Hertz in 1887 succeeded in producing and observing electromagnetic waves of wavelength of the order of 6 m in laboratory.

(c) J.C. Bose in 1894 succeeded in producing and observing electromagnetic waves of much shorter wavelength 25 mm to 5 mm.

(d) In laboratory, G. Marconi in same year succeeded in transmitting the electromagnetic waves up to a few kilometres. Marconi also discovered that if one of the spark gap terminals is connected to an antenna and the other terminal is earthed, the EM waves radiated could go up to several kilometres.

(e) The antenna and the earth wires form the two plates of a capacitor which radiate radio frequency waves. These waves could be received at a large distance by making use of an antenna earth system as detector.

(f) Using these arrangements, Marconi in 1899 first established wireless communication across the English Channel i.e. across a distance of about 50 km.

ELECTROMAGNETIC OSCILLATIONS

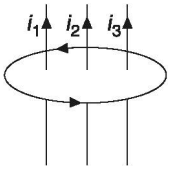
An accelerated charge produces electromagnetic field in the form of radiations. The frequency of oscillations, in LC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

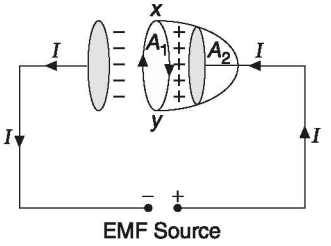
AMPERE CIRCUITAL LAW AND ITS CONTRADICTION

According to Ampere circuital law, the line integral of magnetic field along a closed loop is equal to μ_0 times the sum of steady currents threading the closed loop. So, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (i_1 + i_2 + i_3)$$



Let us consider a parallel plate capacitor which is being charged as shown in Figure.



If at an instant, charge on the capacitor is Q then the instantaneous value of current I in the connecting wire is

$$I = \frac{dQ}{dt}$$

Now, let us consider two surfaces A_1 and A_2 which are bound by a closed loop xy. The surface A_1 lies between the two plates of capacitor and A_2 lies outside the plates of capacitor. The area A_2 is pierced by the current I but the area A_1 is not pierced by this current. So, for the surface A_2, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

But for the surface A_1, we have

$$\oint \vec{B}\cdot d\vec{l} = 0$$

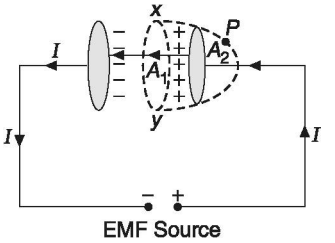
These results create an apparent contradiction in Ampere's Circuital Law (ACL) while applying it to the given situation.

CONCEPT OF DISPLACEMENT CURRENT

The displacement current I_d is a current which is produced due to the rate of change of electric flux with respect to time. Mathematically, the displacement current is given by

$$I_d = \varepsilon_0 \frac{d}{dt}(\phi_E)$$

According to Maxwell, the contradiction in Ampere's law is because of some missing term. This missing term must be such that we must get the same magnetic fields at point P, irrespective of the surface used. This missing term must be related with the changing electric field which passes through area A_1 between the plates of capacitor shown in Figure.



At any instant, if charge on the plates of capacitor is Q, area of plate is A, then electric field between the plates is uniform (but is zero outside the plates) and is given by

$$E = \frac{Q}{A\varepsilon_0} \qquad \ldots(1)$$

This field is perpendicular to area A_1, so the electric flux through area A_1 is

$$\phi_E = EA_1 = \left(\frac{Q}{A\varepsilon_0}\right)A$$

where, $A = A_1$
Also, according to Gauss Law, we have

$$\phi_E = \frac{Q}{\varepsilon_0} \qquad \ldots(2)$$

If charge on the capacitor plate changes with time then

$$I_d = \frac{dQ}{dt}$$

Using equation (2), we get

$$I_d = \frac{dQ}{dt} = \frac{d}{dt}(\varepsilon_0\phi_E)$$

$$\Rightarrow \quad I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

This current I_d (the missing term in the Ampere's law) passes through the surface A_1 and is known as Maxwell displacement current.
The following inferences can be drawn from the above discussion.

(a) The displacement current arises due to the rate of change of electric flux or the electric field between the plates of the capacitor.
(b) The displacement current is equal to the conduction current when both are present in different parts of the circuit.
(c) The conduction current and the displacement current are individually discontinuous, however the currents together possess the property of continuity through any closed electric circuit.
(d) Just like the conduction current, the displacement current acts as the source of varying magnetic field.

Conceptual Note(s)

(a) For consistency of Ampere circuital law, there must be a current between the plates of capacitor. It is called displacement (I_d) current because it is produced by changing electric field or electric displacement.

(b) The current in conductor is due to the flow of charge carrier and hence is called conduction current I_c.
(c) In **Modified Ampere Circuital Law**, the line integral of magnetic field along a closed loop in free space is equal to μ_0 times the total current (sum of conduction and displacement) threading the loop.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_c + I_d)$$

ILLUSTRATION 1

A parallel plate capacitor consists of two circular plates each of radius 0.1 m separated by a distance of 0.5 mm. If electric field between the capacitor plates changes at rate of 5×10^{13} $\text{Vm}^{-1}\text{s}^{-1}$, find the displacement current between the plates.

SOLUTION

Area of each capacitor plate is

$$A = \pi r^2 = 3.14\times(0.1)^2 \text{ m}^2$$

The rate of change of electric field is

$$\frac{dE}{dt} = 5\times10^{13} \text{ Vm}^{-1}\text{s}^{-1}$$

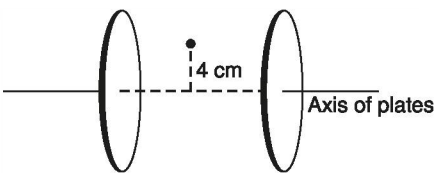
Since, $I_d = \frac{d\phi_E}{dt} = \varepsilon_0 A\left(\frac{dE}{dt}\right)$ $\{\because \phi_E = EA\}$

$$\Rightarrow \quad I_d = (8.85\times10^{-12})(3.14)(0.1)^2(5\times10^{13}) \text{ A}$$

$$\Rightarrow \quad I_d = 13.9 \text{ A}$$

ILLUSTRATION 2

Two circular plates each of radius 0.1 m are used to form a parallel plate capacitor. If displacement current between the plates is 2π ampere, then calculate the magnetic field produced by displacement current 4 cm from the axis of the plates.



SOLUTION

Current density of the displacement current is

$$J_d = \frac{2\pi}{\pi(0.1)^2} = \frac{2}{0.01} = 200 \text{ Am}^{-2}$$

Since, $J_d = \frac{I_d}{A}$

$\Rightarrow \quad i_d = J_d \times \pi(0.04)^2$

$\Rightarrow \quad i_d = 200 \times \pi \times 16 \times 10^{-4}$

$\Rightarrow \quad i_d = 32 \times \pi \times 10^{-2}$ A

$\Rightarrow \quad i_d \approx 1$ A

Since, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{enclosed}}$

$\Rightarrow \quad B \times 2\pi(0.04) = 4\pi \times 10^{-7} \times 1$

$\Rightarrow \quad B = \frac{2 \times 10^{-7}}{4 \times 10^{-2}} = 0.5 \times 10^{-5}$

$\Rightarrow \quad B = 5 \times 10^{-6}$ T

MAXWELL'S EQUATIONS

Maxwell's equations govern the basics laws of electricity and magnetism by giving the complete idea of all electromagnetic interactions. These equations relate the electric field E and magnetic field B to their sources which are electric charges and current respectively. Maxwell predicted the existence of electromagnetic waves on the basis of these equations. *Just like Newton's Laws being fundamental laws for Mechanics, these equations are same way fundamental equations for electromagnetics.*

In free space, the Maxwell's equations are

(a) Gauss Law For Electrostatics (Maxwell's First Equation)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

According to Gauss's Law for electrostatics, "the net electric flux associated with any closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface". This law relates electric field with the charge distribution. The Gauss law is in accordance with the Coulomb's inverse square law which has experimentally been confirmed.

(b) Gauss Law for Magnetism (Maxwell's Second Equation)

$$\oint \vec{B} \cdot d\vec{A} = 0$$

According to Gauss Law for magnetism, "the net magnetic flux associated with a closed surface is zero". This simply implies that the number of magnetic field lines entering the closed surface is equal to number of magnetic field lines leaving the closed surface due to which we donot have isolated magnetic monopoles.

(c) Faraday's Laws of EMI (Maxwell's Third Equation)

$$\oint \vec{E} \cdot d\vec{l} = \frac{d\phi_B}{dt}$$

According to Faraday's Laws of EMI, "the line integral of electric field along a closed path is equal to the rate of change of magnetic flux through the surface". This law relates electric field with a changing magnetic flux and vice versa. The induced current in a conducting loop placed in a time varying magnetic field confirms this equation.

(d) Ampere-Maxwell's Circuital Law (Maxwell's Fourth Equation)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

This is also called as Modified Ampere's Circuital Law, according to which "the line integral of magnetic field along a closed loop is equal to μ_0 times the total current threading the surface bound by the closed loop". This law describes how a magnetic field can be produced by both changing electric flux and a conduction current.

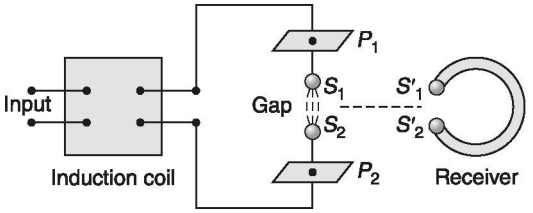
Problem Solving Technique(s)

Important consequence of the Maxwell's equations is that these can be used to derive the Law of Conservation of Charge.

EXPERIMENTAL SETUP FOR PRODUCING ELECTROMAGNETIC WAVES

Hertz experiment was based on the fact that when an oscillating charge is accelerating continuously, then it will radiate electromagnetic waves continuously.

The metallic plates P_1 and P_2 act as a capacitor and the wires connecting spheres S_1 and S_2 to the plates provide a low inductance as shown in Figure.



When a high voltage is applied across the metallic plates, then these plates get discharged by sparking across the narrow gap. This spark gives rise to oscillations which in turn send out electromagnetic waves having frequency given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The succession of sparks sends out a train of such waves which are received by the receiver.

SOURCES OF ELECTROMAGNETIC WAVES

The waves that are produced by accelerated charged particles and composed of electric and magnetic field vibrating transversely and sinusoidally perpendicular to each other and to the direction of propagation are called electromagnetic waves or electromagnetic radiations. These waves are produced in the following physical phenomena.

(a) An accelerating charge produces both electric field and magnetic field which varies with space and time which forms electromagnetic wave.

(b) An accelerating charge (in case of LC oscillation) emits electromagnetic wave of same frequency as frequency of accelerating charge (i.e., frequency of oscillating LC circuit)

(c) An electron orbiting in a stationary orbit around the nucleus does not emit an electromagnetic wave. However, during a transition of an electron from a higher orbit to a lower orbit, it emits electromagnetic waves of some definite frequency governed by Bohr's Transition Rule.

(d) Electromagnetic wave (X-ray) is produced when high speed electron enters into a target of high atomic weight.

(e) Electromagnetic wave (γ-rays) is produced during de-excitation of nucleus in radioactivity.

However, please note that electromagnetic waves will not be produced when

(a) an electric charge is at rest, because it only produces electrostatic field around it.

(b) a charge is moving with uniform velocity (i.e. steady current), because then it produces both electric and magnetic field. However, this magnetic field does not change with time and hence it does not produce time varying electric field.

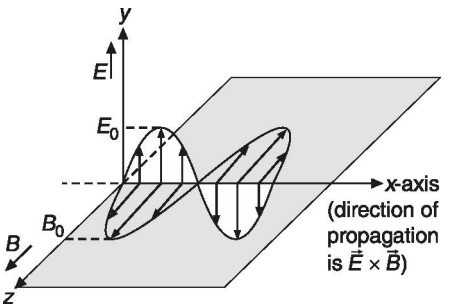
PLANE PROGRESSIVE EM WAVE

In an EM wave, at any point in space the electric field E and the magnetic field B are sinusoidal functions of time t and at any instant, the spatial variation (variation with x) of these fields is also sinusoidal.

If at any instant the fields are uniform over any plane perpendicular to the direction of wave propagation, then the wave is called a plane wave.

Also, it is observed that $\vec{E}$ and $\vec{B}$ are at right angles to the direction of propagation of EM wave. So, electromagnetic waves are transverse in nature.

A sinusoidal electromagnetic wave travelling in the $+x$ direction as shown in Figure.



The $\vec{E}$ and $\vec{B}$ vectors are shown for few points on the x-axis. The point where $\vec{E}$ is in the $+y$ direction has B is in the $+z$ direction and the point where E is in $-y$ direction has B is in $-z$ direction.

Let E and B represent the instantaneous values, E_0 and B_0 represent the peak values of corresponding fields. The equation of the travelling electromagnetic wave is then given by

$$\vec{E} = E_0 \sin(\omega t - kx)\,\hat{j}$$

$$\vec{B} = B_0 \sin(\omega t - kx)\,\hat{k}$$

where, $\omega = 2\pi f$ is the angular frequency, $k = \frac{2\pi}{\lambda}$ is the propagation constant of the EM wave.

The speed of the EM wave in vacuum is

$$c = \frac{\omega}{k} = f\lambda$$

The speed of electromagnetic wave in vacuum is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

The speed of electromagnetic wave in medium is

$$c_{\text{medium}} = v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

where, $\sqrt{\mu_r\varepsilon_r} = \frac{c}{v} = n$ is called the refractive index of medium. E_0 and B_0 are related to each other according to equation

$$B_0 = \frac{E_0}{c}$$

The speed of electromagnetic wave is given by

$$c = \frac{E_0}{B_0}$$

ILLUSTRATION 3

An electromagnetic wave is propagating in vacuum along x-axis, which is produced by oscillating charge of frequency 3×10^{10} Hz. The amplitude of magnetic field (B_0) is 1×10^{-7} T along z-axis. Calculate the wavelength of the EM wave, its propagation constant and the equation for oscillating electric field and magnetic field.

SOLUTION

Since we know that

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^{10}} = 1\times10^{-2} \text{ m}$$

$$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2(3.14)}{10^{-2}} = 6.28\times10^2 = 628 \text{ m}^{-1}$$

The equation for oscillating electric field is

$$E = cB_0 \sin(2\pi\times3\times10^{10}t - 628x)$$

$$\Rightarrow \quad \vec{E} = 30\sin(6\pi\times10^{10}t - 628x)\hat{j} \text{ NC}^{-1} \text{ and}$$

$$\vec{B} = 1\times10^{-7}\sin(6\pi\times10^{10}t - 628x)\hat{k} \text{ T}$$

ENERGY OF AN EM WAVE

The energy possessed by an EM wave is equally divided between the electric and magnetic fields. Since we are already aware that the energy density associated with an electric field is given by

$$u_e = \frac{1}{2}\varepsilon_0 E^2$$

and the energy density associated with a magnetic field is given by

$$u_m = \frac{B^2}{2\mu_0}$$

The total energy per unit volume possessed by an EM wave is

$$u = u_e + u_m = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Since we know that $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E}{B}$

$$\Rightarrow \quad u_e = u_m = \frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\Rightarrow \quad u = u_e + u_m = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

The average energy density associated by the electric field is

$$\langle u_e \rangle = \left\langle \frac{1}{2}\varepsilon_0 E^2 \right\rangle = \frac{1}{2}\varepsilon_0 \langle E^2 \rangle$$

Since, we know that $E = E_0 \sin(\omega t - kx)$

$$\Rightarrow \quad \langle u_e \rangle = \frac{1}{2}\varepsilon_0 \langle E^2 \rangle = \frac{1}{2}\varepsilon_0 E_0^2 \langle \sin^2(\omega t - kx) \rangle$$

Also, we are aware that

$$\langle \sin^2(\omega t - kx) \rangle = \langle \cos^2(\omega t - kx) \rangle = \frac{1}{2}$$

$$\Rightarrow \quad \langle u_e \rangle = \frac{1}{2}\varepsilon_0 \left(\frac{E_0^2}{2} \right) = \frac{1}{4}\varepsilon_0 E_0^2 \qquad \ldots(1)$$

Similar arguments when applied to the magnetic part of the EM wave also give us

$$\langle u_m \rangle = \frac{1}{2\mu_0}\langle B^2 \rangle = \frac{B_0^2}{4\mu_0} \qquad \ldots(2)$$

Since, we know that $c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

So, from (1) and (2), we get

$$\langle u_e \rangle = \langle u_m \rangle = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}$$

$$\Rightarrow \quad \langle u \rangle = \langle u_e \rangle + \langle u_m \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

Conceptual Note(s)

Average energy density of electromagnetic wave is equal to average value of total instantaneous energy density in one time period.

$$\langle u \rangle = \frac{1}{T}\int_0^T u\,dt$$

$$\Rightarrow \quad \langle u \rangle = \frac{1}{T}\int_0^T \varepsilon_0 E^2\,dt$$

$$\Rightarrow \quad \langle u \rangle = \frac{\varepsilon_0 E_0^2}{T}\int_0^T \sin^2(kx - \omega t)\,dt$$

Since, $\int_0^T \sin^2(kx - \omega t)\,dt = \frac{T}{2}$

$$\Rightarrow \quad \langle u \rangle = \left(\frac{\varepsilon_0 E_0^2}{T}\right)\left(\frac{T}{2}\right)$$

$$\Rightarrow \quad \langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

ILLUSTRATION 4

In an electromagnetic wave, the amplitude of electric field is 1 Vm^{-1}. What is

(a) the amplitude of magnetic field?
(b) average energy density of electric field?
(c) average energy density of magnetic field?
(d) average energy density of wave?

SOLUTION

(a) As $\frac{E_0}{B_0} = c$

$$\Rightarrow \quad B_0 = \frac{E_0}{c}$$

$$\Rightarrow \quad B_0 = \frac{1}{3\times 10^8} = 3.33 \text{ nT}$$

(b) $U_E = \frac{1}{4}\varepsilon_0 E^2 = \frac{1}{4}\times 8.85\times 10^{-12}\times 1^2$

$$\Rightarrow \quad U_E = 2.2125\times 10^{-12} \text{ Jm}^{-3}$$

(c) $U_B = U_E = 2.2125\times 10^{-12} \text{ Jm}^{-3}$

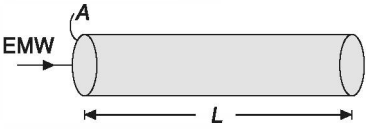
(d) $U = U_E + U_B = 4.425\times 10^{-12} \text{ Jm}^{-3}$

INTENSITY OF AN EM WAVE

The intensity of an EM wave is defined as the amount of energy crossing per unit time per unit area of a surface held normally to the direction of propagation of electromagnetic wave. Mathematically, we have

$$I = \frac{U}{A\Delta t} \qquad \ldots(1)$$

where U is the total energy of the EM wave.
Let us consider a cylindrical region of length L, area of cross section A through which EM wave is passing.



Volume of cylinder is

$$V = AL$$

Since, the intensity of the EM wave is

$$I = \frac{U}{A\Delta t}$$

$$\Rightarrow \quad I = \left(\frac{U}{AL}\right)\left(\frac{L}{\Delta t}\right) = uc$$

where, $u = \frac{U}{V}$ is the total energy density of the EM wave and $c = \frac{L}{\Delta t}$ is the speed of the EM wave.

Total energy of EM wave in the cylinder is

$$E = \langle u \rangle (AL)$$

$$\Rightarrow \quad I_{av} = \langle u \rangle c$$

Since, $\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$

$$\Rightarrow \quad I_{av} = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c$$

Also, we know that

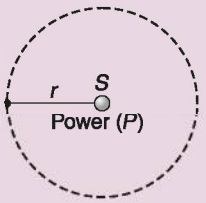
$$E_{rms} = \frac{E_0}{\sqrt{2}}$$

$$\Rightarrow \quad I_{av} = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c = \varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 c = \varepsilon_0 E_{rms}^2 c$$

Conceptual Note(s)

INTENSITY DUE TO A POINT SOURCE

For a point source S having power P, the intensity of EM wave at a distance r from the source is



$$I = \frac{U}{A\Delta t} = \left(\frac{U}{\Delta t}\right)\frac{1}{A} = \frac{P}{A} = \frac{P}{4\pi r^2} \text{ Wm}^{-2}$$

Since, $I_{av} = \langle u \rangle c$

$$\Rightarrow \quad \frac{P}{4\pi r^2} = \langle u \rangle c \text{, where } \langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$$

$$\Rightarrow \quad \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{P}{2\pi r^2 c\varepsilon_0}} \text{ and } B_0 = \frac{E_0}{c} = \sqrt{\frac{P\mu_0}{2\pi r^2 c}}$$

ILLUSTRATION 5

A point source of electromagnetic radiation has average power output of 800 W, then calculate the maximum value of electric field at a distance 3.5 m from the source, maximum value of magnetic field and the average energy density at 3.5 m from the source.

SOLUTION

The intensity of EM wave is

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad I = \frac{800}{4 \times 3.14 \times (3.5)^2}$$

$$\Rightarrow \quad I = 5.2 \text{ Wm}^{-2}$$

Since, we know that

$$I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times I}{\varepsilon_0 \times c}}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times 5.2}{8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$$\Rightarrow \quad E_0 = 0.626 \times 10^2 = 62.6 \text{ NC}^{-1}$$

$$\Rightarrow \quad B_0 = \frac{E_0}{c} = \frac{62.6}{3 \times 10^8} = 20.87 \times 10^{-8}$$

$$\Rightarrow \quad B_0 = 2.087 \times 10^{-7} \text{ T}$$

Since, we know that

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{8.85 \times 10^{-12}}{2} \times (62.6)^2$$

$$\Rightarrow \quad \langle u \rangle = 1.74 \times 10^{-8} \text{ Jm}^{-3}$$

ILLUSTRATION 6

Calculate the electric and magnetic field amplitude in an electromagnetic wave radiated by a 200 W bulb at a distance 2 m from it. Assume that the efficiency of bulb is 5% and it behaves like a point source.

SOLUTION

Effective power output of the bulb is

$$P = \frac{5}{100} \times 200 = 10 \text{ watt}$$

Intensity at distance r is given by

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad I = \frac{10}{4 \times 3.14 \times 2^2} \text{ Wm}^{-2}$$

$$\Rightarrow \quad I = \frac{10}{16 \times 3.14}\ \text{Wm}^{-2} = 0.199\ \text{Wm}^{-2}$$

$$\Rightarrow \quad I \approx 0.2\ \text{watt m}^{-2}$$

Since, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$$\Rightarrow \quad E_0^2 = \frac{2I}{c \times \varepsilon_0}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2I}{c \times \varepsilon_0}}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times 0.2}{3 \times 10^8 \times 8.85 \times 10^{-12}}}$$

$$\Rightarrow \quad E_0 = 12.27\ \text{NC}^{-1}$$

$$\Rightarrow \quad B_0 = \frac{E_0}{c} = \frac{12.27}{3 \times 10^8} = \frac{12.27}{3} \times 10^{-8}\ \text{T}$$

$$\Rightarrow \quad B_0 = 4.09 \times 10^{-8}\ \text{T}$$

MOMENTUM POSSESSED BY AN EM WAVE

It is interesting to note that, though EM waves do not possess mass, yet they have finite momentum p given by

$$p = \frac{U}{c}$$

If an EM wave is incident on a perfectly absorbing surface, then the momentum delivered by the wave to the surface is equal to the change in momentum of the EM wave. So, we have

$$p_{\substack{\text{imparted to} \\ \text{the surface}}} = \Delta p_{\text{wave}} = \frac{U}{c}$$

Similarly, if an EM wave is incident on a perfectly reflecting surface, then the momentum delivered by the wave to the surface is also equal to the change in momentum of the EM wave. So, we have

$$p_{\substack{\text{imparted to} \\ \text{the surface}}} = \Delta p_{\text{wave}} = \frac{2U}{c}$$

RADIATION PRESSURE OF AN EM WAVE

When the EM wave is incident on a surface, perfectly absorbing or perfectly reflecting, then the momentum delivered by the wave to the surface is equal to the change in momentum of the EM wave. Due to this change in momentum, the surface experiences a force and hence pressure called as radiation pressure given by

$$P = \frac{F}{A} = \frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right)$$

For a perfectly absorbing surface, we have

$$\Delta p = \frac{U}{c}$$

$$P = \frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right) = \frac{1}{A}\left(\frac{U}{c\Delta t}\right) = \frac{1}{c}\left(\frac{U}{A\Delta t}\right) = \frac{I}{c}$$

Similarly, for a perfectly reflecting surface, we have

$$\Delta p = \frac{2U}{c}$$

$$P = \frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right) = \frac{2}{A}\left(\frac{U}{c\Delta t}\right) = \frac{2}{c}\left(\frac{U}{A\Delta t}\right) = \frac{2I}{c}$$

ILLUSTRATION 7

An electromagnetic wave of intensity $10\ \text{Wm}^{-2}$ strikes a small mirror of area $20 \times 10^{-4}\ \text{m}^2$, held perpendicular to the approaching wave. What is the radiation force on the mirror?

SOLUTION

Radiation force is the momentum transferred per second by electromagnetic wave to the mirror.

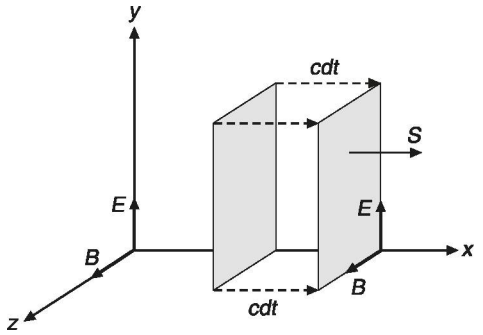
$$F = PA = \frac{2IA}{c} = \frac{2 \times (10) \times 20 \times 10^{-4}}{3 \times 10^8}$$

$$\Rightarrow \quad F = 1.33 \times 10^{-10}\ \text{N}$$

POYNTING VECTOR FOR AN EM WAVE

The concept of Poynting vector $\vec{S}$ was introduced by the British physicist John Poynting (1852-1914). It is defined as the flow of energy in the direction of the propagation of a wave per unit time through a unit cross-sectional area perpendicular to the propagation direction.

Consider a plane EM wave front of area A travelling with speed c along the $+x$ direction as shown in Figure.



In time dt, this wave front moves through a distance $dx = cdt$. The energy in the space between these two positions of the wave front is the product of energy density (u) of the EM wave and volume (dV) of the space between these two positions. So, we have

$$dU = udV, \text{ where } dV = Adx = Acdt$$

$$\Rightarrow \quad dU = udV = \left(\varepsilon_0 E^2\right)(Acdt) \qquad \ldots(1)$$

The energy flowing per unit time per unit area is called the Poynting Vector given by

$$S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 cE^2 = uc$$

$$\Rightarrow \quad S = \varepsilon_0 cE^2 = \varepsilon_0\left(\frac{1}{\sqrt{\varepsilon_0\mu_0}}\right)E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}}E^2$$

Since, we know that $\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \quad S = \sqrt{\frac{\varepsilon_0}{\mu_0}}E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}}E(Bc) = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\frac{EB}{\sqrt{\mu_0\varepsilon_0}}\right) = \frac{EB}{\mu_0}$$

Vectorially, $\vec{S}$ is parallel to x-axis, cdt is also parallel to x-axis, $\vec{E}$ is along y axis and $\vec{B}$ is along z axis. So, we can also say that the Poynting vector $\vec{S}$ describes both the magnitude and direction of energy flow rate. For vacuum, the Poynting vector is given by

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

Its SI unit is Jsm^{-2} or Wm^{-2}

The magnitude $\frac{EB}{\mu_0}$ gives the flow of energy through a cross section perpendicular to the propagation direction, per unit area per unit time at an instant. The electric and magnetic fields at any point vary with time, so the Poynting vector at any point is also function of time.

The average value of Poynting vector at any point is energy flowing on an average in unit time from unit area. This is also called intensity of the radiation at that point. We can also write

$$|\vec{S}| = S = \frac{E_0B_0}{\mu_0}\sin^2(\omega t - kx)$$

$$\Rightarrow \quad \langle S\rangle = I = \frac{E_0B_0}{\mu_0}\left\langle\sin^2(\omega t - kx)\right\rangle = \frac{E_0B_0}{2\mu_0}$$

$$\Rightarrow \quad I = S_{av} = \frac{E_0B_0}{2\mu_0} = \frac{E_0H_0}{2} \qquad \left\{\because H_0 = \frac{B_0}{\mu_0}\right\}$$

$$\Rightarrow \quad I = \frac{E_0B_0}{2\mu_0} = \left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{B_0}{\sqrt{2}}\right)\frac{1}{\mu_0} = \frac{E_{rms}B_{rms}}{\mu_0}$$

Problem Solving Technique(s)

(a) The average value of Poynting vector represents the intensity (I) of the EM wave.

(b) Poynting vector $\vec{S}$ gives the direction of EM wave propagation.

ILLUSTRATION 8

The oscillating magnetic field in a plane electromagnetic wave is given as

$$B_y = 8\times10^{-6}\sin\left(5000\pi x - 3\times10^{11}\pi t\right)\text{ T}$$

Calculate the frequency, wavelength, speed of EM wave, electric field amplitude and the expression for oscillating electric field.

SOLUTION

Comparing given equation with standard equation i.e. $B_y = B_0(kx - \omega t)$, we get

$$\omega = 3\times10^{11}\pi$$

$$\Rightarrow \quad 2\pi f = 3\times10^{11}\pi$$

$$\Rightarrow \quad f = 1.5\times10^{11}\text{ Hz}$$

Since, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi}{5000\pi} = 0.4\times10^{-3}\text{ m} = 0.4\text{ mm}$$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{3\times10^{11}\times\pi}{5000\pi} = 0.6\times10^{8} = 6\times10^{7}\text{ ms}^{-1}$$

Since, $v = \frac{E_0}{B_0}$

$\Rightarrow \quad E_0 = vB_0 = (6\times10^7)(8\times10^{-6}) = 480\ \text{Vm}^{-1}$

Since the wave is travelling along the x-direction, so $\vec{E}\times\vec{B}$ should be along x-axis. Since $-(\hat{k}\times\hat{j}) = \hat{i}$, hence we have

$$E_z = 480\sin(500\pi x - 3\times10^{11}\ \pi t)\ \text{Vm}^{-1}$$

SUMMARY OF IMPORTANT CHARACTERISTICS AND NATURE OF ELECTROMAGNETIC WAVES

Electromagnetic waves are transverse waves made up of oscillating electric and magnetic fields, which oscillate perpendicular to each other as well as to the direction of propagation of the wave. They possess the following properties.

(a) The fundamental sources of electromagnetic waves are accelerating electric charges. For examples radio waves emitted by an antenna arise from the continuous oscillations (and hence acceleration) of charges within the antenna structure.

(b) Electromagnetic waves obey the principle of superposition.

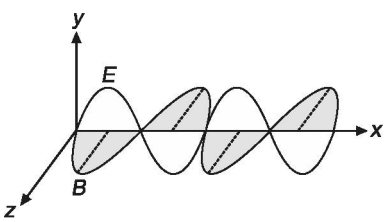
(c) The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x-direction can also be written as

$$E = E_0\sin(kx - \omega t)$$
$$B = B_0\sin(kx - \omega t)$$

where ω is the angular frequency of the wave and k is angular wave number or propagation constant which are given by

$$\omega = 2\pi f \ \text{ and } \ k = \frac{2\pi}{\lambda}$$

(d) In an EM wave, $\vec{E}$ and $\vec{B}$ vary sinusoidally. $\vec{E}$ and $\vec{B}$ become maximum at same place and at the same time, are perpendicular to each other as well as to the direction of propagation. So, the phase difference between the two fields is zero.



(e) EM Waves travel in free space with speed equal to $3\times10^8\ \text{ms}^{-1}$ which is given by $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

(f) Electromagnetic waves travel with speed of light. In vacuum their speed is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

In isotropic medium, their speed is

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_r\varepsilon_r}}\cdot\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{n}$$

where $n = \sqrt{\mu_r\varepsilon_r}$
is refractive index of medium.
μ_r = relative permeability of medium and
ε_r = relative permittivity of medium or electric dielectric constant.

(g) The velocity of electromagnetic wave in a medium is decided by electric and magnetic properties of medium and not by the amplitude of electric and magnetic field vector. The speed of electromagnetic wave in a medium is

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ is the permeability of the medium and ε is the permittivity of the medium.

(h) EM Waves do not require material medium for their propagation.

(i) The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2E}{\partial t^2}$$
$$\frac{\partial^2B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2B}{\partial t^2}$$

(j) The amplitude of electric field and the magnetic field in EM wave are related to each other as

$$c = \frac{E_0}{B_0}.$$

(k) The direction of propagation of an EM wave is determined by $\vec{E}\times\vec{B}$.

(l) The electric field vector of an electromagnetic wave produces optical effects and hence it is also called as Light vector.

(m) Electromagnetic waves are not deflected by electric and magnetic field because EM waves are made up of uncharged particles called photons.

(n) The energy carried by an electromagnetic wave is equally divided between the electric field and magnetic field. Total average energy density is

$$u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

(o) The intensity of an electromagnetic wave is defined as energy crossing per second per unit area of a surface normal to the direction of propagation of electromagnetic wave.
Average intensity is given by

$$I_{av} = \langle I \rangle = \frac{1}{2}\varepsilon_0 E_0^2 c = \varepsilon_0 E_{rms}^2 c$$

(in terms of electric field)

$$I_{av} = \left(\frac{B_0^2}{2\mu_0}\right)c = \left(\frac{B_{rms}^2}{\mu_0}\right)c$$

(in terms of magnetic field)

(p) The electromagnetic waves carry both energy as well as momentum. The momentum of an EM wave is

$$p = \frac{U}{c}$$

where, U is the energy carried by an EM wave in free space and c is the speed of an EM wave in free space.

(q) The radiation pressure is defined as force exerted by an EM Wave on a unit area of a surface. Nichols and Hull measured radiation pressure of visible light and found it to be of the order of 7×10^{-6} Nm^{-2}.

(r) For an EM wave of intensity I falling normally on a perfectly absorbing surface, the radiation pressure is

$$P = \frac{I}{c}$$

(s) For an EM wave of intensity I falling normally on a perfectly reflecting surface, the radiation pressure is

$$P = \frac{2I}{c}$$

For all other surfaces, the radiation pressure lies between

$$\frac{I}{c} < P < \frac{2I}{c}$$

(t) The **Poynting vector** $\vec{S} = \vec{E} \times \vec{H}$ represents the direction of energy flow per unit area per sec along the direction of wave propagation. Its unit is Wm^{-2}.

(u) The intensity of a sinusoidal plane electromagnetic wave is defined as the **average value of Poynting vector** taken over one cycle.

$$S_{av} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c}{2\mu_0}B_0^2 \qquad \left\{\because c = \frac{E_0}{B_0}\right\}$$

(v) The medium in which EM wave is travelling, offers hindrance to the propagation of wave. Such a hindrance is called **Wave Impedance** (Z) and its value in a medium is given by

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

because, $\mu = \mu_r \mu_0$ and $\varepsilon = \varepsilon_r \varepsilon_0$

where μ_r and ε_r are relative permeability and relative permittivity of medium.
For vacuum or free space

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6\ \Omega$$

ELECTROMAGNETIC SPECTRUM

The electromagnetic spectrum has a very wide range-from very large to very small wavelengths. Different methods are used for producing them and they have different properties. However, the boundaries of different types of electromagnetic waves are not sharp and they overlap.

Name	Discoverer	Approximate Wavelength Range	Generated by	Properties	Applications
Radio waves	Marconi	0.1 m – 10^4 m	Oscillating circuits	Exhibit waves like properties more than particle like properties	Radio Communication
Microwaves	Hertz (Heinrich Rudolph Hertz)	10^3 m – 0.1 m	Special electronic devices such as klystron tube	Phenomena of reflection, refraction and diffraction.	(a) Radar and Tele-communication. (b) Analysis of fine details of molecular structure.
Infra-Red	William Herschell	8000 Å to 10^7 Å	(a) Rearrangement of outer orbital electrons in atoms and molecules. (b) Change of molecular vibrational and rotational energies. (c) By bodies at high temperature.	(a) Thermal effect (b) All properties similar to those of light except λ	(a) Used in industry, medicine and astronomy. (b) Useful for fog or haze photography. (c) Elucidating molecular structure.
Visible light and Sub parts of visible spectrum	Newton		Rearrangement of outer orbital electrons in atoms and molecules, e.g., gas discharge tube, incandescent solids and liquids.	Sensitive to human eye	(a) To see objects. (b) To study molecular structure.
(a) Violet		3990 Å to 4550 Å			
(b) Blue		4550 Å to 4920 Å			
(c) Green		4920 Å to 5770 Å			
(d) Yellow		5770 Å to 5970 Å			
(e) Orange		5970 Å to 6220 Å			
(f) Red		6220 Å to 7880 Å			
Ultraviolet	Ritter	10 Å to 1000 Å	Rearrangement of orbital electrons of atoms, as in high voltage gas discharge tube, arc, the sun and the mercury vapour lamp.	(a) All properties of light. (b) Photoelectric effect.	Sterilization of water due to its destructive action on bacteria. To detect adulteration, writing and signature.

(*Continued*)

Name	Discoverer	Approximate Wavelength Range	Generated by	Properties	Applications
X-rays	Roentgen	0.1 Å to 10 Å	(a) Bombardment of high energy electrons on a high atomic number target. (b) Energy changes of innermost orbital electrons.	(a) Low penetrating power. (b) Other properties similar to γ-rays except wavelength.	(a) Medical diagnosis and treatment. (b) Study of crystal structure. (c) Industrial radiography.
Gamma Rays	Henry Becquerel and Madam Curie	0.006 Å to 1 Å	(a) Change of nuclear energy levels. (b) Radioactive substances.	(a) High penetrating power. (b) Uncharged. (c) Low ionising power.	(a) Gives information on nuclear structure. (b) Medical treatment.

SUB PARTS OF RADIO SPECTRUM

Name	Approximate Wavelength Range	Applications
(a) SHF (Super High Frequency)	0.001 m to 0.1 m	Radar, Radio and satellite communication (Microwaves) Radar and television broadcast, short distance communication, Television communication.
(b) UHF (Ultra High Frequency)	0.1 m to 1 m	
(c) VHF (Very High Frequency)	1 m to 10 m	
(d) HF (High Frequency)	10 m to 100 m	Medium distance communication, Telephone communication, Marine and navigation use, Long range communication, Long distance communication.
(e) MF (Medium Frequency)	100 m to 1000 m	
(f) LF (Low Frequency)	1000 m to 10000 m	
(g) VLF (Very Low Frequency)	10000 m to 30000 m	

ATMOSPHERE AND VARIOUS PARTS OF ELECTROMAGNETIC SPECTRUM

(a) Atmosphere is transparent to visible radiations.

(b) Atmosphere absorbs most of infrared radiations.

(c) The ozone layer which is somewhere 30 km – 50 km above the ground (mesosphere) absorbs ultraviolet radiations, thus protects us from harmful radiations of sun. Practically all radiations of wavelength less than $3000\,\text{Å}\left(3\times10^{-7}\ \text{m}\right)$ are absorbed by ozone layer.

(d) Earth is heated by sun's infrared radiations. The earth also emits radiations most of which are in infrared region. These radiations are reflected back by atmosphere. These reflected radiations keep the earth's surface warm at night. This phenomenon is called **Green House Effect**.

(e) Waves of wavelength 1 mm or higher are broadly called **Radiowaves**. They include radio, television and microwaves. Waves having wavelength 10 cm or more (frequencies less than 30 MHz) are termed as **Amplitude Modulated Waves**. These waves are transmitted in lower atmosphere but are reflected by top-most region of atmosphere called **Ionosphere**. These waves further are of two types.

(i) **Ground waves:** These follow the surface of earth the ground waves are attenuated depending on frequency. The communication upto 1500 kHz may be via **ground waves**.

(ii) **Sky waves:** They are reflected back by Ionosphere. For frequencies above 1500 kHz the communication is via **Sky Waves**.

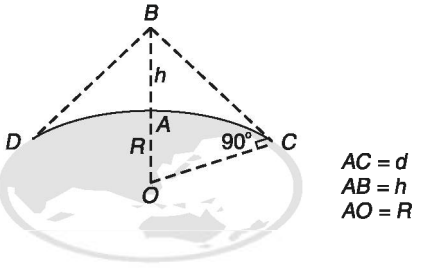
These two regions of waves of Amplitude modulated waves are termed as **Medium Waves (MW)** and **Short Waves (SW)** respectively.

For waves above 40 MHz, the Ionosphere deviates incident waves but does not reflect back to earth.

T.V. Signal waves of frequency 100 MHz – 200 MHz called **Frequency Modulated Wave** neither follow the curvature of earth nor get reflected by Ionosphere, their reception is possible either by large antenna or by geostationary satellites.

COVERING RANGE OF A TRANSMITTER OR T.V. ANTENNA

Consider a TV tower of height h. If $d = AC$ is the covering range of transmitter, then from triangle OBC.



$$(OB)^2 = (OC)^2 + (CB)^2$$

We have

$$OB = R + h$$

$$OC = R$$

and $BC = AC = d$

$$\Rightarrow \quad (R+h)^2 = R^2 + d^2$$

$$\Rightarrow \quad d^2 = (R+h)^2 - R^2$$

$$\Rightarrow \quad d^2 = 2hR + h^2$$

$$\Rightarrow \quad d = \sqrt{2hR + h^2}$$

For $h \ll R$ covering range is given by

$$d = \sqrt{2hR}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A wave is propagating in a medium of electric dielectric constant 2 and relative magnetic permeability 50. The wave impedance of such a medium is

(A) 5 Ω (B) 376.6 Ω
(C) 1883 Ω (D) 3776 Ω

2. The electric field component of a monochromatic radiation is given by $\vec{E} = 2E_0\hat{i}\cos kz\cos\omega t$

Its magnetic field is then given by

(A) $\frac{2E_0}{c}\hat{j}\cos kz\cos\omega t$ (B) $\frac{2E_0}{c}\hat{j}\sin kz\cos\omega t$

(C) $\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$ (D) $-\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$

3. Electromagnetic waves travel in a medium which has relative permeability 1.3 and relative permittivity 2.14. Then the speed of the electromagnetic wave in the medium will be

(A) 13.6×10^{6} ms^{-1} (B) 1.8×10^{2} ms^{-1}
(C) 3.6×10^{8} ms^{-1} (D) 1.8×10^{8} ms^{-1}

4. To which region of electromagnetic spectrum, the frequency 1 BHz correspond?

(A) ultraviolet rays (B) radio waves
(C) visible radiation (D) *X*-rays

5. The oscillating electric and magnetic vectors of an electromagnetic wave are oriented along

(A) the same direction but differ in phase by 90°
(B) the same direction and are in phase
(C) mutually perpendicular directions and are in phase
(D) mutually perpendicular directions and differ in phase by 90°

6. An LC resonant circuit contains a 400 pF capacitor and a 100 μH inductor. It is sent into oscillations coupled to an antenna. The wavelength of the radiated electromagnetic wave is

(A) 377 mm (B) 377 cm
(C) 377 m (D) 3.77 cm

7. The rms value of the electric field of the light coming from the sun is 720 NC^{-1}. The average total energy density of the electromagnetic wave is

(A) 3.3×10^{-3} Jm^{-3} (B) 4.58×10^{-6} Jm^{-3}
(C) 6.37×10^{-9} Jm^{-3} (D) 81.35×10^{-12} Jm^{-3}

8. The relation between electric field *E* and magnetic field *H* in an electromagnetic wave is

(A) $E = H$ (B) $E = \frac{\mu_0}{\varepsilon_0}H$

(C) $E = \sqrt{\frac{\mu_0}{\varepsilon_0}}H$ (D) $E = \sqrt{\frac{\varepsilon_0}{\mu_0}}H$

9. A parallel plate capacitor of plate separation 2 mm is connected in an electric circuit having source voltage 400 V. If the plate area is 60 cm^2, then the value of displacement current for 10^{-6} s will be

(A) 1.062 A (B) 1.062×10^{-2} A
(C) 1.062×10^{-3} A (D) 1.062×10^{-4} A

10. The ratio of electric field vector *E* and magnetic field vector *H* i.e., $\left(\frac{E}{H}\right)$ has the dimensions of

(A) resistance
(B) inductance
(C) capacitance
(D) product of inductance and capacitance

11. A monochromatic beam of light has a frequency $\nu = \frac{3}{2\pi}\times10^{12}$ Hz and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is

(A) $\frac{E_0}{c}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r}+(3\times10^{12})t\right)$

(B) $\frac{E_0}{c}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right)$

(C) $\frac{E_0}{c}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cos\left(10^4\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right)$

(D) $\frac{E_0}{c}\hat{k}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r}+(3\times10^{12})t\right)$

12. In an electromagnetic wave, the direction of the magnetic field induction $\vec{B}$ is
(A) parallel to electric field $\vec{E}$
(B) perpendicular to electric field $\vec{E}$
(C) antiparallel to Poynting vector $\vec{S}$
(D) random

13. Electromagnetic waves are transverse in nature is evident by
(A) polarization (B) interference
(C) reflection (D) diffraction

14. The wave associated with 2.7 K belongs to
(A) radio waves (B) microwaves
(C) ultraviolet rays (D) infrared waves

15. A parallel plate capacitor consists of two circular plates each of radius 2 cm, separated by a distance of 0.1 mm. If voltage across the plates is varying at the rate of 5×10^{13} Vs^{-1}, then the value of displacement current is
(A) 5.50 A (B) 5.56×10^{2} A
(C) 5.56×10^{3} A (D) 2.28×10^{4} A

16. A radio receiver antenna that is 2 m long is oriented along the direction of the electromagnetic wave and receives a signal of intensity 5×10^{-16} Wm^{-2}. The maximum instantaneous potential difference across the two ends of the antenna is
(A) 1.23 μV (B) 1.23 mV
(C) 1.23 V (D) 12.3 mV

17. To double the covering range of a T.V. transmitter tower, its height should be made
(A) 2 times (B) 4 times
(C) $\sqrt{2}$ times (D) 8 times

18. In an electromagnetic wave, the electric and magnetising fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is
(A) 26.5 Wm^{-2} (B) 36.5 Wm^{-2}
(C) 46.7 Wm^{-2} (D) 765 Wm^{-2}

19. Maxwell's equation $\oint\vec{B}\cdot d\vec{A}=0$ is a statement of
(A) Faraday's Law of Induction
(B) Modified Ampere's Law
(C) Gauss's Law of Electricity
(D) Gauss's Law of Magnetism

20. In an electromagnetic wave passing through vacuum the electric and magnetic fields are described by $E=E_0\sin(kx-\omega t)$ and $B=B_0\sin(kx-\omega t)$, then
(A) $E_0k=B_0\omega$ (B) $E_0\omega=B_0k$
(C) $E_0B_0=\omega k$ (D) None of these

21. In an electromagnetic wave the electric field vector E and the magnetic field vector B are
(A) perpendicular to each other
(B) parallel to each other
(C) at 45° to each other
(D) can have any angle between them

22. An electromagnetic wave is represented by the electric field $\vec{E}=E_0\hat{n}\sin\left[\omega t+(6y-8z)\right]$. Taking unit vectors in x, y and z directions to be $\hat{i}$, $\hat{j}$, $\hat{k}$, the direction of propagation $\hat{s}$, is
(A) $\hat{s}=\left(\dfrac{-3\hat{j}+4\hat{k}}{5}\right)$ (B) $\hat{s}=\left(\dfrac{3\hat{i}-4\hat{j}}{5}\right)$
(C) $\hat{s}=\left(\dfrac{-4\hat{k}+3\hat{j}}{5}\right)$ (D) $\hat{s}=\left(\dfrac{4\hat{j}-3\hat{k}}{5}\right)$

23. The ionosphere is mainly composed of
(A) nitrogen and oxygen
(B) ozone
(C) electrons and positive ions
(D) None of the above

24. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 10 m from the lamp will be
(A) 1.34 Vm^{-1} (B) 2.68 Vm^{-1}
(C) 5.36 Vm^{-1} (D) 9.37 Vm^{-1}

25. The transmitting antenna of a radio station is mounted vertically. At a point 10 km due north of the transmitter the peak electric field is 10^{-3} Vm^{-1}. The magnitude of the radiated magnetic field is
(A) 3.33×10^{-10} T (B) 3.33×10^{-12} T
(C) 10^{-3} T (D) 3×10^{5} T

26. The 21 cm radio wave emitted by hydrogen in interstellar space is due to the interaction called the hyperfine interaction is atomic hydrogen. The energy of the emitted wave (in joule) is nearly
(A) 10^{-17} (B) 1
(C) 7×10^{-8} (D) 10^{-24}

27. The energy of electromagnetic wave in vacuum is given by the relation
(A) $\dfrac{E^2}{2\varepsilon_0}+\dfrac{B^2}{2\mu_0}$ (B) $\dfrac{1}{2}\varepsilon_0E^2+\dfrac{1}{2}\mu_0B^2$
(C) $\dfrac{E^2+B^2}{c}$ (D) $\dfrac{1}{2}\varepsilon_0E^2+\dfrac{B^2}{2\mu_0}$

28. The magnetic field of plane electromagnetic wave is given by $\vec{B} = \vec{B}_0\hat{i}\cos(kz - \omega t) + B_1\hat{j}\cos(kz + \omega t)$

where $B_0 = 3\times10^{-5}$ T and $B_1 = 2\times10^{-6}$ T.

The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to

(A) 0.6 N (B) 0.9 N

(C) 3×10^{-2} N (D) 0.1 N

29. The law which states that the variation of electric field causes magnetic field is

(A) Faraday's Law

(B) Biot Savart Law

(C) Modified Ampere's Law

(D) Lenz's Law

30. Infrared radiation is detected by

(A) spectrometer (B) pyrometer

(C) nanometer (D) photometer

31. The average value of electric energy density in an electromagnetic wave is (E_0 is peak value)

(A) $\frac{1}{2}\varepsilon_0 E_0^2$ (B) $\frac{E_0^2}{2\varepsilon_0}$

(C) $\varepsilon_0 E_0^2$ (D) $\frac{1}{4}\varepsilon_0 E_0^2$

32. A plane electromagnetic wave of wave intensity 6 Wm^{-2} strikes a small mirror area 40 cm^2, held perpendicular to the approaching wave. The momentum transferred by the wave to the mirror each second will be

(A) 6.4×10^{-7} $kgms^{-2}$ (B) 4.8×10^{-8} $kgms^{-2}$

(C) 3.2×10^{-9} $kgms^{-2}$ (D) 1.6×10^{-10} $kgms^{-2}$

33. An electromagnetic wave is propagating along Y-axis. Then

(A) Oscillating electric field is along X-axis and oscillating magnetic field is along Y-axis

(B) Oscillating electric field is along Z-axis and oscillating magnetic field is along X-axis

(C) Both oscillating electric and magnetic fields are along Y-axis, but phase difference between them is 90°

(D) Both oscillating electric and magnetic fields are mutually perpendicular in arbitrary directions

34. The magnetic field of an electromagnetic wave is given by

$\vec{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15}t)(2\hat{i} + \hat{j})$ Wbm^{-2}

The associated electric field will be

(A) $\vec{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)(-\hat{i} + 2\hat{j})$ Vm^{-1}

(B) $\vec{E} = 4.8\times10^2\cos(2\times10^7 z - 6\times10^{15}t)(2\hat{i} + \hat{j})$ Vm^{-1}

(C) $\vec{E} = 4.8\times10^2\cos(2\times10^7 z - 6\times10^{15}t)(-2\hat{j} + \hat{i})$ Vm^{-1}

(D) $\vec{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)(\hat{i} - 2\hat{j})$ Vm^{-1}

35. The Maxwell's equation $\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i + \varepsilon_0\frac{d\phi_E}{dt}\right)$ is a statement of

(A) Faraday's Law of Induction

(B) Modified Ampere's Law

(C) Gauss's Law of Electricity

(D) Gauss's Law of Magnetism

36. The S.I. unit of displacement current is

(A) henry (B) coulomb

(C) ampere (D) farad

37. Which of the following are not electromagnetic waves?

(A) cosmic rays (B) gamma rays

(C) β-rays (D) X-rays

38. If $\vec{E}$ and $\vec{B}$ be the electric and magnetic field vectors of an electromagnetic wave, then the propagation of the wave is along the direction of

(A) $\vec{E}$ (B) $\vec{B}$

(C) $\vec{E}\times\vec{B}$ (D) $\vec{B}\times\vec{E}$

39. If c is the speed of electromagnetic waves in vacuum, its speed in a medium of dielectric constant K and relative permeability μ_r is

(A) $v = \frac{1}{\sqrt{\mu_r K}}$ (B) $v = c\sqrt{\mu_r K}$

(C) $v = \frac{c}{\sqrt{\mu_r K}}$ (D) $v = \frac{K}{\sqrt{\mu_r C}}$

40. The speed of electromagnetic waves in vacuum is given by

(A) $\mu_0\varepsilon_0$ (B) $\sqrt{\mu_0\varepsilon_0}$

(C) $\frac{1}{\mu_0\varepsilon_0}$ (D) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

41. An electromagnetic wave of frequency $f = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then

(A) wavelength is doubled and the frequency remains unchanged

(B) wavelength is doubled and frequency becomes half

(C) wavelength is halved and frequency remains unchanged

(D) wavelength and frequency both remain unchanged

42. An electromagnetic wave travels along Z-axis. Which of the following pairs of space and time varying fields would generate such a wave?

(A) E_x, B_y (B) E_y, B_x

(C) E_z, B_x (D) E_y, B_z

43. A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then

(A) $p=0,\ E=0$ (B) $p\neq 0,\ E\neq 0$

(C) $p\neq 0,\ E=0$ (D) $p=0,\ E\neq 0$

44. Maxwell's equation $\oint \vec{E}\cdot d\vec{l} = -\dfrac{\partial \vec{B}}{\partial t}$ is a statement of

(A) Ampere's Law

(B) Faraday's Law of Induction

(C) Gauss's Law of Electricity

(D) Gauss's Law of Magnetism

45. A long straight wire of resistance R, radius a and length l carries a constant current I. The Poynting vector for the wire will be

(A) $\dfrac{IR}{2\pi al}$ (B) $\dfrac{IR^2}{al}$

(C) $\dfrac{I^2R}{al}$ (D) $\dfrac{I^2R}{2\pi al}$

46. The direction of propagation of electromagnetic waves is given by

(A) $\vec{E}\cdot\vec{E}$ (B) $\vec{E}$

(C) $\vec{B}$ (D) $\vec{E}\times\vec{B}$

47. An electromagnetic wave, going through vacuum is described by $E=E_0\sin(kx-\omega t)$. Which of the following is independent of wavelength?

(A) k (B) ω

(C) $\dfrac{k}{\omega}$ (D) $k\omega$

48. If R is the radius of earth and h is height of T.V. transmitting tower, then the covering range of transmitter will be

(A) h^2+R^2 (B) R

(C) $\sqrt{R^2+h^2}$ (D) $\sqrt{2Rh}$

49. In an electromagnetic wave, the amplitude of electric field is 1 Vm^{-1}. The frequency of wave is 5×10^{14} Hz. The wave is propagating along z-axis. The average energy density of electric field, in Jm^{-3}, will be

(A) 1.1×10^{-11} (B) 2.2×10^{-12}

(C) 3.3×10^{-13} (D) 4.4×10^{-14}

50. The relation between electric field E and magnetic field induction B in an electromagnetic wave

(A) $E=\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\,B$ (B) $E=cB$

(C) $E=\dfrac{B}{c}$ (D) $E=\dfrac{B}{c^2}$

51. Radiations of intensity 0.5 Wm^{-2} are striking a metal plate. The pressure on the plate is

(A) 0.166×10^{-8} Nm^{-2} (B) 0.332×10^{-8} Nm^{-2}

(C) 0.111×10^{-8} Nm^{-2} (D) 0.083×10^{-8} Nm^{-2}

52. The wave impedance of free space is

(A) ZERO (B) 376.6 Ω

(C) 33.66 Ω (D) 3.76 Ω

53. In an apparatus, the electric field was found to oscillate with an amplitude of 18 Vm^{-1}. The magnitude of the oscillating magnetic field will be

(A) 4×10^{-6} T (B) 6×10^{-8} T

(C) 9×10^{-9} T (D) 11×10^{-11} T

54. An LC current contains inductance $L=1\ \mu$H and capacitance $C=0.01\ \mu$F. The wavelength of electromagnetic wave generated is nearly

(A) 0.5 m (B) 5 m

(C) 188 m (D) 30 m

55. A laser beam can be focussed on an area equal to the square of its wavelength. A He-Ne laser radiates energy at the rate of 1 mW and its wavelength is 632.8 nm. The intensity of focussed beam is

(A) 1.5×10^{13} Wm^{-2} (B) 2.5×10^{9} Wm^{-2}

(C) 3.5×10^{17} Wm^{-2} (D) None of these

56. Displacement current is same as

(A) conduction current due to flow of free electrons.

(B) conduction current due to flow of positive ions.

(C) conduction current due to flow of both positive and negative free charge carriers.

(D) is not a conduction current but is caused by time varying electric field.

57. The electromagnetic waves do not transport

(A) energy (B) charge

(C) momentum (D) information

58. A radio wave has a maximum electric field intensity of 10^{-4} Vm^{-1} on arrival at a receiving antenna. The maximum magnetic flux density of such a wave is

(A) ZERO (B) 3×10^{4} T

(C) 5.8×10^{-9} T (D) 3.3×10^{-13} T

59. TV waves have a wavelength range of 1 m to 10 m. Their frequency range in megahertz is
(A) 30 to 300 (B) 3 to 30
(C) 300 to 3000 (D) 3 to 3000

60. A lamp radiates power P_0 uniformly in all directions, the amplitude of electric field strength E_0 at a distance r from it is
(A) $E_0 = \dfrac{P_0}{2\pi\varepsilon_0 cr^2}$ (B) $E_0 = \sqrt{\dfrac{P_0}{2\pi\varepsilon_0 cr^2}}$
(C) $E_0 = \sqrt{\dfrac{P_0}{4\pi\varepsilon_0 cr^2}}$ (D) $E_0 = \sqrt{\dfrac{P_0}{8\pi\varepsilon_0 cr}}$

61. A point source of electromagnetic radiation has an average power output of 800 W. The maximum value of electric field at a distance 4 m from the source is
(A) 64.7 Vm^{-1} (B) 57.8 Vm^{-1}
(C) 56.72 Vm^{-1} (D) 54.77 Vm^{-1}

62. The expression $\sqrt{\mu_0\varepsilon_0}$ has the dimensional formula
(A) $[LT^{-1}]$ (B) $[L^{-1}T]$
(C) $[LT^{-2}]$ (D) $[L^2T^{-1}]$

63. If a source is transmitting electromagnetic wave of frequency 8.2×10^6 Hz, then wavelength of the electromagnetic waves transmitted from the source is
(A) 36.6 m (B) 40.5 m
(C) 42.3 m (D) 50.9 m

64. "Green house effect" is due to
(A) absorption of ultraviolet radiation by ozone layer
(B) transmission of visible light by atmosphere
(C) good weather
(D) reflection of infrared rays emitted by earth by atmosphere

65. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of EM waves then the direction of propagation of EM wave is along the direction of
(A) $\vec{E}$ (B) $\vec{B}$
(C) $\vec{E}\times\vec{B}$ (D) None of these

66. In electromagnetic wave the phase difference between electric and magnetic field vectors $\vec{E}$ and $\vec{B}$ is
(A) ZERO (B) $\dfrac{\pi}{2}$
(C) π (D) $\dfrac{\pi}{4}$

67. The frequency 1057 MHz of radiation arising from two close energy levels in hydrogen belongs to
(A) Radio waves (B) Infrared waves
(C) Microwaves (D) γ-rays

68. A TV tower has a height of 100 m. The average population density around the tower is 1000 per km^2. The radius of the earth is 6.4×10^6 m. The population covered by the tower is
(A) 2×10^6 (B) 3×10^6
(C) 4×10^6 (D) 6×10^6

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1. **[Online April 2019]**
The magnetic field of an electromagnetic wave is given by

$$\vec{B} = 1.6\times10^{-6}\cos\left(2\times10^7 z + 6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\ \text{Wbm}^{-2}$$

The associated electric field will be
(A) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z + 6\times10^{15}t\right)\left(-\hat{i}+2\hat{j}\right)\ \text{Vm}^{-1}$
(B) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z - 6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\ \text{Vm}^{-1}$
(C) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z - 6\times10^{15}t\right)\left(-2\hat{j}+\hat{i}\right)\ \text{Vm}^{-1}$
(D) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z + 6\times10^{15}t\right)\left(\hat{i}-2\hat{j}\right)\ \text{Vm}^{-1}$

2. **[Online April 2019]**
A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\ \text{Vm}^{-1}$ along y-direction. Its corresponding magnetic field component, B would be
(A) 2×10^{-8} T along y-direction
(B) 6×10^{-8} T along z-direction
(C) 2×10^{-8} T along z-direction
(D) 6×10^{-8} T along x-direction

3. [Online April 2019]

The magnetic field of plane electromagnetic wave is given by $\vec{B} = \vec{B}_0\hat{i}\cos(kz-\omega t) + B_1\hat{j}\cos(kz+\omega t)$ where $B_0 = 3\times10^{-5}$ T and $B_1 = 2\times10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to

(A) 0.6 N (B) 0.9 N

(C) 3×10^{-2} N (D) 0.1 N

4. [Online April 2019]

The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t)$. The corresponding magnetic field $\vec{B}$ is then given by

(A) $\vec{B} = \frac{E_0}{c}\hat{j}\sin(kz)\cos(\omega t)$

(B) $\vec{B} = \frac{E_0}{c}\hat{j}\cos(kz)\sin(\omega t)$

(C) $\vec{B} = \frac{E_0}{c}\hat{j}\sin(kz)\sin(\omega t)$

(D) $\vec{B} = \frac{E_0}{c}\hat{k}\sin(kz)\cos(\omega t)$

5. [Online April 2019]

An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin\left[\omega t + (6y-8z)\right]$. Taking unit vectors in x, y and z directions to be $\hat{i}$, $\hat{j}$, $\hat{k}$, the direction of propagation $\hat{s}$, is

(A) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$ (B) $\hat{s} = \left(\frac{3\hat{i}-4\hat{j}}{5}\right)$

(C) $\hat{s} = \left(\frac{-4\hat{k}+3\hat{j}}{5}\right)$ (D) $\hat{s} = \left(\frac{4\hat{j}-3\hat{k}}{5}\right)$

6. [Online April 2019]

A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the Electric field is 60 Vm^{-1}. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(A) $\vec{B} = 60\sin(0.5\times10^3 x + 1.5\times10^{11}t)\hat{k}$

(B) $\vec{B} = 2\times10^{7}\sin(0.5\times10^3 z + 1.5\times10^{11}t)\hat{i}$

(C) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^3 z - 1.5\times10^{11}t)\hat{i}$

(D) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^2 x + 0.5\times10^{11}t)\hat{j}$

7. [2018]

An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left(2\pi\nu\left(\frac{z}{c}-t\right)\right)$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z-ct)\right]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic, If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(A) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (B) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$

(C) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$ (D) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$

8. [Online 2018]

A monochromatic beam of light has a frequency $\nu = \frac{3}{2\pi}\times10^{12}$ Hz and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is

(A) $\frac{E_0}{c}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r} + (3\times10^{12})t\right)$

(B) $\frac{E_0}{c}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r} - (3\times10^{12})t\right)$

(C) $\frac{E_0}{c}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cos\left(10^4\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\cdot\vec{r} - (3\times10^{12})t\right)$

(D) $\frac{E_0}{c}\hat{k}\cos\left(10^4\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\cdot\vec{r} + (3\times10^{12})t\right)$

9. [Online 2018]

A plane polarized monochromatic EM wave is travelling in vacuum along z direction such that at $t = t_1$, it is found that the electric field is zero at a spatial point z_1. The next zero that occurs in its neighbourhood is at z_2. The frequency of the electromagnetic wave is

(A) $\frac{1}{t_1 + \frac{|z_2-z_1|}{3\times10^8}}$ (B) $\frac{3\times10^8}{|z_2-z_1|}$

(C) $\frac{6\times10^8}{|z_2-z_1|}$ (D) $\frac{1.5\times10^8}{|z_2-z_1|}$

10. [Online 2018]

A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive y-direction. The allowed expressions for the electric and magnetic fields are given by

(A) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 c}}\cos\left(\frac{2\pi}{\lambda}(y-ct)\right)\hat{i}\ ;\ \vec{B}=\frac{1}{c}E\hat{k}$

(B) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left(\frac{2\pi}{\lambda}(y+ct)\right)\hat{k}\ ;\ \vec{B}=\frac{1}{c}E\hat{i}$

(C) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left(\frac{2\pi}{\lambda}(y-ct)\right)\hat{k}\ ;\ \vec{B}=\frac{1}{c}E\hat{i}$

(D) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 c}}\cos\left(\frac{2\pi}{\lambda}(y-ct)\right)\hat{k}\ ;\ \vec{B}=\frac{1}{c}E\hat{i}$

11. [Online 2017]

Magnetic field in a plane electromagnetic wave is given by $\vec{B}=B_0\sin(kx+\omega t)\hat{j}$ T

Expression for corresponding electric field will be (Where c is speed of light.)

(A) $\vec{E}=-B_0 c\sin(kx+\omega t)\hat{k}\ \text{Vm}^{-1}$

(B) $\vec{E}=B_0 c\sin(kx-\omega t)\hat{k}\ \text{Vm}^{-1}$

(C) $\vec{E}=\frac{B_0}{c}\sin(kx+\omega t)\hat{k}\ \text{Vm}^{-1}$

(D) $\vec{E}=B_0 c\sin(kx+\omega t)\hat{k}\ \text{Vm}^{-1}$

12. [Online 2017]

The electric field component of a monochromatic radiation is given by $\vec{E}=2E_0\hat{i}\cos kz\cos\omega t$

Its magnetic field is then given by

(A) $\frac{2E_0}{c}\hat{j}\cos kz\cos\omega t$ (B) $\frac{2E_0}{c}\hat{j}\sin kz\cos\omega t$

(C) $\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$ (D) $-\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$

13. [2016]

Arrange the following electromagnetic radiations per quantum in the order of increasing energy

A : Blue light B : Yellow light
C : X-ray D : Radio wave

(A) D, B, A, C (B) A, B, D, C
(C) C, A, B, D (D) B, A, D, C

14. [Online 2016]

Microwave oven acts on the principle of

(A) giving rotational energy to water molecules
(B) giving translational energy to water molecules
(C) giving vibrational energy to water molecules
(D) transferring electrons from lower to higher energy levels in water molecule.

15. [Online 2016]

Consider an electromagnetic wave propagating in vacuum. Choose the correct statement.

(A) For an electromagnetic wave propagating in $+y$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{z}$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_z(x,t)y$

(B) For an electromagnetic wave propagating in $+y$ direction the electric fields is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{y}$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(x,t)\hat{z}$

(C) For an electromagnetic wave propagating in $+x$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(y,z,t)(\hat{y}+\hat{z})$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(y,z,t)(\hat{y}+\hat{z})$

(D) For an electromagnetic wave propagating in $+x$ direction the electric field is $E=\frac{1}{\sqrt{2}}E_{yz}(x,t)(\hat{y}-\hat{z})$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(x,t)(\hat{y}+\hat{z})$

16. [2015]

A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

(A) 5.48 Vm^{-1} (B) 7.75 Vm^{-1}
(C) 1.73 Vm^{-1} (C) 2.45 Vm^{-1}

17. [Online 2015]

An electromagnetic wave travelling in the x-direction has frequency of 2×10^{14} Hz and electric field amplitude of 27 Vm^{-1}. From the options given below, which one describes the magnetic field for this wave?

(A) $\vec{B}(x,t)=(3\times10^{-8}\ \text{T})\hat{j}\sin[2\pi(1.5\times10^{-8}x-2\times10^{14}t)]$

(B) $\vec{B}(x,t)=(9\times10^{-8}\ \text{T})\hat{k}\sin[2\pi(1.5\times10^{-6}x-2\times10^{14}t)]$

(C) $\vec{B}(x,t)=(9\times10^{-8}\ \text{T})\hat{i}\sin[2\pi(1.5\times10^{-8}x-2\times10^{14}t)]$

(D) $\vec{B}(x,t)=(9\times10^{-8}\ \text{T})\hat{j}\sin(1.5\times10^{-6}x-2\times10^{14}t)$

18. [Online 2015]

For plane electromagnetic waves propagating in the z direction, which one of the following combinations gives the correct possible direction for field respectively?

(A) $(\hat{i}+2\hat{j})$ and $(2\hat{i}-\hat{j})$

(B) $(-2\hat{i}-3\hat{j})$ and $(3\hat{i}-2\hat{j})$

(C) $\left(2\hat{i}+3\hat{j}\right)$ and $\left(\hat{i}+2\hat{j}\right)$

(D) $\left(3\hat{i}+4\hat{j}\right)$ and $\left(4\hat{i}-3\hat{j}\right)$

19. [2014]

Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists:

List-I	List-II
(P) Infrared waves	(i) To treat muscular strain
(Q) Radio waves	(ii) For broadcasting
(R) X-rays	(iii) To detect fracture of bones
(S) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	P	Q	R	S
(A)	(i)	(ii)	(iii)	(iv)
(B)	(iv)	(iii)	(ii)	(i)
(C)	(i)	(ii)	(iv)	(iii)
(D)	(iii)	(ii)	(i)	(iv)

20. [2014]

During the propagation of electromagnetic waves in a medium

(A) both electric and magnetic energy densities are zero

(B) electric energy density is double of the magnetic energy density

(C) electric energy density is half of the magnetic energy density

(D) electric energy density is equal to the magnetic energy density

21. [2013]

The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT . The peak value of electric field strength is

(A) $12\ \text{Vm}^{-1}$ (B) $3\ \text{Vm}^{-1}$

(C) $6\ \text{Vm}^{-1}$ (D) $9\ \text{Vm}^{-1}$

22. [2012]

An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(A) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{E} \times \vec{B}$

(B) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{E} \times \vec{B}$

(C) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{B} \times \vec{E}$

(D) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{B} \times \vec{E}$

ANSWER KEYS–PRACTICE EXERCISES

Single Correct Choice Type Questions

1. C	2. C	3. D	4. B	5. C	6. C	7. B	8. C	9. B	10. A
11. B	12. B	13. A	14. B	15. C	16. A	17. B	18. A	19. D	20. A
21. A	22. A	23. C	24. A	25. B	26. D	27. D	28. A	29. C	30. B
31. D	32. D	33. B	34. A	35. B	36. C	37. C	38. C	39. C	40. D
41. C	42. A	43. B	44. B	45. D	46. D	47. C	48. D	49. B	50. B
51. A	52. B	53. B	54. C	55. B	56. D	57. B	58. D	59. A	60. B
61. D	62. B	63. A	64. D	65. C	66. A	67. A	68. C		

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1. A	2. C	3. A	4. C	5. A	6. C	7. C	8. B	9. D	10. C
11. D	12. C	13. A	14. A	15. D	16. D	17. *	18. B	19. A	20. D
21. C	22. A								

* No given option is correct

HINTS AND EXPLANATIONS

CHAPTER 1: MAGNETIC EFFECTS OF CURRENT

Test Your Concepts-I (Based on Force and Fleming's Left Hand Rule)

1. Since $v = \dfrac{E}{B}$

$\Rightarrow \left[\dfrac{E}{B}\right] = [\text{Velocity}] = LT^{-1}$

2. Since $\vec{F} = q(\vec{v} \times \vec{B})$

By properties of cross product, we know that

$\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$. Hence

(a) $\vec{v} \cdot (\vec{v} \times \vec{B}) = \vec{v} \cdot \left(\dfrac{\vec{F}}{q}\right) = 0$

(b) $\vec{B} \cdot (\vec{v} \times \vec{B}) = \vec{B} \cdot \left(\dfrac{\vec{F}}{q}\right) = 0$

(c) $\vec{v} \times (\vec{v} \times \vec{B}) = (\vec{v} \cdot \vec{v})\vec{B} - (\vec{v} \cdot \vec{B})\vec{v}$

Since $\vec{v} \cdot \vec{B} = 0$

$\Rightarrow \quad \vec{v} \times (\vec{v} \times \vec{B}) = v^2 \vec{B}$

3. Since we have read, $\vec{F}_m \perp \vec{B}$ i.e., the acceleration $\vec{a} \perp \vec{B}$ or $\vec{a} \cdot \vec{B} = 0$

$\Rightarrow \quad (x\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} - 10\hat{k}) = 0$

$\Rightarrow \quad 3x + 8 - 20 = 0$

$\Rightarrow \quad x = 4$

4. $\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 \times 10^4 & 0 \\ 1.63 & 0.98 & 0 \end{vmatrix}$

$\Rightarrow \quad \vec{F} = q\left[\hat{i}(0) - \hat{j}(0) + \hat{k}(0 - (3 \times 10^4)(1.63))\right]$

$\Rightarrow \quad \vec{F} = -(1.22 \times 10^{-8})(3 \times 10^4)(1.63)\hat{k}$

$\Rightarrow \quad \vec{a} = \dfrac{\vec{F}}{m} = \dfrac{(1.22 \times 10^{-8})(3 \times 10^4)(1.63)}{1.81 \times 10^{-3} \text{ kg}}$

$\Rightarrow \quad \vec{a} = -(0.33)\hat{k} \text{ ms}^{-2}$

5. (a) $\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & -1.25 \end{vmatrix}$

$\Rightarrow \quad \vec{F} = q\left[\hat{i}(-1.25v_y) - \hat{j}(-1.25v_x) + \hat{k}(0 - 0)\right]$

$\Rightarrow \quad \vec{F} = q\left[-1.25v_y\hat{i} + 1.25v_x\hat{j}\right] \qquad \ldots(1)$

However

$\vec{F} = \left(-3.4 \times 10^{-7}\hat{i} + 7.4 \times 10^{-7}\hat{j}\right) \text{ N} \qquad \ldots(2)$

From (1) and (2), by equating the components, we get

$\Rightarrow \quad -3.4 \times 10^{-7} = -1.25qv_y$

$\Rightarrow \quad v_y = \dfrac{-3.4 \times 10^{-7}}{-(1.25)(-5.6 \times 10^{-9})} = -48.6 \text{ ms}^{-1}$

Similarly, we get

$7.4 \times 10^{-7} = 1.25qv_x$

$\Rightarrow \quad v_x = \dfrac{7.4 \times 10^{-7}}{(1.25)(-5.6 \times 10^{-9})} = -106 \text{ ms}^{-1}$

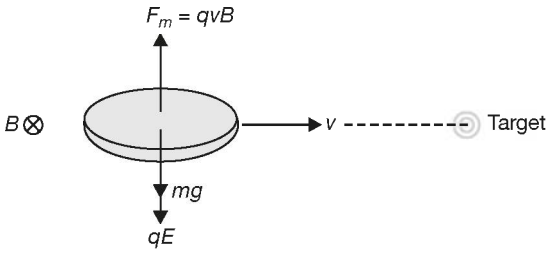
(b) Yes, $v_z = 0$

(c) Since, we know that $\vec{F}_m \perp \vec{v}$ as well as $\vec{B}$. So

$\vec{v} \cdot \vec{F} = 0$

$\Rightarrow$ Angle between $\vec{v}$ and $\vec{F}$ is $\dfrac{\pi}{2}$.

6. The direction of B must be perpendicular to the initial velocity of the coin as shown.



For equilibrium, we must have

$qvB = mg + qE$

$\Rightarrow \quad B = \dfrac{mg + qE}{qv}$

$\Rightarrow \quad B = \dfrac{(2500 \times 10^{-6})(27.5) + (5 \times 10^{-3})(9.8)}{(2500 \times 10^{-6})(12.8)}$

$\Rightarrow \quad B = 3.7 \text{ T}$

7. Given that

$m = 2 \text{ g} = 2 \times 10^{-3} \text{ kg}$, $v = 5 \text{ ms}^{-1}$,

$B = 0.5 \text{ T}$, $q = 1 \text{ mC} = 10^{-3} \text{ C}$

Since the magnetic force is given by

$\vec{F} = q(\vec{v} \times \vec{B})$

$\Rightarrow \quad F = qvB\sin\theta$

$\Rightarrow \quad F = (10^{-3})(5)(0.5)\sin(90°)\ \text{N}$

$\Rightarrow \quad F = 2.5\times10^{-3}\ \text{N}$

The acceleration a is given by

$$a = \frac{F}{m}$$

$$\Rightarrow \quad a = \frac{2.5\times10^{-3}}{2\times10^{-3}}\ \text{ms}^{-2}$$

$$\Rightarrow \quad a = 1.25\ \text{ms}^{-2}$$

8. In the second part of the question, it is given that magnetic force is along x-axis when velocity is along z-axis. Hence magnetic field should be along negative y-direction. As in case of positive charge (here proton).

$$\vec{F}_m \uparrow\uparrow \vec{v}\times\vec{B}$$

So, let $\vec{B} = -B_0\hat{j}$ where B_0 = positive constant

Now applying $\vec{F}_m = q(\vec{v}\times\vec{B})$ we can find value of B_0 from the first part of the question. Substituting proper values in,

$$\vec{F}_m = q(\vec{v}\times\vec{B}) \qquad \ldots(1)$$

Since $\vec{v} = (2\hat{i}+3\hat{j})\times10^6\ \text{ms}^{-1}$ and $\vec{B} = -B_0\hat{j}$

$$\Rightarrow \quad \vec{v}\times\vec{B} = -(2B_0\times10^6)\hat{k}$$

Also, it is given that $\vec{F}_m = -(1.28\times10^{-13}\hat{k})\ \text{N}$ and charge on the proton is $q = 1.6\times10^{-19}\ \text{C}$, so on substituting these values in equation (1), we get

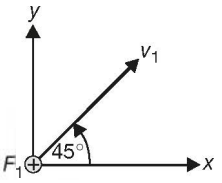
$$-(1.28\times10^{-13}\hat{k}) = (1.6\times10^{-19})(-2B_0\times10^6)\hat{k}$$

$$\Rightarrow \quad 1.28 = 1.6\times2\times B_0$$

$$\Rightarrow \quad B_0 = \frac{1.28}{3.2} = 0.4$$

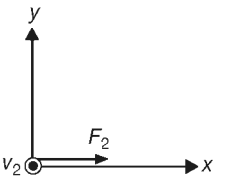
Therefore, the magnetic field is, $\vec{B} = (-0.4\hat{j})\ \text{T}$

9. (a) The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in figure



Since, $\vec{F} = q\vec{v}\times\vec{B}$ so, $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in figure



$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Both pieces of information taken together will give $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$

To calculate F_y, we have

$$\vec{F}_2 = F_2\hat{i},\ \vec{v}_2 = v_2\hat{k},\ \vec{B} = B_y\hat{j}$$

$$\vec{F}_2 = q\vec{v}_2\times\vec{B}$$

$$\Rightarrow \quad F_2\hat{i} = qv_2B_y\hat{k}\times\hat{j} = qv_2B_y(-\hat{i}) \text{ and}$$

$$F_2 = -qv_2B_y$$

$$B_y = -\frac{F_2}{(qv_2)} = -\frac{F_2}{(qv_1)}$$

$\vec{B}$ has the magnitude $\dfrac{F_2}{(qv_1)}$ and is in the $-y$-direction

(b) $F_1 = qvB\sin\phi = \dfrac{qv_1|B_y|}{\sqrt{2}} = \dfrac{F_2}{\sqrt{2}}$

10. The acceleration of the particle is given by

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}(\vec{v}\times\vec{B})$$

Given that, $\dfrac{q}{m} = 0.2\ \text{Ckg}^{-1}$

$$\vec{v} = (2\hat{i}-3\hat{j})\ \text{ms}^{-1}$$

$$\vec{B} = (5\hat{i}+2\hat{j})\ \text{T}$$

$$\Rightarrow \quad \vec{a} = 0.2(2\hat{i}-3\hat{j})\times(5\hat{i}+2\hat{j})\ \text{ms}^{-2}$$

$$\Rightarrow \quad \vec{a} = 0.2(4\hat{k}+15\hat{k})\ \text{ms}^{-2}$$

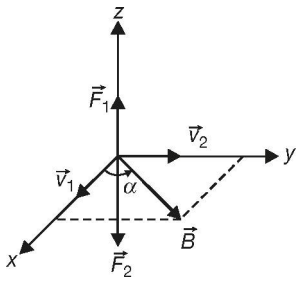
$$\Rightarrow \quad \vec{a} = 3.8\hat{k}\ \text{ms}^{-2}$$

11. Applying Flemings Left Hand Rule to the first proton, we get $\vec{B}$ towards the $+y$-axis from $\vec{v}_1$, while when applied to the second proton requires $\vec{B}$ lie towards the $+x$-axis from $\vec{v}_2$. Thus $\vec{B}$ lies in the first quadrant of the xy-plane. The force on each proton is

$$F_1 = qv_1B\sin\alpha$$

$$F_2 = qv_2B\sin(90°-\alpha) = qv_1B\cos\alpha$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{v_1}{v_2}\tan\alpha$$



$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{2\times10^{6}\ \text{ms}^{-1}}{10^{6}\ \text{ms}^{-1}}\cdot\frac{1.2\times10^{-16}\ \text{N}}{4.16\times10^{-16}\ \text{N}}\right) = 30°$$

The magnetic field strength is thus

$$B = \frac{F_1}{qv_1\sin\alpha} = \frac{1.2\times10^{-16}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(10^{6}\ \text{ms}^{-1})\sin 30°}$$

$$B = 1.5\times10^{-3}\ \text{T}$$

$$\Rightarrow \quad \vec{B} = (1.5\ \text{mT}, 30°\ \text{CCW from}\ +x\text{-axis})$$

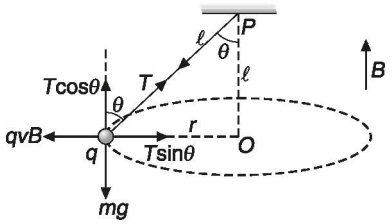
12. Do yourself

$$F_x = 0$$

$$F_y = 2.49\times10^{-3}\ \text{N}$$

$$F_z = 1.32\times10^{-3}\ \text{N}$$

13. The situation described in problem is shown in figure.



If T be the tension in the string then, we have

$$T\cos\theta = mg \qquad \ldots(1)$$

and $$T\sin\theta - qvB = \frac{mv^2}{r} \qquad \ldots(2)$$

From equation (1), we get

$$T = \frac{mg}{\cos\theta}$$

Also, if ω be the angular speed of the ball then

$$v = r\omega\ \text{, where}\ \omega = \frac{2\pi}{T_0}$$

So, from (2), we get

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta - qB(r\omega) = mr\omega^2$$

From figure, we have

$$\sin\theta = \frac{r}{l}\ \text{and}\ \cos\theta = \frac{\sqrt{l^2-r^2}}{l}$$

$$\Rightarrow \quad \frac{mgr}{\sqrt{l^2-r^2}} - qBr\omega = mr\omega^2$$

$$\Rightarrow \quad \frac{1}{\sqrt{l^2-r^2}} = \frac{\omega^2}{g} + \frac{qB\omega}{mg}$$

$$\Rightarrow \quad \frac{1}{l^2-r^2} = \left(\frac{\omega^2}{g} + \frac{qB\omega}{mg}\right)^2$$

$$\Rightarrow \quad \frac{1}{l^2-r^2} = \left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2$$

$$\Rightarrow \quad l^2 - r^2 = \frac{1}{\left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2}$$

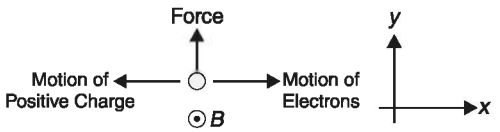
$$\Rightarrow \quad r^2 = l^2 - \frac{1}{\left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2}$$

$$\Rightarrow \quad r = \sqrt{l^2 - \frac{1}{\left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2}}$$

Test Your Concepts-II (Based on Charged Particle in a Magnetic Field)

1. Since $\vec{F} = q(\vec{v}\times\vec{B})$ and the beam consists of charged particles. So, the particles must experience some force. However, if the beam of particles enters parallel or anti parallel to the field, then either $\theta = 0°$ or $\theta = 180°$. Hence the charged particles will not experience any force.

2. According to Fleming's Left Hand Rule, we get the direction of $\vec{B}$ to be inwards along the $+z$ -axis.



3. Yes, No (Think Why?)

4. $r = \frac{mv}{qB}$ and $f = \frac{qB}{2\pi m}$

(a) For an electron, $r_e = \frac{m_e v}{eB}$

For a proton, $r_p = \frac{m_p v}{eB}$

Since $m_p > m_e$, so $r_p > r_e$ i.e., an electron describes a smaller circle

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(b) $f_e = \dfrac{eB}{2\pi m_e}$ and $f_p = \dfrac{eB}{2\pi m_p}$

Since $m_p > m_e$, so $f_e > f_p$ i.e., frequency of electron will be more

5. (a) The kinetic energy K is given by

$$K = \frac{1}{2}mv^2 = 6 \text{ MeV}$$

$$\Rightarrow \quad K = (6\times10^6)(1.6\times10^{-19}) \text{ J}$$

$$\Rightarrow \quad K = 9.6\times10^{-13} \text{ J} = \frac{1}{2}mv^2$$

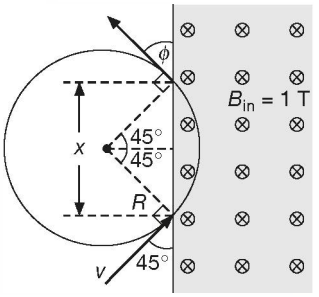
$$v = \sqrt{\frac{2(9.6\times10^{-13} \text{ J})}{1.67\times10^{-27} \text{ kg}}} = 3.39\times10^7 \text{ ms}^{-1}$$

Since, $F = qvB = \dfrac{mv^2}{R}$

$$\Rightarrow \quad R = \frac{mv}{qB}$$

$$\Rightarrow \quad R = \frac{(1.67\times10^{-27} \text{ kg})(3.39\times10^7 \text{ ms}^{-1})}{(1.6\times10^{-19} \text{ C})(1 \text{ T})}$$

$$\Rightarrow \quad R = 0.354 \text{ m}$$



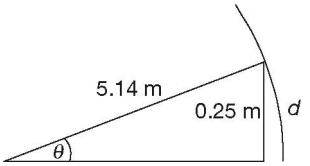
From the diagram, we get

$$x = 2R\sin(45°) = 2(0.354 \text{ m})\sin(45°) = 0.501 \text{ m}$$

(b) From the diagram, observe that $\phi = 45°$

6. (a) $R = \dfrac{mv}{qB} = \dfrac{(3.2\times10^{-11} \text{ kg})(1.45\times10^5 \text{ ms}^{-1})}{(2.15\times10^{-6} \text{ C})(0.42 \text{ T})}$

$$\Rightarrow \quad R = 5.14 \text{ m}$$



(b) The distance along the curve, d, is given by

$$d = R\theta = (5.14)\sin^{-1}\left(\frac{0.25}{5.14}\right) = 0.25 \text{ m}$$

Since, $t_1 = \dfrac{d}{v} = \dfrac{0.25 \text{ m}}{1.45\times10^5 \text{ ms}^{-1}}$

$$\Rightarrow \quad t_1 = 1.72\times10^{-6} \text{ s}$$

(c) $\Delta x_1 = d\tan\left(\dfrac{\theta}{2}\right) = (0.25 \text{ m})\tan\left(\dfrac{2.79°}{2}\right)$

$$\Rightarrow \quad \Delta x_1 = 6.08\times10^{-3} \text{ m}$$

(d) $\Delta x = \Delta x_1 + \Delta x_2$

$$\Rightarrow \quad \Delta x = 6.08\times10^{-3} \text{ m} + (0.5 \text{ m})\tan(2.79°)$$

$$\Rightarrow \quad \Delta x = 0.0304 \text{ m}$$

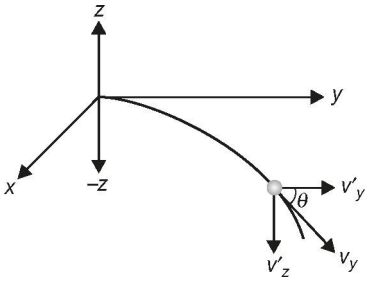
7. Since, $\vec{F} = q(\vec{v}\times\vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ B & 0 & 0 \end{vmatrix}$

$$\Rightarrow \quad \vec{F} = -(qBv_y)\hat{k}$$

The component v_x of velocity will remain unaltered, because $\vec{F}$ acting along z-direction.

Since the particle will follow a circular path in the $(y, -z)$ plane, therefore

$$\vec{v} = v_x'\hat{i} + v_y'\hat{j} - v_z'\hat{k}$$



where $v_x' = v_x$

$$v_y' = v_y\cos\theta = v_y\cos(\omega t) \text{ and}$$

$$v_z' = v_y\sin\theta = v_y\sin(\omega t)$$

where $\omega = \dfrac{eB}{m}$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\cos\left(\frac{eBt}{m}\right)\hat{j} - v_y\sin\left(\frac{eBt}{m}\right)\hat{k}$$

8. $\dfrac{1}{2}mv^2 = q(\Delta V)$

$$\Rightarrow \quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

Since, $r = \dfrac{mv}{qB}$

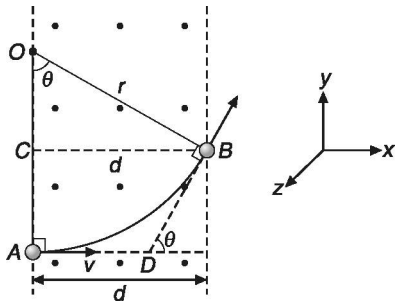
$$\Rightarrow \quad r = \frac{m\sqrt{\frac{2q(\Delta V)}{m}}}{qB}$$

$$\Rightarrow \quad r^2 = \frac{m}{q}\frac{2(\Delta V)}{B^2} \text{ and } (r')^2 = \frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$$

$$\Rightarrow \quad m = \frac{qB^2r^2}{2(\Delta V)} \text{ and } (m') = \frac{(q')B^2(r')^2}{2(\Delta V)}$$

$$\Rightarrow \quad \frac{m'}{m} = \left(\frac{q'}{q}\right)\left(\frac{r'}{r^2}\right)^2 = \left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2 = 8$$

9. The figure shows the particle entering the region at point A and leaving the region at point B.



We draw normal to the circular path followed by the particle, these normal meet at point O, centre of the circle. Applying geometry, we see that the deviation $(\angle ADB)$ of particle is same as the angular displacement of the particle about centre O i.e., θ.

$$\sin\theta = \frac{d}{r} = \frac{qBd}{mv}$$

So, the deviation is given by

$$\theta = \sin^{-1}\left(\frac{qBd}{mv}\right)$$

10. Since every electron passes through the same potential difference and the angle ϕ is small so, they all require the same time interval to travel the axial distance d. Now, all the electrons are all fired from the electron gun with the same speed v. Hence, we have

$$U_i = K_f$$

$$\Rightarrow \quad qV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad (-e)(-\Delta V) = \frac{1}{2}m_e v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2e\Delta V}{m_e}}$$

For ϕ small, $\cos\phi$ is nearly equal to 1. The time T of passage of each electron in the chamber is given by

$$d = vT$$

$$\Rightarrow \quad T = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

Each electron moves in a different helix, around a different axis. If each completes just one revolution within the chamber, it will be in the right place to pass through the exit port. Its transverse velocity component $v_\perp = v\sin\phi$ swings around according to $F_\perp = ma_\perp$

$$qv_\perp B\sin 90° = \frac{mv_\perp^2}{r}$$

$$\Rightarrow \quad eB = \frac{m_e v_\perp}{r} = m_e\omega = m_e\frac{2\pi}{T}$$

$$\Rightarrow \quad T = \frac{m_e 2\pi}{eB} = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

$$\Rightarrow \quad \frac{2\pi}{B}\left(\frac{m_e}{e}\right)^{1/2} = \frac{d}{(2\Delta V)^{1/2}}$$

$$\Rightarrow \quad B = \frac{2\pi}{d}\left(\frac{2m_e\Delta V}{e}\right)^{1/2}$$

11. $q(\Delta V) = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

Since $qvB = \frac{mv^2}{r}$

$$\Rightarrow \quad r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{2m(\Delta V)}{qB^2}}$$

$$\Rightarrow \quad r_p^2 = \frac{2m_p(\Delta V)}{eB^2}$$

$$\Rightarrow \quad r_d^2 = \frac{2m_d(\Delta V)}{q_d B^2} = \frac{2(2m_p)(\Delta V)}{eB^2}$$

$$\Rightarrow \quad r_d^2 = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2 \text{ and}$$

$$r_\alpha^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2}$$

$$\Rightarrow \quad r_\alpha^2 = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

$$\Rightarrow \quad r_\alpha = r_d = \sqrt{2}r_p$$

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12. For each electron, $|q|vB\sin(90°) = \frac{mv^2}{r}$

$$v = \frac{eBr}{m}$$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic.

$$K = \frac{1}{2}mv_{1i}^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$$\Rightarrow \quad K = \frac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right)$$

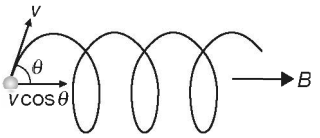
$$\Rightarrow \quad K = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$\Rightarrow \quad K = \frac{\left(1.6\times10^{-19}\right)^2(0.044)^2}{2\left(9\times10^{-31}\right)}\left[(0.01)^2 + (0.024)^2\right]$$

$$\Rightarrow \quad K = 115 \text{ keV}$$

13. $\frac{1}{2}mv^2 = qV$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$



This particle will follow a helical path of pitch p, where

$$p = \frac{2\pi mv\cos\theta}{qB}$$

If another identical particle is launched, at the same moment in the direction of B, then this particle will not experience any force due to B. Hence, if it is launched with a horizontal velocity $v\cos\theta$ then it will meet the first particle again and again during the motion. So,

$$v_2 = v\cos\theta = \left(\sqrt{\frac{2qV}{m}}\right)\cos\theta$$

They will meet just at the completion of first revolution i.e.,

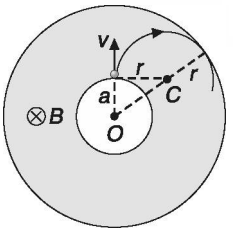
$$T_{MIN} = \frac{2\pi m}{qB}$$

Horizontal distance travelled by the second particle during this time is

$$x = v_2T_{MIN} = \left(\sqrt{\frac{2qV}{m}}\cos\theta\right)\left(\frac{2\pi m}{qB}\right)$$

$$\Rightarrow \quad x = \frac{2\pi\cos\theta}{B}\sqrt{\frac{2mV}{q}}$$

14. The electron follows a circular path with centre C such that it just escapes the outer shell tangentially and does not collide with it as shown in Figure.



The radius of the circular path followed by electron is given by

$$r = \frac{mv}{eB} \qquad \ldots(1)$$

Also, we observe that the length OC is given by

$$b - r = \sqrt{a^2 + r^2}$$

$$\Rightarrow \quad b^2 + r^2 - 2br = a^2 + r^2$$

$$\Rightarrow \quad r = \frac{b^2 - a^2}{2b} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\frac{mv}{eB} = \frac{b^2 - a^2}{2b}$$

$$\Rightarrow \quad v = \frac{eB\left(b^2 - a^2\right)}{2mb}$$

15. (a) $qvB = \frac{mv^2}{R}$

$$\Rightarrow \quad qRB = mv$$

But $L = mvR = qR^2B$

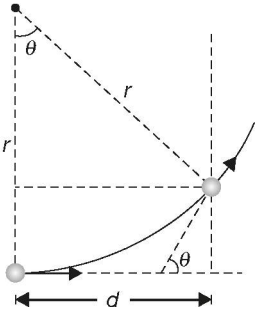
$$\Rightarrow \quad R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4\times10^{-25}\text{ J}\cdot\text{s}}{(1.6\times10^{-19}\text{ C})(1\times10^{-3}\text{ T})}}$$

$R = 0.05 \text{ m} = 5 \text{ cm}$

(b) Since, $v = \frac{L}{mR} = \frac{4\times10^{-25}\text{ J}\cdot\text{s}}{\left(9\times10^{-31}\text{ kg}\right)(0.05\text{ m})}$

$$\Rightarrow \quad v = 9\times10^6 \text{ ms}^{-1}$$

16. $\frac{1}{2}mv^2 = qV$



$$\Rightarrow \quad v=\sqrt{\frac{2qV}{m}}$$

$$\sin\theta=\frac{d}{r}, \text{ where } r=\frac{mv}{qB}=\frac{1}{B}\sqrt{\frac{2mV}{q}}$$

Since $\theta=\omega t$, where $\omega=\frac{qB}{m}$

$$\Rightarrow \quad \theta=\left(\frac{qB}{m}\right)t$$

where $\theta=\sin^{-1}\left(\frac{d}{R}\right)=\sin^{-1}\left(Bd\sqrt{\frac{q}{2mV}}\right)$

$$\Rightarrow \quad t=\left(\frac{m}{qB}\right)\theta$$

$$\Rightarrow \quad t=\left(\frac{m}{qB}\right)\sin^{-1}\left(Bd\sqrt{\frac{q}{2mV}}\right)$$

17. The component of velocity along the field is

$$v_{\parallel}=2\times10^5\cos(60°)=10^5 \text{ ms}^{-1}$$

and the component of velocity perpendicular to the field is given by

$$v_{\perp}=2\times10^5\sin(60°)=\sqrt{3}\times10^5 \text{ ms}^{-1}$$

Proton will describe a circle in a plane perpendicular to the magnetic field. If r be the radius of the helix then

$$r=\frac{mv_{\perp}}{qB}=\frac{(1.67\times10^{-27})(\sqrt{3}\times10^5)}{(1.6\times10^{-19})(0.3)}$$

$$\Rightarrow \quad r=6\times10^{-3} \text{ m}=0.6 \text{ cm}$$

Time taken to complete one revolution is

$$T=\frac{2\pi r}{v_{\perp}}=\frac{2\times3\cdot14\times0.006}{\sqrt{3}\times10^5} \text{ s}$$

Due to the component of velocity parallel to the magnetic field the protons will also move in the direction of magnetic field. The pitch (p) of the helical path is then given by

$$p=v_{\parallel}T=10^5\left(\frac{2\times3.14\times0.006}{\sqrt{3}\times10^5}\right)$$

$$\Rightarrow \quad p=0.022 \text{ m}=2.2 \text{ cm}$$

18. $$v=\sqrt{\frac{2qV}{m}}=\sqrt{\frac{(2)(1.6\times10^{-19})(1000)}{9.1\times10^{-31}}}$$

$$\Rightarrow \quad v=18.8\times10^6 \text{ ms}^{-1}$$

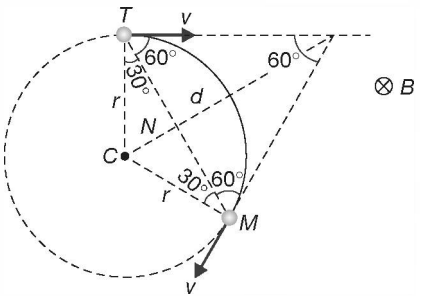
(a) In triangle CMT

$$r\cos30=\frac{d}{2}$$

$$\Rightarrow \quad r\frac{\sqrt{3}}{2}=\frac{d}{2}$$

$$\Rightarrow \quad r=\frac{d}{\sqrt{3}}=\frac{5}{\sqrt{3}} \text{ cm}$$

Since $r=\frac{mv}{qB}$



$$\Rightarrow \quad \frac{0.05}{\sqrt{3}}=\frac{9.1\times10^{-31}\times18.7\times10^6}{1.6\times10^{-19} \text{ B}}$$

$$\Rightarrow \quad B=\frac{(9.1\times10^{-31})(18.8\times10^6)(\sqrt{3})}{(1.6\times10^{-19})(0.05)}$$

$$\Rightarrow \quad B=3.7\times10^{-3} \text{ T}=3.7 \text{ mT}$$

(b) In this case, B is parallel to TM at 60° with the velocity of electrons. So, we have the electrons following a helical path with origin at T and will hit M if it lies on the helix at $TM=n(\text{pitch})$

$$\Rightarrow \quad TM=n\left(\frac{2\pi mv\cos\theta}{qB}\right)$$

$$\Rightarrow \quad B=\frac{n(2\pi mv\cos\theta)}{q(TM)}$$

For B to be MINIMUM, $n=1$

$$\Rightarrow \quad B_{MIN}=\frac{(2\pi)(mv\cos\theta)}{q(TM)}$$

where $TM=\frac{5}{100}$ cm

$$\Rightarrow \quad B_{MIN}=\frac{(2\pi)(9.1\times10^{-31})(18.8\times10^6)(0.5)}{(1.6\times10^{-19})(0.05)}$$

$$\Rightarrow \quad B_{MIN}=6.7\times10^{-3} \text{ T}=6.7 \text{ mT}$$

Test Your Concepts-III (Based on Charged Particle in Magnetic and Electric Field)

1. (a) The net force is the Lorentz Force given by

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{F}=(3.2\times10^{-19})$$

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$$\left[\left(4\hat{i}-1\hat{j}-2\hat{k}\right)+\left(2\hat{i}+3\hat{j}-1\hat{k}\right)\times\left(2\hat{i}+4\hat{j}+1\hat{k}\right)\right]\text{ N}$$

Carrying out the indicated operations, we find

$$\vec{F}=\left(3.52\hat{i}-1.6\hat{j}\right)\times10^{-18}\text{ N}$$

(b) $\theta=\cos^{-1}\left(\frac{F_x}{F}\right)=\cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2+(1.6)^2}}\right)=24.4°$

2. $F_B=F_e$

$\Rightarrow\quad qvB=qE$

where $v=\sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron

$\Rightarrow\quad E=vB=\sqrt{\frac{2K}{m}}B=\sqrt{\frac{2(750)(1.6\times10^{-19})}{9.11\times10^{-31}}}(0.015)$

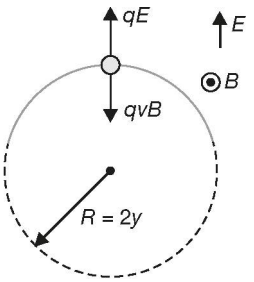
$\Rightarrow\quad E=244\text{ kVm}^{-1}$

3. (a) The maximum speed occurs at the top of the cycloidal path and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y=0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W=Fd=qEd=qEy=\frac{1}{2}mv^2$

$\Rightarrow\quad v=\sqrt{\frac{2qyE}{m}}$

(c) At the top, the net force on the particle $(qvB-qE)$ provides the necessary centripetal force to the particle to be in a circle of radius



$R=2y$

$\Rightarrow\quad qvB-qE=\frac{mv^2}{R}$

$\Rightarrow\quad qvB-qE=\frac{m}{2y}\left(\sqrt{\frac{2qyE}{m}}\right)^2$

$\Rightarrow\quad qvB-qE=qE$

$\Rightarrow\quad qvB=2qE$

$\Rightarrow\quad v=\frac{2E}{B}$

4. Let q be the charge and m the mass of the particle. At $(x_0,\ 0,\ 0)$ speed of the particle is $\sqrt{4^2+3^2}=5$ units

Using Work-Energy Theorem, we get

$$(qE_0)x_0=\frac{1}{2}mv^2=\frac{25m}{2}$$

$\Rightarrow\quad x_0=\frac{25m}{2qE_0}$

5. Since $\vec{E}$ and $\vec{B}$ are parallel to each other and $\vec{v}$ is perpendicular to both, so, the path of the particle is a helix of increasing pitch. The speed of particle at any time t is

$$v=\sqrt{v_x^2+v_y^2+v_z^2}\qquad\ldots(1)$$

Since the magnetic force acts along $-z$ axis, so the particle will have a circular track in yz plane, where

$$v_y^2+v_z^2=v_0^2$$

Due to the electric field along the x-axis, the x velocity of the particle keeps on increasing with time t as

$$v_x=\left(\frac{qE}{m}\right)t\qquad\left\{\because\ a=\frac{qE}{m}\right\}$$

According to the problem, we have

$$v=2v_0$$

Substituting the values in equation (1), we get

$$t=\sqrt{3}\left(\frac{mv_0}{qE}\right)$$

6. The net electric field

$$E=\vec{E}_1+\vec{E}_2$$

$$E=\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$$

The net force acting on the electron is zero because it moves with constant velocity, due to its motion on straight line.

$\Rightarrow\quad \vec{F}_{\text{net}}=\vec{F}_e+\vec{F}_m=0$

$\Rightarrow\quad |\vec{F}_e|=|\vec{F}_m|$

$\Rightarrow\quad eE=evB$

$\Rightarrow\quad v=\frac{E}{B}=\frac{\sigma}{\varepsilon_0 B}$

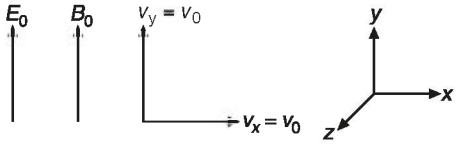
So, the time of motion inside the capacitor is

$t=\frac{\ell}{v}=\frac{\varepsilon_0\ell B}{\sigma}$

7. Since the electric and magnetic field both are parallel to the velocity of the charged particle, so $F_m = 0$ and $F_e = qE_0$. Due to the only electrostatic force acting on the charged particle, it will move along a straight line with uniform acceleration i.e. its speed keeps on increasing with time.

8. The particle will follow circular path in magnetic field due to v_x. So, its time period is

$$T = \frac{2\pi m}{qB_0}$$



However, the pitch of particle in absence of E_0 is

$$p = v_y T = v_0 T$$

But due to an electric field E_0 acting along y-direction the new pitch will be

$$p' = v_0 T + \frac{1}{2} a_y T^2, \text{ where } a_y = \frac{qE_0}{m}$$

$$\Rightarrow \quad p' = T\left(v_0 + \frac{a_y T}{2}\right)$$

$$\Rightarrow \quad p' = \frac{2\pi m}{qB_0}\left[v_0 + \frac{1}{2}\left(\frac{qE_0}{m}\right)\left(\frac{2\pi m}{qB_0}\right)\right]$$

$$\Rightarrow \quad p' = \frac{2\pi m}{qB_0}\left(v_0 + \frac{\pi E_0}{B_0}\right)$$

9. By Work-Energy Theorem, we have

Work done = Change in K.E.

$$\Rightarrow \quad \left(qE\hat{i}\right)\cdot\left(2a\hat{i}\right) = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2aqE = \frac{3}{2}mv^2$$

$$\Rightarrow \quad E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

Rate of work done by $\vec{E}$ at P is $\vec{F}\cdot\vec{v}$

$$\Rightarrow \quad \frac{dW}{dt} = \left(qE\hat{i}\right)\cdot\left(v\hat{i}\right)$$

$$\Rightarrow \quad \frac{dW}{dt} = qEv = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Rate of work done by $\vec{E}$ at Q is zero, because at Q, $\vec{F} \perp \vec{v}$.

Rate of work done by $\vec{B}$ at both the points P and Q is zero.

10. No change in velocity implies no acceleration i.e. no net force is acting on the charged particle under the joint influence of electric and magnetic field. This thing is possible under the following situations.

Situation 1:

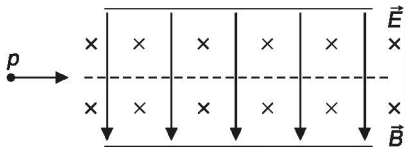
$E = 0$, $B = 0$, i.e. no field exists in the region

Situation 2:

$E = 0$ i.e. no electrostatic force. $B \neq 0$, but the charge particle enters parallel to the field, so that net force equals to zero.

Situation 3:

$E \neq 0$, $B \neq 0$ and both shown in Figure.



Because in such a situation

$F_m = qvB$ (upwards) and

$F_e = qE$ (downwards)

and if both are equal in magnitude then also the charged particle will suffer no change in its velocity and will continue to move along the dotted line as shown. In such a situation, the velocity is $v = \frac{E}{B}$.

Situation 4:

If $E \neq 0$, $B = 0$, then the charged particle must experience an electrostatic force qE and hence must accelerate. So, it will not have a constant velocity.

Hence, we conclude that for the charged particle to move with constant velocity in the region having simultaneous electric and magnetic fields, we may have,

(i) $E = 0$, $B = 0$ (as discussed in Situation 1)
(ii) $E = 0$, $B \neq 0$ (as discussed in Situation 2)
(iii) $E \neq 0$, $B \neq 0$ (as discussed in Situation 3)

Test Your Concepts-IV (Based on Biot Savart's Law and Applications)

1. (a) Since, $B_{\text{centre}} = \frac{\mu_0 I}{2R}$

and $R = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1 \text{ m}$

$$\Rightarrow \quad B_{\text{centre}} = \frac{\left(4\pi \times 10^{-7}\right)(1)}{2(0.1)} = 6.3 \times 10^{-6} \text{ T} = 6.3 \ \mu\text{T}$$

(b) Also, $B_{\text{axis}} = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$

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where $R = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1 \text{ m}$

and $x = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1 \text{ m}$

So, we observe that $x = R = 10^{-5}$ m

$$B_{axis} = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}}$$

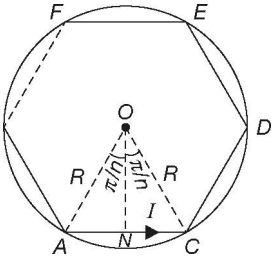
$$B_{axis} = \frac{\mu_0 I R^2}{2R^3 2^{3/2}} = \frac{\mu_0 I}{2R(2\sqrt{2})} = \frac{\mu_0 I}{4\sqrt{2}R}$$

$$B_{axis} = \frac{\mu_0 (1)}{4\sqrt{2} \times (0.1)}$$

$$B_{axis} = \frac{4\pi \times 10^{-7}}{4(1.414)(0.1)} = 2.3 \times 10^{-6} \text{ T}$$

$$\Rightarrow \quad B_{axis} = 2.3 \ \mu\text{T}$$

2. $ON = R\cos\left(\frac{\pi}{n}\right)$



Due to the wire AC, field at the centre O, at perpendicular distance ON is

$$B_{AC} = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi(ON)}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi R\cos\left(\frac{\pi}{n}\right)}\left[2\sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_{AC} = \left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right), \odot$$

Similarly, all wires will give an outward equal field at O. Hence, for an n sided current carrying polygon, we get

$$B_{\text{total}} = nB_{AC} = n\left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right)$$

When the polygon has got infinite sides, then $n \to \infty$ and the polygon approaches the shape of a circle. The magnetic field due to a circular current carrying coil at the centre is $B_{\text{centre}} = \frac{\mu_0 I}{2R}$.

This can also be derived mathematically by taking the limit $n \to \infty$ on B_{total}.

$$B_{\text{total}} = \lim_{n\to\infty} n\left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right)$$

$$\Rightarrow \quad B_{total} = \lim_{n\to\infty}\left(\frac{\mu_0 I\pi}{2\pi R}\right)\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}$$

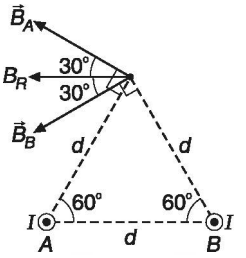
$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{2R}\lim_{n\to\infty}\left[\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}\right]$$

The expression $\lim_{n\to\infty}\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 1$

$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{2R}$$

3. **CASE-1: When the wires carry current in same direction**

The resultant magnetic field B_R is calculated from the given figure. So



$$B_R = 2B\cos(30°) = \sqrt{3}B$$

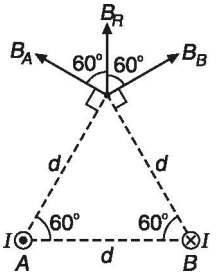
where $|\vec{B}_A| = |\vec{B}_B| = B = \frac{\mu_0 I}{2\pi d} = 10^{-7}\left(\frac{2\times 10}{0.1}\right)$

$$\Rightarrow \quad B = 2 \times 10^{-5} \text{ T}$$

So, $B_R = 2\sqrt{3} \times 10^{-5}$ T

CASE-2: When the wires carry currents in the opposite direction

The resultant magnetic field B_R is calculated from the given figure. So

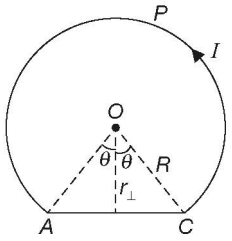


$B_R = 2B\cos(60°) = B$

$\Rightarrow \quad B_R = 2\times10^{-5}$ T

4. $B_{arc} = \left(\frac{\mu_0 I}{4\pi R}\right)\theta$, where θ is in radian

So, $B_{arc} = \frac{\mu_0 I}{4\pi R}(2\pi - 2\theta)$, $\odot$



Due to the straight wire AC,

$$B_{AC} = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\theta + \sin\theta)$$

where $r_\perp = R\cos\theta$

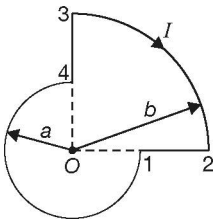
$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi(R\cos\theta)}(2\sin\theta)$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{2\pi R}\tan\theta, \odot$$

So, $B_{total} = B_{arc} + B_{AC}$, $\odot$

$$\Rightarrow \quad B_{total} = \frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta), \odot$$

5. $B_{arc\,41} = \frac{\mu_0 I}{4\pi a}\theta$



$$\Rightarrow \quad B_{arc\,41} = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right) = \frac{3\mu_0 I}{8a}, \otimes$$

$B_{12} = 0$ {$\because$ O lies at extended part of wire 12}

$B_{43} = 0$ {$\because$ O lies at extended part of wire 34}

$$B_{arc\,32} = \frac{\mu_0 I}{4\pi b}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad B_{arc\,32} = \frac{\mu_0 I}{8b}, \otimes$$

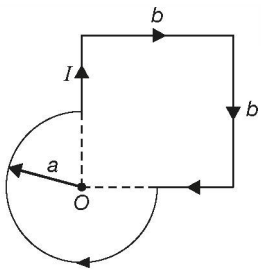
So, $B_{total} = B_{arc\,41} + B_{arc\,32} = \frac{\mu_0 I}{8}\left(\frac{3}{a} + \frac{1}{b}\right)$, $\otimes$

6. $B_{12} = B_{45} = 0$

$$B_{arc\,51} = \frac{3\mu_0 I}{8a}, \otimes$$

$$B_{23} = \frac{\mu_0 I}{4\pi b}(\sin 0 + \sin 45)$$

$$\Rightarrow \quad B_{23} = \frac{\mu_0 I}{(4\sqrt{2})\pi b}, \otimes$$



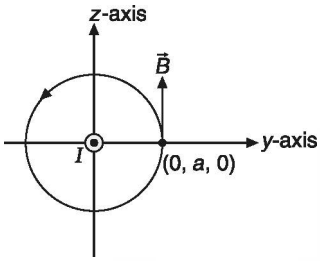
Similarly, $B_{34} = \frac{\mu_0 I}{(4\sqrt{2})\pi b}$, $\otimes$

$$\Rightarrow \quad B_{total} = B_{12} + B_{23} + B_{34} + B_{arc}51, \otimes$$

$$\Rightarrow \quad B_{total} = \frac{\mu_0 I}{4\pi}\left(\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right), \otimes$$

7. **CASE-1: At (0, *a*, 0)**

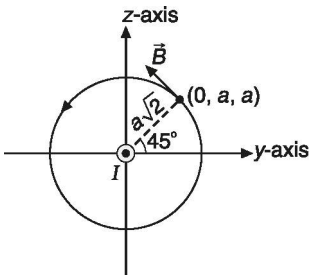
The direction of magnetic field in this case is shown in figure.



Mathematically, we have $\vec{B} = \left(\frac{\mu_0 I}{2\pi a}\right)\hat{k}$

CASE-2: At (0, *a*, *a*)

The direction of magnetic field in this case is shown in figure.



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Since the point lies in y-z plane, so unit vector along the direction of magnetic field $\vec{B}$ is given by

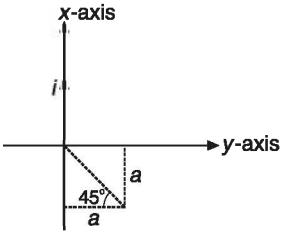
$$\hat{n} = -\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi a\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{8\pi a}\left(-\hat{j} + \hat{k}\right)$$

CASE-3: At $(-a, a, 0)$

In this case we re-align the axis as shown in figure.



The magnetic field at the point $(-a, a, 0)$ is

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(\sin 90° - \sin 45°)\hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi a}\left(1 - \frac{1}{\sqrt{2}}\right)\hat{k}$$

8. Let B_1 be the magnetic field due to circular loop and B_2 be the magnetic field due to straight current carrying wire. Then

$$B_1 = \frac{\mu_0 I}{2r}, \text{ inwards and}$$

$$B_2 = \frac{\mu_0 (4I)}{2\pi(2r)} = \frac{\mu_0 I}{\pi r}, \text{ outwards}$$

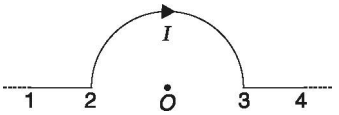
Since $B_1 > B_2$, so the net field B is

$$B = B_1 - B_2 = \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{\pi r}, \text{ inwards}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{r}\left(\frac{1}{2} - \frac{1}{\pi}\right), \text{ inwards}$$

9. The magnetic field due to the wires 12 and 34 at the point O is zero. So

$$B_{12} = B_{34} = 0$$



$$B_{\text{arc }23} = \frac{\mu_0 I}{4R}, \otimes$$

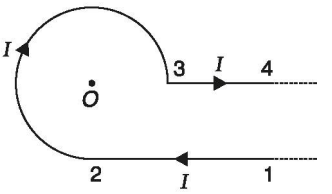
$$\Rightarrow \quad B_{\text{total}} = B_{12} + B_{\text{arc }24} + B_{34} = \frac{\mu_0 I}{4R}, \otimes$$

10. The magnetic field due to the different sections of the conductor are

$$B_{34} = 0$$

$$B_{\text{arc }23} = \frac{3\mu_0 I}{8R}, \otimes$$

$$B_{12} = \frac{\mu_0 I}{4\pi R}(\sin 0° + \sin 90°)$$

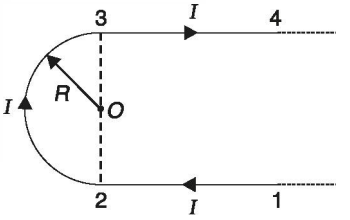


$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi R}, \otimes$$

$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{4\pi R}\left(1 + \frac{3\pi}{2}\right), \otimes$$

11. The magnetic field due to the different sections of the conductor are

$$B_{12} = \frac{\mu_0 I}{4\pi R}\left(\sin 0 + \sin\frac{\pi}{2}\right)$$



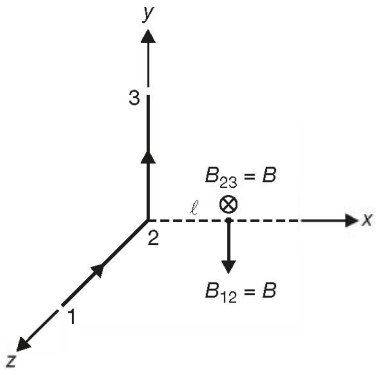
$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi R}, \otimes$$

$$B_{23} = \frac{\mu_0 I}{4R}, \otimes \text{ and } B_{34} = \frac{\mu_0 I}{4\pi R}, \otimes$$

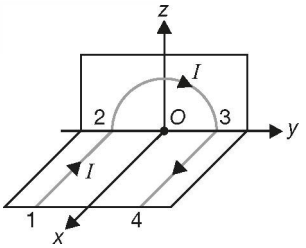
$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{4\pi R}(2 + \pi), \otimes$$

12. $B_{\text{total}} = \sqrt{B^2 + B^2}$

$$\Rightarrow \quad B_{\text{total}} = \sqrt{2}B = \sqrt{2}\left(\frac{\mu_0 I}{4\pi \ell}\right)$$



13. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{\mu_0 I}{4R}(-\hat{i})$$

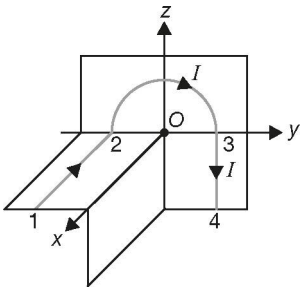
$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\Rightarrow \quad \vec{B} = \vec{B}_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)(\pi\hat{i} + 2\hat{k})$$

$$\Rightarrow \quad |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{\pi^2 + 4}$$

14. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{\mu_0 I}{4R}(-\hat{i})$$

$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{i})$$

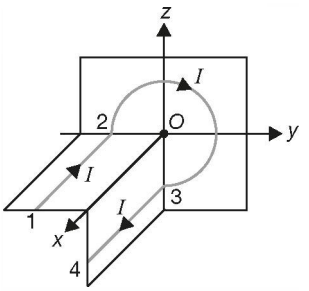
$$\Rightarrow \quad \vec{B} = B_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)[(1+\pi)\hat{i} + \hat{k}]$$

$$\Rightarrow \quad B = |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{(1+\pi)^2 + 1}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 I}{4\pi R}\right)\sqrt{2 + 2\pi + \pi^2}$$

15. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{3\mu_0 I}{8R}(-\hat{i})$$

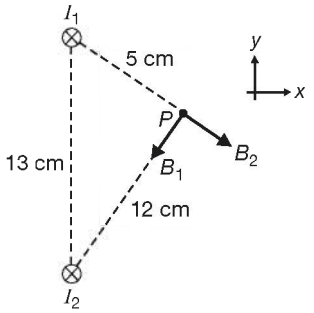
$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{j})$$

$$\Rightarrow \quad \vec{B} = \vec{B}_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)\left[\left(\frac{3\pi}{2}\right)\hat{i} + \hat{j} + \hat{k}\right]$$

$$\Rightarrow \quad B = |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{\frac{9\pi^2}{4} + 2}$$

16. Take the x-direction to the right and the y-direction up in the plane of the paper.



Magnetic field at P due to current I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{(2\times10^{7}\ \text{Tm})(3\ \text{A})}{A(0.05\ \text{m})}$$

$$\Rightarrow \quad B_1 = 12\ \mu\text{T}$$

Downwards and leftwards, at angle 67.4° below the $-x$ axis.

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$$\Rightarrow \quad \vec{B}_2 = (12\ \mu\text{T})\left(-\hat{i}\cos 67.4^\circ - \hat{j}\sin 67.4^\circ\right)$$

Magnetic field at P due to current I_2 is

$$B_2 = \frac{(2\times 10^{-7}\ \text{Tm})(3\ \text{A})}{A(0.12\ \text{m})}$$

$$\Rightarrow \quad B_2 = 5\ \mu\text{T}$$

Clockwise perpendicular to 12 cm to the right and down at angle -22.6°

$$\Rightarrow \quad \vec{B}_2 = (5\ \mu\text{T})\left(\hat{i}\cos 22.6^\circ - \hat{j}\sin 22.6^\circ\right)$$

The total magnetic field $\vec{B}$ is given by

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\Rightarrow \quad \vec{B} = (12)\left(-\hat{i}\cos 67.4^\circ - \hat{j}\sin 67.4^\circ\right) + (5)\left(\hat{i}\cos 22.6^\circ - \hat{j}\sin 22.6^\circ\right)$$

$$\Rightarrow \quad \vec{B} = (-11.1\ \mu\text{T})\hat{j} - (1.92\ \mu\text{T})\hat{j} = (-13\ \mu\text{T})\hat{j}$$

17. Since $I = \dfrac{q}{T} = \dfrac{q}{\left(\dfrac{2\pi r}{v}\right)}$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\left(\frac{qv}{2\pi r}\right)$$

Substituting values, we get

$$B = 12.5\ \text{T}$$

18. Wire 1 creates at the magnetic field at the origin, given by

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r}, \text{ upwards along } +y \text{ axis.}$$

So, $\vec{B} = \left(\dfrac{\mu_0 I}{2\pi a}\right)\hat{j}$

(a) If the total field at the origin is upwards, then

$$\frac{2\mu_0 I_1}{2\pi a}\hat{j} = \frac{\mu_0 I_1}{2\pi a}\hat{j} + \vec{B}_2$$

So, the second wire creates a field given by

$$\vec{B}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{j} = \frac{\mu_0 I_2}{2\pi(2a)}, \odot$$

$$\Rightarrow \quad I_2 = 2I_1 \text{ out of the paper}$$

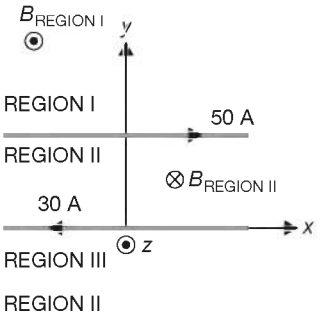
(b) The other possibility is

$$\vec{B}_1 + \vec{B}_2 = \frac{2\mu_0 I_1}{2\pi a}\left(-\hat{j}\right) = \frac{\mu_0 I_1}{2\pi a}\hat{j} + \vec{B}_2$$

$$\Rightarrow \quad \vec{B}_2 = \frac{3\mu_0 I_1}{2\pi a}\left(-\hat{j}\right) = \frac{\mu_0 I_2}{2\pi(2a)}, \otimes$$

$$\Rightarrow \quad I_2 = 6I_1 \text{ into the paper}$$

19. (a) Above the pair of wires, in Region I, both the wires give field out of the page. Between the wires, in Region II, both produce fields into the page.



They can only add to zero below the wires, in Region III, at coordinate y. Here the total field is

$$\vec{B}_{\text{Region III}} = \frac{\mu_0 I_2}{2\pi r}\otimes + \frac{\mu_0 I_1}{2\pi r}, \odot$$

$$\Rightarrow \quad 0 = \frac{\mu_0}{2\pi}\left[\frac{50}{\left(y+\frac{2}{3}\right)} - \frac{30\ \text{A}}{y}\right]$$

$$\Rightarrow \quad 50y = 30\left(y + \frac{2}{3}\right)$$

$$\Rightarrow \quad 50y = 30y + 20$$

$$\Rightarrow \quad 20y = 20$$

$$\Rightarrow \quad y = 1\ \text{m}, \text{ below wire 1}$$

(b) At $y = 0.1$ m the total field is $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\otimes + \dfrac{\mu_0 I}{2\pi r}\otimes$

$$\Rightarrow \quad \vec{B} = \frac{4\pi\times 10^{-7}}{2\pi}\left(\frac{50}{\left(\frac{2}{3}-\frac{1}{3}\right)}\left(-\hat{k}\right) + \frac{30}{\frac{1}{3}}\left(-\hat{k}\right)\right)$$

$$\Rightarrow \quad \vec{B} = 4.8\times 10^{-5}\ T\left(-\hat{k}\right)$$

The force on the particle is

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-2\times 10^{-6}\ \text{C})(150\times 10^{6})(i)\times (4.8\times 10^{-5})(-\hat{k}) = 14.4\times 10^{-3}\ \text{N}\left(-\hat{j}\right)$$

20. On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

where in this case $I = \dfrac{Q}{\left(\dfrac{2\pi}{\omega}\right)}$

$$\Rightarrow \quad B = \frac{\mu_0 \omega R^2 Q}{4\pi\left(x^2+R^2\right)^{3/2}}$$

when $x = \dfrac{R}{2}$

$$\Rightarrow \quad B = \frac{\mu_0 \omega R^2 Q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \frac{2\mu_0 Q\omega}{5\sqrt{5}\,\pi R}$$

21. Let the current I be to the right. It creates a field $B=\frac{\mu_0 I}{2\pi d}$ at the proton's location. And we have an equilibrium between the weight of the proton and the magnetic force $\vec{F}$, where

$$\vec{F}=q\left[v(-\hat{i})\times\left(\frac{\mu_0 I}{2\pi d}\right)\hat{k}\right]$$

$$\Rightarrow\quad mg(-\hat{j})+qv(-\hat{i})\times\frac{\mu_0 I}{2\pi d}(\hat{k})=0 \quad \text{at a distance } d \text{ from the wire}$$

$$\Rightarrow\quad d=\frac{qv\mu_0 I}{2\pi mg}$$

22. There is no magnetic field at O due to the straight portions of the current carrying loop.
Field due to an arc of radius r subtending an angle θ at the centre of the arc is given by

$$B=\frac{\mu_0 I}{4\pi R}\theta$$

Due to arc of radius $4R$, field is

$$B_1=\left(\frac{\mu_0 I}{4\pi(4R)}\right)\left(\frac{\pi}{2}\right)=\frac{\mu_0 I}{32R}$$

Due to arch of radius $2R$, field is

$$B_2=\left(\frac{\mu_0 I}{4\pi(2R)}\right)(\pi)=\frac{\mu_0 I}{8R}$$

Due to arc of radius R, field is

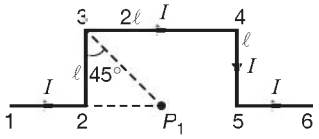
$$B_3=\left(\frac{\mu_0 I}{4\pi R}\right)\left(\frac{\pi}{2}\right)=\frac{\mu_0 I}{8R}$$

Since the current is anticlockwise in all three arcs, therefore, all the fields are outwards towards the reader. So, the net magnetic field is given by

$$B_{\text{net}}=B_1+B_2+B_3$$

$$\Rightarrow\quad B_{\text{net}}=\frac{\mu_0 I}{R}\left(\frac{1}{32}+\frac{1}{8}+\frac{1}{8}\right)=\frac{9\mu_0 I}{32R}$$

23. For the figure (a), we have



$$B_{12}=B_{56}=0$$

$$B_{23}=\frac{\mu_0 I}{4\pi\ell}\left[\sin(0)+\sin\left(\frac{\pi}{4}\right)\right]=\frac{\mu_0 I}{4\sqrt{2}\pi\ell},\ \otimes$$

$$B_{45}=B_{23}=\frac{\mu_0 I}{4\sqrt{2}\pi\ell},\ \otimes$$

$$B_{34}=\frac{\mu_0 I}{4\pi\ell}\left[\sin\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\right]=\frac{\sqrt{2}\mu_0 I}{4\pi\ell},\ \otimes$$

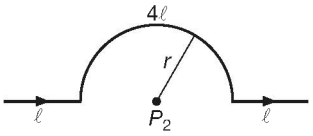
So, field at P_1 is B_1 given by

$$\Rightarrow\quad B_1=\frac{\mu_0 I}{4\pi\ell}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right),\ \otimes \qquad \ldots(1)$$

For figure (b), we observe that if r is the radius of the semicircle, then

$$\pi r=4\ell \qquad \{\because \text{ total length}=6\ell\}$$

$$\Rightarrow\quad r=\frac{4\ell}{\pi} \qquad \ldots(2)$$



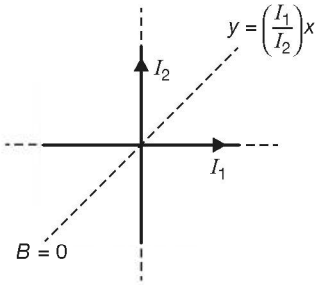
So, field at P_2 is given by,

$$B_2=\frac{\mu_0 I}{4r}=\frac{\mu_0 I}{4\left(\frac{4\ell}{\pi}\right)}=\pi\left(\frac{\mu_0 I}{16\ell}\right),\ \otimes$$

$$\frac{B_1}{B_2}=\frac{4}{\pi^2}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)$$

24. Let the field be zero at the point $P(x,\ y)$, then

$$\frac{\mu_0 I_1}{2\pi y}\odot+\frac{\mu_0 I_2}{2\pi x}\otimes=0$$



$$\Rightarrow\quad \frac{I_1}{y}-\frac{I_2}{x}=0$$

$$\Rightarrow\quad y=\left(\frac{I_1}{I_2}\right)x$$

25. The net field is the vector sum of the fields due to the circular loop and to the long straight wire, both at the centre of the loop.
For the long wire, $B=\frac{\mu_0 I}{2\pi D}$, and for the loop, $B=\frac{\mu_0 I_0}{2R}$
At the center of the circular loop the current I_0 generates a magnetic field that is into the page, so the current I must point to the right. For complete cancellation the two fields must have the same magnitude.

$$\vec{B}_{\text{loop}} + \vec{B}_{\text{wire}} = \vec{0}$$

$$\Rightarrow \quad \left|\vec{B}_{\text{wire}}\right| = \left|\vec{B}_{\text{loop}}\right|$$

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi D} = \frac{\mu_0 I_0}{2R}$$

$$\Rightarrow \quad I = \left(\frac{\pi D}{R}\right) I_0$$

Test Your Concepts-V (Based on Ampere's Circuital Law and Applications)

1. Apply Ampere's Law to a circle of radius r. The current within a radius r is $I = \int \vec{J} \cdot d\vec{A}$, where the integration is over a disk of radius r.

(a) $I_0 = \int \vec{J} \cdot d\vec{A}$

$$\Rightarrow \quad I = \int \left(\frac{b}{r} e^{(r-a)/\delta}\right) r\,dr\,d\theta$$

$$\Rightarrow \quad I = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b\delta e^{(r-a)/\delta}\Big|_0^a$$

$$\Rightarrow \quad I = 2\pi b\delta\left(1 - e^{-a/\delta}\right)$$

(b) For $r \geq a$, we have

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl}} = \mu_0 I_0$$

$$\Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$

(c) For $r \leq a$, we have

$$I(r) = \int \vec{J} \cdot d\vec{A} = \int_0^r \int_0^{2\pi} \left(\frac{b}{r} e^{(r-a)/\delta}\right) r\,dr\,d\theta$$

$$\Rightarrow \quad I(r) = 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b\delta e^{(r-a)/\delta}\Big|_0^r$$

$$\Rightarrow \quad I(r) = 2\pi b\delta\left(e^{(r-a)/\delta} - e^{-a/\delta}\right)$$

$$\Rightarrow \quad I(r) = 2\pi b\delta e^{-a/\delta}\left(e^{r/\delta} - 1\right)$$

$$\Rightarrow \quad I(r) = I_0 \frac{\left(e^{r/\delta} - 1\right)}{\left(e^{a/\delta} - 1\right)}$$

(d) For $r \leq a$, we have

$$\oint \vec{B} \cdot d\vec{\ell} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \frac{\left(e^{r/\delta} - 1\right)}{\left(e^{a/\delta} - 1\right)}$$

$$\Rightarrow \quad B = \frac{\mu_0 I_0 \left(e^{r/\delta} - 1\right)}{2\pi r\left(e^{a/\delta} - 1\right)}$$

2. Apply Ampere's Law to a circular path of radius r
Let us assume the current is uniform over the cross-section of the conductor

(a) For $r < a$ we have $I_{\text{encl}} = 0$

$$\Rightarrow \quad B = 0$$

(b) For $a < r < b$ we have

$$I_{\text{encl}} = I\left(\frac{A_{a\to r}}{A_{a\to b}}\right) = I\left(\frac{\pi\left(r^2 - a^2\right)}{\pi\left(b^2 - a^2\right)}\right)$$

$$\Rightarrow \quad I_{\text{encl}} = I\frac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

From Ampere's Circuital Law, we get

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I \frac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

$$\Rightarrow \quad B = \frac{\mu_0 I\left(r^2 - a^2\right)}{2\pi r\left(b^2 - a^2\right)}$$

(c) For $r > b$ we have

$$I_{\text{encl}} = I$$

From Ampere's Circuital Law, we get

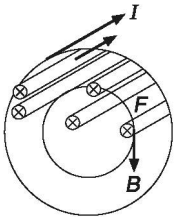
$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I \quad \text{and} \quad B = \frac{\mu_0 I}{2\pi r}$$

3. (a) Force on a single wire due to the field of the other ninety-nine is given by

$$B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

$$\Rightarrow \quad B = \frac{\left(4\pi \times 10^{-7}\right)(99)(2)\left(0.2 \times 10^{-2}\right)}{2\pi\left(0.5 \times 10^{-2}\right)^2}$$

$$\Rightarrow \quad B = 3.17 \times 10^{-3}\ \text{T}$$

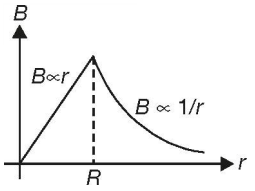


This field points tangentially to a circle of radius 0.2 cm and exerts force $\vec{F} = I(\vec{\ell} \times \vec{B})$ towards the center of the bundle, on the single hundredth wire

$$\frac{F}{\ell} = IB\sin\theta = (2\ \text{A})\left(3.17 \times 10^{-3}\ \text{T}\right)\sin 90°$$

$$\Rightarrow \quad \frac{F}{\ell} = 6.34\ \text{mNm}^{-1}$$

So, $\frac{F_B}{\ell} = 6.34 \times 10^{-3}\ \text{Nm}^{-1}$, inwards



(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is greatest at the outer surface.

4. When seen from the top, left side portion of the Amperian loop is traversed counter clockwise, whereas the other portion is traversed clockwise. So, by applying the sign convention, we see that I_1 is positive, I_2 is positive and I_3 is negative.

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 (I_1 + I_2 - I_3)$$

5. Use Ampere's Law,

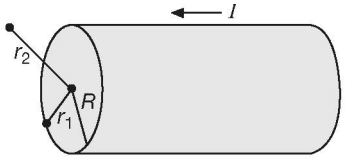
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$$

For current density $\vec{J}$, this becomes

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \int \vec{J}\cdot d\vec{A} \qquad \left\{\because I = \int \vec{J}\cdot d\vec{A}\right\}$$

(a) For $r_1 < R$, we get

$$B(2\pi r_1) = \mu_0 \int_0^{r_1} (br)(2\pi r dr)$$



$$\Rightarrow \quad B = \frac{\mu_0 b r_1^2}{3}$$

(for $r_1 < R$ or inside the cylinder)

(b) When $r_2 > R$, Ampere's Law gives

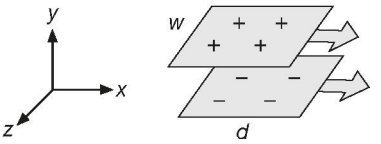
$$B(2\pi r_2)\mu_0 \int_0^{R} (br)(2\pi r dr) = \frac{2\pi\mu_0 bR^3}{3}$$

$$\Rightarrow \quad B = \frac{\mu_0 bR^3}{3r_2}$$

(for $r_2 > R$ or outside the cylinder).

6. Let J_s be the current per unit length, then the upper sheet creates field $\vec{B} = \frac{\mu_0 J_s}{2}\hat{k}$ above it and $\frac{\mu_0 J_s}{2}(-\hat{k})$ below it.

Consider a patch of the sheet of width w parallel to the z-axis and length d parallel to the x-axis as shown in Figure.



The charge on it, σwd passes a point in time $\frac{d}{v}$ so the current it constitutes is $\frac{q}{t} = \frac{\sigma wvd}{d}$ and the linear current density is $J_s = \frac{\sigma wv}{w} = \sigma v$. So, the magnitude of the magnetic field created by the upper sheet is $\frac{1}{2}\mu_0\sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\frac{1}{2}\mu_0\sigma v(-\hat{k})$ above it and $\frac{1}{2}\mu_0\sigma v\hat{k}$ below it.

(a) Between the plates, their fields add to $\mu_0\sigma v(-\hat{k})$ i.e. $\mu_0\sigma v$ away from you horizontally.

(b) Above both sheets and below both the sheets, their equal magnitude fields add to zero.

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0\sigma v(-\hat{k})$ will exert a magnetic force on the current in the section of area $w \times d$ given by

$$\vec{F}_m = I(\vec{\ell}\times\vec{B}) = (\sigma wvd)\hat{i}\times\left(\frac{1}{2}\mu_0\sigma v\right)(-\hat{k})$$

$$\Rightarrow \quad \vec{F}_m = I(\vec{\ell}\times\vec{B}) = \left(\frac{1}{2}\mu_0\sigma^2 v^2 wd\right)\hat{j}$$

The magnetic force per area is

$$\frac{\vec{F}_m}{A} = \frac{1}{2}\left(\frac{\mu_0\sigma^2 v^2 wd}{wd}\right)\hat{j}$$

$$\Rightarrow \quad \frac{F_m}{A} = \frac{1}{2}\mu_0\sigma^2 v^2, \text{ upwards}$$

(d) The electrical force on the considered section of the upper plate is

$$\vec{F}_e = q\vec{E}_{\text{lower}} = (\sigma\ell w)\left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{j})$$

$$\Rightarrow \quad \vec{F}_e == \left(\frac{\ell w\sigma^2}{2\varepsilon_0}\right)(-\hat{j})$$

The electrical force per area is

$$\frac{F_e}{A} = \frac{\ell w\sigma^2}{2\varepsilon_0 \ell w} = \frac{\sigma^2}{2\varepsilon_0}, \text{ downwards}$$

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The magnetic force balances the electric force acting on the plate when we have

$$F_m = F_e$$

$$\Rightarrow \quad \frac{1}{2}\mu_0\sigma^2 v^2 = \frac{\sigma^2}{2\varepsilon_0}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{(4\pi\times10^{-7})(8.85\times10^{-12})}}$$

$$\Rightarrow \quad v = 3\times10^8 \text{ ms}^{-1}$$

This is the speed of light which cannot be a speed that can be attained by a metal plate.

7. (a) According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 (I_{\text{inside}})$$

For $r_1 < R$

$$I_{\text{inside}} = 0$$

$$\Rightarrow \quad B_{\text{inside}} = 0$$

(b) According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 (I_{\text{inside}})$$

For $r_2 > R$

$$I_{\text{inside}} = NI$$

$$\Rightarrow \quad B(2\pi r_2) = \mu_0 NI$$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi r_2}$$

(c) $$B = \frac{(2\times10^{-7})(100)(10)}{5\times10^{-2}} = 4\times10^{-3} \text{ T}$$

$$\Rightarrow \quad B = 4 \text{ mT}$$

8. Apply Ampere's Law to a circle of radius r in each case.

Assume that the currents are uniform over the cross sections of the conductors.

(a) $r < a$

$$\Rightarrow \quad I_{\text{encl}} = I\left(\frac{A_r}{A_a}\right) = I\left(\frac{r^2}{a^2}\right)$$

From Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl.}} = \mu_0 I\left(\frac{r^2}{a^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 Ir}{2\pi a^2}$$

When $r = a$ we have, $B = \dfrac{\mu_0 I}{2\pi a}$

(b) $b < r < c$

$$\Rightarrow \quad I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

From Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl}}$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = \mu_0 I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$$

When $r = b$, $B = \dfrac{\mu_0 I}{2\pi b}$ and

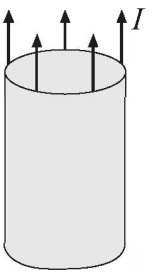
When $r = c$ then $B = 0$

9. We assume the current to be vertically upwards. Consider a circle of radius r slightly less than R. It encloses no current so from Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{inside}}$$

$$\Rightarrow \quad B(2\pi r) = 0$$

We conclude that the magnetic field is zero.



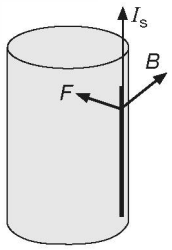
Now let the r be barely larger than R. Ampere's Law becomes

$$B(2\pi R) = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R}$$

The field's direction is tangent to the wall of the cylinder in a counter clockwise sense.

Consider an infinitesimal strip of the wall of width dx and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.



The current carried by the strip is $I_s = \dfrac{Idx}{2\pi R}$ up.

The force on it is

$$\vec{F} = I_s(\vec{\ell} \times \vec{B}) = \frac{Idx}{2\pi R}\left(\ell \frac{\mu_0 I}{2\pi R}\right)$$

$$\vec{F} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2}, \text{ radially inwards}$$

The pressure on the strip and everywhere on the cylinder is

$$P = \frac{F}{A} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2 \ell dx} = \frac{\mu_0 I^2}{4\pi^2 R^2} \text{ inwards}$$

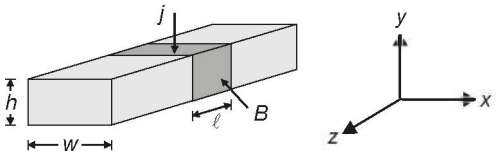
Due to the phenomenon of pinch effect, an empty aluminium can will crush itself when it carries a large current along its length.

10. In case of a long solenoid, the magnetic field is zero outside the solenoid and $B = \mu_0 nI$ inside the solenoid. So, the magnetic pressure on the surface of solenoid is given by

$$P_m = \frac{B^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 I^2$$

Test Your Concepts-VI (Based on Force on Current Carrying Conductor)

1. (a) Let us define vector $\vec{h}$ to have the downward direction of the current and vector $\vec{\ell}$ to be along the pipe into the page as shown. The electric current experiences a magnetic force $I(\vec{h} \times \vec{B})$ in the direction of $\vec{\ell}$



(b) The sodium consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $j \times (\text{area}) = j\ell w$.

The current then feels a magnetic force

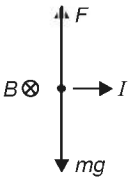
$$I|\vec{h} \times \vec{B}| = j\ell whB \sin 90°$$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{j\ell whB}{hw} = j\ell B$$

2. The magnetic force $\vec{F}$ must be upward and equal to mg. The direction of $\vec{F}$ is determined by the direction of I in the circuit.

$F = I\ell B \sin\theta$ with $\theta = 90°$



From Ohm's Law, we have

$$I = \frac{V}{R}, \text{ where } V \text{ is the battery voltage}$$

(a) The forces are shown in figure. The current I in the bar must be to the right to produce $\vec{F}$ upwards. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F = mg$

$$\Rightarrow \quad I\ell B = mg$$

$$\Rightarrow \quad m = \frac{I\ell B}{g} = \frac{V\ell B}{Rg} = \frac{(175\text{ V})(0.6\text{ m})(1.5\text{ T})}{(5\ \Omega)(9.8\text{ ms}^{-2})}$$

$$m = 3.21 \text{ kg}$$

3. $\sum F_y = 0$

$$\Rightarrow \quad N - mg = 0$$

$$\sum F_x = 0$$

$$\Rightarrow \quad -\mu_k N + IB \sin 90° = 0$$

$$\Rightarrow \quad B = \frac{\mu_k mg}{Id} = \frac{0.1(0.2\text{ kg})(9.8\text{ ms}^{-2})}{(10\text{ A})(0.5\text{ m})} = 39.2\text{ mT}$$

4. Call the length of the rod L and the tension in each wire alone $\dfrac{T}{2}$. Then, at equilibrium

$$\sum F_x = T\sin\theta - ILB\sin 90° = 0$$

$$\Rightarrow \quad T\sin\theta = ILB$$

$$\sum F_y = T\cos\theta - mg = 0$$

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$\Rightarrow \quad T\cos\theta = mg$

$\Rightarrow \quad \tan\theta = \dfrac{ILB}{mg} = \dfrac{IB}{\left(\dfrac{m}{L}\right)g}$

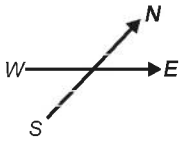
$\Rightarrow \quad B = \dfrac{\left(\dfrac{m}{L}\right)g}{I}\tan\theta = \dfrac{\lambda g}{I}\tan\theta$

5. $F = BIL\sin\theta$

For equilibrium, we have $F = mg$

$\Rightarrow \quad mg = BIL\sin\theta$

$\Rightarrow \quad \dfrac{m}{L}g = IB\sin\theta$



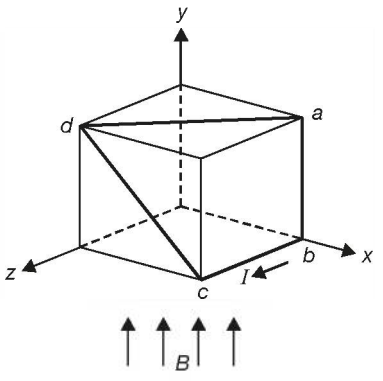
Since, $I = 2$ A and $\dfrac{m}{L} = 0.5\ \text{gcm}^{-1} = 5\times10^{-3}\ \text{kgm}^{-1}$

$\Rightarrow \quad (5\times10^{-2})(9.8) = (2)B\sin(90°)$

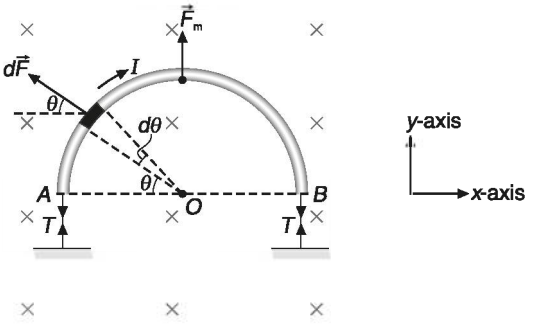
$\Rightarrow \quad B = 0.245$ T and from Fleming's Left Hand Rule the direction is eastward.

6. Since $\vec{B} = B\hat{j}$

Segment	ℓ	$\vec{F} = I(\vec{\ell}\times\vec{B})$
ab	$-\ell\hat{j}$	0
bc	$\ell\hat{k}$	$BI\ell(-\hat{i})$
cd	$-\ell\hat{i}+\ell\hat{j}$	$BI\ell(-\hat{k})$
da	$\ell\hat{i}-\ell\hat{k}$	$BI\ell(\hat{k}+\hat{i})$



7. To find the force on the conductor, consider a small arc of length $dl = Rd\theta$ as shown in Figure.



This arc can be considered as a straight conductor of length dl. The force on this arc is given by

$dF = BIdl$

$\Rightarrow \quad d\vec{F} = BIdl\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

$\Rightarrow \quad dF = BI(Rd\theta)\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

Net force on the semi circular rod is given by

$$\vec{F}_m = \int_0^{\pi} d\vec{F}$$

$$\Rightarrow \quad \vec{F}_m = BIR\left(-\int_0^{\pi}\cos\theta d\theta\hat{i} + \int_0^{\pi}\sin\theta d\theta\hat{j}\right)$$

$\Rightarrow \quad \vec{F}_m = BIR(0+2)\hat{j}$

$\Rightarrow \quad \vec{F}_m = (2BIR)\hat{j}$

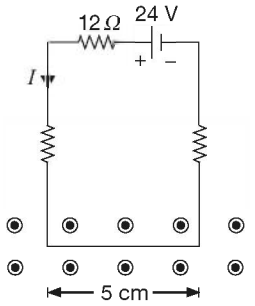
The force experienced by semi-circular wire AB is $2BiR$, perpendicular to its length AB.
For equilibrium, we must have

$2T = F_m = 2BIR$

$\Rightarrow \quad T = BIR$

8. Let x_1 be the elongation due to the weight of the wire and let x_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2kx_2$ where k is the force constant of the spring and can be determined from $k = \dfrac{mg}{2x_1}$. (The factor 2 is included since there are 2 springs in parallel). Combining these two equations, we get

$$F = F_{\text{magnetic}} = 2\left(\frac{mg}{2x_1}\right)x_2 = \frac{mgx_2}{x_1}$$



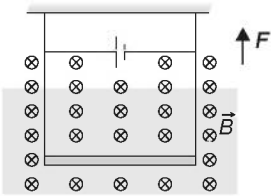
But $|\vec{F}| = I|\vec{L}\times\vec{B}| = BIL$

where $I = \dfrac{24\text{ V}}{12\ \Omega} = 2\text{ A}$

$$\Rightarrow \quad B = \frac{mg\,x_2}{ILx_1} = \frac{(0.1)(9.8)(3\times10^{-3})}{(2)(0.05)(5\times10^{-3})} = 0.588\text{ T}$$

9. $\dfrac{|F|}{\ell} = \dfrac{mg}{\ell} = \dfrac{I|\vec{\ell}\times\vec{B}|}{\ell}$

$$\Rightarrow \quad I = \frac{mg}{B\ell} = \frac{(0.04\text{ kgm}^{-1})(9.8\text{ ms}^{-2})}{3.6\text{ T}} = 0.109\text{ A}$$



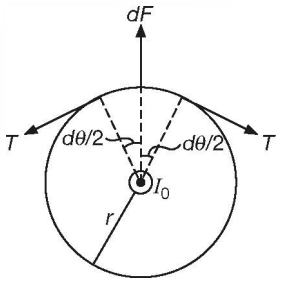
The direction of I in the bar is to the right.

10. TRY YOURSELF
Same as Illustration 71

$$B = \frac{2\lambda g}{\sqrt{3}I}$$

11. Considering the magnetic forces between I_0 and the differential current dI we obtain

$$dF = \frac{\mu_0 I_0}{2\pi r}(dI)\left(dI = \frac{Id\theta}{2\pi}\right)$$



where dF is the force per unit length.

If T is the tension per unit length, then for the elementary portion to be in equilibrium, we have

$$2T\sin\left(\frac{d\theta}{2}\right) = dF$$

$$\Rightarrow \quad Td\theta = dF \quad \left[\text{as } d\theta \text{ is very small, } \sin\frac{d\theta}{2} \cong \frac{d\theta}{2}\right]$$

$$\Rightarrow \quad Td\theta = \frac{\mu_0 I_0}{2\pi r}\left(\frac{Id\theta}{2\pi}\right)$$

$$\Rightarrow \quad T = \frac{\mu_0 I_0 I}{4\pi^2 r}$$

12. METHOD I: Using the basic formula

The conductor $ACDEFB$ consists of 5 straight sections i.e. AC, CD, DE, EF and FB each having length a. The respective forces on these sections of conductor are

$$\vec{F}_{AC} = I\left(a\hat{i}\times B_0\hat{k}\right) = -(B_0Ia)\hat{j}$$

$$\vec{F}_{CD} = I\left(a\hat{j}\times B_0\hat{k}\right) = (B_0Ia)\hat{i}$$

$$\vec{F}_{DE} = I\left(a\hat{i}\times B_0\hat{k}\right) = -(B_0Ia)\hat{j}$$

$$\vec{F}_{EF} = I\left(-a\hat{j}\times B_0\hat{k}\right) = -(B_0Ia)\hat{i}$$

$$\vec{F}_{FB} = I\left(a\hat{i}\times B_0\hat{k}\right) = -(B_0Ia)\hat{j}$$

So, the net force is given by

$$\vec{F} = \vec{F}_{AC} + \vec{F}_{CD} + \vec{F}_{DE} + \vec{F}_{EF} + \vec{F}_{FB}$$

$$\Rightarrow \quad \vec{F} = -3B_0Ia\hat{j}$$

METHOD II: Using the concept of equivalent length

Instead of finding the force on individual wires, we can directly find the net force by finding effective length of the wire. The line joining A and B has length $3a$ thus net force equals.

$$\vec{F} = I\left(3a\hat{i}\times B_0\hat{k}\right)$$

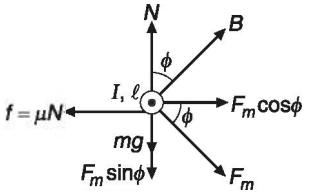
$$\Rightarrow \quad \vec{F} = 3aB_0I\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{F} = -3aB_0I\hat{j}$$

13. The magnetic force is $F_m = IlB\sin\theta$, where θ is the angle between current and magnetic field. In this case $\theta = 90°$.

$$\Rightarrow \quad F_m = BIl$$

Since F_m acts at an angle ϕ with horizontal as shown in figure (according to Fleming left hand rule or right palm rule)



CHAPTER 1

Since the rod is in equilibrium, so the net force along x and y axis are given by

$$F_y = N - (mg + F_m \sin\phi) = 0$$

$$\Rightarrow \quad N = mg + F_m \sin\phi \qquad \ldots(1)$$

$$F_x = f - F_m \cos\phi = 0$$

$$\Rightarrow \quad f = F_m \cos\phi \qquad \ldots(2)$$

Also, we know that the frictional force f is given by

$$f = \mu N \qquad \ldots(3)$$

Substituting (1) and (2) in (3), we get

$$\mu(mg + F_m \sin\phi) = F_m \cos\phi$$

$$\Rightarrow \quad F_m = \frac{\mu mg}{\cos\phi - \mu \sin\phi}$$

$$\Rightarrow \quad BIl = \frac{\mu mg}{\cos\phi - \mu \sin\phi}$$

$$\Rightarrow \quad I = \frac{\mu mg}{Bl(\cos\phi - \mu \sin\phi)}$$

Test Your Concepts-VII (Based on Magnetic Moment and Torque)

1. $|\tau| = MB\sin 90 = BIA$, where the effective current due to the orbiting electrons is given by

$$I = \frac{\text{Charge Circulating}}{\text{Period of one Revolution}} = \frac{q}{T}$$

and the period of the motion is

$$T = \frac{2\pi R}{v}$$

For an electron orbiting the nucleus, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = q\sqrt{\frac{1}{4\pi\varepsilon_0 mR}}$$

Substituting this expression for v into the equation for T, we get

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{q^2}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(9\times10^{-31})(0.53\times10^{-10})^3}{(1.6\times10^{-19})^2(9\times10^9)}}$$

$$\Rightarrow \quad T = 1.52\times10^{-16} \text{ s}$$

Therefore, $|\tau| = BIA = B\left(\frac{q}{T}\right)A$

$$\Rightarrow \quad |\tau| = \frac{1.6\times10^{-19}}{1.52\times10^{-16}}\pi(0.53\times10^{-10})^2(0.4)$$

$$\Rightarrow \quad |\tau| = 3.7\times10^{-24} \text{ Nm}$$

2. Work done in rotating the loop from $\theta_1 = 60°$ to $\theta_2 = 120°$ is

$$W = \Delta U = U_f - U_i$$

Since, we know that the change in potential energy does not depend on choice of reference zero for potential energy, so we have

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow \quad W = -MB(\cos 120° - \cos 60°)$$

$$\Rightarrow \quad W = MB$$

Since we know that the result $U = -MB\cos\theta$ has been obtained for $\theta = 90°$ as the reference, because $U_{90°} = 0$. So, for this zero reference the potential energy at $\theta = 0°$ and $\theta = 45°$ respectively are

$$U_{0°} = -MB \text{ and } U_{45°} = -\frac{MB}{\sqrt{2}}$$

The potential energy difference between position $\theta_1 = 0°$ and $\theta_2 = 45°$ is

$$\Delta U = -MB\cos 45° - (-MB\cos 0°)$$

$$\Rightarrow \quad \Delta U = MB\left(1 - \frac{1}{\sqrt{2}}\right)$$

For $\theta = 0°$ as the new reference, potential energy difference between the same positions should also be the same (because the change in potential energy is independent of the choice of reference level). So, if $U'_{45°}$ and $U'_{0°}$ be the respective potential energies with respect to the new reference, then we have

$$\Delta U = \underbrace{U'_{45°} - U'_{0°}}_{\text{reference at } 0°} = \underbrace{(U_{45°} - U_{0°})}_{\text{reference at } 90°}$$

Since we are given that $U'_{0°} = 0$

$$\Rightarrow \quad \underbrace{U'_{45°} - 0}_{\text{reference at } 0°} = \underbrace{(U_{45°} - U_{0°})}_{\text{reference at } 90°}$$

$$\Rightarrow \quad U'_{45°} = MB\left(1 - \frac{1}{\sqrt{2}}\right)$$

3. (a) $2\pi r = 2$ m

$$\Rightarrow \quad r = 0.318 \text{ m}$$

The magnetic moment M is given by

$$M = IA = (17\times10^{-3} \text{ A})[\pi(0.318)^2 \text{ m}^2]$$

$$M = 5.41\times10^{-3} \text{ Am}^2$$

(b) $\vec{\tau} = \vec{M}\times\vec{B}$

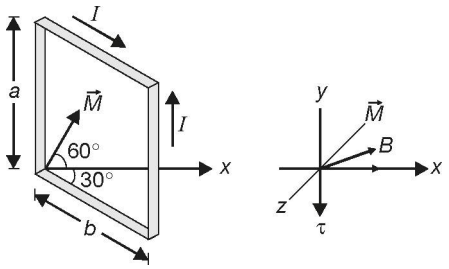
$$\Rightarrow \quad \tau = (5.41\times10^{-3} \text{ Am}^2)(0.8 \text{ T})$$

$$\Rightarrow \quad \tau = 4.33\times10^{-3} \text{ Nm}.$$

4. $\tau = NBIA\sin\theta$

$\Rightarrow \quad \tau = 100(0.8\text{ T})(0.4\times0.3\text{ m}^2)(1.2\text{ A})\sin(60°)$

$\Rightarrow \quad \tau = 9.98\text{ Nm}$



Note that θ is the angle between the magnetic moment and the magnetic field. The loop will rotate so as to align the magnetic moment with the field. Looking down along the y-axis, the loop will rotate in a clockwise direction.

5. (a) Let θ represent the unknown angle, L, the total length of the wire and d, the length of one side of the square coil. Then, using the definition of magnetic moment and the right hand rule in figure, we find

$\mu = (NI)A$

$\Rightarrow \quad \mu = \left(\frac{L}{4d}\right)d^2I$ at angle θ with the horizontal

At equilibrium, $\sum\vec{\tau} = (\vec{\mu}\times\vec{B}) - (\vec{r}\times\vec{mg}) = 0$

$\Rightarrow \quad \left(\frac{BILd}{4}\right)\sin(90°-\theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0$

and $\left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{BILd}{4}\right)\cos\theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{BIL}{2mg}\right)$

$\Rightarrow \quad \theta = \tan^{-1}\left\{\frac{(0.01\text{ T})(3.4\text{ A})(4\text{ m})}{2(0.1\text{ kg})(9.8\text{ ms}^{-2})}\right\} \simeq 4°$

(b) $\tau_m = \left(\frac{BILd}{4}\right)\cos\theta$

$\tau_m = \frac{1}{4}(3.4\text{ A})(4\text{ m})(0.01\text{ T})(0.1\text{ m})\cos4°$

$\tau_m = 3.39\times10^{-3}\text{ Nm}$

6. From $\vec{\tau} = \vec{M}\times\vec{B} = (I\vec{A})\times\vec{B}$, the magnitude of the torque is $BIA\sin(90°)$

(a) Each side of the triangle is $\frac{40\text{ cm}}{3}$

Its altitude is $\sqrt{13.3^2 - 6.67^2}\text{ cm} = 11.5\text{ cm}$ and its area is

$A = \frac{1}{2}(11.5\text{ cm})(13.3\text{ cm}) = 7.7\times10^{-3}\text{ m}^2$

$\Rightarrow \quad \tau = (0.52)(20)(7.7\times10^{-3})$

$\Rightarrow \quad \tau = 80\times10^{-3}\text{ Nm}$

(b) Each side of the square is 10 cm and its area is $100\text{ cm}^2 = 10^{-2}\text{ m}^2$

$\Rightarrow \quad \tau = (0.52\text{ T})(20\text{ A})(10^{-2}\text{ m}^2) = 0.104\text{ Nm}$

(c) $r = \frac{0.4\text{ m}}{2\pi} = 0.0637\text{ m}$

$\Rightarrow \quad A = \pi r^2 = 1.27\times10^{-2}\text{ m}^2$

$\Rightarrow \quad \tau = (0.52)(20\text{ A})(1.27\times10^{-2}\text{ m}^2)$

$\Rightarrow \quad \tau = 0.132\text{ Nm}$

(d) The circular loop experiences the largest torque

7. (a) $\vec{\tau} = \vec{M}\times\vec{B}$

$\Rightarrow \quad |\vec{\tau}| = |\vec{M}\times\vec{B}| = MB\sin\theta = NIAB\sin\theta$

$\Rightarrow \quad \tau_{max} = NIAB\sin(90°)$

$\Rightarrow \quad \tau_{max} = 1(5\text{ A})[\pi(0.05\text{ m})^2](3\times10^{-3}\text{ T})$

$\Rightarrow \quad \tau_{max} = 118\ \mu\text{Nm}$

(b) $U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$

$U_{MIN} = -MB$, when $\theta = 0°$

$U_{MAX} = MB$, when $\theta = 180°$

$\Rightarrow \quad -MB \le U \le MB$, where

$MB = (NIA)B = 1(5)[\pi(0.05)^2](3\times10^{-3})$

$MB = 118\ \mu\text{J}$

the range of the potential energy is

$-118\ \mu\text{J} \le U \le +118\ \mu\text{J}$

8. The magnetic dipole moment of the loop is initially directed along the z-axis due to which the torque exerted by the field turns it about x-axis so as to finally align its magnetic moment along the field i.e. y-axis. Since there is no external force or torque, so the total mechanical energy is of the system is conserved. Hence

$K + U = \text{constant}$, where $U = -MB\cos\theta$

$\Rightarrow \quad (K+U)_{\text{initial}} = (K+U)_{\text{final}}$

$\Rightarrow \quad 0 + 0 = \frac{1}{2}I\omega^2 - MB\cos0° \qquad \ldots(1)$

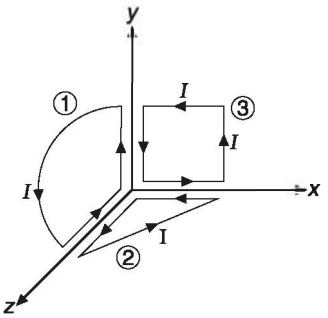
where I is the moment of inertia of the ring about its diameter i.e. $I = \frac{1}{2}mR^2$

From (1), we get

$$\omega = \sqrt{\frac{2MB}{I}} = \sqrt{\frac{2i(\pi R^2 B)}{\frac{1}{2}mR^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4\pi iB}{m}}$$

9. Let us split the given loop in three separate loops such that when overlapped at the three coordinate axes, the opposite currents in the wire segments in these three loops along the axes will cancel each other as shown in Figure.



This arrangement of three loops is equivalent to the single loop arrangement asked in the question. The magnetic moment of the loop is the vector sum of the three individual loops. So

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3$$

$$\Rightarrow \quad \vec{M} = I\left(\frac{\pi R^2}{4}\right)\hat{i} + I\left(\frac{R^2}{2}\right)\hat{j} + I(R^2)\hat{k}$$

10. For a moving coil galvanometer, we have

$$i = \left(\frac{C}{NAB}\right)\theta$$

The torsional constant of the spring is given as

$$C = \frac{NABi}{\theta}$$

Substituting the values in SI units we get

$$C = \frac{(60)(5.0\times10^{-4})(9\times10^{-3})(0.2\times10^{-3})}{18\times\frac{\pi}{180}}$$

$$\Rightarrow \quad C = 1.71\times10^{-7} \text{ Nmrad}^{-1}$$

11. $M = IA$

where $I = \dfrac{\text{Charge Circulating}}{\text{Period of one Revolution}}$

$$I = \frac{e}{\left(\frac{2\pi r_n}{v_n}\right)}$$

where $r_n =$ radius of the nth orbit

$v_n =$ velocity of the electron in the nth orbit

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2} \text{ and } v_n = \frac{e^2}{2h\varepsilon_0 n}$$

$$M = \left(\pi r_n^2\right)\left(\frac{ev_n}{2\pi r_n}\right)$$

$$\Rightarrow \quad M = \frac{ev_n r_n}{2}$$

$$\Rightarrow \quad M = \frac{e}{2}\left(\frac{n^2h^2\cancel{\varepsilon_0}}{\pi m\cancel{e^2}}\right)\left(\frac{\cancel{e^2}}{2h\cancel{\varepsilon_0}n}\right)$$

$$\Rightarrow \quad M = \frac{neh}{4\pi m}, \text{ where } m = \text{mass of electron}$$

$$B = \frac{\mu_0 I_n}{2r_n}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{2r_n}\right)\left(\frac{ev_n}{2\pi r_n}\right)$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 e}{4\pi}\right)\left(\frac{v_n}{r_n^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 e}{4\pi}\left(\frac{e^2}{2h\varepsilon_0 n}\right)\left(\frac{\pi^2m^2e^4}{n^4h^4\varepsilon_0^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0\pi m^2e^7}{8\varepsilon_0h^5n^5}$$

12. DO YOURSELF

$$\vec{F}_{AB} = -\vec{F}_{CD} = -N(BI\ell\sin\theta)\hat{k}$$

$$\vec{F}_{BC} = -\vec{F}_{DA} = N(BIb\sin\theta)\hat{j}$$

$$\vec{\tau} = \sqrt{2}(NBI\ell b\sin\theta\cos\theta)\hat{k}$$

or $\vec{\tau} = (MB\cos\theta)\hat{k}$ where $M = \sqrt{2}(NI\ell b)\sin\theta$

13. Magnitude of torque is given by

$$|\vec{\tau}| = MB\sin\theta$$

Here, $M = NIA = (1)(1)(\pi)(0.2)^2 = (0.04\,\pi)$ Am^2

$B = 2$ T and $\theta =$ angle between $\vec{M}$ and $\vec{B}$ is 90°

$$\Rightarrow \quad |\vec{\tau}| = (0.04\,\pi)(2)\sin 90° = 0.25 \text{ Nm}$$

Vectorially, we have

$$\vec{B} = 2\cos(45°)\hat{i} + 2\sin(45°)\hat{j} = \sqrt{2}\left(\hat{i}+\hat{j}\right) \text{ T}$$

and $\vec{M} = -(0.04\pi)\hat{k}$ Am^2

Since, we know that $\vec{\tau} = \vec{M} \times \vec{B} = (0.04\sqrt{2}\pi)(-\hat{j} + \hat{i})$

$\Rightarrow \quad \vec{\tau} = 0.18(\hat{i} - \hat{j})$ Nm

14. (a) The magnetic field intensity inside the solenoid is

$B = \mu_0 nI$

The magnetic moment of the coil is $\mu = NIA$

$\Rightarrow \quad \mu = NI_0(\pi r^2) = \pi I_0 r^2 \qquad \{\because N = 1\}$

The maximum torque $= \tau_{max} = |\vec{\mu} \times \vec{B}|_{max} = \mu B$

$\Rightarrow \quad \tau = (I_0 \pi r^2)(\mu_0 nI) = \mu_0 \pi n I I_0 r^2$

(b) For small displacement from mean position, $\tau = \mu B\theta$

$\therefore$ Motion will be angular SHM

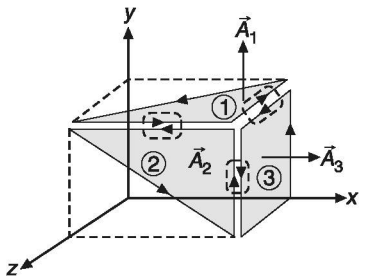
Period of oscillation, $T = 2\pi\sqrt{\dfrac{I}{\mu B}}$

$$T = 2\pi\sqrt{\frac{\left(\frac{2}{5}\right)mr^2}{\pi r^2 I_0 \mu_0 nI}} = 2\pi\sqrt{\frac{2m}{5\pi\mu_0 n I I_0}}$$

15. The net force on a current carrying loop of any arbitrary shape placed in a uniform magnetic field is zero.

Hence, $\vec{F}_{net} = 0$.

The given loop can be considered to be a superposition of three loops (1, 2 and 3) as shown in Figure.



The respective area vectors (in square metre) of the three loops 1, 2 and 3 are

$$\vec{A}_1 = \frac{1}{2}\left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{j} = (0.005)\hat{j}$$

$$\vec{A}_2 = \frac{1}{2}\left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{k} = (0.005)\hat{k} \text{ and}$$

$$\vec{A}_3 = \left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{i} = (0.01)\hat{i}$$

Magnetic moment vector, $\vec{M} = i\vec{A}$

$\Rightarrow \quad \vec{M} = 10(0.01\hat{i} + 0.005\hat{j} + 0.005\hat{k})$ Am^2

$\Rightarrow \quad \vec{M} = (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})$ Am^2

The torque $\vec{\tau}$ acting on the loop is given by

$\vec{\tau} = \vec{M} \times \vec{B}$

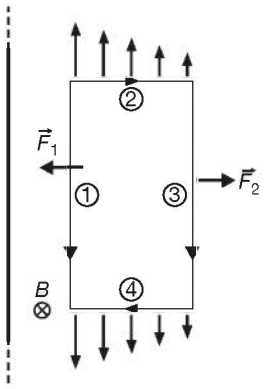
$\vec{\tau} = (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1 & 0.05 & 0.05 \\ 2 & -3 & 1 \end{vmatrix} = (-0.1\hat{i} - 0.4\hat{k}) \text{ Nm}$$

Test Your Concepts-VIII (Based on Force between Current Carrying Conductors)

1. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{i}$$



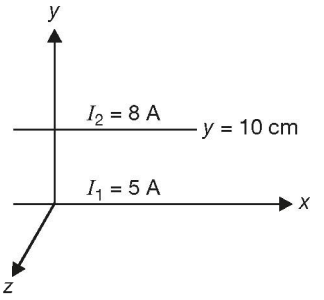
$$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{i}$$

$$\Rightarrow \quad \vec{F} = -\frac{(4\pi \times 10^{-7})(5)(10)(0.45)}{2\pi}\left(\frac{0.15}{(0.1)(0.25)}\right)\hat{i}$$

$\Rightarrow \quad \vec{F} = -(2.7 \times 10^{-5}\hat{i})$ N

$\Rightarrow \quad \vec{F} = 2.7 \times 10^{-5}$ N toward the left

2. Consider both wires to carry current in the x direction, the first at $y = 0$ and the second at $y = 10$ cm.



CHAPTER 1

(a) $\vec{B}=\left(\frac{\mu_0 I}{2\pi r}\right)\hat{k}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{mA}^{-1})(5\ \text{A})}{2\pi(0.1\ \text{m})}\hat{k}$

$\vec{B}=1\times10^{-5}\ \text{T}$ out of the page

(b) $\vec{F}=I_2(\vec{\ell}\times\vec{B})=(8\ \text{A})\left[(1\ \text{m})\hat{i}\times(1\times10^{-5}\ \text{T})\hat{k}\right]$

$\vec{F}=(8\times10^{-5}\ \text{N})(-\hat{j})$

$\vec{F}=8\times10^{-5}\ \text{N}$ toward the first wire

(c) $\vec{B}=\left(\frac{\mu_0 I}{2\pi r}\right)(-\hat{k})=\frac{(4\pi\times10^{-7})(8)}{2\pi(0.1)}(-\hat{k})$

$\vec{B}=(1.6\times10^{-5}\ \text{T})(-\hat{k})$

$\vec{B}=1.6\times10^{-5}\ \text{T}$ into the page

(d) $\vec{F}=I_1(\vec{\ell}\times\vec{B})=(5)\left[(1)\hat{i}\times(1.6\times10^{-5})(-\hat{k})\right]$

$\vec{F}=(8\times10^{-5}\ \text{N})(+\hat{j})$

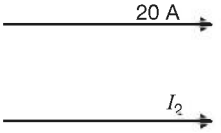
$\vec{F}=8\times10^{-5}\ \text{N}$ towards the second wire

3. To attract, both currents must be to the right. The attraction is described by

$$F=BI_2\ell\sin90^\circ=I_2\ell\frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow\quad I_2=\frac{F}{\ell}\frac{2\pi r}{\mu_0 I_1}=(320\times10^{-6})\left(\frac{2\pi(0.5)}{4\pi\times10^{-7}(20)}\right)$$

$$\Rightarrow\quad I_2=40\ \text{A}$$



Let y represent the distance of the zero field point below the upper wire.

Then $\vec{B}=\frac{\mu_0 I}{2\pi r}\otimes+\frac{\mu_0 I}{2\pi r}\odot$

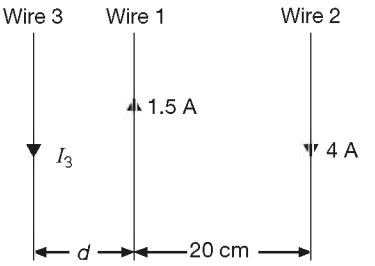
$$\Rightarrow\quad 0=\frac{\mu_0}{2\pi}\left(\frac{20\ \text{A}}{y}-\frac{40\ \text{A}}{(0.5-y)}\right)$$

$$\Rightarrow\quad 20(0.5-y)=40y$$

$$\Rightarrow\quad 20(0.5\ \text{m})=60y$$

$$\Rightarrow\quad y=0.167\ \text{m}\ \text{ below the upper wire}$$

4. Since wire 1 and 2 carry currents in opposite directions, so they repel each other.
If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero.



If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero.
Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2.
For the equilibrium of wire 3 we have

$$F_{31}=F_{32}$$

$$\Rightarrow\quad \frac{\mu_0(1.5\ \text{A})I_3}{2\pi d}=\frac{\mu_0(4\ \text{A})I_3}{2\pi(20\ \text{cm}+d)}$$

$$\Rightarrow\quad 1.5(20\ \text{cm}+d)=4d$$

$$\Rightarrow\quad d=\frac{30\ \text{cm}}{2.5}=12\ \text{cm}\ \text{ to the left of wire 1}$$

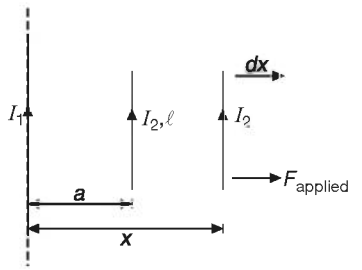
For the equilibrium of wire 1,

$$\frac{\mu_0 I_3(1.5\ \text{A})}{2\pi(12\ \text{cm})}=\frac{\mu_0(4\ \text{A})(1.5\ \text{A})}{2\pi(20\ \text{cm})}$$

$$\Rightarrow\quad I_3=\frac{12}{20}4\ \text{A}=2.4\ \text{A}\ \text{ down}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

5. Let the separation between the conductors be increased through a distance b as shown in Figure.



The force acting on the conductor of length l when it is at a distance x from the infinite conductor is given by

$$F=\left(\frac{\mu_0 I_1 I_2}{2\pi x}\right)l$$

The minimum work required to move it away by small displacement dx is by applying an external force $F_{applied}$ such that the conductor moves very slowly and hence the applied force must be equal to the magnetic force i.e.

$$F_{applied} = F = \left(\frac{\mu_0 I_1 I_2}{2\pi x}\right) l$$

If dW be the small work done in displacing the conductor through dx, then

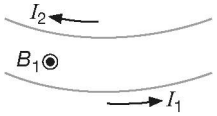
$$dW = Fdx$$

$$\Rightarrow \quad dW = \left(\frac{\mu_0 I_1 I_2 l}{2\pi}\right)\frac{dx}{x}$$

$$\Rightarrow \quad W = \frac{\mu_0 I_1 I_2 l}{2\pi}\int_a^{a+b}\frac{dx}{x} = \left(\frac{\mu_0 I_1 I_2 l}{2\pi}\right)\ln x\Big|_a^{a+b}$$

$$\Rightarrow \quad W = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left(\frac{a+b}{a}\right)$$

6. Let us arrange two wires as straight parallel wires as shown.



(a) $F = \dfrac{\mu_0 I^2 \ell}{2\pi r}$

$$\Rightarrow \quad F = \frac{(4\pi\times10^{-7})(140)^2(2\pi)(0.1)}{2\pi(1\times10^{-3})}$$

$$\Rightarrow \quad F = 2.46 \text{ N upward}$$

(b) Since $F - mg = ma$, where m = mass of the loop a = acceleration of the loop.

$$\Rightarrow \quad a = \frac{F - mg}{m}$$

$$\Rightarrow \quad a = \frac{2.46 - (0.021)(10)}{0.021}$$

$$\Rightarrow \quad a \cong 107 \text{ ms}^{-2}, \text{ upwards}$$

7. At equilibrium, $\dfrac{F}{\ell} = \dfrac{\mu_0 I_A I_B}{2\pi r} = \dfrac{mg}{\ell}$

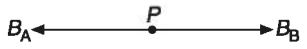
$$\Rightarrow \quad I_B = \frac{2\pi r\left(\frac{m}{\ell}\right)g}{\mu_0 I_A}$$

$$\Rightarrow \quad I_B = \frac{2\pi(0.025 \text{ m})(0.01 \text{ kgm}^{-1})(9.8 \text{ ms}^{-2})}{(4\pi\times10^{-7} \text{ TmA}^{-1})(150 \text{ A})}$$

$$\Rightarrow \quad I_B = 81.7 \text{ A}$$

$$\Rightarrow \quad I_B \cong 82 \text{ A}$$

8. At the point P, the field due to the wires A and B is shown in Figure.



For the net magnetic field at P to be zero, the direction of current at B should be perpendicular to paper outwards. Let current in the wire B be I_B. Then,

$$\frac{\mu_0}{2\pi}\frac{I_A}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{I_B}{\left(\frac{10}{11}\right)}$$

$$\Rightarrow \quad \frac{I_B}{I_A} = \frac{10}{32}$$

$$\Rightarrow \quad I_B = \frac{10}{32}\times I_A = \frac{10}{32}\times 9.6 = 3 \text{ A}$$

Force per unit length on wire B is given by

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_A I_B}{r} \qquad \{\because r = AB = 2 \text{ m}\}$$

$$\Rightarrow \quad \frac{F}{l} = \frac{(2\times10^{-7})(9.6\times3)}{2}$$

$$\Rightarrow \quad \frac{F}{l} = 2.88\times10^{-6} \text{ Nm}^{-1}$$

Single Correct Choice Type Questions

1. $B = \dfrac{\mu_0 q}{4\pi}\left(\dfrac{\vec{v}\times\vec{r}}{r^3}\right)$

$\vec{v}$ is the velocity of charge with respect to observer. Let the observers A and B have velocities $\vec{v}_A$ and $\vec{v}_B$ and since both see that a point charge produces same magnetic field at the same point, so

$$\vec{v}_{CA}\times\vec{r} = \vec{v}_{CB}\times\vec{r}$$

where $\vec{v}_{CA}$ is velocity of charge w.r.t. A and $\vec{v}_{CB}$ is velocity of charge w.r.t. B.

$$\Rightarrow \quad (\vec{v}_C - \vec{v}_A)\times\vec{r} = (\vec{v}_C - \vec{v}_B)\times\vec{r}$$

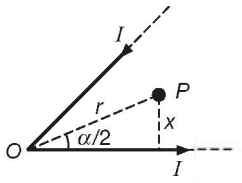
$$\Rightarrow \quad (\vec{v}_A - \vec{v}_B)\times\vec{r} = 0$$

$$\Rightarrow \quad (\vec{v}_A - \vec{v}_B) \parallel \vec{r}$$

So, the relative velocity of observers is parallel to $\vec{r}$

Hence, the correct answer is (A).

2.



CHAPTER 1

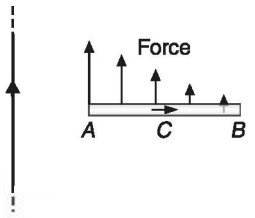
$$\Rightarrow \quad B_P = 2\left(\frac{\mu_0 I}{4\pi x}\right)\left[\sin\left(90° - \frac{\alpha}{2}\right) + \sin 90°\right]$$

Since $x = r\sin\left(\frac{\alpha}{2}\right)$

$$\Rightarrow \quad B_P = \frac{\mu_0 I}{2\pi r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\left(\sin\frac{\alpha}{2}\right)} = \frac{\mu_0 I}{2\pi r}\left[\operatorname{cosec}\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\alpha}{2}\right)\right]$$

Hence, the correct answer is (C).

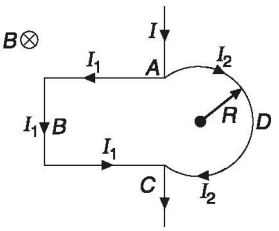
3. Variation of magnetic force on wire ACB is as shown in Figure.



Point of application of net force lies somewhere between A and C.

Hence, the correct answer is (D).

4.



Since $F = BIl_{\text{eff}}$

$\Rightarrow \quad F_{ABC} = BI_1 l$ (towards right)

Also, $F_{ADC} = BI_2 l$ (towards right)

$\Rightarrow \quad F_{\text{loop}} = F_{ABC} + F_{ADC}$

$\Rightarrow \quad F_{\text{loop}} = B(I_1 + I_2)l$

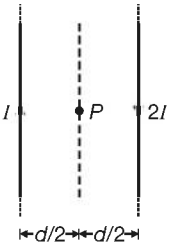
$\Rightarrow \quad F_{\text{loop}} = BIl \qquad \{\because I_1 + I_2 = I\}$

Hence, the correct answer is (B).

5. Consider the gas as a thick conductor carrying a uniform current, use Ampere's Law to find the magnetic field and then apply the left-hand rule to find the direction of the Ampere force.

Hence, the correct answer is (B).

6. The magnetic field at point P is perpendicular to paper inwards due to both the wires. Since, the charged particle is also projected in the same direction. So, force on charged particle is zero.



$$\Rightarrow \quad \vec{F}_m = q(\vec{v}\times\vec{B}) = \vec{0} \qquad \{\because \vec{v} \parallel \vec{B}\}$$

Hence, the correct answer is (D).

7. Since $\frac{F_{\text{magnetic}}}{F_{\text{electrostatic}}} = \frac{v^2}{c^2}$

So, the ratio of electric field to magnetic field is independent of r.

Hence, the correct answer is (D).

8. $\vec{F} = I(\vec{l}_{\text{eff}} \times \vec{B})$

where $\vec{l}_{\text{eff}} = 2\left[\cos(45°)\hat{i} + \sin(45°)\hat{j}\right]$

$$\Rightarrow \quad \vec{l}_{\text{eff}} = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$$

$$\vec{F} = \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ \sqrt{2} & \sqrt{2} & 0 \\ 3 & 4 & 1 \end{vmatrix}$$

$$\Rightarrow \quad \vec{F} = \sqrt{2}\hat{i} - \sqrt{2}\hat{j} + \sqrt{2}\hat{k}$$

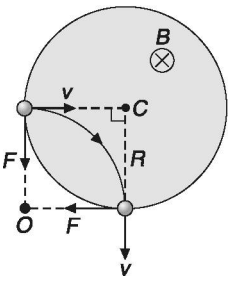
$$\Rightarrow \quad \vec{F} = \sqrt{2}\left(\hat{i} - \hat{j} + \hat{k}\right)$$

Hence, the correct answer is (B).

9. Net force on a current carrying loop in a uniform magnetic field is zero. So magnetic force cannot balance its weight.

Hence, the correct answer is (D).

10. The particle will move along a circle of radius that equals the radius of circular region in which magnetic field is present (as shown in Figure).



Radius of the circular path followed by the particle is

$$R = \frac{mv}{qB}$$

$$\Rightarrow \quad v = \frac{qBR}{m}$$

$$\Rightarrow \quad v = \frac{5\times10^{-6}\times4\times(0.1)}{2\times10^{-3}} = 10^{-3}\ \text{ms}^{-1} = 1\ \text{mms}^{-1}$$

Hence, the correct answer is (D).

11. The pitch p of the helical path is given by

$$p = \frac{2\pi mv\cos\theta}{qB}$$

$$\Rightarrow \quad p = \frac{2\pi m}{qB}(v\cos45°) = \frac{2\pi m}{qB}(v\sin45°)$$

Since $r = \frac{mv\sin\theta}{qB}$

$$\Rightarrow \quad r = \frac{mv\sin45°}{qB} = \frac{p}{2\pi}$$

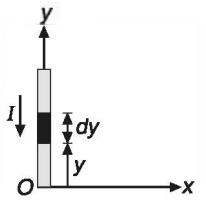
Hence, the correct answer is (D).

12. Since $d\vec{F} = I\left(d\vec{l}\times\vec{B}\right)$

where $I = 2\times10^{-3}$ A

$$Id\vec{l} = -(Idy)\hat{j}$$

$$\vec{B} = 0.3y\hat{i} + 0.4y\hat{j}$$



$$\Rightarrow \quad d\vec{F} = I\left(d\vec{l}\times\vec{B}\right) = 0.3Iydy\hat{k}$$

$$\Rightarrow \quad \vec{F} = \int d\vec{F} = 0.3I\left(\frac{y^2}{2}\Big|_0^1\right)\hat{k} = \left(\frac{0.3I}{2}\right)\hat{k}$$

$$\Rightarrow \quad \vec{F} = \left(3\times10^{-4}\right)\hat{k}$$

Hence, the correct answer is (D).

13. $B_{\text{centre}} = 0$

$$\Rightarrow \quad \frac{\mu_0 I}{4\pi b}\odot + \frac{\mu_0 I}{4\pi b}\odot + \frac{\mu_0 I}{2b}\odot + \frac{\mu_0 I}{2a}\otimes = 0$$

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi b} + \frac{\mu_0 I}{2b} - \frac{\mu_0 I}{2a} = 0$$

$$\Rightarrow \quad \frac{1}{2\pi b} + \frac{1}{2b} = \frac{1}{2a}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{\pi}{\pi+1}$$

Hence, the correct answer is (B).

14. Let the capacitor discharge in time Δt, then a current will flow in the conducting bar due to which magnetic force will act on bar for small time

Impulse due to force is $J = F\Delta t$

$$\Rightarrow \quad J = BIl\Delta t$$

$$\Rightarrow \quad J = B\left(\frac{\Delta q}{\Delta t}\right)l\Delta t$$

$$\Rightarrow \quad J = B(\Delta q)l$$

where, Δq is the total charge flowing through capacitor in time Δt.

Change in momentum equals impulse, so

$$\Delta p = mv = \text{Impulse}$$

$$\Rightarrow \quad mv = B(\Delta q)l$$

Since $\Delta q = CV_0$

$$\Rightarrow \quad mv = B(CV_0)l \qquad \ldots(1)$$

Also, two springs are in parallel, so the equivalent stiffness constant is $k_{\text{eq}} = 2k$

By Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}(2k)A^2, \text{ where } A \text{ is amplitude}$$

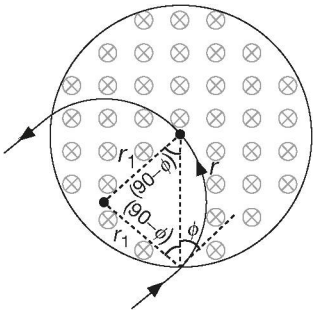
$$\Rightarrow \quad A = \frac{BlCV_0}{\sqrt{2km}}$$

Hence, the correct answer is (A).

15. $r_1 = \frac{mV}{qB} = \frac{5\times10^{-5}\times\left(\frac{1}{\sqrt{3}}\right)}{5\times10^{-5}\times1} = \frac{1}{10\sqrt{3}}$

By sine rule, we have

$$\frac{r_1}{\sin(90°-\phi)} = \frac{r}{\sin2\phi}$$



CHAPTER 1

$$\Rightarrow \quad \frac{1}{10\sqrt{3}\cos\phi}=\frac{0.1}{2\sin\phi\cos\phi}$$

$$\Rightarrow \quad \sin\phi=\frac{0.1\times10\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \phi=60°$$

Hence, the correct answer is (C).

16. The field at A is directed into the page.

Hence, the correct answer is (A).

17. Since $\vec{E}$ and $\vec{B}$ are parallel to each other and $\vec{v}$ is perpendicular to both, so, the path of the particle is a helix of increasing pitch. The speed of particle at any time t is

$$v=\sqrt{v_x^2+v_y^2+v_z^2} \qquad ...(1)$$

Since the magnetic force acts along $-z$ axis, so the particle will have a circular track in yz plane, where

$$v_y^2+v_z^2=v_0^2$$

Due to the electric field along the x-axis, the x velocity of the particle keeps on increasing with time t as

$$v_x=\left(\frac{qE}{m}\right)t \qquad \left\{\because\ a=\frac{qE}{m}\right\}$$

According to the problem, we have

$$v=2v_0$$

Substituting the values in equation (1), we get

$$t=\sqrt{3}\left(\frac{mv_0}{qE}\right)$$

Hence, the correct answer is (D).

18. To enter region II, radius of particle in region I should be greater than d, so we have

$$R>d$$

$$\Rightarrow \quad \frac{mv}{qB}>d$$

$$\Rightarrow \quad v>\frac{qBd}{m}$$

$$\Rightarrow \quad v>\frac{1.6\times10^{-19}\times0.001\times5\times10^{-2}}{9\times10^{-31}}$$

$$\Rightarrow \quad v>\frac{8}{9}\times10^{7}\ \text{ms}^{-1}$$

To come out of region II, we must have

$$2R>d$$

$$\Rightarrow \quad \frac{2mv}{qB}>d$$

$$\Rightarrow \quad v>\frac{qBd}{2m}$$

$$\Rightarrow \quad v>\frac{1.6\times10^{-19}\times0.002\times5\times10^{-12}}{2\times9\times10^{-31}}$$

$$\Rightarrow \quad v>\frac{8}{9}\times10^{7}\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

19. $W_E\neq0$, because a charged particle placed in a field experiences a force parallel (if positive) or antiparallel (if negative) to the field.

When a charged particle moves in a magnetic field, the force experienced by it is $\vec{F}=q(\vec{v}\times\vec{B})$. The $\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$. So $W_M=0$.

Hence, the correct answer is (C).

20. Along the wire $d\vec{l}\times\vec{r}=0$

$$\Rightarrow \quad B=0$$

Hence, the correct answer is (D).

21. Since we know that

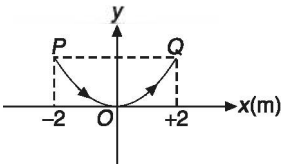
$$\vec{F}=q(\vec{v}\times\vec{B})=q\left[2\hat{i}\times\left(\hat{i}+2\hat{j}+3\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}=2q\left(-3\hat{j}+2\hat{k}\right)$$

So, this force lies in the yz plane.

Hence, the correct answer is (D).

22. In uniform field,



Magnetic force on POQ is equal to magnetic force on straight wire PQ having the same current. Hence,

$$\vec{F}=i\left(\vec{l}_{\text{eff}}\times\vec{B}\right)=i\left(\overrightarrow{PQ}\times\vec{B}\right)$$

$$\Rightarrow \quad \vec{F}=2\left[\left(4\hat{i}\right)\times\left(-0.02\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}=\left(0.16\hat{j}\right)$$

$$\Rightarrow \quad \vec{a}=\frac{\vec{F}}{m}=\frac{\left(0.16\hat{j}\right)}{0.1}=1.6\hat{j}$$

Hence, the correct answer is (C).

23. Since $\vec{B}=\frac{\mu_0}{4\pi}\left(\frac{q\vec{v}\times\vec{r}}{r^3}\right)$ and $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$

$$\Rightarrow \quad \vec{B}=\mu_0\varepsilon_0(\vec{v}\times\vec{E})=\frac{\vec{v}\times\vec{E}}{c^2}$$

Now $\vec{v}\times\vec{E}=\begin{vmatrix}\overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}}\\ 1 & 3 & 0\\ 0 & 0 & 2\end{vmatrix}=6\hat{i}-2\hat{j}$

$\Rightarrow \quad \vec{B}=\dfrac{6\hat{i}-2\hat{j}}{c^2}$

Hence, the correct answer is (A).

24. Let $\vec{B}=\left(x\hat{i}+y\hat{j}+z\hat{k}\right)T$. Since $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$, so we get

$$e\left(-\hat{j}+\hat{k}\right)=e\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 0\\ x & y & z\end{vmatrix}=e\left(-z\hat{j}+y\hat{k}\right)$$

$\Rightarrow \quad z=1,\ y=1$

Also, we have

$$e\left(\hat{i}-\hat{k}\right)=e\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 0\\ x & y & z\end{vmatrix}=e\left(z\hat{i}-x\hat{k}\right)$$

$\Rightarrow \quad z=1,\ x=1$

$\Rightarrow \quad \vec{B}=\hat{i}+\hat{j}+\hat{k}$

Hence, the correct answer is (D).

25. $\omega=10^8\times 2\pi$ rads^{-1}

Also, $\omega=\dfrac{qB}{m}=\dfrac{eB}{m}$

$\Rightarrow \quad 2\pi\times10^8=\dfrac{1.6\times10^{-19}B}{9.1\times10^{-31}}$

$\Rightarrow \quad B=\left(\dfrac{2\pi\times9.1\times10^{-23}}{1.6\times10^{-19}}\right)\text{T}$

For a solenoid $B=\mu_0 ni$

$\Rightarrow \quad \dfrac{2\pi\times9.1\times10^{-23}}{1.6\times10^{-19}}=\left(4\pi\times10^{-7}\right)n(2)$

$\Rightarrow \quad n\approx 1420$ turns/m

Hence, the correct answer is (B).

26. Since $F=BI\ell$

$\Rightarrow \quad \left[\dfrac{F}{I\ell}\right]=\dfrac{MLT^{-2}}{AL}=MT^{-2}A^{-1}$

Since $[\phi]=[BA]=MT^{-2}A^{-1}L^2$

$\Rightarrow \quad [\phi]=ML^2T^{-2}A^{-1}$

Hence, the correct answer is (C).

27. $F_m=qvB$, and directed radially outward. Since,

$$N-mg\sin\theta-qvB=\frac{mv^2}{R}$$

$\Rightarrow \quad N=\dfrac{mv^2}{R}+mg\sin\theta+qvB$

Hence at $\theta=\dfrac{\pi}{2}$, we get the maximum value of N as

$\Rightarrow \quad N_{\max}=\dfrac{2mgR}{R}+mg+qB\sqrt{2gR}$

$\Rightarrow \quad N_{\max}=3mg+qB\sqrt{2gR}$

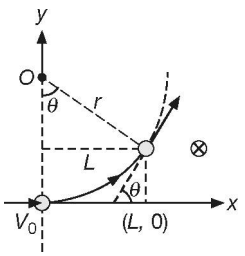
Hence, the correct answer is (C).

28. Since net force on the current carrying closed loop placed in uniform magnetic field is zero.

So, $F_{PQ}=\sqrt{\left(F_3-F_1\right)^2+F_2^2}$

Hence, the correct answer is (D).

29. $\sin\theta=\dfrac{L}{r}=\dfrac{L}{\frac{mv}{qB}}$



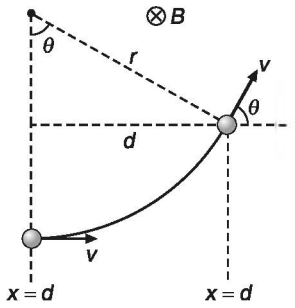
$\Rightarrow \quad v\propto\dfrac{1}{\sin\theta}$

$\Rightarrow \quad \dfrac{v}{v_0}=\dfrac{\sin60^\circ}{\sin30^\circ}$

$\Rightarrow \quad v=v_0\sqrt{3}$

Hence, the correct answer is (D).

30. Figure below shows the path of motion of the particle.



CHAPTER 1

Deviation (θ) angle of particle is given by

$$\sin\theta = \frac{d}{r} = \frac{d}{\frac{mv}{Bq}}$$

$$\Rightarrow \quad \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

Hence, the correct answer is (D).

31. The electric field at the axis is

$$E = E_{axis} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

At $x = R$, $E = \frac{qR}{4\pi\varepsilon_0(2R^2)^{3/2}}$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 R^2(2)^{3/2}}$$

The magnetic field at the axis is

$$B = B_{axis} = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}$$

where, $i = qf = q\left(\frac{v}{2\pi R}\right)$

At $x = R$, $B = \frac{\mu_0 iR^2}{2R^3(2)^{3/2}} = \frac{\mu_0 i}{2R(2)^{3/2}}$

$$\Rightarrow \quad B = \frac{\mu_0\left(\frac{qv}{2\pi R}\right)}{2R(2)^{3/2}} = \frac{\mu_0 qv}{4\pi R^2(2)^{3/2}}$$

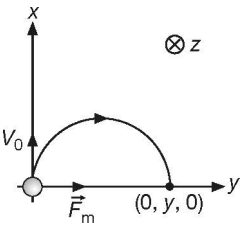
$$\Rightarrow \quad \frac{E}{B} = \frac{1}{\mu_0\varepsilon_0 v}$$

Since $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \quad \frac{E}{B} = \frac{c^2}{v}$$

Hence, the correct answer is (A).

32.



$$y = 2r = \frac{2mv_0}{qB_0} = \frac{2v_0}{B_0\alpha}$$

where the specific charge α equals $\frac{q}{m}$

Hence, the correct answer is (D).

33. For the charge to be prevented from colliding the plate, we have

$$r = d$$

$$\Rightarrow \quad \frac{\sqrt{2mK}}{qB} = d$$

$$\Rightarrow \quad B = \sqrt{\frac{2mK}{q^2d^2}}$$

$$\Rightarrow \quad B = \sqrt{\frac{2\times1.6\times10^{-26}\times2\times10^3\times e}{e^2\times10^{-4}}}$$

$$\Rightarrow \quad B = \sqrt{\frac{2\times1.6\times10^{-26}\times2\times10^3}{1.6\times10^{-19}\times10^{-4}}}$$

$$\Rightarrow \quad B = 2\text{ T}$$

Hence, the correct answer is (A).

34. Since the magnetic field is uniform and the particle is projected in a direction perpendicular to the field hence, it will describe a circular path. The particle will not hit the y-z plane only if the radius of the circle happens to be smaller than d. For the maximum value of v the radius is just equal to d.

So, $r = d = \frac{mv_0}{qB}$

$$\Rightarrow \quad v = \frac{qBd}{m}$$

Hence, the correct answer is (C).

35. Pitch p of helical path is,

$$p = (v\cos\theta)\times\frac{2\pi m}{qB} = \frac{2\pi mv\cos(45°)}{qB}$$

$$\Rightarrow \quad p = \frac{2\pi mv}{\sqrt{2}qB}$$

Also, radius of helical path is,

$$r = \frac{mv\sin\theta}{qB} = \frac{mv}{\sqrt{2}qB}$$

$$\Rightarrow \quad \frac{r}{p} = \left(\frac{mv}{\sqrt{2}qB}\right)\left(\frac{\sqrt{2}qB}{2\pi mv}\right) = \frac{1}{2\pi}$$

$$\Rightarrow \quad r = \frac{p}{2\pi}$$

Hence, the correct answer is (C).

36. $B_{\text{centre}} = B_c = \dfrac{\mu_0 I}{2a}$, $B_{\text{axis}} = B_a = \dfrac{\mu_0 I a^2}{2(a^2+9a^2)^{3/2}}$

So, the desired ratio is

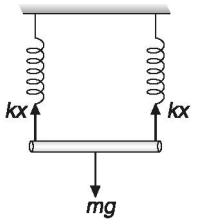
$$\frac{B_c}{B_a} = \frac{\frac{\mu_0 I}{2a}}{\frac{\mu I a^2}{2(10a^2)^{3/2}}} = (10)^{3/2}$$

$$\Rightarrow \quad \frac{B_c}{B_a} = 10\sqrt{10}$$

Hence, the correct answer is (C).

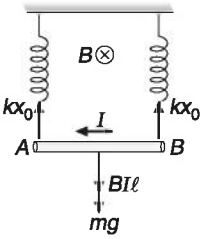
37. Initially, when B does not exist, then we have

$$mg = (2k)x \qquad \ldots(1)$$



When B is switched on, then

$$mg + BIl = 2kx_0 \qquad \ldots(2)$$



From equations (1) and (2), we get

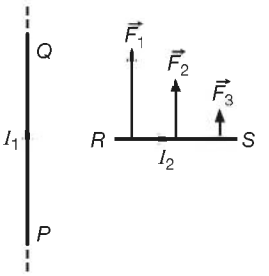
$$\frac{mg + BIl}{mg} = \frac{x_0}{x}$$

$$\Rightarrow \quad BIl = mg\left(\frac{x_0}{x} - 1\right)$$

$$\Rightarrow \quad B = \frac{mg}{Il}\left(\frac{x_0 - x}{x}\right)$$

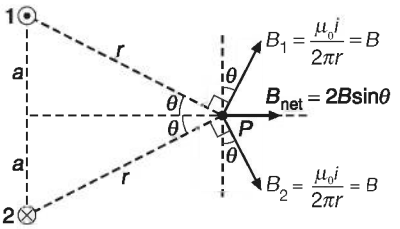
Hence, the correct answer is (D).

38. Since, the magnetic field due to wire PQ is non uniform. Neither the force nor the torque on wire RS is zero. Therefore, it will have both translational as well as rotational motion.



Hence, the correct answer is (C).

39. Net magnetic field at P due to wires 1 and 2 at point P is



$$B_{\text{net}} = 2B\sin\theta = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{a}{r}\right)$$

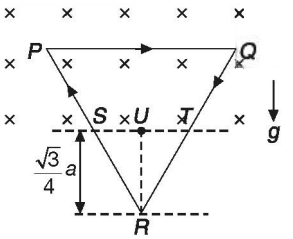
$$\Rightarrow \quad B_{\text{net}} = \frac{\mu_0 i a}{\pi r^2}$$

Hence, the correct answer is (D).

40. The force on section $MPQN$ is

$$\vec{F}_{MPQN} = \vec{F}_{MN}$$

and this force should be upwards to balance the weight.



$$\Rightarrow \quad IlB = mg \text{, where } l = ST = 2(UT)$$

$$\Rightarrow \quad l = 2\left[\frac{\sqrt{3}a}{4}\tan(30°)\right] = \frac{a}{2}$$

$$\Rightarrow \quad I\left(\frac{a}{2}\right)B = Mg$$

$$\Rightarrow \quad I = \frac{2Mg}{aB}$$

Force is upwards if current in loop is clockwise.

Hence, the correct answer is (B).

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41. Since the point $P(0, 0, -a)$ lies on z-axis. Therefore, magnetic field due to current along z-axis is zero and due to other two wires is $\frac{\mu_0}{2\pi}\frac{I}{a}$ in mutually perpendicular directions along positive y-direction and negative x-direction. So,

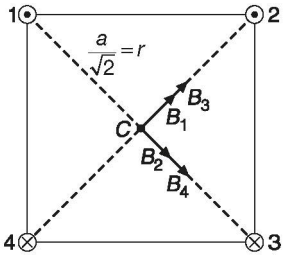
$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{a}\left(-\hat{i} + \hat{j}\right)$$

Hence, the correct answer is (C).

42. The given point lies on the wire, so $B = 0$.

Hence, the correct answer is (D).

43.



Since $B_1 = B_2 = B_3 = B_4 = B = \frac{\mu_0 I}{2\pi r}$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi\left(\frac{a}{\sqrt{2}}\right)} = \frac{\sqrt{2}\mu_0 I}{2\pi a}$$

Resultant magnetic field abc is

$$B_{\text{net}} = \sqrt{(2B)^2 + (2B)^2}$$

$$\Rightarrow \quad B_{\text{net}} = 2B\sqrt{2}$$

$$\Rightarrow \quad B_{\text{net}} = 2\sqrt{2}\frac{\mu_0 I}{2\pi\left(\frac{a}{\sqrt{2}}\right)}$$

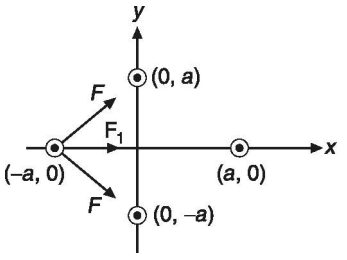
$$\Rightarrow \quad B_{\text{net}} = \frac{2\mu_0 I}{\pi a}$$

Hence, the correct answer is (A).

44. Forces acting on the loop, due to I_1 act in the plane of the loop and hence give zero torque.

Hence, the correct answer is (A).

45.

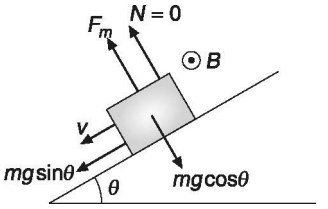


Net force is along $+x$-axis

Hence, the correct answer is (C).

46. At break-off from the surface, we have $N = 0$

So, the magnetic force on block is given by



$$F_m = mg\cos\theta$$

$$\Rightarrow \quad qvB = mg\cos\theta$$

Since $v = at$

$$\Rightarrow \quad v = (g\sin\theta)t$$

$$\Rightarrow \quad q(gt\sin\theta)B = mg\cos\theta$$

$$\Rightarrow \quad t = \frac{m\cot\theta}{qB}$$

Hence, the correct answer is (C).

47. $B = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\theta_2 - \sin\theta_1)$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}\left[\sin\frac{\pi}{2} - \sin\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}(1 - \cos\alpha)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r\left(2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)}\left(2\sin^2\left(\frac{\alpha}{2}\right)\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r}\tan\left(\frac{\alpha}{2}\right)$$

Hence, the correct answer is (D).

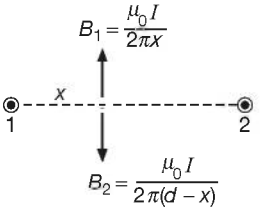
48. $F_{AB} = F_{BC}$

$$\Rightarrow \quad \frac{\mu_0}{2\pi}\frac{(i)(2i)}{d_1} = \frac{\mu_0}{2\pi}\frac{(2i)(3i)}{d_2}$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1}{3}$$

Hence, the correct answer is (A).

49.



$$B_{net} = B = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(d-x)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)$$

The above equation is best represented by (D).

Hence, the correct answer is (D).

50. At point A, the magnetic field due to both the cylinders is along $+y$ direction. The magnitude of field due to one cylinder is given by

$$B_1 = \frac{\mu_0 Jd}{4}$$

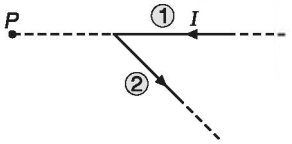
So, net magnetic field at point A is given by

$$B = B_1 + B_2$$

$$\Rightarrow \quad B = \frac{\mu_0 Jd}{2}, \text{ along the } +y \text{ direction}$$

Hence, the correct answer is (A).

51. Magnetic field at P due to 1 is zero. So, magnetic field is only due to the straight current carrying wire 2, which is given by

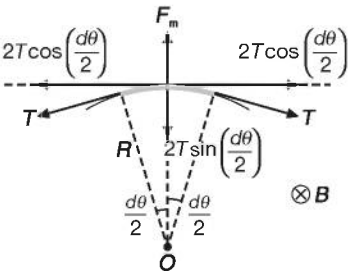


$$B = \frac{\mu_0 I}{4\pi\left(\frac{R}{\sqrt{2}}\right)}(\sin 90° + \sin 135°)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi R}(\sqrt{2} - 1)$$

Hence, the correct answer is (A).

52.



Magnetic force on the circular arc is $F_m = BId\ell$

$$F_m = BI(Rd\theta)$$

For the arc to be in equilibrium, $F = 2T\sin\left(\frac{d\theta}{2}\right)$

For small angle $d\theta$, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$\Rightarrow \quad 2Td\theta = BIRd\theta$$

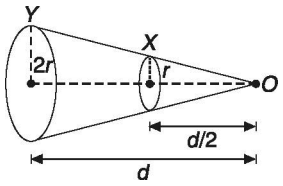
$$\Rightarrow \quad T = BIR$$

Since $L = 2\pi R$

$$\Rightarrow \quad T = BIR = \frac{BIL}{2\pi}$$

Hence, the correct answer is (C).

53.



The two coils subtend the same solid angle at O and since

$$\Rightarrow \quad \text{Solid angle} = \frac{\text{Area}}{(\perp \text{distance})^2}$$

Hence area of coil Y is 4 times the area of coil X

$\Rightarrow$ Radius of coil Y is 2 times the radius of coil X

Since the magnetic field at the axis of the coil is

$$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_Y = \frac{\mu_0}{4\pi} \times \frac{2\pi I(2r)^2}{\left[(2r)^2 + (d^2)\right]^{\frac{3}{2}}}$$

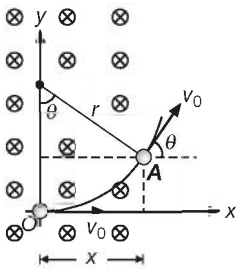
$$\Rightarrow \quad B_X = \frac{\mu_0}{4\pi} \times \frac{2\pi I(r)^2}{\left[r^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}$$

$$\Rightarrow \quad \frac{B_Y}{B_X} = \frac{4}{(4r^2 + d^2)^{\frac{3}{2}}} \times \left[\frac{4r^2 + d^2}{4}\right]^{\frac{3}{2}}$$

$$\Rightarrow \quad \frac{B_Y}{B_X} = \frac{4}{(4)^{\frac{3}{2}}} = \frac{4}{8} = \frac{1}{2}$$

Hence, the correct answer is (C).

54. $r = \frac{mv_0}{qB_0}$



CHAPTER 1

Since, $\frac{x}{r}=\frac{\sqrt{3}}{2}=\sin\theta$

$\Rightarrow \quad \theta=60°$

$\Rightarrow \quad t_{OA}=\frac{T}{6}=\frac{m\pi}{3qB_0}$

Therefore, x co-ordinate of particle at any time

$t>\frac{m\pi}{3qB_0}$ is

$$x=\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+v_0\left(t-\frac{m\pi}{3qB_0}\right)\cos(60°)$$

$$\Rightarrow \quad x=\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+\frac{v_0}{2}\left(t-\frac{m\pi}{3qB_0}\right)$$

Hence, the correct answer is (D).

55. $KE=\frac{1}{2}mv^2=eV$

$\Rightarrow \quad mv=\sqrt{2emV}$

The electron will be refocused after travelling a distance equal to pitch of helix. So

$$p=\frac{2\pi mv\cos\theta}{qB}$$

Since θ is small, so $\cos\theta\approx 1$

$$\Rightarrow \quad p=\frac{2\pi mv}{eB}=\frac{2\pi\sqrt{2emV}}{eB}$$

$$\Rightarrow \quad p=\sqrt{\frac{8\pi^2 mV}{eB^2}}$$

Hence, the correct answer is (A).

56. Using Ampere's Circuital Law

For $r\le a$, $B=0$

For $a\le r\le b$, $B=\frac{\mu_0 I}{2\pi(b^2-a^2)}\left(\frac{r^2-a^2}{r}\right)$

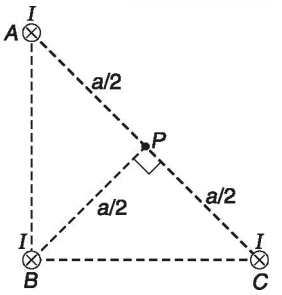
and for $r\ge b$, $B=\frac{\mu_0}{2\pi}\frac{I}{r}$

The corresponding B-r graph will be as shown in OPTION (C).

Hence, the correct answer is (C).

57. At P, net magnetic field is given by

$$B_{net}=B_A+B_B+B_C$$

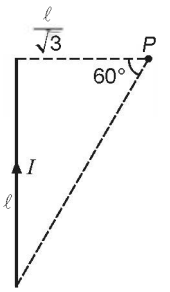


Since B_A and B_C cancel so, net field at P is only due to wire B given by

$$B_{net}=B=\frac{\mu_0 I}{2\pi r_\perp}=\frac{\mu_0 I}{2\pi\left(\frac{a}{2}\right)}=\frac{\mu_0 I}{\pi a}$$

Hence, the correct answer is (A).

58. Since, $B=\frac{\mu_0}{4\pi}\frac{I}{r_\perp}(\sin\alpha+\sin\beta)$ where $r_\perp=\frac{\ell}{\sqrt{3}}$



In this case, $\alpha=0°$ and $\beta=60°$

$$\Rightarrow \quad B=\frac{\mu_0 I}{4\pi\frac{\ell}{\sqrt{3}}}(\sin 0°+\sin 60°)=\frac{3\mu_0 I}{8\pi\ell}$$

Hence, the correct answer is (D).

59. $B=\frac{\mu_0 I}{2r}\left[1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}+\ldots\right]$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2r}\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right]$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2r}\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{3r}$$

Hence, the correct answer is (D).

60. $\vec{F}_{CAD} = \vec{F}_{CD} = \vec{F}_{CED} = BI\ell_{\text{eff}} = BI(2a)$

Net force on the frame is $\vec{F} = 3\vec{F}_{CD}$

$\Rightarrow \quad |\vec{F}| = 6BIa \qquad \{F = I\ell B\}$

Hence, the correct answer is (A).

61. When electron is projected in an electric field, then we have

$$\frac{mv_0^2}{r_1} = eE \qquad \ldots(1)$$

When electron is projected in magnetic field, then we have

$$\frac{mv_0^2}{r_2} = ev_0B \qquad \ldots(2)$$

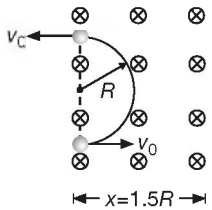
Dividing equation (1) and (2), we get

$$\frac{r_2}{r_1} = \frac{eE}{ev_0B}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{Bv_0}{E}$$

Hence, the correct answer is (D).

62. From the figure it is clear that deviation is 180° i.e., π radian (π^c), for $x > R$.



Hence, the correct answer is (D).

63. Magnetic induction at point P due current I_1 is given by

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(AP)}\hat{k} = \frac{4\pi\times10^{-7}\times2}{2\pi\times1\times10^{-2}}\hat{k}$$

$$\Rightarrow \quad \vec{B}_1 = (4\times10^{-5}\ \text{T})\hat{k}$$

Magnetic induction at point P due current I_2 is given by

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(BP)}\hat{j} = \frac{4\pi\times10^{-7}\times3}{2\pi\times2\times10^{-2}}\hat{j}$$

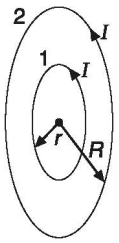
$$\Rightarrow \quad \vec{B}_2 = (3\times10^{-5}\ \text{T})\hat{j}$$

So net magnetic induction at point P is given as

$$\vec{B} = (3\times10^{-5}\ \text{T})\hat{j} + (4\times10^{-5}\ \text{T})\hat{k}$$

Hence, the correct answer is (B).

64.



Work done in rotating the smaller I coil about any of its diameter through an angle π is given by

$$W = -M_1B_2(\cos\pi - \cos0) = 2M_1B_2$$

where $M_1 = n(\pi r^2)I$ and $B_2 = N\left(\frac{\mu_0 I}{2R}\right)$

$$\Rightarrow \quad W = 2M_1B_2 = \frac{\mu_0\pi nNI^2r^2}{R}$$

Hence, the correct answer is (C).

65. $$\vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\ldots.\right)\hat{k} - \left(\frac{1}{2}+\frac{1}{8}+\ldots.\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{B} = \frac{2}{3}\frac{\mu_0}{2\pi}\hat{k} = \left(\frac{2}{3}\times2\times10^{-7}\right)\hat{k}$$

$$\Rightarrow \quad \vec{B} = 1.33\times10^{-7}\hat{k}$$

Hence, the correct answer is (C).

66. Force on the wires parallel to x-axis will be obtained by integration (as $B \propto x$ and x coordinates vary along these wires). But on a loop, there are two such wires. Force on them will be equal and opposite. However, forces on two wires parallel to y-axis can be obtained directly (without integration) as value of B is same along these wires. But their values will be different as x-coordinate and therefore B is different.

$$F_{\text{net}} = \Delta F \ \text{(on two wires)}$$

$$\Rightarrow \quad F_{\text{net}} = Ia(\Delta B) = Ia(B_0)(\Delta x)$$

$$\Rightarrow \quad F_{\text{net}} = IaB_0(a) = IB_0a^2$$

This is independent of x

$$\Rightarrow \quad F_1 = F_2 = IBla^2 \neq 0$$

Hence, the correct answer is (D).

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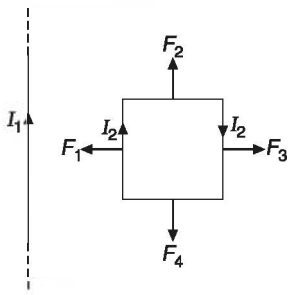
67.



F_2 and F_4 cancel each other and $F_1 > F_3$

$$\Rightarrow \quad F_{\text{net}} = F_1 - F_3 = \left(\frac{\mu_0 I_1 I_2}{2\pi a}\right)a - \left(\frac{\mu_0 I_1 I_2}{4\pi a}\right)a$$

$$\Rightarrow \quad F_{\text{net}} = \frac{\mu_0 I_1 I_2}{4\pi}$$

Hence, the correct answer is (B).

68. Electrostatic force on electron should be equal and opposite to magnetic force, i.e.,

$$\vec{F}_e = -\vec{F}_m$$

$$\Rightarrow \quad q\vec{E} = -q(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad -e\vec{E} = -(-e)(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad \vec{E} = -(\vec{v} \times \vec{B})$$

So, $\vec{B}$ should be along negative z-axis.

Hence, the correct answer is (C).

69. Work done is

$W = U_f - U_i$, where

$U_f = -M_f B$ and $U_i = -M_i B$

Since $M_i = Ia^2$

Keeping the length same, the shape of loop is changed from square to an equilateral triangle of side l (say), so we have

$$4a = 3l$$

$$\Rightarrow \quad l = \frac{4a}{3}$$

Since area of an equilateral triangle of side l is

$$A = \frac{\sqrt{3}}{4}l^2$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}\left(\frac{16a^2}{9}\right) = \frac{4\sqrt{3}a^2}{9}$$

$$\Rightarrow \quad M_f = IA = \frac{4\sqrt{3}Ia^2}{9}$$

$$\Rightarrow \quad W = U_f - U_i = -M_f B - (-M_i B)$$

$$\Rightarrow \quad W = B(M_i - M_f)$$

$$\Rightarrow \quad W = BIa^2\left(1 - \frac{4\sqrt{3}}{9}\right)$$

Hence, the correct answer is (A).

70. Since we know that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$. Also, we are aware of the fact that the ratio of electric field to magnetic field equals velocity. So, we have

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E}{B}$$

$$\Rightarrow \quad \left[\mu_0 \varepsilon_0 \left(\frac{B}{E}\right)^2\right] = M^0 L^0 T^0$$

Hence, the correct answer is (C).

71. Magnetic field due to wire

$$B = \frac{\mu_0 I}{2\pi r} = \left(\frac{4\pi \times 10^{-7}}{2\pi}\right)\left(\frac{30}{2 \times 10^{-2}}\right)$$

$$\Rightarrow \quad B = 3 \times 10^{-4} \text{ T}$$

This magnetic field will be perpendicular to external magnetic field. So, net magnetic field is

$$B_{\text{net}} = \sqrt{B^2 + B_0^2}$$

$$\Rightarrow \quad B_{\text{net}} = \sqrt{(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2}$$

$$\Rightarrow \quad B_{\text{net}} = 5 \times 10^{-4} \text{ T}$$

Hence, the correct answer is (C).

72. Conceptual

Hence, the correct answer is (A).

73. If I_1 is current in $ACDB$ then

$$\frac{\mu_0}{4}\frac{I_1}{r} = \frac{\mu_0}{4}\frac{I_2}{4r}$$

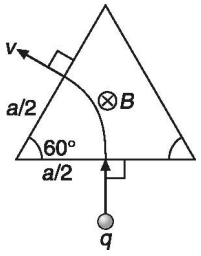
$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{4}$$

$$\Rightarrow \quad I_1 = \frac{I}{5}$$

$$\Rightarrow \quad \frac{I_1}{I} = \frac{1}{5}$$

Hence, the correct answer is (D).

74. The charged particle moves in a circle of radius $\frac{a}{2}$



$$\Rightarrow \quad qvB = \frac{mv^2}{\frac{a}{2}}$$

$$\Rightarrow \quad B = \frac{2mv}{qa}$$

Hence, the correct answer is (B).

75. Let N be the number of turns and R be the radius of the coil. Then

$$\ell = 2\pi RN$$

$$\Rightarrow \quad R = \frac{\ell}{2\pi N} \qquad \ldots(1)$$

The magnetic moment of the coil is

$$M = NIA = NI(\pi R^2)$$

$$\Rightarrow \quad M = (NI\pi)\left(\frac{\ell^2}{4\pi^2 N^2}\right) = \frac{I\ell^2}{4\pi N}$$

Maximum value of M can be only when $N = 1$, then

$$M_{max} = \frac{I\ell^2}{4\pi}$$

So, $\tau_{max} = \tau_0 = M_{max} B \sin 90° = \frac{BI\ell^2}{4\pi}$

Hence, the correct answer is (C).

76. $\tau = MB\sin(90-\theta) = C\theta$

$$\Rightarrow \quad (10)(0.025)\cos\theta = (1)\theta$$

$$\Rightarrow \quad \frac{1}{4}\sqrt{1-\sin^2\theta} = \theta$$

Since θ is small, so $\sin\theta \approx \theta$

$$\Rightarrow \quad \frac{1}{4}\sqrt{1-\theta^2} = 4\theta$$

$$\Rightarrow \quad 1-\theta^2 = 16\theta^2$$

$$\Rightarrow \quad 17\theta^2 = 1$$

$$\Rightarrow \quad \theta = \frac{1}{\sqrt{17}} \text{ radian} = 0.24 \text{ radian}$$

Hence, the correct answer is (A).

77. The particles will not collide, when

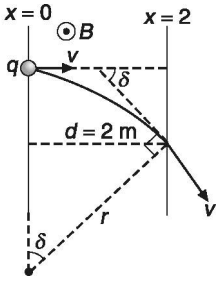
$$\ell > 2(r_1 + r_2)$$

$$\Rightarrow \quad \ell > 2\left(\frac{mv_1}{qB} + \frac{mv_2}{qB}\right)$$

$$\Rightarrow \quad \ell > \frac{2m}{qB}(v_1 + v_2)$$

Hence, the correct answer is (D).

78.



$$\sin\delta = \frac{d}{r}$$

where $r = \frac{mv}{qB} = \frac{p}{qB}$

$$\Rightarrow \quad \sin(60°) = \frac{qBd}{p}$$

$$\Rightarrow \quad p = \frac{qBd}{\sin(60°)} = \frac{2\times 2\times 10^{-6}\times 2\sqrt{3}\times 10^{-3}\times 2}{\sqrt{3}}$$

$$\Rightarrow \quad p = 10\times 10^{-9} = 1.6\times 10^{-8} \text{ kgms}^{-1}$$

Hence, the correct answer is (B).

79. Since, every current carrying loop behaves like a magnetic dipole. If it lies in the plane of paper and current in it is in clockwise direction magnetic field at all points lying within the loop is perpendicular to paper inwards and at points outside the loop magnetic field is perpendicular to paper in outward direction.

For $\theta < 180°$, the centre O lies outside the loop and current is clockwise. Therefore, magnetic field is perpendicular to paper in outward direction.

Hence, the correct answer is (B).

80. The magnified view of the arrangement is shown in Figure.

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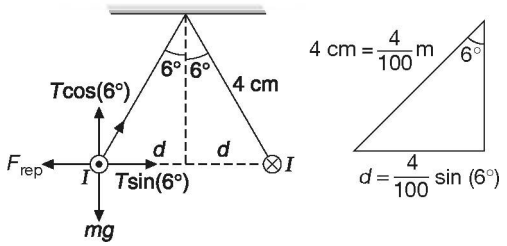


$$T\cos(6°) = mg \qquad \ldots(1)$$

$$T\sin(6°) = \frac{\mu_0 I^2 l}{2\pi(2d)} \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\tan(6°) = \frac{\mu_0 I^2 l}{4\pi mgd} = \frac{\mu_0 I^2}{4\pi\lambda gd}$$

where $\lambda = \frac{m}{l} = 0.0125\ \text{kgm}^{-1}$ and

$$\tan(6°) \approx \sin(6°) = \frac{6\times\pi}{180} \approx 0.1$$

$$\Rightarrow \quad 0.1 = \frac{10^{-7} I^2}{(0.0125)(10)\left(\frac{4}{100}\times 0.1\right)}$$

$$\Rightarrow \quad I^2 = 500$$

$$\Rightarrow \quad I = 22.3\ \text{A}$$

Hence, the correct answer is (B).

81. $B = \frac{\mu_0 I_{in}}{2\pi x}$ where

$$I_{in} = I - \left[\frac{I}{\pi(c^2-b^2)}\right]\left[\pi(x^2-b^2)\right]$$

$$\Rightarrow \quad I_{in} = I\left(\frac{c^2-x^2}{c^2-b^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi x}\left(\frac{c^2-x^2}{c^2-b^2}\right)$$

Hence, the correct answer is (C).

82. The path of the particle will be a helix of time period

$$T = \frac{2\pi m}{qB_0} = \frac{2\pi}{B_0\alpha} \qquad \left\{\because \alpha = \frac{q}{m}\right\}$$

The given time $t = \frac{\pi}{B_0\alpha} = \frac{T}{2}$

So, the co-ordinates of particle at time $t = \frac{T}{2}$ are $\left(\frac{v_x T}{2},\ 0,\ -2r\right)$

where $r = \frac{mv_y}{qB_0} = \frac{v_0}{B_0\alpha}$

Hence the required co-ordinates are $\left(\frac{v_0\pi}{B_0\alpha},\ 0,\ \frac{-2v_0}{B_0\alpha}\right)$

Hence, the correct answer is (C).

83. On a horizontal rough surface, minimum force required to just slide the wire of mass m is

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad B_{min} iL = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad B_{min} = \frac{\mu mg}{iL\sqrt{1+\mu^2}}$$

Hence, the correct answer is (A).

84. $B = 2\left(\frac{\mu_0}{4\pi}\frac{I}{\frac{d}{\sqrt{2}}}\right)(\sin 90° - \sin 45°)$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\sqrt{2}\pi d}\left(1-\frac{1}{\sqrt{2}}\right),\ \otimes$$

Hence, the correct answer is (B).

85. According to Ampere's law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

For loop A, $\oint \vec{B}\cdot d\vec{l} = \mu_0(0) = 0$

For loop B, $\oint \vec{B}\cdot d\vec{l} = \mu_0(I+I-I) = \mu_0 I$

For loop C, $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$

For loop D, $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$

$$\Rightarrow \quad B > A > C = D$$

Hence, the correct answer is (C).

86. Just like electric field is negative gradient of electric potential, we can also say that magnetic field is negative gradient of magnetic scalar potential. So, we have

$$B = -\frac{\Delta V_B}{\Delta x_\perp}$$

where $\Delta x_\perp$ is the perpendicular separation between the equipotential surfaces drawn for magnetic scalar potential.

$$\Rightarrow \quad |B| = \frac{2\times10^{-5} - 1\times10^{5}}{(0.1)\sin 30^\circ} = 2\times10^{-4}\ \text{T}$$

Hence, the correct answer is (D).

87. $$x = \frac{B}{M} = \left(\frac{\mu_0 I}{2r}\right)\left(\frac{1}{I(\pi r^2)}\right)$$

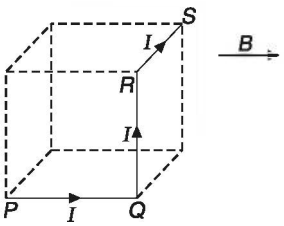
$$\Rightarrow \quad x \propto \frac{1}{r^3}$$

So, x becomes $\frac{x}{8}$ when radius and current both are doubled.

$$\Rightarrow \text{ Decrease in value of } x \text{ is } \left(x - \frac{x}{8}\right) = \frac{7x}{8}$$

Hence, the correct answer is (C).

88.



Let the field be directed along the side PQ, then

$$F_{PQ} = 0$$

$$F_{QR} = BIl$$

$$F_{RS} = BIl$$

Since, F_{QR} and F_{RS} are perpendicular to each other, so

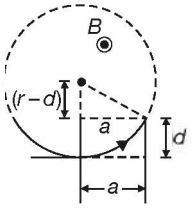
$$F_{\text{net}} = \sqrt{(BIl)^2 + (BIl)^2} = \sqrt{2}BIl$$

Hence, the correct answer is (C).

89. $$qvB\sin 90 = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv = qBr \qquad \ldots(1)$$

Since $a^2 + (r-d)^2 = r^2$



$$\Rightarrow \quad a^2 = r^2 - (r-d)^2$$

$$\Rightarrow \quad a^2 = (r - r + d)(r + r - d)$$

$$\Rightarrow \quad a^2 = (d)(2r - d)$$

$$\Rightarrow \quad 2r - d = \frac{a^2}{d}$$

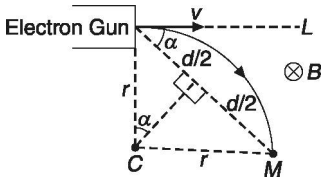
$$\Rightarrow \quad r = \frac{a^2}{2d} + \frac{d}{2} \qquad \ldots(2)$$

Put (2) in (1), we get

$$p = mv = qBr = \frac{qB}{2}\left(\frac{a^2}{d} + d\right)$$

Hence, the correct answer is (A).

90. For electron (negatively charged) to hit the target M, magnetic field B must be directed inwards



Since $r = \frac{mv}{eB}$

$$\Rightarrow \quad r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2mqV}}{qB}$$

$$\Rightarrow \quad r = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

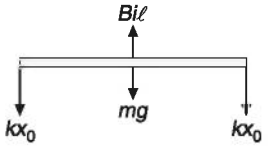
Also, $r = \frac{d}{2\sin\alpha}$

$$\Rightarrow \quad \frac{d}{2\sin\alpha} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

$$\Rightarrow \quad B = 2\sqrt{\frac{2mV}{e}}\,\frac{\sin\alpha}{d}$$

Hence, the correct answer is (A).

91. If the springs are extended through x_0, then for equilibrium of rod, we have



CHAPTER 1

$$2kx_0 + mg = ilB$$

$$\Rightarrow \quad x_0 = \frac{ilB - mg}{2K}$$

Length of spring now becomes

$$l = l_0 + x_0$$

$$\Rightarrow \quad l = l_0 + \frac{ilB - mg}{2k}$$

Hence, the correct answer is (B).

92. Using Ampere's Circuital Law, we get the desired results.

Hence, the correct answer is (B).

93. At the centre of loop, magnetic field is

$$B_{\text{centre}} = \frac{\mu_0 I}{2a}$$

At axial point, the magnetic field is

$$B_{\text{axis}} = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

$$\frac{B_{\text{axis}}}{B_{\text{centre}}} = \frac{a^3}{\left(a^2 \times x^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = B_{\text{centre}} \frac{a^3}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = 0.5 \times 10^{-4} \times \frac{(12)^3}{(144 + 25)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = 0.5 \times 10^{-4} \times \left(\frac{12}{13}\right)^3$$

$$\Rightarrow \quad B_{\text{axis}} = 3.9 \times 10^{-5}\ \text{T}$$

Hence, the correct answer is (A).

94. Since $B = \mu_0 nI$

Also, we know that the magnetic energy density equals pressure. So,

$$P = \frac{B^2}{2\mu_0} = \frac{\left(\mu_0 nI\right)^2}{2\mu_0} = \frac{\mu_0 n^2 I^2}{2}$$

Hence, the correct answer is (B).

95. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$

$$\Rightarrow \quad r_p = \frac{\sqrt{2m_p K}}{eB}$$

$$\Rightarrow \quad r_d = \frac{\sqrt{2\left(2m_p\right)K}}{eB}$$

$$\Rightarrow \quad r_\alpha = \frac{\sqrt{2\left(4m_p\right)K}}{(2e)B}$$

$$\Rightarrow \quad r_p : r_d : r_\alpha = 1 : \sqrt{2} : 1$$

Hence, the correct answer is (A).

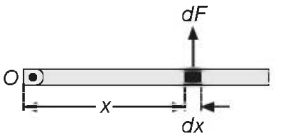
96. The force acting on the elementary portion of the current carrying conductor is given as,

$$dF = I(dx)B\sin 90°$$

$$\Rightarrow \quad dF = BIdx$$

The torque applied by dF about $O = d\tau = xdF$

$\Rightarrow$ The total torque about O is τ given by



$$\tau = \int d\tau = \int x(BIdx)$$

$$\Rightarrow \quad \tau = IB\int_0^L xdx = \frac{IBL^2}{2}$$

The angular acceleration

$$\alpha = \frac{\tau}{\left(\begin{matrix}\text{Moment of Inertia}\\ \text{of Rod about O}\end{matrix}\right)} = \frac{\tau}{\left(\frac{1}{3}mL^2\right)}$$

$$\Rightarrow \quad \alpha = \frac{\left(\frac{BIL^2}{2}\right)}{\left(\frac{mL^2}{3}\right)}$$

$$\Rightarrow \quad \alpha = \frac{3IB}{2m}$$

Hence, the correct answer is (C).

97. Current divides in the inverse ratio of resistances, so we get

$$i_1 = \frac{20}{47}\ \text{A}\ ,\ i_2 = \frac{15}{47}\ \text{A}\ ,\ i_3 = \frac{12}{47}\ \text{A}$$

For force on wire 2 to be zero, we have

$$\frac{i_1 i_2}{d_1} = \frac{i_2 i_3}{d_2}$$

$$\Rightarrow \quad \frac{20\times 15}{d_1}=\frac{15\times 12}{d_2}$$

$$\Rightarrow \quad \frac{d_1}{d_2}=\frac{5}{3}$$

Hence, the correct answer is (B).

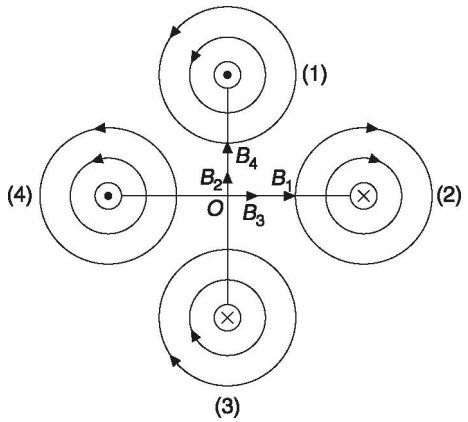
98. Diameter, $D=2r=\dfrac{2mv}{qB}$

$$\Rightarrow \quad D\propto \frac{m}{q}$$

Here $\dfrac{m}{q}$ is maximum for C^+. So, it will hit the farthest.

Hence, the correct answer is (B).

99. The field at origin O due to the wires is shown in Figure.



$$\Rightarrow \quad \vec{B}_{\text{origin}}=\vec{B}_0=2B\hat{i}+2B\hat{j}$$

$$\Rightarrow \quad \left|\vec{B}_0\right|=2\sqrt{2}B$$

Hence, the correct answer is (C).

100. The central wire creates field $\vec{B}=\dfrac{\mu_0 I_1}{2\pi R}$ counter clockwise. The curved portions of the loop experience no force since $\vec{\ell}\times\vec{B}=\vec{0}$. The straight portions both experience a force $I(\vec{\ell}\times\vec{B})$ to the right, amounting to

$$\vec{F}=I_2(2L)\left(\frac{\mu_0 I_1}{2\pi R}\right)=\frac{\mu_0 I_1 I_2 L}{\pi R} \text{ to the right.}$$

Hence, the correct answer is (C).

101. $mg=kx_0$ and $mg+BiL=2(kx_0)$

$$\Rightarrow \quad mg+B\left(\frac{E}{R}\right)L=2mg$$

$$\Rightarrow \quad B=\frac{mgR}{EL}$$

Hence, the correct answer is (B).

102. $\dfrac{F_{AB}}{\ell}=\dfrac{\mu_0 I I_0}{2\pi\left(\frac{\ell}{2}\right)}$ {towards XY}

$$\Rightarrow \quad F_{AB}=\frac{\mu_0 I I_0}{\pi} \quad \{\text{towards } XY\}$$

Similarly, $\dfrac{F_{CD}}{\ell}=\dfrac{\mu_0 I I_0}{2\pi\left(\frac{3\ell}{2}\right)}$ {away from XY}

$$\Rightarrow \quad F_{CD}=\frac{\mu_0 I I_0}{3\pi} \quad \{\text{away from } XY\}$$

$$\Rightarrow \quad F_{\text{net}}=\frac{\mu_0 I I_0}{\pi}-\frac{\mu_0 I I_0}{3\pi}=\frac{2}{3}\left(\frac{\mu_0 I I_0}{\pi}\right) \quad \{\text{towards } XY\}$$

Hence, the correct answer is (D).

103. $\vec{F}=I(\vec{l}\times\vec{B})$, where

$\vec{l}=10^{-2}\hat{i}$ and $\vec{B}=\left(0.74\hat{j}-0.3\hat{k}\right)$ T

$$\Rightarrow \quad \vec{F}=3.5\left[10^{-2}\hat{i}\times\left(0.74\hat{j}-0.36\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}=\left(2.59\hat{k}+1.26\hat{j}\right)\times 10^{-2} \text{ N}$$

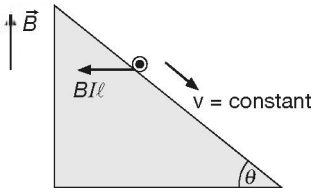
Hence, the correct answer is (A).

104. $\left|\vec{F}_{ABC}\right|=\left|\vec{F}_{AC}\right|=BIl_{\text{eff}}$

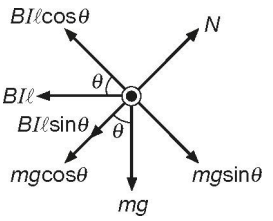
$$\Rightarrow \quad F_{ABC}=(2)(5)(2)=20 \text{ N}$$

Hence, the correct answer is (B).

105. Magnetic force $\left|\vec{F}_m\right|=BI\ell$ acts in the direction shown in Figure.



Since the rod moves downwards with constant velocity, hence the net force on it is zero.



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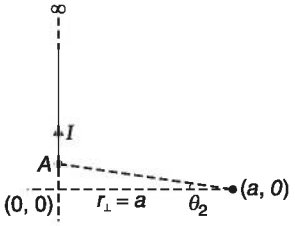
$\Rightarrow \quad F_m \cos\theta = mg\sin\theta$

$\Rightarrow \quad BI\ell\cos\theta = mg\sin\theta$

$\Rightarrow \quad B = \left(\frac{mg}{I\ell}\right)\tan\theta$

Hence, the correct answer is (B).

106.



Since $B = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 - \sin\theta_2)$

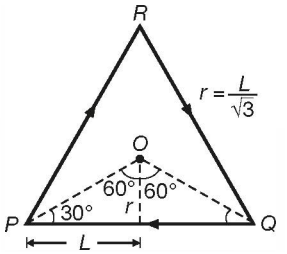
where $\theta_1 = 90°$ and $\sin\theta_2 = \frac{b}{\sqrt{a^2+b^2}}$

$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}\left(1 - \frac{b}{\sqrt{a^2+b^2}}\right)$

Hence, the correct answer is (B).

107. Magnetic field at O is

$B = 3B_{PQ} = 3\left[\frac{\mu_0}{4\pi}\frac{I}{r}(\sin 60° + \sin 60°)\right]$



$\Rightarrow \quad B = 3\left(\frac{\mu_0 I}{4\pi}\right)\left(\frac{\sqrt{3}}{L}\right)(\sqrt{3})$

$\Rightarrow \quad B = \frac{9\mu_0 I}{4\pi L}$

Hence, the correct answer is (D).

108. Velocity magnitude will not change as magnetic field does not work on charge. So tangential acceleration (i.e., rate of change of speed) is zero.
Hence, the correct answer is (D).

109. $\overrightarrow{DA} = -2\cos(30°)\hat{i} - 2\sin(30°)\hat{k} = \left(-\sqrt{3}\hat{i} - \hat{k}\right)$ and $\overrightarrow{AB} = 2\hat{j}$

Since, $\vec{M} = I(\overrightarrow{DA} \times \overrightarrow{AB})$

$\Rightarrow \quad \vec{M} = \frac{1}{2}\left[\left(-\sqrt{3}\hat{i} - \hat{k}\right) \times \left(2\hat{j}\right)\right]$

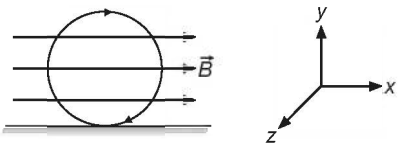
$\Rightarrow \quad \vec{M} = -\sqrt{3}\hat{k} + \hat{i}$

$\Rightarrow \quad \vec{M} = \left(\hat{i} - \sqrt{3}\hat{k}\right)\ \text{Am}^2$

Hence, the correct answer is (C).

110. $\vec{M} = IA\left(-\hat{k}\right) = -(4)(\pi)(0.5)^2\hat{k}$

$\Rightarrow \quad \vec{M} = -\pi\hat{k}\,(Am^2)$



Since, $\vec{\tau} = \vec{M} \times \vec{B} = \left(-\pi\hat{k}\right) \times \left(10\hat{i}\right) = -(10\pi)\hat{j}$

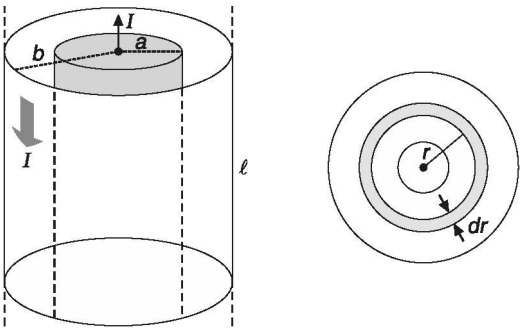
The axis of rotation is along $\vec{\tau}$ i.e., axis of rotation is the y-axis, moment of inertia about which is

$I = \frac{1}{2}mR^2 = \frac{1}{2}(2)(0.5)^2 = \frac{1}{4}\ \text{kgm}^2$

$\Rightarrow \quad \alpha = \frac{|\vec{\tau}|}{I} = \frac{10\pi}{\left(\frac{1}{4}\right)} = 40\pi\ \text{rads}^{-2}$

Hence, the correct answer is (D).

111. Consider an infinitesimal element of radius r, thickness dr, length l such that volume of element is $dV = (2\pi r dr)l$



The magnetic field at this element is $B = \frac{\mu_0 I}{2\pi r}$

The magnetic energy density u_m is given by

$u_m = \frac{B^2}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$

If dU be the energy associated with this infinitesimal element, then

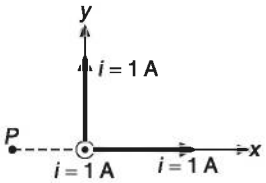
$$dU = u_m dV = \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right)(2\pi r dr)l$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 I^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad U = \frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (A).

112.



No magnetic field at P due to wire along x-axis

Magnetic field at P due to wire along y-axis is

$$\vec{B}_1 = \left(\frac{\mu_0 I}{4\pi a}\right)\hat{k}$$

Magnetic field at P due to wire along z-axis is

$$\vec{B}_2 = \left(\frac{\mu_0 I}{4\pi a}\right)\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{B}_P = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi a}\left(-\hat{j} + \hat{k}\right)$$

Since $a = 2$ m

$$\Rightarrow \quad \vec{B}_P = \frac{\mu_0 I}{8\pi}\left(-\hat{j} + \hat{k}\right)$$

Hence, the correct answer is (C).

113. Let q be the charge and m the mass of the particle.

At $(x_0, 0, 0)$ speed of the particle is $\sqrt{4^2 + 3^2} = 5$ units

Using Work-Energy Theorem, we get

$$(qE_0)x_0 = \frac{1}{2}mv^2 = \frac{25m}{2}$$

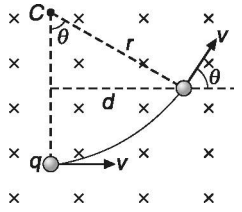
$$\Rightarrow \quad x_0 = \frac{25m}{2qE_0}$$

Hence, the correct answer is (C).

114. For $qBd \ll mv$, we have

$$d \ll \frac{mv}{qB}$$

$$\Rightarrow \quad d \ll r$$



Since impulse is equal to change in momentum. The initial and final momentum are

$$p_i = mv\hat{i}$$

$$\vec{p}_f = mv\cos\theta\hat{i} + mv\sin\theta\hat{j}$$

Since $\sin\theta = \frac{d}{r}$ and $d \ll r$, so θ is very small and hence

$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1$$

$$\Rightarrow \quad \vec{p}_f \approx mv\hat{i} + mv\theta\hat{j}$$

$$\Rightarrow \quad \Delta\vec{p} = mv\hat{i} + mv\theta\hat{j} - mv\hat{i}$$

$$\Rightarrow \quad \Delta\vec{p} = (mv\theta)\hat{j}$$

Since, $\theta = \frac{d}{r}$

$$\Rightarrow \quad |\Delta\vec{p}| = (mv)\frac{d}{r} \text{ where } r = \frac{mv}{qB}$$

$$\Rightarrow \quad |\Delta\vec{p}| = qBd$$

Hence, Impulse $J = qBd$

Hence, the correct answer is (C).

115. The following forces act on the particle.

- Force T acting radially inwards.
- Centrifugal force $\frac{mv^2}{r}$ acting radially outwards.
- Magnetic force qvB acting radially inwards.

$$\Rightarrow \quad T + qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad \frac{mv^2}{r} - qvB - T = 0$$

$$\Rightarrow \quad v^2 - \left(\frac{qBr}{m}\right)v - \frac{Tr}{m} = 0$$

$$\Rightarrow \quad v = \frac{1}{2}\left[\frac{qBr}{m} + \sqrt{\frac{q^2B^2r^2}{m^2} + \frac{4Tr}{m}}\right]$$

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$$\Rightarrow \quad v=\frac{r}{2}\left[\frac{qB}{m}+\sqrt{\frac{q^2B^2}{m^2}+\frac{4T}{mr}}\right]$$

Hence, the correct answer is (D).

116. Energy gained in one movement across the gap is 100 KeV. So, the energy gained in one turn is 200 KeV

$$\Rightarrow \quad N=\frac{20\times10^6}{200\times10^3}=100$$

Hence, the correct answer is (D).

117. $B_c=\dfrac{\mu_0 NI}{2r}$ and

$$B_a=\frac{\mu_0 NIr^2}{2\left(r^2+h^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_a=\frac{\mu_0 NI}{2r\left(1+\dfrac{h^2}{r^2}\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_a=B_c\left(1+\frac{h^2}{r^2}\right)^{-\frac{3}{2}}$$

$$\Rightarrow \quad B_a\simeq B_c\left(1-\frac{3h^2}{2r^2}\right) \qquad \{\because h^2\ll r^2\}$$

$$\Rightarrow \quad \frac{B_c-B_a}{B_c}=\frac{3h^2}{2r^2}$$

Hence, the correct answer is (D).

119. $\left[\dfrac{B^2}{2\mu_0}\right]=\left[\dfrac{\text{Energy}}{\text{Volume}}\right]=ML^{-1}T^{-2}$

$$[R^2C^2]=[\text{Time}]^2=T^2$$

$$\left[\frac{B^2R^2C^2}{2\mu_0}\right]=ML^{-1}$$

Since the velocity of transverse wave in a string is given by $v=\sqrt{\dfrac{T}{\lambda}}$ where T is the tension in the string of mass per unit length λ. So, we have

$$[v]=\left[\sqrt{\frac{T}{\lambda}}\right]=\left[\sqrt{\frac{F}{\lambda}}\right]$$

$$\Rightarrow \quad [v^2]=\left[\frac{F}{\lambda}\right]$$

$$\Rightarrow \quad [\lambda]=\left[\frac{F}{v^2}\right]=\left[\frac{qvB}{v^2}\right]=\left[\frac{qB}{v}\right]$$

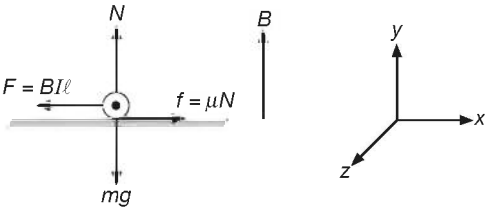
Hence, the correct answer is (B).

120. $B_P=\dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{2}-\dfrac{1}{3}\right)=\dfrac{\mu_0 I}{12\pi}$

Hence, the correct answer is (C).

121. $F_m=(2)(1)(1)=2\text{ N}$

$f=\mu N=(0.2)(2)(10)=4\text{ N}$



Since, the applied force i.e., the magnetic force is less than the force of friction, so the wire will not move at all.

Hence, the correct answer is (D).

122. $B=\dfrac{\mu_0 Ni}{2r}$

Since $l=(2\pi r)N$

$$\Rightarrow \quad r=\frac{l}{2\pi N}$$

$$\Rightarrow \quad B=\frac{\mu_0 Ni}{2\left(\dfrac{l}{2\pi N}\right)}$$

$$\Rightarrow \quad B\propto N^2$$

$$\Rightarrow \quad B=9B_0$$

Hence, the correct answer is (B).

123. The torque on the loop must be equal to the gravitational torque exerted about an axis that is tangential the loop.

The gravitational torque is given by

$$\tau_1=mgr \qquad \ldots(1)$$

Since $\vec{M}=(IA)\hat{k}$ and only B_x causes a torque. Therefore, torque due to the magnetic field on the current loop is

$$\tau_2 = |\vec{M} \times \vec{B}| = MB\sin 90° = \pi r^2 I B_x \quad \ldots(2)$$

Equations (1) and (2), we get

$$I = \frac{mg}{\pi r B_x}$$

Hence, the correct answer is (B).

124. Since we know that $B = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$

So, magnetic field due to wire 1 at P is given by

$$B_1 = \frac{\mu_0 i}{4\pi(a\cos 30°)}(\sin 30° + \sin 30°)$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 i}{2\pi\sqrt{3}a}, \text{ outwards}$$

Due to wire 2, magnetic field at point P is given by

$$B_2 = \frac{\mu_0 i}{2\pi\sqrt{3}(2a)}, \text{ inwards}$$

Total magnetic field at point P is given by

$$B = \frac{\mu_0 i}{2\pi\sqrt{3}a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots\right]$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 i 2\ln 2}{4\pi\sqrt{3}a} = \frac{\mu_0 I \ln 4}{4\pi\sqrt{3}a}\hat{k}$$

Hence, the correct answer is (B).

125. Charge on a ring of radius x and width dx

$$dq = (2\pi x dx)\sigma$$

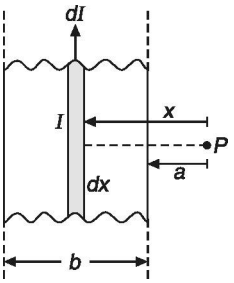
Current, $dI = \frac{dq}{dt} = \frac{2\pi x\sigma dx}{dt} = \omega\sigma x\, dx$

$$dB = \frac{\mu_0 dI r^2}{2\left(x^2 + y^2\right)^{3/2}}$$

$$B = \frac{\mu_0\sigma\omega}{2}\left(\frac{r^2 + 2y^2}{\sqrt{r^2 + y^2}} - 2y\right)$$

Hence, the correct answer is (C).

126. Consider an infinitesimal thin very long element carrying a current dI, at a distance x from it as shown in Figure. Then



$$dI = \frac{I}{b}dx$$

If dB be the magnetic field due to this strip at point P, then

$$dB = \frac{\mu_0(dI)}{2\pi x} = \frac{\mu_0 I}{2\pi b}\frac{dx}{x}$$

$$\Rightarrow \quad B = \int dB = \frac{\mu_0 I}{2\pi b}\int_a^{a+b}\frac{dx}{x}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi b}\log_e\left(\frac{a+b}{a}\right)$$

$$\Rightarrow \quad B = \frac{2\mu_0 I}{4\pi b}\log_e\left(1 + \frac{b}{a}\right)$$

Hence, the correct answer is (B).

127. Since, we have studied that

$$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m}$$

$$\Rightarrow \quad \text{Magnetic moment} = (\text{Angular Momentum})\frac{q}{2m}$$

$$\Rightarrow \quad \text{Magnetic moment} = (I\omega)\frac{q}{2m}$$

$$\Rightarrow \quad \text{Magnetic moment} = \left(\frac{1}{2}mR^2\right)(\omega)\left(\frac{q}{2m}\right) = \frac{1}{4}q\omega R^2$$

Hence, the correct answer is (D).

128. In time $t = \frac{\pi m}{qB} = \frac{T}{2}$, which is half the revolution time, the velocity of the particle will get reversed in yz plane, so we have

$$\vec{v} = 2\hat{i} - 3\hat{j} - 4\hat{k}$$

Since, net force on the particle is zero, so we have

$$q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$$

$$\Rightarrow \quad \vec{E} = -(\vec{v} \times \vec{B}) = -\left(2\hat{i} - 3\hat{j} - 4\hat{k}\right) \times 2\hat{i}$$

$$\Rightarrow \quad \vec{E} = -6\hat{k} + 8\hat{j}$$

Hence, the correct answer is (C).

129. $\frac{\mu_0 I_1}{4\pi(a+x)} = \frac{\mu_0 I_2}{4\pi(a-x)}$

$$\Rightarrow \quad \frac{a-x}{a+x} = \frac{I_2}{I_1}$$

$$\Rightarrow \quad I_1 a - I_1 x = I_2 a + I_2 x$$

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$$\Rightarrow \quad x=\left(\frac{I_1-I_2}{I_1+I_2}\right)a$$

Hence, the correct answer is (C).

130. For a closed current carrying loop placed in a uniform magnetic field,

$$F_{\text{net}} = \text{zero}\ .$$

Hence, the correct answer is (D).

131. Since we know (have already calculated) the magnetic field due to the cross sectional (half current carrying) cylinder at its centre is

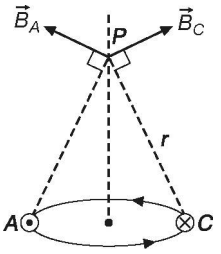
$$B=\frac{\mu_0 I}{\pi^2 R}$$

Further, $F = BI\ell$, so for 1 metre of length, we have

$$F=\frac{\mu_0 I^2}{\pi^2 R}$$

Hence, the correct answer is (C).

132. The magnetic field vectors at a point P on axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instants the point charge is at A and C respectively as shown in Figure.



Since the particles rotates in circle, only magnitude of magnetic field remains constant at the point on axis P but it's direction changes. We can also think that, the magnetic field at point on the axis due to charged particle moving along a circular path is given by

$$\vec{B}=\frac{\mu_0}{4\pi}\left(\frac{q\vec{v}\times\vec{r}}{r^3}\right)$$

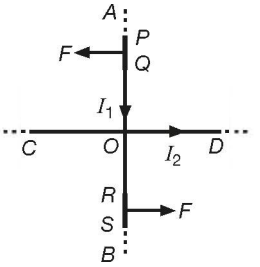
So, it can be seen that the magnitude of the magnetic field at a point on the axis remains constant. But the direction of the field keeps on changing, because at any instant $\vec{B}$ must be perpendicular to $\vec{r}$ (as shown)

Hence, the correct answer is (A).

133. Force on BC and DE are equal and opposite and hence cancel out. So, the force on CD is BIL

Hence, the correct answer is (A).

135. Consider two elements PQ and RS at equal distance point O. Magnetic field at PQ due to I_2 is perpendicular to paper outwards ($\odot$) and at RS perpendicular to paper inwards ($\otimes$). Therefore, magnetic force on these elements will be in the directions shown in Figure.



Hence, net force on wire AB will be zero but it will have an anticlockwise torque.

So, $\vec{F}=\vec{0}$ and $\vec{\tau}\neq\vec{0}$ (CCW)

Hence, the correct answer is (B).

137. Magnetic field due to ADB is

$$B_1=\left(\frac{\mu_0 I}{4\pi a}\right)\theta, \odot$$

and magnetic field due to ACB is

$$B_2=\frac{\mu_0 I}{4\pi a}(2\pi-\theta), \otimes$$

$$\Rightarrow \quad B_{\text{net}}=B_2-B_1=\frac{\mu_0 I}{4\pi a}(2\pi-2\theta)=\frac{\mu_0 I}{2\pi a}(\pi-\theta), \otimes$$

Hence, the correct answer is (C).

138. Since, $R=\dfrac{\sqrt{2mK}}{qB}$ and $R_d = R_p$

$$\Rightarrow \quad \frac{\sqrt{2m_dK_d}}{q_dB}=\frac{\sqrt{2m_pK_p}}{q_pB}$$

$$\Rightarrow \quad \frac{\sqrt{2(2m_p)K_d}}{eB}=\frac{\sqrt{2m_pK_p}}{eB}$$

$$\Rightarrow \quad K_p = 2K_d = 2\times 40\ \text{MeV} = 80\ \text{MeV}$$

Hence, the correct answer is (B).

139. Using Ampere's Circuital Law, we get the desired results.

Hence, the correct answer is (D).

140. Electron velocity v is directed along magnetic field lines, so

$$F = 0$$

Hence, the correct answer is (D).

141. $\vec{B}=\frac{\mu_0 I}{2R}(\pm\hat{i})+\frac{\mu_0 I}{2R}(\pm\hat{j})+\frac{\mu_0 I}{2R}(\pm\hat{k})$

$\Rightarrow \quad |\vec{B}|=\frac{\mu_0 I}{2R}\sqrt{3}$

Hence, the correct answer is (B).

142. Since $|\vec{\tau}|=MB\sin\theta$

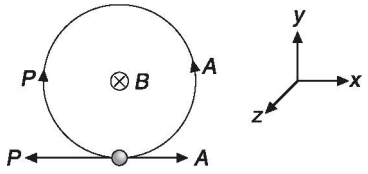
$\Rightarrow \quad W=\int dW=\int \tau d\theta=\int_0^{\theta} MB\sin\theta d\theta=MB(-\cos\theta)\Big|_0^{\theta}$

$\Rightarrow \quad W=MB(1-\cos\theta)$

Hence, the correct answer is (D).

143. When A emits P, then both have charge of equal magnitude. Also, A and P will have the momentum same in magnitude, but they will move in opposite directions.

Since $r=\frac{mv}{qB}=\frac{p}{qB}$, so A and P will move in the circle of same radius and the same centre but in opposite sense as shown.



If they meet after time t, then we have

$\omega_A t+\omega_P t=2\pi$

$\Rightarrow \quad t=\frac{2\pi}{\omega_A+\omega_P}=\frac{2\pi}{\frac{2eB}{4m}+\frac{2eB}{(A-4)m}}$

$\Rightarrow \quad t=\frac{4(A-4)m\pi}{eBA}$

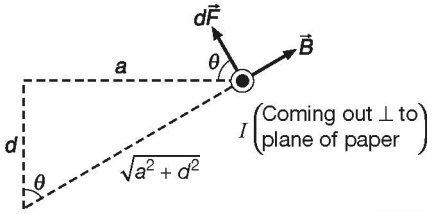
Since $\theta=\omega t$

$\Rightarrow \quad \theta_A=\omega_A t=\frac{2eB}{4m}\times\frac{4m(A-4)}{eBA}$

$\Rightarrow \quad \theta_A=\frac{2(A-4)\pi}{A}=\frac{48\pi}{25}$ radian

Hence, the correct answer is (D).

144. The situation, for the cross-sectional view of the wire is shown in figure in which the current is coming out perpendicular to the plane of paper.



Force on the current element ds is $dF=BIds$.

By symmetry, the forces in the plane of the current loop gives zero when contributions around the loop are added. But vertically net force F is calculated as

$F=I(B\sin\theta)$ (circumference of loop)

$\Rightarrow \quad F=(IB\sin\theta)(2\pi a)$

$\Rightarrow \quad F=2\pi aIB\sin\theta$

Hence, the correct answer is (C).

145. Since $\vec{F}=I(\vec{l}\times\vec{B})$

where, $\vec{Il}=(Il)\hat{i}$ and $\vec{B}=B_0(\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow \quad \vec{F}=B_0 Il\begin{vmatrix}\overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}}\\ 1 & 0 & 0\\ 1 & 1 & 1\end{vmatrix}$

$\Rightarrow \quad \vec{F}=B_0 Il(-\hat{j}+\hat{k})$

$\Rightarrow \quad |\vec{F}|=B_0 Il\sqrt{2}=\sqrt{2}B_0 Il$

Hence, the correct answer is (D).

146. The work done is given by

$W=U_f-U_i$

where, $U=$ potential energy

$\Rightarrow \quad W=(-M_f B\cos 0°)-(-M_i B\cos 0°)$

$\Rightarrow \quad W=IB(A_i-A_f)$

where $A_i=a^2$ and $A_f=\pi R^2$

Since, $2\pi R=4a$

$\Rightarrow \quad R=\frac{2a}{\pi}$

$\Rightarrow \quad A_f=\frac{4a^2}{\pi}$

$\Rightarrow \quad W=BIa^2\left(1-\frac{4}{\pi}\right)$

Hence, the correct answer is (C).

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147. At the point A, field due to both the wire and the loop is inwards, so

$$B_{net} = B_{wire} + B_{loop}$$

So, net field is more than the field due to loop/coil.

Hence, the correct answer is (B).

148. Maximum magnetic field is at surface of the wire and is given by

$$B = \frac{\mu_0}{2\pi}\frac{I}{R} \qquad \{\text{using Ampere's Circuital Law}\}$$

$$\Rightarrow \quad I = \frac{BR}{\frac{\mu_0}{2\pi}}$$

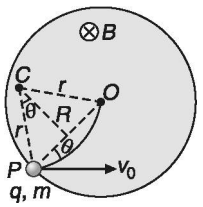
$$\Rightarrow \quad I = \frac{(5\times10^{-3})\left(\frac{3.2}{2}\times10^{-3}\right)}{2\times10^{-7}}$$

$$\Rightarrow \quad I = 40 \text{ A}$$

Hence, the correct answer is (A).

149. Since the particle enters perpendicular to the field, so it follows a circular path of radius r given by

$$r = \frac{mv_0}{qB}$$



For particle to pass through O, P and O must lie on the perimeter of this circle of radius $r = \frac{mv_0}{qB}$.

Since, $R = 2r\sin\theta$

$$\Rightarrow \quad R = 2\frac{mv_0}{qB}\sin\theta$$

$$\Rightarrow \quad v_0 = \frac{qBR}{2m\sin\theta}$$

Hence, the correct answer is (B).

150. If R be the radius of the loop, then

$$A = \pi R^2$$

$$\Rightarrow \quad R = \sqrt{\frac{A}{\pi}}$$

Since, $B = \frac{\mu_0 I}{2R}$

$$\Rightarrow \quad I = \frac{2BR}{\mu_0}$$

By definition $M = IA = \left(\frac{2BR}{\mu_0}\right)A = \left(\frac{2BA}{\mu_0}\right)\sqrt{\frac{A}{\pi}}$

Hence, the correct answer is (C).

151. B at the centre of smallest square is given by

$$B_1 = 4\left[\frac{\mu_0 i}{4\pi\left(\frac{a}{2}\right)}2\sin(45°)\right]$$

$$\Rightarrow \quad B_1 = \frac{2\sqrt{2}\mu_0 i}{\pi a}, \text{ inside}$$

Similarly, we get

$$B_2 = \frac{2\sqrt{2}\mu_0 i}{\pi(2a)}, \text{ outside}$$

$$B_3 = \frac{2\sqrt{2}\mu_0 i}{\pi(3a)}, \text{ inside}$$

and so on.

Hence, net magnetic field at the centre of arrangement is

$$B_{net} = B_1 + B_2 + B_3 + B_4 + B_5 + ...$$

$$\Rightarrow \quad B_{net} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} .....\right)$$

$$\Rightarrow \quad B_{net} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\log_e(2)$$

Hence, the correct answer is (B).

152. For $\vec{B}$ to be zero at the centre, sum of field due to the ring and that due to the semi-infinite straight section should be equal and opposite

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi R} - \frac{\mu_0 I}{2\pi R}\left(\frac{\theta}{2}\right) = 0$$

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi R}\left(1 - \frac{\theta}{2}\right) = 0$$

$$\Rightarrow \quad \theta = 2 \text{ rad}$$

Hence, the correct answer is (C).

153. Magnetic field due to circle is

$$B_1 = \frac{\mu_0 I}{2R}$$

Since, $L = 2\pi R$

$$\Rightarrow \quad 2R = \frac{L}{\pi}$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 \pi I}{L} \approx 3.14\left(\frac{\mu_0 I}{L}\right)$$

Magnetic field due to square coil is

$$B_2 = 2\sqrt{2}\left(\frac{\mu_0 I}{\pi x}\right)$$

Since $L = 4x$

$$\Rightarrow \quad x = \frac{L}{4}$$

$$\Rightarrow \quad B_2 = 8\sqrt{2}\frac{\mu_0 I}{\pi L} \approx 3.60\left(\frac{\mu_0 I}{L}\right)$$

$$\Rightarrow \quad B_1 < B_2$$

Hence, the correct answer is (B).

154. Consider an element of length dx at a distance x from the wire.

Field at the element is $B = \frac{\mu_0 i}{2\pi x}$

If dF is the force experienced by the element then

$$dF = Bl\,dx$$

$$\Rightarrow \quad dF = \frac{\mu_0 il}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 il}{2\pi}\int_{L/2}^{3L/2}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 il}{2\pi}\ell nx\Big|_{L/2}^{3L/2}$$

$$\Rightarrow \quad F = \frac{\mu_0 il}{2\pi}\left[\ell n\frac{3L}{2} - \ell n\frac{L}{2}\right]$$

$$\Rightarrow \quad F = \frac{\mu_0 il}{2\pi}\ell n3$$

Hence, the correct answer is (C).

155. Since Impulse = change in linear momentum, so we have

$$\int F\,dt = mv$$

$$\Rightarrow \quad \int BI\ell\,dt = mv$$

$$\Rightarrow \quad B\ell\int Idt = mv$$

$$\Rightarrow \quad B\ell q = mv \qquad \left\{\because \int Idt = q\right\}$$

$$\Rightarrow \quad q = \frac{mv}{B\ell}$$

Hence, the correct answer is (B).

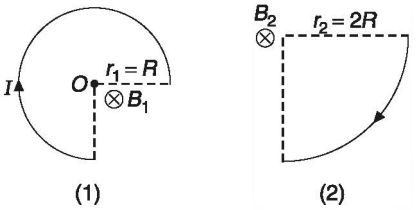
156. Because current in both the loops is zero, so $B = 0$

Hence, the correct answer is (A).

157. Field at centre of an arc subtending an angle θ is

$$B = \left(\frac{\mu_0 I}{4\pi r}\right)\theta$$

Net magnetic field B is



$$B = B_1 + B_2$$

where $B_1 = \frac{3\mu_0 I}{8R}$ and $B_2 = \frac{\mu_0 I}{8(2R)}$

$$\Rightarrow \quad B = \frac{3}{8}\frac{\mu_0 I}{R} + \frac{\mu_0 I}{2(2R)} \times \frac{1}{4}$$

$$\Rightarrow \quad B = \frac{7}{16}\left(\frac{\mu_0 I}{R}\right)\otimes$$

Hence, the correct answer is (A).

158. Applying Ampere's Law to the Rectangular Loop 12341 shown, we get

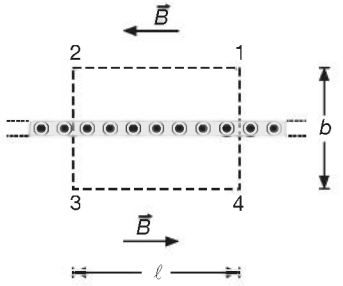
$$\oint \vec{B}\cdot d\vec{\ell} = \int_1^2 \vec{B}\cdot d\vec{\ell} + \int_2^3 \vec{B}\cdot d\vec{\ell} + \int_3^4 \vec{B}\cdot d\vec{\ell} + \int_4^1 \vec{B}\cdot d\vec{\ell}$$

For 14 and 32 (or 41 and 23),

$$\vec{B} \perp d\vec{\ell} \text{ and hence } \int \vec{B}\cdot d\vec{\ell} = 0$$

For 1 to 2 and 3 to 4 $\vec{B} \parallel d\vec{\ell}$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = 2B\ell$$



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According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\Rightarrow \quad 2B\ell = \mu_0(\lambda\ell)$$

$$\Rightarrow \quad B = \frac{\mu_0\lambda}{2}$$

Hence, the correct answer is (B).

159. Due to symmetry of the circuit, it is clear that for each current carrying wire there is another wire that cancels the field of the first.
For example, the field at P due to wire AB is cancelled by field of wire GF. Field of wire CD is cancelled by wire HE. Similarly, AH and CF, DG and BE, BC and HG, AD and EF fields are cancelled and hence net field is zero at P.
Hence, the correct answer is (C).

160. Since, kinetic energy $K = q\Delta V$

$$\Rightarrow \quad K = QU$$

For the particle to move in a circle, we have

$$R = \frac{mv}{QB} = \frac{\sqrt{2mK}}{QB} = \frac{\sqrt{2mQU}}{QB}$$

Since the magnetic moment M is

$$M = iA = i(\pi R^2)$$

$$\Rightarrow \quad M = \frac{Q}{T}(\pi R^2) = \frac{Q}{\left(\frac{2\pi m}{QB}\right)}\pi\left(\frac{2mQU}{Q^2B^2}\right)$$

$$\Rightarrow \quad M = \frac{Q^2B}{2\pi m}\frac{2\pi mQU}{Q^2B^2} = \frac{QU}{B}$$

If U is doubled and B is also doubled, then M remains same
Hence, the correct answer is (D).

161. Since magnetic field at the centre of an arc is equal to

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

Hence net

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r} - \frac{1}{4r} + \frac{1}{5r} - \frac{1}{6r}\right]\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(\frac{60 - 30 + 20 - 15 + 12 - 10}{60}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(\frac{37}{60}\right) = \frac{37\mu_0 I\theta}{240\pi r}$$

Hence, the correct answer is (B).

162. When the current in the loop flows clockwise, then only the loop will be attracted towards the conductor because AB is attracted strongly towards conductor and CD is repelled weakly away from it (Think Why?). So, net force will attract the loop towards the conductor.
Hence, the correct answer is (D).

163. $$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$

$$F' = \frac{\mu_0}{2\pi}\frac{\left(\frac{i_1}{3}\right)\left(\frac{i_2}{3}\right)}{3r} = \frac{F}{27}$$

Hence, the correct answer is (D).

164. When the plane of the loop aligns itself normally to the field, then torque on it becomes zero, because angle between $\vec{M}$ and $\vec{B}$ is $0°$. For this to happen the loop must turn through an angle $\left(\frac{\pi}{2} + \theta\right)$.
Hence, the correct answer is (B).

165. τ due to B_0 is $\tau_B = MB_0 = I(\pi R^2)B_0$

τ due to mg is $\tau_{mg} = mgR$

For equilibrium, $\tau_B = \tau_{mg}$

$$\Rightarrow \quad I(\pi R^2)B_0 = mgR$$

$$\Rightarrow \quad I = \frac{mg}{\pi B_0 R}$$

Hence, the correct answer is (A).

166. From Ampere's Circuital Law, we get

$$B_1(2\pi x) = \mu_0 I$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 I}{2\pi x}$$

Again, using Ampere's Circuital Law, we get

$$B_2(2\pi)(2x) = \mu_0(I + I)$$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I}{2\pi x} = B_1$$

Hence, the correct answer is (C).

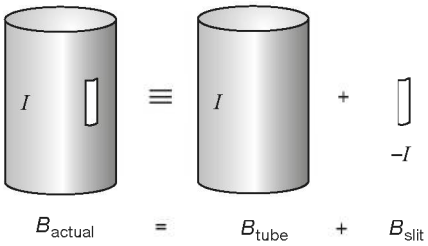
167. The magnetic field due to a lengthy thin walled tube at any internal point is zero. When a longitudinal slit of width w is removed, then a net field will exist

inside. The tube with a slit can be visualised as equivalent to a full tube carrying a current I and a slit carrying current $-I$ (i.e. current equal in magnitude and opposite in direction). So, the net field inside the tube is due to the slit carrying current I in opposite direction.

Since

$$B_{\text{tube}} = 0$$

$$\Rightarrow \quad B_{\text{actual}} = B_{\text{slit}} = \frac{\mu_0 I_{\text{slit}}}{2\pi R}$$



(Since width of slit is very small, so field due to slit is equal to that of a thin wire).

Further

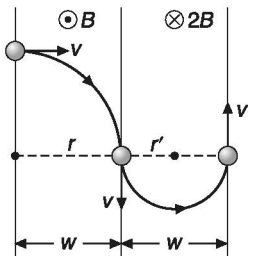
$$I_{\text{slit}} = \left(\frac{I}{2\pi R}\right) w$$

$$\Rightarrow \quad B_{\text{actual}} = \frac{\mu_0\left(\frac{I}{2\pi R}\right) w}{2\pi R} = \frac{\mu_0 I w}{4\pi^2 R^2}$$

Hence, the correct answer is (C).

168. For the first, $r = \frac{mv}{qB}$

For the second region, $r' = \frac{mv}{q(2B)} = \frac{r}{2}$



This makes us conclude that the particle will cover a quarter circle in the first region and a semicircle in the second region, so that it can come out of the magnetic field. Hence, we have

$$t = \frac{T}{4} + \frac{T'}{2}$$

$$\Rightarrow \quad t = \frac{1}{4}\left(\frac{2\pi m}{qB}\right) + \frac{1}{2}\left[\frac{2\pi m}{q(2B)}\right]$$

$$\Rightarrow \quad t = \frac{\pi m}{2qB} + \frac{\pi m}{2qB} = \frac{\pi m}{qB}$$

Hence, the correct answer is (B).

169. Since we know that the GYROMAGNETIC RATIO equals,

$$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m} = \frac{e}{2m}$$

So, magnetic moment is

$$M = \left(\frac{e}{2m}\right)$$

(angular momentum of the electron in n^{th} orbit)

From Bohr's Quantisation Rule, we have

$$L = \frac{nh}{2\pi}$$

$$\Rightarrow \quad M = \left(\frac{e}{2m}\right)\left(\frac{nh}{2\pi}\right) = \frac{neh}{4\pi m}$$

Hence, the correct answer is (B).

170. Magnetic field is non zero only in the region between the two solenoids, where $B = \mu_0 n_2 i_2$

So, energy stored per unit volume is

$$u_2 = \frac{B^2}{2\mu_0} = \frac{\mu_0 n_2^2 i_2^2}{2}$$

The energy stored per unit length is the energy per unit volume times the area of cross section in which $B \neq 0$

$$\Rightarrow \quad e = \frac{\mu_0 n_2^2 i_1^2}{2}\left[\pi\left(r_2^2 - r_1^2\right)\right]$$

Since $n_1 i_1 = n_2 i_2$

$$\Rightarrow \quad e = \frac{\mu_0 n_1^2 i_1^2}{2}\left[\pi\left(r_2^2 - r_1^2\right)\right]$$

$$\Rightarrow \quad e = \frac{\mu_0 \pi}{2} n_1^2 i_1^2 \left(r_2^2 - r_1^2\right)$$

Hence, the correct answer is (A).

172. For rotational equilibrium of sphere about point P, we have

$$(mg\sin\theta)R = M(B\sin(180-\theta))$$

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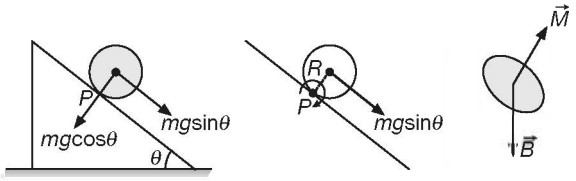
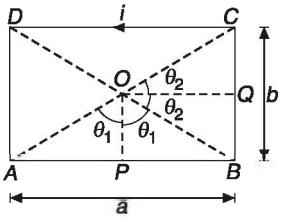


$$\Rightarrow \quad (mgR\sin\theta) = I(\pi R^2)B\sin\theta$$

$$\Rightarrow \quad B = \frac{mg}{\pi IR}$$

Hence, the correct answer is (D).

173. Due to symmetry, $B_{AB} = B_{CD}$ and $B_{BC} = B_{DA}$



$$B_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{b}{2}\right)}(\sin\theta_1 + \sin\theta_1)$$

$$\Rightarrow \quad B_{AB} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{b}\right)\left(\frac{a}{\sqrt{a^2+b^2}}\right)$$

Similarly, $B_{BC} = B_{DA} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{a}\right)\left(\frac{b}{\sqrt{a^2+b^2}}\right)$

$$\Rightarrow \quad B = B_{AB} + B_{BC} + B_{CD} + B_{DA} = 2(B_{AB} + B_{CD})$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{\sqrt{a^2+b^2}}\right)\left(\frac{a}{b}+\frac{b}{a}+\frac{a}{b}+\frac{b}{a}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{\pi\sqrt{a^2+b^2}}\left(2\frac{a}{b}+2\frac{b}{a}\right)$$

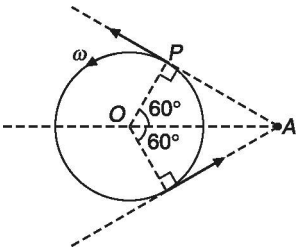
$$\Rightarrow \quad B = \frac{2\mu_0 i}{\pi\sqrt{a^2+b^2}}\left(\frac{a^2+b^2}{ab}\right)$$

$$\Rightarrow \quad B = \frac{2\mu_0 i}{\pi ab}\sqrt{a^2+b^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{4\pi}\left(\frac{8\sqrt{a^2+b^2}}{ab}\right)$$

Hence, the correct answer is (A).

174. The point A shall record zero magnetic field (due to α particle) when the α particle is at position P and Q as shown in Figure.



The time taken by α-particle to go from P to Q is

$$t = \frac{1}{3}\left(\frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad \omega = \frac{2\pi}{3t}$$

Hence, the correct answer is (B).

175. The net electric field

$$E = \vec{E}_1 + \vec{E}_2$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The net force acting on the electron is zero because it moves with constant velocity, due to its motion on straight line.

$$\Rightarrow \quad \vec{F}_{\text{net}} = \vec{F}_e + \vec{F}_m = 0$$

$$\Rightarrow \quad |\vec{F}_e| = |\vec{F}_m|$$

$$\Rightarrow \quad eE = evB$$

$$\Rightarrow \quad v = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B}$$

The time of motion inside the capacitor is

$$t = \frac{\ell}{v} = \frac{\varepsilon_0 \ell B}{\sigma}$$

Hence, the correct answer is (B).

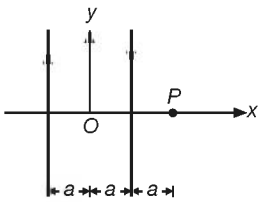
176. $F = BIl\sin\theta$

$$\Rightarrow \quad F_{PQ} = BIl\sin 0° = 0$$

$$\Rightarrow \quad F_{QR} = BIl\sin 90° = BIl$$

Hence, the correct answer is (C).

177. If the currents are in the directions shown in Figure.



Then $\vec{B}_0 = -2\left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)\hat{k} = -\left(\frac{\mu_0 I}{\pi a}\right)\hat{k}$

$$\vec{B} = \left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)\hat{k} - \left(\frac{\mu_0}{2\pi}\frac{I}{3a}\right)\hat{k}$$

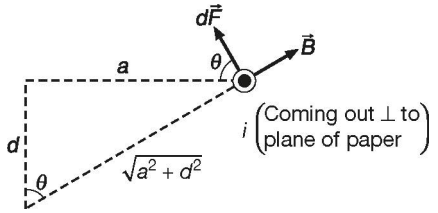
$$\Rightarrow \quad \vec{B} = \left(\frac{\mu_0}{3\pi}\frac{I}{a}\right)\hat{k} = -\frac{B_0}{3}\hat{k}$$

$$\Rightarrow \quad B = \frac{B_0}{3} \text{ along } -z \text{ axis}$$

$$\Rightarrow \quad B = \frac{B_0}{3}, \otimes$$

Hence, the correct answer is (C).

178. The situation, for the cross-sectional view of the wire is shown in Figure in which the current is coming out perpendicular to the plane of paper.



Force on the current element ds is $dF = Bids$.
By symmetry, the forces in the plane of the current loop gives zero when contributions around the loop are added. But vertically net force F is calculated as

$$F = i(B\sin\theta) \text{ (circumference of loop)}$$

$$\Rightarrow \quad F = (iB\sin\theta)(2\pi a)$$

$$\Rightarrow \quad F = 2\pi aiB\sin\theta$$

Since $\sin\theta = \frac{a}{\sqrt{a^2 + d^2}}$

$$\Rightarrow \quad F = \frac{2\pi a^2 iB_0}{\sqrt{a^2 + d^2}}$$

Hence, the correct answer is (B).

179. $\vec{F}_{aob} = \vec{F}_{ab} = I\left(\vec{\ell}_{ab} \times \vec{B}\right)$

where $\ell_{ab} = 2\sqrt{(2)(2)} = 4$ m

$$\Rightarrow \quad \vec{F}_{ab} = 2\left[\left(-4\hat{j}\right) \times \left(-4\hat{k}\right)\right] = 32\hat{i}$$

Hence, the correct answer is (B).

180. According to Modified Work-Energy Theorem, we get,

$$W = qE\Delta x = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 - 0$$

$$\Rightarrow \quad (4\times10^{-6})(4)(x) = \frac{1}{2}m(4^2 + 3^2)$$

$$\Rightarrow \quad (16\times10^{-6})x = \frac{1}{2}\times10\times10^{-6}\times25$$

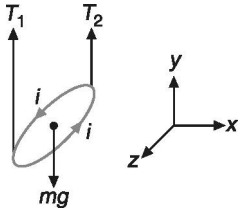
$$\Rightarrow \quad x = \frac{250}{32} \text{ m} = \frac{125}{16} \text{ m}$$

Hence, the correct answer is (B).

181. For equilibrium, we have

$$\Sigma F = 0 \text{ and } \Sigma\tau = 0$$

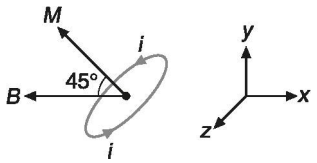
$$\Rightarrow \quad T_1 + T_2 = mg \qquad \ldots(1)$$



Torque on ring due to current is

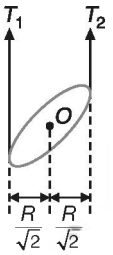
$$\tau_i = MB\sin\theta = i\pi R^2 B\sin(45°)$$

$$\Rightarrow \quad \tau_i = \frac{i\pi R^2 B}{\sqrt{2}}, \text{ counter clockwise in } xy \text{ plane}$$



Since $\Sigma\tau = 0$

$$\Rightarrow \quad \tau_i + \tau_{T_1} + \tau_{T_2} = 0 \qquad \{\text{about } 0\}$$



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$$\Rightarrow \underbrace{\frac{i\pi R^2 B}{\sqrt{2}}}_{CCW} + \underbrace{T_1\left(\frac{R}{\sqrt{2}}\right)}_{CW} + \underbrace{T_2\left(\frac{R}{\sqrt{2}}\right)}_{CCW} = 0$$

Taking CW as positive, we get

$$\frac{T_1 R}{\sqrt{2}} = \frac{T_2 R}{\sqrt{2}} + \frac{i\pi R^2 B}{\sqrt{2}}$$

$$\Rightarrow \quad T_1 = T_2 + \pi iRB$$

$$\Rightarrow \quad T_1 - T_2 = \frac{mg}{4} \qquad \ldots(2)$$

Using equations (1) and (2), we get

$$T_1 = \frac{5mg}{8},\ T_2 = \frac{3mg}{8}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{5}{3}$$

Hence, the correct answer is (B).

182. Since, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ ...(1)

$$\Rightarrow \quad \Delta V = V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{\lambda}{2\pi\varepsilon_0} = \frac{V}{\ln\left(\frac{a}{b}\right)}$$

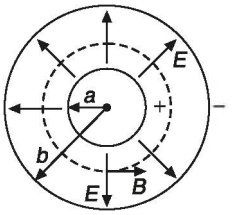
From (1) and (2), we get

$$E = \frac{V}{r\ln\left(\frac{a}{b}\right)}$$

B varies with r as

$$B = \frac{\mu_0 I}{2\pi r}$$

Since, B and E are perpendicular to each other, so for the electron to travel undeviated parallel to axis in the evacuated region, we have



$$F_m = F_e$$

$$\Rightarrow \quad qv_0 B = qE$$

$$\Rightarrow \quad v_0 = \frac{E}{B} = \frac{2\pi V}{\mu_0 I \ln\left(\frac{a}{b}\right)}$$

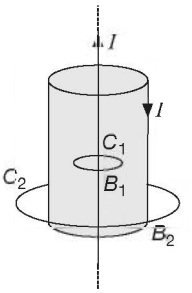
Hence, the correct answer is (B).

183. By Right hand thumb rule, the field due to both the segments is out of the plane that is along $+ve$ z axis.

Hence, the correct answer is (B).

Multiple Correct Choice Type Questions

1. We apply Ampere's Law to the co-axial circular loops C_1 and C_2 with corresponding magnetic fields B_1 and B_2 respectively. For C_1 and C_2 we have the OPTION (B) correct. On the other hand, when the currents are in same direction, $B_1 \neq 0$ and $B_2 \neq 0$. Hence, the OPTIONS (C) and (D) are also correct.



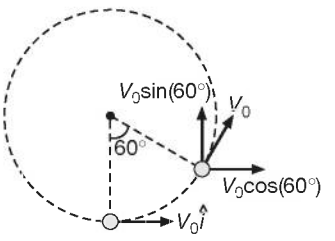
Hence, (B), (C) and (D) are correct.

2. $B_P = 0$, because current in upper part and lower part is the same due to which the magnetic field for upper part is into the page and for the lower part the field is out of the page. $B_Q \neq 0$, because current in upper and lower section is not same. Hence net field at Q will not be zero. $B_R = 0$, because current in upper and lower section is same.

Hence, (A) and (C) are correct.

3. $\vec{v} \perp \vec{B}$

Therefore, path of the particle is a circle. In magnetic field speed of particle remains constant. Therefore, distance moved by the particle in time $t = \dfrac{\pi}{\alpha B_0}$ is $v_0 t$ or $\dfrac{\pi v_0}{\alpha B_0}$



Since $T=\frac{2\pi m}{qB_0}=\frac{2\pi}{\alpha B_0}$

$\Rightarrow \quad t=\frac{\pi}{3\alpha B_0}=\frac{T}{6}$

$\Rightarrow \quad \vec{v}=v_0\left(\cos(60°)\hat{i}+\sin(60°)\hat{j}\right)$

$\Rightarrow \quad \vec{v}=\frac{v_0}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

Hence, (B) and (C) are correct.

Please note that since the magnitude of velocity remains same in the magnetic field. This is true but will not help us to conclude whether (D) is correct or false.

4. $T=\frac{2\pi m}{qB}$

$\Rightarrow \quad a=\frac{T_1}{T_2}=1$

$r=\frac{mv\sin\theta}{qB}$

$\Rightarrow \quad b=\frac{r_1}{r_2}=\frac{\sin 30°}{\sin 60°}=\frac{1}{\sqrt{3}}$

$p=(T)(v\cos\theta)=\frac{2\pi mv\cos\theta}{qB}$

$\Rightarrow \quad c=\frac{p_1}{p_2}=\frac{\cos 30°}{\cos 60°}=\sqrt{3}$

From above, we get

$abc=1$, $a=bc$ and $c=3ab$

Hence, (B), (C) and (D) are correct.

5. At the axis of the coil, we have

$$B_x=\frac{\mu_0 iR^2}{2\left(R^2+x^2\right)^{\frac{3}{2}}} \qquad \text{\{OPTION (A)\}}$$

Let us now calculate $\int_0^\infty B_x dx$

$\Rightarrow \quad \int_0^\infty B_x dx=\frac{\mu_0 iR^2}{2}\int_0^\infty \frac{dx}{\left(R^2+x^2\right)^{\frac{3}{2}}}$

Substituting $x=R\tan\theta$, we get

$dx=R\sec^2\theta d\theta$

When $x=0$, $\theta=0$ and when $x\to\infty$, $\theta\to\frac{\pi}{2}$

$\Rightarrow \quad \int_0^\infty B_x dx=\frac{\mu_0 iR^2}{2}\int_0^{\frac{\pi}{2}} \frac{R\sec^2\theta d\theta}{R^3\sec^3\theta}$

$\Rightarrow \quad \int_0^\infty B_x dx=\frac{\mu_0 i}{2}\int_0^{\frac{\pi}{2}}\cos\theta d\theta$

$\Rightarrow \quad \int_0^\infty B_x dx=\frac{\mu_0 i}{2}\left(\sin\frac{\pi}{2}-\sin 0°\right)$

$\Rightarrow \quad \int_0^\infty B_x dx=\frac{\mu_0 i}{2}$ **{OPTION (B)}**

Similarly, $\int_{-\infty}^{+\infty} B_x dx=\frac{\mu_0 i}{2}\left(\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\right)$

$\Rightarrow \quad \int_{-\infty}^{\infty} B_x dx=\left(\frac{\mu_0 i}{2}\right)(2)=\mu_0 i$ **{OPTION (C)}**

Hence, (A), (B) and (C) are correct.

6. $F=Bqv$

$\Rightarrow \quad B=\frac{F}{qv}=\frac{N}{\text{Cms}^{-1}}=\text{NA}^{-1}\text{m}^{-1} \qquad \{\because 1\ \text{Cs}^{-1}=1\ \text{A}\}$

Further $B=\frac{\mu_0 I}{2R}$

$\Rightarrow \quad \mu_0=\frac{2BR}{I}=\frac{(\text{NA}^{-1}\text{m}^{-1})\text{m}}{A}=\text{NA}^{-2}$

Since $\phi_B=BA=(\text{NA}^{-1}\text{m}^{-1})(\text{m}^2)=\text{NmA}^{-1}$

Since we know that

$\left[\frac{L}{R}\right]=[CR]$

$\Rightarrow \quad [R]=\left[\sqrt{\frac{L}{C}}\right]$

So, $\sqrt{\frac{L}{C}}$ will have units same as that of resistance VA^{-1}.

Hence, (A), (B) and (D) are correct.

7. $B=\frac{\mu_0 NIr^2}{2\left(r^2+x^2\right)^{3/2}}$ along x-axis

$\Rightarrow \quad B=\frac{\mu_0 NI}{2r}\frac{r^3}{\left(r^2+x^2\right)^{\frac{3}{2}}}$

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$$\Rightarrow \quad B = \frac{\mu_0 NI}{2r}\sin^3\theta$$

According to which, the coil with lesser radius is to make more contribution.

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{1}{2}$$

Hence, (B) and (D) are correct.

8. $\vec{B} \perp \vec{v}$, so, it may along y-axis, so, **OPTION (A)** is correct

$\vec{F} \perp \vec{v}$,

$\Rightarrow \quad \vec{a} \perp \vec{v} = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{v} = 0$

$\Rightarrow \quad a_1 b_1 + a_2 b_2 = 0$, so, **OPTION (B)** is correct

Since $\vec{B} \perp \vec{v}$, so for B along x-axis this cannot be possible. Hence, **OPTION (C)** is incorrect.

Magnetic force cannot change the kinetic energy of a particle. Hence, **OPTION (D)** is also correct.

Hence, (A), (B) and (D) are correct.

9. Under the state of equilibrium,

$$\begin{pmatrix}\text{Electrostatic force}\\ \text{acting downward}\end{pmatrix} = \begin{pmatrix}\text{Magnetic force}\\ \text{acting upward}\end{pmatrix}$$

$$\Rightarrow \quad qE = qvB\sin 90°$$

$$\Rightarrow \quad v = \frac{E}{B}$$

and under this state, the particle must hit at O

Hence, (C) and (D) are correct.

10. $$B_a = \frac{\mu_0 i}{4\pi R} + \frac{3\mu_0 i}{8R} = \frac{\mu_0 i}{4R}\left(\frac{1}{\pi} + \frac{3}{2}\right)$$

$$\Rightarrow \quad B_b = \frac{3\mu_0 i}{8R} - \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{2R}\left(\frac{3}{4} - \frac{1}{\pi}\right)$$

$$\Rightarrow \quad B_c = \frac{\mu_0 i}{4\pi R} + \frac{3\mu_0 i}{8R} - \frac{\mu_0 i}{4\pi R} = \frac{3\mu_0 i}{8R}$$

Hence, (A), (B) and (D) are correct.

12. $$\text{Areal velocity} = \frac{\text{Area Swept}}{\text{Period of one Revolution}} = \frac{\pi r^2}{T}$$

$$\Rightarrow \quad \text{Areal velocity} = \frac{\pi\left(\frac{mv}{qB}\right)^2}{\left(\frac{2\pi m}{qB}\right)} = \frac{mv^2}{2qB} = \frac{K}{qB}$$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{K}{qB} = \frac{p^2}{2mqB}$$

Hence, (A), (B), (C) and (D) are correct.

13. Since, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

So, for the proton to suffer deflection along $+x$ axis, the force acting on it must be along $+x$ axis. For that to happen, we must/may have

CASE-1: $\vec{E}$ along x-axis ($\vec{B}$ is switched off)

CASE-2: $\vec{v}$ along $+y$ axis and $\vec{B}$ along $+z$ axis, so that $\vec{v} \times \vec{B}$ is along $+x$ axis ($\vec{E}$ switched off)

CASE-3: $\vec{E}$ along x-axis, $\vec{B}$ along $+y$ axis

$\vec{v} = \pm v_y \hat{j} - v_z \hat{k}$

$\vec{v} \times \vec{B}$ is along $+x$ axis

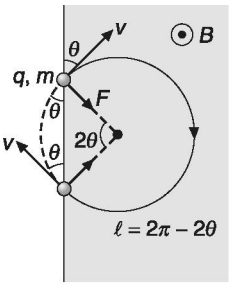
CASE-4: $\vec{E}$ along $+x$ axis $\vec{B}$ along $-y$ axis

$\vec{v} = \pm v_y \hat{j} + v_z \hat{k}$

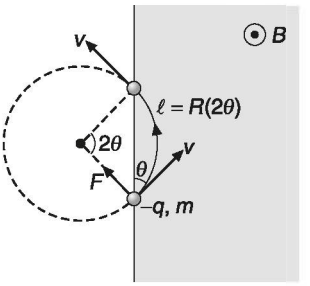
$\vec{v} \times \vec{B}$ is along $+x$ axis

Hence, (B), (C) and (D) are correct.

14. Distance described by the positively charged particle in the magnetic field is



(For +q)



(For −q)

$$l = 2(\pi - \theta)R$$

$$\Rightarrow \quad l = 2(\pi - \theta)\frac{mv}{qB}$$

Time taken by the positive charge particle in the magnetic field is

$$t = \frac{l}{v} = \frac{2(\pi - \theta)}{v}\frac{mv}{qB}$$

$$\Rightarrow \quad t = \frac{2m(\pi - \theta)}{qB} \qquad \ldots(1)$$

For the case of negative charge particle entering the field, we have

$$l = R(2\theta), \text{ where } R = \frac{mv}{qB}$$

$$\Rightarrow \quad t = \frac{l}{v} = \frac{2R\theta}{v} = \frac{2mv\theta}{vqB}$$

$$\Rightarrow \quad t = \frac{2m\theta}{qB} \qquad \ldots(2)$$

Hence, for $q_1 = q_2$, $t_1 = t_2$, **OPTION (B)** is correct.

for $q_1 > 0$, $q_2 < 0$, $t_1 > t_2$, **OPTION (C)** is incorrect.

for $q_1 < 0$, $q_2 > 0$, $t_1 < t_2$, **OPTION (D)** is correct.

Hence, (B) and (D) are correct.

16. Restoring torque $\tau = -MB\sin\theta$

$$\Rightarrow \quad \tau = -\left(\pi r^2 I_2\right)\left(\frac{\mu_0 I_1}{2R}\right)\theta \quad \{\because \text{ for small } \theta,\ \sin\theta \cong \theta\}$$

$$\Rightarrow \quad \left(\frac{1}{2}mr^2\right)\alpha = -\left(\frac{\mu_0 \pi l_1 I_2 r^2}{2R}\right)\theta$$

$$\Rightarrow \quad \alpha = -\left(\frac{\mu_0 \pi l_1 I_2}{mR}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{\mu_0 \pi I_1 I_2}{mR}\right)\theta = 0$$

Comparing with standard equation of SHM, i.e., $\ddot{\theta} + \omega^2\theta = 0$, we get

$$\omega = \sqrt{\frac{\mu_0 \pi l_1 I_2}{mR}}$$

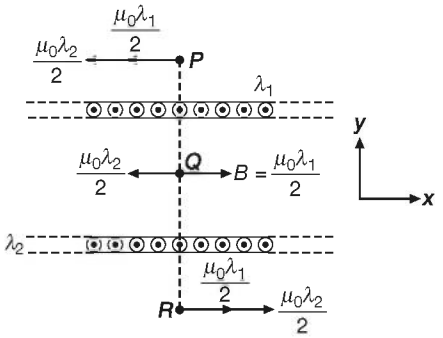
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{mR}{\mu_0 \pi I_1 I_2}}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{I_1 I_2}},\ T \propto \sqrt{m},\ T \propto \sqrt{R}$$

and is independent of r

Hence, (A), (B) and (D) are correct.

17. If λ is the current per unit length, then



$$B = \frac{\mu_0 \lambda}{2}$$

$$B_Q = \frac{\mu_0 \lambda_1}{2} - \frac{\mu_0 \lambda_2}{2}$$

$$B_R = \frac{\mu_0 \lambda_1}{2} + \frac{\mu_0 \lambda_2}{2} = B_P$$

Hence, (B) and (C) are correct.

18. In REGION 1, the charged particle passes undeviated (See Figure given with PROBLEM), so we conclude that the electrostatic force must be balanced by magnetic force. Hence

$$qE = qvB$$

$$\Rightarrow \quad v = \frac{E}{B} \qquad \textbf{\{OPTION (A)\}}$$

Since in both regions, the charged particle has $\vec{F}$ and $\vec{s}$ perpendicular to each other, so work done in both regions is zero. **{OPTION (B)}**

Further, REGION 2, has got only the magnetic field, so the charged particle will move in a circle of radius

$$r = \frac{mv}{qB_0} \qquad \{\because\ B_0 \text{ is the field in REGION 2}\}$$

$$\Rightarrow \quad r = \frac{m}{qB_0}\frac{E}{B}$$

$$\Rightarrow \quad r = \frac{E}{(q/m)BB_0} = \frac{E}{sBB_0} \qquad \textbf{\{OPTION (C)\}}$$

For $\ell_2 > r$ i.e. $\ell_2 > \dfrac{E}{sBB_0}$ the charged particle will complete the semicircle to emerge out from region with a velocity directed opposite to the initial velocity. Hence $\vec{v}' = -\vec{v}$ **{OPTION (D)}**

Hence, (A), (B), (C) and (D) are correct.

19. Since $\vec{v} \perp \vec{B}$, so the particle is performing circular motion in xy plane with radius of circle given by

$$r = \frac{mv}{qB} = \frac{1 \times 10}{2} = 5 \text{ m}$$

The radius of the circle given by the equation

$$x^2 + y^2 - 4x - 21 = 0 \text{ is } 5 \text{ m}$$

Also, the radius of circle given by the equation

$$x^2 + y^2 = 25 \text{ is } 5 \text{ m}$$

The time period of revolution of particle is given by

$$T = \frac{2\pi m}{qB} = \frac{2\pi \times 1}{1 \times 2} = 3.14 \text{ s}$$

Hence, (A), (B) and (D) are correct.

21. Torque acting on loop is given by

$$\tau = ilbB_0\sin\theta$$

The torque direction can be given by right hand thumb rule according to which τ acts along $-y$ direction and hence it has a tendency to decrease θ.

Hence, (A), (B) and (D) are correct.

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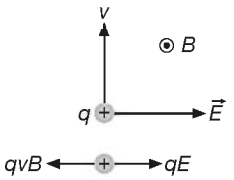
22. A uniform magnetic field B and a uniform electric field E exist perpendicular to each other and the particle moves along a direction perpendicular to both of these fields, then forces exerted by these two fields may be opposite to each other as shown in Figure. If magnitudes of these forces are equal, then resultant force on the particle will become equal to zero.



Hence, particle will move with constant velocity. Hence, the OPTION (A) is correct, obviously, the OPTION (B) is wrong.

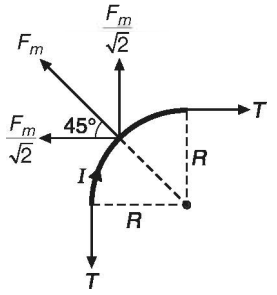
If E is equal to zero, then the particle will experience a force due to magnetic field alone. But the force exerted by the magnetic field is always perpendicular to the direction of its motion, hence, no power is associated with this force. In other words, no work is done by the magnetic field on the particle. Therefore, KE of the particle will remain constant. Hence, the OPTION (C) is also correct.

Hence, (A) and (C) are correct.

23. To find the Ampere force on a conductor of any shape just replace the conductor by an imaginary straight conductor joining the two ends of the given conductor having effective length $\ell_{\text{eff}} = \lambda$.

Hence, (A) and (C) are correct.

25. Since the wire is held in equilibrium, so net force on the wire is zero.



Net magnetic force on the wire is $F = BIl_{\text{eff}} = BI(\sqrt{2}R)$

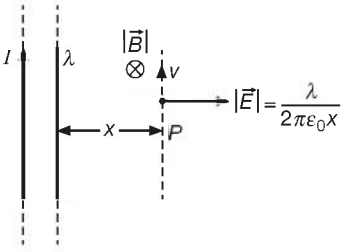
$\Rightarrow \quad F_m = \sqrt{2}BIR$

From the figure it is clear that

$$T = \frac{F_m}{\sqrt{2}} = BIR$$

Hence, (A), (B) and (C) are correct.

26. At P, electric field $E = \dfrac{\lambda}{2\pi\varepsilon_0 x}$ (to the right), and magnetic field $B = \dfrac{\mu_0 I}{2\pi x}$ (into the paper).



For no deflection, $qE = qvB\sin(90°)$

$\Rightarrow \quad v = \dfrac{E}{B}$

$\Rightarrow \quad v = \left(\dfrac{\lambda}{2\pi\varepsilon_0 x}\right)\left(\dfrac{2\pi x}{\mu_0 I}\right) = \dfrac{\lambda}{I}\dfrac{1}{\varepsilon_0\mu_0} = \dfrac{\lambda c^2}{I}$

Hence, (A) and (D) are correct.

27. The pairs $\vec{F}$ and $\vec{v}$ and $\vec{F}$ and $\vec{B}$ are always at right angles to each other, because $\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $\vec{v}$. Vectors $\vec{B}$ and $\vec{v}$ have any angle between them.

Hence, (A) and (B) are correct.

28. Magnetic field at the centre of first coil is

$$B_1 = \frac{\mu_0 N_1 i_1}{2R_1} = \frac{(4\pi\times10^{-7})(50)(2)}{2(5\times10^{-2})}$$

$\Rightarrow \quad B_1 = 4\pi\times10^{-4}$ T

Magnetic field at the centre of second coil is

$$B_2 = \frac{\mu_0 N_2 i_2}{2R_2} = \frac{(4\pi\times10^{-7})(100)(2)}{(2)(10\times10^{-2})}$$

$\Rightarrow \quad B_2 = 4\pi\times10^{-4}$ T

When currents are in the same sense, then we have

$B_{\text{net}} = B_1 + B_2 = 8\pi\times10^{-4}$ T

When currents are in the opposite sense, then we have

$B_{\text{net}} = B_1 - B_2 = 0$

Hence, (A) and (C) are correct.

29. Pitch $= \left(\dfrac{2\pi m}{QB}\right)v\cos\theta$

Also, $r = \dfrac{mv\sin\theta}{QB}$

Maximum distance of the particle from the x-axis is

$d_{\max} = 2r$

$$\Rightarrow \left(\frac{2\pi m}{QB}\right)v\cos\theta = 2\frac{mv}{QB}\sin\theta$$

$$\Rightarrow \tan\theta = \pi$$

Hence, (A), (B) and (C) are correct.

31. Magnetic field at the centre of a circular coil is given by

$$B = \frac{\mu_0 I}{2R}$$

Since we know that $c^2 = \frac{1}{\mu_0\varepsilon_0}$

$$\Rightarrow \mu_0 = \frac{1}{c^2\varepsilon_0}$$

$$\Rightarrow B = \frac{I}{2c^2\varepsilon_0 R}$$

Hence, (A) and (C) are correct.

32. When electric force qE and magnetic force qvB balance each other, then the particle will move in a straight line.

$$\Rightarrow qE = qv_2B$$

$$\Rightarrow v_2 = \frac{E}{B}$$

If electric field force is not balanced by the magnetic field force, then either $qE > qvB$ OR $qE < qvB$

When $qE > qv_3B$, then $v_3 < \frac{E}{B}$ and when $qE < v_1Bq$, then $v_1 > \frac{E}{B}$.

Hence, (A), (B) and (C) are correct.

33. Magnetic force is always perpendicular to the velocity of the particle i.e., $\vec{v} \perp \vec{a}$

$$\Rightarrow \vec{v}\cdot\vec{a} = 0$$

$$\Rightarrow 12 + 4x = 0$$

$$\Rightarrow x = -3$$

Further since $\vec{F} = q(\vec{v}\times\vec{B})$, so the magnetic field is perpendicular to the plane of velocity i.e., xy plane. Hence the magnetic field is along z-direction.

Also, we know that work done by a magnetic force is zero so from Work Energy Theorem, i.e., $W = \Delta K$, we get

$$K_{\text{final}} = K_{\text{initial}}$$

Hence, (A), (C) and (D) are correct.

34. Apply Ampere's Circuital Law we get

$$B(r) = 0 \qquad \{\text{inside the tube}\}$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow B \propto \frac{1}{r}$$

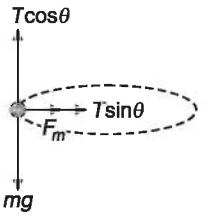
Hence, (A) and (C) are correct.

35. For the conical pendulum, force equations on the charge can be written as

$$T\sin\theta + F_m = mr\omega^2, \text{ where } r = L\sin\theta$$

$$\Rightarrow F_m = qvB = q(L\sin\theta)\omega B$$

$$\Rightarrow T\cos\theta = mg$$



$$\Rightarrow \frac{mg}{\cos\theta}\sin\theta + q(\omega L\sin\theta)B = m\omega^2 L\sin\theta$$

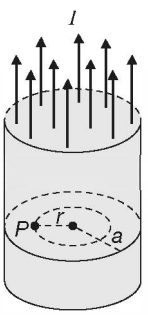
$$\Rightarrow B = \frac{m\omega^2 L\sin\theta}{(\omega L\sin\theta)q} - \frac{\frac{mg\sin\theta}{\cos\theta}}{(\omega L\sin\theta)q}$$

$$\Rightarrow B = \frac{m\omega}{q} - \frac{mg}{q\omega L\cos\theta} = \frac{1}{\beta}\left(\omega - \frac{g}{\omega L\cos\theta}\right)$$

Rate of change of angular momentum i.e., torque of the bob about point of suspension is not constant.

Hence, (A) and (D) are correct.

37. $dI = JdA$



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$$\Rightarrow \quad dI = kr^2(2\pi r\, dr)$$

$$\Rightarrow \quad I = 2\pi k\int_0^a r^3 dr$$

$$\Rightarrow \quad I = \frac{1}{2}\pi k a^4$$

Further, field for $r > a$ is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 k a^4}{4r}$$

and field for $r < a$ is

$$B = \frac{\mu_0\left(\frac{\pi k r^4}{2}\right)}{2\pi r} = \frac{1}{4}\mu_0 k r^3$$

Hence, (A), (C) and (D) are correct.

38. Time period of motion is given as

$$T = \frac{2\pi m}{qB} = \frac{2\pi}{\alpha B_0}$$

At $t = \frac{\pi}{\alpha B_0} = \frac{T}{2}$, the velocity of particle is

$$\vec{v}' = -v_0\hat{i} + v_0\hat{k}$$

Speed of charge in magnetic field always remains constant, so we have

$$v' = v = v_0\sqrt{2}$$

At $t = \frac{2\pi}{\alpha B_0} = T$, displacement is equal to pitch. So, displacement is given by

$$\Delta x = v_0 T = \frac{2\pi v_0}{\alpha B_0}$$

At $t = \frac{2\pi}{\alpha B_0} = T$, the distance (l) is given as product of speed and time, so we have

$$l = vT = \frac{2\sqrt{2}v_0\pi}{\alpha B_0}$$

Hence, (A) and (C) are correct.

40. Velocity vector is perpendicular to magnetic field. Therefore, path of the particle is a circle or radius

$$r = \frac{mv_0}{qB}$$

$v = v_0$ as the speed of the particle does not change in magnetic field.

Centre of the circle is C.

$$CP = CQ$$

$$\Rightarrow \quad \angle CPQ = \angle CQP$$

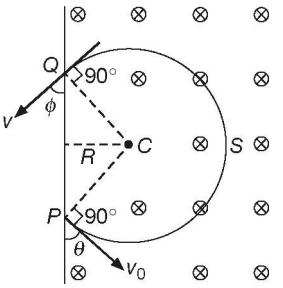
$$\Rightarrow \quad 90° - \theta = 90° - \phi$$

$$\Rightarrow \quad \theta = \phi$$

Further $PQ = 2PR = 2r\cos(90° - \theta)$

$$\Rightarrow \quad PQ = 2r\sin\theta$$

$$\Rightarrow \quad PQ = \frac{2mv_0\sin\theta}{qB}$$



Also, $\angle PCR = \angle RCQ = \theta$

$$\Rightarrow \quad \angle PCQ = 2\theta$$

$$\Rightarrow \quad \overset{\frown}{PSQ} = (2\pi - 2\theta)r = \frac{2mv_0(\pi - \theta)}{qB}$$

$$\Rightarrow \quad t_{PSQ} = \frac{\overset{\frown}{PSQ}}{v_0} = \frac{2m(\pi - \theta)}{qB}$$

Hence, (A), (B), (C) and (D) are correct.

42. Let d = distance of the target T from the point of projection. P will strike T if d is an integral multiple of the pitch.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right)v\cos\theta$$

Here, m, Q and θ are constant

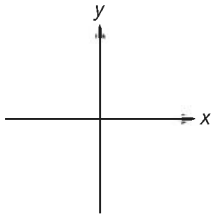
$$\therefore \quad \text{Pitch} = k\left(\frac{v}{B}\right)$$

where k = constant

$$\text{Initially, } d = k\left(\frac{v_0}{B_0}\right)$$

Hence, (A) and (B) are correct.

43. The velocity of proton makes an angle of $45°$ with the direction of magnetic field. Therefore, path of the proton is a helix. The plane of the circle of this helix is the plane formed by negative x and positive z-axis. Therefore, x-coordinate can never be positive. Further, x and z co-ordinates will become zero simultaneously after every pitch and y-coordinate of proton at any time t is



$y = v_0 t$

$\Rightarrow \quad y \propto t$

Hence, (B) and (D) are correct.

45. When $\alpha = \beta$ then $\vec{v} \parallel \vec{B}$

$\Rightarrow \quad \vec{F} = \vec{0}$

So, **OPTION (A)** is correct

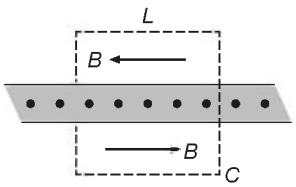
Since, $\vec{F} = q(\vec{v} \times \vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 0 \\ \beta & \alpha & 0 \end{vmatrix}$

$\Rightarrow \quad \vec{F} = q(\alpha^2 - \beta^2)\hat{k} \qquad \ldots(1)$

$\Rightarrow \quad |\vec{F}| \propto (\alpha^2 - \beta^2)$

So, **OPTION (B)** is also correct.
Also, from (1), we get **OPTION (C)** to be correct.
Hence, (A), (B) and (C) are correct.

47. Consider only one plate as shown in Figure.



The field on both sides of plate is shown in Figure.
Applying Ampere's Circuital Law to the contour C, we get

$2BL = \mu_0(jL)$

$B = \frac{\mu_0 j}{2}$

Superimposing, the field due to two plates we get at P both fields cancel each other and at Q, they add to give $B_Q = \mu_0 j$

Hence, (A) and (D) are correct.

48. $B(2\pi r) = \mu_0 I_{\text{enclosed}}$

$B(2\pi r) = \mu_0\left(\frac{I}{\pi a^2}\pi r^2\right)$

$B = \frac{\mu_0 I r}{2\pi a^2}$ for $0 < r \le a$,

$B(r) = 0$ at $r = 0$,

$B(r) \propto \frac{1}{r}$ for $r > a$ and

$B(r)$ is maximum at the surface i.e., at $r = a$

Hence, (A), (B), (C) and (D) are correct.

50. $a_e = \frac{F}{m_e}$, $a_p = \frac{F}{m_p}$

Since, $m_e \ll m_p$

So, $a_e \gg a_p$

$\Rightarrow \quad d = \frac{1}{2}a_e t_1^2$, $d = \frac{1}{2}a_p t_2^2$

$\Rightarrow \quad \frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$

Hence, (A) and (C) are correct.

Reasoning Based Questions

2. $\frac{mv^2}{r} = qvB$

$\Rightarrow \quad a = \frac{v^2}{r} = \frac{qvB}{m}$, is called a centripetal acceleration

If the particle moves parallel or antiparallel to the field then acceleration is zero.
Hence, the correct answer is (A).

3. Work done by magnetic force on moving charge is zero.
Hence, the correct answer is (A).

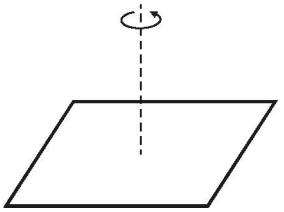
4. $v = \frac{E}{B}$

Hence, the correct answer is (A).

6. $|\vec{B}|$ is proportional to number of magnetic field lines per unit area (area should be normal to field).
Hence, the correct answer is (D).

CHAPTER 1

7. $U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$



Since, M, B as well as θ remains constant, U does not change.
Hence, the correct answer is (A).

8. $r = \dfrac{mv}{qB}$

Since $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$

So, we observe T is independent of r. Hence Statement-1 is incorrect.
Hence, the correct answer is (D).

9. The winding of helix carry currents in same direction. Hence, they experience an attractive force pulling the lower and out of mercury. As a result of this the circuit breaks and so the force of attraction vanishes and the helix comes back to its initial position, completing the circuit again.
Hence, the correct answer is (B).

11. See Theory.
The correct answer is (C).

12. Initially moment $M = I\pi r^2$

And afterwards $M' = I\pi(2r)^2 = 4I(\pi r^2) = 4M$

So magnetic moment becomes four times when radius is doubled.
Hence, the correct answer is (A).

13. $\vec{F} = q\vec{v}\times\vec{B}$

$\Rightarrow \quad \vec{F}\perp\vec{v}$

So, power produced by magnetic force is zero
$\Rightarrow$ Kinetic energy of particle will remain conserved
Hence, the correct answer is (A).

14. Use $\vec{F} = \oint Id\vec{\ell}\times B = 0$

$\vec{\tau} = MB\sin\theta = 0$
Hence, the correct answer is (B).

15. The magnetic lines of force due to current carrying straight solenoid is same as that of bar magnet.
Hence, the correct answer is (A).

18. The period of a charged particle in a magnetic field is given by

$$T = \frac{2\pi m}{qB}$$

Hence, the correct answer is (A).

21. Force on the loop is not zero; because magnetic field is not constant.
Hence, the correct answer is (D).

22. Due to both positive and negative charges the wire is electrically neutral and hence no electric field is present and only magnetic field is created.
Hence, the correct answer is (D).

23. Magnetic force is always perpendicular to magnetic field and small element. Also, the force depends on the current direction.
Hence, the correct answer is (B).

25. $F = 0$ in both case.
Hence, the correct answer is (C).

26. As we know every atom of a magnet act as a dipole. So, poles cannot be separated. When magnet is broken into two equal pieces. Magnetic moment of each part will be half of the original magnet.
Hence, the correct answer is (B).

27. For a solenoid $B_{\text{ends}} = \dfrac{1}{2}B_{\text{inside}}$. Also, for a long solenoid magnetic field is uniform within it but this reason is not explaining the statement (1).
Hence, the correct answer is (B).

28. Velocity is a vector quantity even if direction changes, velocity is said to be changing, no matter speed remains same or different.
Hence, the correct answer is (C).

29. $C\phi = BINA$

$\Rightarrow \quad \phi = \left(\dfrac{BNA}{C}\right)I$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence sensitivity increases. So, Statement-1 is true.
Statement-2 is false as we know soft iron can be easily magnetized or demagnetized.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. When the ends of the wire are in contact with the mercury and current flows in the wire, the magnetic field exerts an upward force and the wire has an upward

acceleration. After the ends leave the mercury the electrical connection is broken and the wire is in free fall.

After the wire leaves the mercury its acceleration is g, downward. The wire travels upward a total distance of 0.35 m from its initial position. Its ends lose contact with the mercury after the wire has travelled 0.025 m, so the wire travels upward 0.325 m after it leaves the mercury. Consider the motion of the wire after it leaves the mercury. Take $+y$ to be upward and take the origin at the position of the wire as it leaves the mercury

$$a_y = -9.8\ \text{ms}^{-2},\ y - y_0 = +0.325\ \text{m},\ v_y = 0$$

(at maximum height), $v_{0y} = ?$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.8\ \text{ms}^{-2})(0.325\ \text{m})}$$

$$v_{0y} = 2.52\ \text{ms}^{-1} \simeq 2.5\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

2. Now consider the motion of the wire while it is in contact with the mercury. Take $+y$ to be upward and the origin at the initial position of the wire. Calculate the acceleration

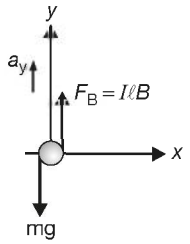
$$y - y_0 = +0.025\ \text{m},\ v_{0y} = 0 \text{ (starts from rest)},$$

$$v_y = +2.52\ \text{ms}^{-1} \text{ (from part (a))},\ a_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$a_y = \frac{v_y^2}{2(y - y_0)} = \frac{(2.52\ \text{ms}^{-1})^2}{2(0.025\ \text{m})} = 127\ \text{ms}^{-2}$$

The free body diagram for the wire is given in figure



$$\sum F_y = ma_y$$

$$\Rightarrow\ F_B - mg = ma_y$$

$$\Rightarrow\ I\ell B = m(g + a_y)$$

$$\Rightarrow\ I = \frac{m(g + a_y)}{\ell B}$$

ℓ is the length of the horizontal section of the wire,

$$\ell = 0.15\ \text{m}$$

$$I = \frac{(5.4 \times 10^{-5}\ \text{kg})(9.8\ \text{ms}^{-2} + 127\ \text{ms}^{-2})}{(0.15\ \text{m})(0.0065\ \text{T})} = 7.58\ \text{A}$$

$$\Rightarrow\ I \cong 7.6\ \text{A}$$

Hence, the correct answer is (A).

3. Use Ohm's Law

$$V = IR \text{ so } R = \frac{V}{I} = \frac{1.5\ \text{V}}{7.58\ \text{A}} = 0.198\ \Omega$$

The current is large and the magnetic force provides a large upward acceleration. During this upward acceleration the wire moves a much shorter distance as it gains speed than the distance is moves while in free-fall with a much smaller acceleration, as it loses the speed it gained. The large current means the resistance of the wire must be small.

Hence, the correct answer is (C).

4. Since $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow\ \vec{F} = 10^6 \times q(8\hat{i} - 6\hat{j} + 4\hat{k}) \times (-0.4)\hat{k}$$

$$\Rightarrow\ \vec{F} = 10^6 \times q(+3.2\hat{j} + 2.4\hat{i})$$

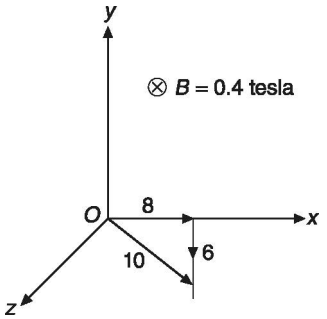
$$\Rightarrow\ |\vec{F}| = q \times 10^6\sqrt{(3.2)^2 + (2.4)^2}$$

$$\Rightarrow\ 1.6 = q \times 4 \times 10^6$$

$$\Rightarrow\ q = 0.4 \times 10^{-6} = 0.4\ \mu\text{C}$$

Hence, the correct answer is (C).

5.



Resultant velocity is 10×10^6 in x-y plane. The circular motion of the particle is in x-y plane and translational component along z-axis.

Hence, the correct answer is (C).

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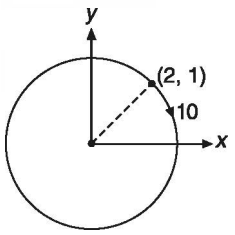
6.



After time $3\ \text{T}$, its x and y coordinate will remain same, whereas z coordinate is

$$z = 4 \times 3\ \text{T} = 12\ \text{T}$$

Since $T = \dfrac{2\pi m}{qB}$

$$\Rightarrow \quad T = \frac{2 \times \pi (4 \times 10^{-15})}{0.4 \times 10^{-6} \times 0.4} = 0.157 \times 10^{-6}\ \text{s}$$

So, z-coordinate of particle is

$$z = v_z t = (4 \times 10^6)(3\ \text{T})$$

$$\Rightarrow \quad z = (4 \times 10^6)(3 \times 0.157 \times 10^{-6})$$

$$\Rightarrow \quad z = 1.884\ \text{m}$$

Hence, the correct answer is (C).

7. $\vec{\tau} = \vec{\mu} \times \vec{B}_0$ where $\vec{\mu}$ is the magnetic moment of the loop

$$\Rightarrow \quad \vec{\tau} = I_0 \pi R^2 \hat{k} \times \left(\frac{B_0}{\sqrt{2}} \hat{i} + \frac{B_0}{\sqrt{2}} \hat{j} \right)$$

$$\Rightarrow \quad \vec{\tau} = \frac{I_0 \pi R^2 B_0}{\sqrt{2}} (\hat{j} - \hat{i})$$

$$\Rightarrow \quad |\vec{\tau}| = I_0 \pi R^2 B_0$$

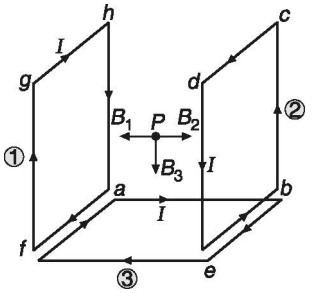
Hence, the correct answer is (A).

8. $\theta = \dfrac{1}{2} \alpha (\Delta t)^2$

$$\Rightarrow \quad \theta = \frac{\pi I_0 B_0}{M} (\Delta t)^2$$

Hence, the correct answer is (D).

9. The current path *abcdefgha* can be treated as superposition of three loops *fghaf*, *fabef* and *ebcde* each carrying a current I.



Hence, the correct answer is (A).

10. The magnetic field at centre P of cube cancels due to two square loops (1) and (2) each carrying current I as shown earlier. So, the magnetic field at centre P of cube is only due to loop 3. Hence the magnetic field at centre of cube is in the negative y-direction.

Hence, the correct answer is (B).

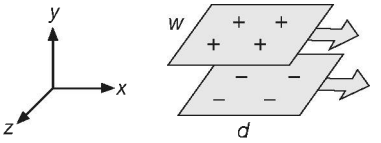
11. The dipole moment of the loop is also along negative y direction.

Since $\vec{B}_{\text{ext}} = B_0 \hat{j}$, so $\vec{\tau} = \vec{M} \times \vec{B} = \vec{0}$

Hence, the correct answer is (D).

12. If J_S be the current per unit length, then the upper sheet creates field $\vec{B} = \dfrac{\mu_0 J_s}{2} \hat{k}$ above it and $\dfrac{\mu_0 J_s}{2}(-\hat{k})$ below it.

Consider a patch of the sheet of width w parallel to the z-axis and length d parallel to the x-axis as shown in Figure.



The charge on it, $\sigma w d$ passes a point in time $\dfrac{d}{v}$ so the current it constitutes is $\dfrac{q}{t} = \dfrac{\sigma w v d}{d}$ and the linear current density is $J_s = \dfrac{\sigma w v}{w} = \sigma v$. So, the magnitude of the magnetic field created by the upper sheet is $\dfrac{1}{2}\mu_0 \sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\dfrac{1}{2}\mu_0 \sigma v(-\hat{k})$ above it and $\dfrac{1}{2}\mu_0 \sigma v \hat{k}$ below it.

Between the plates, their fields add to $\mu_0 \sigma v(-\hat{k})$ i.e. $\mu_0 \sigma v$ away from you horizontally.

Hence, the correct answer is (A).

13. Above both sheets and below both the sheets, their equal magnitude fields add to zero.

Hence, the correct answer is (D).

14. The upper plate exerts no force on itself. The field of the lower plate, $\dfrac{1}{2}\mu_0 \sigma v(-\hat{k})$ will exert a magnetic force on the current in the $w \times d$ section, given by

$$\vec{F} = I(\vec{l} \times \vec{B}) = (\sigma w v d)\hat{i} \times \left(\frac{1}{2}\mu_0 \sigma v \right)(-\hat{k})$$

$$\Rightarrow \quad \vec{F} = I(\vec{\ell} \times \vec{B}) = \left(\frac{1}{2}\mu_0 \sigma^2 v^2 w d \right)\hat{j}$$

The magnetic force per area is

$$\frac{F}{A}=\frac{1}{2}\left(\frac{\mu_0\sigma^2v^2wd}{wd}\right)\hat{j}=\frac{1}{2}\mu_0\sigma^2v^2, \text{ upwards}$$

Hence, the correct answer is (D).

15. The electrical force on the considered section of the upper plate is

$$\vec{F}_e=q\vec{E}_{\text{lower}}=(\sigma\ell w)\left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{j})$$

$$\Rightarrow \quad \vec{F}_e=\left(\frac{\ell w\sigma^2}{2\varepsilon_0}\right)(-\hat{j})$$

The electrical force per area is

$$\frac{F_e}{A}=\frac{\ell w\sigma^2}{2\varepsilon_0\ell w}=\frac{\sigma^2}{2\varepsilon_0}, \text{ downwards}$$

Hence, the correct answer is (B).

16. The magnetic force balances the electric force, when we have

$$\frac{1}{2}\mu_0\sigma^2v^2=\frac{\sigma^2}{2\varepsilon_0}$$

$$\Rightarrow \quad v=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=c$$

This is the speed of light which cannot be a speed that can be attained by a metal plate.

Hence, the correct answer is (C).

17. **The correct answer is (C).**

18. **The correct answer is (A).**

Combined Solution to 17 and 18

Since we know that $\vec{F}=q(\vec{v}\times\vec{B})$

For the first case, we get

$$-(5\sqrt{2}\times10^{-3})\hat{k}=(10^{-5})\left(\frac{10^6}{\sqrt{2}}\right)(\hat{i}+\hat{j})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$\Rightarrow \quad -(5\sqrt{2}\times10^{-3})\hat{k}=\frac{10}{\sqrt{2}}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&0\\B_x&B_y&B_z\end{vmatrix}$$

$$\Rightarrow \quad -(5\sqrt{2}\times10^{-3})\hat{k}=\frac{10}{\sqrt{2}}\left[B_z\hat{i}-B_z\hat{j}+(B_y-B_x)\hat{k}\right]$$

$$\Rightarrow \quad B_z=0 \text{ and } B_y-B_x=-10^{-3}\text{ T} \qquad \ldots(1)$$

For the second case, we have

$$F_2\hat{j}=(10^{-5})(10^6)\hat{k}\times(B_x\hat{i}+B_y\hat{j})$$

$$\Rightarrow \quad F_2\hat{j}=10\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&0&1\\B_x&B_y&0\end{vmatrix}$$

$$\Rightarrow \quad F_2\hat{j}=10B_x\hat{j}-10B_y\hat{i} \qquad \ldots(2)$$

From (2), we get

$$B_y=0$$

$$\Rightarrow \quad B_x=10^{-3}\text{ T} \text{ and } F_2=10B_x=10^{-2}\text{ N}$$

Hence $\vec{B}=(10^{-3}\text{ T})\hat{i}$ and $F_2=10^{-2}\text{ N}$

19. $$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\text{ TmA}^{-1})(25\text{ A})}{2\pi(0.0125\text{ m})}$$

$$\Rightarrow \quad B=4\times10^{-4}\text{ T}=400\ \mu\text{T}$$

Hence, the correct answer is (D).

20. At point C, conductor AB produces a field

$$B=\frac{B_0}{2}=\frac{1}{2}(4\times10^{-4}\text{ T})(-\hat{j}),$$

conductor DE produces a field of $B=\frac{B_0}{2}=\frac{1}{2}(4\times10^{-4}\text{ T})(-\hat{j})$, BD produces no field, and AE produces negligible field. The total field at C is $B_C=\frac{B_0}{2}+\frac{B_0}{2}=B_0=400\ \mu\text{T}$

Hence, the correct answer is (A).

21. $$\vec{F}_B=I(\vec{\ell}\times\vec{B})=(25\text{ A})(0.025\text{ m}\hat{k})\times\left[5(4\times10^{-4}\text{ T})(-\hat{j})\right]$$

$$\vec{F}_B=(1.25\times10^{-3}\text{ N})\hat{i}$$

Hence, the correct answer is (B).

22. $$a=\frac{\sum F}{m}=\frac{(1.25\times10^{-3}\text{ N})\hat{i}}{5\times10^{-3}\text{ kg}}$$

$$\Rightarrow \quad a=(0.25\text{ ms}^{-2})\hat{i}=(25\text{ cms}^{-2})\hat{i}$$

Hence, the correct answer is (C).

23. $v_f^2=v_i^2+2ax=0+2(0.25\text{ ms}^{-2})(1.2\text{ m})$, so

$$v_f=0.6\text{ ms}^{-1}=60\text{ cms}^{-1}\,\hat{i}$$

Hence, the correct answer is (D).

24. The free body diagram of the arrangement is shown in Figure.

CHAPTER 1

For particle to move in a circle of radius L, we have

$$T + qvB = \frac{mv^2}{L}$$

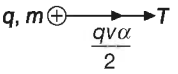
For $T = 0$, we get

$$q\left(\frac{q\alpha L}{m}\right)B = \frac{mv^2}{L}$$

$$\Rightarrow \quad B = \alpha$$

Hence, the correct answer is (B).

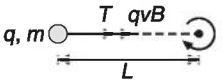
25. Since, $T + \frac{qv\alpha}{2} = \frac{mv^2}{L}$



$$\Rightarrow \quad T = \frac{m(q\alpha L)^2}{m^2 R} - \left[\frac{q(q\alpha L)}{m}\right]\frac{\alpha}{2} = \frac{q^2\alpha^2 L}{2m}$$

Hence, the correct answer is (C).

26. $T + qvB = \frac{mv^2}{L}$



$$\Rightarrow \quad qvB = \frac{mv^2}{L} - T$$

$$\Rightarrow \quad q\left(\frac{q\alpha L}{m}\right)B = \frac{m}{L}\left(\frac{q^2\alpha^2 L^2}{m^2}\right) - \frac{3}{4}\left(\frac{q^2\alpha^2 L}{m}\right)$$

$$\Rightarrow \quad B = \frac{\alpha}{4}$$

When the string breaks due to this magnetic field, then the radius of the path followed by the particle is

$$R = \frac{mv}{qB} = \frac{mv}{q\left(\frac{\alpha}{4}\right)} = \left(\frac{4m}{q\alpha}\right)v$$

$$\Rightarrow \quad R = \left(\frac{4m}{q\alpha}\right)\left(\frac{q\alpha L}{m}\right) = 4L$$

So, maximum separation between the particles is

$$r_{max} = (2R + 2R) - 2L$$

$$\Rightarrow \quad r_{max} = 4R - 2L = 16L - 2L$$

$$\Rightarrow \quad r_{max} = 14L$$

Hence, the correct answer is (C).

27. $\vec{\tau} = \vec{M} \times \vec{B} = \left(3\hat{i} - 4\hat{j}\right) \times \left(4\hat{i} + 3\hat{j}\right) = \left(25\hat{k}\right)$ Nm

Moment of inertia about an axis, parallel to $\vec{\tau}$ and passing through centre of mass is

$$I = \frac{2 \times 10^{-2}}{I} = 10^{-2} \text{ kgm}^2$$

$$\Rightarrow \quad \alpha = \frac{\vec{\tau}}{I} = \frac{25}{10^{-2}} = 2500 \text{ rads}^{-2}$$

Hence, the correct answer is (C).

28. At time $t = 0$, $\vec{M} \cdot \vec{B}$ i.e., $\vec{M} \perp \vec{B}$. Angular velocity will have a maximum value when $\vec{M} \parallel \vec{B}$, as a result of which magnetic potential energy will decrease and rotational kinetic energy will increase. Hence,

$$\frac{1}{2}I\omega^2 = U_i - U_f$$

$$\Rightarrow \quad \frac{1}{2}I\omega^2 = -MB\cos 90° - (-MB\cos 0°)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2MB}{I}} = \sqrt{\frac{(2)(5)(5)}{10^{-2}}} = 50\sqrt{2} \text{ rads}^{-1}$$

Hence, the correct answer is (B).

29. For neutrons charge, $q = 0$

Hence, the correct answer is (D).

31. Due to extremely small mass of an electron, the frequency of revolution of electrons is very high and oscillators of such high frequencies are not easily obtainable.

Hence, the correct answer is (A).

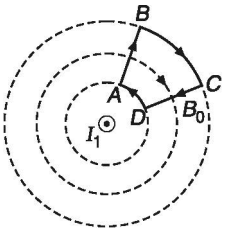
32. $B_{AD} = \frac{\mu_0 I\alpha}{4\pi R} = \frac{\mu_0 I\pi}{4\pi a \times 6} = \frac{\mu_0 I}{24a}$, $\odot$

$B_{BC} = \frac{\mu_0 I\alpha}{4\pi R} = \frac{\mu_0 I\pi}{4\pi b \times 6} = \frac{\mu_0 I}{24b}$, $\otimes$

$$\Rightarrow \quad B_{net} = \frac{\mu_0 I}{24}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I(b-a)}{24ab}, \odot$$

Hence, the correct answer is (B).

33. Since magnetic field lines and current carrying wires are parallel hence no force is exerted on wire DA and BC, because $\vec{I}$ and $\vec{B}$ are collinear for BC and DA.

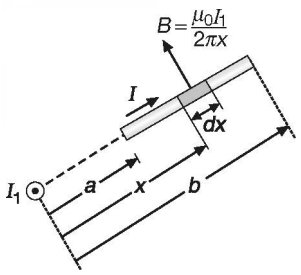


Similarly, F_{AB} and F_{CD} are equal in magnitude but opposite in direction, so

$$F_{net} = \text{zero}$$

Hence, the correct answer is (B).

34.



Force on small element dx is

$$dF = B_1 I dx = \frac{\mu_0 I I_1}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \int dF = \frac{\mu_0 I I_1}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 I I_1}{2\pi}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (B).

35. Since the electric field is in y-direction, therefore, electrostatic force will provide an acceleration to the particle in y-direction. y component of its velocity will go on increasing. Since, magnetic field is also in y-direction, so magnetic force $\vec{F}_m$ will act in x-z plane i.e. the particle will rotate in circle with its plane in x-z plane. Hence, path of particle will be helix with increasing pitch.

Hence, the correct answer is (C).

36. $a_y = \frac{F_y}{m} = \frac{qE_0}{m} = E_0\alpha$

$$\Rightarrow \quad y = \frac{1}{2}a_y t^2 = \frac{1}{2}E_0\alpha t^2$$

Particle will touch the y-axis for nth time, when the time (t) equals integral multiple of T i.e.

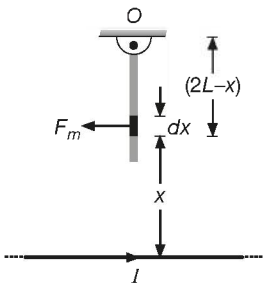
$$t = nT = n\left(\frac{2\pi m}{qB_0}\right) = \frac{2\pi n}{B_0\alpha}$$

$$\Rightarrow \quad y_n = \frac{1}{2}\alpha E_0\left(\frac{2n\pi}{B_0\alpha}\right)^2$$

$$\Rightarrow \quad y_n = \frac{2\pi^2 n^2 E_0}{B_0^2\alpha}$$

Hence, the correct answer is (B).

37. Consider an element of length dx (of the suspended wire) at a distance x from the horizontal wire as shown in Figure.



If $d\tau$ is the torque on this element due to the magnetic force F_m, then

$$\tau_0 = \int_{x=L}^{x=2L} F_m(2L-x) = \int_L^{2L}\left(\frac{\mu_0}{2\pi}\frac{I}{x}\right)(I)dx(2L-x)$$

$$\Rightarrow \quad \tau_0 = \frac{\mu_0 I^2}{2\pi}\left(2L\log_e x - x\right)\Big|_L^{2L}$$

$$\Rightarrow \quad \tau_0 = \frac{\mu_0 I^2}{2\pi}\left(2L\log_e(2) - L\right) = \frac{0.4\mu_0 I^2 L}{2\pi} \qquad \ldots(1)$$

$$\Rightarrow \quad \tau_0 = (0.4)(2\times10^{-7})(2) = 3.2\times10^{-7}\ \text{Nm}$$

Since the moment of inertia of the wire is I_0 given by

$$I_0 = \frac{mL^2}{3} = \frac{(0.1)(1)^2}{3} = \frac{1}{30}\ \text{kgm}^2$$

$$\Rightarrow \quad \alpha = \frac{\tau_0}{I_0} = 9.6\times10^{-6}\ \text{rads}^{-2}$$

Hence, the correct answer is (D).

38. Net torque about O should be zero. Magnetic field on the suspended wire due to already placed wire is perpendicular to paper outwards. Hence, magnetic field on suspended wire due to new wire should be perpendicular to paper inwards.

Hence, the correct answer is (B).

39. Torque about O due to the new wire will be,

$$\tau = \left(\frac{\mu_0}{2\pi}\frac{I}{r}\right)(L)(I)\left(\frac{L}{2}\right) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I^2L^2}{2r}\right)$$

Since $\tau = \tau_0$

$$\Rightarrow \quad \frac{L}{2r} = 0.386$$

$$\Rightarrow \quad r = \frac{L}{2\times0.4} = \frac{1}{2\times0.4} = 1.25\ \text{m}$$

Hence, the correct answer is (C).

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40. When the currents in two parallel wires are in opposite direction, then they repel each other.
Hence, the correct answer is (B).

41. $\left(\frac{\mu_0 i_1 i_2}{2\pi h}\right) l = mg$

$\Rightarrow \quad \frac{\mu_0 i_1 i_2}{2\pi h} = \left(\frac{m}{l}\right) g$

$\Rightarrow \quad \frac{\mu_0 i_1 i_2}{2\pi h} = \lambda g$

$\Rightarrow \quad h = \frac{\mu_0 i_1 i_2}{2\pi \lambda g}$

Hence, the correct answer is (A).

42. At equilibrium, we have

$$\frac{\mu_0 i_1 i_2}{2\pi h} = \lambda g$$

After displacing it dh upwards net force per unit length now becomes

$$F' = \lambda g - \frac{\mu_0 i_1 i_2}{2\pi (h + dh)}$$

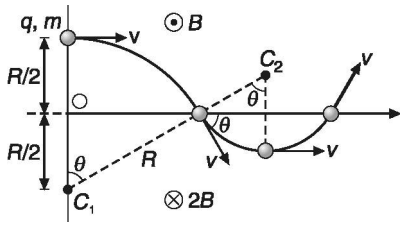
$\Rightarrow \quad F' = \lambda g - \frac{\lambda g}{\left(1 + \frac{dh}{h}\right)} = \frac{\lambda g dh}{h}$

Hence, the correct answer is (D).

43. **The correct answer is (B).**

44. **The correct answer is (C).**

Combined Solution to 43 and 44



From Figure, we have

$$\cos\theta = \frac{R/2}{R} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 60° = \frac{\pi}{3}$ radian

So, time to hit x-axis is

$$t_1 = \frac{\theta}{\omega}, \text{ where } \omega = \frac{qB}{m}, \ \theta = \frac{\pi}{3}$$

$\Rightarrow \quad t_1 = \frac{\pi/3}{qB/m} = \frac{\pi m}{3qB}$

Let t_2 be time when its velocity becomes parallel to x-axis first time after entering the magnetic field, then

$$t_2 = t_1 + \frac{\theta}{\omega'}, \text{ where } \omega' = \frac{q(2B)}{m}$$

$\Rightarrow \quad t_2 = \frac{\pi m}{3qB} + \frac{\pi/3}{\frac{q(2B)}{m}}$

$\Rightarrow \quad t_2 = \frac{\pi m}{3qB} + \frac{\pi m}{6qB}$

$\Rightarrow \quad t_2 = \frac{\pi m}{2qB}$

45. $\vec{\tau} = \vec{M} \times \vec{B}$, where $\vec{M}$ is the dipole moment

$\Rightarrow \quad \vec{\tau} = nI(\pi r^2)\hat{j} \times B\hat{i}$

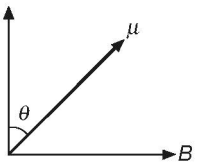
$\Rightarrow \quad \vec{\tau} = -(\pi n I r^2 B)\hat{k}$

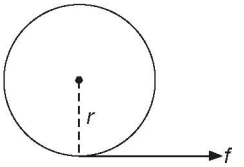
Hence, the correct answer is (A).

46. According to Newton's Second Law for rotational motion, we have

$$\Sigma\tau = I_{cm}\alpha$$

$\Rightarrow \quad \mu B\cos\theta - fr = I_{cm}\alpha \qquad \ldots(1)$





where I_{cm} is the moment of inertia of the ball about the centre of mass. (Do not confuse I_{cm} with the current I) and

$$f = ma_{cm} = mr\alpha \qquad \ldots(2)$$

$\Rightarrow \quad \alpha = \frac{5}{7}\left(\frac{\pi n I B}{m}\right)\cos\theta$

Hence, the correct answer is (A).

47. $\omega\frac{d\omega}{d\theta} = \frac{5}{7}\left(\frac{\pi n I B}{m}\right)\cos\theta$

$\Rightarrow \quad \frac{\omega^2}{2} = \frac{5}{7}\left(\frac{\pi n I B}{m}\right)\sin\theta$

$\Rightarrow \quad \omega = \sqrt{\frac{10}{7}\left(\frac{\pi n I B}{m}\right)\sin\theta}$

Hence, the correct answer is (A).

48. Since, $f = mr\alpha$

$\Rightarrow \quad \mu mg = mr\left(\frac{5}{7}\frac{\pi n I B}{m}\right)$

$$\mu = \frac{5\pi}{7g}\left(\frac{nIB}{m}\right)r$$

Hence, the correct answer is (B).

49. Since there is no friction, there is no force acting on the sphere, which is responsible for the translation motion of the ball.

Hence, the correct answer is (A).

50. The component of velocity of charged particle along the magnetic field does not change. The component of velocity of charged particle normal to magnetic field only changes in direction but always remains normal to magnetic field. So, the angle between velocity and magnetic field remains the same.

Hence, the correct answer is (A).

51. $f = \frac{qB}{2\pi m} = \frac{10^3}{13} \times \frac{26}{2\pi}$

$\Rightarrow \quad f = \frac{1000}{\pi} = 318 \text{ Hz}$

Hence, the correct answer is (B).

52. Pitch $p = v_{\parallel} T$

where $v_{\parallel}$ is the component of velocity along the field i.e.

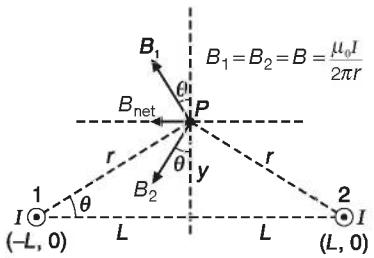
$$v_{\parallel} = \frac{\vec{v}\cdot\vec{B}}{|\vec{B}|} = \frac{36}{26} = \frac{16}{13} \text{ ms}^{-1}$$

$$\Rightarrow \quad p = \left(\frac{16}{13}\right)\left(\frac{\pi}{1000}\right) \approx \frac{50}{13\times 1000}$$

$$\Rightarrow \quad p \approx \frac{1}{260} \text{ m}$$

Hence, the correct answer is (C).

53. At point $P(0, y, 0)$, net magnetic field is



$B_{net} = 2B\sin\theta$, where

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\sqrt{L^2+y^2}} \text{ and } \sin\theta = \frac{y}{\sqrt{L^2+y^2}}$$

$$\Rightarrow \quad B_{net} = \frac{2\mu_0 I y}{2\pi\left(L^2+y^2\right)} = \frac{\mu_0 I}{\pi}\left(\frac{y}{L^2+y^2}\right)$$

Hence, the correct answer is (B).

54. For B to be maximum, we have

$$\frac{dB}{dy} = 0$$

$$\Rightarrow \quad \frac{2\mu_0 I}{2\pi}\left[\frac{\left(L^2+y^2\right)-2y^2}{L^2+y^2}\right] = 0$$

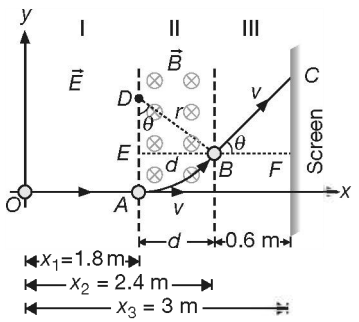
$$\Rightarrow \quad y = \pm L$$

Hence, the correct answer is (D).

55. $B_{\text{maximum}} = B|_{y=\pm L} = \frac{\mu_0 I}{2\pi L}$

Hence, the correct answer is (B).

56.



In REGION (I), we have only the electric field. So, velocity with which the particle enters REGION (II) is given by

$$v^2 = 2\left(\frac{qE}{m}\right)x_1$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{qE}{m}\right)x_1}$$

$$\Rightarrow \quad v = \sqrt{\frac{2\times 1\times 10\times 1.8}{1}} \qquad \{\because q = 1\text{ C},\ m = 1\text{ kg}\}$$

$$\Rightarrow \quad v = 6 \text{ ms}^{-1}$$

In magnetic field speed does not change. Hence, particle will collide with 6 ms^{-1}.

Hence, the correct answer is (C).

57. In magnetic field path of the particle is circle. Radius of circular particle is

$$r = \frac{mv}{Bq} = \frac{(1)(6)}{(5)(1)} = 1.2 \text{ m}$$

$$d = (2.4 - 1.8) \text{ m} = 0.6 \text{ m}$$

Since, $d < r$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{d}{r}\right) = \sin^{-1}\left(\frac{0.6}{1.2}\right) = 30°$$

Now, $AE = r(1-\cos\theta) = 1.2\left(1-\frac{\sqrt{3}}{2}\right)$

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$$\Rightarrow \quad AE = 0.6\left(2-\sqrt{3}\right) \text{ and } FC = BF\tan\theta = \frac{0.6}{\sqrt{3}}$$

So, y-coordinate is

$$y = AE + FC = 0.6\left(2-\sqrt{3}+\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad y = \frac{1.2\left(\sqrt{3}-1\right)}{\sqrt{3}} \text{ m}$$

Hence, the correct answer is (B).

58. Total time is given by $t = t_{OA} + t_{AB} + t_{BC}$

Where $t_{OA} = \sqrt{\frac{2x_1}{a}} = \sqrt{\frac{2x_1}{\left(\frac{qE}{m}\right)}} = \sqrt{\frac{2mx_1}{qE}}$

$$\Rightarrow \quad t_{OA} = \sqrt{\frac{2\times1\times1.8}{(1)(10)}}$$

$$\Rightarrow \quad t_{OA} = \frac{3}{5} \text{ s}$$

$$t_{AB} = \left(\frac{30^\circ}{360^\circ}\right)T = \left(\frac{1}{12}\right)\left(\frac{2\pi m}{Bq}\right)$$

$$\Rightarrow \quad t_{AB} = \frac{(2\pi)(1)}{(12)(5)(1)} = \frac{\pi}{30} \text{ s}$$

$$t_{BC} = \frac{BC}{v} = \frac{0.6\sec\theta}{v} = \frac{(0.6)\left(\frac{2}{\sqrt{3}}\right)}{6}$$

$$\Rightarrow \quad t_{BC} = \frac{1}{5\sqrt{3}} \text{s}$$

$$\Rightarrow \quad t = \frac{1}{5}\left(3+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right) \text{s}$$

Hence, the correct answer is (A).

59. Let us consider a length (dx) of the wire at a distance x from the long straight conductor. Magnetic field at this point is

$$B = \frac{\mu_0 i}{2\pi x}$$

Force acting on the length (dx) of the wire is

$$dF = \left(\frac{\mu_0 i i_0}{2\pi x}\right)dx$$

This force $d\vec{F}$ will be in the upward direction parallel to the current i.
So, the disc will start rotating in anticlockwise direction about its centre in its plane.

Torque due to this force $d\vec{F}$ is

$$d\tau = (dF)r_\perp$$

$$\Rightarrow \quad d\tau = \left(\frac{\mu_0 i i_0}{2\pi x}\right)dx(x-r) \qquad \{\because\ r_\perp = x-r\}$$

Total initial torque is

$$\tau = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\frac{(x-r)}{x}dx = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\left(1-\frac{r}{x}\right)dx$$

$$\Rightarrow \quad \tau = \frac{\mu_0 i i_0}{2\pi}\left[x\Big|_r^{R+r} - r\log_e x\Big|_r^{R+r}\right]$$

$$\Rightarrow \quad \tau = \frac{\mu_0 i i_0}{2\pi}\left[R - r\log_e\left(\frac{r+R}{r}\right)\right]$$

Since $\tau = I\alpha$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\mu_0 i i_0}{2\pi\left(\frac{1}{2}mR^2\right)}\left[R - r\log_e\left(\frac{r+R}{r}\right)\right]$$

$$\Rightarrow \quad \alpha = \frac{\mu_0 i i_0}{\pi m R^2}\left[R - r\log_e\left(1+\frac{R}{r}\right)\right]$$

Hence, the correct answer is (B).

60. Since the C.M. of the (disc + wire) is at rest, so net force on (wire + disc) = 0

$$\Rightarrow \quad N - F = 0$$

$$\Rightarrow \quad N = F = \int dF$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\frac{dx}{x}$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\log_e x\Big|_r^{R+r}$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\log_e\left(\frac{R+r}{r}\right) = \frac{\mu_0 i i_0}{2\pi}\log_e\left(1+\frac{R}{r}\right)$$

Hence, the correct answer is (D).

61. $\vec{\tau} = \vec{\mu}\times\vec{B} = 0.75\left(-\hat{k}\right)\times 0.2\hat{i}$

$$\vec{\tau} = -0.15\,\hat{j}$$

Hence, the correct answer is (B).

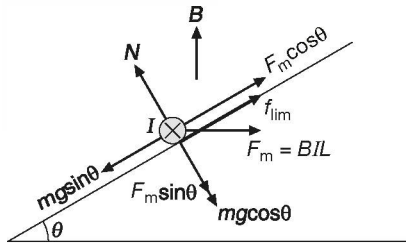
63. $W = MB\cos 0^\circ - MB\cos\pi$

$$W = 2\,\mu B$$

$$W = 0.3 \text{ J}$$

Hence, the correct answer is (C).

64. When magnetic field is minimum, then the rod tends to slide down the incline and hence $f_{\lim}$ acts up the incline as shown in Figure.



For equilibrium, $mg\sin\theta = BIL\cos\theta + f_{lim}$

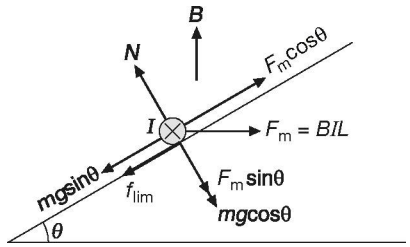
$\Rightarrow \quad mg\sin\theta = BIL\cos\theta + \mu N$

$\Rightarrow \quad mg\sin\theta = BIL\cos\theta + \mu\left(mg\cos\theta + BIL\sin\theta\right)$

$\Rightarrow \quad B_{min} = \dfrac{mg}{IL}\left(\dfrac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)$

Hence, the correct answer is (B).

65. When magnetic field is maximum, then the rod tends to slide up the inclined plane and hence f_{lim} acts down the incline as shown in Figure.



For equilibrium of the rod, we have

$mg\sin\theta + f_{lim} = BIL\cos\theta$

$\Rightarrow \quad mg\sin\theta + \mu N = BIL\cos\theta$

$\Rightarrow \quad mg\sin\theta + \mu\left(BIL\sin\theta + mg\cos\theta\right) = BIL\cos\theta$

$\Rightarrow \quad B_{max} = \dfrac{mg}{IL}\left(\dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$

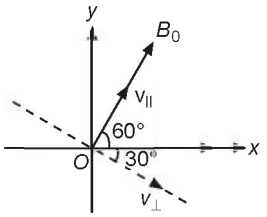
This is the maximum value of the required magnetic field.

Hence, the correct answer is (C).

66. Since the particle enters the field and has a velocity component parallel and perpendicular to the field, so it will follow a helical path.

Hence, the correct answer is (B).

67. Let us show the situation discussed diagrammatically. We observe that $\vec{B}$ makes an angle of $60°$ with x-axis. Let us now resolve $\vec{v}$ along $\vec{B}$ and perpendicular to $\vec{B}$ and label them as $v_{||}$ and $v_{\perp}$ respectively. Then



$v_{||} = v_0\cos 60° = \dfrac{v_0}{2}$ and

$v_{\perp} = v_0\sin 60° = \dfrac{\sqrt{3}v_0}{2}$

Since pitch of the helix is

$p = v_{||} \times T = \dfrac{v_0}{2}\left(\dfrac{2\pi m}{qB_0}\right)$

$\Rightarrow \quad p = \dfrac{\pi m v_0}{qB_0}$

Hence, the correct answer is (B).

68. $v_z = v_{\perp} = \dfrac{\sqrt{3}v_0}{2}$ after $t = \dfrac{T}{4}$

and $v_z = -v_{\perp} = -\dfrac{\sqrt{3}v_0}{2}$ after $t = \dfrac{3T}{4}$

Hence, the correct answer is (B).

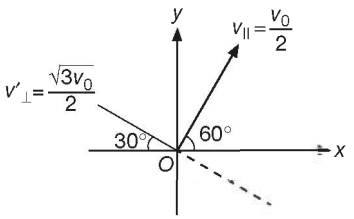
69. $Z_{max} = 2r = \dfrac{2mv_{\perp}}{qB_0} = \dfrac{\sqrt{3}mv_0}{qB_0}$

Hence, the correct answer is (C).

70. When z-coordinate has its maximum value, $v_{||}$ will remain as it is whereas $v_{\perp}$ will change its direction by $180°$.

$\Rightarrow \quad v_x = \dfrac{v_0}{2}\cos 60° - \dfrac{\sqrt{3}v_0}{2}\cos 30° = -\dfrac{v_0}{2}$ and

$v_y = \dfrac{v_0}{2}\sin 60° + \dfrac{\sqrt{3}v_0}{2}\sin 30° = \dfrac{\sqrt{3}}{2}v_0$



Hence, the correct answer is (C).

71. At origin, $F_m = qv_0B_0$, along $+y$ direction and $\vec{F}_e = q\vec{E} = qE_0\left(-\hat{j} + \hat{k}\right)$.

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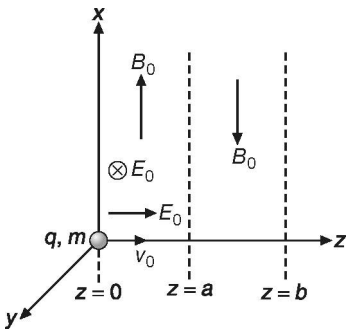
For particle to move undeviated, $(F_m)_y = (F_e)_y$

$\Rightarrow \quad qv_0B_0 = qE_0$

$\Rightarrow \quad E_0 = B_0v_0$



Hence, the correct answer is (B).

72. For particle to reverse its direction of motion, it should move in the magnetic field (from $z = a$ to $z = b$) in a circle of radius $r = b - a$.

Let v be the speed of the particle as it enters the magnetic field at $z = a$. Then by Work-Energy Theorem we have

$W = \Delta K$

$\Rightarrow \quad (qE_0)a = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$\Rightarrow \quad v = \sqrt{v_0^2 + \frac{2qE_0a}{m}}$

$\Rightarrow \quad v = \sqrt{v_0^2 + \frac{2qE_0a}{\frac{2}{3}\left(\frac{qE_0a}{v_0^2}\right)}}$

$\Rightarrow \quad v = 2v_0$

Since we have

$$r = \frac{mv}{qB} = \left(\frac{2}{3}\frac{qE_0a}{v_0^2}\right)\left(\frac{2v_0}{qB_0}\right)$$

$\Rightarrow \quad b - a = \frac{4}{3}\left(\frac{E_0a}{B_0v_0}\right)$

$\Rightarrow \quad b = a\left(1 + \frac{4E_0}{3B_0v_0}\right)$

Hence, the correct answer is (B).

73. After all the fields are switched off, the velocity remains constant, so

$$t = \frac{a}{v} = \frac{a}{2v_0}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (r)
C → (s)
D → (s)

At (a, a), B due to both conductors is along $+z$ direction. At $(-a, -a)$, B due to both conductors is along $-z$ direction

At $(a, -a)$ and $(-a, a)$ B due to both conductors is zero.

2. A → (p, q)
B → (q)
C → (p, q, r)
D → (q, s)

A charge always has an electric field associated with it, whether moving or stationary.

So, (A) → (p, q)

However magnetic field is associated with current. Which can be due to a moving charge only.

So, (B) → (q)

A charged particle, whether moving or stationary will experience an electric force which will accelerate it or decelerate it depending upon the nature of charge and the direction of electric field. So, the velocity of a charge and hence the kinetic energy of a charge changes when it is in the influence of electrostatic force.

So, (C) → (p, q, r)

Since $\vec{F} = q(\vec{v} \times \vec{B})$, so the magnetic force acts on a moving charged particle (for $0° < \theta < 180°$). This force is perpendicular to the velocity of the particle (at any instant) as well as to $\vec{B}$. Also work done by a magnetic force is zero (as studied) so the kinetic energy of a charged particle does not change when it is under the influence of a magnetic force.

Hence, (D) → (q, s)

3. A → (q)
B → (p, t)
C → (s)
D → (r)

According to Amperes Circuital Law, we have

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

where $I_{enc} = I = \int \vec{J} \cdot d\vec{A}$

$\Rightarrow \quad I = \int_0^R br(2\pi r dr) = \frac{2}{3}\pi bR^3$

So, for $r < R$ we have $I_{\text{inside}} = \frac{2}{3}\pi br^3$

Hence, (D) → (r)

Now, for $r < R$, we have

$$\mu_0 I_{\text{enc}} = \oint \vec{B} \cdot d\vec{\ell}$$

$$\mu_0\left(\frac{2}{3}\pi br^3\right) = B(2\pi r)$$

$$\Rightarrow \quad B_{\text{inside}} = \frac{\mu_0 br^2}{3}$$

Hence, (A) → (q)

Also, at the surface we shall observe the B to be maximum and continuous.

So, (B) → (p, t)

Now, for $r > R$ i.e., outside we have

$$\mu_0\left(\frac{2\pi bR^3}{3}\right) = B(2\pi r)$$

$$\Rightarrow \quad B_{\text{outside}} = \frac{\mu_0 bR^3}{3r}$$

Hence, (C) → (s)

4. A → (r)
B → (q)
C → (p)
D → (r)

For (A): $\vec{F} = q(\vec{v} \times \vec{B})$

Since, q is negative, $\vec{v}$ is along $+\hat{i}$ and $\vec{B}$ along $+\hat{j}$. Therefore, $\vec{F}$ is along negative z.

For (B): $\vec{F} = q(\vec{v} \times \vec{B})$ Since q is negative, v along $+\hat{i}$ and $\vec{B}$ along $+\hat{k}$, so $\vec{F}$ is along $+y$ axis.

For (C): $\vec{B}$ is parallel to $\vec{v}$. So, magnetic force is zero. Charge is negative so electric force is opposite to $\vec{E}$.

For (D): Charge is negative. So, electrostatic force is in opposite direction of $\vec{E}$.

5. Done already, see theory
A → (s)
B → (q)
C → (p)
D → (r)

6. Done already, see theory
A → (r)
B → (p)
C → (q, r)
D → (s)

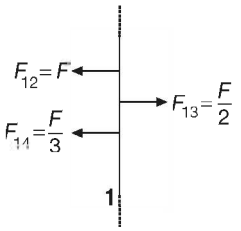
7. A → (q)
B → (r)
C → (q)
D → (r)

The force (F) per unit length is

$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r}$$

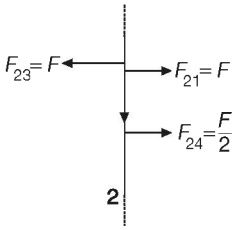
Current in same direction means attraction and current in opposite direction means repulsion.

Force on wire 1 due to other wires 2, 3 and 4 is shown in Figure.



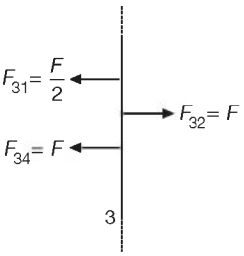
So, force on 1 is leftwards

Force on wire 2 due to other three wires 1, 3 and 4 is shown in Figure.

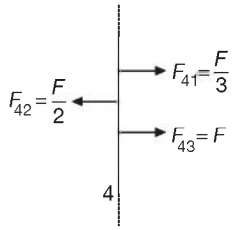


So, net force on 2, i.e. F_2 is rightwards.

Force on wire 3 due to others is also leftwards



Force on wire 4 due to others is rightwards



CHAPTER 1

8. Done already, see theory
A → (p, t)
B → (p, q, r)
C → (p)
D → (s)

9. A → (p, s)
B → (p, q)
C → (p, r)
D → (p, s)
Force on a current carrying loop is zero for all angles. τ is maximum when angle between $\vec{M}$ and $\vec{B}$ is 90°. Potential energy is minimum when $\theta = 0°$. Positive Potential energy is positive for obtuse angle. The direction of $\vec{M}$ is obtained by right hand thumb rule.

10. Done already, see theory
A → (q, r)
B → (p)
C → (q, r)
D → (q)

11. Done already, see theory
A → (s)
B → (t)
C → (q)
D → (p)

12. A → (r)
B → (s)
C → (q)
D → (p)
For direction of magnetic force applying Fleming's left hand rule, according to which w and x are positively charged particles and y and z negatively charged particle.
Also, we know that

$$r = \frac{\sqrt{2mK}}{Bq}$$

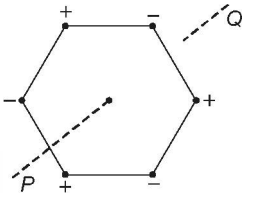
Since K i.e. the kinetic energy is same for all particles, so

$$r \propto \frac{\sqrt{m}}{q}$$

13. Done already, see theory
A → (q)
B → (p)
C → (s, t)
D → (q)

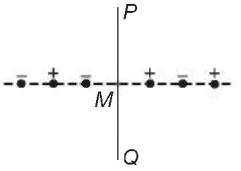
14. A → (p, r, s)
B → (r, s)
C → (p, q, t)
D → (r, s)

For (p)



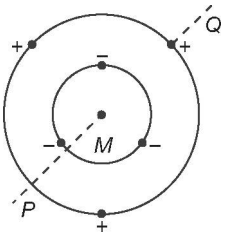
By symmetry $E = 0$, $V = 0$, $B = 0$
and $\mu = NIA$ but $I_{\text{effective}} = 0$
So $\mu = 0$

For (q)



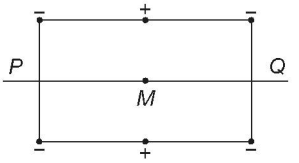
$E \neq 0$, $V = 0$
Since, $I_{\text{effective}} = 0$
$\Rightarrow \quad B = 0$ and $\mu = 0$

For (r)



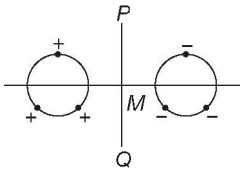
$E = 0$ (by symmetry)
$V \neq 0$ (since distances are different)
$B \neq 0$ (since radius is different)
$\mu \neq 0$

For (s)



$E = 0$ (by symmetry)
$V \neq 0$ (since distances are different)
$B \neq 0$
$\mu \neq 0$

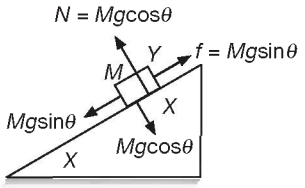
For (t)



$E \neq 0$, $V = 0$, $\mu = 0$, $B = 0$ and given each rotating charge to be equivalent to a steady current so $B = 0$ so $(t \to C)$ and if it was not given that each rotating charge to be equivalent to steady current then $B \neq 0$

15. A → (p, t)
B → (q, s, t)
C → (p, r, t)
D → (q)

For (p)



Net force on Y due to

$$X = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2} = Mg$$

(B) As the inclined is fixed. So, gravitational P.E. of X is constant.

(C) As K.E. is constant and P.E. of Y is decreasing. So mechanical energy of $(X + Y)$ is decreasing.

For (q)

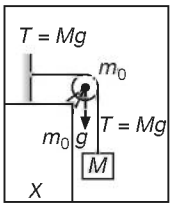
(A) Force on Y due to X will be greater than Mg which is equal to $(Mg + \text{repulsion force})$

(B) As the system is moving up, P.E. of X is increasing

(C) Mechanical energy of $(X + Y)$ is increasing

(D) Torque of the weight of Y about point $P = 0$

For (r)



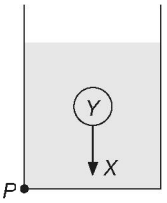
(A) Force on Y due to $X = \sqrt{\left[(M + m_0)g\right]^2 + (Mg)^2}$

(B) As the system moves down, gravitational P.E. of X decreases

(C) As the system moves down, total mechanical energy of $(X + Y)$ also decreases

(D) $\tau_P \neq 0$

For (s)



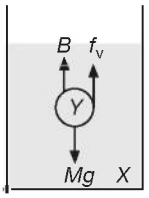
(A) Force on Y due to X = Buoyancy force which is less than mg

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X increases.

(C) As there is no non-conservative force, so total mechanical energy of $X + Y$ remains conserved.

(D) $\tau_P \neq 0$

For (t)



(A) As the sphere is moving with constant velocity

$$B + f_v = Mg$$

so force on Y due to X is $B + f_v = Mg$

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X will increase

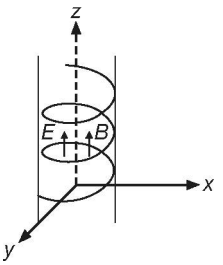
(C) Increase in mechanical energy $= w_{fr} = -ve$

(D) $\tau_P = 0$

Integer/Numerical Answer Type Questions

1. The Z component of the force $\vec{F}$ is a constant electrical force. It produces a constant acceleration a along Z-axis given as

$$Z = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2$$



The other component of the force F is a magnetic force provides the necessary centripetal force to keep the proton in a circular path of radius, r say. The period of revolution of the proton

$$T=\frac{2\pi r}{v_0} \text{ where}$$

$$F_{\text{c.p.}}=\frac{mv_0^2}{r}=ev_0B$$

$$\Rightarrow \quad v_0=\frac{eBr}{m}$$

$$\Rightarrow \quad T=\frac{2\pi m}{eB}$$

Since the particle (proton) moves in circular path having a period of revolution T in x, y plane and moves along Z-axis with a constant acceleration $a=\frac{eE}{m}$, the path of the proton is an helix.

After three revolutions, putting $t=3T$ we obtain

$$z=\frac{1}{2}\left(\frac{eE}{m}\right)(3T)^2=\frac{9}{2}\frac{eET^2}{m}$$

Putting $T=\frac{2\pi m}{eB}$ we get

$$z=\frac{18\pi^2Em}{eB^2}=37 \text{ m}$$

2. Let at time t, particle bet at point $P(x, y)$ and its velocity be

$$\vec{v}=\left(v_x\hat{i}+v_y\hat{j}\right)$$

Since, $|v|=|v_0|$

$$\Rightarrow \quad v_0^2=v_x^2+v_y^2$$

Since work done by magnetic field is always zero, so there is no change in magnitude of velocity, hence

$$|\vec{v}|=|\vec{v}_0|$$

$$\Rightarrow \quad v_0^2=v_x^2+v_y^2$$

Then, magnetic force on the particle at point $P(x, y)$ is

$$\vec{F}=q\left(v_x\hat{i}+v_y\hat{j}\right)\times B_0\left(1+\frac{y}{d}\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}=qv_xB_0\left(1+\frac{y}{d}\right)\hat{j}-qv_yB_0\left(1+\frac{y}{d}\right)\hat{i}$$

$$\Rightarrow \quad F_x=\frac{mdv_x}{dt}=-qv_yB_0\left(1+\frac{y}{d}\right), \text{ where } v_d=\frac{dy}{dt}$$

$$\Rightarrow \quad mdv_x=-qB_0\left(1+\frac{y}{d}\right)dy \qquad \ldots(1)$$

When the particle leaves the field at $y=d$, then let the velocity in x-direction be v_x. Integrating equation (1), we get

$$\int_{v_0}^{v_x}dv=-\frac{qB_0}{m}\int_0^d\left(1+\frac{y}{d}\right)dy$$

$$\Rightarrow \quad v_x-v_0=-\frac{qB_0}{m}\left(d+\frac{d^2}{2d}\right)=-\frac{3qB_0d}{2m}$$

$$\Rightarrow \quad v_x=v_0-\frac{3qB_0d}{2m}$$

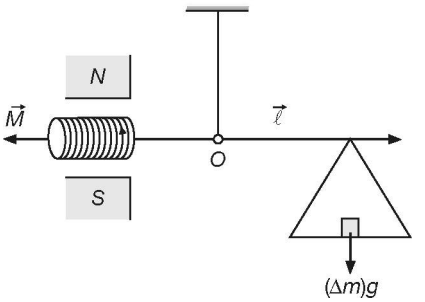
$$\Rightarrow \quad k=1.5$$

3. We know that the torque acting on a magnetic dipole is

$$\vec{\tau}=\vec{M}\times\vec{B}$$

where $\vec{M}=(IA)\hat{n}$, where $\hat{n}$ is the normal on the plane of the loop and the direction of which is given by Right Hand Thumb Rule.

On passing a current through the coil, this torque acting on the magnetic dipole, is counterbalanced by the moment of additional weight, about O. Hence, the direction of current in the loop must be in the direction, shown in the figure.



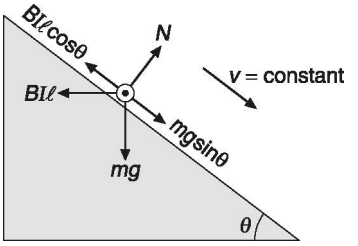
$$\vec{M}\times\vec{B}+\vec{\ell}\times(\Delta m)\vec{g}=\vec{0}$$

$$\Rightarrow \quad NBIA=(\Delta m)g\ell$$

$$\Rightarrow \quad B=\frac{(\Delta m)g\ell}{NIA}=0.4 \text{ T}$$

$$\Rightarrow \quad B=400 \text{ mT}$$

4. The forces acting on conductor are shown in Figure.



For conductor to move down the incline with constant velocity, we have

$$mg\sin\theta = BIl\cos\theta$$

$$\Rightarrow \quad B = \frac{mg}{IL}\tan\theta$$

$$\Rightarrow \quad B = \frac{\left(\frac{50}{1000}\right)(10)}{(2.5)\left(\frac{50}{100}\right)}\frac{3}{4}$$

$$\Rightarrow \quad B = 0.3\ \text{T} = 300\times10^{-3}\ \text{T}$$

$$\Rightarrow \quad B = 300\ \text{mT}$$

5. Let the wire connected to the 25 Ω resistor be designated as WIRE 2 and the wire connected to the 10 Ω resistor be designated as WIRE 1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton B_2 is ⊗ and B_1 is ⊙, where

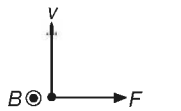
$$B_1 = \frac{\mu_0 I_1}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r}, \text{ with } r = 0.025\ \text{m}$$

$$I_1 = (100\ \text{V})(10\ \Omega) = 10\ \text{A} \text{ and}$$

$$I_2 = (100\ \text{V})(25\ \Omega) = 4\ \text{A}$$

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = 8\times10^{-5}\ \text{T},\ B_2 = \frac{\mu_0 I_2}{2\pi r} = 3.2\times10^{-5}\ \text{T}$$

$$\Rightarrow \quad B = B_1 - B_2 = 4.8\times10^{-5}\ \text{T} \text{ and in the direction } \odot$$



Force is to the right

Since $F = qvB = (1.6\times10^{-19})(650\times10^{3})(4.8\times10^{-5})$

$$\Rightarrow \quad F = 4.99\times10^{-18}\ \text{N} \cong 5\times10^{-18}\ \text{N}$$

$$\Rightarrow \quad x = 5$$

6. Since $R = \frac{mv}{qB}$

Also, $v^2 = \frac{2q\Delta V}{m}$

$$\Rightarrow \quad \frac{m}{q} = \frac{2\Delta V}{v^2}$$

$$\Rightarrow \quad \frac{m}{q} = \frac{2(12\times1000)}{10^{12}}$$

$$\Rightarrow \quad \frac{m}{q} = \frac{24\times10^{3}}{10^{12}}$$

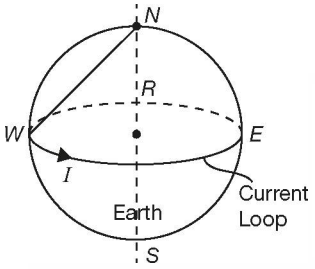
$$\Rightarrow \quad R = \frac{mv}{qB} = \frac{24\times10^{3}\times10^{6}}{10^{12}\times0.2}$$

$$\Rightarrow \quad R = 12\times10^{-2}\ \text{m}$$

$$\Rightarrow \quad R = 12\ \text{cm}$$

7. Since $B_{axis} = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$

Here $x = R$ = Radius of Earth



$$B_{axis} = 7\times10^{-5}\ \text{T}$$

Substituting values, we get

$$I = 2\times10^{9}\ \text{A}$$

$$\Rightarrow \quad I = 2000\ \text{MA}$$

8. Maximum tension that the ring can bear is

$$T = BIR$$

$$\Rightarrow \quad 1.5 = B(10)\left(\frac{15}{100}\right)$$

$$\Rightarrow \quad B = 1\ \text{T}$$

9. We observe here that, reducing the normal force will reduce the friction force $F = BIL$

$$\Rightarrow \quad B = \frac{F}{IL}$$

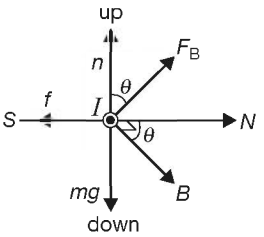
When the wire is just able to move,

$$\sum F_y = N + F\cos\theta - mg = 0$$

$$\Rightarrow \quad N = mg - F\cos\theta \text{ and since}$$

$$f = \mu N$$

$$\Rightarrow \quad f = \mu(mg - F\cos\theta)$$



Also, $\sum F_x = F\sin\theta - f = 0$

$$\Rightarrow \quad F_B\sin\theta = f$$

$$\Rightarrow \quad F\sin\theta = \mu(mg - F\cos\theta)$$

$$\Rightarrow \quad F = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$$

We minimize B by minimizing F

$$\Rightarrow \quad \frac{dF}{d\theta} = 0$$

$$\Rightarrow \quad (\mu mg)\frac{\cos\theta - \mu\sin\theta}{(\sin\theta + \mu\cos\theta)^2} = 0$$

CHAPTER 1

$\Rightarrow \quad \mu\sin\theta = \cos\theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}\left(\frac{1}{0.75}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

for the smallest field, and

$\Rightarrow \quad B = \frac{F}{IL} = \left(\frac{\mu g}{I}\right)\left(\frac{\left(\frac{m}{L}\right)}{\sin\theta + \mu\cos\theta}\right)$

$\Rightarrow \quad B_{\min} = \left[\frac{(0.75)(10)}{2\ A}\right]\frac{0.1}{\sin(53°) + (0.75)\cos(53°)}$

$\Rightarrow \quad B_{\min} = 0.128\ \text{T}$

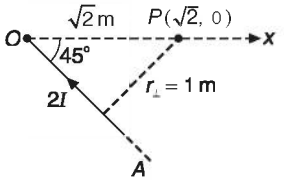
Pointing north at an angle of 78.7° below the horizontal

$\Rightarrow \quad B_{\min} = \frac{(0.75)(10)(0.1)}{(2)\left(\frac{4}{5} + \left(\frac{3}{4}\right)\left(\frac{3}{5}\right)\right)}$

$\Rightarrow \quad B_{\min} = \frac{0.75}{(2)\left(\frac{5}{4}\right)}$

$\Rightarrow \quad B_{\min} = \frac{3}{10} = 0.3\ \text{T} = 300\ \text{mT}$

10. Magnetic field at P due to section OA is



$B_1 = \frac{\mu_0(2I)}{4\pi(1)}(\sin 45° + \sin 90°)$

$\Rightarrow \quad B_1 = \frac{\mu_0 I}{4\pi}(\sqrt{2} + 2)$

Magnetic field at P due to section OB is

$B_2 = \frac{\mu_0 I}{4\pi(1)}(\sin 45° + \sin 90°)$

$\Rightarrow \quad B_2 = \frac{\mu_0 I}{4\pi}\left(\frac{1}{\sqrt{2}} + 1\right)$

Magnetic field at P due to section OC is

$B_3 = \frac{\mu_0 I}{4\pi\sqrt{2}}(\sin 0° + \sin 90°)$

$\Rightarrow \quad B_3 = \frac{\mu_0 I}{4\pi\sqrt{2}}$

Net magnetic field B is given by

$B = B_1 + B_2 + B_3$

$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}\left(\sqrt{2} + 2 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}(2\sqrt{2} + 2 + 1)$

$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}(\sqrt{2} + 1)^2$

$\Rightarrow \quad B = \frac{2\mu_0 I}{8\pi}(\sqrt{2} + 1)^2$

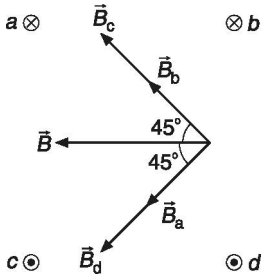
$\Rightarrow \quad x = 2$

11. The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the Right Hand Rule.

For situations (a) and (b), $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

For (c) the fields due to each wire are shown in figure.



$\Rightarrow \quad B = B_a\cos 45° + B_b\cos 45° + B_c\cos 45° + B_d\cos 45°$

$\Rightarrow \quad B = 4B_a\cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°$

where $r = \sqrt{(10\ \text{cm})^2 + (10\ \text{cm})^2}$

$r = 10\sqrt{2}\ \text{cm} = (0.1)\sqrt{2}\ \text{m}$

$\Rightarrow \quad B = 4\frac{(4\pi\times 10^{-7}\ \text{TmA}^{-1})(100\ \text{A})}{2\pi(0.1\sqrt{2}\ \text{m})}\cos 45°$

$B = 4\times 10^{-4}\ \text{T}$, to the left

$\Rightarrow \quad B = 400\ \mu\text{T}$

12. Since $E_y = -E$, $E_x = 0$

$\Rightarrow \quad a_y = -\frac{qE}{M}$, $a_x = 0$

Also, $v_x = u_x + a_x t$ and $v_y = u_y + a_y t$

$\Rightarrow \quad v_x = \frac{v}{\sqrt{2}}$ and $v_y = \frac{v}{\sqrt{2}} - \left(\frac{qE}{M}\right)t$

At $x = 0.5$ m, we have

$0.5 = v_x t$

$\Rightarrow \quad t = \dfrac{0.5}{v/\sqrt{2}} = \dfrac{0.5\sqrt{2}}{v}$

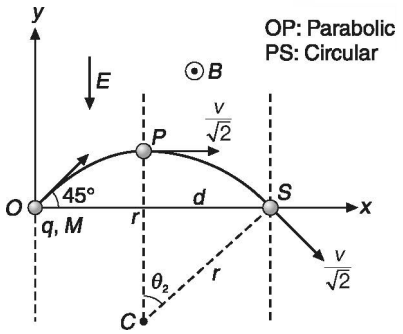
$\Rightarrow \quad v_y = \dfrac{v}{\sqrt{2}} - v^2\left(\dfrac{0.5\sqrt{2}}{v}\right) = \dfrac{v}{\sqrt{2}} - \dfrac{v}{\sqrt{2}}$

$\Rightarrow \quad v_y = 0$

i.e., deviation suffered by particle is $\theta_1 = 45°(\text{CW})$

So, at $x = 0.5$ m, the charged particle enters the magnetic field with a velocity $\dfrac{v}{\sqrt{2}}$, perpendicular to the field so, it will follow a circular path of radius

$r = \dfrac{M(v/\sqrt{2})}{qB} = \dfrac{Mv}{\sqrt{2}qB}$ as shown in Figure.



So, deviation suffered by particle is

$\sin\theta_2 = \dfrac{d}{r} = \dfrac{1}{2}$

$\Rightarrow \quad \theta_2 = 30°(\text{CW})$

So, total deviation is $\theta = \theta_1 + \theta_2 = 75°(\text{CW})$

$\Rightarrow \quad \theta = 75 \times \dfrac{\pi}{180}$ radian

$\Rightarrow \quad \theta = \dfrac{5\pi}{12}$ radian

$\Rightarrow \quad x = 5, \; y = 12$

13. (a) $BI\ell = mg$

$\Rightarrow \quad I = \dfrac{mg}{\ell B} = \dfrac{(0.75 \text{ kg})(10 \text{ ms}^{-2})}{(0.5 \text{ m})(0.5 \text{ T})} = 30 \text{ A}$

Since $\varepsilon = IR = (30 \text{ A})(25\ \Omega) = 750 \text{ V}$

(b) $R = 2\ \Omega$

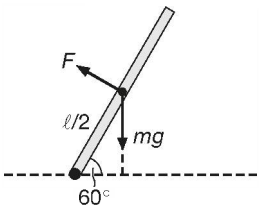
From Ohm's Law, we have

$I = \dfrac{\varepsilon}{R} = \dfrac{(750 \text{ V})}{(2\ \Omega)} = 375 \text{ A}$

$F_1 = BI\ell = 93.75$ N

$\Rightarrow \quad a = \dfrac{(F_1 - mg)}{m} = \dfrac{93.75 - 7.5}{0.75} = 115 \text{ ms}^{-2}$

14. The contact at a will break if the bar rotates about b. The magnetic field is directed out of the page, so the magnetic torque is counter clockwise, whereas the gravity torque is clockwise in the figure in the problem. The maximum current corresponds to zero net torque, in which case the torque due to gravity is just equal to the torque due to the magnetic field.



The magnetic force F is perpendicular to the bar and has moment arm $\dfrac{\ell}{2}$, where $\ell = 1$ m is the length of the bar. The gravity torque is $mg\left(\dfrac{\ell}{2}\cos(60°)\right)$ and the torque to the magnetic field is

$\tau_B = (BI\ell\sin 90)\dfrac{\ell}{2}$

Since, $\tau_{\text{gravity}} = \tau_B$

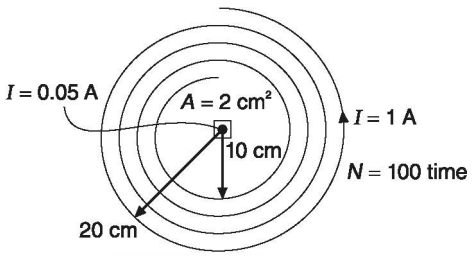
$\Rightarrow \quad mg\left(\dfrac{\ell}{2}\right)\cos(60°) = BI\ell\sin(90°)\dfrac{\ell}{2}$. This gives

$I = \dfrac{mg\cos(60°)}{\ell B\sin 90°}$

$\Rightarrow \quad I = \dfrac{(0.5)(10)(0.5)}{(1)(0.5)}$

$\Rightarrow \quad I = 5$ A

15. Magnetic field at centre of spiral loop is



$B = \dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$

$\Rightarrow \quad B = \dfrac{\mu_0(100)(1)}{2\times 10\times 10^{-2}}\ln(2)$

$\Rightarrow \quad B = \frac{\mu_0}{2}(10^3)\ln(2)$

$\Rightarrow \quad B = 500\mu_0 \ln 2$

Magnetic dipole moment of the loop is

$$M = IA = 0.05 \times 2 \times 10^{-4} = 1 \times 10^{-5} \text{ Am}^{-2}$$

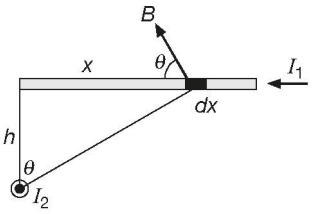
Since $U = -\vec{M} \cdot \vec{B}$

$\Rightarrow \quad (U) = MB\cos\theta$

$\Rightarrow \quad U = \frac{\mu_0}{2}(10^3)(\ln 2)(10^{-5}) = \frac{\mu_0}{200}\ln(2)$

$\Rightarrow \quad x = 200$

16. At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{B} = \frac{\mu_0 I_2}{2\pi\sqrt{h^2 + x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \frac{x}{h}$. This field exerts force on an element of the rod of length dx



$$d\vec{F} = I_1(d\vec{\ell} \times \vec{B}) = I_1 \frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \sin\theta$$

$\Rightarrow \quad d\vec{F} = \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \frac{x}{\sqrt{h^2 + x^2}}$, into the page

$\Rightarrow \quad d\vec{F} = \frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2 + x^2)}(-\hat{k})$

The net force is the sum of the forces on all of the elements of the bar. So,

$$\vec{F} = \int_{x=0}^{\ell} \frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2 + x^2)}(-\hat{k})$$

$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi} \int_0^{\ell} \frac{2x dx}{h^2 + x^2}$

$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi} \log_e(h^2 + x^2)\Big|_0^{\ell}$

$\Rightarrow \quad \vec{F} = \frac{\mu_0 I_1 I_2 (-\hat{k})}{4\pi}\left[\log_e(h^2 + \ell^2) - \log_e h^2\right]$

$\Rightarrow \quad \vec{F} = 10^{-7}(100)(200)(-\hat{k})\log_e\left[\frac{(0.5)^2 + (10)^2}{(0.5)^2}\right]$

$\Rightarrow \quad \vec{F} = 2 \times 10^{-3} \text{ N}(-\hat{k})\log_e(401) = 1.2 \times 10^{-2} \text{ N}(-\hat{k})$

$\Rightarrow \quad |\vec{F}| = 12 \text{ mN}$

17. Magnetic induction at O is given by

$$\vec{B} = \frac{\mu_0 i}{4\pi a}\left[\left(1 - \frac{1}{\sqrt{2}}\right)(-\hat{j}) + \left(\frac{1}{\sqrt{2}}\right)\hat{j} + \hat{k}\right]$$

Comparing with equation given in problem, we get

$x = 2$

18. Force of magnetic interaction is

$$\vec{F}_{\text{mag}} = e(\vec{v} \times \vec{B})$$

where $\vec{B} = \frac{\mu_0}{4\pi}\frac{e(\vec{v} \times \vec{r})}{r^3}$

$\Rightarrow \quad \vec{F}_{\text{mag}} = \frac{\mu_0}{4\pi}\frac{e^2}{r^3}[\vec{v} \times (\vec{v} \times \vec{r})]$

$\Rightarrow \quad \vec{F}_m = \frac{\mu_0}{4\pi}\frac{e^2}{r^3}[(\vec{v} \cdot \vec{r}) \times \vec{v} - (\vec{v} \cdot \vec{v}) \times \vec{r}]$

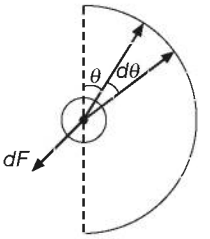
$\Rightarrow \quad \vec{F}_m = \frac{\mu_0}{4\pi}\frac{e^2}{r^3}(-v^2\vec{r})$

and $\vec{F}_{\text{ele}} = \frac{e^2\vec{r}}{4\pi\varepsilon_0|\vec{r}|^3}$

$\Rightarrow \quad \frac{|\vec{F}_{\text{mag}}|}{|\vec{F}_{\text{electric}}|} = -v^2\mu_0\varepsilon_0 = \left(\frac{v}{c}\right)^2 = 1 \times 10^{-6}$

$\Rightarrow \quad x = 1$

19. Magnetic field due to small ring at distance R from the centre



$$B = \frac{\mu_0}{4\pi}\frac{M}{R^3} \text{ where } M = I\pi a^2$$

$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{I\pi a^2}{R^3} = \frac{\mu_0 I a^2}{4R^3}$

$\Rightarrow \quad dF = BI_0 d\ell = BI_0(Rd\theta) = I_0 Rd\theta\frac{\mu_0 I a^2}{4R^3}$

$\Rightarrow \quad dF_x = dF\sin\theta$

$\Rightarrow \quad dF_x = \dfrac{\mu_0 I I_0 a^2 \sin\theta d\theta}{4R^2}$

$\Rightarrow \quad F_x = \dfrac{\mu_0 I I_0 a^2}{4R^2}\int_0^{\pi}\sin\theta d\theta$

$\Rightarrow \quad F_x = \left(\dfrac{\mu_0 I I_0 a^2}{4R^2}\right)2$

$\Rightarrow \quad F_x = \dfrac{\mu_0 I I_0 a^2}{2R^2}$

and $F_y = 0$

$\Rightarrow \quad F_{\text{net}} = F_x = \dfrac{\mu_0 I I_0 a^2}{2R^2} = 8$ newton

20. Force on a small element of length dx of wire is given by

$d\vec{F} = Id\vec{x}\times\vec{B}$

$\Rightarrow \quad d\vec{F} = \left[IB_0\left(1+\dfrac{x}{a}\right)dx\right](\hat{i}\times\hat{k})$

$\Rightarrow \quad d\vec{F} = IB_0\left(1+\dfrac{x}{a}\right)dx(-\hat{j})$

So, magnitude of total force $\vec{F}$ on wire will be

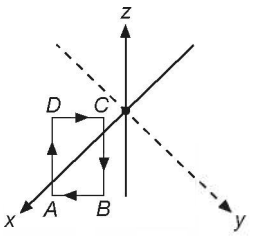
$$F = \int_a^{2a} dF = \int_a^{2a} IB_0\left(1+\frac{x}{a}\right)dx$$

$\Rightarrow \quad F = IB_0\left(x^2+\dfrac{x^2}{2a}\right)\Big|_a^{2a}$

$\Rightarrow \quad F = IB_0\left(\dfrac{5a}{2}\right)$, along negative y-axis

$\Rightarrow \quad k = 5$

21. $\vec{m} = I\vec{S}$, where $\vec{S}$ is the area of the loop.



$\Rightarrow \quad \vec{S} = \overrightarrow{BA}\times\overrightarrow{AD}$

$\Rightarrow \quad \overrightarrow{BA} = 2d\hat{i} - 2a\hat{j}$

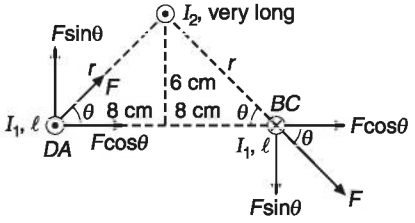
$\Rightarrow \quad \overrightarrow{AD} = 2b\hat{k}$

$\Rightarrow \quad \vec{S} = 2\left(d\hat{i} - a\hat{j}\right)\times 2b\hat{k}$

$\Rightarrow \quad \vec{M} = -4bI\left(d\hat{j} + a\hat{i}\right)$

$\Rightarrow \quad |\vec{M}| = 4bI\sqrt{d^2+a^2} = 2$ Am^2

22. For wires AB and CD, the force due to field of very long wire P on the wires (AB and CD) is zero. However force on wire DA will be attractive (towards P) and that on wire BC will be repulsive (away from P), as shown in Figure.



Let F_0 be force on wire DA due to P, then

$$F_{DA} = F_{BC} = F_0 = \left(\frac{\mu_0 I_1 I_2}{2\pi r}\right)l$$

The components of force i.e. $F\sin\theta$ and $F\sin\theta$ cancel, so net force on the loop is

$$F_{\text{net}} = 2F_0\cos\theta = \left[2\left(\frac{\mu_0 I_1 I_2}{2\pi r}\right)l\right]\cos\theta$$

where $\cos\theta = \dfrac{8}{\sqrt{8^2+b^2}} = \dfrac{4}{5}$

$\Rightarrow \quad F_{\text{net}} = \dfrac{(4\times10^{-7})(10)(5)\left(\frac{16}{100}\right)}{10}\times\dfrac{4}{5}$

$\Rightarrow \quad F_{\text{net}} = 256\times10^{-7}$ N

$\Rightarrow \quad F = 256$

23. $\left(\vec{B}_1+\vec{B}_2+\vec{B}_3+\vec{B}_4\right)_{\text{at the centre}} = \vec{0}$

$$\frac{\mu_0 I_1}{2\pi a}\otimes + \vec{B}_2 + \frac{\mu_0 I_3}{2\pi a}\odot + \frac{\mu_0 I_4}{2\pi a}\otimes = \vec{0} \qquad \ldots(1)$$

Assume inward direction to be positive, then on substituting $a = \dfrac{20}{100}m = 0.2$ m in (1), we get

$$\vec{B}_2 = \frac{\mu_0}{2\pi a}(I_3 - I_1 - I_4)$$

$$\Rightarrow \quad \vec{B}_2 = \frac{\mu_0}{2\pi a}(20-10-8) = \frac{\mu_0}{2\pi a}(2\text{ A}) \qquad \ldots(2)$$

Since $\vec{B}_2$ is $\oplus$, so we conclude that the field due to wire 2 must be directed inwards $\otimes$. For the field due to wire 2 to be directed inwards the current in the wire 2 must be in the downward direction. Hence, we get, from (1),

CHAPTER 1

$$\frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{2\pi a}(2\text{ A})$$

$\Rightarrow$ $I = 2$ A (downwards)

24. Magnetic induction at origin is due to one semi infinite wire and two quarter circle of radii R and $2R$.

$$\vec{B}_0 = \frac{\mu_0 I}{8R}\hat{k} + \frac{\mu_0 I}{8(2R)}\hat{k} + \frac{\mu_0 I}{4\pi R}\hat{j}$$

$$\Rightarrow \quad \vec{B}_0 = \frac{3\mu_0 I}{16R}\hat{k} + \frac{\mu_0 I}{4\pi R}\hat{j}$$

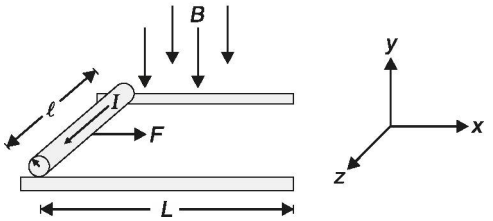
$$\Rightarrow \quad \vec{B}_0 = \frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k} + \frac{1}{\pi}\hat{j}\right)$$

$\Rightarrow$ $a = 3$, $b = 1$

$\Rightarrow$ $ab = 3$

25. The magnetic force acting on the rod is given by

$$\vec{F} = I(\vec{\ell}\times\vec{B}) = I\ell(\hat{k})\times B(-\hat{j}) = I\ell B(\hat{i})$$



From Work Energy Theorem we have

$$(K_{\text{trans}} + K_{\text{rot}})_{\text{initial}} + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_{\text{final}}$$

$$0 + 0 + Fs\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, \text{ where } I = \frac{1}{2}mR^2$$

$$\Rightarrow \quad I\ell BL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow \quad I\ell BL = \frac{3}{4}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{4I\ell BL}{3m}}$$

$$\Rightarrow \quad v = \sqrt{\frac{4(48\text{ A})(0.12\text{ m})(0.24\text{ T})(0.45\text{ m})}{3(0.72\text{ kg})}}$$

$$\Rightarrow \quad v = 1.07\text{ ms}^{-1}$$

$$\Rightarrow \quad v = 107\text{ cms}^{-1}$$

26. Here torque on the magnetic dipole because of magnetic force must be balanced by torque by weight

So, $\tau_{mg} = mg\sin\theta \times R$ (about point of contact)

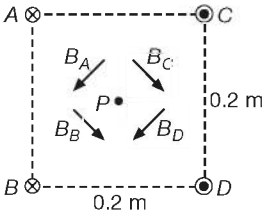
$$|\vec{M}\times\vec{B}| = mg\sin\theta\times R$$

$$\Rightarrow \quad i\times 2R\times L\times B\times\sin\theta = mg\sin\theta\times R$$

$$\Rightarrow \quad i = \frac{mg}{2BL} = \frac{1\times 10\times 2\times 10}{4\times 2\times 1\times 1} = \frac{100}{4} = 25\text{ A}$$

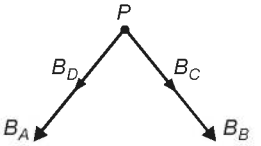
27. Distance of the point P from each of the wires is

$$r = \ell\cos(45°) = \frac{\ell}{\sqrt{2}}$$



Each wire produces a field at P of magnitude say B, then

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I\sqrt{2}}{2\pi\ell}$$

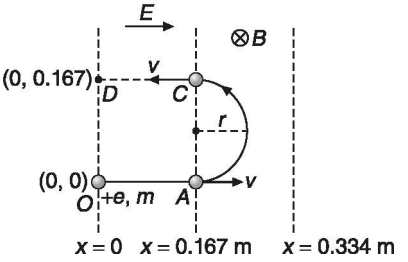


The field directions due to wires A, B, C and D are shown.

$$|\vec{B}_A| = |\vec{B}_B| = |\vec{B}_C| = |\vec{B}_D| = \frac{\sqrt{2}\mu_0 I}{2\pi\ell}$$

$B_P = 4B\sin(45°) = \dfrac{4\sqrt{2}\mu_0 I}{2\pi\ell} = 20\ \mu\text{T}$ toward the bottom of the page.

28. The situation described in the problem is shown in Figure.



Since the electric field is along x-axis, so proton will be accelerated by the electric field and will enter the magnetic field at A (i.e., $x = 0.167$, $y = 0$) with velocity v along x-axis such that

$$\frac{1}{2}mv^2 = W = q\Delta V = q(Ed)$$

$$\Rightarrow \quad v = \sqrt{\frac{2qEd}{m}}$$

$$\Rightarrow \quad v = \left[\frac{2\times 1.6\times 10^{-19}\times 100\times 0.167}{1.67\times 10^{-27}}\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad v = 4\sqrt{2}\times 10^4\text{ ms}^{-1}$$

Now as proton is moving perpendicular to magnetic field, so it will describe a circular path in the magnetic field with radius r such that

$$r = \frac{mv}{qB}$$

For proton to strike the point $(0,\ 0.167)$ m, we have $2r = 0.167$

$$\Rightarrow \quad 2\left(\frac{mv}{qB}\right) = 0.167$$

$$\Rightarrow \quad B = \frac{2 \times 1.67 \times 10^{-27} \times 4\sqrt{2} \times 10^{4}}{1.6 \times 10^{-19} \times 0.167}$$

$$\Rightarrow \quad B = \frac{1}{\sqrt{2}} \times 10^{-2}$$

$$\Rightarrow \quad B = 7.07 \text{ mT}$$

$$\Rightarrow \quad B \approx 7 \text{ millitesla}$$

29. From Ampere's Law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{(4\pi \times 10^{-7})(1 \text{ A})}{2\pi(1 \times 10^{-3} \text{ m})}$$

$$\Rightarrow \quad B_a = 200\ \mu\text{T}, \text{ towards top of page}$$

Similarly at point $b: B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1 \text{ A} - 3 \text{ A} = -2$ A

$$\Rightarrow \quad I_b = 2 \text{ A into the page}$$

$$\Rightarrow \quad B_b = \frac{(4\pi \times 10^{-7})(2 \text{ A})}{2\pi(3 \times 10^{-3} \text{ m})}$$

$$\Rightarrow \quad B_b = 133\ \mu\text{T}, \text{ towards bottom of page}$$

30. Using Ampere's law, we have

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 \int J dA, \text{ where } dA = 2\pi r dr$$

For $r > R$, we have

$$B\int dl = \mu_0 \int_0^R (br)(2\pi r dr)$$

$$B(2\pi r) = \frac{2\pi\mu_0 bR^3}{3}$$

$$\Rightarrow \quad B = \frac{\mu_0 bR^3}{3r}$$

$$\Rightarrow \quad x = 3 \text{ and } y = 3$$

$$\Rightarrow \quad \frac{x}{y} = 1$$

31. At equilibrium, we have

$$\frac{F}{\ell} = \frac{\mu_0 I_A I_B}{2\pi r} = \frac{mg}{\ell}$$

$$\Rightarrow \quad \frac{\mu_0 I_A I_B}{2\pi r} = \left(\frac{m}{\ell}\right) g$$

where $\frac{m}{\ell} = 0.15 \text{ gcm}^{-1} = \left(0.15 \times \frac{100}{1000}\right) \text{kgm}^{-1}$

$$\Rightarrow \quad \frac{(4\pi \times 10^{-7})(100)(I)}{(2\pi)\left(\frac{2}{100}\right)} = \left(0.15 \times \frac{100}{1000}\right)(9.8)$$

$$\Rightarrow \quad 10^{-3} I = 0.147$$

$$\Rightarrow \quad I = 0.147 \times 10^{3}$$

$$\Rightarrow \quad I = 147 \text{ A}$$

32. (a) $|\vec{\tau}| = |\vec{M} \times \vec{B}| = NBIA$

$$\Rightarrow \quad \tau_{\max} = 1000(10^{-2})(0.025 \times 0.04)(0.8)\sin 90°$$

$$\Rightarrow \quad \tau_{\max} = 8 \times 10^{-3} \text{ Nm} = 8 \text{ mNm}$$

(b) $\mathbf{P}_{\max} = \tau_{\max}\omega = (8 \times 10^{-3})\left(3600 \times \frac{2\pi}{60}\right) = 3$ W

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -MB\cos 180° - (-MB\cos 0°)$$

$$\Rightarrow \quad W = 2MB$$

$$\Rightarrow \quad W = 2NBIA = 2(8 \times 10^{-3}) = 16 \times 10^{-3} \text{ J}$$

In one full revolution

$$W = 2(16 \times 10^{-3} \text{ J}) = 32 \times 10^{-3} \text{ J}$$

$$\Rightarrow \quad W = 32 \text{ mJ}$$

(d) $\mathbf{P}_{\text{avg}} = \frac{W}{t} = \frac{32 \times 10^{-3} \text{ J}}{\left(\frac{1}{60}\right) \text{s}}$

$$\Rightarrow \quad \mathbf{P}_{\text{avg}} = 1920 \times 10^{-3} \text{ W} = 1920 \text{ mW}$$

(e) $\frac{\text{Peak Power}}{\text{Average Power}} = \frac{3}{1.92} = \frac{300}{192} = \frac{100}{64} = \frac{25}{16} = \frac{(5)^2}{16}$

So, $* = 5$

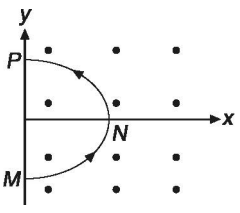
33. At $x = 0$,

$$y = \pm 2m$$

Since, $\vec{F}_{\text{MNP}} = \vec{F}_{\text{MP}} = i[\overrightarrow{MP} \times \vec{B}]$

$$\Rightarrow \quad \vec{F}_{\text{MNP}} = 3\left[\left(4\hat{j}\right) \times \left(5\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_{\text{MNP}} = 60\hat{i}$$



CHAPTER 1

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1. $F = -kx - IlB = -kx - \left(\frac{Blv}{R}\right)(lB)$

$\Rightarrow \quad F = -kx - \left(\frac{B^2l^2}{R}\right)v$

So, it is case of damped oscillation, hence

$$A = A_0 e^{-\frac{bt}{2m}}$$

Given that $A = \frac{A_0}{e}$

$\Rightarrow \quad \frac{A_0}{e} = A_0 e^{-\frac{bt}{2m}}$

$\Rightarrow \quad t = \frac{2m}{\left(\frac{B^2l^2}{R}\right)} = \frac{2 \times 50 \times 10^{-3} \times 10}{0.01 \times 0.01} = 10000 \text{ s}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} \approx 2 \text{ s}$

So, the number N is

$$N = \frac{10000}{2} = 5000$$

Hence, the correct answer is (B).

2. $|\vec{\tau}| = |\vec{\mu} \times \vec{B}|$

$\Rightarrow \quad \tau = (NIA)(B)$

Since, $A = \pi r^2$

$\Rightarrow \quad \tau = NI(\pi r^2)B$

Hence, the correct answer is (C).

3. $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi d}(\hat{k} - \hat{k}) = 0$

Hence, the correct answer is (A).

4. The force on loop is given by

$$F = I_2 a(B_1 - B_2)$$

where

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I_1}{4\pi a}$$

$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}$

Hence, the correct answer is (A).

5. $NBIA = C\theta$

$\Rightarrow \quad (175)B(1 \times 10^{-3})(1 \times 10^{-4}) = \frac{10^{-6} \times \pi}{180}$

$\Rightarrow \quad B = \frac{\pi}{180} \times \frac{10}{175} \approx 9.97 \times 10^{-4} \text{ T}$

$\Rightarrow \quad B = 10^{-3} \text{ T}$

Hence, the correct answer is (D).

6. $\Sigma F = 0$

$\Rightarrow \quad \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x - d)}$

As both wires A and B carry currents in opposite directions, so $x > d$

$\Rightarrow \quad I_1 x - I_1 d = I_2 x$

$\Rightarrow \quad x = \frac{I_1 d}{I_1 - I_2}$

Hence, the correct answer is (A).

7. Radius of circular path (r) in a perpendicular uniform magnetic field is $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

For proton, electron and helium nucleus i.e. α-particle, we have

$m_\alpha = 4m_p$ and $m_p \gg m_e$

Also, $q_\alpha = 2q_p$ and $q_p = q_e$

Since kinetic energy (KE) of all the particles is same, so

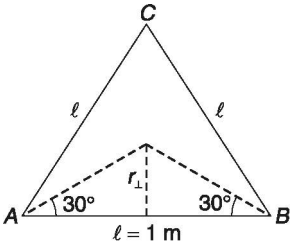
$$r \propto \frac{\sqrt{m}}{q}$$

$\Rightarrow \quad r_\alpha = r_p > r_e$

Hence, the correct answer is (C).

8. The magnetic field due to wire AB is

$$B_{AB} = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$$



where

$$\tan 30° = \frac{r_\perp}{l/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_\perp}{l/2}$$

$$\Rightarrow r_\perp = \frac{l}{2\sqrt{3}}$$

$$\Rightarrow r_\perp = \frac{1}{2\sqrt{3}} \text{ m}$$

Also, $\theta_1 = \theta_2 = 60°$

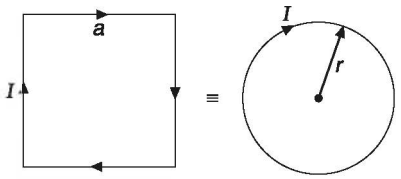
$$\Rightarrow B_{AB} = \frac{\mu_0 I}{4\pi r_\perp} 2\sin(60°) = \frac{\mu_0 I}{4\pi r_\perp}\sqrt{3}$$

$$\Rightarrow B_{AB} = \frac{6\mu_0 I}{4\pi}$$

$$\Rightarrow B_{\text{total}} = 3B_{AB} = \frac{18\mu_0 I}{4\pi} = 18\ \mu\text{T}$$

Hence, the correct answer is (A).

9. The square loop is changed to a circular loop. So, we have



$$2\pi r = 4a$$

$$\Rightarrow r = \left(\frac{2a}{\pi}\right)$$

For Square loop,

$$m = Ia^2$$

For Circular loop,

$$m' = I(\pi r^2)$$

$$\Rightarrow m' = I\pi\left(\frac{4a^2}{\pi^2}\right)$$

$$\Rightarrow m' = \frac{4Ia^2}{\pi}$$

$$\Rightarrow m' = \frac{4m}{\pi}$$

Hence, the correct answer is (B).

10. $B = \frac{\mu_0 I}{2a}$ and $I = \frac{q\omega}{2\pi}$

$$\Rightarrow B = \left(\frac{\mu_0}{2a}\right)\left(\frac{q\omega}{2\pi}\right)$$

$$\Rightarrow B = \frac{(10^{-7})(40)}{0.1}q\pi$$

$$\Rightarrow q = 3\times10^{-5}\ \text{C}$$

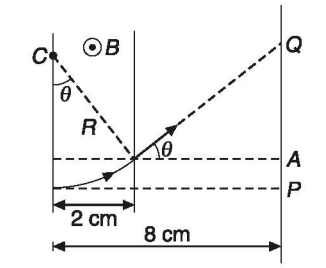
Hence, the correct answer is (B).

11. Radius of path R is given by

$$R = \frac{mv}{eB} = \frac{\sqrt{2m(KE)}}{eB}$$

$$\Rightarrow R = \frac{\sqrt{2\times9.1\times10^{-31}\times100e}}{e\times1.5\times10^{-3}}$$

$$\Rightarrow R = \frac{\sqrt{2\times9.1\times10^{-29}}}{\sqrt{1.6\times10^{-19}}\times1.5\times10^{-3}}\ \text{m}$$



$$\Rightarrow R = \frac{3.37\times10^{-5}}{1.5\times10^{-3}}\times100\ \text{cm} = 2.25\ \text{cm}$$

Since, $\sin\theta = \frac{2}{2.25} = \frac{8}{9}$

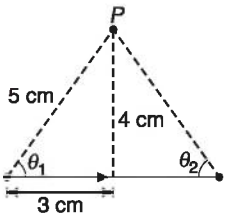
$$\Rightarrow PQ = PA + AQ$$

$$\Rightarrow PQ = 2.25(1-\cos\theta) + 11.64$$

$$\Rightarrow PQ = 1.22 + 11.64 = 12.86\ \text{cm}$$

Hence, the correct answer is (B).

12.



$$B_P = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

$$\Rightarrow B_P = \frac{(10^{-7})(5)}{0.04}\left(2\times\frac{3}{5}\right) = 1.5\times10^{-5}\ \text{T}$$

Hence, the correct answer is (B).

13. For shifting of loop along x-direction, the potential energy is

$$U(x) = -\vec{M}\cdot\vec{B}$$

$$\Rightarrow U(x) = -[(\pi a^2)i]\left(\frac{\mu_0 I_0}{2\pi x}\right)$$

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$$\Rightarrow \quad U(x) = -\frac{\mu_0 i I_0 a^2}{2x}$$

PE decreases as it comes closer to wire.

So, attractive force $F(x)$ is given by

$$F(x) = -\frac{dU}{dx} = \frac{\mu_0 i I_0 a^2}{2}\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \quad F(x) = \frac{\mu_0 i I_0 a^2}{2d^2} \text{ (Attractive)}$$

$$\Rightarrow \quad F \propto \frac{a^2}{d^2}$$

Hence, the correct answer is (B).

14. $B = \frac{\mu_0 I}{4\pi R}\theta$ where $\theta = \frac{\pi}{4}$ radian

$$\Rightarrow \quad B = \frac{\mu_0 I}{16R}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{16}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{\mu_0 I}{120}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{120} = 1.047 \times 10^{-7} \approx 1.0 \times 10^{-7} \text{ T}$$

Hence, the correct answer is (D).

15. Since $2\pi r_1 = L$

$$\Rightarrow \quad r_1 = \frac{L}{2\pi}$$

Also, magnetic field at centre of loop is

$$B_L = \frac{\mu_0 I}{2r_1}$$

Now, $L = (2\pi r_2)N$

$$\Rightarrow \quad r_2 = \frac{L}{2N\pi}$$

Magnetic field at centre of coil is

$$B_C = \frac{N\mu_0 I}{2r_2}$$

$$\Rightarrow \quad \frac{B_L}{B_C} = \frac{1}{N^2}$$

Hence, the correct answer is (D).

16. For particle to move in a straight line, we have

$eE = evB$

Since $R = \frac{mv}{eB}$

$$\Rightarrow \quad v = \frac{eBR}{m}$$

$$\Rightarrow \quad E = \left(\frac{eBR}{m}\right)B$$

$$\Rightarrow \quad m = \frac{eB^2R}{E}$$

$$\Rightarrow \quad m = \frac{1.6 \times 10^{-19} \times (0.5)^2 \times 0.5 \times 10^{-2}}{100}$$

$$\Rightarrow \quad m = 2.0 \times 10^{-24} \text{ kg}$$

Hence, the correct answer is (D).

17. Since, $E = vB$

$$\Rightarrow \quad E = 0.6 \text{ Vm}^{-1}$$

Since, $V = Ed$

$$\Rightarrow \quad V = 0.6 \times 2 \times 10^{-2}$$

$$\Rightarrow \quad V = 12 \text{ mV}$$

Hence, the correct answer is (A).

18. Since, $r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{Bq}$

$$\Rightarrow \quad r = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times V}}{B\sqrt{q}}$$

$$\Rightarrow \quad r = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 500}}{100 \times 10^{-3}\sqrt{1.6 \times 10^{-19}}}$$

$$\Rightarrow \quad r = \frac{1}{100 \times 10^{-3}} \frac{\sqrt{2 \times 9.1 \times 500 \times 10^{-12}}}{1.6}$$

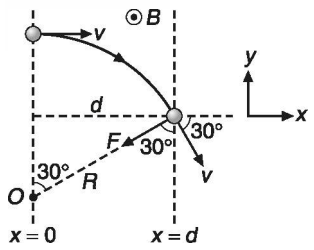
$$\Rightarrow \quad r = \frac{75.4 \times 10^{-6}}{100 \times 10^{-3}} = 7.5 \times 10^{-4} \text{ m}$$

Hence, the correct answer is (D).

19. ***The data given in the question is incorrect.***

However, if the magnetic field $\vec{B} = B\hat{k}$ given in the question would have been between $x = 0$ and $x = d$ instead of $y = 0$ and $y = d$, then we solve the question as given below but we still do not get the matching answer.

So, let us solve the problem assuming that the particle enters the magnetic field that exists in the region $x = 0$ to $x = d$. On entering the magnetic field, the charge particle follows a circular path of radius $R = \frac{mv}{qB}$ as shown in Figure.



Deviation suffered by the charged particle when it leaves the field at $x = d$ is

$$\sin\theta = \frac{d}{R} = \frac{mv/2qB}{mv/qB} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 30°$

At the point of leaving the field, force acting on the particle is towards the centre of the circle i.e. O and has a magnitude given by

$$F = qvB$$

This force is making an angle of $30°$ with the vertical, so

$$\vec{F} = -qvB\left(\sin 30°\hat{i} + \cos 30°\hat{j}\right)$$

$$\Rightarrow \quad \vec{F} = -\frac{qvB}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = -\frac{qvB}{2m}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

*So, none of the OPTIONS given is correct.

20. The straight path from origin to $P(x = 1,\ y = 1)$ is $y = x$

Since work is only done by electric force, so we have

$$W = q\int \vec{E}\cdot d\vec{r} = q\int_0^1 2dx + q3\int_0^1 dy$$

$\Rightarrow \quad W = 2q + 3q = 5q$

Hence, the correct answer is (B).

21. Since, $r = \frac{mv}{qB} = \frac{\sqrt{2m\times(qV)}}{qB}$

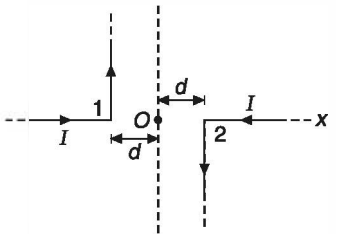
$\Rightarrow \quad r \propto \sqrt{\frac{m}{q}}$

$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}\times\frac{q_\alpha}{q_p}}$

$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{1}{4}\times\frac{2}{1}} = \frac{1}{\sqrt{2}}$

Hence, the correct answer is (D).

22. At O, $\vec{B} = \vec{B}_1 + \vec{B}_2$



where, $\vec{B}_1 = \vec{B}_2 = \frac{\mu_0 I}{4\pi d}$

$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi d} = 10^{-4}$

$\Rightarrow \quad \frac{2\times10^{-7}\times I}{4\times10^{-2}} = 10^{-4}$

$\Rightarrow \quad I = \frac{2}{10^{-1}} = 20\ \text{A}$

Hence, the correct answer is (B).

23. Radius of circular path followed by a charged particle in a uniform magnetic field (B) is given by

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB}$$

For electron, $r_e = \frac{\sqrt{2m_eK}}{eB}$

For proton, $r_p = \frac{\sqrt{2m_pK}}{eB}$

For α particle, $r_\alpha = \frac{\sqrt{2m_\alpha K}}{2eB} = \frac{\sqrt{2\left(4m_pK\right)}}{2eB} = \frac{\sqrt{2m_pK}}{eB}$

Since, $m_p > m_e$, hence $r_\alpha = r_p > r_e$

Hence, the correct answer is (B).

24. Initially, the dipole moment of circular loop is

$$M = IA = I(\pi R^2)$$

and magnetic field is

$$B_1 = \frac{\mu_0 I}{2R}$$

Finally, the dipole moment becomes double, keeping the current constant, so radius of the loop must have become $\sqrt{2}R$. Hence the magnetic field is given is

$$B_2 = \frac{\mu_0 I}{2(\sqrt{2}R)} = \frac{B_1}{\sqrt{2}}$$

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$$\Rightarrow \quad \frac{B_1}{B_2} = \sqrt{2}$$

Hence, the correct answer is (C).

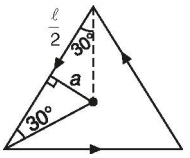
25. Required magnetic field is given by

$$B = 2\left(\frac{\mu_0 NIR^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}}\right) = \frac{\mu_0 NIR^2}{\frac{5^{\frac{3}{2}}}{8}} = \frac{8\mu_0 NI}{5^{\frac{3}{2}} R}$$

Hence, the correct answer is (C).

26. The magnetic field at the centre of the triangle is

$$B_{\text{net}} = 3\left[\frac{\mu_0 I}{4\pi a}(\cos 30° + \cos 30°)\right]$$



Since, from the figure, we see that

$$\tan 30° = \frac{a}{\left(\frac{l}{2}\right)}$$

$$\Rightarrow \quad a = \frac{l}{2}\tan 30° = \frac{l}{2\sqrt{3}}$$

$$\Rightarrow \quad B_{\text{net}} = \frac{3\times(10^{-7})\times 1}{(4.5\times 10^{-2})}\left(2\times\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad B_{\text{net}} = \frac{2\times 9\times 10^{-5}}{4.5} = 4\times 10^{-5}\ \text{Wbm}^{-2}$$

Hence, the correct answer is (A).

27. Magnetic moment is given by $M = iA = \frac{q}{T}(\pi r^2)$

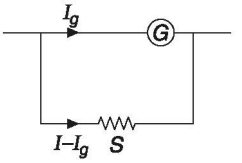
Also, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad M = \frac{q}{\left(\frac{2\pi}{\omega}\right)}(\pi r^2)$$

$$\Rightarrow \quad M = \frac{1}{2}q\omega r^2$$

Hence, the correct answer is (A).

28. Since we know that



$$I_g G = (I - I_g)S$$

As the current through galvanometer is very small, so we have

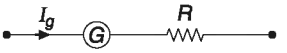
$$S \approx \frac{10^{-3}\times 25}{2}$$

$$\Rightarrow \quad S \approx 12.5\times 10^{-3} = 1.25\times 10^{-2}\ \Omega$$

Hence, the correct answer is (B).

29. Given that $I_g = 5$ mA , $G = 15\ \Omega$

Let R be the resistance to be connected in series with the galvanometer as shown in Figure.



Since $V = I_g(R+G)$

$$\Rightarrow \quad 10 = 5\times 10^{-3}(R+15)$$

$$\Rightarrow \quad 2000 = R + 15$$

$$\Rightarrow \quad R = 1985\ \Omega = 1.985\times 10^3\ \Omega$$

Hence, the correct answer is (A).

30. Here, $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$, $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$

Since, $\vec{F} = \vec{F}_e + \vec{F}_m = \vec{0}$

$$\Rightarrow \quad \vec{F}_e = -\vec{F}_m$$

$$\Rightarrow \quad \vec{F}_e = -q(\vec{v}\times\vec{B})$$

$$\Rightarrow \quad \vec{F}_e = -qv_0B_0\left[(3\hat{i} - \hat{j} + 2\hat{k})\times(\hat{i} + 2\hat{j} - 4\hat{k})\right]$$

$$\Rightarrow \quad \vec{F}_e = -qv_0B_0(14\hat{j} + 7\hat{k})$$

The electric field produced by the charge q, will be given by

$$\vec{E} = \frac{\vec{F}_e}{q} = -\frac{qv_0B_0(14\hat{j} + 7\hat{k})}{q}$$

$$\Rightarrow \quad \vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$$

Hence, the correct answer is (A).

31. When a magnetic dipole of dipole moment M is placed in a uniform magnetic field, it will experience a torque,

$$\tau = MB\sin\theta$$

Torque is maximum when $\theta = 90°$

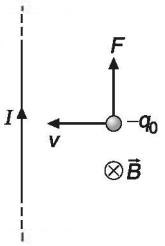
$$\tau_{max} = MB\sin 90° = MB$$

Potential Energy of a magnetic dipole in a uniform magnetic field is,

$$U = -MB\cos\theta = -MB\cos 90° = 0$$

Hence, the correct answer is (A).

32. Given situation is shown in Figure.



Since we know that

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$\Rightarrow \quad \vec{F} = -q_0(\vec{v}\times\vec{B})$$

According to the problem, force acting on the electron is parallel to the direction of current. Hence the motion of test charge is towards the wire.

Hence, the correct answer is (D).

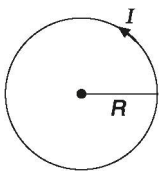
33. Magnetic potential energy of the dipole in a magnetic field is

$$U = -\vec{M}\cdot\vec{B}$$

For stable equilibrium, M and B must be parallel to each other and in this case the potential energy should be minimum.

Hence, the correct answer is (D).

34. Wire A is bent into a circle of radius R as shown, then

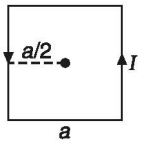


$$l = 2\pi R$$

$$\Rightarrow \quad R = \frac{l}{2\pi}$$

$$\Rightarrow \quad B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\times\left(\frac{l}{2\pi}\right)} = \frac{\mu_0\pi I}{l}$$

Wire B is bent into a square of side a as shown, then



$$l = 4a$$

$$\Rightarrow \quad a = \frac{l}{4}$$

$$\Rightarrow \quad B_B = 4\times\left[\frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}(\sin 45° + \sin 45°)\right]$$

$$\Rightarrow \quad B_B = \frac{2\mu_0 I}{\pi a}\times\frac{2}{\sqrt{2}} = \frac{16\mu_0 I}{\sqrt{2}\pi l}$$

$$\Rightarrow \quad \frac{B_A}{B_B} = \frac{\mu_0\pi\left(\frac{I}{l}\right)}{16\mu_0\left(\frac{I}{\sqrt{2}\pi l}\right)} = \frac{\pi^2}{8\sqrt{2}}$$

Hence, the correct answer is (D).

35. Given that $i_g = 1$ mA, $G = 100\ \Omega$, $i = 10$ A, $S = ?$

Since, $(i - i_g)S = i_g G$

$$\Rightarrow \quad S = \frac{i_g G}{i - i_g} = \frac{1\times10^{-3}\times100}{(10 - 10^{-3})}$$

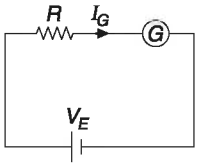
$$\Rightarrow \quad S \approx 10^{-2}\ \Omega$$

$$\Rightarrow \quad S = 0.01\ \Omega$$

Hence, the correct answer is (A).

36. **CASE-1:**

This situation is shown in Figure.

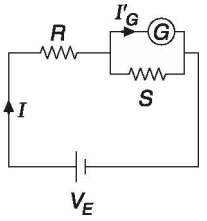


$$I_G = \frac{V_E}{R+G} \qquad \ldots(1)$$

CASE-2:

In this case a shunt is connected across the galvanometer as shown in Figure.

CHAPTER 1



The current in the circuit is given by

$$I = \frac{V_E}{R + \frac{GS}{G+S}} \qquad \ldots(2)$$

According to the problem,

$$I'_G = \frac{I_G}{2} = \frac{IS}{G+S} \qquad \ldots(3)$$

From (1), (2) and (3), we get

$$\frac{V_E}{2(R+G)} = \frac{V_E}{R + \frac{GS}{G+S}} \times \frac{S}{G+S}$$

$$\Rightarrow \quad \frac{1}{2(R+G)} = \frac{(G+S)}{(RG+RS+GS)} \times \frac{S}{(G+S)}$$

$$\Rightarrow \quad RG + RS + GS = 2RS + 2GS$$

$$\Rightarrow \quad RS + GS = RG$$

$$\Rightarrow \quad S(R+G) = RG$$

Hence, the correct answer is (A).

37. Current in the circuit without ammeter is

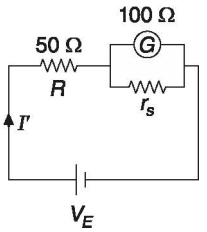
$$I = \frac{V}{R} = \frac{5\text{ V}}{50\ \Omega} = 0.1\text{ A}$$

So, the allowed current with ammeter is

$$I' = 0.099\text{ A}$$

Also, $I' = \frac{V}{R_{eq}}$ where $R_{eq} = 50 + \frac{100 r_s}{100 + r_s}$

$$\Rightarrow \quad 0.099 = \frac{5}{50 + \frac{100 r_s}{100 + r_s}}$$



$$\Rightarrow \quad 50 + \frac{100 r_s}{100 + r_s} = \frac{5}{0.099}$$

$$\Rightarrow \quad \frac{100 r_s}{100 + r_s} = 0.5$$

$$\Rightarrow \quad 100 r_s = 50 + 0.5 r_s$$

$$\Rightarrow \quad r_s = \frac{50}{99.5} = 0.5\ \Omega$$

Hence, the correct answer is (D).

38. Magnetic force on electron in the metal sheet,

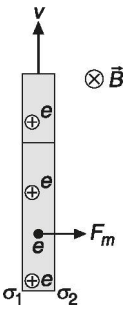
$$\vec{F}_m = -e(\vec{v} \times \vec{B})$$

At equilibrium,

$$F_m = F_e = eE \text{ (induced)}$$

Since $\sigma_2 = -\sigma_1$ and assuming $\sigma_1 = \sigma$, we get

$$evB = e\left(\frac{\sigma}{\varepsilon_0}\right)$$

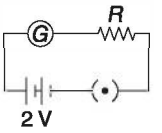


$$\Rightarrow \quad \sigma = \sigma_1 = \varepsilon_0 vB$$

$$\Rightarrow \quad \sigma_2 = -\sigma_1 = -\varepsilon_0 vB$$

Hence, the correct answer is (B).

39. The circuit given in the question is incorrect. The given figure shows the correct circuit.



Let the current which produces full scale deflection in the galvanometer be I_g.

Then according to question

$$\frac{4}{5} I_g = \frac{V}{G+R} = \frac{2}{G+2400} \qquad \ldots(1)$$

$$\frac{2}{5} \mathrm{I}_g = \frac{2}{G+4900} \qquad \ldots(2)$$

From equations (1) and (2)

$$\frac{4}{2}=\frac{G+4900}{G+2400}$$

$\Rightarrow \quad G=100\ \Omega$

Putting G in equation (1)

$$\frac{4}{5}I_g=\frac{2}{100+2400}$$

$$\Rightarrow \quad I_g=\frac{2\times5}{4\times2500}=1\text{ mA}$$

For a deflection of 10 divisions

$$\frac{1}{5}I_g=\frac{V}{G+R}$$

$$\Rightarrow \quad \frac{1}{5}\times10^{-3}=\frac{2}{100+R}$$

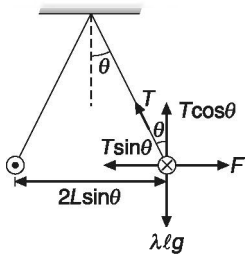
$\Rightarrow \quad R=9900\ \Omega$

Now, current sensitivity is

$$\frac{I_g}{n}=\frac{1\text{ mA}}{50\text{ div}}=20\ \mu\text{A/division}$$

Hence, the correct answer is (A).

40. Let the length of right wire be ℓ, then its mass is $\lambda\ell$.



Forces acting on this wire are tension (T), weight $(\lambda\ell g)$ and force of repulsion due to other wire (F).

From figure, we get

$$T\cos\theta=\lambda\ell g \qquad ...(1)$$

$$T\sin\theta=F \qquad ...(2)$$

where, $F=\dfrac{\mu_0}{2\pi}\dfrac{I^2\ell}{(2L\sin\theta)}$

From (2), we get

$$T\sin\theta=\frac{\mu_0}{2\pi}\frac{I^2\ell}{(2L\sin\theta)}$$

From (1), we get

$$\Rightarrow \quad \frac{\lambda\ell g}{\cos\theta}\sin\theta=\frac{\mu_0}{2\pi}\frac{I^2\ell}{(2L\sin\theta)}$$

$$\Rightarrow \quad I=\sqrt{\frac{4\pi L\lambda g\sin^2\theta}{\mu_0\cos\theta}}=2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

Hence, the correct answer is (D).

41. Since, $I=12$ A, $\vec{B}=0.3\hat{k}$ T

Also, $A=10\times5\text{ cm}^2=50\times10^{-4}\text{ m}^2$

$$\Rightarrow \quad \vec{M}=(IA)\hat{n}=12\times50\times10^{-4}\hat{n}\text{ Am}^2$$

$$\Rightarrow \quad M=6\times10^{-2}\text{ Am}^2$$

Here, $\vec{M}_1=6\times10^{-2}\hat{i}\text{ Am}^2$, $\vec{M}_2=6\times10^{-2}\hat{k}\text{ Am}^2$

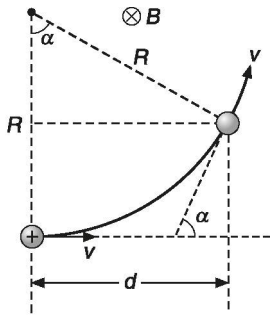
$$\vec{M}_3=-6\times10^{-2}\hat{j}\text{ Am}^2,\ \vec{M}_4=-6\times10^{-2}\hat{k}\text{ Am}^2$$

Since $\vec{M}_2$ is parallel to $\vec{B}$, it means potential energy is minimum, therefore in orientation (ii) the loop is in stable equilibrium. $\vec{M}_4$ is antiparallel to $\vec{B}$, it means potential energy is maximum, therefore in orientation (iv) the loop is in unstable equilibrium.

Hence, the correct answer is (A).

43. Energy of proton is $\dfrac{1}{2}mv^2=qV$

$$\Rightarrow \quad v=\sqrt{\frac{2qV}{m}}$$



Magnetic force, $qvB\sin90°=\dfrac{mv^2}{R}$

$$\Rightarrow \quad R=\frac{mv}{qB}$$

Since, $\sin\alpha=\dfrac{d}{R}=\dfrac{qBd}{mv}=\dfrac{qBd}{m}\sqrt{\dfrac{m}{2qV}}$

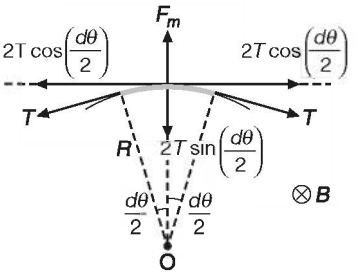
$$\Rightarrow \quad \sin\alpha=Bd\sqrt{\frac{q}{2mV}}$$

Hence, the correct answer is (C).

44. When the currents are parallel, $I_1 I_2$ is positive and the force between them is attractive (i.e. negative). Similarly, when currents are antiparallel, $I_1 I_2$ is negative and the force between them is repulsive (i.e. positive). So, option (D) satisfies the condition.

Hence, the correct answer is (D).

45.



Magnetic force on the circular arc is $F_m = BId\ell$

$$F_m = BI(Rd\theta)$$

For the arc to be in equilibrium, $F = 2T\sin\left(\frac{d\theta}{2}\right)$

For small angle $d\theta$, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$\Rightarrow \quad 2Td\theta = BIRd\theta$$

$$\Rightarrow \quad T = BIR$$

Hence, the correct answer is (A).

46. Force on conductor, 0.325 m

$+y$ $a_y = -9.8\ \text{ms}^{-2}$

$y-$ $v_y = 0$ along $v_{0y} = ?$

Work done v_y^2 on the conductor in moving along $y-$ is

$$v_{0y} = 2.52\ \text{ms}^{-1} \simeq 2.5\ \text{ms}^{-1}$$

$+y$

$v_{0y} = 0$

$y-$

So, average power is $v_y = +2.52$ m

a_y :

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\sum F_y = ma_y$$

Hence, the correct answer is (C).

48. The radius of the circular path of a charged particle in the magnetic field is given by $r = \frac{mv}{Bq}$

Kinetic energy of a charged particle is

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

$$\Rightarrow \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Since K and B are constants, so we have

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

Hence, the correct answer is (A).

49. $dB = \frac{\mu_0(dq)}{2r}\left(\frac{\omega}{2\pi}\right)$

$$\Rightarrow \quad B = \int dB = \frac{\mu_0\omega}{4\pi}\left(\frac{Q}{\pi R^2}\right)2\pi\int_0^R \frac{rdr}{r}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0\omega Q}{2\pi R^2}\right)R = \frac{\mu_0\omega Q}{2\pi R}$$

$$\Rightarrow \quad B \propto \frac{1}{R}$$

Hence, the correct answer is (D).

50. Let the current I in the wire be inwards then

$$\lambda = \frac{l}{\pi R}$$

If dI be an infinitesimal current due to an element subtending angle $d\theta$ at centre, then

$$dI = \lambda d\ell = \lambda(Rd\theta)$$

Magnetic field dB due to this infinitesimal element is

$$dB = \frac{\mu_0 dI}{2\pi R}$$

$$B_x = \int dB_x = \int dB\sin\theta$$

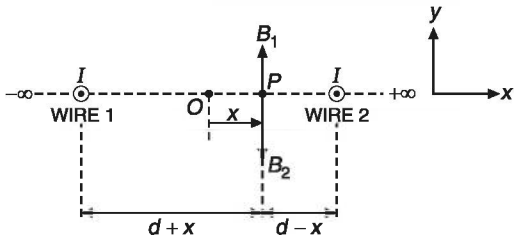
$$\Rightarrow \quad B_x = \frac{\mu_0 \lambda}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta d\theta = 0$$

$$\text{and } B_y = \frac{\mu_0 \pi}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta = \frac{\mu_0 \lambda}{\pi} = \frac{\mu_0 I}{\pi^2 R}$$

Hence, the correct answer is (A).

51. Taking upward direction as positive, the magnetic field at a point P between the wires at point P to the right of origin at a distance x from O is

$$B_p = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d-x} - \frac{1}{d+x}\right)$$



The field will remain negative as we move from $-\infty$ to Wire 1, has a zero value at ∞ and becomes very large near the Wire 1.

As we move from Wire 1 to Wire 2 the field changes sign from positive to negative, becoming zero at O and then again increasing as we approach Wire 2.

As we move from Wire 2 to $+\infty$, field is positive everywhere, very large near Wire 2 and then decreases to zero at $+\infty$.

Hence, the correct answer is (B).

CHAPTER 1

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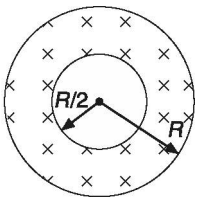
Single Correct Choice Type Problems

1.



Let r be the distance of a point from centre, then

For $r \le \frac{R}{2}$, using Ampere's circuital law,

$$\oint \vec{B}\cdot d\vec{l}$$

$$\Rightarrow \quad Bl = \mu_0 (I_{in})$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 (I_{in})$$

$$\Rightarrow \quad B = \frac{\mu_0}{2\pi}\frac{I_{in}}{r} \qquad \ldots(1)$$

Since, $I_{in} = 0$

$$\Rightarrow \quad B = 0$$

If J be the current per unit area, then

for $\frac{R}{2} \le r \le R$, we have

$$I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]J$$

Substituting in equation (1), we have

$$B = \frac{\mu_0}{2\pi}\frac{\left(\pi r^2 - \pi\frac{R^2}{4}\right)J}{r}$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $r = \frac{R}{2}$, $B = 0$

At $r = R$, $B = \frac{3\mu_0 JR}{8}$

For $r \ge R$, we have

$$I_{in} = I_{Total} = I(\text{say})$$

Therefore, substituting in equation (1), we have

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \quad B \propto \frac{1}{r}$$

Hence, the correct answer is (D).

2. Area A of the given loop is equal to area of two circles of radius $\frac{a}{2}$ and area of a square of side a

$$\Rightarrow \quad A = 2\pi\left(\frac{a}{2}\right)^2 + a^2 = \left(\frac{\pi}{2}+1\right)a^2$$

Since, $|\vec{M}| = IA = \left(\frac{\pi}{2}+1\right)a^2 I$

From Right Hand Thumb Rule, direction of $\vec{M}$ is outwards or in positive z-direction.

$$\Rightarrow \quad \vec{M} = \left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$$

Hence, the correct answer is (B).

3. The radial width $(b-a)$ is having N turns. So, number of turns per unit radial width is $n = \frac{N}{b-a}$.

Consider a circular coil of radius x, radial thickness dx. If dN is the number of turns in it, then we have

$$dN = \frac{N dx}{b-a}$$

If dB is the field due to this element at the centre, then

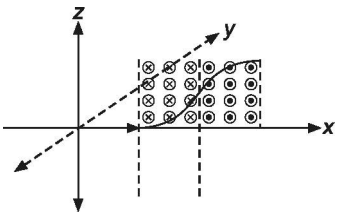
$$dB = \frac{\mu_0 NI\, dx}{2(b-a)x}$$

$$\Rightarrow \quad B = \int_a^b dB = \frac{\mu_0 NI}{2(b-a)}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (A).

4. When $a < x < 2a$

$\vec{F}_m$ acts along positive z-direction



When $2a < x < 3a$

$\vec{F}_m$ acts along negative z-direction.

Hence, the correct answer is (A).

5. Magnetic force does not change the speed of charged particle. Hence $v = u$. Further magnetic field in the given condition is along negative z-axis in the starting. Or it describes a circular path in clockwise direction. The direction is found by using Fleming's Left Hand Rule. Hence when it exits from the field, $y < 0$.

Hence, the correct answer is (D).

6. $U = -\vec{M}\cdot\vec{B}$

Hence, the correct answer is (C).

7. Because of magnetic force acting radially outwards on the loop, it has a tendency to expand.

Hence, the correct answer is (B).

8. Plane of motion must be perpendicular to at least one of the component of the magnetic field.

Hence, the correct answer is (B).

9. $r > (b-a)$

$$\Rightarrow \quad \frac{mv}{qB} > (b-a)$$

$$\Rightarrow \quad v > \frac{q(b-a)B}{m}$$

$$\Rightarrow \quad v_{\min} = \frac{q(b-a)B}{m}$$

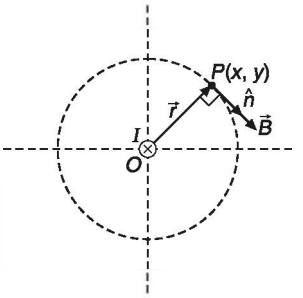
Hence, the correct answer is (B).

10. Outside a bar magnet, field lines go from N to S, whereas inside a magnet they go from S to N. Also, we note that, magnetic field lines can form closed loops.

Hence, the correct answer is (D).

11. In magnitude, field at point $P(x, y)$ is

$$B = \frac{\mu_0 I}{2\pi r}$$



However, vectorially the field is given by

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi r}\right)\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}$.

In this case, $\vec{r} = x\hat{i} + y\hat{j}$

A unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}$ is given by

$$\hat{n} = \frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$$

Because only then, we have $\vec{r}\cdot\hat{n} = 0$

$$\Rightarrow \quad \vec{B} = \left(\frac{\mu_0 I}{2\pi\sqrt{x^2+y^2}}\right)\left(\frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i} - x\hat{j}}{x^2+y^2}\right)$$

Hence, the correct answer is (A).

12. Since $r_A > r_B$

$$\Rightarrow \quad m_A v_A > m_B v_B \qquad \left\{\because r = \frac{mv}{qB}\right\}$$

Hence, the correct answer is (B).

13. The radial width $(b-a)$ of the coil is having N turns.

So, number of turns per unit radial width is $n = \dfrac{N}{b-a}$.

Consider a circular coil of radius x, radial thickness dx. If dN is the number of turns in it, then we have

$$dN = \frac{N dx}{b-a}$$

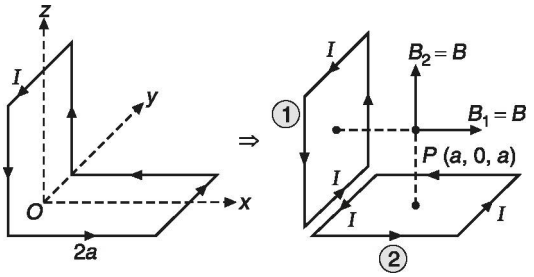
If dB is the field due to this element at the centre, then

$$dB = \frac{\mu_0 NI\, dx}{2(b-a)x}$$

$$\Rightarrow \quad B = \int_a^b dB = \frac{\mu_0 NI}{2(b-a)} \log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (C).

14. Consider the bigger loop to be made up of two loops 1 and 2 as shown in Figure.



Magnetic field due to loop 1 and 2 at point P has same value say B. So,

$$\vec{B}_p = B\hat{i} + B\hat{k} = B(\hat{i} + \hat{k})$$

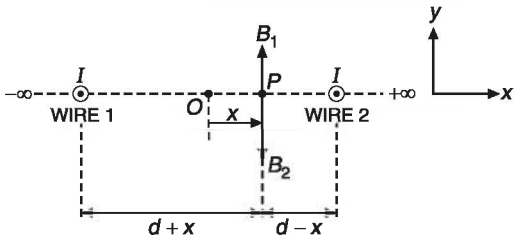
So, magnetic field points along the direction $\hat{n}$ given by

$$\hat{n} = \frac{\vec{B}_p}{|\vec{B}_p|} = \frac{B(\hat{i}+\hat{k})}{\sqrt{2}B} = \frac{\hat{i}+\hat{k}}{\sqrt{2}}$$

Hence, the correct answer is (D).

15. Taking upward direction as positive, the magnetic field at a point P between the wires at point P to the right of origin at a distance x from O is

$$B_p = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d-x} - \frac{1}{d+x}\right)$$



The field will remain negative as we move from $-\infty$ to Wire 1, has a zero value at ∞ and becomes very large near the Wire 1.

As we move from Wire 1 to Wire 2 the field changes sign from positive to negative, becoming zero at O and then again increasing as we approach Wire 2.

As we move from Wire 2 to $+\infty$, field is positive everywhere, very large near Wire 2 and then decreases to zero at $+\infty$.

Hence, the correct answer is (B).

16. Magnetic moment $= iA = \dfrac{q}{(2\pi/\omega)}$

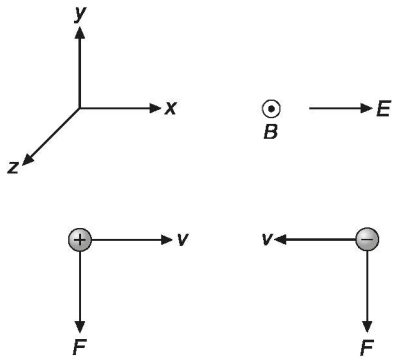
$$\Rightarrow \quad M = \frac{1}{2} q\omega r^2$$

Further angular momentum $L = I\omega = mr^2\omega$

$$\Rightarrow \quad \frac{M}{L} = \frac{q}{2m}$$

Hence, the correct answer is (C).

17. According to Fleming's left-hand rule, both positive and negative experience force along $-y$ axis.



Hence, the correct answer is (C).

18. $H_1 = \dfrac{\mu_o I}{4\pi(QM)}$ and

$$H_2 = \frac{\mu_o I}{4\pi(QM)} + \frac{\mu_o (I/2)}{4\pi(QM)}$$

CHAPTER 1

$$\Rightarrow \quad H_2 = H_1 + \frac{1}{2}H_1$$

$$\Rightarrow \quad H_1 = \left(1+\frac{1}{2}\right)H_1 = \frac{3}{2}H_1$$

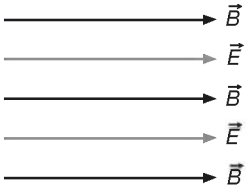
$$\Rightarrow \quad \frac{H_2}{H_1} = \frac{2}{3}$$

Hence, the correct answer is (C).

19. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.

Hence, the correct answer is (D).

20.



Since the charged particle is at rest, so initially, the magnetic field will not make it move. But initially, the charged particle will experience an electrostatic force in the direction of electric field as a result of which it gains a velocity parallel to E or parallel to B and hence even after the motion of the particle it will not experience a magnetic force i.e. its trajectory is a straight line.

Hence, the correct answer is (A).

21. Since both wires carry currents in opposite directions, so B at midpoint of the two wires is normal to the plane containing the wires and hence no net magnetic force is exerted on the charge q.

Hence, the correct answer is (D).

22. Magnetic Moment $M = iA$

$$\Rightarrow \quad M = \frac{2q}{\left(\frac{2\pi}{\omega}\right)}(\pi R^2)$$

$$\Rightarrow \quad M = q\omega R^2$$

Further, angular momentum L is

$L = I\omega$, where

I = Moment of Inertia of system

$$\Rightarrow \quad L = (mR^2 + mR^2)\omega$$

$$\Rightarrow \quad L = 2mR^2\omega$$

$$\Rightarrow \quad \frac{M}{L} = \frac{q\omega R^2}{2mR^2\omega}$$

$$\Rightarrow \quad \frac{M}{L} = \frac{q}{2m}$$

Hence, the correct answer is (A).

23. Radius of the circular path is given by

$$r = \frac{mv}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, K is the kinetic energy to the particle.

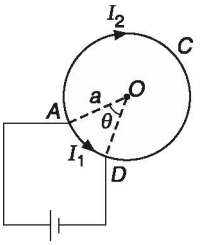
Therefore, $r \propto \frac{\sqrt{m}}{q}$ if K and B are same.

$$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

Hence, $r_\alpha = r_p < r_d$

Hence, the correct answer is (A).

24. For a current flowing in the circular arc of radius a shown in Figure the magnetic induction at the centre is



$$B = \left(\frac{\mu_o I_1}{4\pi a}\right)\theta - \frac{\mu_0 I_2}{4\pi a}(2\pi - \theta) = \frac{\mu_0}{4\pi a}\left(I_1\theta - I_2(2\pi - \theta)\right)$$

$$\Rightarrow \quad B \propto I\theta \qquad \ldots(1)$$

In the given problem, the total current is divided into two arcs such that it divides in the inverse ratio of resistances of arcs i.e.

$$\frac{I_1}{I_2} = \frac{\lambda a(2\pi - \theta)}{\lambda a\theta}$$

$$\Rightarrow \quad I_1\theta = I_2(2\pi - \theta)$$

Substituting in equation (1), we get

$$B = 0$$

Hence, the correct answer is (D).

25. Using Ampere's Circuital Law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.
Hence, the correct answer is (B).

26. $R = \dfrac{\sqrt{2qVm}}{Bq}$

$\Rightarrow \quad R \propto \sqrt{m}$

$\Rightarrow \quad \dfrac{R_1}{R_2} = \sqrt{\dfrac{m_X}{m_Y}}$

$\Rightarrow \quad \dfrac{m_X}{m_Y} = \left(\dfrac{R_1}{R_2}\right)^2$

Hence, the correct answer is (C).

27. Force per unit length between two wires carrying currents I_1 and I_2 at distance r is given by

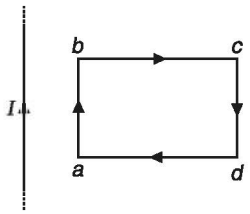
$$\frac{F}{\ell} = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{r}$$

Here, $I_1 = I_2 = I$ and $r = b$

$$\Rightarrow \quad \frac{F}{\ell} = \frac{\mu_o I^2}{2\pi b}$$

Hence, the correct answer is (B).

28. Straight wire will produce a non-uniform field to the right of it. $\vec{F}_{bc}$ and $\vec{F}_{dc}$ will be calculated by integration but these two forces will cancel each other. Further force on wire ab will be towards the long wire and on wire cd will be away from the long wire. But since the wire ab is nearer to the long wire, force of attraction towards the long wire will be more. Hence, the loop will move towards the wire.



Hence, the correct answer is (C).

29. Magnetic force on a current carrying loop in uniform magnetic field is zero.
Hence, the correct answer is (D).

30. In non-uniform magnetic field, the needle will experience both a force and a torque.
Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. Magnetic field due to loop at origin is

$$\frac{\mu_0 I R^2}{2.8R^3}(-\hat{k}) = \frac{\mu_0 I}{16R}(-\hat{k})$$

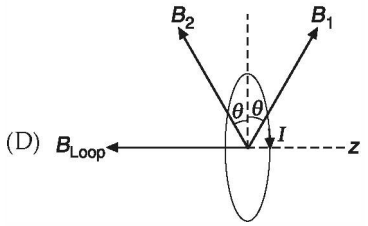
Magnetic field at origin due to wires

$$= \left(\frac{\mu_0 I_1}{2\pi R} - \frac{\mu_0 I_2}{2\pi R}\right)\hat{k}$$

(A) If $I_1 = I_2$ then $\vec{B}_0 = \dfrac{\mu_0 I}{16R}(-\hat{k})$

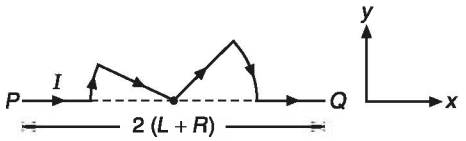
(B) It can be zero if $I_1 > 0$, $I_2 < 0$

(D)



Hence, (A), (B) and (D) are correct.

2.



Force on the complete wire equals the force on straight wire PQ carrying a current I.

$$\vec{F} = I\left(\overrightarrow{PQ} \times \vec{B}\right) = I\left[\{2(L+R)\hat{i}\} \times \vec{B}\right]$$

This force is zero, if $\vec{B}$ is along $\hat{i}$ direction or x-direction. If magnetic field is along $\hat{j}$ direction or $\hat{k}$ direction, then

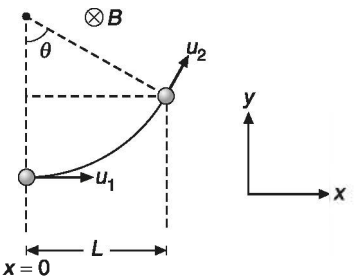
$$|\vec{F}| = F = (I)(2)(L+R)B\sin(90°)$$

$\Rightarrow \quad F = 2I(L+R)B$

$\Rightarrow \quad F \propto (L+R)$

Hence, (A), (B) and (C) are correct.

3. $\vec{u} = 4\hat{i}$ and $\vec{v} = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$



CHAPTER 1

According to the figure, magnetic field should be in $\otimes$ direction, or along $-z$ direction.

Further, $\tan\theta = \frac{v_y}{v_x} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 30°$

$\Rightarrow \quad \frac{\pi}{6} =$ Angle of $\vec{v}$ with x-axis is $\theta = \frac{\pi}{6}$

Since $\theta = \omega t$, where $\omega = \frac{QB}{M}$ and $t = 10^{-2}$ s

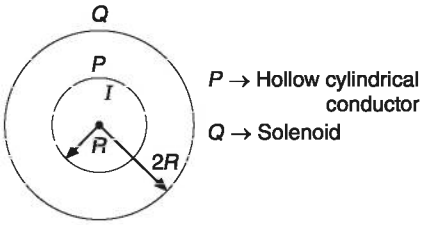
$\Rightarrow \quad \frac{\pi}{6} = \left(\frac{QB}{M}\right)t$

$\Rightarrow \quad B = \frac{\pi M}{6Qt} = \frac{\pi M}{6Q(10^{-2})}$

$\Rightarrow \quad B = \frac{50\pi M}{3Q}$

Hence, (A) and (C) are correct.

4.



In the region, $0 < r < R$

$B_P = 0$,

$B_Q \neq 0$, along the axis

So, $B_{net} \neq 0$

In the region, $R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

So, $B_{net} \neq 0$ and is neither in the directions mentioned in options (B) or (C).

In region, $r > 2R$

$B_P \neq 0$

$B_Q \neq 0$

So, $B_{net} \neq 0$

Hence, (A) and (D) are correct.

5. **For (A) and (B)**

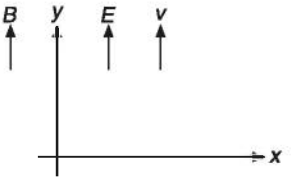
Magnetic field will rotate the particle in a circular path (in x-z plane or perpendicular to B). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical, hence the options (A) and (B) both are incorrect.

For (C)

$v_\perp$ and $\vec{B}$ will rotate the particle in a circular path in x-z plane (or perpendicular to $\vec{B}$). Further $v_\parallel$ and $\vec{E}$ will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along $\vec{B}$ and $\vec{E}$). Therefore, option (C) is correct.

For (D)

Magnetic force is zero, as θ between $\vec{B}$ and $\vec{v}$ is zero.



But electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction). So, option (D) is correct.

Hence, (C) and (D) are correct.

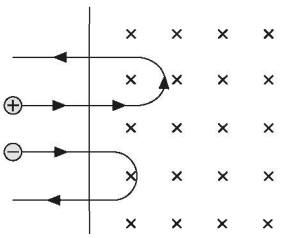
6. $r = \frac{mv}{Bq}$

$\Rightarrow \quad r \propto m$

$\Rightarrow \quad r_e < r_p$ because, $m_e < m_p$

Further, $T = \frac{2\pi m}{Bq}$

$\Rightarrow \quad T \propto m$



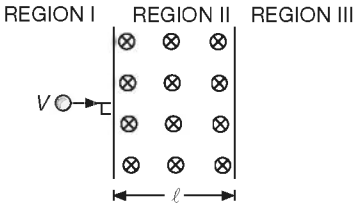
$\Rightarrow \quad T_e < T_p$

Since, $t_e = \frac{T_e}{2}$ and $t_p = \frac{T_p}{2}$

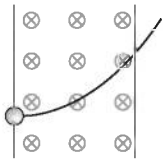
$\Rightarrow \quad t_e < t_p$

Hence, (B) and (D) are correct.

7. The radius of circle of path of charged particle is $R = \frac{mv}{qB}$



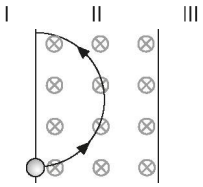
For particle to enter REGION III $R > \ell$ or $\frac{mv}{qB} > \ell$



For path length of particle in REGION II to be maximum

$\ell = R$

$\Rightarrow \quad V = \frac{q\ell B}{m}$



The period of revolution of charged particle is $\omega = \frac{qB}{m}$

The time spent in REGION II is $t = \frac{\pi}{\omega} = \frac{\pi m}{qB}$, which is same for all the cases when it returns to REGION II.

Hence, (A), (C) and (D) are correct.

8. Since, $\vec{F}_{BA} = 0$, because magnetic lines are parallel to this wire.

Also, $\vec{F}_{CD} = 0$, because magnetic lines are antiparallel to this wire.

Now, we see that $\vec{F}_{CB}$ is perpendicular to paper inwards and $\vec{F}_{AD}$ is perpendicular to paper outwards. These two forces (although calculated by integration) will cancel each other but produce a torque which tend to rotate the loop in clockwise direction about an axis OO'.

Hence, (A) and (C) are correct.

9. $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$

$\Rightarrow \quad \text{Deflection} \propto \frac{q}{\sqrt{m}}$

Hence, (A) and (C) are correct.

10. By Work-Energy Theorem, we have

Work done = Change in K.E.

$\Rightarrow \quad (qE\hat{i}) \cdot (2a\hat{i}) = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$

$\Rightarrow \quad 2aqE = \frac{3}{2}mv^2$

$\Rightarrow \quad E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

Rate of work done by $\vec{E}$ at P is $\vec{F} \cdot \vec{v}$

$\Rightarrow \quad \frac{dW}{dt} = (qE\hat{i}) \cdot (v\hat{i})$

$\Rightarrow \quad \frac{dW}{dt} = qEv = \frac{3}{4}\left(\frac{mv^3}{a}\right)$

Rate of work done by $\vec{E}$ at Q is zero, because at Q, $\vec{F} \perp \vec{v}$.

Hence, (A), (B) and (D) are correct.

11. No change in velocity implies no acceleration i.e. no net force is acting on the proton, even under the joint influence of electric and magnetic field. This thing is possible under the following situations.

OPTION (A):

$E = 0$, $B = 0$, i.e. no field exists in the region

OPTION (B):

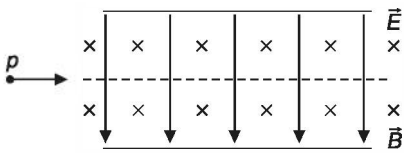
$E = 0$ i.e. no electrostatic force. $B \neq 0$, but the charge particle enters parallel to the field, so that net force equals to zero.

OPTION (C):

$E \neq 0$, i.e. the charged particle proton must experience an electrostatic force eE and hence must accelerate.

OPTION (D):

$E \neq 0$, $B \neq 0$ and both shown in Figure



Because in such a situation

$F_m = evB$ (upwards) and

$F_e = eE$ (downwards)

and if both are equal in magnitude then even the proton will suffer no change in its velocity and will continue to move along the dotted line as shown in Figure. In such a situation, the velocity is $v = \frac{E}{B}$.

Hence, (A), (B) and (D) are correct.

Reasoning Based Questions

1. $C\phi = BINA$

$$\Rightarrow \quad \phi = \left(\frac{BNA}{C}\right) I$$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence sensitivity increases. So, Statement-1 is true.
Statement-2 is false as we know soft iron can be easily magnetized or demagnetized.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. $F_M = Bev = Be\left(\frac{I}{nAe}\right) = \frac{BI}{nA}$ and $F_e = eE$

Since, $F_e = F_m$

$$\Rightarrow \quad eE = \frac{BI}{nA}$$

$$\Rightarrow \quad E = \frac{B}{nAe}$$

Also, $V = Ed$

$$\Rightarrow \quad V = \left(\frac{BI}{nAe}\right) w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{d_2}{d_1}$$

So, if $w_1 = 2w_2$ and $d_1 = d_2$ then

$V_1 = V_2$

Hence, (A) and (D) are correct.

2. Since, $V = \frac{BI}{ned}$

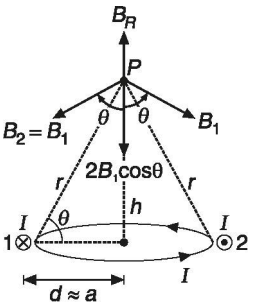
$$\Rightarrow \quad \frac{V_1}{V_2} = \left(\frac{B_1}{B_2}\right)\left(\frac{n_2}{n_1}\right)$$

If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

Hence, (A) and (C) are correct.

3. Let the magnetic field due to the ring, wire 1 and wire 2 be B_R, B_1 and B_2 respectively.



In magnitudes, we have

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\sqrt{a^2 + h^2}}$$

Resultant of B_1 and B_2 is

$$2B_1\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r}\right)\left(\frac{a}{r}\right) = \frac{\mu_0 Ia}{\pi r^2}$$

Also, $B_R = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{\frac{3}{2}}} \approx \frac{2\mu_0 I\pi a^2}{4\pi r^3}$

Since $d \approx a$, so $a^2 + h^2 = r^2$

For zero magnetic field at P, we have

$$2B_1\cos\theta = B_R$$

$$\Rightarrow \quad \frac{\mu_0 Ia}{\pi r^2} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$$

$$\Rightarrow \quad \pi a = 2r = 2\sqrt{a^2 + h^2}$$

$$\Rightarrow \quad \pi^2 a^2 = 4a^2 + 4h^2$$

$$\Rightarrow \quad \frac{(\pi^2 - 4)a^2}{4} = h^2$$

$$\Rightarrow \quad h^2 = 1.46a^2$$

$$\Rightarrow \quad h \approx 1.2a$$

Hence, the correct answer is (C).

4. Magnetic field at mid-point of two wires is

$B = 2$ (magnetic field due to one wire)

$$\Rightarrow \quad B = 2\left(\frac{\mu_0}{2\pi}\frac{I}{d}\right) = \frac{\mu_0 I}{\pi d} \otimes$$

Magnetic moment of loop is

$$M = IA = I(\pi a^2)$$

Torque on loop is

$$\tau = MB\sin 150°$$

$$\Rightarrow \quad \tau = \frac{\mu_0 I^2 a^2}{2d}$$

Hence, the correct answer is (B).

5. If $B_2 > B_1$, critical temperature, (at which resistance of semiconductors abruptly becomes zero) in case-2 will be less than compared to case-1.
Hence, the correct answer is (A).

6. With increase in temperature, T_C is decreasing.

$$T_C(0) = 100 \text{ K}$$

$$T_C = 75 \text{ K at } B = 7.5 \text{ T}$$

Hence, at $B = 5$ T, T_C should lie between 75 K and 100 K.

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial velocity $\neq 0$) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity $= 0$)
Hence, the correct answer is (C).

2. $\vec{F}_{\text{net}} = \vec{F}_e + \vec{F}_m$

$$\Rightarrow \quad \vec{F}_{\text{net}} = q\vec{E} + q\vec{v} \times \vec{B}$$

For particle to move in straight line with constant velocity, we have

$$\vec{F}_{\text{net}} = \vec{0}$$

$$\Rightarrow \quad q\vec{E} + q\vec{v} \times \vec{B} = 0$$

Hence, the correct answer is (A).

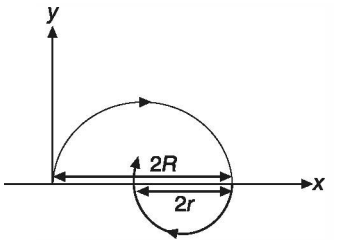
3. For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.
For the given set of options, only option (C) satisfy the condition. Path is helical with increasing pitch.
Hence, the correct answer is (C).

4. Done already, see theory
A → (q, r)
B → (p)
C → (q, r)
D → (q)

5. A → (q)
B → (r, s)
C → (s)
D → (p, q, r)

Integer/Numerical Answer Type Questions

1. The particle will follow the path as shown



$$\text{Average speed} = \frac{\frac{2mv}{qB} + \frac{2mv}{4qB}}{\frac{\pi m}{qB} + \frac{\pi m}{4qB}} = 2.00 \text{ ms}^{-1}$$

2. $\tau = BANi_m = k\theta$

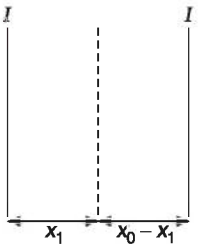
$$i_m = \frac{K\theta}{BAN} = \frac{10^{-4} \times 0.2}{0.02 \times 2 \times 10^{-4} \times 50}$$

$$\Rightarrow \quad i_m = \frac{0.2}{2} = 0.1 \text{ A}$$

$$0.1 \times 50 = 0.9 \text{ S}$$

$$\Rightarrow \quad S = \frac{50}{9}\ \Omega = 5.56\ \Omega$$

3.



When currents are in same direction, then

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

When currents are in opposite direction, then

$$B_2 = \frac{\mu_0 I}{2\pi x_1} + \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

CHAPTER 1

Substituting $x_1 = \frac{x_0}{3}$, we get

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$\Rightarrow \quad R_1 = \frac{mv}{qB_1}$$

and $B_2 = \frac{9\mu_0 I}{4\pi x_0}$

$$\Rightarrow \quad R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

4. If B be the magnetic field due to the given cylinder, B_F be the magnetic field due to full cylinder and B_C be the magnetic field due to removed cylinder i.e. cavity, then

$$B = B_F - B_C$$

$$\Rightarrow \quad B = \frac{\mu_0 I_F}{2a\pi} - \frac{\mu_0 I_C}{2\left(\frac{3a}{2}\right)\pi} \qquad \ldots(1)$$

where, $I_F = J(\pi a^2)$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in equation (1), we get

$$B = \frac{\mu_0}{a\pi}\left(\frac{I_F}{2} - \frac{I_C}{3}\right)$$

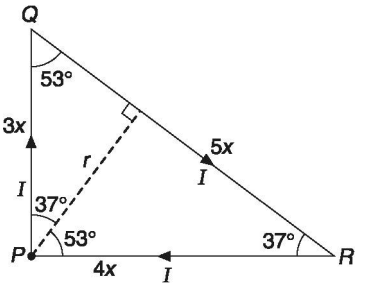
$$\Rightarrow \quad B = \frac{\mu_0}{a\pi}\left(\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right) = \frac{5\mu_0 aJ}{12}$$

$$\Rightarrow \quad N = 5$$

5. Magnetic field at point P due to wires RP and RQ is zero. Only wire QR will produce magnetic field at P. Since

$$r_\perp = r = 3x\cos 37°$$

$$\Rightarrow \quad r_\perp = (3x)\left(\frac{4}{5}\right) = \frac{12x}{5}$$



Also, $B = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{I}{\frac{12x}{5}}[\sin 37° + \sin 53°]$$

$$\Rightarrow \quad B = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

CHAPTER 2: MAGNETISM AND MATTER

Test Your Concepts-I
(Based on Bar Magnet and Properties)

1. Since we know that

$W_1 = MB(\cos 0° - \cos 90°)$

$\Rightarrow \quad W_1 = MB(1-0) = MB$

Similarly, $W_2 = MB(\cos 0° - \cos\theta)$

$\Rightarrow \quad W_2 = MB(1-\cos\theta)$

Since, $W_1 = 2W_2$

$\Rightarrow \quad MB = 2MB(1-\cos\theta)$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

2. Since $L_g = \frac{6}{5}L_m$

So, the geometric length of the magnet is

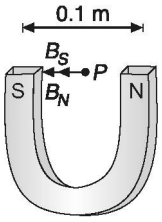
$L_g = \frac{6}{5}\times 10 = 12 \text{ cm}$

3. 8×10^{-7} T

Net magnetic field at mid-point P is $B = B_N + B_S$, where B_N is the magnetic field due to N-pole

B_S is the magnetic field due to S-pole

Since, $B_N = B_S = \frac{\mu_0}{4\pi}\frac{m}{r^2}$

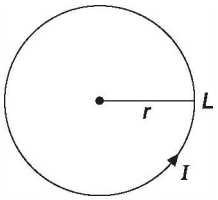


$\Rightarrow \quad B_N = B_S = 10^{-7}\times\frac{0.01}{\left(\frac{0.1}{2}\right)^2} = 4\times 10^{-7}$ T

$\Rightarrow \quad B_{net} = 2B_N = 2B_S = 8\times 10^{-7}$ T

4. Let a wire of length L be bent in a circular loop of radius r. Then, $2\pi r = L$

$\Rightarrow \quad r = \frac{L}{2\pi}$



The magnetic dipole moment of a circular loop is

$M = IA$, where $A = \pi r^2$

$\Rightarrow \quad M = I(\pi r^2)$

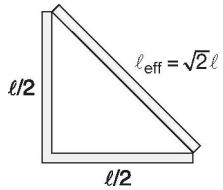
$\Rightarrow \quad M = I\pi\left(\frac{L}{2\pi}\right)^2$

$\Rightarrow \quad M = I\pi\left(\frac{L^2}{4\pi^2}\right)$

$\Rightarrow \quad M = \frac{IL^2}{4\pi}$

5. If m is strength of each pole, then magnetic moment $M = m\times l$

When the wire is bent into L shape, the effective distance between the poles is



$l_{eff} = \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$

However, the magnetic monopole strength m remains the same.

So, new magnetic moment is given by

$M' = ml_{eff} = m\left(\frac{l}{\sqrt{2}}\right) = \frac{M}{\sqrt{2}}$

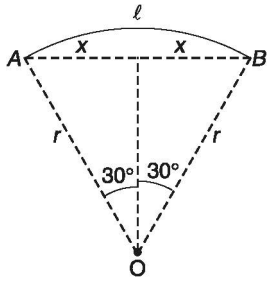
6. Since $l = r\theta$

and $\theta = \frac{\pi}{3}$

$\Rightarrow \quad l = \frac{\pi r}{3}$

Also, $M = ml$

From figure, we get



$$x = r\sin(30°) = \frac{r}{2}$$

Hence, new magnetic moment

$$\vec{M}' = m(2x) = mr = m\left(\frac{3l}{\pi}\right)$$

$$\Rightarrow \quad M' = \frac{3}{\pi}(ml)$$

$$\Rightarrow \quad M' = \frac{3M}{\pi}$$

7. When $\vec{M}$ is parallel to $\vec{B}$, then the magnet is in stable equilibrium, so $\theta = 0°$

PE in this case is minimum and is given by,

$$U_{min} = -\vec{M}\cdot\vec{B} = -MB\cos(0°)$$

$$\Rightarrow \quad U_{min} = -0.32 \times 0.15 \times 1 = -0.048 \text{ J}$$

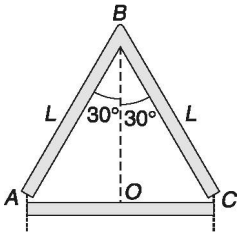
When $\vec{M}$ is anti-parallel to $\vec{B}$, then the magnet will be in unstable equilibrium, so $\theta = 180°$

Thus, potential energy in this case is maximum and is given by

$$U_{max} = -\vec{M}\cdot\vec{B} = -MB\cos(180°)$$

$$\Rightarrow \quad U_{max} = -0.32 \times 0.15 \times (-1) = +0.048 \text{ J}$$

8. On bending the magnet, the new length of the magnetic dipole is



$$AC = AO + OC = L\sin(30°) + L\sin(30°)$$

$$\Rightarrow \quad AC = 2L\sin(30°) = 2L\left(\frac{1}{2}\right) = L$$

9. Let N be the number turns and R radius of the coil. Then, $l = N(2\pi R)$

$$\Rightarrow \quad R = \frac{l}{2\pi N} \qquad \ldots(1)$$

The magnetic moment of the coil is

$$M = NiA = Ni(\pi R^2)$$

$$\Rightarrow \quad M = (Ni\pi)\left(\frac{l}{4\pi^2N^2}\right) = \frac{il^2}{4\pi N}$$

For maximum value of $\vec{M}$, we have

$$N = N_{min} = 1$$

$$\Rightarrow \quad M_{max} = \frac{il^2}{4\pi}$$

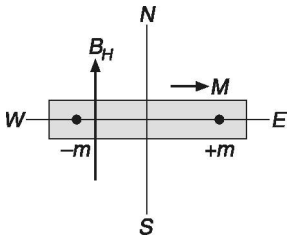
$$\Rightarrow \quad \tau_{max} = M_{max}B\sin 90° = \frac{iBl^2}{4\pi}$$

10. According to Law of Conservation of Energy, we know that the loss in potential energy of the magnet equals the gain in its kinetic energy, so we have

$$\Delta K = U_i - U_f, \text{ where } U = -MB_H\cos\theta$$

$$\Rightarrow \quad \Delta K = -MB_H(\cos\theta_1 - \cos\theta_2)$$

Initially the magnet is along east-west direction, so the angle between $\vec{M}$ and B_H is 90° i.e. $\theta_1 = 90°$ as shown in Figure.



Finally, when the magnet takes the north-south position, then it aligns along the earth's magnetic field, so the angle between $\vec{M}$ and B_H is 0° i.e. $\theta_2 = 0°$

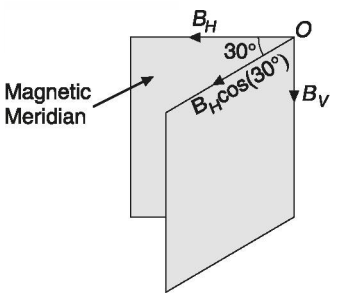
$$\Rightarrow \quad \Delta K = -MB_H(\cos 90° - \cos 0°) = MB_H$$

$$\Rightarrow \quad \Delta K = MB_H = 4(25 \times 10^{-6}) = 100 \times 10^{-6} \text{ J}$$

$$\Rightarrow \quad \Delta K = 100 \text{ μJ}$$

Test Your Concepts-II (Based on Earth's Magnetism)

1. In a vertical plane at 30° from the magnetic meridian, the horizontal component is



$$B'_H = B_H\cos(30°)$$

However, vertical component is still B_V. Therefore, apparent dip will be given by

$$\tan\phi' = \frac{B_V}{B'_H} = \frac{B_V}{B_H\cos(30°)}.$$

Since, $\frac{B_V}{B_H} = \tan\phi$, where ϕ is the true angle of dip.

$$\Rightarrow \quad \tan\phi' = \frac{\tan\phi}{\cos(30°)}$$

It is given that $\phi' = 45°$

$$\Rightarrow \quad \tan\phi = \tan(45°)\cos(30°) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \tan\phi = 0.866$$

$$\Rightarrow \quad \phi \approx 41°$$

2. At neutral point

$$\left|\begin{matrix}\text{Magnetic field due}\\ \text{to magnet}\end{matrix}\right| = \left|\begin{matrix}\text{Horizontal Magnetic}\\ \text{field due to earth}\end{matrix}\right|$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\left(\frac{2M}{d^3}\right) = 5\times10^{-5}$$

$$\Rightarrow \quad 10^{-7}\times\frac{2\times6.75}{d^3} = 5\times10^{-5}$$

$$\Rightarrow \quad d = 0.3 \text{ m} = 30 \text{ cm}$$

3. Let ϕ_1 and ϕ_2 be the angles of dip in two arbitrary planes which are perpendicular to each other, then $\phi_1 = 45°$ and $\phi = 30°$. Since, we know that

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

where, ϕ is true dip. So, we get

$$\cot^2 30° = \cot^2 45° + \cot^2\phi_2$$

$$\Rightarrow \quad \cot^2\phi_2 = 3 - 1 = 2$$

$$\Rightarrow \quad \cot\phi_2 = 1.414$$

$$\Rightarrow \quad \phi_2 = 35.2°$$

4. A compass needle in stable equilibrium position points towards magnetic north i.e., along the horizontal component B_H of earth's magnetic field. When it is turned through the angle of declination θ, so as to point geographical north, then it experiences a torque of magnitude $MB_H\sin\theta$

$$\Rightarrow \quad MB_H\sin\theta = 1.2\times10^{-3} \text{ Nm}$$

where, $M = 60 \text{ Am}^2$

$$B_H = 40\times10^{-6} \text{ Wbm}^{-2}$$

$$\Rightarrow \quad \sin\theta = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = 0.5$$

$$\Rightarrow \quad \theta = 30°$$

5. Since, $B_H = B_e\cos\phi$

$$\Rightarrow \quad B_e = \frac{B_H}{\cos\phi} = \frac{0.16}{\cos(60°)} = \frac{0.16}{\left(\frac{1}{2}\right)}$$

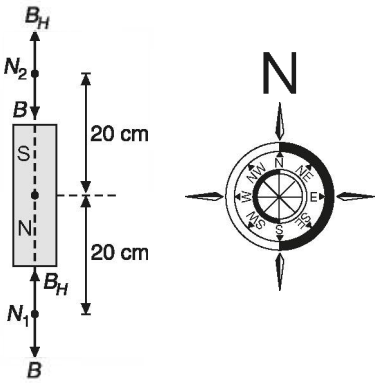
$$\Rightarrow \quad B_e = 0.16\times2 = 0.32 \text{ G}$$

$$\Rightarrow \quad B_e = 0.32\times10^{-4} \text{ T} \qquad \{\because 1 \text{ G} = 10^{-4} \text{ T}\}$$

$$\Rightarrow \quad B_e = 32\ \mu\text{T}$$

Direction of B_e: The earth's magnetic field lies in a vertical plane 12° west of geographic meridian at an angle of 60° above the horizontal line.

6. At neutral point, we have the magnetic field due to bar magnet must be cancelled by the horizontal component of earth's magnetic field as shown in Figure.



So, at the neutral point, we have

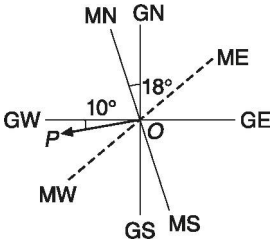
$$|B_H| = |B_{\text{axial}}|$$

$$\Rightarrow \quad \frac{2M}{(20)^3} = 0.3$$

$$\Rightarrow \quad M = 1.2\times10^3 \text{ e.m.u.}$$

CHAPTER 2

7. Since the ship has to reach a place $10°$ south of west i.e., along OP, so it should be steered west of magnetic north at angle of $90° - 18° + 10° = 82°$



8. Since, $B_H = 0.22$ G, $B_v = 0.38$ G

and $\tan\phi = \dfrac{B_V}{B_H} = \dfrac{0.38}{0.22} = 1.7272$

So, angle of dip is $\phi = 59°\ 56'$

Resultant magnetic field of the earth is

$$B = \sqrt{B_V^2 + B_H^2}$$

$$\Rightarrow \quad B = \sqrt{(0.38)^2 + (0.22)^2} = 0.427 \text{ G}$$

9. Since $\tan\phi = \dfrac{B_V}{B_H}$ ...(1)

If apparent dip is ϕ', then

$$\tan\phi' = \frac{B_V}{B_H'} = \frac{B_V}{B_H\cos(30°)} = \frac{B_V}{B_H\left(\dfrac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow \quad \tan\phi' = \left(\frac{2}{\sqrt{3}}\right)\tan\phi \qquad \left\{\because \tan\phi = \frac{B_V}{B_H}\right\}$$

$$\Rightarrow \quad \tan\phi' = \left(\frac{2}{\sqrt{3}}\right)\tan 60° = 2$$

$$\Rightarrow \quad \phi' = \tan^{-1}(2)$$

10. Since we know that

$$\tan\phi = \frac{B_V}{B_H}$$

Given that $B_H = 0.26$ G and $\phi = 60°$

$$\Rightarrow \quad B_V = B_H\tan\phi = (0.26)\tan(60°) = 0.45 \text{ G}$$

Also, we have $B_H = B_e\cos\phi$

$$\Rightarrow \quad B_e = \frac{B_H}{\cos\phi} = \frac{0.26}{\cos(60°)} = 0.52 \text{ G}$$

Test Your Concepts-III (Based on Tangent Law, Tangent Galvanometer and Vibration Magnetometer)

1. Since, $B_H = B\cos\phi$

Then, $H_1 = B_1\cos\phi_1$ and $H_2 = B_2\cos\phi_2$

Further, $T_1 = 2\pi\sqrt{\dfrac{I}{MB_{H_1}}} = 2\pi\sqrt{\dfrac{I}{MB_1\cos\phi_1}}$

and $T_2 = 2\pi\sqrt{\dfrac{I}{MB_{H_2}}} = 2\pi\sqrt{\dfrac{I}{MB_2\cos\phi_2}}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \frac{B_2\cos\phi_2}{B_1\cos\phi_1}$$

$$\Rightarrow \quad \cos\phi_2 = \frac{B_1}{B_2}\times\frac{T_1^2}{T_2^2}\cos\phi_1$$

$$\Rightarrow \quad \cos\phi_2 = \frac{B_1}{B_2}\times\left(\frac{\nu_2}{\nu_1}\right)^2\cos\phi_1$$

$$\Rightarrow \quad \cos\phi_2 = \left(\frac{0.5}{0.6}\right)\left(\frac{20}{15}\right)^2\cos 60° = 0.74$$

$$\Rightarrow \quad \phi_2 = \cos^{-1}(0.74) = 42.2°$$

2. In case of tangent galvanometer as

$$i = k\tan\theta$$

Differentiating both side w.r.t. θ

$$\frac{di}{d\theta} = k\sec^2\theta$$

$$\Rightarrow \quad di = k\sec^2\theta d\theta$$

$$\Rightarrow \quad \frac{di}{i} = \frac{d\theta}{\sin\theta\cos\theta} = \frac{2d\theta}{\sin(2\theta)}$$

Hence, the error in the measurement will be least when we have

$$\sin(2\theta) = \text{MAX} = 1$$

$$\Rightarrow \quad 2\theta = 90°$$

$$\Rightarrow \quad \theta = 45°$$

3. (a) Moment of inertia of magnet is given by,

$$I = \frac{m(l^2 + b^2)}{12}$$

where m is the mass of magnet

$$\Rightarrow \quad I = \frac{250(5^2 + 3^2)\times 10^{-4}\times 10^{-3}}{12}$$

$$\Rightarrow \quad I = 7.08\times 10^{-5} \text{ kgm}^2$$

Also, $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$\Rightarrow \quad M = \frac{4\pi^2 I}{B_H T^2}$

$\Rightarrow \quad M = \frac{4\times(3.14)^2\times(7.08\times10^{-5})}{20\times10^{-6}\times5\times5}$

$\Rightarrow \quad M = 5.58 \text{ Am}^2$

(b) New moment of inertia is given by

$$I' = \frac{m(l^2+h^2)}{12}$$

Since, $T' = 2\pi\sqrt{\frac{I'}{MB_H}}$

$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{I'}{I}} = \sqrt{\frac{l^2+h^2}{l^2+b^2}} = \sqrt{\frac{5^2+(0.5)^2}{5^2+3^2}}$

$\Rightarrow \quad \frac{T'}{T} = 0.86$

$\Rightarrow \quad T' = T\times0.86 = 5\times0.86 = 4.30 \text{ s}$

4. We have

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

If B is resultant earth's magnetic field and θ is angle of dip, then $B_H = B\cos\theta$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB\cos\theta}}$

In first case

$$T_1 = \frac{60}{20} s = 3 \text{ s}, \ \theta_1 = 30°, \ B = B_1$$

In second case

$$T_2 = \frac{60}{15} \text{ s} = 4 \text{ s}, \ \theta_2 = 60°, \ B = B_2$$

Since $T_1 = 2\pi\sqrt{\frac{I}{MB_1\cos\theta_1}}$ and

$$T_2 = 2\pi\sqrt{\frac{I}{MB_2\cos\theta_2}}$$

$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$

$\Rightarrow \quad \frac{B_1}{B_2} = \left(\frac{T_2}{T_1}\right)^2\frac{\cos\theta_2}{\cos\theta_1}$

$\Rightarrow \quad \frac{B_1}{B_2} = \left(\frac{4}{3}\right)^2\frac{\cos60°}{\cos30°}$

$\Rightarrow \quad \frac{B_1}{B_2} = \left(\frac{4}{3}\right)^2\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{16}{9\sqrt{3}}$

5. For the first galvanometer, we have

$$i_1 = K_1\tan\theta_1 = K_1\tan60° = K_1\sqrt{3}$$

For the second galvanometer, we have

$$i_2 = K_2\tan\theta_2 = K_2\tan45°K_2$$

When connected in series, we have

$$i_1 = i_2$$

$\Rightarrow \quad K_1\sqrt{3} = K_2$

$\Rightarrow \quad \frac{K_1}{K_2} = \frac{1}{\sqrt{3}}$

Also, we know that

$$K \propto \frac{1}{N}$$

$\Rightarrow \quad \frac{K_1}{K_2} = \frac{N_2}{N_1}$

$\Rightarrow \quad \frac{N_1}{N_2} = \frac{\sqrt{3}}{1}$

6. Since $I \propto \tan\theta$

$\Rightarrow \quad \frac{I_2}{I_1} = \frac{\tan\theta_2}{\tan\theta_1}$

Given that $I_1 = 10 \text{ mA}, \ \theta_1 = 31°, \ \theta_2 = 42°$

$\Rightarrow \quad I_2 = I_1\frac{\tan\theta_2}{\tan\theta_1} = 10\times\frac{\tan42°}{\tan31°}$

$\Rightarrow \quad I_2 = \frac{10\times0.9}{0.6} = 15 \text{ mA}$

Percentage increase in current is

$$\left(\frac{I_2-I_1}{I_1}\right)\times100\% = \frac{(15-10)}{10}\times100 = 50\%$$

7. For $\tan A$ position, we have

$$\frac{\mu_0}{4\pi}\frac{2M}{r^3} = B_H\tan\theta$$

$\Rightarrow \quad \tan\theta \propto \frac{1}{r^3}$

CHAPTER 2

$$\Rightarrow \quad \frac{\tan\theta_2}{\tan\theta_1} = \left(\frac{r}{r(3)^{\frac{1}{3}}}\right)^3 = \frac{1}{3}$$

$$\Rightarrow \quad \tan\theta_2 = \frac{\tan\theta_1}{3} = \frac{\tan 60°}{3}$$

$$\Rightarrow \quad \tan\theta_2 = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta_2 = 30°$$

8. Here, $T_1 = 3$ s , $T_2 = \frac{1}{12}$ min $= \frac{60 \text{ s}}{12} = 5$ s

Since, $\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{5^2 + 3^2}{5^2 - 3^2} = \frac{34}{16} = \frac{17}{8}$$

9. By sum and difference method, we have

$$\frac{M_1}{M_2} = \frac{v_s^2 + v_d^2}{v_s^2 - v_d^2}$$

$$\Rightarrow \quad \frac{13}{5} = \frac{(15)^2 + v_d^2}{(15)^2 - v_d^2}$$

$$\Rightarrow \quad v_d = 10 \text{ oscillation/min}$$

Test Your Concepts-IV (Based on Magnetic Properties of Materials)

1. The bar magnet has coercivity 4×10^3 Am^{-1} i.e. it requires a magnetic intensity $H = 4\times10^3$ Am^{-1} to get demagnetised. Let i be the current carried by solenoid having n number of turns per metre length, then by definition

$$H = ni$$

Here $H = 4\times10^3$ Amp turn metre^{-1}

Since $n = \frac{N}{l} = \frac{60}{0.12} = 500$ turn metre^{-1}

$$\Rightarrow \quad i = \frac{H}{n} = \frac{4\times10^3}{500} = 8.0 \text{ A}$$

2. Given, magnetic moment, $M = 1.2$ Am^2

Volume, $V = (15\times2\times1)\times10^{-6}$ m^3 $= 30\times10^{-6}$ m^3

So, intensity of magnetisation is

$$I = \frac{M}{V} = \frac{1.2}{(15\times2\times1)\times10^{-6}} = 4\times10^4 \text{ Am}^{-1}$$

3. $I = \frac{M_{net}}{V}$, where $M_{net} = NM$

$$\Rightarrow \quad I = \frac{NM}{V} = \left(\frac{N}{V}\right)M$$

$$\Rightarrow \quad \frac{MN}{V} = \frac{1.5\times10^{-23}\times2\times10^{26}}{1}$$

$$\Rightarrow \quad I = 3\times10^3 \text{ Am}^{-1}$$

4. The coercivity of 4×10^4 Am^{-1} for the permanent magnet implies that a magnetic intensity $H = 4\times10^4$ Am^{-1} is required to be applied in opposite direction to demagnetise the magnet.

Since, $n = \frac{700}{20 \text{ cm}} = \frac{700}{20\times10^{-2} \text{ m}} = 3500$ turnsm^{-1}

Also, $H = ni$

So, current, $i = \frac{H}{n} = \frac{4\times10^4}{3500} = 11.5$ A

5. Since, we know that

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$

$$\Rightarrow \quad \mu = \mu_0(1+\chi)$$

$$\Rightarrow \quad \mu = 4\pi\times10^{-7}(1+599)$$

$$\Rightarrow \quad \mu = 7.536\times10^{-4} T \text{ m}^{A-1}$$

$$\Rightarrow \quad B = \mu H = 7.536\times10^{-4}\times1200 \ T$$

$$\Rightarrow \quad \phi = BA = 7.536\times10^{-4}\times1200\times0.2\times10^{-4} \text{ Wb}$$

$$\Rightarrow \quad \phi = 1.81\times10^{-5} \text{ Wb}$$

6. Magnetic susceptibility,

$\chi_{m_1} = 0.0075$,

$T_1 = -73$ °C $= (-73 + 273)$ K $= 200$ K

$\chi_{m_2} = ?$,

$T_2 = -173$ °C $= (-173 + 273)$ K $= 100$ K

According to Curie's law, we have

$$\chi_m \propto \frac{1}{T}$$

So, ratio of magnetic susceptibilities is

$$\frac{\chi_{m_2}}{\chi_{m_1}} = \frac{T_1}{T_2} = \frac{200}{100} = 2$$

$$\Rightarrow \quad \chi_{m_2} = 2\chi_{m_1} = 2\times0.0075 = 0.015$$

7. Let Q be the energy dissipated per unit volume per hysteresis cycle in the given sample. Then the total energy lost by the volume V of the sample in time t will be

$$W = Q \times V \times \nu \times t$$

where ν is the number of hysteresis cycles per second. Given that $Q = 300\ \text{Jm}^{-3}\ \text{cycle}^{-1}$, $\nu = 25\ \text{cycle s}^{-1}$, $t = 1\ \text{h} = 3600\ \text{s}$. Also, volume of the iron specimen is

$$V = \frac{\text{Mass}}{\text{Density}} = \frac{15}{7500}\ \text{m}^3$$

So, hysteresis loss is given by

$$W = 300 \times \frac{15}{7500} \times 25 \times 3600\ \text{J} = 54000\ \text{J}$$

8. Since, $M = I \times V$, where V is volume of the cylinder i.e. $V = \pi r^2 l$, where r is the radius and l is the length of the cylinder.
The dipole moment is given by

$$M = I(\pi r^2 l)$$

$$\Rightarrow \quad M = (5.30 \times 10^3) \times \frac{22}{7} \times (0.5 \times 10^{-2})^2 (5 \times 10^{-2})$$

$$\Rightarrow \quad M = 2.08 \times 10^{-2}\ \text{JT}^{-1}$$

9. Since, $\mu_r = 1 + \chi_m$

$$\Rightarrow \quad 5500 = 1 + \chi_m$$

$$\Rightarrow \quad \chi_m = 5500 - 1 = 5499$$

Single Correct Choice Type Questions

1. $\chi_m = (\mu_r - 1)$

$$\Rightarrow \quad \chi_m = (5500 - 1) = 5499$$

Hence, the correct answer is (D).

2. In sum position, we have

$$T_s = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}}$$

In difference position, we have

$$T_d = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B_H}}$$

$$\Rightarrow \quad T_d > T_s$$

Hence, the correct answer is (B).

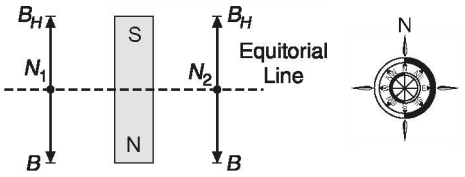
3. Work done $W = MB_H(1 - \cos\theta)$

$$\Rightarrow \quad W = 20 \times 0.3(1 - \cos 30°)$$

$$\Rightarrow \quad W = 6\left(1 - \frac{\sqrt{3}}{2}\right) = 3(2 - \sqrt{3})$$

Hence, the correct answer is (C).

4. N_1 and N_2 are two null points on equitorial line, where B_H (Horizontal component of earth's magnetic field) and B (Magnetic field due to bar magnet) cancel to give neutral points N_1 and N_2 as shown in Figure.

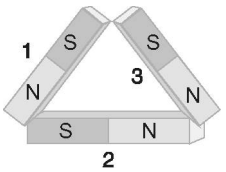


Hence, the correct answer is (A).

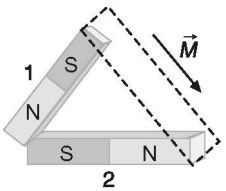
5. Magnetic lines of force are closed curves. Outside the magnet they are directed from north to south while inside the magnet, they are directed from south to north.

Hence, the correct answer is (D).

6. The resultant magnetic moment is $\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3$

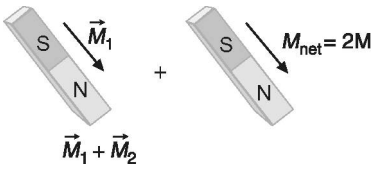


Since $\vec{M}_1 + \vec{M}_2 = M$ as shown in Figure.



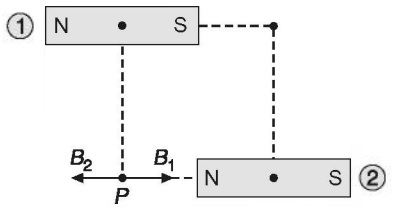
Also, $\vec{M}_3 = \vec{M}$

$$\Rightarrow \quad \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 2\vec{M}$$



Hence, the correct answer is (B).

7. Point P lies on equatorial line of magnet (1) and axial line of magnet (2) as shown in Figure.



So, $B_1 = \dfrac{\mu_0}{4\pi}\dfrac{M}{d^3} = 10^{-7} \times \dfrac{1000}{(0.1)^3} = 0.1\ \text{T}$

$$B_2 = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = 10^{-7} \times \frac{2 \times 1000}{(0.1)^3} = 0.2\ \text{T}$$

$$\Rightarrow \quad B_{\text{net}} = B_2 - B_1 = 0.1\ \text{T}$$

Hence, the correct answer is (A).

8. $\tau = MB_H \sin\theta$

$$\Rightarrow \quad \frac{d\tau}{d\theta} = MB_H \cos\theta$$

This will be maximum, when $\theta = 0°$

Hence, the correct answer is (A).

9. $E = VAnt = \left(\dfrac{m}{\rho}\right)Ant = \dfrac{50 \times 250 \times 10 \times 3600}{7.5 \times 10^3}$

$$\Rightarrow \quad E = 6 \times 10^4\ \text{J}$$

Hence, the correct answer is (A).

10. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2} = \frac{\left(\frac{60}{15}\right)^2}{\left(\frac{60}{10}\right)^2} = \frac{4}{9}$$

Hence, the correct answer is (A).

11. Sensitivity $S = \dfrac{\theta}{i} = \dfrac{\theta}{K\tan\theta}$ where $K = \dfrac{2RB_H}{\mu_0 N}$

For increasing sensitivity K should be decreased and hence number of turns should be increased.

Hence, the correct answer is (B).

12. $F = \dfrac{\mu_0}{4\pi}\dfrac{6M_1M_2}{r^4}$

$$\Rightarrow \quad F \propto \frac{1}{r^4}$$

$$\Rightarrow \quad \frac{F'}{F} = \left(\frac{r}{r'}\right)^4 = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \quad F' = \frac{F}{16} = \frac{4 \cdot 8}{16} = 0.3\ \text{N}$$

Hence, the correct answer is (D).

13. Net magnetic moment

$$M = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

$$\Rightarrow \quad M = \sqrt{M_0^2 + M_0^2 + 2M_0^2\cos 60°}$$

$$\Rightarrow \quad M = \sqrt{3}M_0$$

Hence, the correct answer is (C).

14. Since, $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

For CASE-1, we have

$$T_1 = \frac{60}{10}\ \text{s}$$

$$\frac{60}{10} = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \ldots(1)$$

For CASE-2, we have

$$T = \frac{60}{14}\ \text{s}$$

$$\frac{30}{7} = 2\pi\sqrt{\frac{I}{M(B_H + B)}} \qquad \ldots(2)$$

Divide (2) by (1), we get

$$\frac{\frac{30}{7}}{6} = \sqrt{\frac{B_H}{B_H + B}}$$

$$\Rightarrow \quad B = \left(\frac{24}{25}\right)B_H \qquad \ldots(3)$$

For CASE-3,

Let the magnet make n oscillations per minute, then

$$T = \frac{60}{n}$$

$$\Rightarrow \quad \frac{60}{n} = 2\pi\sqrt{\frac{I}{M(B_H - B)}}$$

$$\Rightarrow \quad \frac{60}{n} = 2\pi\sqrt{\frac{I}{M\left(B_H - \frac{24}{25}B_H\right)}} \qquad \left\{\because B = \frac{24}{25}B_H\right\}$$

$$\Rightarrow \quad \frac{60}{n} = 2\pi \times 5 \times \sqrt{\frac{I}{MB_H}} \qquad \ldots(4)$$

From equations (1), we get

$$2\pi\sqrt{\frac{I}{MB_H}} = 6$$

So, equation (4), becomes,

$$\frac{60}{n} = 5(6)$$

$\Rightarrow \quad n = 2$ vibrations per minute

Hence, the correct answer is (D).

15. Initially magnetic moment of system is

$$M_1 = \sqrt{M^2 + M^2} = \sqrt{2}M \text{ and}$$

moment of inertia is

$$I_1 = I + I = 2I$$

Finally, when one of the magnets is removed then

$$M_2 = M \text{ and } I_2 = I$$

Since, $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2} \times \frac{M_2}{M_1}} = \sqrt{\frac{2I}{I} \times \frac{M}{\sqrt{2}M}}$$

$$\Rightarrow \quad T_2 = \frac{2^{\frac{5}{4}}}{2^{\frac{1}{4}}} = 2 \text{ s}$$

Hence, the correct answer is (C).

16. $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\Rightarrow \quad \frac{T}{T'} = \sqrt{\frac{B'}{B}} = \sqrt{\frac{B}{B_H}}$$

$$\Rightarrow \quad \frac{T}{T'} - \sqrt{\frac{1}{\cos\phi}} - \sqrt{\frac{1}{\cos 60^\circ}} = \sqrt{2}$$

$$\Rightarrow \quad T' = \frac{T}{\sqrt{2}}$$

Hence, the correct answer is (A).

17. $B = B_1 + B_2 = \frac{\mu_0}{4\pi}\left(\frac{m_1}{r_1^2} + \frac{m_2}{r_2^2}\right)$

As $m_1 = m_2$ and $r_1 = r_2$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{2m_1}{r_1^2} = 10^{-7} \times \frac{2 \times 10^{-4}}{(5 \times 10^{-2})^2}$$

$$\Rightarrow \quad B = 8 \times 10^{-9} \text{ T}$$

Hence, the correct answer is (D).

18. Magnetic moment of circular loop carrying current

$$M = iA = i(\pi R^2) = i\pi\left(\frac{L}{2\pi}\right)^2 = \frac{iL^2}{4\pi}$$

$$\Rightarrow \quad L = \sqrt{\frac{4\pi M}{i}}$$

Hence, the correct answer is (B).

19. Since, $B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{r_1}{r_2}\left(\frac{r_2^2 - l^2}{r_1^1 - l^2}\right)^2$$

$$\Rightarrow \quad \frac{12.5}{1} = \frac{10}{20}\left(\frac{400 - l^2}{100 - l^2}\right)^2$$

$$\Rightarrow \quad l = 5 \text{ cm}$$

Hence, length of magnet is $L = 2l = 10$ cm

Hence, the correct answer is (C).

20. Since $W = -MB(\cos\theta_2 - \cos\theta_1)$

When the magnet is rotated from 0° to 60°, then work done is 0.8 J

$$\Rightarrow \quad 0.8 = -MB(\cos 60^\circ - \cos 0^\circ) = \frac{MB}{2}$$

$$\Rightarrow \quad MB = 1.6 \text{ Nm}$$

In order to rotate the magnet further through an angle of 30° i.e., from 60° to 90°, the work done is

$$W' = -MB(\cos 90^\circ - \cos 60^\circ) = -MB\left(0 - \frac{1}{2}\right)$$

$$\Rightarrow \quad W' = \frac{MB}{2} = \frac{1.6}{2} = 0.8 \text{ J} = 0.8 \times 10^7 \text{ erg}$$

Hence, the correct answer is (A).

21. Since $B_H = B_V = B_0$

$$\Rightarrow \quad B = \sqrt{B_H^2 + B_V^2} = \sqrt{B_0^2 + B_0^2}$$

$$\Rightarrow \quad B = \sqrt{2}B_0$$

Hence, the correct answer is (D).

CHAPTER 2

22. Since, $B = B_H \tan\theta$

$$\Rightarrow \quad \frac{\mu_0 Ni}{2r} = B_H \tan\theta$$

$$\Rightarrow \quad i = \frac{2rB_H \tan\theta}{\mu_0 N} = \frac{2\times 0.1\times 4\times 10^{-5}}{10\times 4\pi\times 10^{-7}} = 1.1 \text{ A}$$

Hence, the correct answer is (B).

23. $F = mB$

$$\Rightarrow \quad m = \frac{F}{B} = \frac{1.224\times 10^{-4}}{0.36\times 10^{-4}}$$

$$\Rightarrow \quad m = 4 \text{ Am}$$

Since, $M = m(2\ell)$

$$\Rightarrow \quad 2\ell = \frac{M}{m} = \frac{0.1}{4} \text{ m} = \frac{10}{4} \text{ cm}$$

$$\Rightarrow \quad 2\ell = 2.50 \text{ cm}$$

Hence, the correct answer is (C).

24. Given $B_e = 0.36$ G, $\theta = 60°$

Horizontal component of earth's magnetic field is

$$B_H = B_e \cos 60° = 0.36 \times \cos 60°$$

$$\Rightarrow \quad B_H = 0.36 \times \frac{1}{2} = 0.18 \text{ G}$$

Vertical component of earth's magnetic field is

$$B_V = B_e \sin 60° = 3.36 \sin 60°$$

$$\Rightarrow \quad B_V = 0.36 \times \frac{\sqrt{3}}{2} = 0.18\sqrt{3} \text{ G}$$

Hence, the correct answer is (D).

25. Since $\tan\phi = \frac{B_V}{B_H}$ ...(1)

If apparent dip is ϕ', then

$$\tan\phi' = \frac{B_V}{B_H'} = \frac{B_V}{B_H \cos(30°)} = \frac{B_V}{B_H\left(\frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow \quad \tan\phi' = \left(\frac{2}{\sqrt{3}}\right)\tan\phi \qquad \left\{\because \tan\phi = \frac{B_V}{B_H}\right\}$$

$$\Rightarrow \quad \tan\phi' > \tan\phi$$

$$\Rightarrow \quad \phi' > \phi$$

Hence, the correct answer is (C).

26. Along the axis of magnet, we have

$$B_{\text{axial}} = \frac{2M}{X^3} = 200 \text{ G}$$

$$\Rightarrow \quad B_{\text{equitorial}} = \frac{M}{X^3} = 100 \text{ G}$$

Hence, the correct answer is (A).

27. $\nu = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

$$\Rightarrow \quad \nu \propto \sqrt{M}$$

$$\Rightarrow \quad \frac{\nu_A}{\nu_B} = \sqrt{\frac{M_A}{M_B}}$$

$$\Rightarrow \quad \frac{2}{1} = \sqrt{\frac{M_A}{M_B}}$$

$$\Rightarrow \quad M_A = 4M_B$$

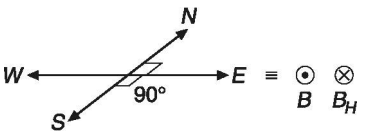
Hence, the correct answer is (C).

28. Magnetic field at the centre of short magnet very long wire is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times 10^{-7}\times 18}{2\pi\times 0.2} T = 18 \ \mu\text{T}$$

Since $T = 2\pi\sqrt{\frac{I}{MB_H}}$ and $T' = 2\pi\sqrt{\frac{I}{M(B_H - B)}}$

Dividing, we get



$$\frac{T'}{T} = \sqrt{\frac{B_H}{B_H - B}}$$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{24}{24-18}} = 2$$

$$\Rightarrow \quad T' = 2\times 0.1 \text{ s} = 0.2 \text{ s}$$

Hence, the correct answer is (B).

29. In two planes at right angles to each other if apparent dips are ϕ_1 and ϕ_2, then true dip at that place is

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

$$\Rightarrow \quad \cot^2\phi = \cot^2(30°) + \cot^2(45°)$$

$$\Rightarrow \quad \cot^2\phi = 4$$

$$\Rightarrow \quad \cot\phi = 2$$

$$\Rightarrow \quad \phi = \cot^{-1}(2)$$

Hence, the correct answer is (C).

30. Since, $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$\Rightarrow \quad T \propto \dfrac{1}{\sqrt{M}}$

$\Rightarrow \quad \dfrac{T_1}{T_2} = \sqrt{\dfrac{M_2}{M_1}}$,

If $M_1 = 100$, then $M_2 = (100 - 36) = 64$

$\Rightarrow \quad \dfrac{T_1}{T_2} = \sqrt{\dfrac{64}{100}} = \dfrac{8}{4}$

$\Rightarrow \quad T_2 = \dfrac{10}{8} T_1 = 1.25 T_1$

So, increase in time period is 25%

Hence, the correct answer is (B).

31. $B_H = B_e \cos\theta$

$\Rightarrow \quad B_e = \dfrac{B_H}{\cos\theta} = \dfrac{0.5}{\cos 30°} = \dfrac{0.5}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

Hence, the correct answer is (C).

32. Torque acting on magnet is

$\tau = MB_H \sin\theta$

$\Rightarrow \quad \tau = 0.1 \times 10^{-3} \times 4\pi \times 10^{-3} \times \sin 30° = 10^{-7} \times 4\pi \times \dfrac{1}{2}$

$\Rightarrow \quad \tau = 2\pi \times 10^{-7}$ Nm

Hence, the correct answer is (A).

33. If ϕ is true angle of dip, then

$$\tan\phi = \frac{B_V}{B_H} \qquad \ldots(1)$$

When the dip circle is rotated in the horizontal plane through an angle θ from the magnetic meridian, the effective horizontal component in the new plane becomes $B'_H = B_H \cos\theta$, while the vertical component remains the same. If ϕ' is apparent dip, then

$$\tan\phi' = \frac{B_V}{B'_H} = \frac{B_V}{B_H \cos\theta} = \frac{\tan\phi}{\cos\theta} \qquad \ldots(2)$$

$\Rightarrow \quad \dfrac{\tan\phi'}{\tan\phi} = \dfrac{1}{\cos\theta}$

Hence, the correct answer is (C).

34. $T_{\text{sum}} = T_s = 2\pi\sqrt{\dfrac{I_1 + I_2}{(M_1 + M_2)B_H}}$

$T_{\text{diff}} = T_d = 2\pi\sqrt{\dfrac{I_1 + I_2}{(M_1 - M_2)B_H}}$

$\Rightarrow \quad \dfrac{T_s}{T_d} = \dfrac{T_1}{T_2} = \sqrt{\dfrac{M_1 - M_2}{M_1 + M_2}} = \sqrt{\dfrac{2M - M}{2M + M}} = \dfrac{1}{\sqrt{3}}$

Hence, the correct answer is (C).

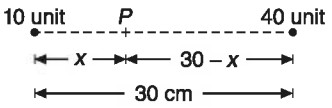
35. Since $\tan\phi' = \dfrac{\tan\phi}{\cos\beta}$

$\Rightarrow \quad \tan\phi' = \dfrac{\tan\phi}{\cos\theta}$

$\Rightarrow \quad \tan\phi' = \tan\phi \sec\theta$

Hence, the correct answer is (B).

37. Suppose magnetic field is zero at point P. Which lies at a distance x from 10 unit pole. Hence, at P



$\dfrac{\mu_0}{4\pi}\dfrac{10}{x^2} = \dfrac{\mu_0}{4\pi}\dfrac{40}{(30-x)^2}$

$\Rightarrow \quad x = 10$ cm

So, from stronger pole distance is 20 cm

Hence, the correct answer is (B).

38. If $M(= m\ell)$ is magnetic moment of original magnet, then magnetic moment of each part is M' given by

$$M' = \left(\frac{m}{2}\right)(\ell) = \frac{m\ell}{2} = \frac{M}{2}$$

Moment of inertia of original magnet,

$I = \dfrac{m_0 \ell^2}{12}$, m_0 = mass of magnet

Moment of inertia of each part $= \dfrac{(m_0/2)\ell^2}{12} = \dfrac{I}{2}$

Time period of original magnet is

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Time period of each part

$$T' = 2\pi\sqrt{\frac{(I/2)}{(M/2)H}} = T$$

i.e. time period remains unchanged.

Hence, the correct answer is (B).

CHAPTER 2

39. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

Moment of inertia (I) is directly proportional to mass so, when mass is quadrupled (made four times), moment of inertia becomes 4 times and so time period T will be doubled.

Hence, the correct answer is (D).

40. As we know for circulating electron magnetic moment

$$M = \frac{1}{2}evr \qquad \ldots(1)$$

And angular momentum

$$J = mvr \qquad \ldots(2)$$

From equation (1) and (2), we get

$$M = \frac{eJ}{2m}$$

Hence, the correct answer is (B).

41. $T = 2\pi\sqrt{\dfrac{I}{MB_H}} = 2$

$$T' = 2\pi\sqrt{\frac{I}{M(B_H + B)}} = 1 \text{ sec}$$

Dividing $2 = \sqrt{\dfrac{B_H + B}{B_H}}$

$$\Rightarrow \quad \frac{B_H}{B} = \frac{1}{3}$$

Hence, the correct answer is (B).

42. $T_1 \propto \dfrac{1}{\sqrt{B_H}}$ and $T_2 \propto \dfrac{1}{\sqrt{B_H + B}}$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{B_H + B}{B_H}}$$

$$\Rightarrow \quad \frac{2}{1} = \sqrt{\frac{B_H + B}{B_H}}$$

$$\Rightarrow \quad B = 3B_H$$

$$\Rightarrow \quad \frac{B_H}{B} = \frac{1}{3}$$

Hence, the correct answer is (B).

43. $B_1 = \dfrac{2M}{x^3}$ and $B_2 = \dfrac{M}{y^3}$

Since $B_1 = B_2$

$$\Rightarrow \quad \frac{2M}{x^3} = \frac{M}{y^3}$$

$$\Rightarrow \quad \frac{x^3}{y^3} = 2$$

$$\Rightarrow \quad \frac{x}{y} = 2^{\frac{1}{3}}$$

Hence, the correct answer is (D).

44. Since $\tan\phi' = \dfrac{\tan\phi}{\cos\beta}$

$$\Rightarrow \quad \tan(45°) = \frac{\tan\phi}{\cos(30°)}$$

$$\Rightarrow \quad \tan\phi = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Hence, the correct answer is (A).

45. The energy lost per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

Hence, the correct answer is (C).

46. Since, $\dfrac{\mu}{4\pi}\left(\dfrac{2M_1}{r_1^3}\right) = \dfrac{\mu_0}{4\pi}\left(\dfrac{2M_2}{r_2^3}\right)$

$$\Rightarrow \quad \frac{M_1}{M_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \quad \frac{1}{2} = \left(\frac{20}{r_2}\right)^3$$

$$\Rightarrow \quad r_2 = 20(2)^{\frac{1}{3}} \text{ cm}$$

Hence, the correct answer is (B).

47. Let ℓ be initial length of magnet. When magnet of magnetic moment $M(= m\ell)$ is cut in two equal parts, the magnetic moment of each part

$$M' = m\left(\frac{\ell}{2}\right) = \frac{m\ell}{2} = \frac{M}{2}$$

Moment of inertia of initial magnet

$$I = \frac{\text{mass} \times (\text{length})^2}{12} = \frac{m_0\ell^2}{12}$$

Moment of inertia of each half part

$$I' = \frac{\left(\dfrac{m_0}{2}\right)\left(\dfrac{\ell}{2}\right)^2}{12} = \frac{1}{8}\left[\frac{m_0\ell^2}{12}\right] = \frac{I}{8}$$

Initial time period is

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Time period of vibration of each half part

$$T' = 2\pi\sqrt{\frac{(I/8)}{(M/2)H}}$$

$$\Rightarrow \quad T' = \frac{1}{2}\left(2\pi\sqrt{\frac{I}{MH}}\right) = \frac{T}{2}$$

i.e. period is halved.

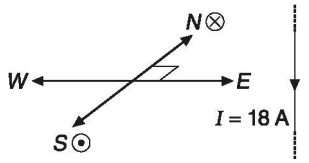
Hence, the correct answer is (A).

48. The horizontal magnetic field due to earth points from South to North i.e. inwards $\otimes$. Due to the wire magnetic field at the magnet is also directed inwards $\otimes$. So, both get added up.

Initially, $T = 2\pi\sqrt{\frac{I}{mB_H}}$ ...(1)

Finally, $T' = 2\pi\sqrt{\frac{I}{m(B+B_H)}}$ ...(2)

where B is the magnetic field due to the wire given by



$$B = \frac{\mu_0 i}{2\pi r} = 18\ \mu\text{T}$$

From (1) and (2), we get

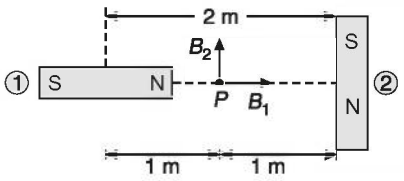
$$\frac{T'}{T} = \sqrt{\frac{B}{B+B_H}}$$

$$\Rightarrow \quad \frac{T'}{0.1} = \frac{24}{18+24}$$

$$\Rightarrow \quad T' = 0.076\ \text{s}$$

Hence, the correct answer is (C).

49. With respect to magnet 1, the point P lies at axial line, so



$$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right), \text{ rightwards}$$

$$\Rightarrow \quad B_1 = 10^{-7} \times \frac{2\times 1}{1} = 2\times 10^{-7}\ T$$

With respect to magnet 2, the point P lies at equitorial line, so

$$B_2 = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right), \text{ upwards}$$

$$\Rightarrow \quad B_2 = \frac{B_1}{2} = 10^{-7}\ \text{T}$$

Since B_1 and B_2 are mutually perpendicular, so the resultant magnetic field is

$$B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{(2\times 10^{-7})^2 + (10^{-7})^2}$$

$$\Rightarrow \quad B_R = \sqrt{5}\times 10^{-7}\ \text{T}$$

Hence, the correct answer is (B).

50. In tangent galvanometer, $i \propto \tan\theta$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow \quad \frac{i_1}{\frac{i_1}{\sqrt{3}}} = \frac{\tan 45^\circ}{\tan\theta_2}$$

$$\Rightarrow \quad \sqrt{3}\tan\theta_2 = 1$$

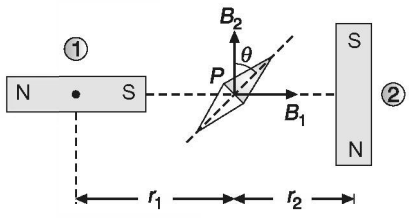
$$\Rightarrow \quad \tan\theta_2 = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta_2 = 30^\circ$$

So, deflection will decrease by $45^\circ - 30^\circ = 15^\circ$

Hence, the correct answer is (B).

51. At equilibrium $B_1 = B_2\tan\theta$



$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{2M}{r_1^3} = \frac{\mu_0}{4\pi}\frac{M}{r_2^3}\tan\theta$$

$$\Rightarrow \quad \frac{r_1}{r_2} = (2\cot\theta)^{\frac{1}{3}}$$

Hence, the correct answer is (C).

52. $n = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

$$\Rightarrow \quad n \propto \sqrt{M}$$

CHAPTER 2

Given $n_A = 2n_B$

$$\Rightarrow \frac{n_A}{n_B} = 2$$

$$\Rightarrow \sqrt{\frac{M_A}{M_B}} = 2$$

$$\Rightarrow \frac{M_A}{M_B} = 4$$

Hence, the correct answer is (C).

53. $K = \dfrac{2RB_H}{\mu_0 N}$

$$\Rightarrow N = \frac{2RB_H}{\mu_0 K} = \frac{2\times 0.1\times 3.6\times 10^{-5}}{4\pi\times 10^{-7}\times 10\times 10^{-3}}$$

$$\Rightarrow N = \frac{1.8\times 10^{3}}{3.14} = 570$$

Hence, the correct answer is (C).

54. $\dfrac{\mu_0}{4\pi}\dfrac{2M}{d^3} = B_H \tan\theta$

$$\Rightarrow M \propto \tan\theta$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{\tan 45^\circ}{\tan 30^\circ}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{1}{1/\sqrt{3}} = \frac{\sqrt{3}}{1}$$

Hence, the correct answer is (B).

55. Given that, $r_1 = 7.5$ cm , $r_2 = 10$ cm

$n_1 = 15$, $n_2 = 10$

$R_1 = 8$, $R_2 = 12$

$\theta_1 = 60^\circ$, $\theta_2 = ?$

Since $\dfrac{\mu_0}{2}\dfrac{n_1}{r_1}\times I_1 = \tan\theta_1$ and ...(1)

$$\frac{\mu_0}{2}\frac{n_2}{r_2}\times I_2 = \tan\theta_2 \qquad ...(2)$$

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{n_2}{n_1}\times\frac{r_1}{r_2}\times\frac{I_2}{I_1} \qquad ...(3)$$

However, $I_1 = \dfrac{R_2}{R_1+R_2}\times\dfrac{V}{R_p}$

$$\Rightarrow I_2 = \frac{R_2}{R_1+R_2}\times\frac{V}{R_p} \qquad ...(4)$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{R_1}{R_2}$$

From equation (3), we get

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{10}{15}\times\frac{75}{10}\times\frac{8}{12} = \frac{1}{3}$$

$$\Rightarrow \tan\theta_2 = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta_2 = 30^\circ$$

Hence, the correct answer is (B).

56. The weight of upper magnet should be balanced by the force of repulsion between the two magnets.

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{m^2}{r^2} = 50 \text{ gwt}$$

$$\Rightarrow 10^{-7}\times\frac{m^2}{(9\times 10^{-6})} = 50\times 10^{-3}\times 9.8$$

$$\Rightarrow m = 6.64 \text{ Am}$$

Hence, the correct answer is (A).

57. Since $\tan\phi' = \dfrac{\tan\phi}{\cos\beta}$

$$\Rightarrow \tan\phi_1 = \frac{\tan\phi}{\cos\beta} \qquad ...(1)$$

and $\tan\phi_2 = \dfrac{\tan\phi}{\cos(90-\beta)} = \dfrac{\tan\phi}{\sin\beta}$ ...(2)

From (1), $\cos\beta = \dfrac{\tan\phi}{\tan\phi_1}$

From (2), $\sin\beta = \dfrac{\tan\phi}{\tan\phi_2}$

Squaring and adding we get

$$\cos^2\beta + \sin^2\beta = \tan^2\phi\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

$$\Rightarrow \cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

Hence, the correct answer is (D).

58. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

where moment of inertia $I = \dfrac{M_0\ell^2}{12}$, M_0 is mass of magnet

When magnet is broken into two halves then length and mass both are halved, so

I becomes $\dfrac{\left(\frac{m_0}{2}\right)\left(\frac{\ell}{2}\right)^2}{12} = \dfrac{I}{8}$ and magnetic moment is also halved.

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{(I/8)}{(M/2)}}$$

$$\Rightarrow \quad T' = 2\pi\frac{2\pi}{2}\sqrt{\frac{I}{MB_H}}$$

$$\Rightarrow \quad \frac{T'}{T} = \frac{1}{2}$$

$$\Rightarrow \quad T' = \frac{T}{2} = \frac{4}{2} = 2 \text{ s}$$

Hence, the correct answer is (B).

59. Since $B \propto \dfrac{1}{x^3}$

$$\Rightarrow \quad \frac{B_1}{B_2} = \left(\frac{x_2}{x_1}\right)^3 = \left(\frac{3x}{x}\right)^3 = \frac{27}{1}$$

Hence, the correct answer is (C).

60. In first case magnet oscillates under total earth's field B_e

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{MB_e}{I}}$$

In second case magnet oscillates under vertical component B_V

$$\Rightarrow \quad n' = \frac{1}{2\pi}\sqrt{\frac{MB_V}{I}}$$

Since $B_V < B_e$, hence $n' < n$

Hence, the correct answer is (C).

61. From the relation $B_V = B\sin\phi$

$$\Rightarrow \quad B = \frac{B_V}{\sin\phi} = \frac{6\times10^{-5}}{\sin\ 40.6°} = \frac{6\times10^{-5}}{0.65} = 9.2\times10^{-5} \text{ T}$$

Hence, the correct answer is (D).

62. For short bar magnet in $\tan A$-position

$$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H\tan\theta \qquad \ldots(1)$$

In $\tan B$-position if deflection is θ', then

$$\frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H\tan\theta' \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{\tan\theta'}{\tan\theta} = \frac{1}{2}$$

$$\Rightarrow \quad \tan\theta' = \frac{\tan\theta}{2} = \frac{\tan 60°}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta' = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = \tan^{-1}(0.866)$$

$$\Rightarrow \quad \theta' = 40.9°$$

Hence, the correct answer is (B).

63. Since, $B^2 = B_V^2 + B_H^2$

$$\Rightarrow \quad B_V = \sqrt{B^2 - B_H^2} = \sqrt{(0.5)^2 - (0.3)^2} = 0.4$$

Also, $\tan\phi = \dfrac{B_V}{B_H} = \dfrac{0.4}{0.3} = \dfrac{4}{3}$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

Hence, the correct answer is (C).

64. Diamagnetic substances are repelled by a magnet. When a bar of diamagnetic material is kept in a magnetic field, it orients itself perpendicular to the field lines.
Hence, the correct answer is (D).

65. Since, $\chi \propto \dfrac{1}{T}$

$$\Rightarrow \quad \chi_1 T_1 = \chi_2 T_2$$

$$\Rightarrow \quad T_2 = \frac{1.2\times10^{-5}\times300}{1.8\times10^{-5}} = 200 \text{ K}$$

Hence, the correct answer is (B).

66. Soft iron has highest susceptibility.
Hence, the correct answer is (C).

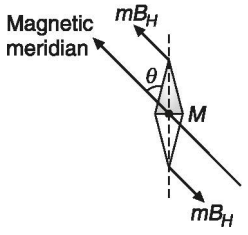
67. For null deflection, we have

$$\frac{M_1}{M_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{40}{50}\right)^3 = \frac{64}{125}$$

Hence, the correct answer is (C).

68. Magnetism of a magnet falls with rise of temperature and becomes practically zero above Curie temperature.
Hence, the correct answer is (C).

69. As the compass needle is free to rotate in a horizontal plane and points along the magnetic meridian,



CHAPTER 2

So, when it is pointing along the geographic meridian, it will experience a torque due to the horizontal component of earth's magnetic field, so

$\tau = MB_H \sin\theta$, where θ is the angle between geographical and magnetic meridian also called angle of declination.

$$\Rightarrow \quad \sin\theta = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Hence, the correct answer is (A).

70. Susceptibility of a paramagnetic substance is independent of magnetising field.

Hence, the correct answer is (A).

71. Since, $i = \frac{2RB_H}{\mu_0 N}\tan\theta$

$$\Rightarrow \quad i = \left(\frac{2\times15\times10^{-2}\times3\times10^{-5}}{4\pi\times10^{-7}\times25}\right)\tan(45°)$$

$$\Rightarrow \quad i = 0.29 \text{ A}$$

Hence, the correct answer is (A).

72. Susceptibility of a ferromagnetic substance falls with rise of temperature and the substance becomes paramagnetic above Curie temperature, so magnetic susceptibility becomes very small above Curie temperature.

Hence, the correct answer is (A).

73. Curie temperature of iron is 770 °C, so below 770 °C it is ferromagnetic and above 770 °C, it is paramagnetic.

Hence, the correct answer is (D).

74. Steel has the greatest retentivity. Retentivity is the property to retain magnetism after the magnet is withdrawn away from the substance to be magnetised.

Hence, the correct answer is (C).

75. $i \propto \tan\theta$

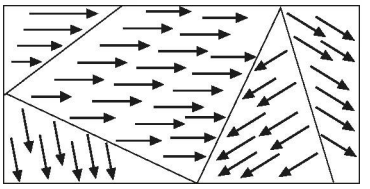
$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow \quad \frac{0.1}{i_2} = \frac{\tan30°}{\tan60°} = \frac{1}{3}$$

$$\Rightarrow \quad i_2 = 0.3 \text{ A}$$

Hence, the correct answer is (B).

76.



On heating, different domains have net magnetisation in them which are randomly distributed. Thus, the net magnetisation of the substance due to various domains decreases to minimum. Magnetisation is inversely proportional to temperature.

Hence, the correct answer is (C).

77. $M = \frac{CB}{T}$ is the Curie's Law.

Hence, the correct answer is (A).

78. $W = -MB(\cos\theta_2 - \cos\theta_1) = -MB(\cos60° - \cos0°)$

$$\Rightarrow \quad W = -MB\left(\frac{1}{2} - 1\right) = \frac{MB}{2}$$

Since $|\tau| = MB\ \sin\theta = MB\ \sin60° = MB\frac{\sqrt{3}}{2}$

$$\Rightarrow \quad \tau = \left(\frac{MB}{2}\right)\sqrt{3}$$

$$\Rightarrow \quad \tau = \sqrt{3}W$$

Hence, the correct answer is (A).

79. When a diamagnetic substance is placed in a non-uniform strong magnetic field, it always aligns itself perpendicular to the lines of induction, hence repulsion. Magnetisation, hence susceptibility is negative.

Hence, the correct answer is (D).

80. For a temporary magnet, the hysteresis loop should be long and narrow.

Hence, the correct answer is (D).

81. When magnet of length ℓ is cut into four equal parts, then we have

$$m' = \frac{m}{2} \text{ and } l' = \frac{l}{2}$$

$$\Rightarrow \quad M' = \frac{m}{2}\times\frac{l}{2} = \frac{ml}{4} = \frac{M}{4}$$

New moment of inertia, if w is mass of original magnet is

$$I' = \frac{wl^2}{12} = \frac{\frac{w}{4}\left(\frac{1}{2}\right)^2}{12} = \frac{1}{16}\frac{wl^2}{12}$$

$$\Rightarrow \quad I' = \frac{I}{16}$$

Time period of each part is given by

$$T' = 2\pi\sqrt{\frac{I'}{M'B_H}}$$

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{\frac{I}{16}}{\left(\frac{M}{4}\right)B_H}} = 2\pi\sqrt{\frac{I}{4MB_H}} = \frac{T}{2}$$

Hence, the correct answer is (C).

82. The bar magnet has coercivity 4×10^3 Am^{-1} i.e. it requires a magnetic intensity $H = 4\times10^3$ Am^{-1} to get demagnetised. Let i be the current carried by solenoid having n number of turns per metre length, then by definition

$$H = ni$$

Here $H = 4\times10^3$ Amp turn metre^{-1}

$$n = \frac{N}{\ell} = \frac{60}{0.12} = 500 \text{ turn metre}^{-1}$$

$$\Rightarrow \quad i = \frac{H}{n} = \frac{4\times10^3}{500} = 8.0 \text{ A}$$

Hence, the correct answer is (D).

83. Since, $\mu_r = 1 + \chi_m$

$$\Rightarrow \quad \mu_r = 1 + \frac{I}{H}$$

$$\Rightarrow \quad I = (\mu_r - 1)H$$

For a solenoid of n-turns per unit length and current i, we have $B = \mu_0 ni$

$$\Rightarrow \quad H = \frac{B}{\mu_0} = ni$$

$$\Rightarrow \quad I = (\mu_r - 1)ni = (1000-1)\times500\times0.5$$

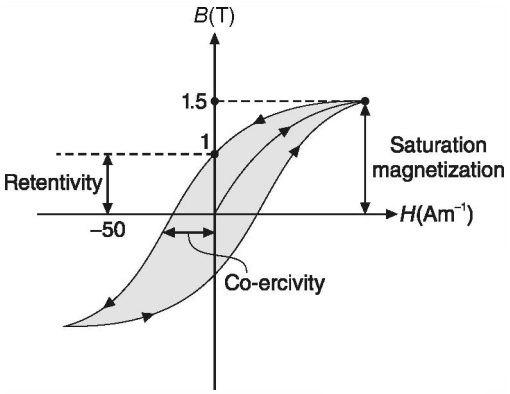
$$\Rightarrow \quad I = 2.5\times10^5 \text{ Am}^{-1}$$

Magnetic moment is given by $M = IV$

$$\Rightarrow \quad M = 2.5\times10^5\times10^{-4} = 25 \text{ Am}$$

Hence, the correct answer is (D).

84.



Retentivity $= 1.0$ T

Co-ercivity $= 50$ Am^{-1}

Saturation $= 1.5$ T

Hence, the correct answer is (B).

85. Since, $B_H = B\cos\phi$ and $B_V = B\sin\phi$

$$\frac{B_V}{B_H} = \tan\phi$$

$$\Rightarrow \quad B_V = B_H\tan\phi$$

$$\Rightarrow \quad B_V = 0.36\times10^{-4}\times\tan60° = 0.623\times10^{-4} \text{ Wbm}^{-2}$$

Hence, the correct answer is (D).

86. Iron is ferromagnetic and is strongly attracted by the magnetic field.

Hence, the correct answer is (A).

87. On passing current through the coil. It acts as a magnetic dipole. Torque acting on magnetic dipole should be counter balanced by the moment of additional weight about position O. Torque acting on a magnetic dipole is

$$\tau = MB\sin\theta = (NiA)B\sin90° = NiAB$$

Also, torque due to additional weight about O is

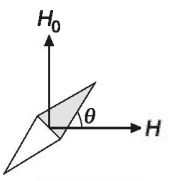
$$\tau = (\Delta m)gl$$

$$\Rightarrow \quad NiAB = (\Delta m)gl$$

$$\Rightarrow \quad B = \frac{(\Delta m)gl}{NiA} = \frac{60\times10^{-3}\times9.8\times30\times10^{-2}}{200\times22\times10^{-3}\times1\times10^{-4}} = 0.4 \text{ T}$$

Hence, the correct answer is (A).

88. In given case, H and H_0 are perpendicular to each other as shown in Figure.



CHAPTER 2

$$\Rightarrow \quad \tan\theta = \frac{H_0}{H}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{H_0}{H}\right)$$

Hence, the correct answer is (A).

89. The bar magnet coercivity is 4×10^3 Am^{-1} i.e., it requires a magnetic intensity $H = 4\times10^3$ Am^{-1} to get demagnetized.

Let i be the current carried by solenoid having n number of turns per metre length, then, we have

$H = ni$, where $H = 4\times10^3$ Am^{-1}

Since, $n = \frac{N}{l} = \frac{60}{0.12} = 500$ turn m^{-1}

$$\Rightarrow \quad i = \frac{H}{n} = \frac{4\times10^3}{500} = 8.0 \text{ A}$$

Hence, the correct answer is (D).

90. Since, $I = \frac{M}{V} = \frac{M}{\text{mass/density}}$,

Given that mass $= 1$ g $= 10^{-3}$ kg and

$$\rho = 5 \text{ gcm}^{-3} = \frac{5\times10^{-3} \text{ kg}}{(10^{-2})\text{ m}^3} = 5\times10^3 \text{ kgm}^{-3}$$

$$\Rightarrow \quad I = \frac{6\times10^{-7}\times5\times10^3}{10^{-3}} = 3$$

Hence, the correct answer is (B).

91. Since $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$$\Rightarrow \quad \frac{T_A}{T_B} = \sqrt{\frac{(B_H)_B}{(B_H)_A}}$$

$$\Rightarrow \quad \frac{\frac{60}{10}}{\frac{60}{20}} = \sqrt{\frac{(B_H)_B}{36\times10^{-6}}}$$

$$\Rightarrow \quad (B_H)_B = 144\times10^{-6} \text{ T}$$

Hence, the correct answer is (C).

92. When a paramagnetic substance is placed in a non-uniform magnetic field, the substance always aligns along the field lines, because by aligning along the lines of induction, magnetisation is maximum.

Hence, the correct answer is (A).

93. Assume the length of magnet to be l, mass be w and magnetic moment be M. When cut in six equal parts length each part becomes $\frac{l}{6}$, mass becomes $\frac{w}{6}$ and magnetic moment also becomes $\frac{M}{6}$. Moment of inertia of each part is

$$I' = \frac{1}{12}w'L^2 = \frac{1}{12}\left(\frac{w}{6}\right)\left(\frac{L}{6}\right)^2 = \frac{I}{6^3}$$

Initially, $T = 2\pi\sqrt{\frac{I}{MB_H}}$

For the arrangement shown,

$$I_{\text{net}} = 6I' = \frac{I}{6^2}$$

$$M_{\text{net}} = \frac{4M}{6} - \frac{2M}{6} = \frac{M}{3}$$

Time period of the arrangement is

$$T_{\text{net}} = \sqrt{\frac{\frac{I}{6^2}}{\left(\frac{M}{3}\right)B_H}} = \frac{T}{\sqrt{12}} = \frac{T}{2\sqrt{3}}$$

Hence, the correct answer is (C).

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1. According to Curie's Law,

$$\chi \propto \frac{1}{T}$$

$$\Rightarrow \quad \chi_1 T_1 = \chi_2 T_2$$

$$\Rightarrow \quad \chi_2 = \frac{2.8\times350}{300}\times10^{-4} = 3.267\times10^{-4}$$

Hence, the correct answer is (D).

2. Magnetic intensity (M) is

$$M = \chi H$$

$$\Rightarrow \quad \chi = \frac{20\times10^{-6}}{60\times10^3\times10^{-6}}$$

$$\Rightarrow \quad \chi = 3.3\times10^{-4}$$

Hence, the correct answer is (C).

3. Time period of hoop is

$$T_h = 2\pi\sqrt{\frac{I_h}{M_h B}}$$

Time period of cylinder is

$$T_c = 2\pi\sqrt{\frac{I_c}{M_c B}}$$

$$\Rightarrow \quad \frac{T_h}{T_c} = \sqrt{\frac{I_h M_c}{I_c M_h}}$$

Since $M_h = 2M_c$

$$\Rightarrow \quad \frac{T_h}{T_c} = \sqrt{\frac{(mR^2)(M_c)}{\left(\frac{mR^2}{2}\right)(2M_c)}} = 1$$

$$\Rightarrow \quad T_h = T_c$$

Hence, the correct answer is (B).

4. Since, $\frac{B_v}{B_H} = \tan(45°)$

$$\Rightarrow \quad B_v = B_H = 18\times10^{-6}\ \text{T}$$

Torque due to vertical component of field is

$$\tau_{B_v} = MB_v$$

where $M = m(2l)$

$$\Rightarrow \quad \tau_{B_v} = \left[(18\times10^{-6})(1.8)(0.12)\right]$$

Torque due to vertical force about centre is

$$\tau_f = F\left(\frac{0.12}{2}\right)$$

Equating both torques, we get

$$F = (18)(1.8\times10^{-6})(2)$$

$$\Rightarrow \quad F \simeq 6.5\times10^{-5}\ \text{N}$$

Hence, the correct answer is (C).

5. Since $\theta = \omega t$

$$\Rightarrow \quad \theta = 0.125\ \text{rad}$$

$$\Rightarrow \quad \theta = \frac{1}{8}\ \text{radian}$$

Initial and final potential energy are given by

$$U_i = -MB\cos\left(\frac{1}{8}\right)$$

$$U_f = MB\cos\left(\frac{1}{8}\right)$$

$$\Rightarrow \quad W = 2MB\cos\left(\frac{1}{8}\right) = 0.0198\ \text{J}$$

*No given option is correct.

6. Coercivity of ferromagnet is $H = 100\ \text{Am}^{-1}$

Since $H = ni = 100$

$$\Rightarrow \quad i = \frac{100}{10^5} = 1\ \text{mA}$$

Hence, the correct answer is (A).

7. Time period of magnetic needle oscillating simple harmonically is given by

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}}$$

$$\Rightarrow \quad T = \frac{2\pi}{10}\times1.05\ \text{s}$$

For 10 oscillations, total time taken

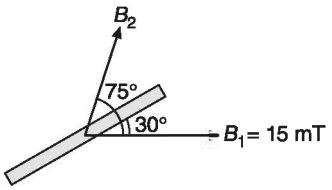
$$T' = 10T = 2\pi\times1.05 \approx 6.65\ \text{s}$$

Hence, the correct answer is (A).

8. For both, the electromagnet and transformer, the magnetic field changes with time. Hence the energy losses must be less in both devices. Hysteresis loop represented in B has less area which means it dissipates less energy.

Hence, the correct answer is (D).

9. The magnetic dipole attains stable equilibrium under the influence of these two fields making an angle $\theta_1 = 30°$ with B_1 and $\theta_2 = 75° - 30° = 45°$ with B_2 as shown in Figure.



For stable equilibrium, net torque acting on dipole must be zero.

$$\Rightarrow \quad \vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$\Rightarrow \quad \tau_1 = \tau_2$$

$$\Rightarrow \quad MB_1\sin\theta_1 = MB_2\sin\theta_2$$

$$\Rightarrow \quad B_2 = B_1\frac{\sin\theta_1}{\sin\theta_2} = (15\ \text{mT})\left(\frac{\sin30°}{\sin45°}\right)$$

$$\Rightarrow \quad B_2 = (15\ \text{mT})\left(\frac{1}{2}\right)\sqrt{2} = 10.6\ \text{mT} \approx 11\ \text{mT}$$

Hence, the correct answer is (B).

CHAPTER 2

10. $\ell = 25$ cm , $r = 2$ cm , $N = 500$, $i = 15$ A

Since, $|\vec{M}| = \dfrac{\text{Magnetic moment}}{\text{Volume}} = \dfrac{NiA}{A\ell} = \dfrac{Ni}{\ell}$

$$\Rightarrow \quad |\vec{M}| = \frac{15 \times 500}{25 \times 10^{-2}} = 30000 \text{ Am}^{-1}$$

Hence, the correct answer is (B).

11. At 30 cm from the magnet on its equatorial plane for the neutral point, we get

$$\left|\vec{B}_{\text{magnet}}\right| = \left|\vec{B}_H\right|$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{M}{r^3} = 3.6 \times 10^{-5}$$

$$\Rightarrow \quad \frac{10^{-7} \times M}{(0.3)^3} = 3.6 \times 10^{-5}$$

$$\Rightarrow \quad M = 3.6 \times 0.027 \times 10^2 = 9.7 \text{ Am}^2$$

Hence, the correct answer is (A).

12. Given that $\dfrac{B}{\mu_0} = 3 \times 10^3$ Am^{-1}

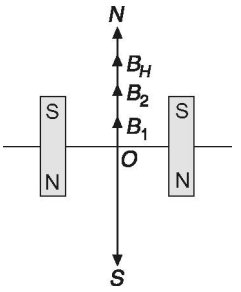
$L = 10$ cm $= 0.1$ m , $N = 100$, $i = ?$

Since, $B = \mu_0 ni = \mu_0 \dfrac{N}{L} i$

$$\Rightarrow \quad i = \frac{B}{\mu_0} \times \frac{L}{N} = 3 \times 10^3 \times \frac{0.1}{100} = 3 \text{ A}$$

Hence, the correct answer is (D).

13. The situation is shown in Figure.



As the point O lies on broad-side position with respect to both the magnets. Therefore, the net magnetic field at point O is

$$B_{net} = B_1 + B_2 + B_H$$

$$\Rightarrow \quad B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3} + B_H$$

$$\Rightarrow \quad B_{\text{net}} = \frac{\mu_0}{4\pi r^3}(M_1 + M_2) + B_H$$

Substituting the given values, we get

$$B_{net} = \frac{4\pi \times 10^{-7}}{4\pi \times (10 \times 10^{-2})^3}(1.2 + 1) + 3.6 \times 10^{-5}$$

$$B_{\text{net}} = \left(\frac{10^{-7}}{10^{-3}}\right)(2.2) + 3.6 \times 10^{-5}$$

$$\Rightarrow \quad B_{\text{net}} = 2.2 \times 10^{-4} + 0.36 \times 10^{-4}$$

$$\Rightarrow \quad B_{\text{net}} = 2.56 \times 10^{-4} \text{ Wbm}^{-2}$$

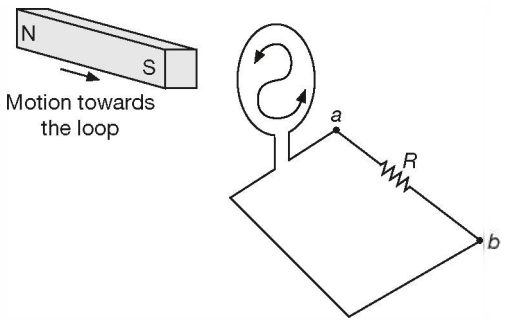
Hence, the correct answer is (C).

CHAPTER 3: ELECTROMAGNETIC INDUCTION

Test Your Concepts-I (Based on Magnetic Flux, Faraday's Laws and Induced EMF)

1. **METHOD I**

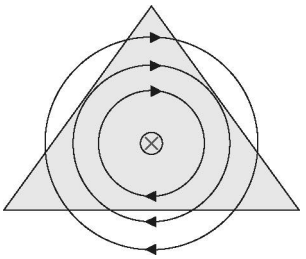
When the S pole moves towards the loop, it must behave as a S pole (as seen from the magnet's side) so, we have the current flowing from b to a and hence $V_a < V_b$ i.e., $V_a - V_b$ is negative



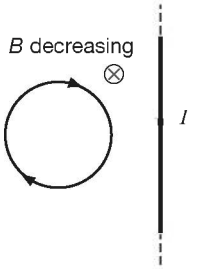
METHOD II

Look in the direction of ba. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counter clockwise current. Therefore, current must flow from b to a through the resistor. Hence, $V_a - V_b$ is negative.

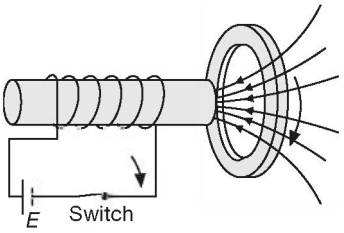
2. Magnetic field lines round the current carrying wire are as shown in figure. Since the lines are tangential to the loop $(\theta = 90°)$ the flux passing through the loop is zero, whether the current is increased or decreased. Change in flux is zero. Therefore, induced current in the loop will be zero.



3. Since I is decreasing in the wire, so the loop lies in an inward decreasing field and hence the induced current must not allow the inward field to decrease.
So, it will be in clockwise sense.

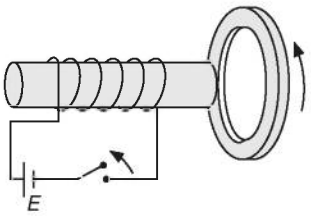


4. (a) At the instant the switch is thrown closed, the situation changes from one in which no magnetic flux exists in the ring to one in which flux exists and the magnetic field is to the left as shown in figure. The counteract this change in the flux, the current induced in the ring must set up a magnetic field directed from left to right in figure. This requires a current directed as shown.



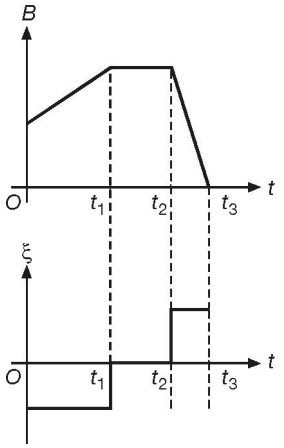
(b) After the switch has been closed for several seconds, no change in the magnetic flux through the loop occurs; hence, the induced current in the ring is zero.

(c) Opening the switch changes the situation from one in which magnetic flux exists in the ring to one in which there is no magnetic flux. The direction of the induced current is as shown in figure because current in this direction produces a magnetic field that is directed right to left and so counteracts the decrease in the flux produced by the solenoid.



5. $\xi = -\dfrac{d\phi_B}{dt}$

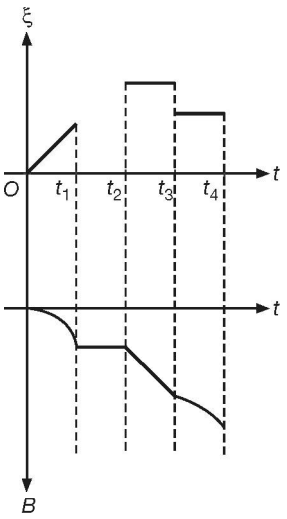
$\Rightarrow \quad \xi = -A\dfrac{dB}{dt}$



6. When the wire is moved to the other side, even though the connections have not changed, bulb 1 goes out and bulb 2 glows. The bulb that is shorted depends on which side of the changing field the switch is positioned! In figure (a), because the branch containing bulb 2 is infinitely more resistant than the branch containing the resistance-free switch, we can imagine removing the branch with the bulb without altering the circuit. Then we have a simple loop containing only bulb 1, which glows.

When the wire is moved, as in figure (b), there are two possible paths for current below points a and b. We can imagine removing the branch with bulb 1, leaving only a single loop with bulb 2.

7.



8. The enclosed flux is

$$\phi_B = BA = B(\pi r^2)$$

$$qvB\sin(90°) = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

$$\Rightarrow \quad \phi_B = \frac{B\pi m^2 v^2}{q^2 B^2}$$

(a) $$v = \sqrt{\frac{\phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{(15\times10^{-6})(30\times10^{-9})^2(0.6)}{\pi(2\times10^{-16})^2}}$$

$$\Rightarrow \quad v = 2.54\times10^5 \text{ ms}^{-1}$$

(b) Since, $q\Delta V = \dfrac{1}{2}\text{ mv}^2$

$$\Rightarrow \quad \Delta V = \frac{mv^2}{2q} = \frac{(2\times10^{-16})(2.54\times10^5)^2}{2(30\times10^{-9})} = 215 \text{ V}$$

9. Due to the current in A a magnetic field is from right to left. When A is moved towards B, magnetic lines passing through B (from right to left) will increase, i.e., magnetic flux passing through B will increase. Therefore, current will be induced in B. The induced current will have such a direction that it gives a magnetic field opposite to that, was passing through B due to current in A. Therefore, induced current in B will be in opposite direction of current in A.

The current in C will decrease due to movement of coil A and this will give rise to an induced current in B in same direction as that of C, but since B is more closer to A, therefore net induced current will be opposite to current in A and C.

10. (a) The flux through the loop at time t is

$$\phi_B = BA\cos\theta = (a+bt)(\pi r^2)\cos0° = \pi(a+bt)r^2$$

At $t = 0$, $\phi_B = \pi a r^2$

(b) $$\xi = -\frac{d\phi_B}{dt} = -\pi r^2\frac{d(a+bt)}{dt} = -\pi b r^2$$

(c) If R be the resistance of the loop, then

$$R = \lambda(2\pi r)$$

$$\Rightarrow \quad I = \frac{\xi}{R} = -\frac{\pi b r^2}{R} = -\frac{\pi b r^2}{\lambda(2\pi r)} = \frac{br}{2\lambda}$$

(d) $$P = \xi I = (-\pi b r^2)\left(-\frac{br}{2\lambda}\right) = \frac{\pi b^2 r^3}{2\lambda}$$

11. $\phi_i = NBA\cos(0°) = NBA$

$\phi_f = NBA\cos(180°) = -NBA$

Since $I = \dfrac{\xi}{R} = \dfrac{1}{R}\dfrac{\Delta\phi}{\Delta t}$

$\Rightarrow \quad I\Delta t = \dfrac{1}{R}\Delta\phi$

$\Rightarrow \quad |Q| = \dfrac{1}{R}(2NBA)$

$\Rightarrow \quad |Q| = \left(\dfrac{1}{50}\right)(2)(500)(0.2)(4\times10^{-4})$

$\Rightarrow \quad |Q| = 1.6 \text{ mC}$

12. The perimeter of the wire and the square loop will be the same, so

$2\pi r = 4\ell$

$\Rightarrow \quad \ell = \dfrac{\pi r}{2}$

Now, initial area of loop is $A_i = \pi r^2$ and final area of the loop is $A_f = \ell^2 = \dfrac{\pi^2 r^2}{4}$

$\xi = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\left|\dfrac{A_f - A_i}{t}\right|$

$\Rightarrow \quad \xi = \left(\dfrac{100}{0.1}\right)\left|\dfrac{\pi^2 r^2}{4} - \pi r^2\right|$

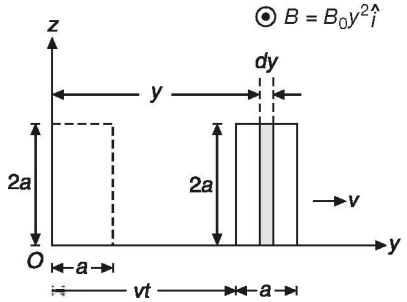
$\Rightarrow \quad \xi = \left(\dfrac{100}{0.1}\right)(0.1)^2\left|\dfrac{\pi^2}{4} - \pi\right|$

$\Rightarrow \quad \xi = (10)(0.675)$

$\Rightarrow \quad \xi = 6.75 \text{ V}$

13. Let at an instant t, the loop is at the position shown in Figure. Consider a small element of length $2a$ and width dy. If dA is the area of the element, then

$dA = 2a\,dy$



The magnetic flux associated with the loop at this instant is

$\phi = \int \vec{B}\cdot d\vec{A} = \int_{vt}^{vt+a} B_0y^2(2a\,dy)$

$\Rightarrow \quad \phi = 2aB_0\left(\dfrac{y^3}{3}\right)\Big|_{vt}^{vt+a}$

$\Rightarrow \quad \phi = 2aB_0\left[\dfrac{(vt+a)^3 - (vt)^3}{3}\right]$

Since $|\xi| = \dfrac{d\phi}{dt}$

$\Rightarrow \quad |\xi| = 2aB_0[(vt+a)^2 - (vt)^2]v$

$\Rightarrow \quad |\xi| = 2aB_0v(a)(2vt+a)$

$\Rightarrow \quad |\xi| = 2a^2B_0v(2vt+a)$

14. $\xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad |\xi| = \left|\dfrac{d\phi}{dt}\right| = NA\left(\dfrac{dB}{dt}\right)$

where $\dfrac{dB}{dt} = (0.01 + 0.08t)$

$\Rightarrow \quad |\xi|_{\text{at } t=5 \text{ s}} = (30)[\pi(0.04)^2](0.01+0.4)$

$\Rightarrow \quad |\xi|_{\text{at } t=5 \text{ s}} = 62 \text{ mV}$

15. The area of one turn of the coil is $(0.18 \text{ m})^2 = 0.0324 \text{ m}^2$. The magnetic flux through the coil at $t = 0$ is zero because $B = 0$ at that time. At $t = 0.8$ s, the magnetic flux through one turn is $\phi_B = BA = (0.5 \text{ T})(0.0324 \text{ m}^2) = 0.0162 \text{ Tm}^2$. Therefore, the magnitude of the induced emf is

$|\xi| = N\left(\dfrac{\Delta\phi_B}{\Delta t}\right) = 200\dfrac{(0.0162 - 0)}{0.8}$

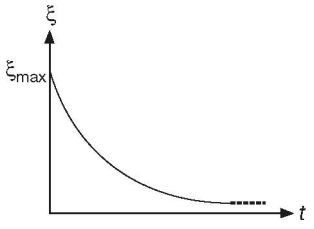
$\Rightarrow \quad |\xi| = 4.1 \text{ Tm}^2\text{s}^{-1} = 4.1 \text{ V}$

You should be able to show that

$1 \text{ Tm}^2\text{s}^{-1} = 1 \text{ V}$

16. Since $\vec{B}$ is perpendicular to the plane of the loop, the magnetic flux through the loop at time $t > 0$ is

$\phi_B = BA\cos(0°) = AB_0e^{-at}$



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Since AB_0 and a are constants, so the induced emf is given by

$$\xi = -\frac{d\phi_B}{dt} = -AB_0\frac{d}{dt}(e^{-at})$$

$$\Rightarrow \quad \xi = aAB_0e^{-at}$$

This expression indicates that the induced emf decays exponentially in time. Note that the maximum emf occurs at $t = 0$, where $\xi_{max} = aAB_0$. The plot of ξ versus t is shown in figure.

17. The upper loop has area $\pi(0.05\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. The induced emf in it is

$$\xi = -N\frac{d}{dt}(BA\cos\theta) = -1A\cos0°\frac{dB}{dt}$$

$$\Rightarrow \quad \xi = -7.85\times10^{-3}\text{ m}^2(2\text{ Ts}^{-1}) = -1.57\times10^{-2}\text{ V}$$

The minus sign indicates that it tends to produce counter clockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$$\xi = -NA\cos\theta\left(\frac{dB}{dt}\right) = -\pi(0.09\text{ m})^2\,2\text{ Ts}^{-1}$$

$$\Rightarrow \quad \xi = -5.09\times10^{-2}\text{ V} = +5.09\times10^{-2}\text{ V}$$

to produce counter clockwise current in the lower loop, which becomes clockwise current in the upper loop.

The net emf for current in this sense around the digit 8 is

$$\xi_{net} = 5.09\times10^{-2} - 1.57\times10^{-2} = 3.52\times10^{-2}\text{ V}$$

This net emf pushes the current in this sense through a series resistance given by

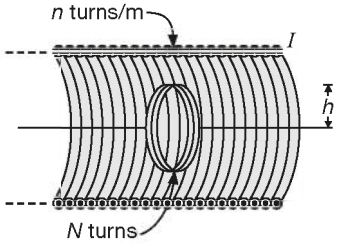
$$R = [2\pi(0.05\text{ m}) + 2\pi(0.09\text{ m})]3\ \Omega\text{m}^{-1} = 2.64\ \Omega$$

The current is $I = \dfrac{\xi_{net}}{R} = \dfrac{3.52\times10^{-2}\text{ V}}{2.64\ \Omega} = 13.3\text{ mA}$

18. $B = \mu_0 nI = \mu_0 n(30)(1-e^{-1.6t})$

Since, $\phi_B = \int BdA = \mu_0 n(30)(1-e^{-1.6t})\int dA$

$$\Rightarrow \quad \phi_B = \mu_0 n(30)(1-e^{-1.6t})\pi R^2$$



From Faraday's Laws, we have

$$\xi = -N\frac{d\phi_B}{dt} = -N\mu_0 n(30)\pi R^2(1.6)e^{-1.6t}$$

$$\Rightarrow \quad \xi = -(250)(4\pi\times10^{-7})(400)(30)$$
$$[\pi(0.06)^2](1.6e^{-1.6t})$$

$$\Rightarrow \quad \xi = (68)e^{-1.6t}\text{ mV}\text{, counter clockwise}$$

19. From Angular Impulse - Angular Momentum Theorem, we have

$$\tau dt = I\omega$$

$$\Rightarrow \quad (MB\sin90°)dt = \frac{2}{3}mR^2\omega$$

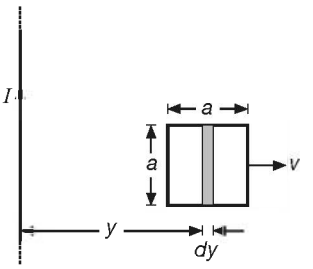
$$\Rightarrow \quad (NIA)Bdt = \frac{2}{3}mR^2\omega$$

$$\Rightarrow \quad (N\pi R^2Bq) = \frac{2}{3}mR^2\omega \qquad \{I\,dt = q\}$$

$$\Rightarrow \quad \omega = \frac{1.5\pi NBq}{m} = \frac{(1.5\pi)(20)(0.5\times10^{-4})(5\times3600)}{10^3}$$
$$\{\text{where } Q = 5\text{ Ahr} = 5\times3600\text{ C}\}$$

$$\Rightarrow \quad \omega = 27\pi\times10^{-3}\text{ rads}^{-1}$$

20. Consider a rectangular strip of the frame as shown. If dA is the area of the strip then, $dA = ady$



Flux associated with this strip is

$$d\phi = BdA = \left(\frac{\mu_0 I}{2\pi y}\right)(a\,dy)$$

Total flux associated with the strip is

$$\phi = \int d\phi = \frac{\mu_0 Ia}{2\pi}\int_x^{x+a}\frac{dy}{y} = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{x+a}{x}\right)$$

Now $\xi = -\dfrac{d\phi}{dt}$

$$\Rightarrow \quad \xi = \frac{\mu_0 Ia}{2\pi}\left(\frac{x}{x+a}\right)\frac{d}{dx}\left(1+\frac{a}{x}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I a^2 x}{2\pi(x+a)} \frac{1}{x^2}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I a^2 v}{2\pi x(x+a)} \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

21. Please note that the loops are connected in a manner such that if current in one is clockwise then current in the other is counter clockwise i.e., the emf in loop b opposes the emf in loop a. If ξ_1 be the emf associated with loop a, then

$$\xi_1 = \frac{d}{dt}(a^2 B) = a^2\left(\frac{dB}{dt}\right) = a^2 B_0 \omega \cos(\omega t) \qquad \ldots(1)$$

If ξ_2 is the emf associated with loop b, then

$$\xi_2 = \frac{d}{dt}(b^2 B) = b^2\left(\frac{dB}{dt}\right) = b^2 B_0 \omega \cos(\omega t) \qquad \ldots(2)$$

So, net emf ξ is

$$\xi = B_0 \omega (a^2 - b^2)\cos(\omega t)$$

Since the resistance of the circuit is $R = 4\lambda(a+b)$, hence the current amplitude is

$$I_0 = \frac{B_0 \omega (a^2 - b^2)}{4\lambda(a+b)}$$

$$\Rightarrow \quad I_0 = \frac{B_0 \omega (a-b)}{4\lambda} = \frac{(10\times10^{-3})(100)\left(\frac{10}{100}\right)}{4(50\times10^{-3})}$$

$$\Rightarrow \quad I_0 = 0.5 \text{ A}$$

22. Since, the induced emf is given by

$$\xi = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K \quad \{\because N = 1\}$$

(a) $Q = C\xi = \pi a^2 CK$

(b) Since $\vec{B}$ into the paper is decreasing, therefore the induced current will attempt to counteract this. So, the positive charge will go to upper plate.

(c) The changing magnetic field through the enclosed area induces an electric field, surrounding the magnetic field that causes the separation of the charges.

23. For a discharging RC circuit

$$q = q_0 e^{-t/RC}$$

$$\Rightarrow \quad |I| = \frac{dq}{dt} = \frac{q_0}{RC} e^{-\frac{t}{RC}} = \frac{CV_0}{RC} e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

where V_0 is the initial voltage across the capacitor. The resistance of the small loop is

$$(25)(0.6 \text{ m})(1\ \Omega\text{m}^{-1}) = 15\ \Omega$$

(a) The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)(20\times10^{-6}\text{ F}) = 200\ \mu\text{s}$. So, the current as a function of time is

$$I = \frac{V_0}{R} e^{-\frac{t}{RC}} = 10 e^{-\frac{t}{200}}$$

At $t = 200\ \mu$s, we obtain $I = (10\text{ A})(e^{-1}) = 3.7$ A

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop then, we get

$$\phi_B = \int_c^{c+a} \frac{\mu_0 Ib}{2\pi r} dr = \frac{\mu_0 Ib}{2\pi} \log_e\left(1 + \frac{a}{c}\right)$$

So, the emf induced in the small loop at $t = 200\ \mu$s is $\xi = -\frac{d\phi}{dt} = -\frac{\mu_0 b}{2\pi}\log_e\left(1+\frac{a}{c}\right)\frac{dI}{dt}$, where $\frac{dI}{dt} = -\frac{I_0}{RC} e^{-\frac{t}{RC}}$

So, at $t = 200\ \mu$s, we get

$$\xi = -\frac{(4\pi\times10^{-7})(0.2\text{ m})}{2\pi}\log_e(3)\left(-\frac{3.7\text{ A}}{200\times10^{-6}\text{ s}}\right)$$

$$\xi = +0.81 \text{ mV}$$

Thus, the induced current in the small loop is

$$I' = \frac{\xi}{R} = \frac{0.81\text{ mV}}{15\ \Omega} = 54\ \mu\text{A}$$

(c) The magnetic field from the large loop is directed out of the page within the small loop. The induced current will act to oppose the decrease in flux from the large loop. Hence, the induced current flows counter clockwise.

(d) Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop. This can be seen by comparing the distance c to the dimensions of the large loop.

24. $$I = \frac{|\xi|}{R} = \frac{1}{R}\left(\frac{d\phi}{dt}\right) = \frac{A}{R}\left(\frac{dB}{dt}\right) \qquad \ldots(1)$$

where A is the area of the loop and R is the resistance of the loop.

Since $m = Vd = (\pi a^2)(2\pi r)d$ $\ldots(2)$

Also, $R = \frac{\rho \ell_{\text{wire}}}{\pi a^2} = \frac{2\pi r}{\sigma(\pi a^2)}$ $\ldots(3)$

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$$\Rightarrow \quad I=\frac{\sigma(\pi r^2)(\pi a^2)}{2\pi r}\left(\frac{dB}{dt}\right)=\frac{\sigma(\pi r)(\pi a^2)}{2\pi}\left(\frac{dB}{dt}\right)$$

Since from (2), we have

$$(\pi a^2)(\pi r)=\frac{m}{2d}$$

$$\Rightarrow \quad I=\left(\frac{\sigma m}{4\pi d}\right)\frac{dB}{dt}$$

This expression is independent of the size of the wire as well as the loop.

Test Your Concepts-II (Based on Faraday's Laws: Motional EMF)

1. $F=BI\ell$ where $I=\frac{\xi}{R}=\frac{B\ell v}{R}$

$$\Rightarrow \quad B=\frac{IR}{\ell v}$$

(a) $F=1\text{ N}=BI\ell$

$$\Rightarrow \quad 1=\left(\frac{IR}{\ell v}\right)I\ell$$

$$\Rightarrow \quad I^2R=v$$

$$\Rightarrow \quad I=\sqrt{\frac{v}{R}}=\sqrt{\frac{2}{8}}=\sqrt{\frac{1}{4}}=0.5\text{ A}$$

This could have been done directly by using the relation, $P=Fv=I^2R$

$$\Rightarrow \quad I=\sqrt{\frac{Fv}{R}}=0.5\text{ A}$$

(b) $P=I^2R=(0.5)^2 8=2\text{ W}$

(c) $P_{\text{mech}}=Fv=(1)(2)=2\text{ W}$

2. When the terminal speed is obtained, then we have

$$Mg=F_B$$

$$\Rightarrow \quad Mg=(BI)w\text{, where } I=\frac{Bwv_T}{R}$$

$$\Rightarrow \quad Mg=\frac{B^2w^2v_T}{R}$$

$$\Rightarrow \quad v_T=\frac{MgR}{B^2w^2}$$

3. $B=\frac{\mu_0 I}{2\pi r}$

$$\xi=B\ell v=\frac{\mu_0 Iv\ell}{2\pi r}$$

$$\Rightarrow \quad I=\frac{\xi}{R}=\frac{\mu_0 Iv\ell}{2\pi rR}$$

4. $I=\frac{\xi}{R_{\text{net}}}$, where $\xi=B\ell v$ and $R_{\text{net}}=R+\frac{R_1R_2}{R_1+R_2}$

$$\Rightarrow \quad I=\frac{B\ell v}{\left(R+\frac{R_1R_2}{R_1+R_2}\right)}$$

5. $mg\sin\alpha-F_m=ma$

$mg\sin\alpha-BI\ell=ma$, where $I=\frac{B\ell v}{R}$

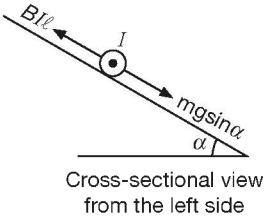
For steady state, $a=0$

$$\Rightarrow \quad mg\sin\alpha=BI\ell$$

$$\Rightarrow \quad mg\sin\alpha=B\left(\frac{B\ell v}{R}\right)\ell$$

$$\Rightarrow \quad mg\sin\alpha=\left(\frac{B^2\ell^2}{R}\right)v$$

$$\Rightarrow \quad v=\frac{mgR\sin\alpha}{B^2\ell^2}$$



6. Potential difference between the two rails :

$$\xi=B\ell v$$

(When $\vec{B}$, $\vec{v}$ and $\vec{l}$ all are mutually perpendicular)

$$\Rightarrow \quad \xi=\left(0.2\times10^{-4}\right)\left(180\times\frac{5}{18}\right)(1)=10^{-3}\text{ V}=1\text{ mV}$$

7. Since the network given, forms a balanced Wheatstone bridge so, the net resistance of the circuit is $3\ \Omega+1\ \Omega=4\ \Omega$.

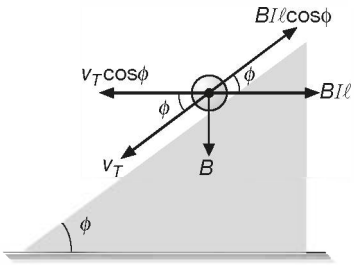
Since the induced emf is given by $B\ell v_o$. Therefore, current in the circuit is given by

$$I=\frac{B\ell v_0}{R}$$

$$\Rightarrow \quad v_o=\frac{IR}{B\ell}=\frac{\left(1\times10^{-3}\right)(4)}{2\times0.1}=0.02\text{ ms}^{-1}$$

Inward magnetic field passing through the loop is decreasing. Therefore, induced current will produced magnetic field in cross direction. So, the direction of induced current is clockwise.

8. (a)



(b) $F_{net} = 0$

$$\Rightarrow \quad mg\sin\phi = BI\ell\cos\phi = B\left(\frac{B\ell v_T\cos\phi}{R}\right)(\ell\cos\phi)$$

$$\Rightarrow \quad v_T = \frac{mgR\sin\phi}{B^2\ell^2\cos^2\phi}$$

(c) $$I = \frac{B\ell v_T\cos\phi}{R} = \frac{mg\sin\phi}{B\ell\cos\phi} = \frac{mg\tan\phi}{B\ell}$$

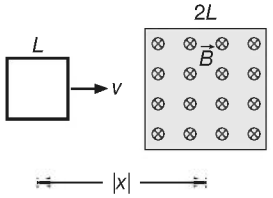
(d) $$P_1 = I^2R = \frac{m^2g^2R\tan^2\phi}{B^2\ell^2}$$

(e) $$P_2 = Fv = (mg\sin\theta)v_T = mgv_T\sin\phi$$

$$\Rightarrow \quad P_2 = \frac{m^2g^2R\tan^2\phi}{B^2\ell^2}$$

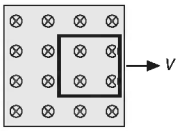
We observe that both are equal.

9. The loop before it starts to enter the magnetic field region is shown in figure.



For $x < -\frac{3L}{2}$ or $x > \frac{3L}{2}$ the loop is completely outside the field region. $\phi_B = 0$, and $\frac{d\phi_B}{dt} = 0$.

Thus $\xi = 0$ and $I = 0$, so there is no force from the magnetic field and the external force F necessary to maintain constant velocity is zero.



When the loop is completely inside the field region i.e., for $-\frac{L}{2} < x < \frac{L}{2}$, we have

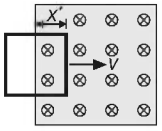
$$\phi_B = BL^2 = \text{constant}$$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = 0$$

$$\Rightarrow \quad \xi = 0 \text{ and } I = 0$$

There is no force $\vec{F} = I\vec{\ell}\times\vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

As the loop enters the magnetic field region i.e., for $-\frac{3L}{2} < x < -\frac{L}{2}$, if x' be the length of the loop that is within the field, then $|\phi_B| = BLx'$ and $\left|\frac{d\phi_B}{dt}\right| = BLv$.

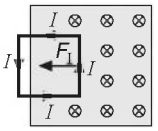


The magnitude of the induced emf is $|\xi| = \left|\frac{d\phi_B}{dt}\right| = BLv$ and the induced current is $I = \frac{|\xi|}{R} = \frac{BLv}{R}$. The current induced in the loop is counterclockwise.

The induced current and magnetic force on the loop are shown in figure for the situation when the loop is entering the field.

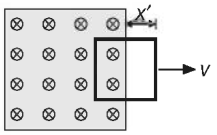
Since, $\vec{F}_1 = I\vec{\ell}\times\vec{B}$, so the force $\vec{F}_1$ exerted on the loop by the magnetic field is to the left and has magnitude

$$F_1 = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}.$$



The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_1$ so is to the right and has this same magnitude.

As the loop leaves the magnetic field region i.e., for $\frac{L}{2} < x < \frac{3L}{2}$ the loop is leaving the field region. Let x' be the length of the loop that lies outside the field.



Then $|\phi_B| = BL(L - x')$ and $\left|\frac{d\phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\xi| = \left|\frac{d\phi_B}{dt}\right| = BLv$ and the induced current is $I = \frac{|\xi|}{R} = \frac{BLv}{R}$. The current induced in the loop is clockwise.

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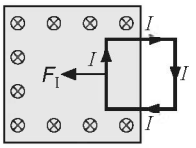
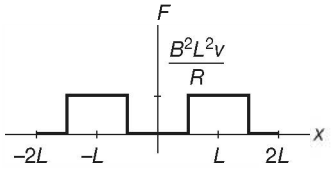
Now, the induced current and magnetic force on the loop are shown in figure for the situation when the loop is leaving the field.

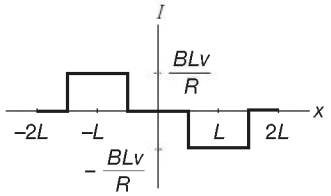
Since, $\vec{F}_I = I\vec{\ell} \times \vec{B}$, so the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude

$$F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$$



The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude. So, the graphs are shown below.



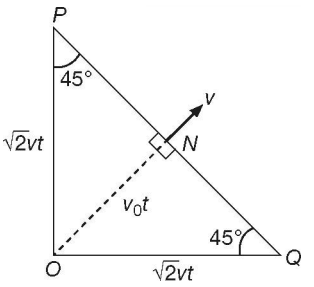


Please note that when the loop is either totally outside or totally inside the magnetic field region the flux is not changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field, the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's Law according to which, the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

10. $ON = vt$

$\Rightarrow \quad OQ = OP = \sqrt{2}\,vt$

$\Rightarrow \quad PQ = \sqrt{OP^2 + OQ^2} = 2vt$



The flux associated with this triangle at time t is

$$\phi = B(\text{Area of triangle at time } t)$$

$$\Rightarrow \quad \phi = B\left(\frac{1}{2}(\sqrt{2}vt)(vt)\right) = Bv^2t^2$$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = Bv^2(2t)$$

$$\xi = 2Bv^2t \qquad \ldots(1)$$

Since $I = \frac{\xi}{R}$

$$\Rightarrow \quad I = \frac{2Bv^2t}{\lambda(\sqrt{2}vt + \sqrt{2}vt + 2vt)}$$

$$\Rightarrow \quad I = \frac{Bv}{\lambda(1+\sqrt{2})} = \text{constant}$$

11. (a) $\phi = BA = B(\ell - 2vt)^2$

$$\xi = -\frac{d\phi}{dt} = -2B(\ell - 2vt)(-2v)$$

$$\Rightarrow \quad \xi = 4Bv(\ell - 2vt)$$

(b) $I = \frac{\xi}{R} = \frac{B\ell v}{\lambda \ell} = \frac{Bv}{\lambda}$

(c) $F = BI(\ell - 2vt)$

$$\Rightarrow \quad F = B\left(\frac{Bv}{\lambda}\right)(\ell - 2vt)$$

$$\Rightarrow \quad F = \frac{B^2v}{\lambda}(\ell - 2vt)$$

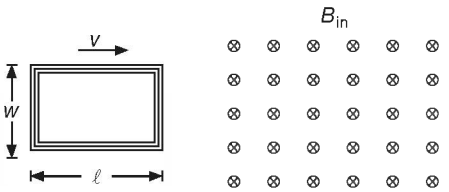
(d) $P = 4Fv = \frac{4B^2v^2}{\lambda}(\ell - 2vt)$

(e) $P_{\text{Thermal}} = \frac{\xi^2}{R_{\text{total}}} = \frac{16B^2v^2(\ell - 2vt)^2}{4(\ell - 2vt)\lambda}$

$$\Rightarrow \quad P_{th} = \frac{4B^2v^2(\ell - 2vt)}{\lambda}$$

12. (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$



The induced emf in the coil is

$$|\xi| = N\frac{d\phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$

So, the current is $I = \frac{|\xi|}{R} = \frac{NBwv}{R}$ counter clockwise

The force on the leading side of the coil is then

$$F = N\left(\frac{NBwv}{R}\right)wB = \frac{N^2B^2w^2v}{R} \text{ to the left}$$

(b) Once the coil is entirely inside the field,

$\phi_B = NBA = \text{constant}$,

$\Rightarrow \quad \xi = 0$, $I = 0$ and hence $F = 0$

(c) As the coil starts to leave the field, the flux decreases at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is

$$F = \frac{N^2B^2w^2v}{R} \text{ to the left again}$$

13. The emf induced is given by $\xi = B\left(\frac{dA}{dt}\right)$

$$\Rightarrow \quad \xi = B\left(\frac{\text{Area Swept by the blades}}{\text{Time of one Revolution}}\right)$$

$$\Rightarrow \quad \xi = \frac{B(\pi r^2)}{\frac{2\pi}{\omega}}$$

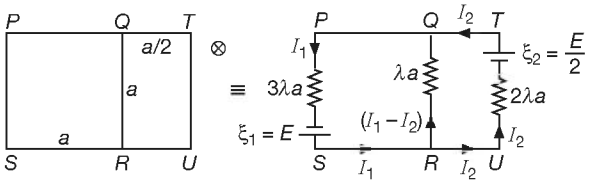
$$\Rightarrow \quad \xi = \frac{1}{2}B\omega r^2$$

$$\Rightarrow \quad \xi = \frac{1}{2}(50\times10^{-6})(4\pi)(9)$$

$$\Rightarrow \quad \xi = 2.83 \text{ mV}$$

14. Let the induced emf for the loop $PQRSP$ be ξ_1 and that for $QTURQ$ be ξ_2. Then $|\xi_1| = A_1\left(\frac{dB}{dt}\right) = E(\text{say})$ and $|\xi_2| = A_2\left(\frac{dB}{dt}\right) = \frac{a^2k}{2} = \frac{E}{2}$

So, the equivalent circuit diagram for the given set up is



For $PQRSP$, we have

$$(\lambda a)(I_1 - I_2) - E + (3\lambda a)I_1 = 0$$

$$\Rightarrow \quad (4\lambda a)I_1 - (\lambda a)I_2 = E \qquad \ldots(1)$$

For $QTURQ$, we have

$$-\frac{E}{2} + (2\lambda a)I_2 - (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow \quad -(\lambda a)I_1 + (3\lambda a)I_2 = \frac{E}{2}$$

$$\Rightarrow \quad -(2\lambda a)I_1 + (6\lambda a)I_2 = E \qquad \ldots(2)$$

Eliminating by using 2 (2) + (1), we get

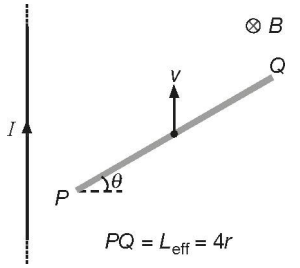
$$(11\lambda a)I_2 = 3E$$

$$\Rightarrow \quad I_2 = \frac{3E}{11\lambda a} = \frac{3a^2k}{11\lambda a} = \frac{3ak}{11\lambda}$$

Solving for others, we get

$$I_1 = \frac{7ak}{22\lambda} \text{ and } (I_1 - I_2) = \frac{ak}{22\lambda}$$

15. As discussed in theory, let us join the end point P and Q and replace the two semicircles by a straight rod of equivalent length $4r$, as shown.



The effective rod is not perpendicular to the direction of motion. Taking the projection of the rod perpendicular to its direction of motion we find the effective length of the conductors is $PN = 4r\cos\theta$.

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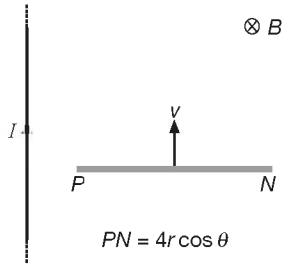
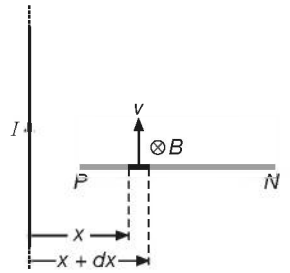


We now have a rod of effective length $PN = 4r\cos\theta$ translating in a non-uniform magnetic field.

Consider an infinitesimal element of the wire of length dx located at a distance x from the wire. The magnetic field at this element is $B = \frac{\mu_0 I}{2\pi x}$.



The potential difference across this element is $d\xi = Bvdx = \frac{\mu_0 I}{2\pi x} v dx$.

The potential difference across the ends of the rod is calculated by integrating over the entire length. Therefore,

$$\xi = \int d\xi = \int_{d-2r\cos\theta}^{d+2r\cos\theta} \frac{\mu_0 I}{2\pi x} v dx$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I v}{2\pi} \log_e \left(\frac{d + 2r\cos\theta}{d - 2r\cos\theta} \right)$$

16. For the suspended mass M, we have

$$Mg - T = Ma \qquad \ldots(1)$$

For the bar connected to the suspended mass M,

$$T - BI\ell = ma \qquad \ldots(2)$$

where $I = \frac{\xi}{R} = \frac{B\ell v}{R}$ $\qquad \ldots(3)$

$$\Rightarrow \quad Mg - BI\ell = (M+m)a$$

$$\Rightarrow \quad Mg - \frac{B^2\ell^2 v}{R} = (M+m)a$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{Mg}{M+m} - \frac{B^2\ell^2}{R(M+m)} v$$

$$\Rightarrow \quad a = \alpha - \beta v \text{ where } \alpha = \frac{Mg}{M+m},\ \beta = \frac{B^2\ell^2}{R(M+m)}$$

$$\Rightarrow \quad \frac{dv}{dt} = \alpha - \beta v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{\beta} \log_e (\alpha - \beta v) \Big|_0^v = t$$

$$\Rightarrow \quad \log_e \left(\frac{\alpha - \beta v}{\alpha} \right) = -\beta t$$

$$\Rightarrow \quad v = \frac{\alpha}{\beta} (1 - e^{-\beta t})$$

$$\Rightarrow \quad v = \frac{MgR}{B^2\ell^2} \left(1 - e^{-\frac{B^2\ell^2 t}{R(M+m)}} \right)$$

At the terminal velocity of the bar, say v_T we have

$$a = 0$$

$$\Rightarrow \quad \frac{Mg}{M+m} - \frac{B^2\ell^2 v_T}{R(M+m)} = 0$$

$$\Rightarrow \quad Mg = \frac{B^2\ell^2}{R} v_T$$

$$\Rightarrow \quad v_T = \frac{MgR}{B^2\ell^2}$$

Test Your Concepts-III (Based on Faraday's Laws: AC Generator)

1. $\phi = BA\cos 0$

$$\Rightarrow \quad \phi = NBA$$

$$\Rightarrow \quad \xi = N\frac{d\phi}{dt} = NA\left(\frac{dB}{dt} \right)$$

where $\frac{dB}{dt} = 4000\cos(1000t)\, mTs^{-1} = 4\cos(1000t)\ \text{Ts}^{-1}$

$$\Rightarrow \quad \xi = (50)(3.14)(7 \times 10^{-3})^2 4\cos(1000t)$$

$$\Rightarrow \quad \xi = 30.8\cos(1000t)\ \text{mV}$$

$$\Rightarrow \quad \xi_{\text{peak}} = 30.8\ \text{mV}$$

2. The flux through the coil is $\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos(\omega t)$. The induced emf is

$$\xi = -N\frac{d\phi_B}{dt} = NBA\omega\sin(\omega t)$$

(a) $\xi_{max} = \xi_0 = NBA\omega = 60(1\text{ T})(0.1\times0.2)(30) = 36\text{ V}$

(b) $\frac{d\phi_B}{dt} = \frac{\xi}{N}$, thus

$$\left|\frac{d\phi_B}{dt}\right|_{max} = \frac{\xi_{max}}{N} = \frac{36}{60} = 0.6\text{ V} = 0.6\text{ Wbs}^{-1}$$

(c) At $t = \frac{\pi}{120}$ s, $\omega t = \frac{\pi}{4}$ radian

$$\Rightarrow \quad \xi = \xi_0\sin\left(\frac{\pi}{4}\right) = \frac{36}{\sqrt{2}}\text{V} = 18\sqrt{2}\text{ V}$$

(d) The torque on the coil at any time is

$$\vec{\tau} = |\vec{M}\times\vec{B}| = |NI\vec{A}\times\vec{B}| = (NAB)I|\sin\omega t|$$

$$\vec{\tau} = \left(\frac{\xi_{max}}{\omega}\right)\left(\frac{\xi_{max}}{R}\right)|\sin\omega t|$$

Now when $\xi = \xi_{max}$, $\sin(\omega t) = 1$

$$\Rightarrow \quad \tau = \frac{\xi_{max}^2}{\omega R} = \frac{(36)^2}{(30)(10\ \Omega)} = 4.32\text{ Nm}$$

3. (a) $\xi = -N\left(\frac{d\phi_B}{dt}\right) = -NA\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad \xi = -NA\frac{d}{dt}(\mu_0 nI) = -\mu_0 NnA\frac{dI}{dt}$$

where A = area of coil

N = number of turns in coil and

n = number of turns per unit length in solenoid.

$$\Rightarrow \quad |\xi| = N\mu_0 An\frac{d}{dt}5\sin(100\pi t)$$

$$\Rightarrow \quad |\xi| = N\mu_0 An500\pi\cos(100\pi t)$$

$$\Rightarrow \quad |\xi| = 160(4\pi\times10^{-7})[\pi(0.05)^2]$$

$$(2\times10^3)(500\pi)\cos(100\pi t)$$

$$\Rightarrow \quad |\xi| = 5\cos(100\pi t)$$

(b) $P = \frac{V^2}{R} = \frac{\xi^2}{R}$

$$\Rightarrow \quad P = \frac{(5)^2\cos^2(100\pi t)}{8}$$

Since $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos2\theta$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2} + \frac{1}{2}\langle\cos(2\theta)\rangle$$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2} + 0 \qquad \{\because \langle\cos(2\theta)\rangle = 0\}$$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2}$$

$$\Rightarrow \quad \langle P\rangle = \frac{1}{2}\frac{(5)^2}{8} = \frac{25}{16} = 1.57\text{ W}$$

4. Outside the solenoid net magnetic field zero and inside the solenoid $B = \mu_o nI$

Induced emf

$$\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}\left(\mu_o nI\pi a^2\right)$$

$$\Rightarrow \quad \xi = -\mu_0 n\pi a^2\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \left(\mu_o n\pi a^2\right)\left(I_o\omega\cos\omega t\right)$$

Resistance of the cylindrical vessel

$$R = \frac{\rho\ell}{s} = \frac{\rho(2\pi R)}{Ld}$$

So, the induced current is given by

$$I = \frac{|\xi|}{R} = \frac{\mu_o Lnda^2 I_o\omega\cos(\omega t)}{2\rho R}$$

5. (a) $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$

$$\Rightarrow \quad \phi = B\left(\frac{\pi r^2}{2}\cos(\omega t) + ab\right)$$

(b) $\xi = -\frac{d\phi}{dt}$

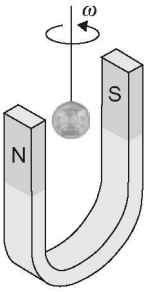
$$\Rightarrow \quad \xi = \frac{\pi Br^2\omega}{2}\sin(\omega t)$$

(c) $I = \frac{\xi}{R} = \frac{\pi Br^2\omega}{2R}\sin(\omega t)$

6. Since $\phi_B = BA\cos\theta = BA\cos(\omega t)$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = -\omega BA\sin\theta$$

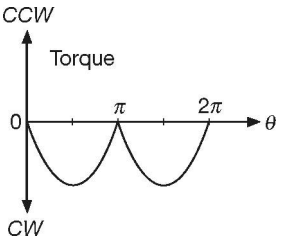
$$\Rightarrow \quad I \propto -\sin\theta$$



Since $|\vec{\tau}| = MB\sin\theta = IAB\sin\theta$

$\Rightarrow \quad \tau \propto IB\sin\theta$

$\Rightarrow \quad \tau \propto -\sin^2\theta$



7. (a) $\xi = NBA\omega\sin\theta = NBA\omega\sin(\omega t)$

$\Rightarrow \quad \xi_{MAX} = NBA\omega = BA\omega = B\left(\frac{\pi R^2}{2}\right)\omega$

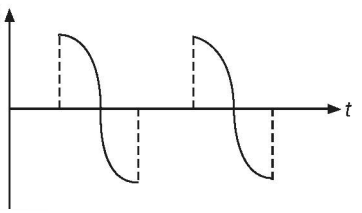
$\Rightarrow \quad \xi_{MAX} = 1.6 \text{ V}$

(b) $\langle\xi\rangle = \dfrac{\int_0^{2\pi}\xi d\theta}{\int_0^{2\pi} d\theta}$

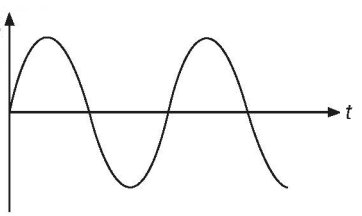
$\Rightarrow \quad \langle\xi\rangle = \dfrac{BA\omega}{2\pi}\int_0^{2\pi}\sin\theta d\theta = 0$

(c) Both remain unchanged.

(d)



(e)



8. (a) $\phi = BA\cos\theta = BA\cos(\omega t) = BA\cos(2\pi ft)$

$\Rightarrow \quad \phi = (0.8)(0.1)^2\cos(2\pi(60)t)$

$\Rightarrow \quad \phi = (8\times10^{-3})\cos(377t)\ Tm^2$

(b) $\xi = -\dfrac{d\phi}{dt} = (8\times10^{-3})(377)\sin(377t)$

$\Rightarrow \quad \xi = 3\sin(377t) \text{ V}$

(c) $I = \dfrac{\xi}{R} = 3\sin(377t) \text{ A}$

(d) $P = I^2R = 9\sin^2(377t) \text{ W}$

(e) $P = Fv = \tau\omega$

$\Rightarrow \quad \tau = \dfrac{P}{\omega} = \dfrac{9\sin^2(377t)}{120\pi}$

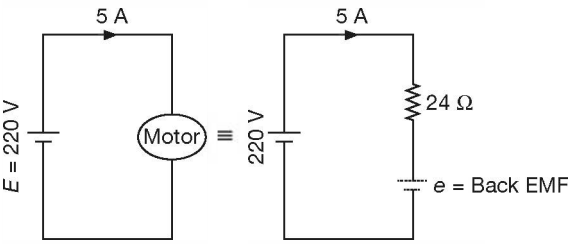
$\Rightarrow \quad \tau \cong 24\times10^{-3}\sin^2(377t) \text{ Nm}$

9. (a) According to Kirchhoff's Law, we have

$220 - (5)(24) - e = 0$

$\Rightarrow \quad e = 220 - 120$

$\Rightarrow \quad e = 100 \text{ V}$



(b) $P = I^2R = (5)^2(24) = 600 \text{ W}$

(c) With no motion, the motor does not function and so we have back emf $e = 0$

$\Rightarrow \quad 220 - I(24) = 0$

$\Rightarrow \quad I' = 9.2 \text{ A}$

Since $P' = I'^2R$

$\Rightarrow \quad P' = (9.2)^2(24) \cong 2 \text{ kW}$

Test Your Concepts-IV (Based on Faraday's Laws: Induced Electric Field)

1. Since $\oint\vec{E}\cdot d\vec{\ell} = \left|\dfrac{d\phi_B}{dt}\right|$

$\Rightarrow \quad (2\pi r)E = (\pi r^2)\left(\dfrac{dB}{dt}\right)$

$\Rightarrow \quad E = \dfrac{r}{2}\left(\dfrac{dB}{dt}\right)$

For a solenoid, we have $B = \mu_0 nI$

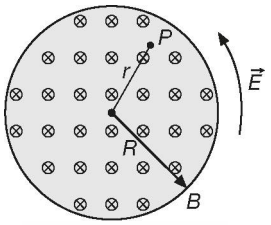
$$\Rightarrow \quad \frac{dB}{dt} = \mu_0 n\left(\frac{dI}{dt}\right), \text{ where } \frac{dI}{dt} = I_0\omega\cos(\omega t)$$

$$\Rightarrow \quad E = \left(\frac{\mu_0 n r\omega I_0}{2}\right)\cos(\omega t)$$

The electric field is always opposite to the increasing B, so if must be clockwise.

2. Since we know that

$$\oint \vec{E}\cdot d\vec{\ell} = \left|\frac{d\phi_B}{dt}\right|$$



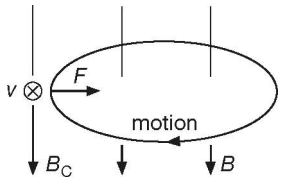
$$\Rightarrow \quad E(2\pi r) = (\pi r^2)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \left(\frac{0.02}{2}\right)(0.06\times 3)$$

$$\Rightarrow \quad E = 1.8\times 10^{-3}\ \text{NC}^{-1}$$

3. Let us assume the field to be acting vertically downwards. Consider an electron that is moving away from us as shown. The force on it is in the radially inward direction, so as to make them circulate clockwise.



(a) Now, when the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed counter clockwise direction (using Right Hand Thumb Rule) carried by negative electrons moving clockwise. Therefore, the original electron motion speeds up.

(b) For the electrons moving in a circle, we have

$$|q|vB_c\sin(90°) = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv = |q|rB_c$$

The increasing magnetic field $\vec{B}_{av}$ in the area enclosed by the orbit produces a tangential electric field given by

$$\left|\oint \vec{E}\cdot d\vec{\ell}\right| = \left|\frac{-d\phi_B}{dt}\right| \left|\frac{d}{dt}\left(\vec{B}_{av}\cdot\vec{A}\right)\right|$$

$$\Rightarrow \quad E(2\pi r) = \pi r^2\frac{dB_{av}}{dt}$$

$$\Rightarrow \quad E = \frac{r}{2}\frac{dB_{av}}{dt}$$

An electron feels a tangential force when $\sum F_t = ma_t$

$$\Rightarrow \quad |q|E = m\frac{dv}{dt}$$

$$\Rightarrow \quad |q|\frac{r}{2}\frac{dB_{av}}{dt} = m\frac{dv}{dt}$$

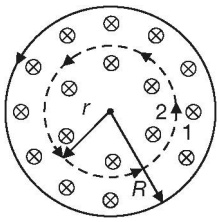
$$\Rightarrow \quad |q|\frac{r}{2}B_{av} = mv = |q|rB_c$$

$$\Rightarrow \quad B_{av} = 2B_c$$

4. Since $\xi = -\frac{d\phi}{dt}$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{\ell} = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{\ell} = \left|A\frac{d\vec{B}}{dt}\right|$$



For $r \le R$:

Using $E\ell = A\left|\frac{dB}{dt}\right|$, we get

$$E(2\pi r) = (\pi r^2)\alpha$$

$$\Rightarrow \quad E = \frac{r\alpha}{2}$$

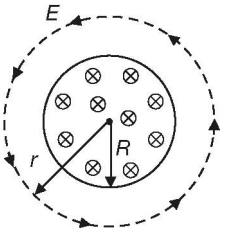
$$\Rightarrow \quad E \propto r$$

i.e., E-r graph is a straight line passing through origin.

At $r = R$, $E = \frac{R\alpha}{2}$

For $r \geq R$:

Using $E\ell = A\left|\frac{dB}{dt}\right|$



$$\Rightarrow \quad E(2\pi r) = (\pi R^2)(\alpha)$$

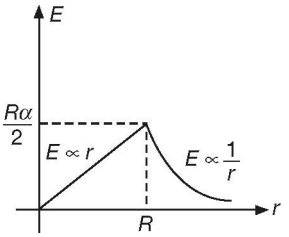
$$\Rightarrow \quad E = \frac{\alpha R^2}{2r}$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

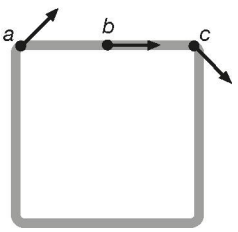
i.e. E-r graph is a rectangular hyperbola.

The E-r graph is as shown in figure.

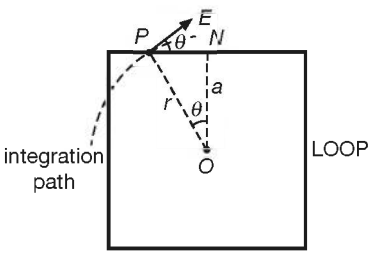
Direction of electric field is shown in figure.



5. (a)



(b) Consider an integration path which is a circle of radius r. By symmetry the induced field is tangent to this path and constant in magnitude at all the points on the path.



For the loop, we have $E_{\text{Loop}} = E\cos\theta$

Since $E(2\pi r) = \xi$ $\qquad \left\{\because |\xi| = \int \vec{E}\cdot d\vec{\ell}\right\}$

$$\Rightarrow \quad E = \frac{\xi}{2\pi r}$$

From triangle OPN

$$r\cos\theta = a$$

$$\Rightarrow \quad E = \frac{\xi\cos\theta}{2\pi a}$$

$$\Rightarrow \quad E_{loop} = E\cos\theta = \frac{\xi\cos^2\theta}{2\pi a} \qquad \ldots(1)$$

But $\xi = \frac{d\phi_B}{dt} = A\left(\frac{dB}{dt}\right) = \pi r^2\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad \xi = \frac{\pi a^2}{\cos^2\theta}\left(\frac{dB}{dt}\right)$$

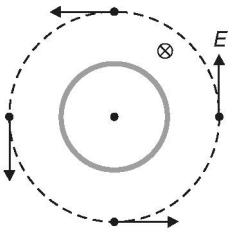
$$\Rightarrow \quad \xi\cos^2\theta = \pi a^2\left(\frac{dB}{dt}\right)$$

But from (1) $\xi\cos^2\theta = E_{\text{loop}}(2\pi a)$

$$\Rightarrow \quad E_{\text{loop}}(2\pi a) = \pi a^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E_{\text{loop}} = \frac{a}{2}\left(\frac{dB}{dt}\right)$$

6. (a) The ring will rotate in the direction of induced electric field $\vec{E}$ as shown in Figure.



$$E(2\pi R) = (\pi R^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E = \left(\frac{R}{2}\right)(B_0)$$

Since, $F = qE = \frac{qB_0R}{2}$ and $\tau = FR = \frac{qB_0R^2}{2}$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\tau}{mR^2} = \frac{qB_0}{2m}$$

Since, α = constant, so we have

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \quad \omega = 0 + \left(\frac{qB_0}{2m}\right)t = \left(\frac{qB_0}{2m}\right)t$$

(b) $$P = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I \cdot \omega \cdot \frac{d\omega}{dt} = I\omega\alpha$$

$$\Rightarrow \quad P = I(\alpha t)\alpha = I\alpha^2 t = (mR^2)\left(\frac{qB_0}{2m}\right)^2 t$$

$$\Rightarrow \quad P = \frac{q^2B_0^2R^2t}{4m}$$

7. The induced electric field is set up as a result of the changing current through the solenoid.
Inside the solenoid i.e. for $r < a$, the induced electric field is

$$E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

where, $B = \mu_0 ni$ for $r < a$ i.e. inside the solenoid

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\frac{d}{dt}(\mu_0 ni) = \frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)$$

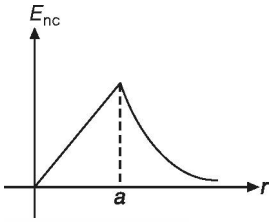
According to the problem, we have

$$\frac{di}{dt} = I$$

$$\Rightarrow \quad E_{nc} = \left(\frac{\mu_0 nI}{2}\right)r$$

Outside the solenoid i.e. for $r > a$, the induced electric field is

$$E_{nc} = \frac{a^2}{2r}\left(\frac{dB}{dt}\right)E = \left(\frac{\mu_0 nIa^2}{2}\right)\frac{1}{r}$$



8. $$\xi = \frac{d\phi}{dt} = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad \xi = -\frac{1}{2}(L)\sqrt{R^2 - \frac{L^2}{4}}\frac{dB}{dt}$$

$$\Rightarrow \quad |\xi| = \left(\frac{L}{2}\sqrt{R^2 - \frac{L^2}{4}}\right)\frac{dB}{dt}$$

9. Induced emf in the loop is

$$|\xi| = A\left(\frac{dB}{dt}\right) = \left(\frac{\pi r^2}{2}\right)(6t^2 + 6t)$$

Net resistance of the loop is

$$R_{net} = R\left(\frac{r}{2} + \frac{r}{2} + 2r + \pi r\right) = rR(3 + \pi)$$

Since the induced emf has opposing nature, so the current in the loop is

$$i = \frac{\text{Net Voltage}}{\text{Net Resistance}} = \frac{E - \xi}{R_{net}}$$

$$\Rightarrow \quad i = \frac{E - \frac{\pi r^2}{2}(6t^2 + 6t)}{rR(3 + \pi)} = \frac{E - 3\pi r^2(t^2 + t)}{rR(3 + \pi)}$$

$$\Rightarrow \quad i = \frac{E - 18\pi r^2}{rR(3 + \pi)}$$

Test Your Concepts-V (Based on Faraday's Laws: Self Induction)

1. $$|\xi| = L\frac{dI}{dt} = L(2t - 6)$$

(a) $$|\xi| = \left|(90 \times 10^{-3})(2 - 6)\right| = 360 \text{ mV}$$

(b) $$|\xi| = \left|(90 \times 10^{-3})(2)\right| = 180 \text{ mV}$$

(c) $$|\xi| = 0$$

$$\Rightarrow \quad t = 3 \text{ s}$$

2. Since $\xi = \xi_0 e^{-kt}$ and $\xi = -L\frac{dI}{dt}$

$$\Rightarrow \quad -L\frac{dI}{dt} = \xi_0 e^{-kt}$$

Since we are to find the current as a function of t, so let the current be I at time t. Then as $t \to \infty$, $I \to 0$ $(\because \xi \to 0)$

$$\Rightarrow \quad \int_I^0 dI = -\frac{\xi_0}{L}\int_t^\infty e^{-kt}dt$$

$$\Rightarrow \quad -I = \frac{\xi_0}{kL}e^{-kt}\Big|_t^\infty$$

$$\Rightarrow \quad -I = \frac{\xi_0}{kL}(e^{-\infty} - e^{-kt})$$

CHAPTER 3

$$\Rightarrow \quad I = \frac{\xi_0}{kL} e^{-kt}$$

Now, we know that $|Q| = \left| \int_0^\infty I dt \right|$

$$\Rightarrow \quad |Q| = \left| \frac{\xi_0}{kL} \int_0^\infty e^{-kt} dt \right|$$

$$\Rightarrow \quad |Q| = \frac{\xi_0}{k^2 L}$$

3. (a) Self induced emf,

$$\xi = -L\frac{dI}{dt} = (-0.54)(-0.03) \text{ V}$$

$$\Rightarrow \quad \xi = 1.62 \times 10^{-2} \text{ V}$$

(b) $V_{ba} = L\frac{dI}{dt} = -1.62 \times 10^{-2}$ V

Since, $V_{ba} (= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

4. P.D. across inductor,

$$V_L = L\frac{dI}{dt} = (5)(-1) = -5 \text{ V}$$

Now, $V_a - IR - V_L - E = V_b$

$$\Rightarrow \quad V_{ab} = V_a - V_b = E + IR + V_L$$

$$\Rightarrow \quad V_{ab} = 20 + (2)(10) - 5 = 35 \text{ V}$$

5. (a) The inductance of a solenoid is given by,

$$L = \mu_0 n^2 A\ell = \frac{\mu_0 N^2 A}{\ell} \qquad \left\{ \because n = \frac{N}{\ell} \right\}$$

Substituting the values, we get

$$L = \frac{(4\pi \times 10^{-7})(300)^2 (4 \times 10^{-4})}{(25 \times 10^{-2})} \text{ H}$$

$$\Rightarrow \quad L = 1.81 \times 10^{-4} \text{ H}$$

(b) $\xi = -L\frac{dI}{dt}$

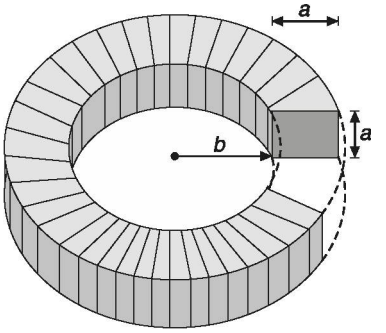
where $\frac{dI}{dt} = -50 \text{ As}^{-1}$

$$\Rightarrow \quad \xi = -(1.81 \times 10^{-4})(-50)$$

$$\Rightarrow \quad \xi = 9.05 \times 10^{-3} \text{ V}$$

$$\Rightarrow \quad \xi = 9.05 \text{ mV}$$

6. If we consider an elemental ring of radius r and width dr as shown in Figure.



The number of turns per unit length in this ring-shaped element is

$$n = \frac{N}{2\pi r}$$

The magnetic induction inside the elemental toroid is

$$B = \mu_r \mu_0 nI = \frac{\mu_r \mu_0 NI}{2\pi r}$$

The magnetic flux through a cross-section of the elemental ring-shaped toroid is

$$d\phi = \left(\frac{\mu_r \mu_0 NI}{2\pi r} \right) (a dr)$$

Magnetic flux linked with all N turns of the toroid in the elemental ring-shaped element is

$$N d\phi = \left(\frac{\mu_r \mu_0 N^2 Ia}{2\pi} \right) \frac{dr}{r}$$

Total magnetic flux linked with the given toroid is given as

$$\phi_{\text{total}} = \int d\phi = \frac{\mu_r \mu_0 N^2 Ia}{2\pi} \int_b^{b+a} \frac{dr}{r}$$

$$\Rightarrow \quad \phi = \frac{\mu_r \mu_0 N^2 Ia}{2\pi} \ln\left(\frac{b+a}{a} \right)$$

Self-inductance of the toroid is

$$L = \frac{\phi}{I}$$

$$\Rightarrow \quad L = \frac{\mu_r \mu_0}{2\pi} N^2 a \ln\left(\frac{b+a}{a} \right)$$

7. (a) $\Delta V_R = IR = (2)(8) = 16$ V

(b) $\Delta V_L + \Delta V_R = E$

$\Rightarrow \quad \Delta V_R = 36 - 16 = 20$ V

(c) $\Delta V_L = E - \Delta V_R = E - IR$

$\Rightarrow \quad \Delta V_L = 36 - (4.5)(8)$

$\Rightarrow \quad \Delta V_L = 36 - 36 = 0$ V

So, at $I = 4.5$ A, voltage across the inductor is zero volt.

8. Magnetic induction due to the solenoid at its centre is given by

$$B = \mu_0 ni$$

Total flux linked with the coil is

$$\phi = NBA = N(\mu_0 ni)A$$

EMF induced in the coil is

$$|\xi| = \frac{d\phi}{dt} = \mu_0 nNA\frac{di}{dt}$$

According to the problem, we have

$$\frac{di}{dt} = \frac{4}{0.05} = 80$$

$$\Rightarrow \quad \xi = (4\pi \times 10^{-7})(2 \times 10^4)(100)(\pi \times 10^{-4})(80)$$

$$\Rightarrow \quad \xi = 6.4\pi^2 \times 10^{-3} \text{ V}$$

$$\Rightarrow \quad \xi \approx 64 \times 10^{-3} \text{ V} = 64 \text{ mV}$$

Charge flown through the coil is

$$q = i\Delta t = \frac{\xi}{R}\Delta t$$

$$\Rightarrow \quad q = \frac{(6.4\pi^2 10^{-3})(0.05)}{10\pi^2}$$

$$\Rightarrow \quad q = 3.2 \times 10^{-5} \text{ C}$$

$$\Rightarrow \quad q = 32\ \mu\text{C}$$

9. Let a be the radius of each wire b be the separation between the wires of double line in which current in the wires is in the opposite direction. If each wire carries a current I but in opposite direction, then the magnetic field between the wires at a distance x from one of the wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{b-x}\right)$$

If we consider an elemental strip of length l, width dx at a distance x from one wire, then magnetic flux associated with this strip is

$$d\phi = BdA = B(ldx)$$

$$\Rightarrow \quad \phi = \int d\phi = \frac{\mu_0 Il}{2\pi}\left(\int_a^{b-a}\frac{dx}{x} + \int_a^{b-a}\frac{dx}{b-x}\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{\pi}\ln\left(\frac{b-a}{a}\right)$$

The self-inductance of this double line is

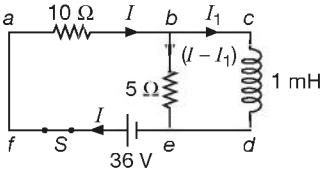
$$L = \frac{\phi}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{b-a}{a}\right)$$

Hence the self-inductance per unit length of this double line is

$$\Rightarrow \quad \frac{L}{l} = \frac{\mu_0}{\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu_0}{\pi}\ln(\eta)$$

Test Your Concepts-VI (Based on Growth and Decay of Current in LR Circuits)

1.



For loop *abefa*, we get

$$-10I - 5(I - I_1) + 36 = 0$$

$$-5I_1 + 15I = 36 \qquad \ldots(1)$$

For loop *bcdeb*, we get

$$-L\frac{dI_1}{dt} + 5(I - I_1) = 0$$

$$-5I_1 + 5I = L\frac{dI_1}{dt} \qquad \ldots(2)$$

Multiply (2) by (3) and subtracting from (1)

$$-5I_1 + 15I_1 = 36 - 3L\frac{dI_1}{dt}$$

$$\Rightarrow \quad 10I_1 = 36 - 3L\frac{dI_1}{dt}$$

$$\Rightarrow \quad 3L\frac{dI_1}{dt} = 36 - 10I_1$$

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$$\Rightarrow \int_0^{I} \frac{dI_1}{36-10I_1} = \frac{1}{3L}\int_0^{t} dt$$

$$\Rightarrow -\frac{1}{10}\log_e(36-10I_1)\Big|_0^{I_1} = \frac{t}{3L}$$

$$\Rightarrow \frac{36-10I_1}{36} = 10e^{-\frac{t}{3L}}$$

$$\Rightarrow 36-10I_1 = 36e^{-\frac{t}{3L}}$$

$$\Rightarrow 10I_1 = 36\left(1-e^{-\frac{t}{3L}}\right)$$

$$\Rightarrow I_1 = 3.6\left(1-e^{-\frac{t}{3L}}\right)A$$

Since $L = 1$ mH

$$\Rightarrow I_1 = 3.6\left(1-e^{-\frac{t}{3\times10^{-1}}}\right)$$

The current through $10\ \Omega$ resistor is I which is not a constant and varies with t.

2. (a) This is the case of growth of current in an LR circuit. Hence, current at time t is given by,

$$I = I_0(1-e^{-t/\tau_L}), \text{ where } \tau_L = \frac{L}{R}$$

$$\Rightarrow I = I_0\left(1-e^{-\frac{Rt}{L}}\right)$$

Rate of increase of current is given by

$$\frac{dI}{dt} = \frac{I_0}{\tau_L}e^{-t/\tau_L}$$

At $t = 0$, $\frac{dI}{dt} = \frac{I_0R}{L} = \frac{R}{L}\left(\frac{E}{R}\right) = \frac{E}{L}$

Substituting the value, we have

$$\frac{dI}{dt} = \frac{200}{0.5} = 400\ \text{As}^{-1}$$

(b) At $t = \frac{L}{R}$, $\frac{dI}{dt} = (400)e^{-1} = (0.37)(400) = 148\ \text{As}^{-1}$

(c) The steady state current in the circuit,

$$I_0 = \frac{E}{R} = \frac{200}{20} = 10\ \text{A}$$

3. (a) $\tau_L = \frac{L}{R} = \frac{2}{10} = 0.2$ s

(b) $I_{\max} = I_0 = \frac{E}{R} = \frac{100}{10} = 10$ A

(c) $I = I_0\left(1-e^{-\frac{t}{\tau_L}}\right) = 10\left(1-e^{-\frac{1}{0.2}}\right)$

$$\Rightarrow I = 10(1-e^{-5}) = 9.93\ \text{A}$$

(d) At steady state $I = I_{\max} = I_0$

$$\Rightarrow U = \frac{1}{2}LI_{\max}^2 = \frac{1}{2}(2)(10)^2 = 100\ \text{J}$$

4. (a) $I = I_0e^{-\left(\frac{R_{\text{total}}}{L}\right)t}$

$$\Rightarrow I = I_0e^{-\left(\frac{R+r}{L}\right)t}$$

At $t = 0$, when the switch is opened, just before that the current in the circuit must be maximum and must have been passing through L only. So, we have

$$I_0 = \frac{E}{r}$$

$$\Rightarrow I = \frac{E}{r}e^{-\left(\frac{R+r}{L}\right)t}$$

(b) $dH = I^2r\,dt$

$$\Rightarrow dH = \frac{E^2}{r}e^{-2\left(\frac{R+r}{L}\right)t}dt$$

$$\Rightarrow H = \int dH = \frac{E^2}{r}\int_0^{\infty} e^{-2\left(\frac{R+r}{L}\right)t}dt$$

$$\Rightarrow H = -\frac{E^2L}{2r(R+r)}e^{-2\left(\frac{R+r}{L}\right)t}\Big|_0^{\infty}$$

$$\Rightarrow H = \frac{E^2L}{2r(R+r)}$$

5. At $t = 0$, when the key is closed, the inductor does not allow current to pass through it. So, the Bulb-1 does not glow but Bulb-2 starts glowing at its full intensity instantly.

After a long time, the current through branch of Bulb-1 is more, as the branch has lesser resistance, so finally, Bulb-1 glows brighter.

When the key is opened, then both bulbs are in series and due to the inductor connected in this series combination of bulbs, the current in both the bulbs decays slowly and hence both bulbs die slowly. However, both bulbs glow with equal intensities at any time after the key is opened.

6. $20-(I_1+I_2)R=5\dfrac{dI_1}{dt}=10\dfrac{dI_2}{dt_2}$ ...(1)

At $t=0$, $I_1=I_2=0$

$\Rightarrow \quad 5I_1=10I_2$ ...(2)

In steady state, we have

$$\frac{dI_1}{dt}=\frac{dI_2}{dt}=0$$

$\Rightarrow \quad I_1+I_2=\dfrac{20}{R}=\dfrac{20}{5}=4\text{ A}$

$\Rightarrow \quad I_1+\dfrac{I_1}{2}=4\text{ A} \qquad \{\because I_1=2I_2\}$

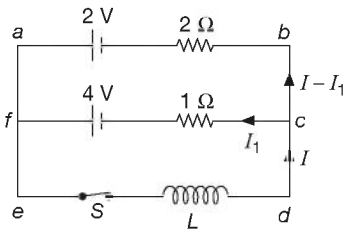
$\Rightarrow \quad 3I_1=8$

$\Rightarrow \quad I_1=\dfrac{8}{3}\text{ A}$

7. For loop $abcfa$, we get

$$-2+2(I-I_1)-I_1+4=0$$

$\Rightarrow \quad -3I_1+2I=-2$ ...(1)



For loop $fcdef$, we get

$$-4+I_1+L\frac{dI}{dt}=0$$

$$I_1+L\frac{dI}{dt}=4$$ ...(2)

Multiply (2) by (3) and adding to (1), we get

$2I+3L\dfrac{dI}{dt}=10$, where $L=10^{-3}$ H

$\Rightarrow \quad 3L\dfrac{dI}{dt}=10-2I$

$\Rightarrow \quad \displaystyle\int_0^I \frac{dI}{10-2I}=\int_0^t \frac{dt}{3L}$

$\Rightarrow \quad -\dfrac{1}{2}\log_e\left(\dfrac{10-2I}{10}\right)=\dfrac{t}{3L}$

$\Rightarrow \quad \left(\dfrac{10-2I}{10}\right)=e^{-\frac{2t}{3L}}$

$\Rightarrow \quad 10-2I=10e^{-\frac{2t}{3L}}$

$\Rightarrow \quad I=5\left(1-e^{-\frac{2000t}{3}}\right)\text{ A}$

8. $R^2=\dfrac{L}{C}$

$\Rightarrow \quad RC=\dfrac{L}{R}$

$\Rightarrow \quad \tau_C=\tau_L$ ...(1)

Now, $I=I_L+I_C$ ...(2)

where $I_L=I_0\left(1-e^{-\frac{t}{\tau_L}}\right)=\dfrac{V}{R}\left(1-e^{-\frac{t}{\tau_L}}\right)$ ...(3)

$$I_C=\frac{dq}{dt}=\frac{d}{dt}\left[q_0\left(1-e^{-\frac{t}{\tau_C}}\right)\right]=\frac{q_0}{\tau_C}e^{-\frac{t}{\tau_C}}$$

Since $\tau_C=RC$ and $q_0=CV$

$\Rightarrow \quad I_C=\dfrac{V}{R}e^{-\frac{t}{\tau_C}}$ ...(4)

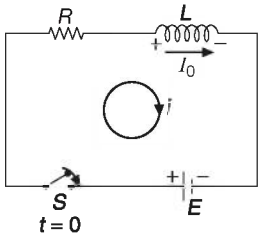
So, $I=\dfrac{V}{R}\left(1-e^{-\frac{t}{\tau_L}}+e^{-\frac{t}{\tau_C}}\right)$

Since $\tau_C=\tau_L$, so, we get

$$I=\frac{V}{R}$$

9. In the given circuit, applying KLL, we get

$$E-iR-L\frac{di}{dt}=0$$



$$\Rightarrow \int_{I_0}^{i}\frac{di}{E-iR}=\int_{0}^{t}\frac{dt}{L}$$

$$\Rightarrow -\frac{1}{R}\ln\left(\frac{E-iR}{E-I_0R}\right)=\frac{t}{L}$$

$$\Rightarrow \frac{E-iR}{E-I_0R}=e^{-\frac{Rt}{L}}$$

$$\Rightarrow i=\frac{1}{R}\left[E-(E-I_0R)e^{-\frac{Rt}{L}}\right]$$

10. After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{net}=\frac{\frac{R}{2}+R}{2}=\frac{3R}{4}$$

Current through the batteries,

$$I=\frac{2E}{\frac{3R}{4}+r_1+r_2}$$

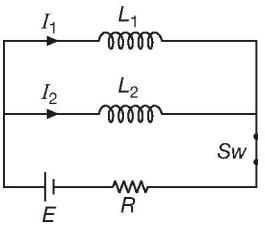
Given that potential across the terminals of cell A is zero, so

$$E-Ir_1=0$$

$$\Rightarrow E-\left(\frac{2E}{\frac{3R}{4}+r_1+r_2}\right)r_1=0$$

$$\Rightarrow R=\frac{4}{3}(r_1-r_2)$$

11. In steady state, the inductors behave like short circuits so the current through battery is



$$I=I_1+I_2=\frac{E}{R}$$

Since the current in inductors connected in parallel combination is divided in inverse ratio of their inductances, so we have currents through inductors given by

$$I_1=\frac{EL_2}{R(L_1+L_2)} \text{ and } I_2=\frac{EL_1}{R(L_1+L_2)}$$

12. Current before closing the switch is

$$I_0=\frac{V}{R+R_0} \qquad ...(1)$$

Let current in the circuit be I at any time t. Applying KVL in the circuit, we get

$$V=L\frac{dI}{dt}+RI$$

$$\Rightarrow \frac{dI}{dt}+\frac{RI}{L}=\frac{V}{L}$$

$$\frac{dI}{dt}=\frac{V-IR}{L}$$

$$\Rightarrow \int_{I_0}^{I}\frac{dI}{V-IR}=\int_{0}^{t}\frac{dt}{L}$$

$$\Rightarrow -\frac{1}{R}\log_e\left(\frac{V-IR}{V-I_0R}\right)=\frac{t}{L}$$

$$\Rightarrow \frac{V-IR}{V-I_0R}=e^{-\frac{Rt}{L}}$$

$$\Rightarrow V-IR=(V-I_0R)e^{-\frac{Rt}{L}}$$

$$\Rightarrow IR=V-(V-I_0R)e^{-\frac{Rt}{L}}$$

where $I_0=\frac{V}{R+R_0}$

$$\Rightarrow I=\frac{V}{R}-\left(V-\frac{VR}{R+R_0}\right)e^{-\frac{Rt}{L}}$$

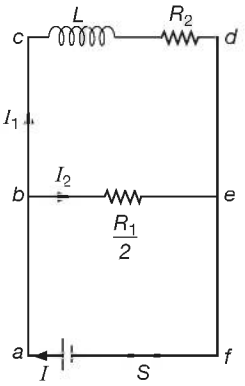
$$\Rightarrow I=\frac{V}{R}-\left(\frac{VR_0}{R+R_0}\right)e^{-\frac{Rt}{L}}$$

13. Applying Kirchhoff's Law in loop *abcdefa*, we get

$$E - L\left(\frac{dI_1}{dt}\right) - I_1R_2 = 0$$

$$\Rightarrow \int_0^{I_1} \frac{LdI_1}{E - I_1R_2} = \int_0^t dt$$

$$\Rightarrow I_1 = \frac{E}{R_2}\left(1 - e^{-\frac{R_2t}{L}}\right)$$



Potential drop across inductor is

$$V_L = L\frac{dI_1}{dt} = Ee^{-\frac{R_2t}{L}} = 10e^{-5t}\ \text{V}$$

in steady state $I_1 = \frac{E}{R_2}$

When the switch S is opened, we have

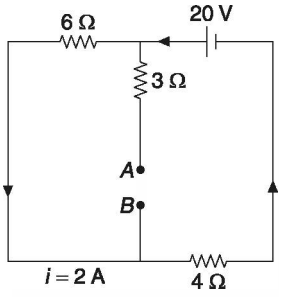
$$I = \left(\frac{E}{R_2}\right)e^{-\left(\frac{R_1}{2}+R\right)\frac{t}{L}}$$

Substituting the values, we get, $I = 5\,e^{-10t}$ A (direction of current is from c to d)

14. Remove the inductor, short the battery and then calculate the equivalent resistance across the open terminals (lets call then A and B) after removing the inductor is

$$R_{AB} = \frac{4\times 6}{4+6} + 3 = 5.4\ \Omega$$

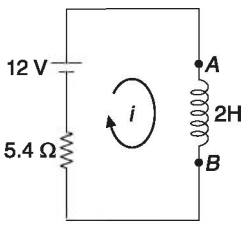
Now calculate the current in the battery circuit at the moment key is closed i.e. when inductor offers infinite resistance path to the flow of current, then from the circuit shown in Figure, the current is



$$I = \frac{20}{6+4} = 2\text{A}$$

So, open circuit potential difference across A and B is

$$V_{AB} = (2)(6) = 12\ \text{V}$$



Hence, current through the inductor is given by

$$I_L = \frac{V_{AB}}{R_{AB}}\left(1 - e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow I_L = \frac{12}{5.4}\left(1 - e^{-\frac{5.4t}{2}}\right) = 2.22\left(1 - e^{-2.7t}\right)\ \text{A}$$

$$\Rightarrow i = 2.22\left(1 - e^{-2.7t}\right)\ \text{A}$$

15. The set up discussed is a series LR circuit with time constant $\tau_L = \frac{L}{R}$. So, we just need to verify that the power delivered by the battery equals the sum of power dissipated as heat in the resistor and the power stored in the inductor i.e.,

$$EI = I^2R + LI\frac{dI}{dt} \qquad \ldots(1)$$

CHAPTER 3

Since at any instant t, current in LR circuit is given by

$$I = I_0\left(1-e^{-\frac{Rt}{L}}\right) = \frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

At $t = \tau_C = \frac{L}{R}$, we have

$$I = \frac{E}{R}\left(1-\frac{1}{e}\right) = \frac{5}{10}\left(1-\frac{1}{2.718}\right) = 0.316 \text{ A}$$

The rate at which energy is delivered by the battery is

$$P_1 = EI = (5)(0.316) \text{ W} = 1.58 \text{ W} \qquad \ldots(2)$$

Energy dissipated as heat in the resistance, given by

$$P_2 = I^2R = (0.316)^2(10) = 0.998 \text{ W}$$

$$\Rightarrow \quad P_2 = 0.998 \text{ W} \qquad \ldots(3)$$

The energy stored in the magnetic field, linked with the inductor is given by the relation

$$P_3 = LI\left(\frac{dI}{dt}\right), \text{ where}$$

$$\frac{dI}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}} = \frac{E}{L}\left(\frac{1}{e}\right) \qquad \left\{\text{at}\left(t=\frac{L}{R}\right)\right\}$$

$$\Rightarrow \quad P_3 = (LI)\left(\frac{E}{L}\right)\left(\frac{1}{e}\right)$$

$$\Rightarrow \quad P_3 = \frac{EI}{e} = \frac{(5)(0.316)}{2.718} = 0.582 \text{ W} \qquad \ldots(4)$$

From equations (1), (2), (3) and (4) we observe that

$$P_1 = P_2 + P_3$$

It is same as required by the Principle of Conservation of Energy.

Test Your Concepts-VII (Based on Magnetic Energy and Magnetic Energy Density)

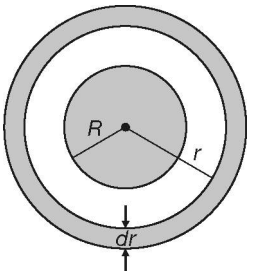
1. We have, $I = 200$ A and $U = 1$ kWh $= 3.6 \times 10^6$ J

Since, $L = \frac{2U}{I^2}$ $\qquad \left\{\because U = \frac{1}{2}LI^2\right\}$

$$\Rightarrow \quad L = \frac{2(3.6\times10^6)}{(200)^2} = 180 \text{ H}$$

2. Since $u = u_m = \frac{B^2}{2\mu_0}$

$$\Rightarrow \quad u = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$$



So, the magnetic energy in a spherical shell of radius r thickness dr is given by

$$dU = udV, \text{ where } dV = 4\pi r^2 dr$$

$$\Rightarrow \quad dU = \left(\frac{B_0^2R^4}{2\mu_0}\right)\frac{4\pi r^2 dr}{r^4}$$

$$U = \int dU = \frac{2\pi B_0^2R^4}{\mu_0}\int_R^\infty r^{-2}dr$$

$$\Rightarrow \quad U = \frac{2\pi B_0^2R^3}{\mu_0} \qquad \left\{\because \int_R^\infty r^{-2}dr = \frac{1}{R}\right\}$$

3. From Ampere's Circuital Law, we have

$$B(2\pi r) = \mu_0(I_{\text{enc}})$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

If dU is the energy associated with the cylindrical element, then

$$dU = u_m dV$$

$$\Rightarrow \quad dU = u_m(2\pi r\ell dr)$$

where $u_m = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

$$\Rightarrow \quad dU = u_m(2\pi r\ell dr) = \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r\ell dr) = \left(\frac{\mu_0 I^2 l}{4\pi}\right)\frac{dr}{r}$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 I^2 l}{4\pi}\int_R^{2R}\frac{dr}{r} = \frac{\mu_0 I^2 l}{4\pi}\ln r\Big|_R^{2R} = \left(\frac{\mu_0 I^2 l}{4\pi}\right)\ln 2$$

$$\Rightarrow \quad \frac{U}{\ell} = \frac{\mu_0 I^2}{4\pi}\ln 2$$

So, we observe that the magnetic energy per unit length within the wire is independent of its radius.

4. $u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.85\times10^{-12})(1000)^2$

$$\Rightarrow \quad u_e = 44.25 \text{ nJm}^{-3}$$

$$u_m = \frac{B^2}{2\mu_0} = \frac{(50\times10^{-6})^2}{8\pi\times10^{-7}} = 995\ \mu\text{Jm}^{-3}$$

For $u_e = u_m$

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\Rightarrow \quad \frac{E}{B} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^8 \text{ ms}^{-1}$$

Yes, this ratio happens to be the velocity of light.

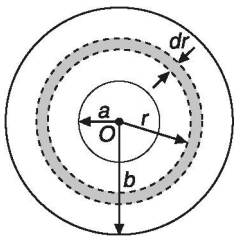
5. The magnetic field B in the space between the two conductors is given by

$$B = \frac{\mu_0 i}{2\pi r}$$

The magnetic energy density in the space between the conductors is

$$u_m = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2 = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

The cross-sectional view of the coaxial cable described in the problem is shown in Figure.



Consider a volume element dV in the form of a cylindrical shell of radii r and $(r+dr)$. Energy stored in this elemental volume is

$$dU = u_m dV = \left(\frac{\mu_0 i^2}{8\pi^2 r^2}\right)2\pi r l dr$$

$$\Rightarrow \quad dU = \frac{\mu_0 i^2 l}{4\pi}\left(\frac{dr}{r}\right)$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 i^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad U = \frac{\mu_0 i^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

6. (a) $|\xi| = L\frac{dI}{dt}$

$$\Rightarrow \quad 10 = L(16) \qquad \left\{\because \frac{dI}{dt} = 16\right\}$$

$$\Rightarrow \quad L = 6.25\times10^{-4} \text{ H}$$

(b) $E = \frac{1}{2}LI^2$

$$\Rightarrow \quad E = \frac{1}{2}(6.25\times10^{-4})(5+16t)^2$$

$$\Rightarrow \quad E|_{\text{at } t=1\text{ s}} = \frac{1}{2}(6.25\times10^{-4})(21)^2 \cong 138 \text{ mJ}$$

Since $P = VI = (10\times10^{-3})(5+16t)$

$$\Rightarrow \quad P|_{\text{at } t=1\text{ s}} = (10\times10^{-3})(21)$$

$$\Rightarrow \quad P|_{\text{at } t=1\text{ s}} = 210 \text{ mW}$$

7. (a) Since $\xi = -L\frac{dI}{dt} = -Lk$

(b) $I = \frac{dQ}{dt}$

$$\Rightarrow \quad Q = \int_0^t I dt = \int_0^t k t dt = \frac{kt^2}{2}$$

Since $V = V_C = \frac{Q}{C}$

$$\Rightarrow \quad |V| = \frac{kt^2}{2C}$$

(c) $\frac{1}{2}CV^2 \geq \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{1}{2}C\left(\frac{k^2t^4}{4C^2}\right) \geq \frac{1}{2}L(k^2t^2)$$

CHAPTER 3

$\Rightarrow \quad t \geq 2\sqrt{LC}$

$\Rightarrow \quad t_{MIN} = 2\sqrt{LC}$

8. $U = \frac{1}{2}LI^2 \Rightarrow U \propto I^2$

U will reach $\frac{1}{4}th$ of its maximum value when current reaches half of its maximum value. In series LR circuit, equation of current growth is

$$I = I_o\left(1 - e^{-t/\tau_L}\right)$$

where, I_o = Maximum value of current $= \frac{E}{R}$ and $= \frac{10\text{ H}}{2\ \Omega} = 5\text{ s}^{-1}$

$\Rightarrow \quad I = \frac{I_0}{2} = I_0\left(1 - e^{-t/5}\right)$

$\Rightarrow \quad \frac{1}{2} = 1 - e^{-t/5}$

$\Rightarrow \quad e^{-t/5} = \frac{1}{2}$

$\Rightarrow \quad -\frac{t}{5} = \ln\left(\frac{1}{2}\right)$

$\Rightarrow \quad \frac{t}{5} = \ln(2) = 0.693$

$\Rightarrow \quad t = (5)(0.693)\,s$

$\Rightarrow \quad t = 3.465\,s$

Test Your Concepts-VIII (Based on Faraday's Laws: Mutual Induction)

1. (a) $\phi_2 = MI_1$

$\Rightarrow \quad (6 \times 10^{-3})(1000) = M(3)$

$\Rightarrow \quad M = 2\text{ H}$

(b) $\xi_2 = -M\frac{dI_1}{dt}$

$\Rightarrow \quad \xi_2 = -(2)\left(\frac{0-3}{0.2}\right)$

$\Rightarrow \quad \xi_2 = 30\text{ V}$

(c) $\phi_1 = LI_1$

$\Rightarrow \quad (600)(5 \times 10^{-3}) = L(3)$

$\Rightarrow \quad L = 1\text{ H}$

2. Since $\xi = M\left(\frac{\Delta I}{\Delta t}\right)$

$\Rightarrow \quad 50 \times 10^{-3} = M\left(\frac{12-4}{0.5}\right)$

$\Rightarrow \quad M = \frac{25 \times 10^{-3}}{8} = 3.125\text{ mH}$

$$\xi_2 = (3.125 \times 10^{-3})\left(\frac{9-3}{0.02}\right) = 937.5\text{ mV}$$

3. The magnetic induction B at the location of coil P due to coil Q is

$$B = \frac{\mu_0 N_Q i}{2R}$$

Flux linked with the coil P is

$$\phi = N_P BA = N_P B(\pi r^2)$$

$\Rightarrow \quad \phi = \left(\frac{\mu_0 \pi r^2 N_Q N_P}{2R}\right) i$

Mutual inductance between the two coils is

$$M = \frac{\phi}{i} = \frac{\mu_0 \pi r^2 N_Q N_P}{2R}$$

$\Rightarrow \quad M = \frac{(4\pi \times 10^{-7})\pi(2 \times 10^{-2})^2 (100)(1000)}{2(0.2)}$

$\Rightarrow \quad M \approx 4 \times 10^{-4}\text{ H}$

The emf induced in the coil P is

$$\xi = M\frac{di}{dt}$$

$\Rightarrow \quad \xi = (4 \times 10^{-4})\left(\frac{5-3}{0.04}\right)$

$\Rightarrow \quad \xi = 0.02\text{ V} = 20\text{ mV}$

Rate of change of flux through coil P is given by

$$\frac{d\phi}{dt} = \xi = 20 \times 10^{-3}\text{ Wbs}^{-1}$$

Charge passing through coil P is given by

$$q = \frac{\Delta\phi}{R}$$

The total change in flux in the coil in this time interval is

$$\Delta\phi = \xi\Delta t = (0.02)(0.04) = 8 \times 10^{-4}\text{ Wb}$$

$\Rightarrow \quad q = \frac{\Delta\phi}{R} = \frac{8 \times 10^{-4}}{8} = 10^{-4}\text{ C}$

$\Rightarrow \quad q = 10^{-4}\text{ C} = 100\ \mu\text{C}$

4. Field produced by the larger coil at the centre of the smaller coil is

$$B_1 = \frac{\mu_0 N_1 I_1 R_1^2}{2\left(x^2 + R_1^2\right)^{3/2}}$$

This field B_1 is normal to area of coil 2 and is nearly uniform over this area. So, it produces a flux,

$$\phi_{12} = N_2 B_1 A_2 = \frac{\mu_0 N_1 N_2 I_1 R_1^2 \left(\pi R_2^2\right)}{2\left(x^2 + R_1^2\right)^{3/2}}$$

When I_1 varies, then

$$\xi_2 = -\frac{d\phi_{12}}{dt} = -\frac{\mu_0 \pi \left(N_1 N_2\right)\left(R_1 R_2\right)^2}{2\left(x^2 + R_1^2\right)^{3/2}}\left(\frac{dI_1}{dt}\right) = -M\frac{dI_1}{dt}$$

$$\Rightarrow \quad M = \frac{\mu_0 \pi \left(N_1 N_2\right)\left(R_1 R_2\right)^2}{2\left(x^2 + R_1^2\right)^{3/2}}$$

5. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current I_1 is given by the relation,

$$B = \mu_0 n_1 I_1$$

The magnetic flux through the secondary of N_2 turns each of area A is given by,

$$N_2\phi_2 = N_2(BA) = \mu_0 n_1 N_2 I_1 A$$

$$\Rightarrow \quad M = \frac{N_2\phi_2}{I_1} = \mu_0 n_1 N_2 A$$

Substituting the values, we get

$$M = \left(4\pi \times 10^{-7}\right)\left(\frac{50}{10^{-2}}\right)(200)\left(4 \times 10^{-4}\right)$$

$$\Rightarrow \quad M = 5 \times 10^{-4}\ \text{H}$$

6. The magnetic field at the location of one loop due to the other loop carrying a current i in it, considering it as a magnetic dipole is

$$B = \frac{\mu_0 i a^2}{2\left(a^2 + l^2\right)^{3/2}}$$

Since, $l \gg a$ so we have $\left(a^2 + l^2\right)^{3/2} \approx l^3$

$$\Rightarrow \quad B = \frac{\mu_0 i a^2}{2l^3}$$

Flux associated with the first loop is

$$\phi = B\left(\pi a^2\right)$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 i a^2}{2l^3}\right)\left(\pi a^2\right)$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 \pi a^4}{2l^3}\right) i$$

If M is the mutual inductance between the two loops, then we have

$$M = \frac{\phi}{i} = \frac{\mu_0 \pi a^4}{2l^3}$$

7. (a) $L_1 = \mu_0 n^2 A \ell_1 = \dfrac{\mu_0 N^2 A}{\ell_1}$

$$\Rightarrow \quad L_1 = \frac{\left(4\pi \times 10^{-7}\right)(1000)^2\left(10^{-4}\right)}{0.5} = 251\ \mu\text{H}$$

(b) $M = \dfrac{N_2\phi_2}{I_1} = \dfrac{N_2\phi_1}{I_1} = \dfrac{N_2 BA}{I_1} = \dfrac{\mu_0 N_1 N_2 A}{\ell_1}$

$$\Rightarrow \quad M = \frac{\left(4\pi \times 10^{-7}\right)(1000)(100)\left(10^{-4}\right)}{0.5}$$

$$\Rightarrow \quad M = 25.1\ \mu\text{H}$$

(c) $\xi_1 = -M\dfrac{dI_2}{dt}$

$$\Rightarrow \quad I_1 R_1 = -M\frac{dI_2}{dt}$$

$$\Rightarrow \quad \frac{dq_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$\Rightarrow \quad q_1 = -\frac{M}{R_1}\int_1^0 dI_2$$

$$\Rightarrow \quad q_1 = \left(\frac{M}{R_1}\right)(1) = \frac{\left(2.51 \times 10^{-5}\right)}{1000}$$

$$\Rightarrow \quad q_1 = 25.1\ \text{nC}$$

8. The magnetic field B inside the solenoid is

$$B = \mu_0 n_s i_s$$

CHAPTER 3

The magnetic flux linked with the coaxial coil placed inside the solenoid is

$$\phi_c = N_c B A_c = N_c(\mu_0 n_s i_s)A_c$$

where A_c is the cross sectional area of coil.

The mutual inductance between solenoid and coil is

$$M = \frac{\phi_c}{i_s} = \mu_0 n_s N_c A_c$$

Substituting the given values, we get

$$M = (4\pi\times10^{-7})(800)(100)(\pi\times10^{-4})\ \text{H}$$

$$\Rightarrow \quad M = 3.2\times10^{-5}\ \text{H}$$

9. (a) $\phi(t) = BA\cos\omega t = \left(\frac{\mu_0 I}{2b}\right)(\pi a^2)\cos(\omega t)$

$$\Rightarrow \quad \phi(t) = \frac{\mu_0\pi a^2 I}{2b}\cos(\omega t)$$

$$\xi = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0\omega\pi a^2 I}{2b}\sin(\omega t)$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{\mu_0\omega\pi a^2 I}{2bR}\sin(\omega t)$$

(b) $\tau = MB\sin(\omega t) = (\pi a^2 i)\left(\frac{\mu_0 I}{2b}\right)\sin(\omega t)$

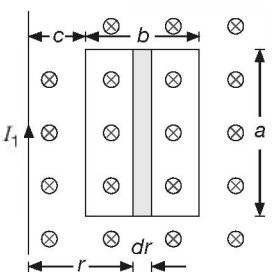
$$\Rightarrow \quad \tau = \frac{\mu_0^2\omega\pi^2 a^4 I^2}{4b^2 R}\sin^2(\omega t)$$

(c) Induced emf in larger loop,

$$|\xi| = \left|-M\frac{di}{dt}\right| = \left(\frac{\mu_0\pi a^2}{2b}\right)\left(\frac{\mu_0\omega^2\pi a^2 I}{2b}\right)\cos(\omega t)$$

$$\Rightarrow \quad |\xi| = \frac{\mu_0^2\pi^2 a^4\omega^2 I}{4b^2}\cos(\omega t)$$

10. The magnetic field due to the straight wire has magnitude $B_1 = \frac{\mu_0 I_1}{2\pi r}$ at a distance r. In accordance with right hand rule B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = a\,dr$. Magnetic flux through area dA, $d\phi_B = B_1(a\,dr)$.



Total flux through the loop,

$$\phi_{B_{12}} = \int B_1 dA_2 = \int_c^{c+b}\frac{\mu_0 I_1}{2\pi r}a\,dr$$

$$\phi_{B_{12}} = \frac{\mu_0 I_1 a}{2\pi}\int_c^{c+b}\frac{dr}{r} = \frac{\mu_0 I_1 a}{2\pi}\log_e\left(\frac{c+b}{c}\right)$$

Therefore, the mutual inductance,

$$M = M_{12} = \frac{\phi_{12}}{I_1} = \frac{\mu_0 a}{2\pi}\log_e\left(1+\frac{b}{c}\right)$$

11. When the given inductor is cut in identical parts then each part will have an inductance $\frac{L}{3}$. When two such parts are connected in series then equivalent self-inductance of the combination is given as

$$L_s = L_1 + L_2$$

$$\Rightarrow \quad L_s = \frac{L}{3}+\frac{L}{3} = \frac{2L}{3}$$

When connected in parallel, then

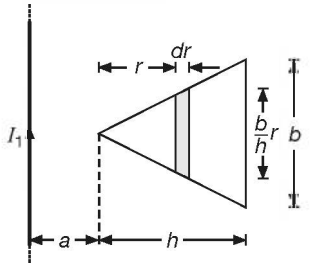
$$\frac{1}{L_p} = \frac{1}{L_1}+\frac{1}{L_2}$$

$$\Rightarrow \quad \frac{1}{L_p} = \frac{1}{\frac{L}{3}}+\frac{1}{\frac{L}{3}}$$

$$\Rightarrow \quad L_p = \frac{L}{6}$$

12. Consider a differential strip of area dA, then from property of similar triangles length of the strip is $\frac{br}{h}$.

$$\Rightarrow \quad dA = \left(\frac{b}{h}r\right)dr$$



Let I_1 be the current in wire then, its magnetic field on the strip is

$$B = \frac{\mu_0 I}{2\pi(a+r)}$$

If $d\phi_{B_{12}}$ is the flux associated with the strip, then

$$d\phi_{B_{12}} = B_1 dA = \frac{\mu_0 I_1}{2\pi h}\frac{br\,dr}{a+r}$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\int_0^h\left[\frac{(a+r)-a}{(a+r)}\right]dr$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\int_0^h\left[1-\frac{a}{a+r}\right]dr$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\left[h - a\log_e\left(\frac{a+h}{a}\right)\right]$$

Therefore, mutual inductance M_{12} is

$$M_{12} = \frac{\phi_{B_{12}}}{I_1} = \frac{\mu_0 b}{2\pi h}\left[h - a\log_e\left(\frac{a+h}{a}\right)\right]$$

Test Your Concepts-IX (Based on LC Oscillations)

1. (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.1)(10^{-6})}} \cong 503$ Hz

(b) $Q = CE = (10^{-6})(12) = 12\ \mu$C

(c) $\dfrac{1}{2}CE^2 = \dfrac{1}{2}LI_{max}^2$

$$\Rightarrow \quad I_{max} = E\sqrt{\frac{C}{L}} = 12\sqrt{\frac{10^{-6}}{0.1}} = 38 \text{ mA}$$

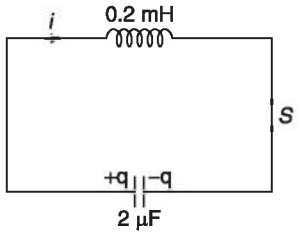
(d) $U = \dfrac{1}{2}CE^2 = \dfrac{1}{2}(10^{-6})(144) = 72\ \mu$J

2. $\dfrac{1}{2}CV^2 = \dfrac{1}{2}LI_0^2$

$$\Rightarrow \quad V = \sqrt{\frac{L}{C}}I_0 = \left(\sqrt{\frac{20\times10^{-3}}{0.5\times10^{-6}}}\right)(0.1)$$

$$\Rightarrow \quad V = 20 \text{ V}$$

3. When switch is closed, current flows in the circuit as shown in Figure.



As per the condition given in the problem, we have

$$U_L = \frac{1}{3}U_C$$

If Q_0 be the initial charge on the capacitor, then, the total energy of system is given by

$$U_L + U_C = \frac{Q_0^2}{2C} \qquad \ldots(1)$$

At any instant, let q be the charge on the capacitor, then we have

$$U_C = \frac{q^2}{2C}$$

So, equation (1) becomes

$$\frac{1}{3}U_C + U_C = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad \frac{4}{3}\frac{q^2}{2C} = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad q = \frac{\sqrt{3}}{2}Q_0$$

For LC oscillations, the angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

Charge on the capacitor as a function of time is given by

$$q = Q_0\cos\omega t$$

CHAPTER 3

$$\Rightarrow \quad \frac{\sqrt{3}}{2}Q_0 = Q_0\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$\Rightarrow \quad \frac{t}{\sqrt{LC}} = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\pi}{6}\sqrt{LC} = \frac{\pi}{6}\sqrt{0.2\times10^{-3}\times2\times10^{-6}}$$

$$\Rightarrow \quad t = \frac{\pi}{6}\times2\times10^{-5} = 10.5\ \mu s$$

4. Initially, the current in the circuit is $I_0 = \frac{E}{R}$. When the switch is shifted to B, then by Law of Conservation of Energy, the magnetic energy stored in the conductor gets converted to the electrostatic energy in the capacitor. So, we have

$$\frac{1}{2}LI_0^2 = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad \frac{1}{2}L\left(\frac{E}{R}\right)^2 = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad Q_0 = \frac{E\sqrt{LC}}{R}$$

$$\Rightarrow \quad I_{max} = V\sqrt{\frac{C}{2L}}$$

5. (a) When current is zero, charge and hence potential across capacitor is maximum. So,

$$V_0 = \frac{q_0}{C} = \frac{5\times10^{-6}}{400\times10^{-6}}$$

$$\Rightarrow \quad V_0 = 12.5\ \text{mV}$$

(b) $$\frac{1}{2}LI_0^2 = \frac{1}{2}CV_0^2$$

$$\Rightarrow \quad I_0 = V_0\sqrt{\frac{C}{L}} = 1.25\times10^{-2}\sqrt{\frac{400\times10^{-6}}{90\times10^{-3}}}$$

$$\Rightarrow \quad I_0 = 8.33\times10^{-4}\ \text{A}$$

(c) $$U_{max} = \frac{1}{2}LI_0^2 = \frac{1}{2}CV_0^2$$

$$\Rightarrow \quad U_{max} = \frac{1}{2}\times(400\times10^{-6})\times(12.5\times10^{-3})^2$$

$$\Rightarrow \quad U_{max} = 3.125\times10^{-8}\ \text{J}$$

(d) $$U_{max} = \frac{1}{2}L\left(\frac{I_0}{2}\right)^2 + \frac{Q^2}{2C}$$

$$3.125\times10^{-8} = \left(\frac{0.09}{2}\right)\left(\frac{8.33\times10^{-4}}{2}\right)^2 + \frac{Q^2}{2(4\times10^{-4})}$$

$$\Rightarrow \quad Q = 4.33\times10^{-6}\ \text{C}$$

$$\Rightarrow \quad U = \frac{1}{2}LI^2 = 7.8\times10^{-7}\ \text{J}$$

6. (a) Since both capacitors are in parallel, so

$$f = \frac{1}{2\pi\sqrt{L(C_1+C_2)}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{(0.5\times10^{-3})(5\times10^{-6})}}$$

$$\Rightarrow \quad f = \frac{1}{(2\pi)(5\times10^{-5})} = \frac{10^4}{\pi}\ \text{Hz}$$

(b) $$\frac{1}{2}LI_0^2 = \frac{1}{2}(C_1+C_2)V^2$$

$$\Rightarrow \quad \frac{1}{2}(0.5\times10^{-3})I_0^2 = \frac{1}{2}(5\times10^{-6})(200)^2$$

$$\Rightarrow \quad I_0^2 = \frac{(5\times10^{-6})(4\times10^4)}{(5\times10^{-4})}$$

$$\Rightarrow \quad I_0^2 = 4\times10^2$$

$$\Rightarrow \quad I_0 = 20\ \text{A}$$

7. Since, $E = \frac{Q'^2}{2C} + \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{Q^2}{2C} = \frac{\left(\frac{Q}{2}\right)^2}{2C} + \frac{1}{2}LI^2$$

$$\Rightarrow \quad I = \frac{Q}{2}\sqrt{\frac{3}{LC}}$$

The flux through each turn of the coil is

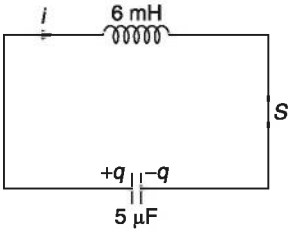
$$\phi = \frac{LI}{N}$$

$$\Rightarrow \quad \phi = \frac{QL}{2N}\sqrt{\frac{3}{LC}}$$

$$\Rightarrow \quad \phi = \frac{Q}{2N}\sqrt{\frac{3L}{C}}$$

8. At $t=0$ when switch is closed capacitor starts discharging through the inductor and current in circuit increases. When charge on capacitor is q and current in circuit is i by conservation of energy we have

$$\frac{q^2}{2C}+\frac{1}{2}Li^2=\frac{Q^2}{2C}$$



Current in circuit in terms of charge on capacitor is given as

$$i=\frac{1}{\sqrt{LC}}\sqrt{Q^2-q^2}=\frac{\sqrt{\left[(100)^2-(50)^2\right]\times10^{-12}}}{\sqrt{3\times10^{-8}}}$$

$$\Rightarrow\quad i=\frac{10^{-6}}{\sqrt{3}\times10^{-4}}\sqrt{7500}$$

$$\Rightarrow\quad i=0.5\text{ A}$$

Maximum energy is stored in inductor when

$$\frac{1}{2}LI_{max}^2=\frac{Q^2}{2C}$$

$$\Rightarrow\quad I_{max}=\frac{Q}{\sqrt{LC}}=\frac{100\times10^{-6}}{\sqrt{3\times10^{-8}}}=\frac{1}{\sqrt{3}}\text{ A}$$

9. At $t=0$, we have $Q=Q_{max}=Q_0$. Since the charge on the capacitor at any time t is

$$Q=Q_0\cos(\omega t),\text{ where }\omega=\frac{1}{\sqrt{LC}}$$

Energy stored in the capacitor at time t is,

$$U=\frac{1}{2}\left(\frac{Q^2}{C}\right)=\frac{Q_0^2}{2C}\cos^2(\omega t)=U_0\cos^2(\omega t)$$

Since, $U=\frac{U_0}{4}$

$$\Rightarrow\quad \frac{1}{4}=\cos^2(\omega t)$$

$$\Rightarrow\quad \omega t=\frac{\pi}{3}$$

$$\Rightarrow\quad \frac{t}{\sqrt{LC}}=\frac{\pi}{3}$$

$$\Rightarrow\quad \frac{t^2}{LC}=\frac{\pi^2}{9}$$

$$\Rightarrow\quad L=\frac{9t^2}{\pi^2C}$$

10. (a) $\omega=2\pi f$

Since $f=10^3$ cps $=10^3$ Hz, so

$$\omega=6.28\times10^3\text{ rads}^{-1}$$

$$T=\frac{1}{f}=10^{-3}\text{ s}$$

(b) $q=q_0\cos(\omega t)$

$$\Rightarrow\quad q=CV_0\cos(\omega t)$$

$$\Rightarrow\quad q=10^{-4}\cos\left(6.28\times10^3\text{t}\right)$$

(c) $\omega^2=\frac{1}{LC}$

$$\Rightarrow\quad 4\pi^2f^2=\frac{1}{LC}$$

$$\Rightarrow\quad (40)(10^6)=\frac{1}{L(10^{-6})}$$

$$\Rightarrow\quad L=\frac{1}{40}=25\text{ mH}$$

(d) Since

$$I=\frac{dq}{dt}=-(10^{-4})(6.28\times10^3)\sin(6.28\times10^3\text{ t})$$

$$\Rightarrow\quad \langle I\rangle=(10^{-4})(6.28\times10^3)\frac{2}{\pi}$$

$$\left\{\because\ \langle\sin(\omega t)\rangle=\frac{2}{\pi}\right\}$$

$$\Rightarrow\quad \langle I\rangle=(10^{-4})(6.28\times10^3)\left(\frac{2}{3.14}\right)$$

$$\Rightarrow\quad \langle I\rangle=0.4\text{ A}$$

CHAPTER 3

11. If q_0 is the maximum charge on capacitor then by Law of Conservation of Energy, we have

$$\frac{1}{2}LI_0^2 = \frac{q_0^2}{2C}$$

$$\Rightarrow \quad q_0 = \sqrt{LC}I_0$$

Charge on capacitor as a function of time is given by

$$q = q_0 \sin(\omega t + \alpha)$$

where $\omega = \frac{1}{\sqrt{LC}}$

Since at $t = 0$ we have $q = 0$, so we get $\alpha = 0$.

Thus, charge on capacitor is given by

$$q = I_0\sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

12. The time period of oscillation is

$$T = 2\pi\sqrt{LC_{eq}}$$

where $C_{eq} = \frac{(C)(C)}{C+C} = \frac{C}{2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{LC}{2}}$$

The initial charge on the first capacitor is $q_1 = CV$ and that on the second capacitor is $q_2 = 2CV$.

When these capacitors are connected along with an inductor, then due to the charge sharing between the capacitors there will be a loss in energy of the capacitor combination. This loss in energy of the capacitor combination is given by

$$\text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$$

$$\Rightarrow \quad \text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{(C)(C)}{C+C}\right)(2V - V)^2$$

$$\Rightarrow \quad \text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C}{2}\right)(V)^2 = \frac{CV^2}{4}$$

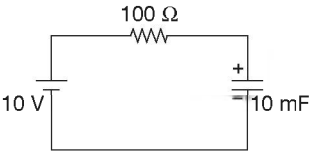
This loss in energy of the capacitor combination will be equal to the gain in magnetic energy in the inductor. So, we have

$$\frac{CV^2}{4} = \frac{1}{2}LI_{max}^2$$

$$\Rightarrow \quad I_{max} = V\sqrt{\frac{C}{2L}}$$

13. (a) Initially, charge on the capacitor is $Q_0 = CV_0$

$$\Rightarrow \quad Q_0 = (10\times10^{-3})(5\text{ V}) = 50\text{ mC}$$



When the capacitor is connected to position 1, then we get from Kirchhoff's Loop Law,

$$E - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad \int_0^t \frac{1}{RC}dt = \int_{Q_0}^{q}\frac{dq}{CE-q}$$

$$\frac{t}{RC} = -\log_e\left(\frac{CE-q}{CE-Q_0}\right)$$

$$\Rightarrow \quad CE - q = (CE - Q_0)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q = CE - (CE - Q_0)e^{-\frac{t}{RC}}$$

Since $CE = 100$ mC and $Q_0 = 50$ mC

$$\Rightarrow \quad q = 100 - (100-50)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q = 100 - 50e^{-t/RC}$$

$$\Rightarrow \quad q = 50\left(2 - e^{-\frac{t}{RC}}\right)$$

At $t = 1$ s, we have

$$q = 50(2 - e^{-1})\text{ mC}$$

$$\Rightarrow \quad q = 81.5\text{ mC}$$

Voltage across capacitor at that time is

$$V = \frac{q}{C} = \frac{81.5 \times 10^{-3}}{10 \times 10^{-3}} = 8.15 \text{ V}$$

(b) $\frac{1}{2} L I_{max}^2 = \frac{1}{2} C V_{max}^2$

$$\Rightarrow \quad I_{max} = \left(\sqrt{\frac{C}{L}}\right) V_{max}$$

$$\Rightarrow \quad I_{max} = \left(\sqrt{\frac{10}{2.5}}\right) 8.15 = 16.3 \text{ A}$$

Frequency, $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1000}{2\pi \times 25}$

$$\Rightarrow \quad f = \frac{10^3}{50\pi} \text{ Hz} = \frac{20}{\pi} \text{ Hz}$$

14. (a) By Law of Conservation of energy, we have

$$\frac{1}{2} L i_0^2 = \frac{1}{2} C V_0^2$$

$$\Rightarrow \quad L = \frac{C V_0^2}{i_0^2}$$

$$\Rightarrow \quad L = \frac{(4 \times 10^{-6})(1.5)}{(50 \times 10^{-3})^2}$$

$$\Rightarrow \quad L = 3.6 \times 10^{-3} \text{ H}$$

(b) Oscillation frequency of the LC circuit

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{(3.6 \times 10^{-3})(4 \times 10^{-6})}}$$

$$\Rightarrow \quad f = 0.133 \times 10^4 \text{ Hz}$$

$$\Rightarrow \quad f = 1.33 \text{ kHz}$$

(c) During oscillations, the time taken for charge to rise from zero to maximum is one fourth of the oscillation period which is given by

$$t = \frac{T}{4} = \frac{1}{4f}$$

$$\Rightarrow \quad t = \frac{1}{4 \times 1.33 \times 10^3} \text{ s}$$

$$\Rightarrow \quad t = 0.188 \times 10^{-3} \text{ s}$$

$$\Rightarrow \quad t = 0.188 \text{ ms}$$

15. $q_1 = 8CV_0$ and $q_2 = CV_0$

$$I = -\frac{dq_1'}{dt} = \frac{dq_2'}{dt} = \frac{dq}{dt}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{d^2q}{dt^2}$$

Applying loop law, we get

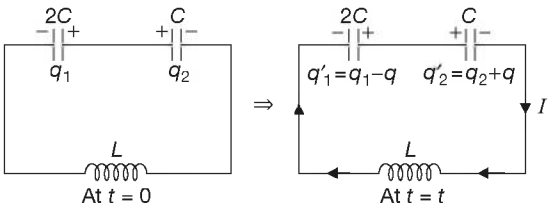
$$\frac{q_1'}{2C} - \frac{q_2'}{C} - L\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \left(\frac{q_1 - q}{2C}\right) - \left(\frac{q_2 + q}{C}\right) - L\frac{d^2q}{dt} = 0$$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} = \left(\frac{q_1 - 2q_2}{2C}\right) - \frac{3q}{2C}$$

The solution of this equation with the condition $q = 0$ at $t = 0$ is, $q = q_0(1 - \cos\omega t)$

where $\omega = \sqrt{\frac{3}{2LC}}$ and $q_0 = \frac{q_1 - 2q_2}{3} = 2CV_0$



(a) $I = \frac{dq}{dt} = q_0\omega \sin\omega t$

$$\Rightarrow \quad I_{max} = q_0\omega \text{ when } \omega t = \frac{\pi}{2}$$

(b) At this instant $q = q_0$, so we get

$$V_{2C} = \frac{q_1'}{2C} = \frac{q_1 - q_0}{2C} = \frac{8CV_0 - 2CV_0}{2C} = 3V_0 \text{ and}$$

$$V_C = \frac{q_2'}{C} = \frac{q_2 + q_0}{C} = \frac{CV_0 + 2CV_0}{C} = 3V_0$$

(c) $I = q_0\omega \sin(\omega t)$

Single Correct Choice Type Questions

1. At the terminal speed, the net force on the loop equals zero. So, we have

$$F_B = mg$$

$$\Rightarrow \quad BI\ell = mg$$

$$\Rightarrow \quad \frac{B^2\ell^2 v_T}{R} = mg \qquad \left\{\because I = \frac{\xi}{R} = \frac{B\ell v}{R}\right\}$$

$$\Rightarrow \quad v_T = \frac{mgR}{B^2\ell^2}$$

Now, $m = \rho V = \rho(4\ell)\pi\left(\frac{d}{2}\right)^2 = \pi\rho\ell d^2$ and

$$R = \frac{\ell_{\text{net}}}{\sigma A} = \frac{4\ell}{\sigma\left(\frac{\pi d^2}{4}\right)} = \frac{16\ell}{\pi\sigma d^2}$$

$$\Rightarrow \quad v_T = \frac{(\pi\rho\ell d^2)g\left(\frac{16\ell}{\pi\sigma d^2}\right)}{B^2\ell^2}$$

$$\Rightarrow \quad v_T = \frac{16\rho g}{\sigma B^2}$$

Hence, the correct answer is (C).

2. $\phi = 6t^2 - 5t + 1$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = -(12t - 5)$$

At $t = 0.25$ sec , $\xi = 2$ V

$$\Rightarrow \quad I = \frac{\xi}{R} = \frac{2}{10} = 0.2 \text{ A}$$

Hence, the correct answer is (D).

3. $U = \frac{1}{2}Li^2$

$$\Rightarrow \quad [L] = \left[\frac{U}{i^2}\right] = \left[\frac{ML^2T^{-2}}{A^2}\right]$$

$$\Rightarrow \quad [L] = [ML^2T^{-2}A^{-2}]$$

Hence, the correct answer is (C).

4. The net flux associated with the loop is

$$\phi = k\pi(a^2 - b^2)$$

The induced current is

$$I = \frac{\xi}{R} = \frac{1}{R}\left|\frac{d\phi}{dt}\right| = \frac{k\pi(a^2 - b^2)}{2\pi(a+b)\lambda} = \frac{k(a-b)}{2\lambda}$$

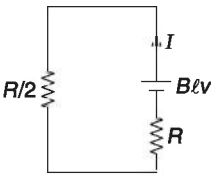
Hence, the correct answer is (C).

5. The voltage across inductance coil is

$$V_L = L\frac{di}{dt}$$

Hence, the correct answer is (C).

6. The wire PQ can be replaced with a battery of emf $\xi = Blv$ and internal resistance R. As the other two resistances are in parallel, their equivalent resistance is $\frac{R}{2}$.



$$\Rightarrow \quad I = \frac{Blv}{R + R/2} = \frac{2Blv}{3R}$$

Hence, the correct answer is (D).

7. $$\frac{\text{Electric Flux}}{\text{Magnetic Flux}} = \frac{EA}{BA} = \frac{E}{B}$$

$$\left[\frac{E}{B}\right] = [\text{Velocity}] = \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

Hence, the correct answer is (C).

8. At the instant the rod crosses point $(a, 0)$, its effective length is $l = 2\sqrt{4a \times a} = 4a$

The induced emf in the rod at this instant is

$$\xi = Blv = 4Bav$$

Hence, the correct answer is (D).

9. Value remains $\frac{1}{4}$th in 20 ms time. Hence, two half-lives are equal to 20 ms . So, one half-life is 10 ms

$$\Rightarrow \quad t_{1/2} = (\ln 2)\tau_C = (\ln 2)\frac{L}{R}$$

$$\Rightarrow \quad R = \frac{(\ln 2)L}{t_{1/2}}$$

$$\Rightarrow \quad R = \frac{(\ln 2)(2)}{10 \times 10^{-3}} = (100 \ln 4)\ \Omega$$

Hence, the correct answer is (C).

10. The peak value of induced emf is

$$\xi_0 = NBA\omega = 30 \times 1 \times (400 \times 10^{-4}) \times \left(\frac{1800 \times 2\pi}{60}\right)$$

$$\Rightarrow \quad \xi_0 = 226 \text{ V}$$

Hence, the correct answer is (B).

11. Induced electric field at a distance R from centre is

$$E = \frac{R}{2}\left(\frac{dB}{dt}\right)$$

Since $B(t) = B_0 + \alpha t$

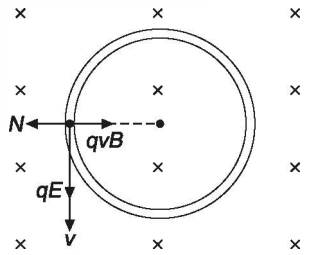
$$\Rightarrow \quad \frac{dB}{dt} = \alpha$$

$$\Rightarrow \quad E = \frac{R}{2}\alpha$$

Speed attained by the bead is

$$v = at \text{, where } a = \frac{qE}{m}$$

$$\Rightarrow \quad v = \left(\frac{qE}{m}\right)t$$



For circular motion of bead, we have

$$qvB - N = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{\alpha q^2 Rt}{4m}(2B_0 + \alpha t)$$

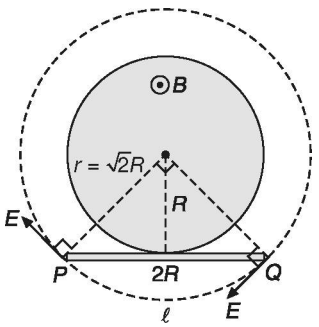
Hence, the correct answer is (B).

12. Consider a circle of radius r passing through points P and Q. The electric field at P and Q is given by

$$E(2\pi r) = A\frac{dB}{dt}$$

$$\Rightarrow \quad E\left[2\pi\left(\sqrt{2}R\right)\right] = \pi R^2\frac{dB}{dt} = \pi\alpha R^2$$

$$\Rightarrow \quad E = \frac{\alpha R}{2\sqrt{2}}$$



Therefore, emf across rod is

$$\xi = El$$

where $l = r\theta$

$$\Rightarrow \quad l = \sqrt{2}R\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \xi = \frac{\alpha R}{2\sqrt{2}} \times \frac{\pi\left(\sqrt{2}R\right)}{2} = \frac{\alpha\pi R^2}{4}$$

Hence, the correct answer is (D).

13. Since, $I = (10t + 5)$ A

$$\Rightarrow \quad \frac{dI}{dt} = 10 \text{ As}^{-1} = \text{constant}$$

At, $t = 0$, $I = 5$ A

Applying KLL, we get

$$V_A - 3 \times 5 - 1 \times 10 + 10 = V_B$$

$$\Rightarrow \quad V_A - V_B = 15 \text{ V}$$

Hence, the correct answer is (A).

15. The mutual inductance due to smaller ring carrying current I is same as the mutual inductance due to larger ring. If the larger ring carries current I, then the field at the location of smaller ring is

$$B = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

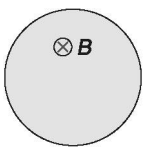
The flux associated with the smaller ring is

$$\phi = B(\pi r^2) = \frac{\mu_0\pi R^2 r^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}I$$

$$\Rightarrow \quad M = \frac{\phi}{I} = \frac{\mu_0\pi R^2 r^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

Hence, the correct answer is (D).

16. If magnetic field in the cylindrical region shown in Figure is changing, then induced electric field exists both inside and outside the cylindrical regions i.e. induced electric field also exists in the region where magnetic field does not exist.



Hence, the correct answer is (C).

CHAPTER 3

17. $q = q_0 \cos(\omega_0 t)$

$$\Rightarrow \quad I = \frac{dq}{dt} = -q_0\omega_0 \sin(\omega_0 t)$$

$$\Rightarrow \quad \frac{dI}{dt} = -q_0\omega_0^2 \cos(\omega_0 t)$$

$$\Rightarrow \quad \left(\frac{dI}{dt}\right)_{max} = q_0\omega_0^2 = \frac{q_0}{LC}$$

We can also think that since,

$$(V_C)_{max} = (V_L)_{max}$$

$$\Rightarrow \quad \frac{q_0}{C} = L\left(\frac{dI}{dt}\right)_{max}$$

$$\Rightarrow \quad \left(\frac{dI}{dt}\right)_{max} = \frac{q_0}{LC}$$

Hence, the correct answer is (A).

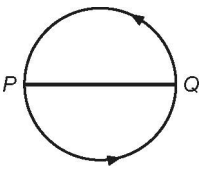
18. For wire ab, velocity vector $\vec{v}$ is parallel to length $\vec{l}$, hence $\xi = 0$

Hence, the correct answer is (C).

19. If the coil is rotated about an axis perpendicular to the plane of coil passing through O, there will be no change in flux and hence no emf will be induced.

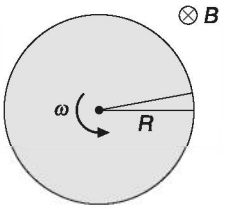
Hence, the correct answer is (D).

20. Since the inward field is decreasing at a constant rate, so the induced current must be set up in the loop so that if does not allow the inward field to decrease. Hence the induced current must set up an outward field. So, the induced current must flow in the loop in the Counter-Clockwise sense. Due to this fact no current flows through the branch PQ.



Hence, the correct answer is (D).

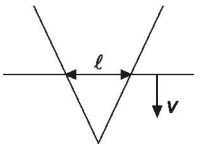
21. Consider a thin section of the disc shown in Figure.



The emf induced across it is $\xi = \frac{BR^2\omega}{2}$. Same emf is induced across all such sections. So, $\xi = \frac{BR^2\omega}{2}$ is the induced emf between the rim and the axis.

Hence, the correct answer is (D).

22. $i = \frac{Bvl}{R}$



Let λ be the resistance per unit length of conducting rod, then

$$i = \frac{Bvl}{\lambda l} = \frac{Bv}{\lambda} = \text{constant}$$

Hence, the correct answer is (C).

23. The effective length of a straight rod joining points P and Q is $l = 2L\sin\left(\frac{\theta}{2}\right)$

The emf induced across it is

$$\xi = BlV = 2BLV\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (C).

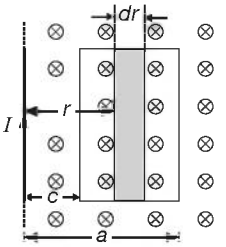
24. $B = \frac{\mu_0 I}{2\pi r}$

Since $\phi_m = \int BdA$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 I}{2\pi}\int \frac{dA}{r}$$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 I}{2\pi} b \int_c^{a+c} \frac{dr}{r}$$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 Ib}{2\pi} \ell n\left(\frac{a+c}{c}\right)$$



Hence, the correct answer is (C).

25. The force acting on the rod is $F = BIL$ towards left. If d is the distance moved by the rod, then by Work-Energy Theorem, we have

$$W_F = \Delta KE$$

$$\Rightarrow \quad -Fd = 0 - \frac{1}{2}mV_0^2$$

$$\Rightarrow \quad d = \frac{mV_0^2}{2F} = \frac{mV_0^2}{2BIL}$$

Hence, the correct answer is (B).

26. Imagine the rod OQ of length $2l$ with P as its centre.

Then, $V_Q - V_O = \frac{1}{2}B(2l)^2\omega$

and $V_P - V_O = \frac{1}{2}Bl^2\omega$

$$\Rightarrow \quad V_Q - V_P = \frac{1}{2}B(2l)^2\omega - \frac{1}{2}Bl^2\omega = \frac{3}{2}Bl^2\omega$$

Hence, the correct answer is (C).

27. The current through inductor is

$$I_1 = \frac{20}{4+6}\left(1 - e^{-(4+6)t/0.005}\right) = 2\left(1 - e^{-2000t}\right)$$

The current through capacitor is

$$I_2 = \frac{20}{5+5}e^{-t/(5+5)(10^{-4})} = 2e^{-1000t}$$

The current through the key is

$$I = I_1 + I_2 = 2\left(1 - e^{-2000t}\right) + 2e^{-1000t}$$

At $t = 10^{-3}\ln 2$,

$$I = \frac{3}{2} + 1 = 2.5 \text{ A}$$

Hence, the correct answer is (C).

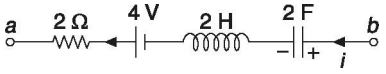
28. $i = \frac{dq}{dt} = (8t)$ A

$$\Rightarrow \quad \frac{di}{dt} = 8 \text{ As}^{-1}$$

At $t = 1$ s, $q = 4$ C, $i = 8$ A

and $\frac{di}{dt} = 8 \text{ As}^{-1}$

Since charge on capacitor is increasing, hence charge on positive plate is also increasing. So, the direction of current is towards left.



Applying KLL from a to b, we get

$$V_a + 2\times 8 - 4 + 2\times 8 + \frac{4}{2} = V_b$$

$$\Rightarrow \quad V_a - V_b = -30 \text{ V}$$

Hence, the correct answer is (B).

29. The change in flux between the initial and final position is $|\Delta\phi| = |0 - BA| = BA$

Time taken to turn through 90° is

$$\Delta t = \frac{T}{4} = \frac{1}{4}\times\frac{2\pi}{\omega} = \frac{\pi}{2\omega}$$

$$\Rightarrow \quad \xi_{av} = \left|-\frac{\Delta\phi}{\Delta t}\right| = \frac{BA}{\pi/2\omega} = \frac{2\omega BA}{\pi}$$

Hence, the correct answer is (D).

30. By short-circuiting the battery, net resistance across inductor is $\frac{R}{2}$, because R and R become parallel.

$$\Rightarrow \quad \tau_{\text{net}} = \frac{L}{R_{\text{net}}} = \frac{2L}{R}$$

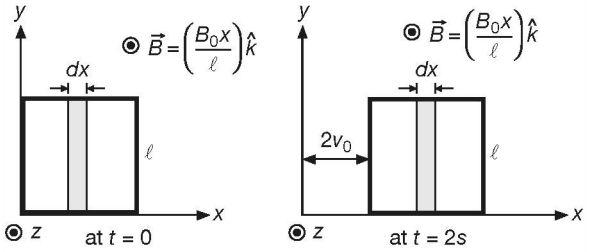
Hence, the correct answer is (B).

31. The motion of charged rod is equivalent to the current flowing along the rod. The field produced due to this current is in the plane of the loop. Hence, the induced emf is zero.

Hence, the correct answer is (D).

32. Total flux linked with the coil at time $t = 0$ is

$$\phi_i = \int_0^{\phi} d\phi = \int_0^{\ell}\left(\frac{B_0 x}{\ell}\right)(\ell\, dx) = \frac{B_0\ell^2}{2}$$



and flux linked with the coil at time $t = 2$ s is

$$\phi_f = \int_{2v_0}^{2v_0+\ell} d\phi$$

$$\Rightarrow \quad \phi_f = \int_{2v_0}^{2v_0+\ell} \frac{B_0 x}{\ell}\cdot(\ell\, dx)$$

$$\Rightarrow \quad \phi_f = \frac{B_0}{2}\left[(2v_0+\ell)^2 - (2v_0)^2\right]$$

$$\phi_f = \frac{B_0}{2}\left(4v_0^2 + \ell^2 + 4v_0\ell - 4v_0^2\right) = \frac{B_0\ell}{2}(\ell + 4v_0)$$

Since $\xi = \frac{\Delta\phi}{\Delta t} = \frac{1}{2}\left(\frac{B_0\ell^2}{2} + \frac{4B_0\ell v_0}{2} - \frac{B_0\ell^2}{2}\right)$

$$\Rightarrow \quad \xi = B_0\ell v_0$$

Hence, the correct answer is (A).

CHAPTER 3

33. The flux associated with the semicircle is

$$\phi = BA\cos\theta = B\frac{\pi r^2}{2}\cos(\omega t)$$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = \frac{B\omega\pi r^2}{2}\sin(\omega t)$$

Since, $P = \frac{\xi^2}{R} = \frac{B^2\omega^2\pi^2 r^4}{4R}\sin^2\omega t$

$$\Rightarrow \quad P_{av} = \langle P\rangle = \frac{1}{T}\int_0^T \frac{B^2\omega^2\pi^2 r^4}{4R}\sin^2(\omega t)\,dt$$

$$\Rightarrow \quad P_{av} = \langle P\rangle = \frac{(B\pi r^2\omega)^2}{8R}$$

Hence, the correct answer is (B).

34. Since all i.e. $\vec{B}$, $\vec{l}$ and $\vec{v}$ are coplanar, so $\xi = 0$.

Hence, the correct answer is (A).

35. The magnitude of electric field at $(r, 0, 0)$ is

$$E_{nc} = \left|\frac{\pi a^2\frac{dB}{dt}}{2\pi r}\right| = \frac{a^2 B_0}{2r}$$

The force on charge q is $F = qE_{nc} = \frac{qB_0a^2}{2r}$

Hence, the correct answer is (B).

36. $\Delta q = \frac{\Delta\phi}{R}$

$$\Rightarrow \quad i\Delta t = \frac{\Delta\phi}{R}$$

$$\Rightarrow \quad \Delta\phi = i(\Delta t)R = (10\times10^{-3})(5)(0.5)$$

$$\Rightarrow \quad \Delta\phi = 25\times10^{-3}\ \text{Wb}$$

Hence, the correct answer is (B).

37. Let E be the emf of batteries and R be the resistances, then, $i_1 = 0$, $i_2 = \frac{E}{R}$, $i_3 = \frac{E}{2R}$

$$\Rightarrow \quad i_2 > i_3 > i_1$$

Hence, the correct answer is (A).

38. Since, $E_{nc}(2\pi r) = A\left(\frac{dB}{dt}\right)$

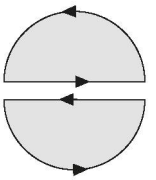
$$\Rightarrow \quad E_{nc}(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E \propto r$$

Hence, the correct answer is (B).

39. On breaking the loop in two parts as shown in Figure, we observe that equal and opposite currents flow through the diameter.



They cancel each other. So, no current flows through diameter.

Hence, the correct answer is (D).

40. $\oint \vec{E}\cdot d\vec{\ell} = \left|\frac{d\phi}{dt}\right|$

$$\Rightarrow \quad E(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r}{2}\left(\frac{dB}{dt}\right), \text{ tangential to the ring}$$

Now

$$\tau = Fr = qEr$$

$$\Rightarrow \quad \tau = q\left(\frac{r}{2}\frac{dB}{dt}\right)r$$

$$\Rightarrow \quad \tau = \frac{1}{2}qr^2\left(\frac{dB}{dt}\right)$$

Hence, the correct answer is (C).

41. There is no change in flux through the ring. Hence, emf induced is zero.

Hence, the correct answer is (A).

42. Since $\xi = \left|\frac{d\phi}{dt}\right| = a\tau - 2at$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{a\tau - 2at}{R}$$

Heat generated in the loop is

$$H = \int_0^\tau i^2 R\,dt = \frac{a^2\tau^3}{3R}$$

Hence, the correct answer is (B).

43. Let the magnetic field be into the paper. Then, if the wire cd moves with velocity v, the current through it is $I = \frac{\xi}{R} = \frac{Blv}{R}$ towards right.

The magnetic force on it is

$$F = IlB = \frac{Blv}{R}lB \text{ upwards.}$$

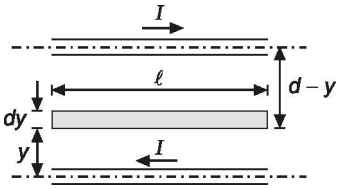
This magnetic force is balanced by the weight mg of rod acting downwards.

$$\Rightarrow \quad \frac{B^2l^2v}{R} = mg$$

$$\Rightarrow \quad v = \frac{mgR}{B^2l^2}$$

Hence, the correct answer is (B).

44. Let us calculate the coefficient of self-induction for this arrangement. For doing this, we need to find the flux between the wires. Let us consider a strip of width dy and length l as shown in Figure.



Magnetic field at the strip due to current in two wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)\otimes$$

Area of the strip, $dA = ldy$

Flux associated with the strip is

$$d\phi = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)ldy$$

Total flux associated with the area between two wires is

$$\phi = \int d\phi = \frac{\mu_0 Il}{2\pi}\left(\int_a^{d-a}\frac{dy}{y} + \int_a^{d-a}\frac{dy}{d-y}\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{d-d+a}{d-a}\right)\right]$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{\pi}\ln\left(\frac{d-a}{a}\right)$$

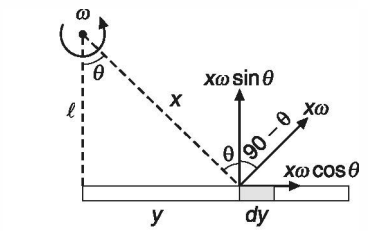
$$\Rightarrow \quad L = \frac{\phi}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Hence the inductance (L) per unit length of this arrangement,

$$\frac{L}{l} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Hence, the correct answer is (A).

45. **METHOD-I**



$$d\xi = B(dy)(x\omega\sin\theta)$$

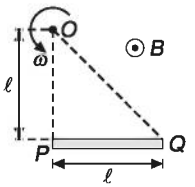
Now $\sin\theta = \frac{y}{x}$

$$\Rightarrow \quad d\xi = B(dy)\left(x\omega\left(\frac{y}{x}\right)\right)$$

$$\Rightarrow \quad \int d\xi = \int_0^l Bydy$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega l^2$$

METHOD-II



Consider two imaginary straight rods OP of length l and OQ of length $\sqrt{2}l$.

Then, $V_P - V_O = \frac{1}{2}B\omega l^2$

and $V_Q - V_O = \frac{1}{2}B\omega(\sqrt{2}l)^2$

$$\Rightarrow \quad V_Q - V_P = \frac{1}{2}B\omega(\sqrt{2}l)^2 - \frac{1}{2}B\omega l^2 = \frac{1}{2}B\omega l^2$$

Hence, the correct answer is (A).

46. $P = i_0^2 R$

$$\Rightarrow \quad i_0^2 = \frac{P}{R}$$

Since $\tau = \frac{L}{R}$

$$\Rightarrow \quad L = \tau R$$

Heat dissipated $= \frac{1}{2}Li_0^2 = \frac{1}{2}(\tau R)\left(\frac{P}{R}\right) = \frac{1}{2}P\tau$

Hence, the correct answer is (B).

CHAPTER 3

47. The magnetic flux associated with the loop is

$$\phi = \int_{c}^{a+c} \frac{\mu_0 I}{2\pi x} b\,dx = \frac{\mu_0 Ib}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

$$\Delta I = 0 - I_0 = -I_0$$

$$\Rightarrow \quad \Delta\phi = -\frac{\mu_0 I_0 b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

$$\Rightarrow \quad Q = -\frac{\Delta\phi}{R} = \frac{\mu_0 I_0 b}{2\pi R} \ln\left(\frac{a+c}{c}\right)$$

Hence, the correct answer is (C).

48. $\tau = R_{eq} C_{eq}$

$\tau = (5)(1)$

$\tau = 5$ s

Hence, the correct answer is (A).

49. According to Lenz's law, the induced current in the ring opposes the motion of magnet. The magnetic field produced in the ring will exert an upward force on the magnet. The acceleration of magnet will be thus, less than acceleration due to gravity.

Hence, the correct answer is (B).

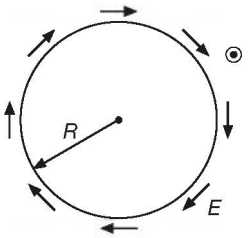
50. Due to the time varying magnetic field, an induced electric field E is set up, so the force on the ring is

$$F = qE$$

If f be the frictional force between the ring and the table, then the ring starts rotating when

$$F = f$$

$$\Rightarrow \quad qE = \mu mg \qquad \ldots(1)$$



Also $\oint \vec{E} \cdot d\vec{\ell} = \left|\frac{d\phi}{dt}\right|$

$$\Rightarrow \quad E(2\pi R) = A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E(2\pi R) = (\pi R^2)(16t) \qquad \left\{\because \frac{dB}{dt} = 16t\right\}$$

$$\Rightarrow \quad E = 8Rt \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$q(8Rt) = \mu mg$$

$$\Rightarrow \quad \mu = \left(\frac{8qR}{mg}\right)t$$

$$\Rightarrow \quad \mu\big|_{t=3} = \frac{24qR}{mg}$$

Hence, the correct answer is (D).

51. The emf induced is

$$\xi = \frac{Bl^2\omega}{2}$$

$$\Rightarrow \quad \omega = \frac{2\xi}{Bl^2}$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{\xi}{\pi Bl^2} = \frac{0.01}{\pi \times 5 \times 10^{-4} \times 1^2}$$

$$\Rightarrow \quad f = \frac{20}{\pi} \text{ rev/sec}$$

Hence, the correct answer is (B).

52. Inductive time constant is

$$\tau_L = \frac{L}{R} = 2 \text{ s}$$

Since, $i_0 = \frac{E}{R} = 3$ A

So, at $t = 2$ s, we get

$$i = i_0\left(1 - e^{-\frac{t}{\tau_L}}\right) = 3\left(1 - e^{-1}\right) \text{ A}$$

Hence, the correct answer is (A).

53. Since $\xi = E - IR$

$$\Rightarrow \quad \xi = E - E\left(1 - e^{-\frac{t}{\tau}}\right) = Ee^{-\frac{t}{\tau}}$$

Therefore, ξ vs t graph is best represented by OPTION (C).

Hence, the correct answer is (C).

54. Already derived in Theory.

The correct answer is (D).

55. The electric field at point P is given by

$$E(2\pi R) = \pi R^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{R}{2}\frac{dB}{dt}$$

Force on charge q is

$$F = qE = \frac{qR}{2}\frac{dB}{dt}$$

$$\Rightarrow \int_{B_0}^{0} dB = \frac{2}{qR}\int_{0}^{t} F dt = \frac{2}{qR} \times (\text{impulse})$$

Since $\int_0^t F dt = mv$

$$\Rightarrow |0 - B_0| = \frac{2}{qR} mv$$

$$\Rightarrow v = \frac{qRB_0}{2m}$$

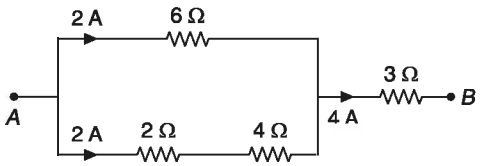
Hence, the correct answer is (C).

56. For the second position, $\Delta\phi = 0$

$$\Rightarrow |Q_2| = \frac{\Delta\phi}{R} = 0$$

Hence, the correct answer is (D).

57. In steady state, inductor acts as a pure conductor. Steady state circuit is shown in Figure.



$$V_A - (2)(6) - (4)(3) - V_B = 0$$

$$\Rightarrow V_A - V_B = 24 \text{ V}$$

Hence, the correct answer is (C).

58. According to Lenz's law, the motion of magnet will always be opposed. So, M will repel R when moving towards it and it will attract R when moving away from it.

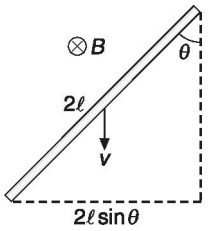
Hence, the correct answer is (A).

59. $$\frac{1}{2}mv_0^2 = \frac{1}{2}Li_{\max}^2$$

$$\Rightarrow i_{\max} = \sqrt{\frac{m}{L}}v_0$$

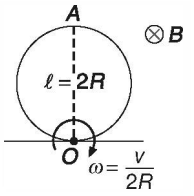
Hence, the correct answer is (B).

60. The projected length of the rod in the direction perpendicular to both $\vec{v}$ and $\vec{B}$ is $2l\sin\theta$. Therefore, potential difference between the two ends of the rod equals $Bv(2l\sin\theta)$



Hence, the correct answer is (B).

61. Let a hypothetical conducting rod of length $l = 2R$ joins O to A, then emf induced across ends of rod is



$$\xi_{OA} = \frac{B\omega l^2}{2} = \frac{(B)\left(\frac{v}{2R}\right)(4R)^2}{2} = 4BvR$$

Hence, the correct answer is (D).

62. Applying Kirchhoff's rule to the path A to B, we get

$$V_A - V_B = L\,di/dt + iR$$

$$\Rightarrow 0.5 = L \times 8 + 0.5 \times 0.2$$

$$\Rightarrow L = 0.05 \text{ H}$$

Hence, the correct answer is (C).

63. Applying Kirchhoff's Loop Law (KLL), we get

$$V_A - IR + E - L\frac{dI}{dt} - V_B = 0$$

According to the problem, we have

$$\frac{dI}{dt} = -10^3 \text{ As}^{-1}$$

$$\Rightarrow V_A - (5)(1) + 15 - \left(5 \times 10^{-3}\right)\left(-10^3\right) - V_B = 0$$

$$\Rightarrow V_A - V_B = 15\,V$$

Hence, the correct answer is (B).

64. Applying Kirchhoff's Loop Law (KLL), we get

$$V_A + IR + E + L\frac{dI}{dt} - V_B = 0$$

According to the problem, we have

$$\frac{dI}{dt} = -10^3 \text{ As}^{-1}$$

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$$\Rightarrow \quad V_A + (5)(1) + 15 + (5\times10^{-3})(-10^3) - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = -15 \text{ V}$$

$$\Rightarrow \quad V_B - V_A = 15 \text{ V}$$

Hence, the correct answer is (C).

65. At mean position, velocity is maximum. Hence motional EMF Bvl is also maximum. Velocity v oscillates simple harmonically so motional emf will also vary simple harmonically. Further, polarity of induced emf will keep on changing.

Hence, the correct answer is (B).

66. The current as a function of time in LR circuit after closing the switch is

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

When energy stored in inductor is half the maximum value, then

$$\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_0^2\right)$$

$$\Rightarrow \quad i = \frac{i_0}{\sqrt{2}} = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{Rt}{L} = \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

Hence, the correct answer is (A).

67. Since $E_{nc}(2\pi r) = A\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad E_{nc}(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\frac{dB}{dt}$$

Force on the charge q is

$$F = qE = \frac{qr}{2}\left(\frac{dB}{dt}\right)$$

Work done is

$$W = Fd = (2\pi r)$$

$$\Rightarrow \quad W = \pi r^2 q\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad W = \left(\frac{22}{7}\right)(1)^2(10^{-6})(2\times10^{-3})$$

$$\Rightarrow \quad W = 2\pi\times10^{-9} \text{ J}$$

Hence, the correct answer is (B).

68. Maximum speed is attained when the entire energy stored in inductor is converted into kinetic energy of rod

$$\Rightarrow \quad \frac{1}{2}L\left(\frac{E}{R}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \left(\frac{E}{R}\right)\sqrt{\frac{L}{m}}$$

Hence, the correct answer is (A).

69. Since both L_1 and L_2 are in parallel, so

$$V_1 = V_2$$

$$\Rightarrow \quad L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$

$$\Rightarrow \quad L_1 dI_1 = L_2 dI_2$$

$$\Rightarrow \quad L_1\int_0^{I_1} dI_1 = L_2\int_0^{I_2} dI_2$$

$$\Rightarrow \quad L_1 I_1 = L_2 I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

Hence, the correct answer is (A).

70. At $t = 0$, potential difference between A and B is V and resistance is $R + R = 2R$.

$$\Rightarrow \quad i_1 = \left(\frac{V}{2R}\right)$$

As $t \to \infty$, potential difference between A and B is V and resistance is R

$$\Rightarrow \quad i_2 = \frac{V}{R}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{1}{2}$$

Hence, the correct answer is (D).

71. Since, $\frac{1}{2}Li^2 = \frac{1}{4}\left(\frac{1}{4}Li_0^2\right)$

$$\Rightarrow \quad i = \frac{i_0}{2}, \text{ half value}$$

$$\Rightarrow \quad t = t_{1/2} = (\ln 2)\tau_L = (\ln 2)\left(\frac{L}{R}\right)$$

Hence, the correct answer is (A).

72. The magnitude of emf induced is $\xi = \frac{d\phi}{dt}$

The current through the circuit is

$$i = \frac{dQ}{dt} = \frac{\xi}{R} = \frac{1}{R}\frac{d\phi}{dt}$$

$$\Rightarrow \quad dQ = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{\Delta\phi}{R}$$

Hence, the correct answer is (D).

73. $\xi = \frac{d\phi}{dt}$ {in magnitude}

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$$

where $A = \pi r^2$ is the area of loop of radius r and R is the resistance of the loop of length $(2\pi r)$ and area of cross-section πa^2.

$$\Rightarrow \quad R = \frac{\rho\ell}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$\Rightarrow \quad i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$

$$\Rightarrow \quad i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt}$$

$$\Rightarrow \quad i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

Hence, the correct answer is (A).

74. When the sides shrink at the rate α, we have

$$\frac{da}{dt} = -\alpha$$

Since, $\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$

$$\Rightarrow \quad \xi = -B\frac{d}{dt}(a^2) = -2Ba\frac{da}{dt} = 2a\alpha B$$

Hence, the correct answer is (A).

75. The induced emf is given by

$$\xi = A\frac{dB}{dt} = \pi r^2 B_0\left[\frac{d}{dt}(e^{-t})\right] = \pi r^2 B_0 e^{-t}$$

$$\Rightarrow \quad P = \frac{V^2}{R} = \left(\frac{\pi^2 r^4 B_0^2}{R}\right)e^{-2t}$$

At $t = 0$, $P = \frac{B_0^2\pi^2 r^4}{R}$

Hence, the correct answer is (D).

76. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right) = I_0(1 - e^{-2t})$

$$\Rightarrow \quad \frac{I(t\to\infty)}{I(t=1)} = \frac{I_0}{I_0(1-e^{-2})} = \frac{e^2}{e^2-1}$$

Hence, the correct answer is (B).

77. $\frac{dI}{dt} = I_0\omega\cos\omega t$

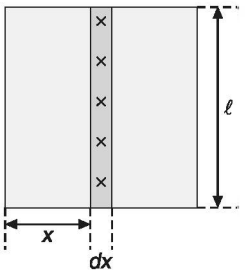
Since $\xi = M\frac{dI}{dt} = MI_0\omega\cos\omega t$

$$\Rightarrow \quad \xi_{max} = MI_0\omega$$

$$\Rightarrow \quad \xi_{max} = 0.005\times 10\times 100\pi = 5\pi \text{ V}$$

Hence, the correct answer is (B).

78. The flux associated with the loop in initial position is



$$\phi_i = \int_0^l Bldx = B_0 l\int_0^l xdx = \frac{B_0 l^3}{2}$$

The flux associated with the loop in final position is

$$\phi_f = -\int_l^{2l} Bldx = -B_0 l\int_l^{2l} xdx = -\frac{3B_0 l^3}{2}$$

$$\Rightarrow \quad |\Delta\phi| = |\phi_i - \phi_f| = 2B_0 l^3$$

Hence, the correct answer is (B).

79. At $t = 0$ i.e. when the key is just pressed, no current exists inside the inductor. So $10\ \Omega$ and $20\ \Omega$ resistors are in series and a net resistance of $(10+20) = 30\ \Omega$ exists across the circuit.

Hence $I_1 = \frac{2}{30} = \frac{1}{15}A$

As $t\to\infty$, the current in the inductor grows to attain a maximum value i.e. the entire current passes through the inductor and no current passes through $10\ \Omega$ resistor

Hence $I_2 = \frac{2}{20} = \frac{1}{10}A$

Hence, the correct answer is (A).

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80. Energy stored in the inductor is

$$U=\frac{1}{2}LI^2=\frac{1}{2}L\left(\frac{E}{R}\right)^2\left(1-e^{-\frac{Rt}{L}}\right)^2$$

The rate at which energy is stored is

$$\frac{dU}{dt}=\frac{E^2}{R}\left(1-e^{-\frac{Rt}{L}}\right)\left(e^{-\frac{Rt}{L}}\right)$$

Its value is maximum, when

$$1-e^{-\frac{Rt}{L}}=e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}}=\frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{dU}{dt}\right)_{\max}=\frac{E^2}{4R}$$

Hence, the correct answer is (A).

81. At time $t=0$, resistance offered by a capacitor is zero and resistance offered by an inductor is ∞.

$$\Rightarrow \quad R_{\text{net}}=\frac{R}{2}+\frac{R}{3}=\frac{5R}{6}=5\ \Omega$$

So, current from the battery is

$$i=\frac{E}{R_{\text{net}}}=\frac{5}{5}=1\ \text{A}$$

Hence, the correct answer is (A).

82. The charge flowing through galvanometer is

$$Q=\frac{|\Delta\phi|}{R}=\frac{AB}{R}$$

Hence, the correct answer is (C).

83. The emf induced in the triangular loop is

$$\xi=\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi=A\frac{dB}{dt}$$

Since area of an equilateral triangle is

$$A=\frac{\sqrt{3}}{4}\times(\text{side})^2$$

$$\Rightarrow \quad A=\frac{\sqrt{3}}{4}(2)^2=\sqrt{3}\ \text{m}^2$$

$$\Rightarrow \quad \xi=\sqrt{3}\times\sqrt{3}=3\ \text{V}$$

Due to symmetry, emf developed across each wire of frame will be 1 V.

The current in the triangular loop is

$$i=\frac{\xi}{R}=\frac{3}{2+2+1}=0.6\ \text{A}$$

Voltage across AB is

$$\Rightarrow \quad V_{AB}=\xi_{AB}-iR_{AB}=1-(0.6)(1)$$

$$\Rightarrow \quad V_{AB}=1-0.6=0.4\ \text{V}$$

Hence, the correct answer is (A).

84. Flux linked with the loop, when it is at a distance r from the wire is

$$\phi=BA\cos 0=\left(\frac{\mu_0 I}{2\pi r}\right)(\pi a^2)$$

$$\Rightarrow \quad \phi=\frac{\mu_0 I a^2}{2r}$$

Since $\xi=-\frac{d\phi}{dt}$

$$\Rightarrow \quad \xi=\frac{\mu_0 I a^2}{2}\frac{d}{dt}(r^{-1})$$

$$\Rightarrow \quad \xi=\frac{\mu_0 I a^2}{2r^2}\left(\frac{dr}{dt}\right)$$

$$\Rightarrow \quad \xi=\left(\frac{\mu_0 I a^2}{2r^2}\right)v \qquad \left\{\because \frac{dr}{dt}=v\right\}$$

Hence, the correct answer is (C).

85. The current through the wire PQ is $I=\frac{\xi}{R}$

When the wire just starts sliding, the force IlB towards right is equal to the force μmg towards left.

$$\Rightarrow \quad \mu mg=IlB=\left(\frac{\xi}{R}\right)lB$$

$$\Rightarrow \quad \mu=\frac{\xi lB}{mgR}=\frac{6\times 4.9\times 10^{-2}\times 0.8}{10^{-2}\times 9.8\times 20}=0.12$$

Hence, the correct answer is (D).

86. Relative velocity $v_r=0$

So, change in flux $\Delta\phi=0$

Hence, the correct answer is (D).

87. At $t=0$, current through L is zero.

So, $i_1=\frac{10}{6+4}=1\ \text{A}$

As $t\to\infty$ effective resistance of circuit is

$$R_{\text{eff}}=6+\frac{4}{2}=8\ \Omega$$

$$\Rightarrow \quad i_2=\frac{10}{8}=\frac{5}{4}\ \text{A}$$

$$\Rightarrow \quad \frac{i_1}{i_2}=0.8$$

Hence, the correct answer is (B).

88. $\frac{L}{R} = 2 \times 10^{-3}$ ...(1)

$$\frac{L}{R+90} = 0.5 \times 10^{-3} \quad ...(2)$$

From (1) and (2), on solving we get

$L = 60$ mH and $R = 30\ \Omega$

Hence, the correct answer is (C).

89. The distance of points P and Q from centre is $2R$. Electric field at these points is given by

$$E(2\pi \times 2R) = \pi R^2 \left(\frac{dB}{dt}\right) = \pi R^2 b$$

$$\Rightarrow \quad E = \frac{Rb}{4}$$

So, potential difference between P and Q is

$$\xi = El = E\left[\frac{2\pi(2R)}{3}\right] = \frac{\pi b R^2}{3}$$

The current induced in loop is

$$I = \frac{\xi}{R_{net}} = \frac{\xi}{(2R + 2R + 2\sqrt{3}R)\lambda}$$

$$\Rightarrow \quad I = \frac{\pi bR}{6\lambda(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \quad I = \frac{\pi Rb(2-\sqrt{3})}{6\lambda}$$

Hence, the correct answer is (A).

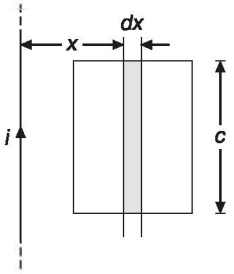
90. At $t = 0$, $V_L = -E$

Hence, the correct answer is (C).

91. The emf induced across wire HE is $\xi = Blv$ and is a constant. Once capacitor attains this potential difference, no current flows in $HKDE$.

Hence, the correct answer is (D).

92. Consider an element of length c, thickness dx at a distance x from the wire as shown in Figure.



Area (dA) of element is

$dA = cdx$

Magnetic field at the element is

$$B = \frac{\mu_0 i}{2\pi x}$$

Since $d\phi = BdA$

$$\Rightarrow \quad d\phi = \frac{\mu_0 ic}{2\pi x} dx$$

$$\Rightarrow \quad \phi = \int_a^b d\phi = \frac{\mu_0 ic}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 c}{2\pi} \ln\left(\frac{b}{a}\right)$$

Hence, the correct answer is (B).

93. The field produced due to induced current is directed into the paper. So, current in outer loop is clockwise and in inner loop is anticlockwise. Hence, current will flow from A to B and D to C.

Hence, the correct answer is (C).

94. For PQ, $\vec{l}$ is parallel to $\vec{v}$. Hence, $\xi = 0$

Hence, the correct answer is (A).

95. The horizontal distance between points P and Q is $l = a$. The potential difference between these points is

$\xi = Blv = Bva$

Hence, the correct answer is (D).

96. The equivalent resistance of five resistors is $3\ \Omega$

{It is a balanced Wheat Stone Bridge}

So $R_{total} = 3 + 1 = 4\ \Omega$

$$\Rightarrow \quad (10^{-3})(4) = (2)(0.1)v \qquad \{\because \xi = IR = B\ell v\}$$

$$\Rightarrow \quad v = 2\ \text{cms}^{-1}$$

Hence, the correct answer is (C).

97. The current through L and C are respectively

$$I = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \text{ and } I = \frac{V}{R} e^{-\frac{t}{RC}}$$

Both are equal when

$$1 - e^{-\frac{Rt}{L}} = e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad R = \sqrt{\frac{L}{C}}$$

$$\Rightarrow \quad \frac{R}{L} = \frac{1}{RC}$$

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$$\Rightarrow \quad 1-e^{-\frac{Rt}{L}}=e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}}=\frac{1}{2}$$

$$\Rightarrow \quad t=CR(\ln 2)$$

Hence, the correct answer is (B).

98. By Law of Conservation of Energy, we get

$$\frac{1}{2}Li^2=\frac{1}{2}CV^2$$

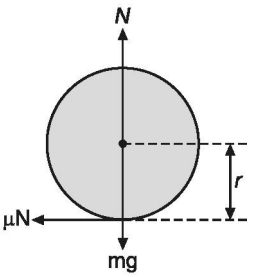
$$\Rightarrow \quad V=\sqrt{\frac{L}{C}}$$

$$\Rightarrow \quad i=\sqrt{\frac{2}{4\times10^{-6}}}(2)=\sqrt{2}\times10^3 \text{ V}$$

Hence, the correct answer is (A).

99. The electric field induced at the ring is

$$E=\frac{\pi r^2\left(\frac{dB}{dt}\right)}{2\pi r}=\frac{r}{2}kt=\frac{rt}{2}$$



Since the magnetic field is increasing with time, so an anticlockwise torque acts on the ring and is given by

$$\tau=Fr_{\perp}=QEr=\frac{Qr^2t}{2}$$

The ring starts motion when

$$\tau=\mu Nr=\mu mgr$$

$$\Rightarrow \quad \frac{Qr^2t}{2}=\mu mgr$$

$$\Rightarrow \quad t=\frac{2\mu mg}{rQ}$$

Hence, the correct answer is (B).

100. $i_0=\frac{E}{R}=\frac{12}{0.3}=40$ A

Energy stored in the inductor is

$$U=\frac{1}{2}Li_0^2=\frac{1}{2}\times50\times10^{-3}(40)^2$$

$$\Rightarrow \quad U=40 \text{ J}$$

Hence, the correct answer is (A).

101. Induced EMF across the rod is

$$\xi=\vec{B}\cdot(\vec{l}\times\vec{v})=\begin{vmatrix}B_x & B_y & B_z\\ l_x & l_y & l_z\\ v_x & v_y & v_z\end{vmatrix}$$

The magnetic field in the region is

$$\vec{B}=3\hat{j}+4\hat{k}$$

The length vector of the rod is

$$\vec{l}=(10\cos53°)\hat{i}+(10\sin53°)\hat{j}=6\hat{i}+8\hat{j}$$

The velocity of the rod is

$$\vec{v}=1\hat{i}$$

So, EMF induced is

$$\xi=\begin{vmatrix}3 & 0 & 4\\ 6 & 8 & 0\\ 1 & 0 & 0\end{vmatrix}$$

$$\Rightarrow \quad \xi=4(0-8)=-32 \text{ V}$$

Hence, the correct answer is (A).

102. $\Delta\phi=0-BA$

$$\Rightarrow \quad \Delta\phi=-BA \text{ and } \Delta t=\frac{T}{4} \text{ for } \theta=\frac{\pi}{2}$$

$$\Rightarrow \quad \xi_{av}=\left|\frac{\Delta\phi}{\Delta t}\right|=\frac{BA}{\left(\frac{T}{4}\right)}$$

$$\Rightarrow \quad \xi_{av}=\frac{BA}{\frac{2\pi}{4\omega}}=\frac{2\omega BA}{\pi}$$

Hence, the correct answer is (D).

103. At any time t, the current in the circuit grows in accordance with formula

$$I=I_0(1-e^{-Rt/L})=I_0(1-e^{-t/\tau})$$

$$\Rightarrow \quad I=\frac{E}{R}(1-e^{-t/\tau})$$

$$\Rightarrow \quad \frac{dq}{dt}=\frac{E}{R}(1-e^{-t/\tau})$$

$$\Rightarrow \quad dq=\frac{E}{R}dt-\frac{E}{R}e^{-t/\tau}dt$$

$$\Rightarrow \quad q=\frac{E}{R}\int_0^{\tau}dt-\frac{E}{R}\int_0^{\tau}e^{-t/\tau}dt$$

$$\Rightarrow \quad q = \frac{E\tau}{R} - \left[-\frac{E\tau}{\text{Re}} + \frac{E\tau}{R} \right] = \frac{E\tau}{\text{Re}}$$

Hence, the correct answer is (C).

104. While moving towards the loop, the flux through the loop will increase and while moving away, the flux will decrease. So, the current will change direction as the electron passes by.

Hence, the correct answer is (D).

105. $M \propto N_1 N_2$

$$\Rightarrow \quad M' = 4M$$

Hence, the correct answer is (A).

106. Efficiency of a d.c. motor is the ratio of the back emf to the applied emf.

Hence, the correct answer is (B).

107. $\xi = L\left|\frac{di}{dt}\right| = L$ (Slope of i-t graph)

Initially, slope $= 0$

$$\Rightarrow \quad \xi = 0$$

In the remaining two regions, slopes are constants but of opposite signs. Hence, induced emfs are constants but of opposite signs.

Hence, the correct answer is (C).

108. Due to increasing field, the induced current in both loops individually must be anticlockwise which actually oppose each other. However, as the emf induced in larger loop is more, the net current in larger loop is anticlockwise and in smaller loop is clockwise. Hence, its direction is B to A and D to C

Hence, the correct answer is (D).

109. Since we know that

$$dq = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{1}{R}(\Delta\phi) = \frac{1}{R}(\phi_f - \phi_i)$$

where $\phi_i = 0$ and $\phi_f = \left(\frac{\mu_0 I}{2b}\right)(\pi a^2)$

$$\Rightarrow \quad \Delta\phi = \frac{\mu_0 \pi I a^2}{2b}$$

So, $Q = \frac{\mu_0 \pi I a^2}{2bR}$

Hence, the correct answer is (D).

110. $\vec{v}$ is parallel to $\vec{l}$, so $\xi = 0$

Hence, the correct answer is (D).

111. On increasing current, the flux through Q increases. An opposing magnetic field is developed due to induced current. The loops will therefore repel each other.

Hence, the correct answer is (B).

112. The time in which the charge decays to 0.368 of its initial value is equal to the time constant of the RC circuit i.e. $\tau = RC$

$$\Rightarrow \quad \tau = 5 \times 10^4 \times 2 \times 10^{-6} s = 10 \times 10^{-2} \text{ s}$$

$$\Rightarrow \quad \tau = 0.1 \text{ s}$$

Hence, the correct answer is (D).

113. For the case of free fall, we have

$$s = \frac{1}{2} g t^2$$

$$\Rightarrow \quad s = \frac{1}{2}(10)(1)^2 = 5 \text{ m}$$

Here due to repulsion from induced effects, we get

$$a < g$$

$$\Rightarrow \quad s < 5 \text{ m}$$

Hence, the correct answer is (C).

114. Applying Kirchhoff's rule to the path A to B, we get

$$V_A - V_B = IR + L dI/dt - 10$$

where, $dI/dt = 10 \text{ As}^{-1}$

At $t = 0$, $I = 5$ A

$$\Rightarrow \quad V_A - V_B = 5 \times 3 + 1 \times 10 - 10 = 15 \text{ V}$$

Hence, the correct answer is (A).

115. $V_C = BLv$

$$\Rightarrow \quad q = CV_C = BvLC = \text{constant}$$

$$\Rightarrow \quad I_C = \frac{dq}{dt} = 0$$

$$\Rightarrow \quad U_C = \frac{1}{2} CV^2 = \frac{1}{2} CB^2 L^2 v^2$$

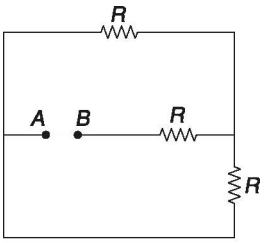
Hence, the correct answer is (C).

116. As the ring and magnetic field remain in same plane, no emf will be induced in all the motions given in different options.

Hence, the correct answer is (D).

117. By short circuiting the battery, the resistance across the open ends of inductor is

CHAPTER 3



$$R_{AB} = R + \frac{R}{2} = \frac{3R}{2}$$

$$\Rightarrow \quad \tau = \frac{L}{R_{AB}} = \frac{2L}{3R}$$

Hence, the correct answer is (D).

118. Magnetic field due to toroid is $B = \mu_0 nI$, where $n = \frac{N}{2\pi r}$ (n is number of turns per unit length)

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi r}$$

Further, flux ϕ is given by NBA

$$\Rightarrow \quad \phi = NBA = \frac{\mu_0 N^2 AI}{2\pi r}$$

Now, by definition of L we have

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

$$\Rightarrow \quad L = \frac{4\pi \times 10^{-7} \times (1200)^2 \times \left(\frac{12}{1000}\right)}{2\pi\left(\frac{15}{100}\right)}$$

$$\Rightarrow \quad L = 2 \cdot 3 \times 10^{-3} \text{ H}$$

$$\Rightarrow \quad L = 2.3 \text{ mH}$$

Hence, the correct answer is (C).

119. Flux across the secondary ϕ_s is

$$\phi_s = N_s BA$$

$$\Rightarrow \quad \phi_s = \frac{\mu_0 N_s NAI}{2\pi r}$$

Since, $\xi = -\frac{d\phi_s}{dt}$

$$\Rightarrow \quad |\xi| = \frac{\mu_0 N_s NA}{2\pi r}\frac{dI}{dt}$$

$$\Rightarrow \quad |\xi| = \frac{(4\pi \times 10^{-7})(300)(1200)\left(\frac{12}{10000}\right)}{2\pi\left(\frac{15}{100}\right)}\left(\frac{2}{0.05}\right)$$

$$\Rightarrow \quad |\xi| = 2.3 \times 10^{-2} \text{ V}$$

$$\Rightarrow \quad |\xi| = 0.023 \text{ V}$$

Hence, the correct answer is (C).

120. The flux associated with the loop is $\phi = B\pi(a^2 + b^2)$
The induced current is

$$I = \frac{\xi}{R} = \frac{1}{R}\left|\frac{d\phi}{dt}\right| = \frac{k\pi(a^2+b^2)}{2\pi(a+b)\lambda} = \frac{k(a^2+b^2)}{2(a+b)\lambda}$$

Hence, the correct answer is (D).

121. Since, $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\Rightarrow \quad i = \frac{E}{R} - \frac{Ee^{-\frac{Rt}{L}}}{R} = i_0 - \left(\frac{V_L}{R}\right) \qquad \left\{\because V_L = Ee^{-\frac{Rt}{L}}\right\}$$

$$\Rightarrow \quad V_L = (i_0 R) - (R)i$$

Hence, V_L versus i graph is a straight line with positive intercept and negative slope.

Hence, the correct answer is (D).

122. Since, $N_P\phi_P = L_P I_P$

$$\Rightarrow \quad 2000 \times 0.8 \times 10^{-3} = L_P \times 1$$

Similarly, $N_S\phi_S = MI_P$

$$\Rightarrow \quad 5000 \times 0.4 \times 10^{-3} = M \times 1$$

$$\Rightarrow \quad \frac{L_P}{M} = \frac{4}{5}$$

Hence, the correct answer is (C).

123. $i = \frac{\xi}{R} = \frac{M}{R}\frac{dI}{dt}$

Since $I = I_0 t$

$$\Rightarrow \quad \frac{dI}{dt} = I_0$$

$$\Rightarrow \quad i = \frac{M}{R}I_0$$

Hence, the correct answer is (B).

124. In small time dt, rod PQ moves by a distance vdt. The increase in area is $lvdt$ and the increase in flux, $d\phi = Blvdt$

The magnitude of induced emf is

$$\xi = \left|\frac{d\phi}{dt}\right| = Blv$$

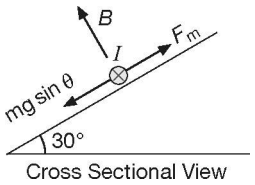
Hence, the correct answer is (A).

125. When current is passed through the straight wire, magnetic lines are circular and tangential to the loop. So, no flux is linked with the loop.

Hence, the correct answer is (B).

126. In the rod, an induced current I is set up in the inward direction $\otimes$, as shown. Due to this I, the rod experiences a magnetic force $F_m = BI\ell$, up the incline. At $v = v_T$, we have

$$mg\sin\theta - F_m = 0$$



$$\Rightarrow \quad mg\sin\theta = BI\ell$$

$$\Rightarrow \quad mg\sin\theta = B\left(\frac{B\ell v_T}{R}\right)\ell \qquad \left\{\because I = \frac{\xi}{R} = \frac{B\ell v_T}{R}\right\}$$

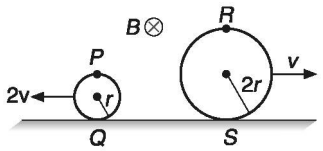
$$\Rightarrow \quad v_T = \frac{mgR\sin\theta}{B^2\ell^2}$$

Since $\theta = 30°$

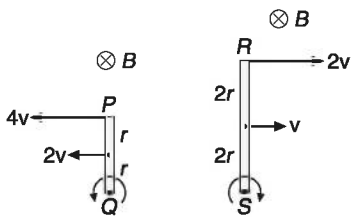
$$\Rightarrow \quad v_T = \frac{mgR}{2B^2\ell^2}$$

Hence, the correct answer is (C).

127. Let the magnetic field be directed into the paper as shown in Figure.



We can solve this problem by taking two conducting rods PQ and RS as shown in Figure.



The emf induced across the rod PQ is

$$\xi_1 = \frac{1}{2}B\omega_1 l_1^2 = \frac{1}{2}B\left(\frac{2v}{r}\right)(2r)^2 = 4Bvr$$

The emf induced across the rod RS is

$$\xi_2 = \frac{1}{2}B\omega_2 l_2^2 = \frac{1}{2}B\left(\frac{v}{2r}\right)(4r)^2 = 4Bvr$$

Using Fleming's Right Hand Rule, we see that points Q and R are at high potential whereas points P and S are at low potential. So

$$V_Q - V_P = \xi_1 = 4Bvr$$

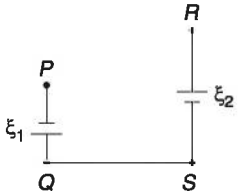
$$V_R - V_S = \xi_2 = 4Bvr$$

Since the horizontal surface is conducting, so we have

$$V_Q = V_S$$

$$\Rightarrow \quad V_R - V_P = 8Bvr$$

A simple circuit diagram for the equivalent batteries in shown in Figure.



Hence, the correct answer is (D).

128. Clearly, $V_P = V_Q$, $V_S = V_R$ and $V_P > V_S$

Hence, the correct answer is (D).

129. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$

At $I = \frac{I_0}{2}$, we have

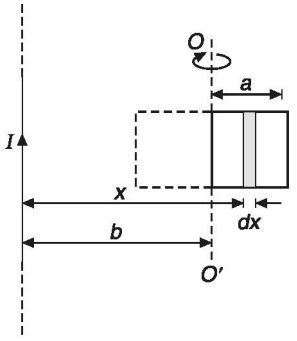
$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln 2 = \frac{300\times10^{-3}}{2}\times 0.693 \approx 0.1\text{ s}$$

Hence, the correct answer is (D).

130. Initial magnetic flux associated with the square loop is calculated by integrating the flux through an elemental strip considered in the square loop as shown in Figure.

$$\phi_i = \frac{\mu_0 ia}{2\pi}\int_b^{b+a}\frac{dx}{x}$$



$$\phi_i = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

Similarly, after 180° rotation, final flux associated with the loop is

$$\phi_f = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b-a}{b}\right)$$

$$\Rightarrow \quad \Delta\phi = |\phi_i - \phi_f| = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b+a}{b-a}\right)$$

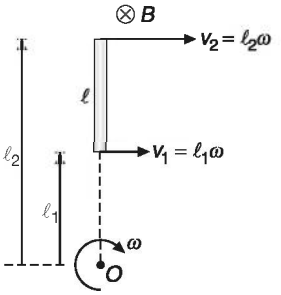
So, charge flown through the loop in process of rotation is

$$\Delta q = \frac{\Delta\phi}{R} = \frac{\mu_0 ia}{2\pi R}\ln\left(\frac{b+a}{b-a}\right)$$

Hence, the correct answer is (D).

131. Using our knowledge of rotational dynamics, we know that

$$\omega = \frac{v_2 - v_1}{l}$$



When this rod rotates with angular velocity ω in the magnetic field B, then

$$\xi = \frac{1}{2}B\omega\left(l_2^2 - l_1^2\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega\left[\left(\frac{v_2}{\omega}\right)^2 - \left(\frac{v_1}{\omega}\right)^2\right]$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega\left(\frac{v_2^2 - v_1^2}{\omega^2}\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\left(\frac{v_2^2 - v_1^2}{\omega}\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\left[\frac{v_2^2 - v_1^2}{\left(\frac{v_2 - v_1}{l}\right)}\right]$$

$$\Rightarrow \quad \xi = \frac{1}{2}Bl\left(v_2 + v_1\right)$$

Hence, the correct answer is (C).

132. The wire ab can be replaced with a battery of emf $\xi = Blv$ and internal resistance R. The current flowing through the battery is I. Therefore, potential difference between a and b is

$$V = \xi - IR = Blv - IR$$

Hence, the correct answer is (C).

133. When the solenoid is cut into two equal parts, then each divided part has a self-inductance $\frac{L}{2}$ and resistance $\frac{R}{2}$. So, time constant τ which is the ratio of self-inductance to the resistance of the circuit remains the same.

$$\Rightarrow \quad \tau_{\text{initial}} = \tau_{\text{final}} = \frac{L_{eq}}{R_{eq}} = \frac{L/4}{R/4}$$

Hence, the correct answer is (D).

134. The steady state current is

$$\Rightarrow \quad I_0 = \frac{E}{R_{\text{equivalent}}} = \frac{E}{R/4}$$

$$\Rightarrow \quad I_0 = \frac{4E}{R}$$

Hence, the correct answer is (B).

135. When X is joined to Y, the time constant is $\tau = \frac{L}{R}$ and the rate of heat produced in steady state is $P = I_0^2 R$

$$\Rightarrow \quad I_0^2 = \frac{P}{R}$$

When X is joined to Z, the total heat produced is

$$H = \frac{1}{2}LI_0^2 = \frac{1}{2}L\frac{P}{R} = \frac{1}{2}\left(\frac{L}{R}\right)P = \frac{1}{2}\tau P$$

Hence, the correct answer is (B).

136. Since we know that

$$q = \frac{1}{R}(\Delta\phi) \qquad \ldots(1)$$

where q is the area under the I-t graph

So, $q=\left(\frac{1}{2}\right)(0.2)(8)$

$\Rightarrow \quad q=0.8\ \text{C}$

Now, from (1), we get

$\Delta\phi = q(R) = (0.8)(10) = 8\ \text{Wb}$

Hence, the correct answer is (D).

137. $\phi_i = BA\cos 0° = 2\ \text{Wb}$ and

$\phi_f = BA\cos 180° = -2\ \text{Wb}$

$\Rightarrow \quad |\Delta\phi| = 4\ \text{Wb}$

Since, $|\Delta q| = \frac{|\Delta\phi|}{R}$

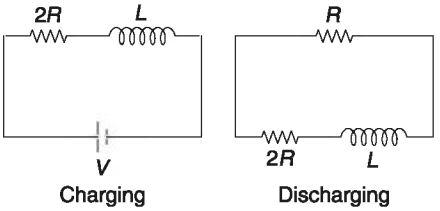
$\Rightarrow \quad |\Delta q| = \frac{4}{10} = 0.4\ \text{C}$

Hence, the correct answer is (A).

138. During charging, $\tau_1 = \frac{L}{2R}$

During discharging, $\tau_2 = \frac{L}{3R}$

$\Rightarrow \quad \tau_1 : \tau_2 = 3:2$



Hence, the correct answer is (B).

139. Even if radius is doubled, flux is not going to change, because B being constant we get ϕ as constant and hence

$\xi = 0$ and $i = 0$

Hence, the correct answer is (C).

140. The emf induced is $\xi = BLv$.

Since, AC is in parallel to $(AB + BC)$, so their combined resistance is

$$R = \frac{\lambda L \times \lambda\sqrt{2}L}{\lambda L + \lambda\sqrt{2}L} = \frac{\sqrt{2}\lambda L}{\sqrt{2}+1}$$

$$\Rightarrow \quad I = \frac{\xi}{R} = \frac{Bv(\sqrt{2}+1)}{\sqrt{2}\lambda}$$

The force required to pull the triangle is

$$F = BIL = \frac{B^2Lv(\sqrt{2}+1)}{\sqrt{2}\lambda}$$

Hence, the correct answer is (A).

141. $i = i_0 e^{-\frac{Rt}{L}}$

$$\Rightarrow \quad \frac{i_0}{\beta} = i_0 e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{L}{R} = \frac{T}{\ln\beta}$$

Hence, the correct answer is (C).

142. $I = I_0\left(1 - e^{-\frac{t}{\tau_L}}\right)$ where $I_0 = \frac{E}{R}$ and $\tau_L = \frac{L}{R}$

$$Q = \int_0^{\tau_L} I\,dt = I_0\int_0^{\tau_L}\left(1 - e^{-\frac{t}{\tau_L}}\right)dt$$

$$\Rightarrow \quad Q = I_0 t\Big|_0^{\tau_L} - \left(\frac{I_0 e^{-\frac{t}{\tau_L}}}{-\frac{1}{\tau_L}}\right)\Bigg|_0^{\tau_L}$$

$$\Rightarrow \quad Q = I_0\tau_L + I_0\tau_L\left[e^{-1} - e^0\right]$$

$$\Rightarrow \quad Q = I_0\tau_L + \frac{I_0\tau_L}{e} - I_0\tau_L$$

$$\Rightarrow \quad Q = \frac{I_0\tau_L}{e} = \frac{1}{e}\left(\frac{E}{R}\right)\left(\frac{L}{R}\right) = \frac{EL}{eR^2}$$

Hence, the correct answer is (C).

143. At $t = 0$, capacitor acts as a pure conductor. The effective resistance of the circuit is R and current $i_1 = \frac{\xi}{R}$

As $t \to \infty$ capacitor acts as a pure insulator and inductor acts as a pure conductor. The effective resistance of the circuit is $\frac{3R \times 6R}{3R + 6R} + R = 3R$

and the current $i_2 = \frac{\xi}{3}R$

$\Rightarrow \quad i_1 : i_2 = 3:1$

Hence, the correct answer is (A).

CHAPTER 3

144. Since $V_{PQ} = L\frac{dI}{dt} + IR$

For the first case, we have

$$I = 4\text{ A},\ \frac{dI}{dt} = 4\text{ As}^{-1} \text{ and } V_{PQ} = 16\text{ V}$$

$$\Rightarrow \quad 16 = L(4) + 4R$$

$$\Rightarrow \quad 4 = L + R \qquad \ldots(1)$$

For the second case, we have

$$I = 2\text{ A},\ \frac{dI}{dt} = -1\text{ As}^{-1} \text{ and } V_{PQ} = 5\text{ V}$$

$$\Rightarrow \quad 5 = -L + 2R \qquad \ldots(2)$$

From (1) and (2), we get

$$3R = 9$$

$$\Rightarrow \quad R = 3\ \Omega \text{ and } L = 1\text{ H}$$

Hence, the correct answer is (A).

145. The relative velocity of approach becomes $2v$ (i.e. doubled), so induced emf is also doubled i.e., becomes 2ξ.

Hence, the correct answer is (C).

146. Since inward magnetic field is increasing, so induced electric lines are circular and anti-clockwise. Hence the negative charge experiences a force opposite to electric field.

Hence, the correct answer is (A).

147. At time $t = 0$, resistance capacitor behaves like short circuit and inductor behaves like open circuit so at this instant circuit resistance across the battery will be

$$R_{\text{net}} = \frac{R}{2} + \frac{R}{3} = \frac{5R}{6} = 5\ \Omega$$

Current through the battery at $t = 0$ is given as

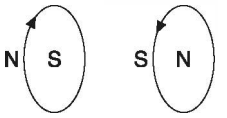
$$i = \frac{E}{R_{\text{net}}} = \frac{5}{5} = 1\text{ A}$$

Hence, the correct answer is (D).

148. $\xi = \frac{B\omega l^2}{2} = \text{constant}$

Hence, the correct answer is (C).

149. When brought closer, the induced effects should produced repulsion. So, currents should increase, because of which that pole strength increases. Hence, repulsion increases.



Hence, the correct answer is (B).

150. $I = \frac{\xi}{R} = -\frac{B}{R}\left(\frac{dA}{dt}\right)$

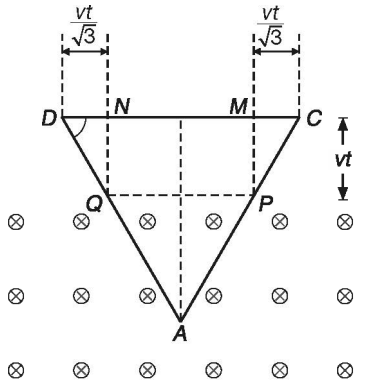
$$DN = CM = \frac{vt}{\sqrt{3}}$$

$$PQ = 2a - (DN + CM) = 2a - \frac{2vt}{\sqrt{3}}$$

At time t, area of the triangle APQ, as a function of t is

$$Ar(\Delta APQ) = Ar(\Delta ACD) - Ar \text{ of Trap } (PQDC)$$

$$\Rightarrow \quad A = \sqrt{3}a^2 - \frac{1}{2}\left(2a + 2a - \frac{2vt}{\sqrt{3}}\right)(vt)$$



$$\Rightarrow \quad A = \sqrt{3}a^2 - 2avt + \frac{v^2t^2}{\sqrt{3}}$$

$$\frac{dA}{dt} = -2av + \frac{2v^2t}{\sqrt{3}}$$

$$\Rightarrow \quad I = \frac{B}{R}\left(2av - \frac{2v^2t}{\sqrt{3}}\right)$$

$$\Rightarrow \quad I = \frac{2vB}{R}\left(a - \frac{vt}{\sqrt{3}}\right)$$

$$\Rightarrow \quad I = \frac{2Bva}{R} - \left(\frac{2Bv^2}{\sqrt{3}R}\right)t$$

Which happens to be a straight line with negative slope, So, the I-t graph is best represented by (C).

Hence, the correct answer is (C).

151. Magnetic field due to infinite wire at a distance x is

$$B = \frac{\mu_0 I}{2\pi x}$$

Consider an infinitesimal element of length dx of conductor EF. EMF induced across this element is

$$d\xi = Bvdx = \frac{\mu_0 I}{2\pi x} vdx$$

$$\Rightarrow \quad \xi = \int_a^b d\xi = \frac{\mu_0 Iv}{2\pi} \ln\left(\frac{b}{a}\right)$$

So, induced current in the conductor EF is

$$i = \frac{\xi}{R} = \frac{\mu_0 Iv}{2\pi R} \ln\left(\frac{b}{a}\right)$$

Force on conductor EF due to field of PQ is

$$dF = Bidx$$

$$\Rightarrow \quad dF = \left(\frac{\mu_0 I}{2\pi x}\right)\left[\frac{\mu_0 Iv}{2\pi R} \ln\left(\frac{b}{a}\right)\right] dx$$

$$\Rightarrow \quad F = \int_a^b dF = \frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi} \log_e\left(\frac{b}{a}\right)\right]^2$$

Hence, the correct answer is (A).

152. The magnetic field produced at the site of wire CD is into the plane of paper. The force on positive charges on the wire CD is in the direction of $\vec{v} \times \vec{B}$ which is towards point C. So, C will be at higher potential.

Hence, the correct answer is (A).

154. The induced emf between points O and Q is $2Brv$. However, as there is no change in magnetic flux through the ring, the net emf induced in the ring is zero and hence, the induced current is zero.

Hence, the correct answer is (A).

155. The back emf equals the applied voltage potential drop across armature coil

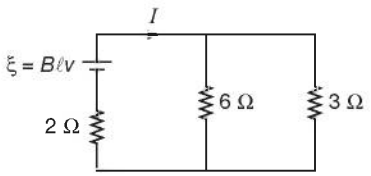
$$\Rightarrow \quad \xi = 200 - iR$$

$$\Rightarrow \quad \xi = 200 - 1.5 \times 20$$

$$\Rightarrow \quad \xi = 170 \text{ V}$$

Hence, the correct answer is (B).

156. The equivalent circuit for this set up is



$$F = BI\ell = B\left(\frac{\xi}{R}\right)\ell$$

$$\Rightarrow \quad F = B\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{R}, \text{ (opposite to } v)$$

where $R = \frac{(6)(3)}{6+3} + 2 = 4\ \Omega$

$$\Rightarrow \quad F = \frac{(2)^2(2)^2(4)}{2} = 32 \text{ N}$$

Hence, the correct answer is (D).

157. $L_{eq} = \frac{(5.5)(5)}{5.5+5} = 2.619\ H$

Hence, the correct answer is (A).

158. In steady state, the entire current passes through the inductor.

Hence, the correct answer is (D).

159. The combined inductance of L and $2L$ in parallel is $2\frac{L}{3}$. The steady state current is independent of inductance and is equal to $\frac{V}{R}$.

Hence, the correct answer is (A).

160. $|V| = L\frac{dI}{dt}$

$$|V| = (1)\frac{d}{dt}(3t\sin t)$$

$$|V| = 3\sin t + 3t\cos t$$

Hence, the correct answer is (B).

161. In horizontal position, the motion of rod in the plane of magnetic field and hence, induced emf is zero. In vertical position,

$$\xi = Blv = 4 \times 10^{-5} \times 3 \times \left(30 \times \frac{5}{18}\right) = 10^{-3} \text{ V}$$

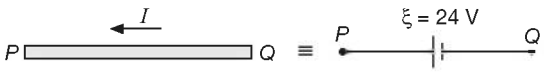
Hence, the correct answer is (B).

162. $\xi = B\ell v_\perp$

$$\Rightarrow \quad \xi = (3)(2)(8\sin 30)$$

$$\Rightarrow \quad \xi = 24 \text{ V}$$

From Fleming's Right Hand Rule, the induced current in the rod PQ is directed from Q to B, thus giving an equivalent emf replacement of the motional emf as



So, P is at a higher potential.

Hence, the correct answer is (B).

CHAPTER 3

Please note that when the moving rod is replaced by its motional emf ξ, then actually the induced current going from Q to P creates a confusion that Q must be at a higher potential. However, do not forget that current goes from higher potential to lower potential in the external circuit but inside a single battery connected in a circuit it always goes from lower potential to higher potential. The most important thing is that the rod is showing the internal part of the battery and not the external circuit. So, it is P that is at a higher potential and not Q.

163. Since $V_A - V_B = L\dfrac{di}{dt}$

$\Rightarrow \quad V_A - V_B = L(-\alpha) = -\alpha L$

Hence, the correct answer is (B).

164. $\vec{A} = (ab)\hat{k}$ (perpendicular to xy-plane)

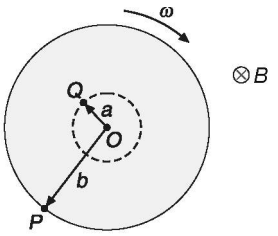
$\phi = \vec{B}\cdot\vec{A} = (50)(ab) = \text{constant}$

$\Rightarrow \quad \dfrac{d\phi}{dt} = 0$

$\Rightarrow \quad \xi = 0$

Hence, the correct answer is (D).

165. If the disc were without the hole then

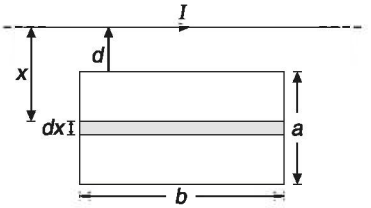


$$V_P - V_O = \frac{1}{2}Bb^2\omega \text{ and } V_Q - V_O = \frac{1}{2}Ba^2\omega$$

$\Rightarrow \quad V_P - V_Q = \dfrac{B\omega}{2}(b^2 - a^2)$

Hence, the correct answer is (B).

166. To calculate the magnetic flux through the rectangular loop, we consider an elemental strip of width dx at a distance x from the wire carrying current I as shown in Figure.



The magnetic flux associated with the strip is

$$d\phi = \frac{\mu_0 I}{2\pi x}bdx$$

Total magnetic flux linked with the loop is

$$\phi = \int d\phi$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Ib}{2\pi}\int_a^{a+b}\frac{dx}{x}$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln x\Big|_a^{a+b} = \frac{\mu_0 Ib}{2\pi}\ln\left(\frac{a+b}{a}\right)$$

The induced emf in the loop is

$$\xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a+b}{a}\right)\frac{dI}{dt}$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a+b}{a}\right)\frac{d}{dt}\left(I_0 e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 bI_0}{2\pi\tau}\ln\left(\frac{a+b}{a}\right)e^{-\frac{t}{\tau}}$$

At $t = \tau$, we have

$$\xi = \frac{\mu_0 bI_0}{2\pi\tau}\ln\left(\frac{a+b}{a}\right)e^{-1}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 bI_0}{2\pi e\tau}\ln\left(\frac{a+b}{a}\right)$$

Hence, the correct answer is (B).

167. Since $\xi = B\ell v$

So, if Q is the charge across the capacitor, then current in the branch containing the capacitor is

$$I = \frac{dQ}{dt}, \text{ where } Q = C\xi = B\ell Cv$$

$$\Rightarrow \quad I = B\ell C\left(\frac{dv}{dt}\right)$$

Since $\dfrac{dv}{dt} = 0$ $\quad \{\because v = \text{constant}\}$

So, $I = 0$

Hence, the correct answer is (D).

168. Current increases with time. So, flux passing through B will increase with time. From Lenz's law, it should have a tendency to move away from the coil to decrease flux.

Hence, the correct answer is (B).

169. Magnetic field through Q (by I_2) is downwards. By decreasing I_1, downward magnetic field through Q will decrease. Hence, induced current in Q should produce magnetic field in same direction.

Hence, the correct answer is (A).

170. $E = L\frac{dI}{dt} + IR$

$\Rightarrow \quad E = \xi + IR$

$\Rightarrow \quad \xi = E - IR$

So, the graph of ξ vs I is a straight line with negative slope.

Hence, the correct answer is (B).

171. For $E \neq 0$, ϕ must change

$\Rightarrow \quad \frac{d\phi}{dt} \neq 0$

Hence, the correct answer is (C).

172. For $r < R$, i.e. inside

$$E_{in}(2\pi r) = \left|\frac{d\phi}{dt}\right| = \pi r^2 \frac{dB}{dt}$$

$\Rightarrow \quad E_{in} = \frac{r\,dB/dt}{2}$

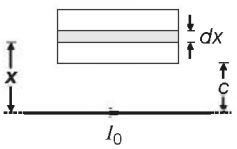
For $r > R$, i.e., outside

$$E_{out}(2\pi r) = \left|\frac{d\phi}{dt}\right| = \pi R^2 \frac{dB}{dt}$$

$\Rightarrow \quad E_{out} = \frac{R^2}{2r}\left(\frac{dB}{dt}\right)$

Hence, the correct answer is (D).

173. Consider an infinitesimal element of length b, thickness dx at a distance x from the wire.



Since $d\phi = BdA$

$\Rightarrow \quad d\phi = \frac{\mu_0 I}{2\pi x}(b\,dx)$

$\Rightarrow \quad \phi = \int d\phi = \frac{\mu_0 Ib}{2\pi}\int_c^{c+a}\frac{dx}{x}$

$\Rightarrow \quad \phi = \frac{\mu_0 bI}{2\pi}\log_e\left(\frac{c+a}{c}\right)$

Given that $I = I_0\left(1 - \frac{t}{\tau}\right)$

$\Rightarrow \quad \phi = \frac{\mu_0 b}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left[I_0\left(1-\frac{t}{\tau}\right)\right]$

Since, $\xi = -\frac{d\phi}{dt}$

$\Rightarrow \quad \xi = -\frac{\mu_0 bI_0}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left(-\frac{1}{\tau}\right)$

$\Rightarrow \quad \xi = \frac{\mu_0 bI_0}{2\pi\tau}\log_e\left(\frac{c+a}{c}\right)$

$\Rightarrow \quad I = \frac{\xi}{R} = \frac{\mu_0 bI_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$

$\Rightarrow \quad \frac{dq}{dt} = \frac{\mu_0 bI_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$

$\Rightarrow \quad dq = \frac{\mu_0 bI_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)dt$

$\Rightarrow \quad q = \frac{\mu_0 bI_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)\int_0^{\tau} dt$

$\Rightarrow \quad q = \frac{\mu_0 bI_0}{2\pi R}\log_e\left(\frac{c+a}{c}\right)$

Hence, the correct answer is (C).

174. At time t side of square is

$$l = l_0 - \alpha t$$

Area of square is

$$A = l^2 = (l_0 - \alpha t)^2$$

At given time, we have

$$\frac{dl}{dt} = -\alpha$$

Since, $\phi = BA = B(l_0 - \alpha t)^2$

$\Rightarrow \quad \xi = \left|\frac{d\phi}{dt}\right| = 2B\alpha(l_0 - \alpha t)$

But, $(l_0 - \alpha t) = a$

$\Rightarrow \quad \xi = 2a\alpha B$

Hence, the correct answer is (A).

175. Consider a point at distance r from centre.

For $r < R$, emf developed is

$$\xi = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt} = \pi r^2 \alpha$$

$\Rightarrow \quad E_{nc}(2\pi r) = \pi r^2 \alpha$

$\Rightarrow \quad E_{nc} \propto r$

For $r > R$, $E_{nc}(2\pi r) = \pi R^2 \alpha$

$\Rightarrow \quad E_{nc} \propto \frac{1}{r}$

Hence, the correct answer is (D).

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176. If I be the current flowing in the larger coil, then field at the centre of coil is $B=\frac{\mu_0 I}{2R}$

Flux linked with smaller coil is

$$\phi=(\text{Area of smaller coil})B$$

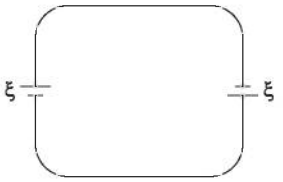
$$\Rightarrow \quad \phi=(\pi r^2)B$$

$$\Rightarrow \quad \phi=\frac{\mu_0\pi r^2}{2R}I$$

$$\Rightarrow \quad M=\frac{\phi}{I}=\frac{\mu_0\pi r^2}{2R}$$

Hence, the correct answer is (B).

177. The left and right arms of the tubes can be replaced by a battery of emf $\xi=Blv$ each. The emf induced in the circuit is $2\xi=2Blv$



Hence, the correct answer is (B).

178. $I=\frac{\xi}{R_{eq}}=\frac{B\ell v}{\left(\frac{R_{\text{Half}}}{2}\right)}$ where

$$R_{\text{Half}}=\pi(2)\left(\frac{2}{\pi}\right)=4\ \Omega$$

$$\Rightarrow \quad I=\frac{(2)(4)(2)}{2}=8\ \text{A}$$

So, if F_m is the magnetic force, then

$$F_m=BI\ell$$

$$\Rightarrow \quad F_m=(2)(8)(4)=64\ \text{N}$$

Hence, the correct answer is (D).

179. Steady state current through inductor in $\frac{E}{R}$.

So, at $t=0$, current in closed loop will remain same.
Hence, the correct answer is (C).

180. At $t=0$, $i=\frac{E}{R}$

Now, this current will decay in closed loop in anti-clockwise direction. So, $|i_2|=i_2=\frac{E}{R}$ in upward or opposite direction.

$$\Rightarrow \quad i_2=-\frac{E}{R}$$

Hence, the correct answer is (B).

181. $\frac{I}{I_0}=10\%=\frac{1}{10}$

Since, $I=I_0\left(1-e^{-t/\tau}\right)$

$$\Rightarrow \quad \frac{1}{10}=1-e^{-t/\tau}$$

$$\Rightarrow \quad \frac{9}{10}=e^{-t/\tau}$$

$$\Rightarrow \quad \frac{t}{\tau}=\log_e\left(\frac{10}{9}\right)$$

$$\Rightarrow \quad t=\tau\log_e(1.1)$$

Hence, the correct answer is (A).

182. 10% less than the steady state value implies a growth of 90%

$$\Rightarrow \quad \frac{I}{I_0}=90\%=\frac{9}{10}$$

Since

$$I=I_0\left(1-e^{-t/\tau}\right)$$

$$\Rightarrow \quad \frac{1}{10}=e^{-t/\tau}$$

$$\Rightarrow \quad t=\tau\log_e(10)$$

Hence, the correct answer is (B).

183. From right hand rule, we can see that points P and Q are at higher potential than the point O.

Hence, the correct answer is (A).

184. $U=\frac{1}{2}LI^2$ and $P=I^2R$

$$\Rightarrow \quad L=\frac{2U}{I^2} \text{ and } R=\frac{P}{I^2}$$

So, $\tau_L=\frac{L}{R}=\frac{2U}{P}$

Hence, the correct answer is (C).

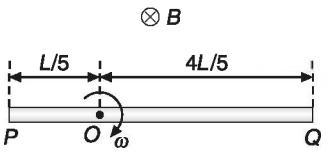
185. When the loops approach, the field between them becomes strong and according to Lenz's law the field must not become strong, so the current decreases in both the loops.

Hence, the correct answer is (A).

186. Magnetic field of ring is also along its axis, or in the direction of velocity of charged particle.
Hence, no magnetic force will act on charged particle. However, due to g velocity of charged particle will increase.
Hence, the correct answer is (C).

187. According to Lenz's Law, induced effects always oppose the cause due to which they are produced. So, when the first loop is moved towards the smaller loop, it will face repulsion.
Hence, the correct answer is (B).

188. From the figure, we have



$$V_Q - V_O = \frac{1}{2}B\omega\left(\frac{4L}{5}\right)^2 = \frac{16}{50}B\omega L^2 \text{ and}$$

$$V_P - V_O = \frac{1}{2}B\omega\left(\frac{L}{5}\right)^2 = \frac{1}{50}B\omega L^2$$

$$\Rightarrow \quad V_Q - V_P = \frac{15}{50}B\omega L^2 = \frac{3}{10}B\omega L^2$$

Hence, the correct answer is (C).

189. Time constant (τ) is the time during which the current grows from zero to 0.632 (or 63.2%) times the maximum value. Since, this is being done in a duration of 1 second, so $\tau = 1\ s$
Hence, the correct answer is (C).

190. Since, the motional emf, ξ, is

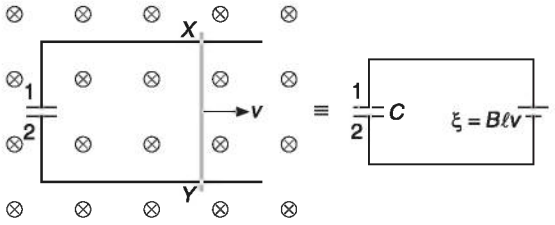
$$\xi = B\ell v$$

$$\Rightarrow \quad \xi = (2)(0.5)(4) = 4\ \text{V}$$

Since $Q = C\xi$

$$\Rightarrow \quad Q = (4)(5) = 20\ \mu\text{C}$$

The equivalent circuit for the arrangement is



So, $q_1 = +20\ \mu\text{C}$ and $q_2 = -20\ \mu\text{C}$
Hence, the correct answer is (D).

191. $|\xi| = \dfrac{d\phi}{dt}$

$$\Rightarrow \quad |\xi| = A\frac{dB}{dt}$$

$$\Rightarrow \quad |\xi| = (d^2)\frac{d}{dt}\left[B_0\left(1+\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad |\xi| = (d^2)B_0\left(0+\frac{1}{a}\frac{dx}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \frac{B_0 d^2}{a}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \frac{B_0 d^2 v_0}{a}$$

Hence, the correct answer is (D).

192. Initial current is

$$I_{\text{initial}} = \frac{10}{10} = 1\ \text{A}$$

$$\Rightarrow \quad \phi_{\text{initial}} = L(I_{\text{initial}}) = 500\ \text{mWb} = 0.5\ \text{Wb}$$

Final current is

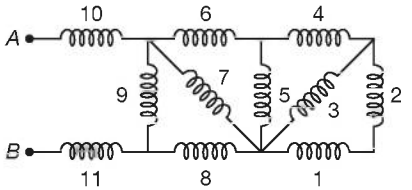
$$I_{\text{final}} = \frac{20}{5} = 4\ \text{A}$$

$$\Rightarrow \quad \phi_{\text{final}} = L(I_{\text{final}}) = (0.5)\times 4 = 2\ \text{Wb}$$

$$\Rightarrow \quad \Delta\phi = 1.5\ \text{Wb}$$

Hence, the correct answer is (B).

193. 1 and 2 in series in parallel with 3. The result of this is in series with 4, which is in parallel with 5, again in series with 6, which again is in parallel with 7, in series with 8 and in parallel with 9. Finally result of above steps, 10 and 11 are in series to get 2.618 H.



Hence, the correct answer is (D).

194. In decay of current through L-R circuit, current cannot remain constant.
Hence, the correct answer is (D).

195. $\dfrac{1}{2} = e^{-t/RC}$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/2}$$

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$$\Rightarrow \quad e^{t/2} = 2$$

$$\Rightarrow \quad \frac{t}{2} = 0.693$$

$$\Rightarrow \quad t = 1.38 \text{ s}$$

Hence, the correct answer is (A).

196. Applying Kirchhoff's law to the circuit starting from E_1, we get

$$E_1 + iR + L\frac{di}{dt} - E_2 = 0$$

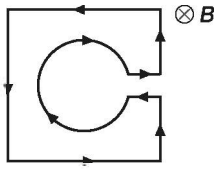
$$\Rightarrow \quad E_2 - E_1 = iR + L\frac{di}{dt}$$

Hence, the correct answer is (B).

197. $|\xi| = \frac{d\phi}{dt} = A\frac{dB}{dt} = (4b^2 - \pi a^2)B_0$

$$i = \frac{|\xi|}{R} = \frac{(4b^2 - \pi a^2)B_0}{R}$$

Inward $\otimes$ magnetic field is increasing. So, an outward $\odot$ magnetic field is produced due to induction.



Hence, the correct answer is (D).

198. Since $U = \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{dU}{dt} = LI\left(\frac{dI}{dt}\right)$$

For the growth of current, we have

$$I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0 R}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{dU}{dt} = L\left(\frac{I_0 R}{L}\right)I_0\left(1 - e^{-\frac{Rt}{L}}\right)e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{dU}{dt} = I_0^2 R\left(e^{-\frac{Rt}{L}} - e^{-\frac{2Rt}{L}}\right)$$

This is best represented by the curve shown in OPTION (C).

Hence, the correct answer is (C).

199. At time t side of square is

$$l = (a + 2v_0 t)$$

Area of square is

$$A = l^2 = (a + 2v_0 t)^2$$

Since $\phi = BA = B(a + 2v_0 t)^2$

$$\Rightarrow \quad \xi = \frac{d\phi}{dt} = 4Bv_0(a + 2v_0 t)$$

The resistance of the square frame at this instant is

$$R = \lambda(4l) = 4\lambda(a + 2v_0 t)$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{Bv_0}{\lambda}$$

Hence, the correct answer is (C).

200. The induced emf is

$$\xi = Bvl = 0.5 \times 4 \times 0.25 = 0.5 \text{ V}$$

Since, $12\ \Omega$ and $4\ \Omega$ are parallel. Hence, their net resistance is $R = 3\ \Omega$.

$$\Rightarrow \quad i = \frac{\xi}{R + r} = \frac{0.5}{3 + 2} = 0.1 \text{ A}$$

Hence, the correct answer is (A).

201. At any instant

$$I = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{E}{R}\frac{R}{L}e^{-t/\tau} \qquad \left\{\because \ \tau = \frac{L}{R}\right\}$$

$$\Rightarrow \quad V_L = L\frac{dI}{dt} = Ee^{-t/\tau}$$

So, potential difference across the coil is $Ee^{-t/\tau}$

Hence, the correct answer is (D).

202. At any instant

$$I = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{E}{R}\frac{R}{L}e^{-t/\tau} \qquad \left\{\because \ \tau = \frac{L}{R}\right\}$$

$$\Rightarrow \quad L\frac{dI}{dt} = Ee^{-t/\tau}$$

Since, potential difference across the inductor coil is

$$V_L = L\frac{dI}{dt}$$

$$\Rightarrow \quad V_L = L\frac{dI}{dt} = Ee^{-t/\tau}$$

Let t be the time when potential difference across coil equals potential difference across the resistor R.

$$\Rightarrow \quad Ee^{-t/\tau} = IR$$

$$\Rightarrow \quad Ee^{-t/\tau} = (I_0R)(1-e^{-t/\tau}) \qquad \{\because I = I_0(1-e^{-t/\tau})\}$$

$$\Rightarrow \quad Ee^{-t/\tau} = E(1-e^{-t/\tau})$$

$$\Rightarrow \quad 2e^{-t/\tau} = 1$$

$$\Rightarrow \quad e^{t/\tau} = 2$$

$$\Rightarrow \quad t = \tau\log_e(2)$$

Hence, the correct answer is (C).

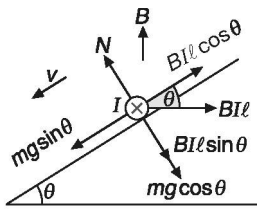
203. In steady state, the impedance of an ideal inductor is zero. In steady state, the entire current will pass throughout the inductor and therefore, the final value of current in 10 Ω resistor is zero.

Hence, the correct answer is (D).

204. For conductor to move with constant speed v, we have

$$mg\sin\theta = IlB\cos\theta$$

where $I = \dfrac{Blv\cos\theta}{R}$



$$\Rightarrow \quad mg\sin\theta = \left(\frac{Blv}{R}\right)lB\cos^2\theta$$

$$\Rightarrow \quad B^2 = \frac{mgR\sin\theta}{vl^2\cos^2\theta}$$

$$\Rightarrow \quad B = \sqrt{\frac{mgR\sin\theta}{vl^2\cos^2\theta}}$$

Hence, the correct answer is (D).

205. The induced emf is

$$\xi = \frac{1}{2}B\omega l^2$$

Since $v = r\omega = \dfrac{l}{2}\omega$

$$\Rightarrow \quad \xi = Blv$$

Hence, the correct answer is (A).

206. At time t, angle rotated by loop is $\theta = \omega t$. This is also the angle between $\vec{B}$ and $\vec{A}$. So, flux associated is

$$\phi = BA\cos\theta$$

$$\Rightarrow \quad \phi = Bb^2\cos\omega t$$

$$\Rightarrow \quad \xi = \left|\frac{d\phi}{dt}\right| = b^2B\omega\sin\omega t$$

Hence, the correct answer is (A).

207. When solenoid is cut in two parts, then

$$L_1 = \left(\frac{\eta}{\eta+1}\right)L, \; R_1 = \left(\frac{\eta}{\eta+1}\right)R$$

$$L_2 = \left(\frac{1}{\eta+1}\right)L, \; R_2 = \left(\frac{1}{\eta+1}\right)R$$

When these parts are connected in parallel, then

$$L_{\text{net}} = \frac{L_1L_2}{L_1+L_2}$$

$$R_{\text{net}} = \frac{R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad \tau_L = \frac{L_{\text{net}}}{R_{\text{net}}} = \frac{L}{R}$$

Hence, the correct answer is (A).

208. In steady state, the inductor has a lower resistance, so the majority of current passes through it. When the switch is suddenly opened, this current now passes through the bulb until it decays to zero.
So, the bulb will glow very brightly for a moment.

Hence, the correct answer is (C).

209. $\tau_L = \dfrac{L}{R} = \dfrac{0.01}{10} = 10^{-3}$ s

$$\tau_C = CR = (0.1\times10^{-3})(10) = 10^{-3} \text{ s}$$

$$\Rightarrow \quad (i_0)_L = \frac{20}{10} = 2 \text{ A}$$

$$\Rightarrow \quad (i_0)_C = \frac{20}{10} = 2 \text{ A}$$

The given time is the half-life time of both the circuits, so

$$i_L = i_C = \frac{2}{2} = 1 \text{ A}$$

Hence, total current is 2 A

Hence, the correct answer is (A).

210. $I_0 = \text{Peak value} = \dfrac{E}{2R}$

Total heat produced across R is $H = \dfrac{1}{2}LI_0^2$

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$$\Rightarrow \quad H = \frac{1}{2}(2L)\frac{E^2}{4R^2}$$

$$\Rightarrow \quad H = \frac{LE^2}{4R^2}$$

Hence, the correct answer is (C).

211. $I = \frac{I_0}{\alpha}$ at $t = t_0$

Since, $I = I_0 e^{-t_0/\tau}$

$$\Rightarrow \quad \frac{1}{\alpha} = e^{-t_0/\tau}$$

$$\Rightarrow \quad \alpha = e^{t_0/\tau}$$

$$\Rightarrow \quad t_0 = \tau \log_e \alpha$$

$$\Rightarrow \quad \tau = \frac{t_0}{\log_e \alpha}$$

Hence, the correct answer is (C).

212. Since the charge moves along the magnetic field of the solenoid, the force acting on it is zero. So, its acceleration is equal to g.

Hence, the correct answer is (C).

213. $V_L = L\frac{dI}{dt}$

$$4t = 2\frac{dI}{dt}$$

$$\Rightarrow \quad dI = 2t\,dt$$

$$\Rightarrow \quad I = \int dI = 2\int_0^4 t\,dt = 2\left(\frac{16}{2}\right) = 16 \text{ A}$$

If U be the energy stored in the coil, then

$$U = \frac{1}{2}LI^2 = \left(\frac{1}{2}\right)(2)(16)^2 = 256 \text{ J}$$

Hence, the correct answer is (B).

214. Since, $\xi = \frac{d\phi}{dt} = \frac{d}{dt}(NBA)$

$$\Rightarrow \quad \xi = \frac{d}{dt}\left[NA(\mu_0 n\text{I})\right]$$

$$\Rightarrow \quad \xi = NA\mu_0 n\left(\frac{dI}{dt}\right)$$

where N is total number of turns in the coil and n is the number of turns per unit length in the solenoid.

$$\xi = (300)(1.2\times10^{-3})(4\pi\times10^{-7})\times\frac{2000}{0.3}\times\frac{4}{0.25}$$

$$\Rightarrow \quad \xi = 4.8\times10^{-2} \text{ V} = 48 \text{ mV}$$

Hence, the correct answer is (D).

215. Let M be the mutual inductance between X and Y. By definition.

$$\xi_y = M\frac{dI_x}{dt}$$

$$\Rightarrow \quad E = M\frac{dI}{dt} = M\dot{I}$$

$$\Rightarrow \quad M = \frac{E}{\dot{I}}$$

The flux linked with X is

$$\phi_X = MI_Y = \left(\frac{E}{\dot{I}}\right)I_0$$

Hence, the correct answer is (B).

216. Let the field be E at a distance x from the centre of the disc. Then

$$eE = mx\omega^2$$

$$\Rightarrow \quad E = \left(\frac{m\omega^2}{e}\right)x$$

Since $\Delta V = \left|\int_0^a E\,dx\right| = \frac{m\omega^2}{e}\int_0^a x\,dx = \frac{m\omega^2 a^2}{2e}$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. When the magnet is above R, it is repelled by the ring and when it is below R, it is attracted by the ring. In both cases, its acceleration is less than g.

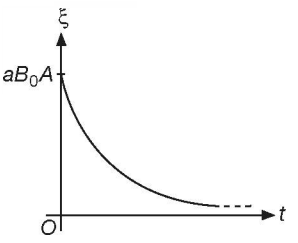
Hence, (A) and (C) are correct.

3. $V = |\xi| = Blv$, where $l = \frac{L}{2}$

The polarity of this motional emf is obtained by using Flemings Right Hand Rule.

Hence, (A) and (C) are correct.

4.



$$\xi = -A\frac{dB}{dt}$$

$$\Rightarrow \quad \xi = AB_0 ae^{-at}$$

Hence, (B) and (D) are correct.

5. $$Q = \frac{1}{R}|\Delta\phi|$$

$$\phi_i = NBA\cos 0 = NBA = 2NB\ell^2$$

$$\phi_f = NBA\cos\theta = 2NB\ell^2\cos\theta$$

$$R = 2\lambda(2\ell + \ell) = 6\lambda\ell$$

$$\Rightarrow \quad Q = \frac{2NB\ell^2}{6\lambda\ell}|1 - \cos\theta|$$

For $\theta = 90°$, $Q = \frac{NB\ell}{3\lambda}$

For $\theta = 180°$, $Q = \frac{2NB\ell}{3\lambda}$

For $\theta = 270°$, $Q = \frac{NB\ell}{3\lambda}$

For $\theta = 360°$, $Q = \frac{NB\ell}{3\lambda}$

Hence, (A) and (B) are correct.

6. In the case of LC oscillations, when the capacitor if fully charged, then we have

$$q = q_0\cos\omega t \quad \text{and} \quad i = -q_0\omega\sin\omega t$$

where, $\omega = \frac{1}{\sqrt{LC}}$

According to the problem, the electric and magnetic fields store equal energy, so

$$\frac{q^2}{2C} = \frac{1}{2}Li^2$$

$$\Rightarrow \quad \frac{q_0\cos^2\omega t}{2C} = \frac{1}{2}Lq_0^2\omega^2\sin^2\omega t$$

$$\Rightarrow \quad \cot^2\omega t = 1$$

$$\Rightarrow \quad \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots.$$

$$\Rightarrow \quad t = \frac{\pi\sqrt{LC}}{4}, \frac{3\pi\sqrt{LC}}{4}, \frac{5\pi\sqrt{LC}}{4}, \frac{7\pi\sqrt{LC}}{4} \ldots.$$

Hence, (A) and (C) are correct.

8. Since $\tau_L = \frac{L}{R} = \frac{2}{2} = 1\text{ s}$

$$\Rightarrow \quad t_{\frac{1}{2}} = (\ln 2)\tau_L = (\ln 2)\text{ s}$$

Hence, the given time is half-life time.

$$\Rightarrow \quad i = \frac{i_0}{2} = \frac{8/2}{2} = 2\text{ A}$$

Rate of energy supplied by battery is

$$P_{\text{supplied}} = Ei = 8 \times 2 = 16\text{ Js}^{-1}$$

Rate of heat dissipated across the resistor is

$$P_R = i^2R = (2)^2(2) = 8\text{ Js}^{-1}$$

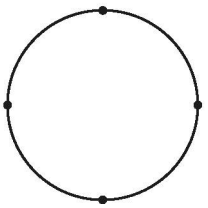
Also, $V_a - V_b = E - iR = 8 - 2 \times 2 = 4\text{ V}$

Hence, (A), (B) and (D) are correct.

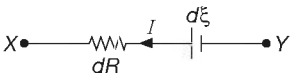
9. $$\xi = \left|\frac{d\phi}{dt}\right| = A\left(\frac{dB}{dt}\right) = \pi r^2 B \qquad \left\{\because \frac{dB}{dt} = B\right\}$$

Consider an infinitesimal element of length $d\ell$ on the ring. If $d\xi$ be emf induced across this element and dR be the resistance of this element, then

$$d\xi = \left(\frac{\xi}{2\pi r}\right)d\ell \text{ and } dR = \left(\frac{R}{2\pi r}\right)d\ell$$



For the arrangement of infinitesimal element and infinitesimal emf source, we have



$$V_X + I(dR) - d\xi - V_Y = 0$$

$$\Rightarrow \quad V_X - V_Y = -Idr + d\xi$$

$$\Rightarrow \quad V_X - V_Y = -\left(\frac{\xi}{R}\right)\left(\frac{R}{2\pi r}\right)d\ell + \left(\frac{\xi}{2\pi r}\right)d\ell = 0$$

So, all the points on the ring are at the same potential.

Hence, (B) and (D) are correct.

10. $$e = -\frac{d\phi}{dt}$$

$$e = -a(n^2 - 1)t^{n^2-2}$$

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$\Rightarrow \quad |e| = a(n^2-1)t^{n^2-2}$

For $n = \pm 1$, $e = 0$

For $n^2 - 1 = 1$ we get $\phi = at$ i.e. $|e| = a$

$\Rightarrow \quad n = \pm\sqrt{2}$ for $|e| = a$

Hence, (A) and (B) are correct.

11. By Law of Conservation of Energy,

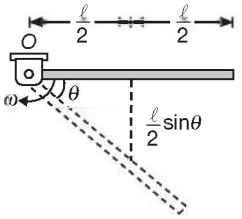
$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy}\end{pmatrix}$$

$$\Rightarrow \quad mg\left(\frac{\ell}{2}\sin\theta\right) = \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mg\left(\frac{\ell}{2}\sin\theta\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g\sin\theta}{\ell}}$$

Further



$$\xi = \frac{1}{2}B\omega\ell^2$$

$$\Rightarrow \quad \xi = \frac{B}{2}\sqrt{\frac{3g\sin\theta}{\ell}}\,\ell^2$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\sqrt{3g}\,(\sin\theta)^{\frac{1}{2}}\ell^{\frac{3}{2}}$$

$$\Rightarrow \quad \xi \propto B$$

$$\Rightarrow \quad \xi \propto \sqrt{\sin\theta}$$

$$\Rightarrow \quad \xi \propto \ell^{3/2}$$

Hence, (A), (C) and (D) are correct.

12. As ba and bc are equal the potential difference of a and c will be same so we have

$$V_a - V_c = 0$$

and $V_a - V_b = V_c - V_b = \dfrac{B\omega l^2}{2}$

Hence, (A) and (C) are correct.

13. When an iron core is inserted inside a coil, its inductance increases. The steady state current, $I_0 = \dfrac{E}{R}$ remains the same.

The time constant, $\tau = \dfrac{L}{R}$ increases.

Hence, (A), (B) and (D) are correct.

14. Due to Lenz's Law

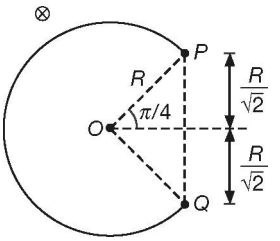
$$s < \frac{1}{2}g(2)^2$$

$$\Rightarrow \quad s < 20\ m$$

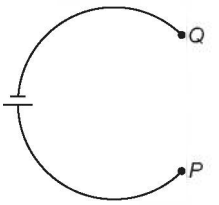
Hence, (A) and (C) are correct.

15. Length $PQ = 2\left(\dfrac{R}{\sqrt{2}}\right) = \sqrt{2}R$

So, $\xi = B(\ell_{PQ})v = \sqrt{2}BRv$



From Fleming's Right Hand Rule, the induced current in the equivalent rod PQ is from Q to P, so that the equivalent circuit diagram is



So, P is positive w.r.t. Q.

Hence, (B) and (C) are correct.

16. The flux initially and finally is

$$\phi_i = BA\cos 0° = (4)(2) = 8\ \text{Wb}$$

$$\phi_f = BA\cos 90° = 0$$

$$\Rightarrow \quad \Delta\phi = 8\ \text{Wb}$$

Since, $|\xi| = \dfrac{\Delta\phi}{\Delta t} = \dfrac{8}{0.1} = 80\ \text{V}$

$$\Rightarrow \quad i = \frac{|\xi|}{R} = 20\ \text{A}$$

Also, $\Delta q = \dfrac{|\Delta\phi|}{R} = 2\ \text{C}$

We cannot find the heat generated without knowing the variation of current with time.

Hence, (B), (C) and (D) are correct.

17. $v < g(t)$

$\Rightarrow \quad v < 2g$ {due to Lenz's Law}

Also, $s < \dfrac{1}{2}gt^2$

$\Rightarrow \quad s < 2g$ {due to Lenz's Law}

Hence, (B) and (D) are correct.

18. At $t = 0$, $I_2 = 0$ and $I_3 = \dfrac{E}{3R}$

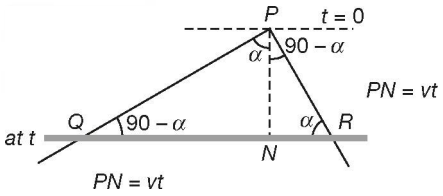
$$\Rightarrow \quad I_1 = I_2 + I_3 = \frac{E}{3R}$$

As $t \to \infty$, $I_3 = 0$, $I_2 = \dfrac{E}{2R}$

$\Rightarrow \quad I_1 = I_2 + I_3 = \dfrac{E}{2R}$

Hence, (B) and (D) are correct.

19.



In ΔPQN

$$\tan(90-\alpha) = \frac{PN}{QN} = \frac{vt}{QN}$$

$\Rightarrow \quad QN = vt\tan\alpha$ ...(1)

In ΔPNR

$$\tan\alpha = \frac{PN}{RN} = \frac{vt}{RN}$$

$\Rightarrow \quad RN = vt\cot\alpha$ ...(2)

Induced emf in the circuit at time t is

$\xi = B(QN + NR)v$

$\Rightarrow \quad \xi = B(vt)(\tan\alpha + \cot\alpha)v$

$\Rightarrow \quad \xi = Bv^2t(\tan\alpha + \cot\alpha)$

$\Rightarrow \quad \xi \propto v^2$ and $\xi \propto t$

Hence, (B) and (C) are correct.

20. $\xi(t) = -\dfrac{d\phi_m}{dt}$

$\Rightarrow \quad \xi(t) = -\dfrac{d}{dt}[Ba]$

$\Rightarrow \quad \xi(t) = -a\dfrac{dB}{dt}$

$\Rightarrow \quad \xi(t) = -a\beta < 0 \qquad \{\because \beta > 0\}$

The emf is constant and negative, so that induced electric field points around the ring F_2 towards F_1. So, face F_1 will develop an excess positive charge.

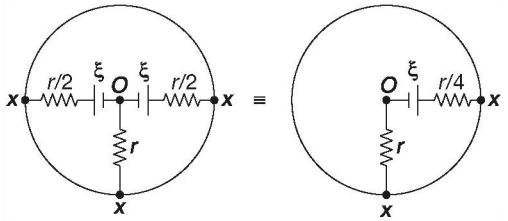
Hence, the correct answer is (A).

21. $E = \dfrac{|\xi|}{\delta} = \dfrac{a\beta}{\delta}$

This expression is independent of R as long as the radius of the ring exceeds the radius $\sqrt{\dfrac{a}{\pi}}$ of the solenoid.

Hence, (A) and (D) are correct.

22. The two halves of the rotating rod produce the motional emf, so the equivalent circuit of the given arrangement is shown in Figure.



Induced emf in each half of rod is

$$\xi = \frac{1}{2}B\omega a^2$$

Let x be the potential of ring and lets take the potential of the point O to be zero, then by making use of Kirchhoff's Loop Law (KLL), we get

$$\left(\frac{x-\xi}{r/4}\right) + \left(\frac{x-0}{r}\right) = 0$$

$\Rightarrow \quad 5x = 4\xi$

$\Rightarrow \quad x = \dfrac{4\xi}{5} = \dfrac{2}{5}B\omega a^2$

$\Rightarrow \quad i = \dfrac{x}{r} = \dfrac{2B\omega a^2}{5r}$

The direction of current in r will be from higher potential to lower potential i.e. from the circumference to the centre O.

Hence, (B) and (D) are correct.

23. The magnitude of emf developed in the ring is

$$\xi = \left|\frac{d\phi}{dt}\right| = \pi a^2\left(\frac{dB}{dt}\right) = \pi a^2\alpha$$

So, OPTION (B) is correct.

If R is the resistance of ring, then

$$I = \frac{\xi}{R} = \frac{\pi a^2\alpha}{R}$$

Consider a small length l of the ring, then

$$\xi' = \frac{\xi}{2\pi a}l = \frac{a\alpha dl}{2}$$

Its resistance is $r = \left(\dfrac{R}{2\pi a}\right)l$

The potential difference across it is

$$V = \xi' - Ir = \frac{a\alpha l}{2} - \left(\frac{\pi a^2\alpha}{R}\right)\left(\frac{R}{2\pi a}\right)l = 0$$

So, all points on the ring are at the same potential.

So, OPTION (A) is correct

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Since, $E(2\pi a)=\xi=\pi a^2\alpha$

$\Rightarrow \quad E=\dfrac{\alpha a}{2}$

So, OPTION (D) is correct.

Hence, (A), (B) and (D) are correct.

24. $\xi=\oint \vec{E}\cdot d\vec{\ell}=-\dfrac{d\phi}{dt}$

Inside $(r<R)$

$\Rightarrow \quad E(2\pi r)=-\dfrac{d}{dt}(B\pi r^2)$

$\Rightarrow \quad E=-\dfrac{1}{2}r\dfrac{dB}{dt}$

Outside $(r>R)$

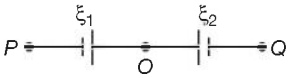
$$E(2\pi r)=-\pi R^2\frac{dB}{dt}$$

$\Rightarrow \quad E=-\dfrac{1}{2}\dfrac{R^2}{r}\dfrac{dB}{dt}$

Hence, (A) and (C) are correct.

25. Using Fleming's Right Hand Rule, we get

$$\xi_1=V_0-V_P=\frac{1}{2}B\omega(3\ell)^2=\frac{9}{2}B\omega\ell^2$$



$$\xi_2=V_0-V_Q=\frac{1}{2}B\omega(5\ell)^2=\frac{25}{2}B\omega\ell^2$$

$\Rightarrow \quad \xi_2-\xi_1=V_0-V_Q-V_0+V_P$

$\Rightarrow \quad \xi_2-\xi_1=V_P-V_Q=8B\omega\ell^2$

Hence, (B), (C) and (D) are correct.

26. $L=\dfrac{N\phi}{i}$

$\Rightarrow \quad \phi=\dfrac{Li}{N}$

So, SI unit of flux is henry-ampere.

$$L=\frac{-\xi}{\Delta i/\Delta t}=-\frac{\xi\Delta t}{\Delta i}$$

Hence, SI unit of L is $\dfrac{\text{V-s}}{\text{ampere}}$

Hence, (A) and (C) are correct.

27. $I=\dfrac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)=5(1-e^{-2t})$

When potential difference across R is 4 V, the current is

$$I=\frac{4}{4}=1\text{ A}$$

$\Rightarrow \quad 1=5(1-e^{-2t})$

$\Rightarrow \quad e^{-2t}=0.8$

At this instant, we have

$$\frac{dI}{dt}=10e^{-2t}=100\times0.8=8\text{ As}^{-1}$$

Power supplied by battery is

$$P=EI=20\times1=20\text{ W}$$

Power consumed by resistor is

$$P_R=I^2R=1^2\times4=4\text{ W}$$

So, power stored in magnetic field is

$$P_m=20\text{ W}-4\text{ W}=16\text{ W}$$

Hence, (A), (B), (C) and (D) are correct.

28. $L_1\dfrac{dI_1}{dt}=L_2\dfrac{dI_2}{dt}$

$\Rightarrow \quad L_1I_1=L_2I_2$

$\Rightarrow \quad \dfrac{I_1}{I_2}=\dfrac{L_2}{L_1}$ ...(1)

Steady state current passing through the resistor is

$$I=I_1+I_2=\frac{E}{R}$$

$\Rightarrow \quad I_1+\left(\dfrac{L_1}{L_2}\right)I_1=\dfrac{E}{R}$

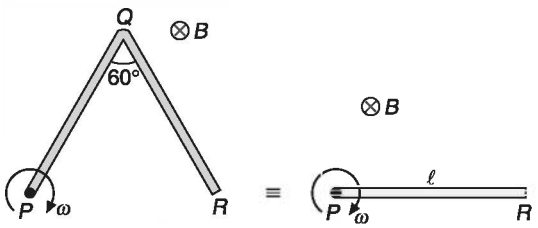
$\Rightarrow \quad I_1=\dfrac{EL_2}{R(L_1+L_2)}$ and $I_2=\dfrac{EL_1}{R(L_1+L_2)}$

Hence, (A) and (D) are correct.

29. The emf induced across the rod PQ is

$$\xi_{PQ}=V_Q-V_P=\frac{1}{2}B\omega l^2 \qquad \text{...(1)}$$

Also, we see that the equivalent conducting rod that fits between the points P and R also has an equivalent length l (because the triangle is equilateral) as shown in Figure.



So, the emf induced across the ends of this rod is

$$\xi_{PR} = V_R - V_P = \frac{1}{2}B\omega l^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$V_Q - V_R = 0$$

$$\Rightarrow \quad V_Q = V_R$$

Hence, (B), (C) and (D) are correct.

30. $\phi = BA = B(\ell + 2vt)^2$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = -4Bv(\ell + 2vt)$$

$$\Rightarrow \quad |\xi| = 4Bv(\ell + 2vt)$$

Further $I = \frac{|\xi|}{R} = \frac{4Bv(\ell + 2vt)}{4\lambda(\ell + 2vt)} = \frac{Bv}{\lambda}$

So, the correct graphs are the ones depicted in (A) and (D).

Hence, (A) and (D) are correct.

31. $\xi = A\frac{dB}{dt} = (\pi a^2)(2)$

$$\Rightarrow \quad \xi = 2\pi a^2$$

Since $I = \frac{dq}{dt} = \frac{\xi}{R} = \frac{2\pi a^2}{R} = \frac{2\pi a^2}{\lambda(2\pi a)} = \frac{a}{\lambda}$

$$Q = It = \left(\frac{a}{\lambda}\right)(3) = \frac{6\pi a^2}{\lambda}$$

$$P = I^2R = \left(\frac{a^2}{\lambda^2}\right)R = \left(\frac{2\pi a^2}{R}\right)^2 R = \frac{4\pi^2 a^4}{R}$$

Hence, (A), (B), (C) and (D) are correct.

32. Just after closing switch, capacitor offers no impedance. So, current through R_3 is zero. Long after closing switch, inductor offers no impedance. So, current through R_3 is zero.

Hence, (A) and (B) are correct.

33. Since the peak value (steady state) current for both 1 and 2 is the same. So, we have

$$\frac{V}{R_1} = \frac{V}{R_2}$$

$$\Rightarrow \quad R_1 = R_2$$

Further 1 grows faster than 2, so we conclude that

$$\tau_{L_2} < \tau_{L_1}$$

$$\Rightarrow \quad L_2 < L_1 \qquad \left\{\because \tau_L = \frac{L}{R}\right\}$$

Hence, (B) and (C) are correct.

34. Given that $q = 2t^2$

$$\Rightarrow \quad i = \frac{dq}{dt} = 4t$$

$$\Rightarrow \quad \frac{di}{dt} = 4 \text{ As}^{-1}$$

At $t = 1$ s, $q = 2$ C, $i = 4$ A and $\frac{di}{dt} = 4$ As^{-1}

$$\Rightarrow \quad V_a - V_b = V_{ab} = L\frac{di}{dt} = 1 \times 4 = 4 \text{ V}$$

$$\Rightarrow \quad V_b - V_c = V_{bc} = \frac{q}{C} = \frac{2}{2} = 1 \text{ V}$$

$$\Rightarrow \quad V_c - V_d = V_{cd} = iR = 4 \times 4 = 16 \text{ V}$$

$$\Rightarrow \quad V_{ad} = V_a - V_d = V_{ab} + V_{bc} + V_{cd} = 21 \text{ V}$$

Hence, (A), (B) and (C) are correct.

35. $R = \sqrt{\frac{L}{C}}$

$$\Rightarrow \quad R^2 = \frac{L}{C}$$

$$\Rightarrow \quad RC = \frac{L}{R}$$

$$\Rightarrow \quad \tau_C = \tau_L = \tau \text{ (say)}$$

Since $I_L = I_0\left(1 - e^{-\frac{t}{\tau_L}}\right) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$ and

$$I_C = \frac{dq}{dt} = \frac{V}{R}e^{-\frac{t}{\tau}}$$

For $I_L = I_C$, we have

$$1 - e^{-\frac{t}{\tau}} = e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad 1 = 2e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad e^{\frac{t}{\tau}} = 2$$

$$\Rightarrow \quad \frac{t}{\tau} = \log_e(2) = \ln(2)$$

$$\Rightarrow \quad t = \tau\ln(2) = RC\ln(2) = \frac{L}{R}\ln(2)$$

Hence, (B) and (C) are correct.

CHAPTER 3

36. Due to current i in the straight conductor, the flux associated with the loop is

$$\phi = \int_{r}^{r+a} \frac{\mu_0 i}{2\pi x} a\,dx = \frac{\mu_0 ia}{2\pi} \ln\left(\frac{r+a}{r}\right)$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi} \ln\left(1+\frac{a}{r}\right)$$

If the loop is pulled with speed v, then

$$\xi = -\frac{d\phi}{dt} = -\left(\frac{d\phi}{dr}\right)\left(\frac{dr}{dt}\right) = \left(\frac{a}{r^2}\frac{\mu_0 ia}{2\pi}\right)\left(\frac{r}{r+a}\right)(v)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 ia^2 v}{2\pi r(r+a)}$$

If i increases, current in loop is induced in anticlockwise direction resulting in repulsion.

Hence, (A), (B) and (D) are correct.

37. The induced emf will first increase and then decrease. It is maximum when the loop is half outside the field. In this case,

$$\xi = B(\sqrt{2}l)v = Blv\sqrt{2}$$

Hence, (A) and (C) are correct.

39. Since there is no change in flux so, no current flows in the ring.

$$V_C = V_E \text{ and } V_D = V_A = B(2R)v$$

Hence, (A), (C) and (D) are correct.

40. By Law of Conservation of Energy, we have

$$P_B = P_R + P_L$$

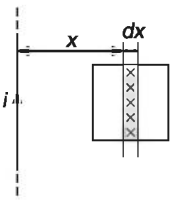
When the circuit is just closed, then we have

$$P_R = i^2R \text{ and } P_L = Li\frac{di}{dt}$$

Since $\frac{di}{dt}$ has greater value at the instant when the circuit is just closed, so we have $P_L > P_R$ and in steady state $P_R > P_L$ because P_L becomes zero as $\frac{dI}{dt} = 0$.

Hence, (A) and (C) are correct.

41. Consider an element of length a, width dx at a distance x from wire as shown in Figure.



Then magnetic field at the element due to wire is

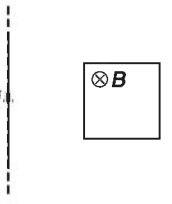
$$B = \frac{\mu_0 i}{2\pi x}$$

Flux associated with this element is

$$d\phi = (B_x)dA = \left(\frac{\mu_0 i}{2\pi x}\right)(a\,dx)$$

$$\Rightarrow \quad \phi = \int_{a}^{2a} d\phi = \frac{\mu_0 ia}{2\pi} \ln 2$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi} \ln 2$$



The wire produces an inward ⊗ magnetic field over the loop. If the loop is brought closer to the wire, ⊗ magnetic field passing through the loop increases. Hence, induced current produces an outward ⊙ magnetic field so, induced current is anti-clockwise.

Hence, (A) and (C) are correct.

42. When inward magnetic field ⊗ increases, then induced electric lines are anti-clockwise. When inward magnetic field ⊗ decreases, then induced electric lines are clockwise (both inside and outside the cylindrical region).

On a positive charge, force is in the direction of $\vec{E}$ and on a negative charge, force is in the direction opposite to $\vec{E}$.

Hence, (B), (C) and (D) are correct.

43. When I increases, field associated with P increases. So Q lies in a field that increases or we can say that apparently Q moves closer to P. Hence P and Q must repel each other. Similarly I decreases, then P attracts Q.

Hence, (A) and (D) are correct.

44. According to Lenz's law, induced effects always oppose the change. Since i_1 and i_2 both are in same direction, so magnetic lines from B due to both currents are from right to left. By bringing A closer to B or increasing i_1 right to left, the magnetic field from B will increase. So, i_2 should decrease.

Hence, (A) and (C) are correct.

46. After time t, the velocity of wire is $v = at$, the emf induced is $\xi = Blv$ and charge stored on capacitor is

$$Q = C\xi = BlCv = (BLC)at$$

The current in the circuit is $I = \frac{dQ}{dt} = BlCa$

The opposing force on wire is

$$F' = BIl = B^2l^2Ca$$

According to Newton's Second Law, we have

$$F - F' = ma$$

$$\Rightarrow \quad ma = F - B^2l^2Ca$$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

Hence, (A) and (C) are correct.

48. L is the analogue of m

Q is analogue of x

$\frac{dQ}{dt} = I$ is analogue of $\frac{dx}{dt} = v$

$\frac{dI}{dt} = I$ is analogue of $\frac{dv}{dt} = \dot{v}$

Hence, (A), (C) and (D) are correct.

49. $\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_0^2$

$$\Rightarrow \quad I_{max} = V_0\sqrt{\frac{C}{L}}$$

Also, we know that for this series LC circuit,

$$q = q_0\cos(\omega t) \text{ and } V = V_0\cos(\omega t) \quad \{\because q = CV\}$$

where $\omega = \frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$

So, potential across the capacitor becomes zero when

$$\omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{\pi}{2}\sqrt{LC}$$

Since at this moment $t = \frac{\pi}{2}\sqrt{LC}$, energy across the capacitor is zero, so energy across the inductor is maximum and has a value $\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_0^2$

Hence, (A), (C) and (D) are correct.

50. Since $\vec{F} = q(\vec{v} \times \vec{B})$, the force on positive charge is towards D and on negative charge is towards A. Accordingly, all options are correct.

Hence, (A), (B), (C) and (D) are correct.

51. The voltage across the inductor is given as

$$V_L = L\frac{di}{dt}$$

$$\Rightarrow \quad V_L = (2)\frac{d}{dt}(10e^{-4t})$$

$$\Rightarrow \quad V_L = -80e^{-4t}$$

Writing Equation of potential drop from point A to B, we get

$$V_A - iR - L\frac{di}{dt} - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = iR + L\frac{di}{dt}$$

$$\Rightarrow \quad V_{AB} = (10e^{-4t})(4) - 80e^{-4t} = -40e^{-4t}$$

Hence, (B) and (D) are correct.

Reasoning Based Questions

1. As the coil rotates, the magnetic flux linked with the coil (being $\vec{B} \cdot \vec{A}$) will change and e.m.f. may be induced in the loop.

Hence, the correct answer is (D).

3. $\Delta W = q(\Delta V)$

Here ΔV = non-zero in a closed loop.

Hence, the correct answer is (A).

4. Lenz's Law is based on conservation of energy and induced e.m.f. always opposes the cause of it i.e., change in magnetic flux.

Hence, the correct answer is (D).

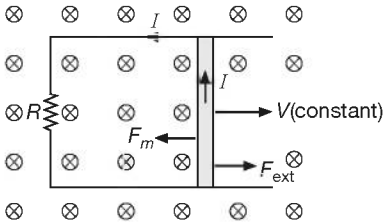
5. L is dependent only upon geometrical parameter.

Hence, the correct answer is (C).

6. $\xi = B\ell V$

$$I = \frac{B\ell V}{R}$$

$$F_B = I\ell B = \frac{B^2\ell^2 V}{R}$$



Hence, the correct answer is (A).

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7. Even though flux through individual lines changes, it remains unchanged for the solenoid as a whole. Therefore no e.m.f. is induced in the long solenoid.
Hence, the correct answer is (D).

8. Electric field generated from time dependent magnetic field obeys Lenz's Law.
Hence, the correct answer is (C).

9. $\phi = BA\cos(\omega t) = 0$

$\Rightarrow \quad \omega t = \dfrac{\pi}{2}$

Since $|\xi| = NBA\omega\sin(\omega t)$

$\Rightarrow \quad |\xi|_{\max} = NBA\omega \qquad \left\{\because \sin(\omega t) = \sin\left(\dfrac{\pi}{2}\right) = 1\right\}$

Hence, the correct answer is (B).

12. Charged particle(s) in motion produce both electric and magnetic field.
So, Statement-2 is false.
Hence, the correct answer is (C).

13. Coefficient of coupling between them

$$M = K^2(L_1L_2)$$

When 2 coils are wound on each other, the coefficient of coupling is maximum and hence mutual inductance between the coils is maximum.
Hence, the correct answer is (C).

15. The current in LR circuit grows exponentially $I = I_0(1 - e^{-Rt/L})$
Hence, the correct answer is (D).

16. Lenz's Law gives the nature or polarity of induced emf or the direction of current.
Hence, the correct answer is (B).

Linked Comprehension Type Questions

1. When mass is moving with constant velocity, then ring is rotating at constant angular velocity.
So, net torque $\tau = 0$
Hence, torque by mg equals the torque by magnetic force

$\Rightarrow \quad \tau_{\text{magnetic force}} = mgr = \left(\dfrac{20}{1000}\right)(10)(0.2) = 0.04$ Nm

Hence, the correct answer is (B).

2. Induced emf in spoke is

$$\xi = \frac{Bvr}{2}$$

$\Rightarrow \quad i = \dfrac{\xi}{R} = \dfrac{Bvr}{2R}$

$\Rightarrow \quad i = 1$ A

Hence, the correct answer is (A).

3. Current through the resistor is

$$i = \frac{B\omega r^2}{2R}$$

$$\omega = \frac{v_T}{r}$$

$$i = \frac{Bv_Tr}{2R}$$

$\Rightarrow \quad i^2R = mgv_T$ (Energy conservation)

$\Rightarrow \quad \dfrac{Bv_T^2r^2}{4R} = mgv_T$

$\Rightarrow \quad v_T = \dfrac{4mgR}{B^2r^2}$

$\Rightarrow \quad v_T = \dfrac{(4)\left(\dfrac{20}{1000}\right)(10)(0.15)}{(0.5)^2(0.2)^2} = 12 \text{ ms}^{-1}$

Hence, the correct answer is (D).

4. The magnetic flux at time t is given by

$$\phi_B = BA = (B_0 + bt)(\pi a^2) = \pi(B_0 + bt)a^2$$

where we have chosen the area vector to point into the page, so that $\phi_B > 0$ g. At $t = 0$, we have

$$\phi_B = \pi B_0 a^2$$

Hence, the correct answer is (B).

5. Using Faraday's Law, the induced emf is

$$\xi = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} = (\pi a^2)\frac{d(B_0 + bt)}{dt} = -\pi ba^2$$

Hence, the correct answer is (A).

6. The induced current is

$$I = \frac{|\xi|}{R} = \frac{\pi ba^2}{R} = \frac{\pi ba^2}{\lambda(2\pi a)} = \frac{ba}{2\lambda}$$

and its direction is counterclockwise by Lenz's Law.
Hence, the correct answer is (D).

7. The power dissipated due to the resistance R is

$$P = I^2R = \left(\frac{\pi ba^2}{R}\right)^2 R = \frac{(\pi ba^2)^2}{R} = \frac{\pi^2 b^2 a^4}{\lambda(2\pi a)} = \frac{\pi b^2 a^3}{2\lambda}$$

Hence, the correct answer is (C).

8. $|\xi| = \frac{d\phi}{dt} = A\frac{dB}{dt}$

$\Rightarrow \quad \xi = (0.2 \times 0.4)(2) = 0.16 \text{ V}$

$\Rightarrow \quad i = \frac{|\xi|}{R} = \frac{0.16}{(1)(40+40+20)\times 10^{-2}}$

$\Rightarrow \quad i = 0.16 \text{ A}$

Outward magnetic field passing through the loop is increasing. So, induced current should produce an inward magnetic field and hence, induced current is clockwise.

Hence, the correct answer is (A).

9. At $t = 2$ s, rod will move 10 cm towards left, so 40 cm side will become 30 cm

$\Rightarrow \quad |\xi| = \xi_1(\text{say}) = A\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad \xi_1 = (0.2 \times 0.3)(2) = 0.12 \text{ V}$

At $t = 2$ s, $B = 4$ T

$\Rightarrow \quad \xi_2 = Bvl$

$\Rightarrow \quad \xi_2 = (4)(5\times 10^{-2})(0.2)$

$\Rightarrow \quad \xi_2 = 0.04 \text{ V}$

So, $\xi_{net} = \xi_1 - \xi_2 = 0.08 \text{ V}$

Hence, the correct answer is (B).

10. $i = \frac{\xi_{net}}{R} = \frac{0.08}{(1)(30+30+20)\times 10^{-2}}$

$\Rightarrow \quad i = 0.1 \text{ A}$

Since $F = Bil$

$\Rightarrow \quad F = (0.1)(0.2)(4) = 0.08 \text{ N}$

Hence, the correct answer is (C).

11. Consider an annulus of radius r, width dr, height b and resistivity ρ. Around its circumference, a voltage is induced according to

$$\xi = -N\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -1\frac{d}{dt}\left[B_0(\cos\omega t)\pi r^2\right]$$

$\Rightarrow \quad \xi = B_0\pi r^2\omega\sin(\omega t)$

The resistance around the loop is

$$\frac{\rho\ell}{A_x} = \frac{\rho(2\pi r)}{bdr}$$

The eddy current in the ring is

$$dI = \frac{\xi}{\text{resistance}} = \frac{B_0\pi r^2\omega\sin(\omega t)bdr}{\rho(2\pi r)}$$

$$dI = \frac{B_0 rb\omega dr\sin(\omega t)}{2\rho}$$

The infinitesimal instantaneous power is

$$dP_i = \xi dI = \frac{B_0^2\pi r^3 b\omega^2 dr\sin^2(\omega t)}{2\rho}$$

$\Rightarrow$ Since, $\langle\sin^2(\omega t)\rangle = \frac{1}{2}$

So, the infinitesimal time-averaged power delivered to the annulus is

$$dP = \frac{B_0^2\pi r^3 b\omega^2 dr}{4\rho}$$

The power delivered to the disk is

$$P = \int dP = \int_0^R \frac{B_0^2\pi b\omega^2}{4\rho} r^3 dr$$

$$\Rightarrow \quad P = \frac{B_0^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \frac{\pi B_0^2 R^4 b\omega^2}{16\rho}$$

Hence, the correct answer is (D).

12. When B_0 gets two times larger, P gets four times larger.

Hence, the correct answer is (B).

13. When f and $\omega = 2\pi f$ is doubled, P gets four times larger.

Hence, the correct answer is (B).

14. When R is doubled, P becomes 2^4 i.e. 16 times larger.

Hence, the correct answer is (D).

15. Since, $\frac{dB}{dt} = (6t^2 + 24) \text{ Ts}^{-1}$

At $t = 2$ s, $\frac{dB}{dt} = 48 \text{ Ts}^{-1}$

Also, $E_{nc}l = \frac{d\phi}{dt} = A\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad E_{nc}(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad F = qE_{nc} = \frac{qr}{2}\left(\frac{dB}{dt}\right)$

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$$\Rightarrow \quad F = \frac{(1.6\times10^{-19})(1.25\times10^{-2})}{2}(48)$$

$$\Rightarrow \quad F = 48\times10^{-21}\ \text{N}$$

Hence, the correct answer is (B).

16. Since $E_{nc} = E = \frac{r}{2}\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad E \propto r$$

So, E-r graph is a straight line passing through origin.

Hence, the correct answer is (C).

17. The inward magnetic field is increasing so, an outward magnetic field is produced by a conducting circular loop placed there. For producing an outward magnetic field, the induced current should be anti-clockwise.
So, direction of induced circular electric lines are also anti-clockwise.

Hence, the correct answer is (B).

18. Since $B_{\text{centre}} = \frac{\mu_0 I}{2a}$

$$\Rightarrow \quad \phi = BA = \left(\frac{\mu_0 I}{2a}\right)(\pi a^2) = \frac{\mu_0 \pi a I}{2}$$

Hence, the correct answer is (A).

19. Since $\phi = \frac{\mu_0 \pi a I}{2}$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad IR = -\frac{d}{dt}\left(\frac{\mu_0 \pi a I}{2}\right)$$

$$\Rightarrow \quad IR = -\frac{\mu_0 \pi a}{2}\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad -\frac{dI}{dt} = \left(\frac{2R}{\mu_0 \pi a}\right)I \qquad \ldots(1)$$

So, the rate of fall of current is $\left(\frac{2R}{\mu_0 \pi a}\right)I$

Hence, the correct answer is (C).

20. From (1), we have

$$\frac{dI}{I} = -\left(\frac{2R}{\mu_0 \pi a}\right)dt$$

$$\Rightarrow \quad \int_{I_0}^{I}\frac{dI}{I} = -\frac{2R}{\mu_0 \pi a}\int_0^t dt$$

$$\Rightarrow \quad \log_e\left(\frac{I}{I_0}\right) = -\left(\frac{2R}{\mu_0 \pi a}\right)t$$

$$\Rightarrow \quad I = I_0 e^{-\left(\frac{2R}{\mu_0 \pi a}\right)t}$$

Hence, the correct answer is (D).

21. Torque due to frictional force F is

$$\tau = FR$$

Since $\tau = NBIA$

$$\Rightarrow \quad NIAB = FR$$

$$\Rightarrow \quad I = \frac{FR}{NAB}$$

$$\Rightarrow \quad I = 23.15\ \text{A}$$

Hence, the correct answer is (B).

22. Back emf is $\xi = NBA\omega\sin(\omega t)$

Since the magnetic field is always parallel to the plane of coil, so

$$\omega t = 90^\circ \text{ i.e., } \sin(\omega t) = 1$$

$$\Rightarrow \quad \xi = NBA\omega$$

Since $\omega = \frac{v}{R}$

$$\Rightarrow \quad \xi = \frac{NBAv}{R}$$

$$\Rightarrow \quad \xi = 90\ \text{V}$$

Hence, the correct answer is (C).

23. Power supplied by battery is $P_{\text{input}} = VI$

$$\Rightarrow \quad P_{\text{in}} = 10\times12\times23$$

$$\Rightarrow \quad P_{\text{in}} = 2760$$

Power generated in the coil is $P_{\text{output}} = \xi I$

$$\Rightarrow \quad P_{\text{out}} = 90\times23.15$$

$$\Rightarrow \quad P_{\text{out}} = 2083$$

Efficiency $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\text{Power generated}}{\text{Power supplied}}$

$$\Rightarrow \quad \eta = \frac{2083}{2760} = 75\%$$

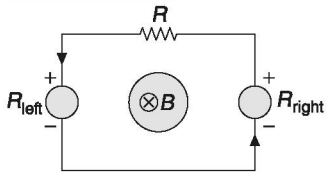
Hence, the correct answer is (A).

26. The correct answer is (B).

Combined Solution to 24, 25 and 26

Given that $V_{\text{left}} = +0.1$ volt

So, current enters through positive terminal and is anticlockwise.



Also, $R_{left} = R_{right}$

$\Rightarrow \quad V_{left} = V_{right} = 0.1$ Volt

$$i = \frac{\Delta V}{R} = \frac{V_{left}}{R_{left}} = \frac{0.1}{10^4} = 10^{-5} \text{ ampere}$$

Induced emf $\xi = i\left(R + R_{left} + R_{right}\right)$

$\Rightarrow \quad \xi = 10^{-5}\left(2.5 \times 10^4\right)$

$\Rightarrow \quad \xi = 0.25$ V

27. $|\xi| = \left|\frac{d\phi}{dt}\right| = A\frac{dB}{dt} = (\pi a^2)B_0$

Hence, the correct answer is (A).

28. Since, $El = \frac{d\phi}{dt}$

$\Rightarrow \quad E(2\pi a) = (\pi a^2)B_0$

$\Rightarrow \quad E = \frac{1}{2}aB_0$

Hence, the correct answer is (C).

29. Since, $F = qE = \frac{1}{2}qaB_0$

$\Rightarrow \quad \tau = Fa = \frac{1}{2}qa^2B_0$

$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\left(\frac{1}{2}qa^2B_0\right)}{ma^2} = \frac{qB_0}{2m}$

Hence, the correct answer is (A).

30. Angular velocity ω is

$$\omega = \alpha t = \left(\frac{qB_0}{2m}\right)t$$

$\Rightarrow \quad P = \tau\omega = \left(\frac{1}{2}qa^2B_0\right)\left(\frac{qB_0}{2m}\right)$

$\Rightarrow \quad P = \frac{q^2B_0^2a^2}{4m}$

Hence, the correct answer is (D).

31. At $v = v_T$, i.e., equilibrium condition, we have

$$mg - BI\ell = 0$$

$\Rightarrow \quad mg = BI\ell$ where $I = \frac{\xi}{R} = \frac{B\ell v_T}{R}$

$\Rightarrow \quad mg = \frac{B^2\ell^2 v_T}{R}$

$\Rightarrow \quad v_T = \frac{mgR}{B^2\ell^2}$

Hence, the correct answer is (C).

32. Energy dissipated as heat $= mgh - \frac{1}{2}mv^2$

$\Rightarrow \quad H = (1)(10)(1) - \left(\frac{1}{2}\right)(1)(4)^2 = 2$ J

Hence, the correct answer is (A).

33. Once the terminal speed is attained, then the energy dissipated in the resistor per unit time is

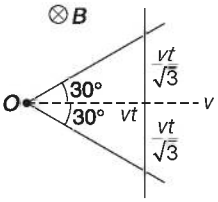
$$H = I^2R = \left(\frac{B^2\ell^2v_T^2}{R^2}\right)R = \frac{B^2\ell^2v_T^2}{R}$$

However, $\frac{B^2\ell^2v_T}{R} = mg$

$\Rightarrow \quad H = mgv_T = \frac{B^2\ell^2v_T^2}{R}$

Hence, (B) and (C) are correct.

34. $A = \frac{1}{2}\left(\frac{2vt}{\sqrt{3}}\right)(vt)$



$\Rightarrow \quad A = \frac{v^2t^2}{\sqrt{3}}$

Since, $\phi = BA = \frac{Bv^2t^2}{\sqrt{3}}$

$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = \frac{2Bv^2t}{\sqrt{3}}$

So, induced emf increases with time.

Hence, the correct answer is (A).

35. Since, $i = \frac{\xi}{R}$

where, $R = \frac{2vt}{\sqrt{3}} + \frac{2vt}{\sqrt{3}} + \frac{2vt}{\sqrt{3}}$

Since resistance per unit length is 1 Ω

$\Rightarrow \quad R = \frac{6vt}{\sqrt{3}}$

$\Rightarrow \quad i = \frac{2Bv}{6} = \frac{Bv}{3} =$ constant

Hence, the correct answer is (C).

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36. Work done

$$W=\int_0^{10} i^2R\,dt=\int_0^{10}\left(\frac{B^2v^2}{9}\right)\left(\frac{6vt}{\sqrt{3}}\right)dt$$

$$\Rightarrow\quad W=\left(\frac{6B^2v^3}{9\sqrt{3}}\right)\left(\frac{10^2}{2}\right)=\frac{6\times0.16\times0.001\times100}{9\sqrt{3}\times2}$$

$$\Rightarrow\quad W=3\times10^{-3}\text{ J}$$

Hence, the correct answer is (A).

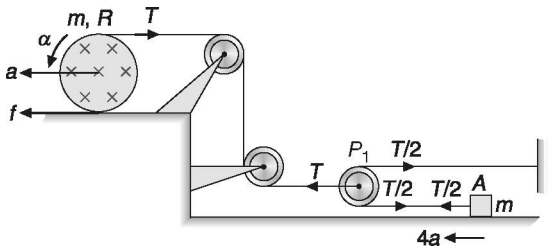
39. **The correct answer is (A).**

Combined Solution to 37, 38 and 39

$$f-T=ma \qquad \ldots(1)$$

$$\frac{T}{2}=m\times4a$$

$$T=8ma \qquad \ldots(2)$$



Torque at disc due to time varying magnetic field is

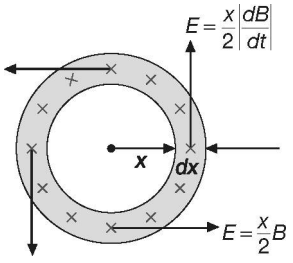
$$\tau=\int_0^R(2\pi x\,dx)\frac{Q}{\pi R^2}\frac{x}{2}B_0x$$

$$\Rightarrow\quad \tau=\frac{QB_0}{R^2}\frac{R^4}{4}=\frac{QB_0R^2}{4}$$

Torque about centre of disc is

$$\frac{QB_0R^2}{4}-TR-fR=\frac{MR^2}{2}\frac{a}{R}$$

$$\Rightarrow\quad \frac{QB_0R}{4}-T-f=\frac{ma}{2} \qquad \ldots(3)$$



Solving these three equations, we get

$$a=\frac{QB_0R}{70m}\text{ ms}^{-2}$$

$$T=\frac{4QB_0R}{35}\text{ N}$$

$$\alpha=\frac{a}{R}=\frac{QB_0}{70m}$$

Acceleration of block is

$$a_{\text{block}}=4a=\frac{2QB_0R}{35m}$$

41. Since, $\phi=BA\cos0°$

As area is changing, so we have

$$\xi=-\frac{d\phi}{dt}$$

$$\Rightarrow\quad \xi=-NB\left(\frac{dA}{dt}\right)$$

$$\Rightarrow\quad \xi=\frac{(NB)(\pi r^2)}{\Delta t}$$

$$\Rightarrow\quad i=\frac{(NB)(\pi r^2)}{\Delta tR}=\frac{100\times4\times\pi\times(0.1)^2}{3\times2}=2.09\text{ A}$$

Hence, the correct answer is (A).

42. Power supplied which is converted into heat is

$$P_{\text{supplied}}=I^2R$$

So, energy supplied is $\Delta U=I^2R\times\Delta t=(2.09)^2\times2\times3$

$$\Rightarrow\quad \Delta U=26\text{ joule}$$

Loop was initially $2r$ wide and it become πr long after it is flattened.

Since, $\Delta U=F\Delta r$

$$\Rightarrow\quad 26=F(\pi r-2r)$$

$$\Rightarrow\quad F=231\text{ N}$$

Hence, the correct answer is (B).

45. **The correct answer is (B).**

Combined Solution to 43, 44 and 45

$$y=2A\sin(kx)\cos(\omega t)$$

$$\Rightarrow\quad v=\frac{dy}{dt}=-2A\omega\sin(kx)\sin(\omega t)$$

$$\Rightarrow\quad v_{\max}=-2A\omega\sin(kx)$$

Induced emf is

$$\xi = \int_0^l B v_{max} dx$$

For third harmonic, $l = \frac{3\lambda}{2}$, so we get

$$\xi = \int_0^l B(-2A\omega \sin(kx))dx$$

$$\Rightarrow \quad \xi = -2AB\omega\left(-\frac{\cos kx}{k}\right) = \frac{2AB\omega}{k}\left(\cos(kx)\Big|_0^l\right)$$

$$\Rightarrow \quad \xi = \frac{2AB\omega}{k}(\cos 3\pi - \cos 0) = -\frac{4BA\omega}{k}$$

$$\Rightarrow \quad \xi \text{ is maximum}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2\omega}$$

For second harmonic $k = \frac{2\pi}{AB}$

47. $\phi = BA = B(\ell^2 + b^2)$

$$\Rightarrow \quad |\xi| = \left|\frac{d\phi}{dt}\right| = (\ell^2 + b^2)\left(\frac{dB}{dt}\right) = \left[\left(\frac{20}{100}\right)^2 + \left(\frac{10}{100}\right)^2\right]10$$

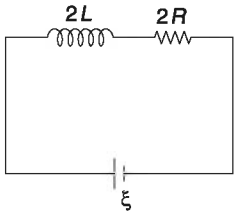
$$\Rightarrow \quad \xi = 0.5 \text{ V}$$

Hence, the correct answer is (D).

48. $I = \frac{\xi}{2R}\left(1 - e^{-\left(\frac{2R}{2L}\right)t}\right)$

$$\Rightarrow \quad I = \frac{0.5}{20}(1 - e^{-t})$$

$$\Rightarrow \quad I = \frac{1}{40}(1 - e^{-t})$$



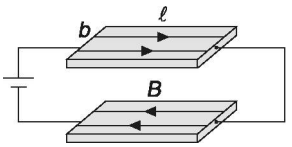
Hence, the correct answer is (B).

49. Magnetic field in the region between the plates is

$$B = \frac{\mu_0\lambda}{2} + \frac{\mu_0\lambda}{2}$$

$$\Rightarrow \quad B = \mu_0\lambda$$

where $\lambda = \frac{i}{b}$



Flux $\phi = BA$

$$\Rightarrow \quad \phi = (\mu_0\lambda)(al)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 ila}{b}$$

Since $\phi = Li$

$$\Rightarrow \quad L = \frac{\phi}{i} = \frac{\mu_0 la}{b}$$

Hence, the correct answer is (A).

50. Voltage at a distance x from shorted end is

$$V_x = L_x\frac{di}{dt} \qquad \ldots(1)$$

Using Kirchhoff's Law, we get

$$V_0 - L\frac{di}{dt} = 0$$

$$\Rightarrow \quad di = \frac{V_0}{L}dt$$

$$\Rightarrow \quad i = \int di = \frac{V_0}{L}\int_0^t dt = \frac{V_0 t}{L}$$

Since, $L_x = \frac{\mu_0 xa}{b}$

So, from (1), we get

$$V_x = \left(\frac{\mu_0 xa}{b}\right)\frac{di}{dt} = \left(\frac{\mu_0 xa}{b}\right)\left(\frac{V_0}{L}\right)$$

Hence, the correct answer is (A).

51. Energy flow rate i.e. $P = V_x \times i$

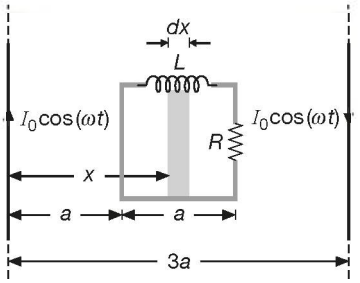
$$\Rightarrow \quad P = \left(\frac{\mu_0 xaV_0}{bL}\right)\frac{V_0 t}{L}$$

$$\Rightarrow \quad P = \frac{\mu_0 xaV_0^2 t}{bL^2}$$

Hence, the correct answer is (B).

52. $d\phi = BdA$, where $B = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}$

$$\Rightarrow \quad d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}\right]adx$$



$$\Rightarrow \quad \phi = \frac{\mu_0 I}{2\pi}\left[\int_a^{2a}\frac{dx}{x}+\int_a^{2a}\frac{dx}{(3a-x)}\right]a$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Ia}{\pi}\log_e(2)$$

Hence, the correct answer is (A).

53. Magnitude of emf in this circuit

$$\xi = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a\log_e(2)}{\pi}\left|\frac{dI}{dt}\right|$$

$$\Rightarrow \quad \xi = \frac{\mu_0 a\log_e(2)}{\pi} I_0\omega\sin(\omega t)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I_0 a\omega\log_e(2)}{\pi}\sin(\omega t)$$

Hence, the correct answer is (A).

54. Since in series LR circuit, current lags behind the voltage by $\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$, so we have

$$I = \frac{\xi_0}{Z}\sin(\omega t-\phi)$$

$$\Rightarrow \quad I = \frac{\mu_0 I_0\omega a\log_e(2)}{\pi\sqrt{R^2+L^2\omega^2}}\sin(\omega t-\phi)$$

Hence, the correct answer is (D).

56. The correct answer is (B).

Combined Solution to 55 and 56

$$Q = C\xi = B\ell Cv$$

$$I = \frac{dQ}{dt} = B\ell C\left(\frac{dv}{dt}\right) \qquad \ldots(1)$$

$$F - BI\ell = ma$$

$$\Rightarrow \quad F-(B^2\ell^2C)a = ma$$

$$\Rightarrow \quad F = (m+B^2\ell^2C)a$$

$$\Rightarrow \quad a = \frac{F}{m+B^2\ell^2C} \qquad \ldots(2)$$

From (2), we get

$$a = \frac{dv}{dt} = \frac{F}{m+B^2\ell^2C}$$

Substituting in (1), we get

$$I = \frac{BC\ell F}{m+B^2\ell^2C}$$

59. The correct answer is (D).

Combined Solution to 57, 58 and 59

At terminal velocity, we have

$$BiL = mg$$

$$\Rightarrow \quad i = \frac{mb}{LB} = \frac{0.2\times 98}{1\times 0.6}$$

$$\Rightarrow \quad i = 3.27 \text{ A} \qquad \ldots(1)$$

If v is the terminal velocity, then

$$\xi = BLv$$

$$\Rightarrow \quad \xi = (0.6)(v)(1)$$

$$\Rightarrow \quad \xi = 0.6v$$

Also, $P_{R_1} = \frac{\xi^2}{R_1}$

$$\Rightarrow \quad 0.76 = \frac{0.36v^2}{R_1} \qquad \ldots(2)$$

Also, $P_{R_2} = \frac{\xi^2}{R_2}$

$$\Rightarrow \quad 1.2 = \frac{0.36v^2}{R_2} \qquad \ldots(3)$$

Since R_1 and R_2 are in parallel, so we have

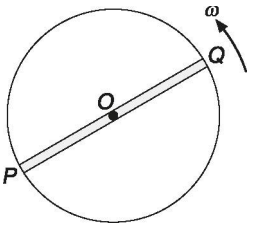
$$R_{net} = \frac{R_1R_2}{R_1+R_2} \qquad \ldots(4)$$

$$\Rightarrow \quad i = \frac{\xi}{R_{net}} = 3.27 \text{ A} \qquad \ldots(5)$$

Solving these five equations, we can get the desired results.

60. Induced emf across OP is $\xi = \frac{1}{2}B\omega\left(\frac{l}{2}\right)^2$

$$\Rightarrow \quad \xi = \frac{B\omega l^2}{8}$$



The current is

$$i = \frac{B\omega l^2}{8R} \qquad \ldots(1)$$

Torque on the rod is

$$\tau = 2\int_0^{\frac{l}{2}} Bixdx = \frac{Bil^2}{4} \qquad \ldots(2)$$

Since $\tau = I\alpha$, where $\alpha = \frac{d\omega}{dt}$

$$\Rightarrow \left(\frac{Ml^2}{12}\right)\frac{d\omega}{dt} = -\frac{B^2\omega l^4}{32R}$$

$$\Rightarrow -\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = \frac{3B^2l^2}{8RM}\int_0^t dt$$

$$\Rightarrow -\log\left(\frac{\omega}{\omega_0}\right) = \left(\frac{3B^2l^2}{8RM}\right)t$$

$$\Rightarrow \omega = \omega_0 e^{-\left(\frac{3B^2l^2}{8RM}\right)t} = \omega_0 e^{-\alpha t}$$

where $\alpha = \frac{3B^2l^2}{8RM}$

So, from equation (1), we get

$$i = \left(\frac{B\omega_0 l^2}{8R}\right)e^{-\alpha t}$$

Hence, the correct answer is (C).

61. $$Q = \int_0^{\infty} idt = \frac{B\omega_0 l^2}{8R}\int_0^{\infty} e^{-\alpha t}dt$$

where $\alpha = \frac{3B^2l^2}{8RM}$

$$\Rightarrow Q = \frac{B\omega_0 l^2}{8R}\left(\frac{e^{-\alpha t}}{-\alpha}\Big|_0^{\infty}\right)$$

$$\Rightarrow Q = -\frac{B\omega_0 l^2}{8\alpha R}\left(e^{-\infty} - e^0\right)$$

$$\Rightarrow Q = \frac{B\omega_0 l^2}{8R\left(\frac{3B^2l^2}{8RM}\right)}$$

$$\Rightarrow Q = \frac{\omega_0 M}{3B}$$

Hence, the correct answer is (B).

62. Heat generated equals the loss in kinetic energy of rod.

So, $\Delta H = |\Delta K| = \frac{1}{2}I\omega_0^2 = \frac{1}{2}\left(\frac{Ml^2}{12}\right)\omega_0^2$

$$\Rightarrow \Delta H = \frac{Ml^2\omega_0^2}{24}$$

Hence, the correct answer is (A).

63. Using Fleming's Right Hand Rule, we see that the induced current in the wire PQ is from Q to P. So, we have

$$V_P - V_Q = \xi = Blv = (4)(1)(2) = 8 \text{ V}$$

Hence, the correct answer is (D).

64. Inductor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

65. Capacitor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

66. Resistor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

67. Since $V_L = 8$ V

$$\Rightarrow L\frac{di_1}{dt} = 8$$

$$\Rightarrow 4\frac{di_1}{dt} = 8$$

$$\Rightarrow \frac{di_1}{dt} = 2 \text{ As}^{-1}$$

Hence, the correct answer is (C).

68. Charge on the capacitor is

$$Q = C\xi = (1)(8) = 8 \text{ coulomb}$$

$$\Rightarrow i_2 = \frac{dQ}{dt} = \frac{d}{dt}(8) = 0$$

Hence, the correct answer is (A).

69. Current through resistor is

$$i_3 = \frac{\xi}{R} = \frac{8}{2} = 4 \text{ A}$$

Hence, the correct answer is (C).

70. Since rate of change of current in inductor is

$$\frac{di_1}{dt} = 2 \text{ As}^{-1}$$

So, current in the inductor at $t = 2$ s is

$$i_1 = \left(\frac{di_1}{dt}\right)\Delta t = (2)(2) = 4 \text{ A}$$

Hence, the correct answer is (C).

CHAPTER 3

71. Since wire PQ is acting as source of emf and supplies current i_1 to inductor, i_2 to capacitor and i_3 to resistor, so current through wire PQ is

$$i = i_1 + i_2 + i_3 = 4 + 0 + 4 = 8 \text{ A}$$

Hence, the correct answer is (D).

72. To keep the wire PQ moving with constant velocity, an external force that equals the magnetic force on the current carrying conductor should be applied on it.

$$\Rightarrow \quad F_{\text{external}} = F_{\text{magnetic}} = Bil = (4)(8)(1) = 32 \text{ N}$$

Hence, the correct answer is (D).

73. Energy supplied per second by the source is

$$P_{\text{supplied}} = \xi i = (8)(8) = 64 \text{ W}$$

Hence, the correct answer is (C).

74. Power generated by the applied external force is

$$P_{\text{external force}} = F_{\text{ext}} v = (32)(2) = 64 \text{ W}$$

Hence, the correct answer is (C).

75. Magnetic energy stored per second in inductor is

$$\frac{dU_m}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li_1^2\right) = \frac{L}{2}\left(2i_1\frac{di_1}{dt}\right) = Li_1\frac{di_1}{dt}$$

Since $\frac{di_1}{dt} = 2 \text{ As}^{-1}$ and $i_1 = 4$ A at $t = 2$ s

$$\Rightarrow \quad \frac{dU_m}{dt} = Li_1\frac{di_1}{dt} = (4)(4)(2) = 32 \text{ W}$$

Hence, the correct answer is (B).

76. Energy dissipated per second in the resistor at $t = 2$ s is

$$P_{\text{resistor}} = i_3^2 R = (4)^2(2) = 32 \text{ W}$$

Hence, the correct answer is (B).

77. Electrostatic stored per second in capacitor at $t = 2$ s is

$$\frac{dU_e}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C}\right) = \frac{1}{2C}\frac{d}{dt}\left(Q^2\right)$$

$$\Rightarrow \quad \frac{dU_e}{dt} = \frac{1}{2C}\left(2Q\frac{dQ}{dt}\right) = \frac{Q}{C}\frac{dQ}{dt} = \frac{Q}{C}i_2$$

Since $i_2 = 0$

$$\Rightarrow \quad \frac{dU_e}{dt} = 0$$

Hence, the correct answer is (A).

80. **The correct answer is (B).**

Combined Solution to 78, 79 and 80

The fan is operating at 200 V, consuming 1000 W

then $I = \frac{1000}{200} = 5$ A

Since the coil resistance is 1 Ω, so power dissipated by internal resistance as heat is

$$P_1 = I^2 R = 25 \text{ W}$$

If V is the net emf across coil, then

$$\frac{V^2}{R} = 25$$

$$\Rightarrow \quad V = 5 \text{ volt}$$

Net Emf = Source Emf – Back Emf

$$\Rightarrow \quad V = V_S - \xi$$

$$\Rightarrow \quad \xi = 195 \text{ V}$$

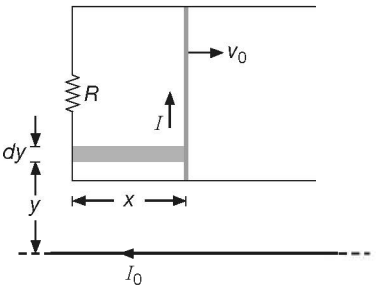
The work done is $P_2 = 1000 - 25 = 975$ W

81. $$\phi = \int_a^b x\frac{\mu_0 I_0}{2\pi y}dy = \frac{\mu_0 I_0 x}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\xi = \frac{d\phi}{dt} = \frac{\mu_0 I_0 v_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad I = \frac{\mu_0 I_0 v_0}{2\pi R}\ln\left(\frac{b}{a}\right)$$

Current induced in the circuit will be anticlockwise to reduce the change in flux.

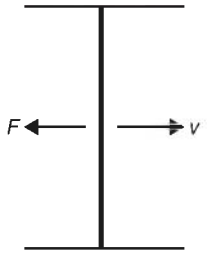


Hence, the correct answer is (C).

82. At any instant when the velocity of the sliding wire is v, current through the wire is

$$I = \frac{\mu_0 I_0 v}{2\pi R}\ln\left(\frac{b}{a}\right)$$

Force on the wire $F = I(dy)B$



$$\Rightarrow \quad F=\int_a^b \frac{\mu_0 I_0 v}{2\pi R}\ln\left(\frac{b}{a}\right)\left(\frac{\mu_0 I_0}{2\pi y}\right)dy$$

$$\Rightarrow \quad F=\frac{\mu_0^2 I_0^2 v}{4\pi^2 R}\left(\ln\frac{b}{a}\right)\ln\left(\frac{b}{a}\right)\frac{\mu_0^2 I_0^2}{4\pi^2 R}\left[\ln\left(\frac{b}{a}\right)\right]^2$$

$$\Rightarrow \quad dv=-\frac{\mu_0^2 I_0^2 v}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2\int_0^t dt$$

$$\Rightarrow \quad \int_{v_0}^{\frac{v_0}{2}}\frac{dv}{v}=-\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log\left(\frac{b}{a}\right)\right)^2\int_0^t dt$$

$$\Rightarrow \quad t=\frac{4\pi^2 mR\ln(2)}{\mu_0^2 I_0^2\left(\ln\left(\frac{b}{a}\right)\right)^2}$$

Hence, the correct answer is (A).

83. $$-\frac{dv}{dt}=-v\frac{dv}{dx}=\frac{\mu_0^2 I_0^2 v}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2$$

$$\Rightarrow \quad \frac{dv}{dx}=-\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2$$

$$\Rightarrow \quad \int_{v_0}^{0} dv=-\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log_e\left(\frac{b}{a}\right)\right)^2\int_0^x dx$$

$$\Rightarrow \quad v_0=\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log_e\left(\frac{b}{a}\right)\right)^2 x$$

$$\Rightarrow \quad x=\left[\frac{4\pi^2 mR}{\left(\mu_0 I_0\ln\left(\frac{b}{a}\right)\right)^2}\right]v_0$$

$$\Rightarrow \quad x\propto v_0$$

Hence, the correct answer is (A).

84. For CASE-1, the total area inside the coils is L^2+l^2 and for CASE-2, it is L^2-l^2.

The corresponding flux are

$(L^2+l^2)B$ and $(L^2-l^2)B$.

Hence, the correct answer is (D).

85. Since the field is decreasing, so the field produced due to induced current is inwards. Hence the current is from b to a and from d to c.

Hence, the correct answer is (C).

86. Since the field is decreasing, so field produced due to induced current is inwards. Hence, the current is from b to a and f to e.

Hence, the correct answer is (B).

87. The induced Current is

$$I=\left|\frac{1}{R}\frac{d\phi}{dt}\right|=\left|\frac{A}{R}\frac{dB}{dt}\right|$$

Since R is same for both and area A inside the coil in CASE-1 is more, so $I_1>I_2$.

Hence, the correct answer is (B).

88. Total initial energy

$$U_{\text{initial}}=\frac{Q_0^2}{2C}=\frac{10^{-2}\times 10^{-2}}{2\times 50\times 10^{-6}}\text{ J}=1\text{ J}$$

This energy shall remain conserved in the absence of resistance.

Hence, the correct answer is (C).

89. Natural frequency, $f=\frac{1}{2\pi\sqrt{LC}}$

$$f=\frac{1}{2\times 3.14\left(20\times 10^{-3}\times 50\times 10^{-6}\right)^{1/2}}\text{ Hz}$$

$$\Rightarrow \quad f=159\text{ Hz}$$

Hence, the correct answer is (B).

90. $Q=Q_0\cos\omega t$

$$\Rightarrow \quad Q=Q_0\cos\left(\frac{2\pi}{T}t\right), \text{ where } T=\frac{1}{f}$$

Energy stored is completely electrical at $t=0$, $\frac{T}{2}$, T, $\frac{3T}{2}$

Hence, the correct answer is (A).

91. Electrical energy is zero i.e., energy stored is completely magnetic at $t=\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$

Hence, the correct answer is (D).

92. At times $t=\frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots..$ we have

$$Q=Q_0\cos\left(\frac{\omega T}{8}\right)=Q_0\cos\left(\frac{\pi}{4}\right)=\frac{Q_0}{\sqrt{2}}$$

CHAPTER 3

Electrical energy at this instant is

$$U_e = \frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C}$$

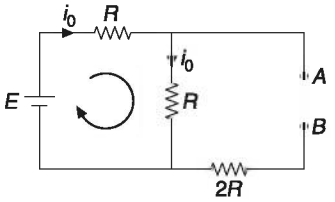
which is half of the total energy

Hence, the correct answer is (C).

93. R damps out the LC oscillations eventually. The entire initial energy 1 J is eventually dissipated as heat.

Hence, the correct answer is (C).

94. Since we know that at $t = 0$, the inductor offers infinite resistance path to the flow of current, so the given circuit can be redrawn as shown in Figure.



Current in the inductor just when the switch is closed is

$$i_0 = \frac{E}{R+R} = \frac{E}{2R}$$

Hence, the correct answer is (D).

95. The internal resistance across terminals A and B can be calculated by shorting the battery which gives

$$R_{AB} = 2R + \frac{R}{2} = \frac{5R}{2}$$

So, the time constant of the circuit is

$$\tau = \frac{L}{R_{AB}} = \frac{2L}{5R}$$

Hence, the correct answer is (C).

96. Due to removal of inductor right loop of circuit is now open and a current flow in the left loop which is given as

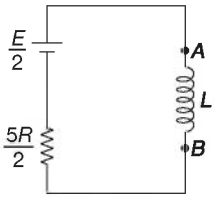
$$i = \frac{E}{2R}$$

The open circuit potential difference across terminals A and B is given as

$$V_{AB} = iR = \frac{E}{2}$$

Hence, the correct answer is (B).

97. The simpler form of the circuit given in the problem is shown in Figure.



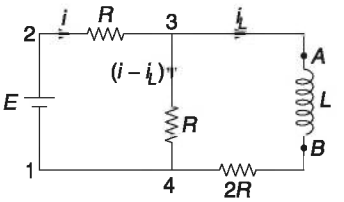
This circuit is now reduced to simple RL circuit and in this case the current as a function of time can be given as

$$i_L = \frac{V_{AB}}{R_{AB}} = \frac{\left(\frac{E}{2}\right)}{\left(\frac{5R}{2}\right)}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

$$\Rightarrow \quad i_L = \frac{E}{5R}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

Hence, the correct answer is (B).

98. To calculate the current supplied by the battery as a function of time, we redraw the circuit as shown in Figure.



Applying KLL, to loop 12341, we get

$$E - iR - (i - i_L)R = 0$$

$$\Rightarrow \quad 2iR = E + i_L R$$

$$\Rightarrow \quad i = \frac{E}{2R} + \frac{i_L}{2}$$

$$\Rightarrow \quad i = \frac{E}{2R} + \frac{E}{10R}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

Hence, the correct answer is (C).

99. On shorting the battery, the resistances across the inductor are 3 Ω and 6 Ω in parallel. Their combined resistance is $R_{net} = 2\ \Omega$. So,

$$\tau = \frac{L}{R_{net}} = \frac{1}{2}\ \text{sec}$$

The current through inductor is

$$i_1 = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = 3\left(1 - e^{-2t}\right)$$

Potential difference across $3\ \Omega$ resistance is equal to Potential difference across inductor, so

$$|V_R| = L\frac{di}{dt} = 6e^{-2t}$$

Hence, the correct answer is (B).

100. Current through $3\ \Omega$ resistor is

$$i_R = \left|\frac{V_R}{3}\right| = 2e^{-2t}$$

Current through inductor is $i_L = 3(1-e^{-2t})$

So, current from battery is

$$i = i_L + i_R = 3 - e^{-2t}$$

Hence, the correct answer is (D).

101. $i_R = i_L$

$$\Rightarrow \quad 2e^{-2t} = 3 - e^{-2t}$$

$$\Rightarrow \quad e^{-2t} = \frac{3}{5}$$

$$\Rightarrow \quad t = \ln\sqrt{\frac{5}{3}}$$

Hence, the correct answer is (A).

102. Current through inductor is towards right and hence positive throughout. Since

$$i_L = 3(1-e^{-2t})$$

At $t = 0$, $i_L = 0$

and at $t \to \infty$,

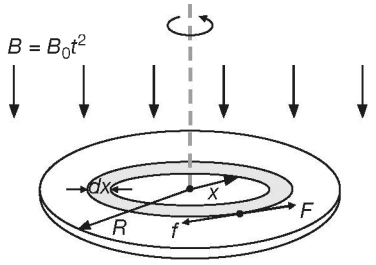
$i_L = 3$ A

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (s)
B → (p)
C → (p, r, t)
D → (q)

Assume the disc to be made of a number of infinitesimal concentric rings. Consider one such ring of radius x, thickness



dx having mass dm. Then the frictional force on this infinitesimal element is

$$df = \mu(dN) = \mu(dm)g \qquad \{\because dN = (dm)g\}$$

where $dm = (2\pi x dx)\frac{M}{\pi R^2} = \left(\frac{2M}{R^2}\right)(x\,dx)$

Infinitesimal torque due to this frictional force is

$$d\tau = (df)x$$

$$\Rightarrow \quad d\tau = \mu\left(\frac{2M}{R^2}x\,dx\right)g\,x$$

$$\Rightarrow \quad d\tau = \frac{2\mu Mg}{R^2}x^2\,dx$$

$$\Rightarrow \quad \tau = \int d\tau = \frac{2\mu Mg}{R^2}\int_0^R x^2 dx$$

$$\Rightarrow \quad \tau = \frac{2\mu Mg}{R^2}\left(\frac{R^3}{3}\right) = \frac{2}{3}(\mu MgR)$$

This expression is independent of t. So
(B) → (p)
Now, let us calculate the torque due to the varying magnetic field.
The varying magnetic field produces an electric field which will be tangential to the disc (or the infinitesimal element). If dq be the charge on the infinitesimal element, then

$$dq = \left(\frac{Q}{\pi R^2}\right)(2\pi x dx) = \left(\frac{2Q}{R^2}\right)x\,dx$$

Electrostatic force on this element in the presence of tangential electric field E_t is

$$dF = (dq)E_t$$

So, if $d\tau_m$ is the torque due to the magnetic field, then

$$\tau_m = \int d\tau_m = \int x\,dF = \int\left(\frac{2Q}{R^2}x^2 dx\right)E_t \qquad \ldots(1)$$

From Faraday's Laws, we know that

$$\left|\oint \vec{E}_t \cdot d\vec{\ell}\right| = \left|\frac{d\phi_B}{dt}\right|$$

$$\Rightarrow \quad (2\pi x)E_t = (\pi x^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E_t = \frac{x}{2}(2B_0 t) = (B_0 t)x$$

$$\Rightarrow \quad \tau_m = \frac{(2QB_0)t}{R^2}\int_0^R x^3 dx$$

$$\Rightarrow \quad \tau_m = \frac{(2QB_0)t}{R^2}\left(\frac{R^4}{4}\right) = \left(\frac{QB_0R^2}{2}\right)t$$

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$$\Rightarrow \quad \tau_m = \left(\frac{QB_0R^2}{2}\right)t \qquad \ldots(2)$$

The disc will start rotating at $t = t_0$, when

$$\tau_m = \tau_f$$

$$\Rightarrow \quad \left(\frac{QB_0R^2}{2}\right)t_0 = \frac{2}{3}\mu MgR$$

$$\Rightarrow \quad t_0 = \frac{4}{3}\left(\frac{\mu Mg}{QB_0R}\right)$$

So, we get (A) → (s)

Also, from (2), we observe that torque due to magnetic field

at $t = 0$ is zero

at $t = t_0$ is $\left(\frac{QB_0R^2}{2}\right)t_0 = \frac{2}{3}\mu MgR$

at $t = 3t_0$ is $\frac{3}{2}\left(QB_0R^2\right)t_0 = 2\mu MgR$

So, (C) → (p, r, t)

Let us calculate the net torque τ on the disc at time $t(>t_0)$

$$\tau = \tau_m - \tau_f = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad I\alpha = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad I\frac{d\omega}{dt} = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad \frac{1}{2}MR^2\frac{d\omega}{dt} = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad d\omega = \frac{QB_0}{M}t\,dt - \frac{4\mu g}{3R}dt$$

$$\Rightarrow \quad \omega = \frac{QB_0}{M}\int_{t_0}^{2t_0} t\,dt - \frac{4\mu g}{3R}\int_{t_0}^{2t_0} dt$$

$$\Rightarrow \quad \omega = \left(\frac{QB_0}{2M}\right)\left(3t_0^2\right) - \left(\frac{4\mu g}{3R}\right)t_0 \qquad \ldots(3)$$

Substituting $t_0 = \frac{4}{3}\left(\frac{\mu Mg}{QB_0R}\right)$ in (3), we get

$$\omega = \frac{8}{9}\left(\frac{M\mu^2g^2}{QB_0R^2}\right)$$

So, (D) → (q)

2. A → (q)
B → (p)
C → (r)
D → (s)

$$V_L = \frac{LdI}{dt} = Ee^{-\frac{t}{\tau_L}} = 10e^{-\frac{t}{\tau_L}}, \text{ where } \tau_L = \frac{L}{R}$$

$$\Rightarrow \quad \tau_L = \frac{L}{R} = 1\text{ s}$$

$$\Rightarrow \quad V_L = 10e^{-t}$$

At $t = 0$, $V_L = 10$ V

At $t = 1$ s, $V_L = \frac{10}{e}$ V

$$\Rightarrow \quad V_R = E - V_L = 10\left(1 - e^{-t}\right)$$

At $t = 0$, $V_R = 0$

At $t = 1$ s, $V_R = 10\left(1 - \frac{1}{e}\right)$ V

3. A → (q)
B → (p)
C → (s)
D → (r)

farad is the unit of capacitance, so

$$[C] = \left[\frac{Q}{V}\right] = \left[\frac{Q^2}{W}\right] = \frac{A^2T^2}{ML^2T^{-2}}$$

$$\Rightarrow \quad [C] = M^{-1}L^{-2}T^4A^2$$

weber is the unit of magnetic flux, so

$$[\phi] = [BA] = \left[\frac{F}{qv}A\right] = \frac{(MLT^{-2})L^2}{(AT)(LT^{-1})} = ML^2T^{-2}A^{-1}$$

ohm is the unit of resistance, so

$$[R] = \left[\frac{V}{I}\right] = \left[\frac{W}{Iq}\right] = \left[\frac{W}{I^2t}\right] = \frac{ML^2T^{-2}}{A^2T} = ML^2T^{-3}A^{-2}$$

henry is the unit of inductance, since

$$\left[\frac{L}{R}\right] = T$$

$$\Rightarrow \quad [L] = (ML^2T^{-3}A^{-2})T = ML^2T^{-2}A^{-2}$$

4. A → (s)
B → (q)
C → (p)
D → (p)

Steady state current through inductor is

$$i_0 = \frac{9}{3} = 3\text{ A}$$

Now, this current decay exponentially across inductor and two resistors. The inductive time constant of the circuit is

$$\tau_L = \frac{L}{R} = \frac{9}{6+3} = 1 \text{ s}$$

$$\Rightarrow \quad t_{1/2} = (\ln 2)\tau_L = (\ln 2) \text{ s}$$

So, given time is half-life time

Hence, current will be $\frac{i_0}{2} = 1.5$ A

Since $i = i_0 e^{-\frac{t}{\tau_L}}$

$$\Rightarrow \quad i = 3e^{-t}$$

$$\Rightarrow \quad \left(-\frac{di}{dt}\right) = 3e^{-t}$$

In the beginning, $\left(-\frac{di}{dt}\right) = 3 \text{ As}^{-1}$ $\{\because t = 0\}$

After one half-life time $\left(-\frac{di}{dt}\right) = 1.5 \text{ As}^{-1}$ $\{\because t = \ln 2\}$

So, $V_L = L\left|-\frac{di}{dt}\right| = 9 \times 1.5 = 13.5$ V

$V_{3\,\Omega} = iR = 1.5 \times 3 = 4.5$ V

$V_{6\,\Omega} = iR = 1.5 \times 6 = 9$ V and

$V_{bc} = V_L - V_{3\,\Omega} = 9$ V

5. A → (q)
B → (s)
C → (p)
D → (r)

For (A) $\tau_C = \frac{R}{2}(C_1 + C_2)$

For (B) $\tau_L = \frac{L_1 L_2}{(L_1 + L_2)(R_1 + R_2)}$

For (C) $\tau_L = \frac{L_1 + L_2}{R_1 + R_2}$

For (D) $\tau_C = R(C_1 + C_2)$

6. A → (s)
B → (q)
C → (s)
D → (p)

Since, $\phi = 2t$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = 2 \text{ V}$$

$$\Rightarrow \quad i = \frac{\xi}{R} = 1 \text{ A} = \text{constant}$$

$$\Rightarrow \quad \Delta q = i\Delta t = 1 \times 2 = 2 \text{ C}$$

$$\Rightarrow \quad H = i^2 R \Delta t = (1)^2 (2)(2) = 4 \text{ J}$$

7. A → (p, r)
B → (p, r)
C → (q, s)
D → (p, r, t)
For case (A)

$$I = \frac{V}{R} \text{ and } U = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

For case (B)

$$I = \frac{V}{R} \text{ and } U = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

For case (C)

$$I = \frac{V}{3R} \text{ and } U = \frac{1}{2}\left(\frac{L}{2}\right)I^2 = \frac{LV^2}{36R^2}$$

For case (D)

$$I = \frac{V}{R} \text{ and } U_1 = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

and $U_2 = \frac{1}{2}\left(\frac{L}{9}\right)I^2 = \frac{LV^2}{18R^2}$

8. A → (q, s)
B → (p, r)
C → (p, r)
D → (q, s)

9. A → (q)
B → (r, s)
C → (s)
D → (p, q, r)

10. A → (r)
B → (s)
C → (p)
D → (t)

In steady state, all the currents and voltages reach their final maximum value. At steady state both the inductors can be shorted so we get current through L_1 is

$$I_1 = \frac{V}{R_1} = \frac{16}{4 \times 10^3} = 4 \times 10^{-3} \text{ A} = 4 \text{ mA}$$

Voltage across L_2 is

$$V_2 = L_2 \frac{dI_2}{dt}$$

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At steady state I_2 = constant

$\Rightarrow \quad V_2 = 0$

Energy stored in inductor L_1 is

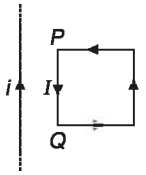
$$E_1 = \frac{1}{2}L_1 I_1^2 = 16\ \mu J$$

Energy stored in inductor L_2 is

$$E_2 = \frac{1}{2}L_2 I_1^2$$

$\Rightarrow \quad E_2 = 24\ \mu J$

11. A → (q, s)
B → (p, r)
C → (p, r)
D → (q, s)
When current is increased, the inward magnetic field passing through loop will increase. So, induced current will produce an outward magnetic field. Hence, induced current is anti-clockwise.



When i and I currents in PQ are in opposite directions, then they will repel each other.

12. A → (q)
B → (p)
C → (t)
D → (r)
Also for 1, the induced current will set up an inward field, so direction of I is from h → g → f → e → d → c → b → a → h and for 2, the induced current will be from h → g → f → e → d → c → b → a → h

13. A → (q)
B → (s)
C → (p)
D → (r)

LC Oscillations (Electrical Oscillations)	Spring Block Oscillations (Mechanical Oscillations)
Q	x
I	v
V	F

(*Continued*)

LC Oscillations (Electrical Oscillations)	Spring Block Oscillations (Mechanical Oscillations)
C	$\frac{1}{k}$
L	m
$I = \frac{dQ}{dt}$	$v = \frac{dx}{dt}$
$\frac{dI}{dt}$	$\frac{dv}{dt}$ = acceleration
$U_L = \frac{1}{2}LI^2$	$K = \frac{1}{2}mv^2$
$U_C = \frac{Q^2}{2C}$	$U = \frac{1}{2}kx^2$

14. A → (r)
B → (p)
C → (q)
D → (q)
At $t = 0$, current through R_1 is zero and current through R_2 is $i_{R_2} = \frac{E}{R_2} = \frac{18}{6} = 3$ A

At $t \to \infty$, current through R_1 is $i_{R_1} = \frac{E}{R_1} = \frac{18}{3} = 6$ A

and current through R_2 is $i_{R_2} = \frac{E}{R_2} = \frac{18}{6} = 3$ A

Integer/Numerical Answer Type Questions

1. (a) Since, according to Faraday's Laws of Electromagnetic Induction, we have

$$\xi = -N\frac{\Delta\phi_B}{\Delta t}, \text{ with } N = 1$$

Given $a = 5 \times 10^{-3}$ m and $h = 0.5$ m,

$$\Delta\phi_B = B_2 A - B_1 A = A(B_2 - B_1)$$

$$\Rightarrow \quad \Delta\phi_B = a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right)$$

$$\Rightarrow \quad \Delta\phi_B = \frac{a^2\mu_0 I}{2\pi}\left(\frac{1}{h+a} - \frac{1}{a}\right) = -\frac{\mu_0 ahI}{2\pi(h+a)}$$

The time for the washer to drop a distance h (from rest) is given by

$$\Delta t = \sqrt{\frac{2h}{g}}$$

From Faraday's Laws, we have

$$\xi = -\frac{\Delta\phi_B}{\Delta t} = \frac{\mu_0 ahI}{2\pi(h+a)\Delta t}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 ahI}{2\pi(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 aI}{2\pi(h+a)}\sqrt{\frac{gh}{2}}$$

$$\Rightarrow \quad \xi = \frac{(4\pi\times10^{-7})(5\times10^{-3})(10)}{2\pi(0.5+0.005)}\sqrt{\frac{(9.8)(0.5\text{ m})}{2}}$$

$$\Rightarrow \quad \xi = 30.9\text{ nV} \cong 31\text{ nV}$$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a clockwise direction.

2. Power dissipated in the loop is

$$P = \frac{\xi^2}{R} \qquad \ldots(1)$$

where R is the resistance of the loop given by

$$R = \lambda(2\pi a) = 2\pi\lambda a$$

where ξ = induced emf in the loop given by

$$\xi = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right| = (\pi a^2)\left|\frac{dB}{dt}\right|$$

where $B(x) = \frac{\mu_0}{2\pi}\frac{I_0}{x} = B$

$$\Rightarrow \quad \left(\frac{dB}{dt}\right) = -\left(\frac{\mu_0 I_0}{2\pi x^2}\right)\left(\frac{dx}{dt}\right) = -\frac{\mu_0 I_0 v}{2\pi x^2} \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

$$\Rightarrow \quad \xi = \frac{\pi a^2 \mu_0 I_0 v}{2\pi x^2} \qquad \ldots(2)$$

From equations (1) and (2)

$$P = \frac{\pi^2 a^4 \mu_0^2 I_0^2 v^2}{4\pi^2 x^4 \cdot R}$$

$$\Rightarrow \quad v = \sqrt{\frac{4Px^4 R}{\mu_0^2 I_0^2 a^4}}$$

$$\Rightarrow \quad v = \frac{2x^2}{\mu_0 I_0 a^2}\sqrt{PR} = \frac{2x^2}{\mu_0 I_0 a^2}\sqrt{2\pi\lambda Pa}$$

$$\Rightarrow \quad v = \frac{x^2}{\mu_0 I_0 a^2}\sqrt{8a\pi\lambda P}$$

$$\Rightarrow \quad k = 8$$

3. Instantaneous flux through loop is

$$\phi = \pi a^2 B\cos0° + \pi b^2 B\cos180°$$

$$\Rightarrow \quad \phi = \pi(a^2 - b^2)B$$

$$\Rightarrow \quad \phi = \pi(a^2 - b^2)B_0\sin(\omega t)$$

Differentiating w.r.t. time

$$\left|\frac{d\phi}{dt}\right| = \pi(a^2 - b^2)B_0\omega\cos(\omega t)$$

$$\Rightarrow \quad I = \frac{\pi(a^2 - b^2)B_0\omega\cos(\omega t)}{R}$$

$I_{\max} = I_0 = \frac{\pi(a^2 - b^2)B_0\omega}{2\pi(a+b)\lambda}$, λ = resistance per unit length

$$I_{\max} = I_0 = \frac{1}{2\lambda}(a-b)B_0\omega$$

$$I_{\max} = I_0 = \frac{1}{2}(20-10)\times10^{-2}\times10\times10^{-3}\times\frac{100}{50\times10^{-3}}$$

$$I_{\max} = I_0 = 1\text{ A}$$

4. Let l_0 be the length of the solenoid, then according to the problem, we have

$$l_0 = 100\text{ cm}$$

Self-inductance of the solenoid is

$$L = \mu_0 n^2 A l_0 = \frac{\mu_0 N^2 \pi r^2}{l_0}$$

If l be the total length of wire used to make the solenoid, then

$$l = N(2\pi r)$$

$$\Rightarrow \quad r = \frac{l}{2\pi N}$$

Substitute this value in equation (1), we get

$$L = \frac{\mu_0 l^2}{4\pi l_0}$$

$$\Rightarrow \quad l = \sqrt{\frac{4\pi l_0 L}{\mu_0}} = \sqrt{(10^7)(1)(10^{-3})} = 100\text{ m}$$

5. Let the current in the outer coil at any instant of time, say t, be $I = \alpha t$ and in the inner coil is $2I = 2\alpha t$, where α is a constant. Because of these currents the magnetic field in the outer coil is $B = \mu_0 n(\alpha t)$, where n is the number of turns per unit length. Because in the annular part (region between two coils) no field will be there due to the inner coil, because the annular region is the region outside the inner coil, so $(B_{\text{annular}})_{\text{inner coil}} = 0$ and $(B_{\text{annular}})_{\text{outer coil}} = \mu_0 nI$

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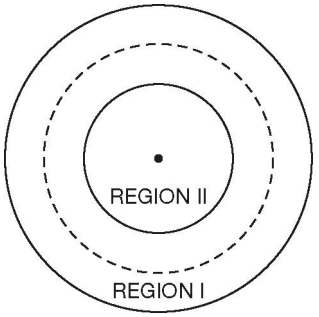


Similarly, the field in the Region II inside the inner coil will be

$$B_{\text{REGION II}} = B_{\text{outer coil}} + B_{\text{inner coil}}$$

$$\Rightarrow \quad B_{\text{REGION II}} = \mu_0 nI + \mu_0 n(2I) = 3\mu_0 nI = 3B$$

The magnetic flux enclosed by the particle's trajectory of radius r is

$$\phi = (\pi R^2)(2B) + (\pi r^2)(B)$$

$$\Rightarrow \quad \phi = (2R^2 + r^2)\pi\mu_0 n\alpha t$$

Since we have

$$\left|\oint \vec{E}\cdot d\vec{\ell}\right| = \frac{d\phi}{dt}$$

$$\Rightarrow \quad E(2\pi r) = \frac{d\phi}{dt}$$

$$\Rightarrow \quad E(2\pi r) = (2R^2 + r^2)\pi\mu_0 n\alpha$$

$$\Rightarrow \quad E = \frac{(2R^2 + r^2)}{r}\frac{\mu_0 n\alpha}{2} \qquad \ldots(1)$$

The charged particle is held in its circular orbit by the magnetic field, and so we have

$$\frac{mv^2}{r} = qvB \qquad \ldots(2)$$

Further, due to the induced electric field the particle gets accelerated along its circular orbit by the tangential component of the net force.

$$\Rightarrow \quad ma_t = qE$$

where m is the mass and q the electric charge of the particle. Further, since the magnitude of the electric field is constant, the speed of the particle increases uniformly with time. So,

$$v = a_t t = \left(\frac{qE}{m}\right)t$$

Substituting the value of E from (1), we get

$$v = \frac{q}{m}\left(\frac{2R^2 + r^2}{r}\frac{\mu_0 n\alpha}{2}\right)t \qquad \ldots(3)$$

Again, substituting this value of v and the value of $B(=\mu_0 n\alpha t)$, into equation (2), we get

$$\frac{m}{r}\frac{(2R^2 + r^2)}{r}\frac{\mu_0 n\alpha}{2}\frac{q}{m}t = q\mu_0 n\alpha t,$$

$$\Rightarrow \quad \frac{(2R^2 + r^2)}{2r^2} = 1$$

$$\Rightarrow \quad r = \sqrt{2}R$$

$$\Rightarrow \quad k = 2$$

6. Induced electric field

$$E(2\pi r) = \frac{d\phi}{dt}$$

$$\Rightarrow \quad E = \frac{\pi a^2 2B_0 t}{2\pi r}$$

Torque due to field about centre of ring

$$\tau_1 = (qE)r = (\lambda(2\pi r))\left(\frac{2\pi a^2 B_0 t}{2\pi r}\right)r$$

ring starts rotating when

$$\tau_{\text{due to electric field}} = \tau_{\text{due to friction}}$$

$$\Rightarrow \quad \tau_1 = (\mu mg)r$$

Solving, we get $t = \dfrac{\mu mg}{2\pi a^2 B_0 \lambda} = 4$ s

7. When the switch is closed for a long time, then $I_{\max} = I_0 = \dfrac{E}{R}$. Now when the switch is opened, the current in the battery and the resistor drops to zero. However, the coil carries this same current because the oscillations begin in the LC loop. So, we have

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_0^2$$

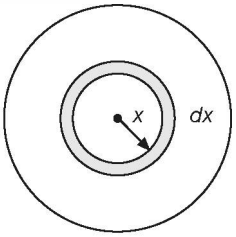
$$\Rightarrow \quad L = \frac{CV^2}{I_0^2} = \frac{(0.5\times 10^{-6})(150)^2(250)^2}{(50)^2} \quad \left\{\because I_0 = \frac{E}{R}\right\}$$

$$\Rightarrow \quad L = 281 \text{ mH}$$

8. $E = \dfrac{x}{2}\dfrac{dB}{dt}$

$$\Rightarrow \quad E = \frac{3Kxt^2}{2}$$

$$\Rightarrow \quad d\tau = (dq)Ex = \left(\frac{3Kxt^2}{2}\right)\left(\frac{2\pi x dx}{\pi r^2}\right)qx$$



$$\Rightarrow \quad \tau = \frac{3Kt^2q}{r^2}\int_0^r x^3 dx$$

$$\Rightarrow \quad \tau = \frac{3Kqt^2}{4}r^2 \qquad \ldots(1)$$

Torque due to friction force

$$d\tau = \mu g x dm = \mu g x(2\pi x dx)\frac{m}{\pi r^2}$$

$$\Rightarrow \quad \tau = 2\mu g\frac{m}{r^2}\int_0^r x^2 dx = \frac{2}{3}\mu mgr \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{3Kqt^2r^2}{4} = \frac{2}{3}\mu mgr$$

$$\Rightarrow \quad t = \sqrt{\frac{8\mu mg}{9Kqr}}$$

Substituting values, we get

$$t = 2 \text{ s}$$

9. $$\xi = \frac{d\phi}{dt} = \frac{d}{dt}(NB\ell^2\cos\theta) = N\ell^2\cos\theta\frac{dB}{dt}$$

$$\ell = \sqrt{\frac{\xi}{N\left(\frac{dB}{dt}\right)\cos\theta}}$$

where $\xi = 80\times10^{-3}$ V,

$N = 50$ and $\frac{dB}{dt} = \left(\frac{600-200}{0.4}\right)\times10^{-6}$ Ts^{-1}

$$\Rightarrow \quad \ell = \sqrt{\frac{80\times10^{-3}}{(50)\left(\frac{400\times10^{-6}}{0.4}\right)\cos(30°)}}$$

$$\Rightarrow \quad \ell = 1.36 \text{ m}$$

$$\Rightarrow \quad \text{Total length} = N(4\ell) = (50)(4)(1.36) = 272 \text{ m}$$

10. Since $\phi = BA = \mu_0 nI(A_{\text{solenoid}})$

$$\Rightarrow \quad \phi = \mu_0 n(\pi r^2_{\text{solenoid}})I, \text{ where } n = 1000$$

According to Faraday's Laws, we have

$$\xi = -\frac{N d\phi}{dt} = -N\mu_0 n(\pi r^2_{\text{solenoid}})\left(\frac{dI}{dt}\right), \text{ where } N = 15$$

Since $I = 4\cos(250t)$

$$\Rightarrow \quad \frac{dI}{dt} = -1000\sin(250t)$$

$$\xi = (15)(4\pi\times10^{-7})(1000)\pi(0.02)^2(1000)\sin(250t)$$

$$\Rightarrow \quad \xi = (60\pi^2)(4\times10^{-4})(10^{-1})\sin(250t)$$

$$\Rightarrow \quad \xi = 600\times4\times10^{-5}\sin(250t)$$

$$\Rightarrow \quad \xi = 2400\times10^{-5}\sin(250)t$$

$$\Rightarrow \quad \xi = (24\times10^{-3})\sin(250t) \text{ volt}$$

$$\Rightarrow \quad \xi_{\text{peak}} = 24 \text{ mV}$$

11. We know the magnetic field at P a distance $x = a$ from the centre of the coil lying on the axis is

$$B = \frac{\mu_0 Ia^2}{2\pi(a^2+a^2)^{3/2}}$$

The current due to revolving charge is

$$I = \frac{q\omega}{2\pi}$$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{q\omega a^2}{(2a^2)^{3/2}} = \frac{\mu_0 q\omega}{8\sqrt{2}\pi a}$$

The energy density due to the magnetic field is

$$u_M = \frac{B^2}{2\mu_0}$$

The electric field strength at an axial point is given as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qa}{(a^2+a^2)^{3/2}} = \frac{q}{8\sqrt{2}\pi\varepsilon_0 a^2}$$

The energy density of the electric field is given as

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Ratio of energy densities of magnetic and electric field is

$$\frac{u_M}{u_E} = \frac{\left(\frac{B}{E}\right)^2}{\mu_0\varepsilon_0} = (\omega a)^2\mu_0\varepsilon_0$$

$$\Rightarrow \quad \frac{u_M}{u_E} = \frac{a^2\omega^2}{c^2} = \frac{(81\times10^{-4})(10^4)}{9\times10^{16}} = 9\times10^{-16}$$

$$\Rightarrow \quad * = 9$$

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12. After reading the statement carefully, we note that the flux inside the tube is constant. So,

$$\phi = B_{\text{initial}}A_{\text{initial}} = B_{\text{final}}A_{\text{final}}$$

$$\Rightarrow \quad B_iA_i = B_fA_f$$

$$\Rightarrow \quad B_f = B_i\left(\frac{A_i}{A_f}\right)$$

$$\Rightarrow \quad B_f = (2.5)\left(\frac{R}{r}\right)^2 \qquad \{\because A_i = \pi R^2 \text{ and } A_f = \pi r^2\}$$

$$\Rightarrow \quad B_f = (2.5)(12)^2$$

$$\Rightarrow \quad B_f = (2.5)(144) = 360 \text{ T}$$

13. (a) $|\vec{F}| = I|\vec{\ell} \times \vec{B}| = BI\ell$, where $I = \frac{\xi}{R} = \frac{B\ell v}{R}$

$$\Rightarrow \quad F = \frac{B^2\ell^2 v}{R} = \frac{(2.5)^2(1.2)^2(2)}{6} = 3 \text{ N}$$

(b) $P = I^2R = Fv = \frac{B^2\ell^2v^2}{R} = 6$ W

14. The magnetic field at the axis of a solenoid is

$$B = \mu ni = (\mu_r\mu_0)ni \qquad \ldots(1)$$

where, μ_r is the relative permeability of the medium.

The magnetic flux associated with the cross-section of the solenoid is 1.6×10^{-3} Wb. This simply means that the flux associated with each turn of the solenoid is 1.6×10^{-3} Wb i.e. Since the magnetic flux is

$$\phi = (1)BA = 1.6 \times 10^{-3}$$

$$\Rightarrow \quad B = \frac{\phi}{A} = \frac{1.6 \times 10^{-3}}{10^{-3}} = 1.6 \text{ T}$$

Substituting the values in Equation (1), we get

$$1.6 = \mu_r(4\pi \times 10^{-7}) \times 400 \times 5$$

$$\Rightarrow \quad \mu_r \approx 637$$

The total number of turns in the solenoid are

$$N = n \times l = 400 \times 0.5 = 200$$

Hence, the total magnetic flux linked with the solenoid is

$$\phi_l = N\phi = 200 \times 1.6 \times 10^{-3} = 0.32 \text{ Wb}$$

$$\Rightarrow \quad L = \frac{N\phi}{i} = \frac{0.35}{5} = 0.064 \text{ H} = 64 \text{ mH}$$

15. (a) $\xi = B\ell v = (2.5)\left(\frac{50}{100}\right)(8) = 10$ V

$$\Rightarrow \quad I_1 = \frac{10}{5/3} = 6 \text{ A} \text{ and } I_2 = \frac{10}{5} = 2 \text{ A}$$

(b) $P = \xi(I_1 + I_2) = (10)(8) = 80$ W

(c) $F = \frac{P}{v} = \frac{80}{8} = 10$ N

16. Since, $\phi_B = 3(at^3 - bt^2) = (6t^3 - 18t^2)$

$$\Rightarrow \quad \xi = -\frac{d\phi_B}{dt} = -18t^2 + 36t$$

ξ is maximum when, $\frac{d\xi}{dt} = -36t + 36 = 0$ which gives

$t = 1$ s

Therefore, the maximum current (at $t = 1$ s) is

$$I = \frac{\xi}{R} = \frac{(-18 + 36) \text{ V}}{3\ \Omega} = 6 \text{ A}$$

17. Magnetic energy stored in inductor is

$$U = \frac{1}{2}Li^2$$

This energy is completely utilised in melting the ice. If L_f be the latent heat of fusion of ice, then we have

$$\frac{1}{2}Li^2 = mL_f$$

$$\Rightarrow \quad m = \frac{Li^2}{2L_f}$$

Substituting the values, we get

$$m = \frac{(42)(20)^2}{2(80 \times 4200)} = 0.025 \text{ kg}$$

$$\Rightarrow \quad m = 25 \text{ g}$$

18. $\frac{1}{2}CV^2 = \frac{1}{2}LI_0^2$

$$\Rightarrow \quad I_0 = \sqrt{\frac{CV^2}{L}}$$

$$\Rightarrow \quad I_0 = \sqrt{\frac{(10^{-6})(1600)}{10 \times 10^{-3}}}$$

$$\Rightarrow \quad I_0 = 0.4 \text{ A} = 400 \text{ mA}$$

19. (a) $I_0 = I_{\max} = \frac{E}{R} = \frac{12 \text{ V}}{12\ \Omega} = 1$ A

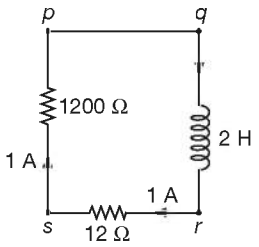
(b) At the instant the switch is thrown quickly from a to b, the current through the circuit is 1 A i.e., current through 12 Ω and now 1200 Ω both is 1 A.

$$\Rightarrow \quad \Delta V_{12\ \Omega} = (1 \text{ A})(12\ \Omega) = 12 \text{ V} \text{ and}$$

$$\Delta V_{1200\ \Omega} = (1 \text{ A})(1200\ \Omega) = 1200 \text{ V}$$

Consider the circuit at the instant immediately after the switch is thrown quickly from a to b. For the loop $pqrs$, we get

$$-(12)(1)-(1200)(1)-\Delta V_L=0$$



$$\Rightarrow \quad \Delta V_L=-1212 \text{ V}$$

$$\Rightarrow \quad |\Delta V_L|=1212 \text{ V}$$

(c) Since $I=I_{\max}e^{-\frac{Rt}{L}}=I_0e^{-\frac{Rt}{L}}$

$$\Rightarrow \quad \frac{dI}{dt}=-\frac{I_0R}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad -L\frac{dI}{dt}=\Delta V_L=(I_0R)e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad 12=(1)(1200+12)e-\frac{(1200+12)t}{2}$$

$$\Rightarrow \quad \frac{12}{1212}=e^{-606t}$$

$$\Rightarrow \quad \log_e\left(\frac{1212}{12}\right)=606t$$

$$\Rightarrow \quad 606t=\log_e(101)$$

$$\Rightarrow \quad 606t=4.6$$

$$\Rightarrow \quad t=76\times10^{-4} \text{ s}$$

$$\Rightarrow \quad x=76$$

20. Let the magnitude of the magnetic field generated by the wire carrying a current I, at a distance x from it be B. Then

$$B=\frac{\mu_0I}{2\pi x}$$

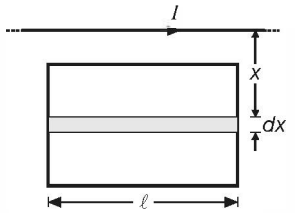
Consider a rectangular strip of width dx, area $dA=\ell dx$

$$d\phi=\vec{B}\cdot d\vec{A}=BdA\cos0$$

$$\Rightarrow \quad d\phi=\frac{\mu_0I}{2\pi x}\ell dx$$

$$\Rightarrow \quad \phi=\int d\phi=\frac{\mu_0I\ell}{2\pi}\int_{0.4\text{ mm}}^{1.7\text{ mm}}\frac{dx}{x}$$

$$\Rightarrow \quad \phi=\frac{\mu_0I\ell}{2\pi}\log_e\left(\frac{1.7}{0.4}\right)=\frac{\mu_0I\ell}{2\pi}\log_e\left(\frac{17}{4}\right)$$



The mutual inductance between the wire and the loop is

$$M=\frac{N_2\phi_{12}}{I_1}=\frac{N_2\mu_0I\ell}{2\pi I}\log_e\left(\frac{17}{4}\right)=\frac{N_2\mu_0\ell}{2\pi}(1.45)$$

$$\Rightarrow \quad M=\frac{(1)(4\pi\times10^{-7})(2.7\times10^{-3})}{2\pi}(1.45)$$

$$\Rightarrow \quad M=783\times10^{-12} \text{ H}$$

$$\Rightarrow \quad M=783 \text{ pH}$$

21. $\tau_C=\tau_L$

$$\Rightarrow \quad CR=\frac{L}{R}$$

$$\Rightarrow \quad R=\sqrt{\frac{L}{C}}=\sqrt{\frac{3}{3\times10^{-6}}}=1000\ \Omega=1\ \text{k}\Omega$$

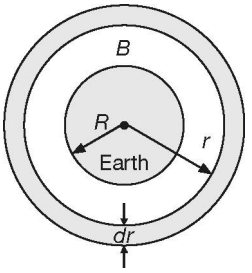
Also, $\tau=RC=(1000)(3\times10^{-6})=3$ ms

22. Since, the magnetic energy density is

$$u_m=\frac{B^2}{2\mu_0}=\frac{B_0^2R^4}{2\mu_0r^4}$$

Consider an infinitesimal spherical shell (concentric with earth) of radius r, thickness dr and volume dV. Then

$$dV=4\pi r^2dr$$



If dU is the energy associated with the shell, then

$$dU=u_mdV$$

$$\Rightarrow \quad dU=\left(\frac{B_0^2R^4}{2\mu_0}\right)\left(\frac{4\pi r^2dr}{r^4}\right)$$

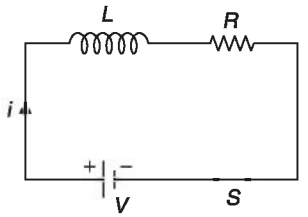
$$\Rightarrow \quad U=\int dU=\frac{2\pi B_0^2R^4}{\mu_0}\int_R^\infty r^{-2}dr$$

$$\Rightarrow \quad U=\frac{2\pi B_0^2R^3}{\mu_0}=\frac{(2\pi)(50\times10^{-6})^2(6\times10^6)^3}{4\pi\times10^{-7}}$$

$$\Rightarrow \quad U=2700\times10^{15}\text{ J}$$

$$\Rightarrow \quad U=2700\text{ peta joule} \qquad \{\because\ 1\text{ peta}=10^{15}\}$$

23. Let at a time instant t, after shorting of switch, current through the circuit be i as shown in Figure.



For the given loop, the current in the circuit as a function of time is

$$i=\frac{V}{R}(1-e^{-Rt/L})=i_0(1-e^{-Rt/L}) \qquad ...(1)$$

where, $i_0=\frac{V}{R}$ is the maximum current in circuit.

The magnetic energy stored in inductor is

$$U=\frac{1}{2}Li^2$$

Given that at an instant the magnetic energy attains one fourth of the maximum value, so we have

$$\frac{1}{2}Li^2=\frac{1}{4}\left(\frac{1}{2}Li_0^2\right)$$

$$\Rightarrow \quad i=\frac{i_0}{2}=\frac{V}{2R}$$

Substituting the values in Equation (1), we get

$$\frac{V}{2R}=\frac{V}{R}(1-e^{-2t/20})$$

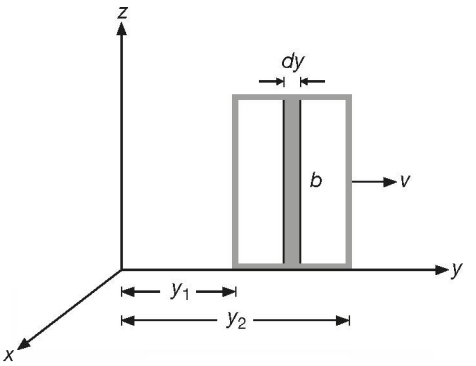
$$\Rightarrow \quad \frac{1}{2}=1-e^{-t/10}$$

$$\Rightarrow \quad e^{-t/10}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{t}{10}=0.7$$

$$\Rightarrow \quad t=7\text{ s}$$

24. (a) Magnetic flux linked with the loop



Consider a strip of length b, width dy, then

$$d\phi=BdA$$

$$\Rightarrow \quad d\phi=(6-y)(b\,dy)$$

$$\Rightarrow \quad \phi=\int_{y_1}^{y_2}(6-y)b\,dy, \text{ where } y_1=vt \text{ and } y_2=vt+\ell$$

$$\Rightarrow \quad \phi=6b(y_2-y_1)-\frac{b}{2}(y_2-y_1)(y_2+y_1) \qquad ...(1)$$

$$\Rightarrow \quad \phi=6\ell b-\frac{\ell b}{2}(\ell+2vt)$$

Induced e.m.f. $\xi=-\frac{d\phi}{dt}=b\ell v$

$$\Rightarrow \quad \xi=(5)(2)(3)\ V$$

$$\Rightarrow \quad \xi=30\text{ V}$$

(b) From (1), we get

$$\phi=6b(y_2-y_1)-\frac{b}{2}(y_2-y_1)(y_2+y_1)$$

Now, in this case we have, if a to be the acceleration, then

$$y_1=\frac{1}{2}at^2 \text{ and } y_2=\frac{1}{2}at^2+\ell$$

$$\Rightarrow \quad y_2-y_1=\ell \text{ and } y_2+y_1=at^2+\ell$$

$$\Rightarrow \quad \phi=6\ell b-\frac{\ell b}{2}(at^2+\ell)$$

Since $\xi=-\frac{d\phi}{dt}$

$$\Rightarrow \quad \xi=(\ell ba)t$$

$$\Rightarrow \quad \xi=(2)(5)(2)=20\text{ V}$$

25. Volume of the balloon at any instant, when radius is r

$$V=\frac{4}{3}\pi r^3$$

Time rate of change of volume

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through the band at the given instant is

$$\phi = B(\pi r^2)$$

Induced emf $\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2)$

$$\Rightarrow \quad \xi = -2\pi r B\frac{dr}{dt}$$

$$\Rightarrow \quad \xi = -2\pi r B\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right)$$

$$\Rightarrow \quad \xi = -\frac{B}{2r}\frac{dV}{dt}$$

Since, the volume of the balloon is decreasing, so $\frac{dV}{dt}$ is negative

$$\Rightarrow \quad \xi = -\frac{(0.04)}{2\times 6\times 10^{-2}}\times(-100\times 10^{-6})$$

$$\Rightarrow \quad \xi = 333\ \mu V$$

26. Flux linked with the loop at any instant

$$\phi = \int_{x}^{x+\ell}\left(\frac{\mu_0 I}{2\pi y}\right)\ell\, dy$$

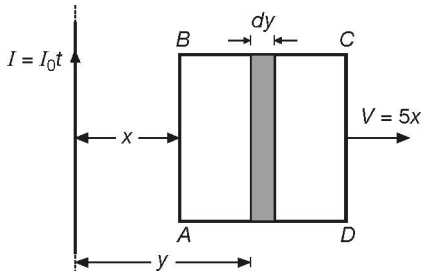
$$\Rightarrow \quad \phi = \frac{\mu_0 I_0 \ell t}{2\pi}\log_e\left(\frac{x+\ell}{x}\right) \qquad \ldots(1)$$

$$\Rightarrow \quad \xi = \frac{-d\phi}{dt}$$

$$\xi = -\left[\frac{\mu_0 I_0 \ell}{2\pi}\log_e\frac{(x+\ell)}{x} + \frac{\mu_0 I_0 \ell t}{2\pi}\left(\frac{x}{x+\ell}\right)\left(-\frac{\ell}{x^2}\right)\left(\frac{dx}{dt}\right)\right]$$

$$\left\{\because \frac{d}{dt}\log_e(u) = \frac{1}{u}\frac{du}{dt}\right\}$$

$$\Rightarrow \quad I = \frac{\xi}{R} = -\frac{\mu_0 I_0 \ell}{2\pi R}\log_e\left(\frac{x+\ell}{x}\right) + \frac{\mu_0 I_0 \ell^2 t}{2\pi R x(x+\ell)}\frac{dx}{dt} \qquad \ldots(2)$$



Since, we have

$$\frac{dx}{dt} = 5x$$

$$\Rightarrow \quad \int_{\frac{\ell}{10}}^{10\ell}\frac{dx}{x} = 5\int_0^t dt$$

$$\Rightarrow \quad t = \frac{\log_e 100}{5}$$

Putting the value of t and $R = \frac{\mu_0 I_0 \ell}{2\pi}$ in equation (2), we get

$$I = -\log_e(1.1) + \frac{1}{11}\log_e(100)$$

$$\Rightarrow \quad I = -0.095 + \frac{4.605}{11}$$

$$\Rightarrow \quad I = 324\ \text{mA}$$

27. (a) Voltage across the inductor is given by

$$V_L = L\frac{dI}{dt}$$

$$\Rightarrow \quad V_L = 10^{-3}(20)\ \text{V}$$

$$\Rightarrow \quad V_L = 20\ \text{mV}$$

(b) Voltage across the capacitor is given by

$$V_C = \frac{q}{C} = \frac{1}{C}\int_0^t I dt$$

$$\Rightarrow \quad V_C = \left(\frac{1}{10^{-6}}\right)\int_0^t (20t)\, dt$$

$$\Rightarrow \quad V_C = (10^6)\frac{20t^2}{2}\ V = (10^7 t^2)\ V$$

So, at $t = 1$ ms, we have

$$V_C = 10^7(10^{-6}) = 10\ \text{V}$$

(c) $\frac{1}{2}CV_C^2 > \frac{1}{2}LI^2$

$$\Rightarrow \quad CV_C^2 > LI^2$$

$$\Rightarrow \quad (10^{-6})(10^7 t^2)^2 > (10^{-3})(20t)^2$$

$$\Rightarrow \quad t > 63.2\times 10^{-6}\ \text{s}$$

$$\Rightarrow \quad t > 63.2\ \mu\text{s}$$

$$\Rightarrow \quad t = 63\ \mu\text{s}$$

CHAPTER 3

28. $I=\frac{dq}{dt}=\frac{\xi}{R}$ where $\xi=-N\frac{d\phi_B}{dt}$

$\Rightarrow \int_0^Q dq=\frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi_B$

$\Rightarrow |Q|=\frac{N}{R}(\phi_2-\phi_1)$

$\Rightarrow Q=\frac{N}{R}\left[BA\cos(0°)-BA\cos\left(\frac{\pi}{2}\right)\right]=\frac{BAN}{R}$

$\Rightarrow B=\frac{RQ}{NA}=\frac{(200)(5\times10^{-4})}{(100)(40\times10^{-4})}=0.25\text{ T}=250\text{ mT}$

29. $I=\frac{\xi}{R}=\frac{B}{R}\left(\frac{\Delta A}{\Delta t}\right)$

$\Rightarrow I\Delta t=Q=\left|\frac{B}{R}(\Delta A)\right|$

$\Rightarrow Q=\left|\frac{B}{R}(A_f-A_i)\right|=\left|\frac{B}{R}(0-\ell^2)\right|=\frac{B\ell^2}{R}$

$\Rightarrow Q=\frac{(25\times10^{-6})(0.04)}{0.5}$

$\Rightarrow Q=2\times10^{-6}C=2\ \mu C$

$\Rightarrow Q=2\ \mu C$

30. Since we know that

$I=-\frac{N}{R}\frac{d\phi}{dt}$

$\Rightarrow Idt=-\frac{N}{R}d\phi$

$\Rightarrow dq=-\frac{N}{R}d\phi$

$\Rightarrow \int dq=-\frac{N}{R}\int d\phi$

$\Rightarrow \Delta q=-\frac{N}{R}\Delta\phi=-\frac{N}{R}(\phi_f-\phi_i)$

$\Rightarrow \Delta q=-\frac{N}{R}(-BA-BA)$

$\{\because \phi_f=BA\cos(180°)\text{ and }\phi_i=BA\cos(0°)\}$

$\Rightarrow \Delta q=\frac{2NBA}{R}$

$\Rightarrow B=\frac{R\Delta q}{2NA}$

$\Rightarrow B=\frac{(40)(4.5\times10^{-6})}{(2)(30)(3\times10^{-6})}$

$\Rightarrow B=1\text{ T}$

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1. Energy dissipated per second by resistor is

$P_R=i^2\times R$

Energy supplied per second by battery is

$P_B=E\times i$

Since $P_B=P_R+P_L$

$\Rightarrow P_L=Ei-i^2R$

$\Rightarrow Ei-i^2R=i^2R$

$\Rightarrow i=\frac{E}{2R}$

Since $i=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$, where $\tau=\frac{L}{R}=2$

$\Rightarrow \frac{E}{2R}=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$

$\Rightarrow t=\tau\ln(2)=\frac{20}{10}\ln(2)=2\ln(2)$

Hence, the correct answer is (B).

2. Since $B=\mu_0 ni$

$\Rightarrow \phi=(\mu_0 ni)(nL)A$

So, inductance $L'=\frac{\phi}{i}\mu_0 n^2AL$

Since $n=\frac{N}{L}$

$\Rightarrow L'=\mu_0\left(\frac{N^2A}{L}\right)$

$\Rightarrow L'\propto\frac{1}{L}$

Hence, the correct answer is (A).

3. $\phi_Q=Mi$

$\Rightarrow 10^{-3}=M(3)$

$\Rightarrow M=\frac{1}{3}\times10^{-3}\text{ H}$

Since $\phi_P=M(2)$

$$\Rightarrow \quad \frac{10^{-3}}{3}=\frac{\phi_P}{2}$$

$$\Rightarrow \quad \phi_P=\frac{20}{3}\times 10^{-4}=6.67\times 10^{-4}\ \text{Wb}$$

Hence, the correct answer is (B).

4. $i=kte^{-\alpha t}$

Since $\phi=BA=(\mu_0 ni)A=\mu_0 n(kte^{-\alpha t})\pi(2R)^2$

$$\Rightarrow \quad \phi=(4\pi R^2\mu_0 nK)te^{-\alpha t}=k'(te^{-\alpha t})$$

Since $\xi=-\left(\frac{d\phi}{dt}\right)=-k'e^{-\alpha t}+k'\alpha te^{-\alpha t}$

$$\Rightarrow \quad \xi=-k'e^{-\alpha t}(1-\alpha t)$$

So, induced current I is given by

$$I=\frac{E}{r}=-k'e^{-\alpha t}(1-\alpha t)$$

At $t=0$, $I=-k'$

Hence, the correct answer is (B).

5. $I=I_{\text{sat}}\left(1-e^{\frac{Rt}{L}}\right)$, where $R=R_L+r=1\ \Omega$

Since $I=0.8I_{\text{sat}}$

$$\Rightarrow \quad 0.8I_{\text{sat}}=I_{\text{sat}}\left(1-e^{\frac{t}{0.01}}\right)$$

$$\Rightarrow \quad \frac{4}{5}=1-e^{-100t}$$

$$\Rightarrow \quad e^{-100t}=\left(\frac{1}{5}\right)$$

$$\Rightarrow \quad 100t=\ln 5$$

$$\Rightarrow \quad t=\frac{1}{100}\ln 5$$

$$\Rightarrow \quad t=0.016\ \text{s}$$

Hence, the correct answer is (D).

6. $BLv=IR_{eq}$

where $R_{eq}=\frac{4}{3}\Omega+1.7=3\ \Omega$

$$\Rightarrow \quad I=\frac{BLv}{R_{eq}}=\frac{(1)(5\times 10^{-2})\times 10^{-2}}{3}$$

$$\Rightarrow \quad I=\frac{5}{3}\times 10^{-4}\ \text{A}\approx 1.7\times 10^{-4}\ \text{A}$$

$$\Rightarrow \quad I=170\ \mu\text{A}$$

Hence, the correct answer is (A).

7. $i=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$ where $\tau=\frac{L}{R}$

$$\Rightarrow \quad q=\int_0^q dq=\frac{E}{R}\int\left(1-e^{-\frac{t}{\tau}}\right)dt$$

$$\Rightarrow \quad q=\frac{E}{R}\left(t+e^{-\frac{t}{\tau}}\times\tau\right)\Big|_0^t$$

$$\Rightarrow \quad q=\frac{E}{R}\left(\frac{L}{R}+\frac{L}{R}e^{-1}-\frac{L}{R}\right)$$

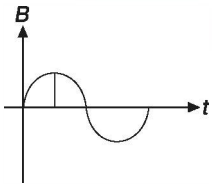
$$\Rightarrow \quad q=\frac{EL}{eR^2}=\frac{EL}{2.7R^2} \qquad \{\because e\approx 2.7\}$$

Hence, the correct answer is (B).

8. Since, $\Delta Q=\frac{\Delta\phi}{R}$

Given that,

$$B(t)=0.4\sin(50\pi t)$$



$$\Rightarrow \quad \frac{2\pi}{T}=50\pi$$

$$\Rightarrow \quad T=\frac{1000}{25}\ \text{ms}$$

$$\Rightarrow \quad T=40\ \text{ms}$$

$$\Rightarrow \quad \Delta Q=\frac{0.4\times 3.5\times 10^{-3}}{10}$$

$$\Rightarrow \quad \Delta Q=1.4\times 10^{-4}\ \text{C}$$

*No given option is correct.

9. Field inside solenoid is

$$B=\mu_0 ni, \text{ where } n=\frac{100\times 10}{2}=500\ \text{m}$$

$$\Rightarrow \quad B=\mu_0\times 500i$$

So, corresponding $H=\frac{B}{\mu_0}=ni$

$$\Rightarrow \quad H=500\times i$$

$$\Rightarrow \quad H=500\times 5.2\ \text{A}$$

$$\Rightarrow \quad H=2600\ \text{Am}^{-1}$$

Hence, the correct answer is (B).

10. Change in energy is

$$\Delta E=\frac{1}{2}LI_f^2-\frac{1}{2}LI_{in}^2$$

Since, $L\frac{dI}{dt}=25$

$$\Rightarrow \quad L\left(\frac{25-10}{1}\right)=25$$

CHAPTER 3

$$\Rightarrow \quad L = \frac{25}{15} = \frac{5}{3}\ \text{H}$$

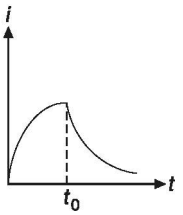
$$\Rightarrow \quad \Delta E = \frac{1}{2}\left(\frac{5}{3}\right)(625 - 100)$$

$$\Rightarrow \quad \Delta E = 437.5\ \text{J}$$

Hence, the correct answer is (A).

11. $i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right),\ t \le t_0$

$$\Rightarrow \quad i(t) = \frac{V}{R} e^{-\frac{R}{L}(t - t_0)},\ t > t_0$$



* No given option is correct

12. The mutual inductance M and self-inductance L are given by

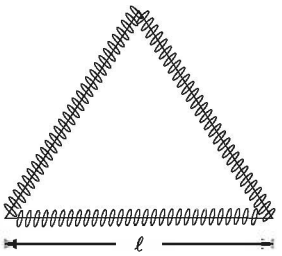
$$M = \mu_0 n_1 n_2 \pi r_1^2 l$$

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\Rightarrow \quad \frac{M}{L} = \frac{n_2}{n_1}$$

Hence, the correct answer is (B).

13.



$\phi = LI = (\mu_0 nl) A (n)(3l)$

where n is the number of turns/length

$$\Rightarrow \quad L \propto l$$

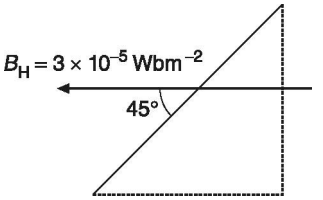
Hence, the correct answer is (A).

14. After long time, the inductor will behave like a wire.

$$I = \frac{15}{\left(\frac{R}{2}\right)} = \frac{30}{5} = 6\ \text{A}$$

Hence, the correct answer is (C).

15. $B_H = 3 \times 10^{-5}\ \text{Wbm}^{-2}$



$$\xi = Blv = 5 \times 3 \times 10^{-5} \times 10 \times \frac{1}{\sqrt{2}} = 1.06 \times 10^{-3}\ \text{V}$$

$$\Rightarrow \quad \xi \approx 1.1 \times 10^{-3}\ \text{V}$$

Hence, the correct answer is (A).

16. Since coil is small, so B is assumed to be constant in this region.

Emf induced in smaller coil is given by

$$\xi = -\frac{d\phi}{dt} = -BA\frac{d}{dt}[\cos(\omega t)]$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I}{2R}\omega\pi r^2 \sin\omega t$$

Hence, the correct answer is (D).

17. Copper rod will acquire terminal velocity when the magnetic force equals the gravitation force.

$$\Rightarrow \quad I\ell B = mg\sin\theta \qquad \ldots(1)$$

$$\text{Also, } I = \frac{\text{induced emf}}{R} = \frac{B\ell v}{R} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{B^2\ell^2 v}{R} = mg\sin\theta$$

$$\Rightarrow \quad v = \frac{mgR\sin\theta}{B^2\ell^2}$$

Hence, the correct answer is (A).

18. EMF induced in the coil is

$$\xi = N_n BA\omega\sin(\omega t)$$

$$\Rightarrow \quad \xi_{\max} = N_n BA\omega$$

Hence, the correct answer is (B).

19. Since $\Delta q = \frac{\Delta\phi}{R}$

$$\Rightarrow \quad \Delta\phi = R\Delta q = R(\text{Area under } I\text{-}t \text{ graph})$$

$$\Rightarrow \quad \Delta\phi = R\left(\frac{1}{2} \times 10 \times 0.5\right) = 100\left(\frac{1}{2}(10)(0.5)\right)$$

$$\Rightarrow \quad \Delta\phi = 250\ \text{Wb}$$

Hence, the correct answer is (C).

20. The induced emf, $\xi = -M\frac{dI}{dt}$ ...(1)

where mutual inductance M is given by

$$M = \frac{\mu_0 \pi a^2}{2b}$$

Since, $I = I_0 \cos(\omega t)$

$$\Rightarrow \quad \xi = \frac{-\mu_0 \pi a^2}{2b}\frac{d}{dt}\left[I_0 \cos(\omega t)\right] = \frac{\mu_0 \pi a^2}{2b} I_0 \omega \sin(\omega t)$$

$$\Rightarrow \quad \xi = \frac{\pi \mu_0 I_0}{2}\frac{a^2}{b}\omega \sin(\omega t)$$

Hence, the correct answer is (C).

21. Given that, $B = B_0 e^{-\frac{t}{\tau}}$

Area of the circular loop is $A = \pi r^2$

Flux linked with the loop at any time t is

$$\phi = BA = \pi r^2 B_0 e^{-\frac{t}{\tau}}$$

Emf induced in the loop is

$$\xi = -\frac{d\phi}{dt} = \left(\pi r^2 B_0\right)\left(\frac{1}{\tau}e^{-\frac{t}{\tau}}\right)$$

Net heat generated in the loop is

$$H = \int_0^\infty \frac{\xi^2}{R}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R}\int_0^\infty e^{-\frac{2t}{\tau}}dt$$

$$\Rightarrow \quad H = \frac{\pi^2 r^4 B_0^2}{\tau^2 R} \times \frac{1}{\left(-\frac{2}{\tau}\right)}\left(e^{-\frac{2t}{\tau}}\right)\Big|_0^\infty$$

$$\Rightarrow \quad H = -\frac{\pi^2 r^4 B_0^2}{2\tau^2 R}\left[\tau(0-1)\right] = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

Hence, the correct answer is (B).

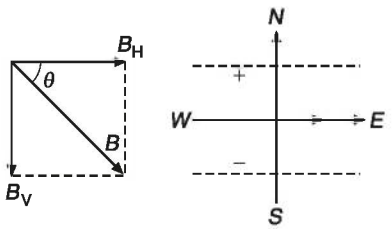
22. Given that, Length of the plane is $\ell = 20$ m

Wing span is $\ell' = 15$ m

Height of plane is $h = 5$ m

Velocity of plane is $v = 240\ \text{ms}^{-1}$, towards east

$$\sin\theta = \frac{2}{3},\ B = 5\times10^{-5}\ \text{T},\ V_B = ?,\ V_W = ?$$



If V_B be the voltage developed between the lower and upper side of the plane, then

$$V_B = vhB\cos\theta$$

$$\Rightarrow \quad V_B = 240\times5\times5\times10^{-5}\times\frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad V_B = 44.72\times10^{-3}\ \text{V} \approx 45\ \text{mV}$$

The vertical component of earth's magnetic field is

$$B_V = B\sin\theta$$

$$\Rightarrow \quad B_V = 5\times10^{-5}\times\frac{2}{3} = \frac{1}{3}\times10^{-4}\ \text{T}$$

If V_W is the voltage developed between tips of the wings, then

$$V_W = B_V \ell' V = \frac{1}{3}\times10^{-4}\times15\times240 = 1200\times10^{-4}$$

$$\Rightarrow \quad V_W = 120\ \text{mV}$$

Hence, the correct answer is (D).

23. When key K_1 is kept closed, a steady current $I_0\left(=\frac{E}{R}\right)$ flows through the circuit.

When K_1 is opened and K_2 is closed, current at any time t in the circuit is

$$I = I_0 e^{-\frac{t}{\tau}} = \frac{E}{R}e^{-\frac{tR}{L}} \qquad \left(\because\ \tau = \frac{L}{R}\right)$$

Given that, $\xi = 15$ V, $R = 0.15\ \text{k}\Omega = 150\ \Omega$

$$L = 0.03\ \text{H},\ t = 1\ \text{ms} = 10^{-3}\ \text{s}$$

$$\Rightarrow \quad I = \frac{15}{150}e^{-\left(\frac{10^{-3}\times150}{0.03}\right)} = \frac{e^{-5}}{10} = \frac{1}{10e^5} = \frac{1}{10\times150}$$

$$\Rightarrow \quad I = 6.67\times10^{-4}\ \text{A} = 0.67\ \text{mA}$$

Hence, the correct answer is (B).

24. At any time t, the equation of the given circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad \text{...(1)}$$

which is equivalent to that of a damped pendulum.

The solution to this equation (1) is

$$q = Q_0 e^{-\frac{Rt}{2L}}\cos\left(\omega' t + \phi\right)$$

where, $\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

The square of maximum charge on capacitor at any time t is

$$Q_{\max}^2 = Q_0^2 e^{-\frac{Rt}{L}}\cos^2\left(\omega' t + \phi\right)$$

CHAPTER 3

It decays exponentially with time

For $L_2 < L_1$, the curve is more steep

Hence, the correct answer is (C).

25. $I_1 = 5$ A, $I_2 = 2$ A

$\Rightarrow \quad \Delta I = 2 - 5 = -3$ A

Also, $\Delta t = 0.1$ s and $\xi = 50$ V

Since, $\xi = -L\dfrac{\Delta I}{\Delta t}$

$\Rightarrow \quad 50 = -L\left(\dfrac{-3}{0.1}\right)$

$\Rightarrow \quad 50 = 30L$

$\Rightarrow \quad L = \dfrac{5}{3} = 1.67$ H

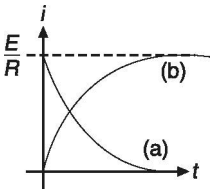
Hence, the correct answer is (B).

26. For RC circuit, we have

$$i = \frac{E}{R}e^{-\frac{t}{RC}}$$

For RL circuit, we have

$$i = \frac{E}{R}\left(1 - e^{-t\left(\frac{L}{R}\right)}\right)$$



Hence, the correct answer is (A).

27. Initially current in the circuit is I_0

After time t, current falls to new value given by

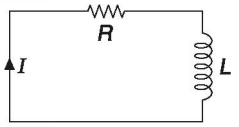
$$I = I_0 e^{\left(-\frac{t}{\tau}\right)}$$

So, voltage drop across the resistance is

$$V_R = IR = V_0 e^{-\frac{t}{\tau}} \qquad \ldots(1)$$

Voltage across the inductor is

$$V_L = L\frac{dI}{dt} = L\left[-\frac{I_0}{\tau}e^{\left(-\frac{t}{\tau}\right)}\right]$$



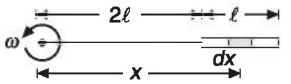
$$\Rightarrow \quad V_L = -I_0 R e^{-\frac{t}{\tau}} = -V_0 e^{-\frac{t}{\tau}} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$\frac{V_R}{V_L} = -1$$

Hence, the correct answer is (D).

28. Consider an element of length dx at a distance x from the fixed end of the string as shown in figure.



Induced emf in the element is

$$d\xi = B(\omega x)dx$$

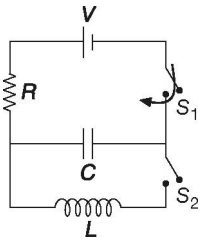
Hence, the emf induced across the ends of the rod is

$$\xi = \int_{2\ell}^{3\ell} B\omega x dx = B\omega\left(\frac{x^2}{2}\right)\Big|_{2\ell}^{3\ell} = \frac{B\omega}{2}\left[(3\ell)^2 - (2\ell)^2\right]$$

$$\Rightarrow \quad \xi = \frac{5B\omega\ell^2}{2}$$

Hence, the correct answer is (A).

29. As switch S_1 is closed and switch S_2 is kept open. Now, capacitor is charging through a resistor R.



Charge on a capacitor at any time t is

$$q = q_0\left(1 - e^{-\frac{t}{\tau}}\right), \text{ where } q_0 = CV \text{ and } \tau = RC$$

$$\Rightarrow \quad q = CV\left(1 - e^{-\frac{t}{\tau}}\right) \qquad [\text{As } q_0 = CV]$$

At $t = \dfrac{\tau}{2}$, we have

$$q = CV\left(1 - e^{-\frac{\tau}{2\tau}}\right) = CV\left(1 - e^{-\frac{1}{2}}\right)$$

At $t = \tau$, we have

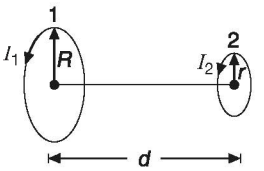
$$q = CV\left(1 - e^{-\frac{\tau}{\tau}}\right) = CV\left(1 - e^{-1}\right)$$

At $t = 2\tau$, we have

$$q = CV\left(1 - e^{-\frac{2\tau}{\tau}}\right) = CV\left(1 - e^{-2}\right)$$

Hence, the correct answer is (D).

30.



As field due to current loop 1 at an axial point

$$\Rightarrow \quad B_1 = \frac{\mu_0 I_1 R^2}{2(d^2 + R^2)^{3/2}}$$

Flux linked with smaller loop 2 due to B_1 is

$$\phi_2 = B_1 A_2 = \frac{\mu_0 I_1 R^2}{2(d^2+R^2)^{3/2}}\pi r^2$$

The coefficient of mutual inductance between the loops is

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0 R^2 \pi r^2}{2(d^2+R^2)^{3/2}}$$

Flux linked with bigger loop 1 is

$$\phi_1 = MI_2 = \frac{\mu_0 R^2 \pi r^2 I_2}{2(d^2+R^2)^{3/2}}$$

Substituting the given values, we get

$$\phi_1 = \frac{4\pi\times10^{-7}(20\times10^{-2})^2\times\pi\times(0.3\times10^{-2})^2\times2}{2\left[(15\times10^{-2})^2+(20\times10^{-2})^2\right]^{3/2}}$$

$\Rightarrow \quad \phi_1 = 9.1\times10^{-11}$ weber

Hence, the correct answer is (B).

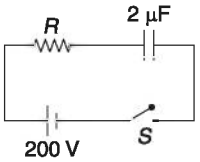
31. Given that $B_H = 5.0\times10^{-5}\ \text{NA}^{-1}\text{m}^{-1}$, $\ell = 2$ m and $v = 1.5\ \text{ms}^{-1}$

Induced emf, $\xi = B_H \ell v = (5\times10^{-5})(2)(1.50)$

$\Rightarrow \quad \xi = 15\times10^{-5}\ \text{V} = 0.15\ \text{mV}$

Hence, the correct answer is (D).

32.



In the case charging of capacitor through the resistance, we have

$$V = V_0\left(1-e^{-\frac{t}{RC}}\right)$$

where, $V = 120$ V, $V_0 = 200$ V, $R = ?$, $C = 2\ \mu$F and $t = 5$ s.

$$\Rightarrow \quad 120 = 200\left(1-e^{-\frac{5}{R}\times2\times10^{-6}}\right)$$

$$\Rightarrow \quad e^{-\frac{5}{R}\times2\times10^{-6}} = \frac{80}{200}$$

Taking the natural logarithm on both sides, we get

$$\frac{-5}{R\times2\times10^{-6}} = \ln(0.4) = -0.916$$

$\Rightarrow \quad R = 2.7\times10^{6}\ \Omega$

Hence, the correct answer is (C).

33. Charge on the capacitor at any instant t is

$$q = q_0\cos\omega_0 t \qquad \ldots(1)$$

Equal sharing of energy means that energy stored in the capacitor is half the total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right)$$

$$\Rightarrow \quad q = \frac{q_0}{\sqrt{2}}$$

From equation (1)

$$\frac{q_0}{\sqrt{2}} = q_0\cos\omega_0 t \text{ , where } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \cos\omega_0 t = \frac{1}{\sqrt{2}}$$

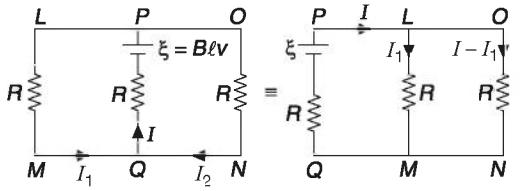
$$\Rightarrow \quad \omega_0 t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega_0} = \frac{\pi}{4}\sqrt{LC}$$

Hence, the correct answer is (B).

34. Emf induced across PQ is $\xi = B\ell v$.

The equivalent circuit diagram is as shown in Figure.



So $I - I_1 = I_1$

$$\Rightarrow \quad I_1 = \frac{I}{2}$$

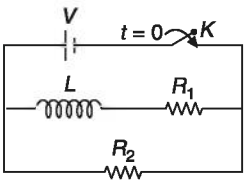
Now $I = \dfrac{\xi}{R_{\text{net}}} = \dfrac{Blv}{R+\dfrac{(R)(R)}{R+R}} = \dfrac{Blv}{3R/2}$

$$\Rightarrow \quad I = \frac{2Blv}{3R}$$

$$\Rightarrow \quad I_1 = \frac{Blv}{3R} \text{ and } I - I_1 = \frac{Blv}{3R}$$

Hence, the correct answer is (C).

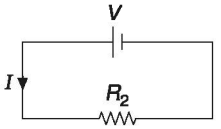
35.



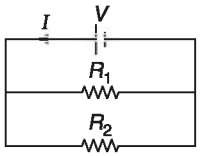
A time $t = 0$, the inductor acts as an open circuit. The corresponding equivalent circuit diagram is as shown in Figure.

The current through battery is

$$I = \frac{V}{R_2}$$



At time $t = \infty$, the inductor acts as a short circuit. The corresponding equivalent circuit diagram is as shown in Figure.



So, the current through the battery is

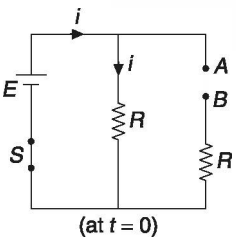
$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1R_2}{R_1 + R_2}}$$

$$\Rightarrow \quad I = \frac{V(R_1 + R_2)}{R_1R_2}$$

Hence, the correct answer is (C).

36. Equivalent time constant of circuit is

$$\tau = \frac{L}{R_{AB}} = \frac{L}{R} \qquad \{\because R_{AB} = R\}$$



Current in the circuit at $t = 0$ is $i = \frac{E}{R}$

So, open circuit voltage across AB is

$$V_{AB} = iR = E$$

Hence current through inductor is

$$i_L = \frac{V_{AB}}{R_{AB}}\left(1 - e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow \quad i_L = \frac{12}{2}\left(1 - e^{-\frac{2t}{0.4}}\right) = 6\left(1 - e^{-5t}\right)$$

$$V_L = L\frac{di_L}{dt}$$

$$\Rightarrow \quad V_L = 0.4 \times 6 \times 5e^{-5t}$$

$$\Rightarrow \quad V_L = 12e^{-5t}$$

Hence, the correct answer is (D).

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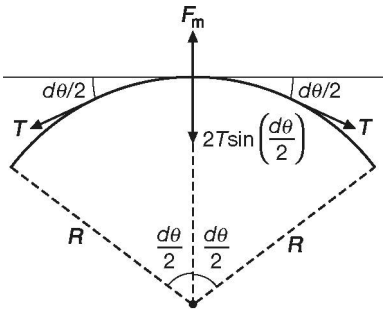
Single Correct Choice Type Problems

1. The induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

Hence, the correct answer is (C).

2.



$$L = 2\pi R$$

$$\Rightarrow \quad R = \frac{L}{2\pi}$$

$$2T\sin\left(\frac{d\theta}{2}\right) = F_m = BI(dl)$$

For small angles, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$2T\left(\frac{d\theta}{2}\right) = BIdl$$

Since $dl = Rd\theta$

$$\Rightarrow \quad Td\theta = BI(Rd\theta)$$

$$\Rightarrow \quad T = BIR$$

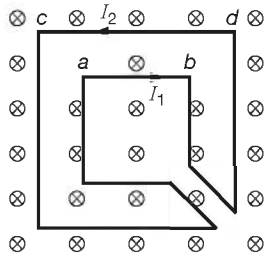
$$\Rightarrow \quad T = BI\left(\frac{L}{2\pi}\right) = \frac{IBL}{2\pi}$$

Hence, the correct answer is (C).

3. Current $I_1 = I_2$

Since magnetic field increases with time

So induced net flux should be outward i.e., current will flow from a to b.



Hence, the correct answer is (D).

4. In uniform magnetic field, change in magnetic flux is zero. Therefore, induced current will be zero.

Hence, the correct answer is (C).

5. Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (B) polarity is changing.

Hence, the correct answer is (B).

6. Power $P=\frac{\xi^2}{R}$

where, ξ is the induced emf given by

$$\xi=-\left(\frac{d\phi}{dt}\right) \text{ and } \phi=NBA$$

$$\Rightarrow \quad \xi=-NA\left(\frac{dB}{dt}\right)$$

The resistance of wire R is

$$R=\frac{\rho l}{A}$$

$$\Rightarrow \quad R\propto\frac{l}{r^2}$$

where, r is radius, ℓ is length of wire.

$$\Rightarrow \quad P\propto\frac{N^2r^2}{l}$$

$$\Rightarrow \quad \frac{P_1}{P_2}=1$$

Hence, the correct answer is (B).

8. Electric field will be induced in both AD and BC.

Hence, the correct answer is (D).

9. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

Hence, the correct answer is (A).

10. Let L be the inductance of coil and R be its resistance. Then current in the coil $I_1(t)$ at time t is given by

$$I_1(t)=I_0\left(1-e^{-\frac{Rt}{L}}\right)=\frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad I_1(t)=k_1\left(1-e^{-k_2t}\right) \text{ where } k_1=\frac{E}{R} \text{ and } k_2=\frac{R}{L}$$

The magnetic field $B(t)$ at the axis of coil is

$$B(t)=\mu_0nI_1(t)$$

$$\Rightarrow \quad B(t)=\mu_0n\left[\frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)\right]$$

$$\Rightarrow \quad B(t)=\frac{\mu_0nE}{R}\left[1-e^{-\frac{Rt}{L}}\right]$$

Assuming $k_3=\frac{\mu_0nE}{R}$, we get

$$B(t)=k_3\left(1-e^{-k_2t}\right) \qquad \ldots(1)$$

If $I_2(t)$ be the current induced in the ring, then

$$I_2(t)=\left|-M\left(\frac{dI_1}{dt}\right)\right|=M\frac{dI_1}{dt}$$

$$\Rightarrow \quad I_2(t)=Mk_1k_2e^{-k_2t}$$

$$\Rightarrow \quad I_2(t)=k_4e^{-k_2t} \qquad \ldots(2)$$

$$\Rightarrow \quad I_2(t)B(t)=k_3k_4\left(1-e^{-k_2t}\right)e^{-k_2t}$$

$$\Rightarrow \quad I_2(t)B(t)\propto\left(1-e^{-k_2t}\right)\left(e^{-k_2t}\right) \qquad \ldots(3)$$

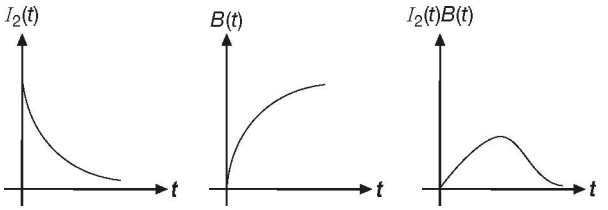
The product, $I_2(t)\ B(t)$ is zero at $t=0$ as well as at $t\to\infty$

So, this product should pass through a maximum value.

Interestingly this product is maximum when

$$e^{-k_2t}=\frac{1}{2} \qquad \text{{try yourself}}$$

The corresponding graphs will be as shown.



Hence, the correct answer is (D).

CHAPTER 3

11. $\oint \vec{E}\cdot d\vec{r} = (2\pi r)E = \xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad 2\pi rE = \pi a^2 - \dfrac{dB}{dt}$

$\Rightarrow \quad E = -\dfrac{1}{2}\dfrac{a^2}{r}\dfrac{dB}{dt}$

Hence, the correct answer is (B).

12. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So, as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.

Hence, the correct answer is (D).

13. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$, where

$I_0 = \dfrac{12}{6}\text{ A} = 2\text{ A}$

$\Rightarrow \quad 1 = 2\left(1 - e^{-\frac{6t}{8.4\times10^{-3}}}\right)$

$\Rightarrow \quad e^{-\frac{6t}{8.4\times10^{-3}}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{6t}{8.4\times10^{-3}} = \ln 2$

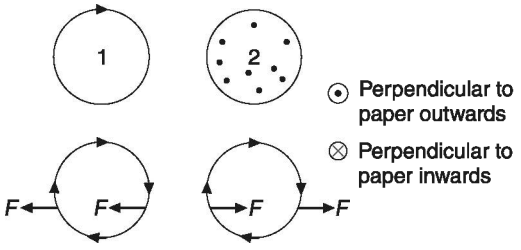
$\Rightarrow \quad t = (1.4\times0.693)\text{ ms}$

$\Rightarrow \quad t = 0.97\text{ ms}$

Hence, the correct answer is (D).

14. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce • magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown in Figure.

The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in the opposite directions.



Hence, the correct answer is (C).

15. $B = \dfrac{2\sqrt{2}\mu_0 I}{\pi L}$

The magnetic flux ϕ_{12} linking big loop with the small square loop of side $l(<<L)$ is

$$\phi_{12} = Bl^2 = \frac{2\sqrt{2}\mu_0 I}{\mu}\left(\frac{l^2}{L}\right)$$

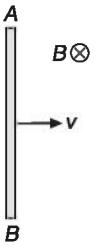
Mutual Inductance is $M_{12} = \dfrac{\phi_{12}}{\text{I}}$

$\Rightarrow \quad M_{12} = \dfrac{2\sqrt{2}\mu_0}{\pi}\left(\dfrac{l^2}{L}\right)$

$\Rightarrow \quad M \propto \dfrac{l^2}{L}$

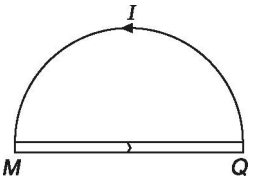
Hence, the correct answer is (B).

16. A motional emf, $\xi = B/v$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



Hence, the correct answer is (B).

17. The induced emf across a conductor does not depend upon its shape but only at the extreme (end) points. As a simple case just replace the actual conductor by an imaginary straight conductor placed in between the end points. Then induced emf is



$\xi = Blv = B(2R)V$

$\Rightarrow \quad \xi = 2BVR$

The direction of induced current in this imaginary conductor MQ is from M to Q. So, in the loop the current flows from Q to N to M thus indicating that Q is at a higher potential.

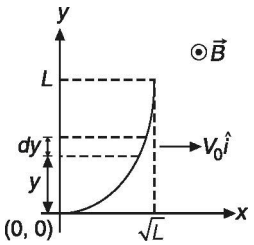
Hence, the correct answer is (D).

18. Since the flux linked with the loop does not change. Hence the induced current is zero.
Hence, the correct answer is (D).

Multiple Correct Choice Type Problems

1. Given that $y = x^2$, $V = V_0\hat{i}$ and $B = B_0\left(1+\left(\frac{y}{2}\right)^{\beta}\right)\hat{k}$

Also, the end points of the conducting wire are $(0, 0)$ and $(\sqrt{L}, L)$. Let us consider an infinitesimal element of length dy at a distance y from the x-axis as shown.



If $d\phi$ be the emf induced across this element, then

$$d\phi = V_0 B\,dy$$

$$\Rightarrow\quad d\phi = V_0 B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)dy$$

$$\Delta\phi = V_0 B_0\left[\left(y+\frac{y^{\beta+1}}{(\beta+1)L^{\beta}}\right)\right]_0^L$$

$$\Rightarrow\quad \Delta\phi = V_0 B_0\left[L+\frac{L^{\beta+1}}{(\beta+1)L^{\beta}}\right]$$

$$\Rightarrow\quad \Delta\phi = B_0 L V_0\left(\frac{\beta+2}{\beta+1}\right)$$

$$\Rightarrow\quad \Delta\phi \propto L$$

When $\beta = 2$, then $\Delta\phi = \frac{4}{3}B_0 L V_0$

When $\beta = 0$, then $\Delta\phi = 2B_0 L V_0$

Hence, (B), (C) and (D) are correct.

2. The net magnetic flux through the loops at time t is

$$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$$

$$\Rightarrow\quad \left|\frac{d\phi}{dt}\right| = B\omega A\sin\omega t$$

So, $\left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \frac{\pi}{2}$

The emf induced in the smaller loop,

$$\xi_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A\sin\omega t$$

So, amplitude of maximum net emf induced in both the loops is equal to amplitude of maximum emf induced in the smaller loop alone.
Hence, (B) and (D) are correct.

3. For right edge of loop from $x = 0$ to $x = L$

$$i = +\frac{BLv}{R}$$

Since, $F = iLB = \frac{vB^2L^2}{R}$ (leftwards)

$$\Rightarrow\quad -mv\frac{dv}{dx} = \frac{vB^2L^2}{R}$$

$$\Rightarrow\quad v(x) = v_0 - \frac{B^2L^2}{mR}x$$

$$\Rightarrow\quad i(x) = \frac{v_0 BL}{R} - \frac{B^3L^3}{mR^2}x \text{ and}$$

$$F(x) = \frac{v_0 B^2L^2}{R} - \frac{B^4L^4}{mR^2}x \text{ (leftwards)}$$

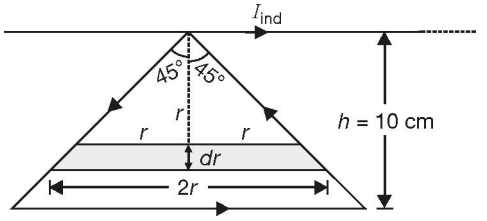
Hence, (C) and (D) are correct.

4. $$\phi = \int_0^h \frac{\mu_0 I}{2\pi r}2r\,dr = \frac{\mu_0 Ih}{\pi}$$

$$\Rightarrow\quad M = \frac{\phi}{I} = \frac{\mu_0 h}{\pi}$$

Now, $\xi = \left|M\frac{dI}{dt}\right| = \frac{\mu_0 h}{\pi}(10)$

$$\Rightarrow\quad \xi = \frac{\mu_0}{\pi}\left(\frac{10}{100}\right)(10) = \frac{\mu_0}{\pi}\text{ volt}$$



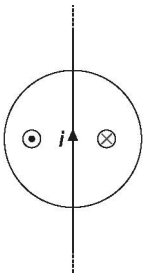
Since, current in triangular loop is increasing at rate of 10 As^{-1}, due to which the outward field also increases. According to Lenz's Law, induced current in conductor should produce an inward field at the triangular loop, so I_{ind} in the conductor must be rightwards due to which a repulsive force acts between the wire and

CHAPTER 3

the loop. Further, due to rotation of the loop there will not be any change in flux through the loop, so there is no extra emf induced in the wire.

Hence, (A) and (D) are correct.

5. Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because in half of the circle, magnetic field is inwards and in other half, magnetic field is outwards.

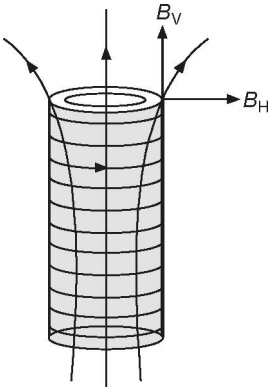


Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.

Hence, (A) and (C) are correct.

6. The horizontal component of magnetic field due to solenoid will exert force on ring in vertical direction

$$F = B_H I(2\pi r)$$



Since $F\Delta t = MV$

$$\Rightarrow \quad I = \frac{\left(\frac{\Delta\phi}{\Delta t}\right)}{\left(\rho\frac{(2\pi r)}{A}\right)}$$

$$B_H I(2\pi r)\Delta t = MV$$

$$V = \frac{B_H \Delta\phi A}{\rho M} = \frac{K}{\rho M}$$

$$h = \frac{V^2}{2g} = \frac{K^2}{\rho^2 M^2}$$

$$h_A > h_B$$

$$\frac{K^2}{\rho_A^2 M_A^2} > \frac{K^2}{\rho_B^2 M_B^2}$$

$$\Rightarrow \quad \rho_B M_B > \rho_A M_A$$

So, we get (B) and (D) as the correct options.

Hence, (B) and (D) are correct.

7. Electrostatic and gravitational field do not make closed loops.

Hence, (B) and (D) are correct.

8. Since $v = L\frac{di}{dt}$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{L_1\left(\frac{di_1}{dt}\right)}{L_2\left(\frac{di_2}{dt}\right)} = \frac{L_1}{L_2} = \frac{8}{2} = 4$$

$$\Rightarrow \quad \frac{v_2}{v_1} = \frac{1}{4}$$

Also, $L_1 i_1 = L_2 i_2$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4} \text{ and}$$

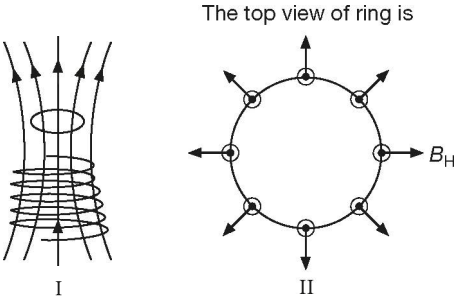
$$\frac{w_2}{w_1} = \frac{L_2 i_2^2}{L_1 i_1^2} = \left(\frac{2}{8}\right)(16) = 4$$

Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

1. The arrow $\rightarrow$ represents horizontal component of magnetic field

$\odot$ Represents vertical component of magnetic field



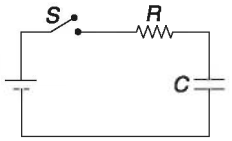
From diagram it is clear that horizontal component of magnetic field will interact with induced current. The resulting magnetic force will act opposite to weight of ring.

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. When switch is closed for a very long-time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor, $E_C = \frac{1}{2}CV^2$ ...(1)



Work done by a battery, $W = qV = (CV)V = CV^2$

Energy dissipated across resistance is

$$E_D = \left(\begin{matrix}\text{Work done}\\ \text{by Battery}\end{matrix}\right) - \left(\begin{matrix}\text{Energy}\\ \text{Stored}\end{matrix}\right)$$

$$\Rightarrow \quad E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2 \qquad ...(2)$$

From equations (1) and (2)

$$E_D = E_C$$

Hence, the correct answer is (B).

2. When a capacitor is charged from v_i to v_f, heat dissipated is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = (\Delta q)V_f - \Delta U_C$$

$$\Rightarrow \quad \Delta H = C(V_f - V_i)V_f - \frac{1}{2}C\left(V_f^2 - V_i^2\right) = \frac{1}{2}C(V_f - V_i)^2$$

So, total heat dissipated is

$$\Delta H = \frac{1}{2}C\left[\left(\frac{V_0}{3} - 0\right)^2 + \left(\frac{2V_0}{3} - \frac{V_0}{3}\right)^2 + \left(V_0 - \frac{2V_0}{3}\right)^2\right]$$

$$\Rightarrow \quad \Delta H = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

Hence, the correct answer is (A).

3. The induced electric field is given by,

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad El = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E(2\pi R) = -(\pi R^2)(B)$$

$$\Rightarrow \quad E = -\frac{BR}{2}$$

Hence, the correct answer is (B).

4. Since the Gyro-Magnetic Ratio is

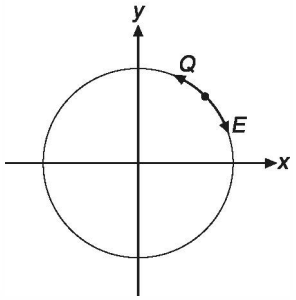
$$\frac{|\vec{M}|}{|\vec{L}|} = \frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}| = \left(\frac{Q}{2m}\right)|\vec{L}|$$

$$\Rightarrow \quad |\vec{M}| \propto |\vec{L}|, \text{ where } \gamma = \frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}| = \left(\frac{Q}{2m}\right)(I\omega) = \left(\frac{Q}{2m}\right)(mR^2\omega)$$

$$\Rightarrow \quad |\vec{M}| = \frac{Q\omega R^2}{2}$$



Since induced electric field is opposing in nature, so

$$\omega' = \omega - \alpha t$$

where, $\alpha = \frac{\tau}{I} = \frac{(QE)R}{mR^2}$

$$\Rightarrow \quad \alpha = \frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2} = \frac{QB}{2m}$$

Since we have $t = 1$ s (see paragraph)

$$\Rightarrow \quad \omega' = \omega - \left(\frac{QB}{2m}\right)1 = \omega - \frac{QB}{2m}$$

$$\Rightarrow \quad M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\Rightarrow \quad \Delta M = M_f - M_i = -\frac{Q^2BR^2}{4m}$$

$$\Rightarrow \quad \Delta M = -\gamma\frac{QBR^2}{2} \qquad \left\{\because \gamma = \frac{Q}{2m}\right\}$$

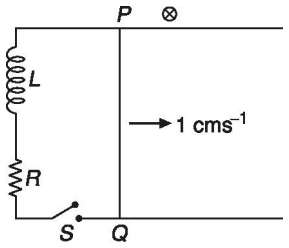
Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p)
B → (p, q, s)
C → (q, s)
D → (q, r, s)

Integer/Numerical Answer Type Questions

1.



Since $B = 1\ \text{T}$, $l = 10\ \text{cm}$, $v = 1\ \text{cms}^{-1}$

$$\Rightarrow \quad \xi = Blv = (1)\left(\frac{1}{10}\right)\left(\frac{1}{100}\right) = 1\times10^{-3}\ \text{V}$$

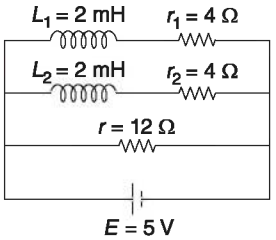
Since, $\tau = \dfrac{L}{R} = \dfrac{1\times10^{-3}}{1} = 10^{-3}\ \text{sec}$

$$\Rightarrow \quad I = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad I = \frac{10^{-3}}{1}\left(1 - e^{-1}\right)$$

$$\Rightarrow \quad I = 10^{-3}(1 - 0.37) = 0.63\ \text{mA}$$

2.



Initially i.e. at $t = 0$ current in the circuit is maximum, so

$$I_{\max} = \frac{E}{R} = \frac{5}{12}\ \text{A}$$

Long time after i.e. in steady state, current in the circuit is minimum, so

$$I_{\min} = \frac{E}{R_{eq}} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right)$$

$$\Rightarrow \quad I_{\min} = 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3}\ \text{A}$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = 8$$

3. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + z^2\right)^{\frac{3}{2}}}$$

Substituting the value of $z = \sqrt{3}R$, we get

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi = NBA\cos\theta = (2)\left(\frac{\mu_0 I}{16R}\right)a^2\cos45°$$

Mutual inductance is given by

$$M = \frac{\phi}{I} = \frac{\mu_0 a^2}{2^{\frac{7}{2}}R}$$

$$\Rightarrow \quad p = 7$$

4. Flux through the ring is

$$\phi = BA = (\mu_0 nI)(\pi r^2)$$

Since $n = \dfrac{1}{l}$ and $I = I_0\cos(300t)$

$$\Rightarrow \quad \phi = \frac{\mu_0}{L}(\pi r^2)I_0\cos(300t)$$

EMF induced in the loop is

$$\xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = \frac{300\mu_0 I_0(\pi r^2)}{l}\sin(300t)$$

Induced current in the loop is

$$i = \frac{\xi}{R} = \frac{300\mu_0 I_0(\pi r^2)}{lR}\sin(300t)$$

Magnetic moment of the loop is

$$M = iA = i(\pi r^2)$$

$$\Rightarrow \quad M = \frac{300\mu_0 I_0\pi^2 r^4}{lR}\sin(300t)$$

Comparing with $M = N\mu_0 I_0\sin(300t)$, we get

$$N = \frac{300\pi^2 r^4}{lR}$$

Taking $\pi^2 \approx 10$, we get

$$N = \frac{(300)(10)(0.1)^4}{(0.005)(10)}$$

$$\Rightarrow \quad N = \frac{(300)(10)(10^{-4})}{5\times10^{-2}}$$

$$\Rightarrow \quad N = \frac{3\times10^{-1}}{5\times10^{-2}} = \frac{30}{5} = 6$$

CHAPTER 4: ALTERNATING CURRENTS

Test Your Concepts-I (Based on AC)

1. For an ac, $I = I_0 \sin(\omega t)$

Therefore, instantaneous value of heat produced in time dt across a resistance R is,

$$dH = I^2 R dt = I_0^2 R \sin^2(\omega t)\, dt$$

Average value of heat produced during a cycle

$$H_{av} = \frac{\int_0^T dH}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} \left(I_0^2 R \sin^2(\omega t)\right) dt}{\int_0^{2\pi/\omega} dt}$$

$$\Rightarrow \quad H_{av} = \frac{I_0^2}{2} R\left(\frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad H_{av} = I_{\text{rms}}^2 RT$$

i.e., ac produces same heating effects as produced by dc of value $I = I_{\text{rms}}$

2. Since hot wire voltmeter reads only rms value we will have to find rms value of the given voltage. Considering one complete cycle,

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 d\theta} \text{ where } \theta = \omega t$$

$$V_{\text{rms}}^2 = \frac{2}{2\pi}\int_0^{2\pi} \left(200\sin\theta + 100\sin(3\theta) + 50\sin(5\theta)\right)^2 d\theta$$

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{2\pi} \left(200^2\sin^2\theta + 100^2\sin^2 3\theta + 50^2\sin^2 5\theta\right) d\theta$$

The integral of all cross terms comes to be zero over one complete cycle. So,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\left(\frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2}\right) 2\pi = 26,250$$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{26250} = 162 \text{ V}$$

3. By definition, $I_{\text{rms}}^2 = \dfrac{\int_0^T I^2 dt}{T}$

$$\Rightarrow \quad I_{\text{rms}}^2 = \frac{\int_0^2 I^2 dt}{2} = \frac{1}{2}\int_0^2 (kt)^2\, dt = \frac{k^2}{2}\left(\frac{t^3}{3}\right)_0^2 = \frac{4k^2}{3}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{2k}{\sqrt{3}}$$

4. For the given time function as shown in problem, the RMS value of the voltage is given by

$$E_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} E_0^2 dt}$$

$$\Rightarrow \quad V_{\text{rms}} = E_0\sqrt{\frac{1}{T}\left(\frac{T}{2}\right)}$$

$$\Rightarrow \quad E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

5. $V_{\text{rms}} = \sqrt{\dfrac{\int_0^T V^2 dt}{T}} = \sqrt{\langle V^2 \rangle}$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{\left\langle V_0^2 + V_1^2\cos^2\omega t + 2V_0V_1\cos\omega t \right\rangle}$$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{\langle V_0^2\rangle + V_1^2\langle\cos^2\omega t\rangle + 2V_0V_1\langle\cos\omega t\rangle}$$

Since, $\langle\cos\omega t\rangle = 0$ and $\langle\cos^2\omega t\rangle = \dfrac{1}{2}$ for one cycle

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{V_0^2 + \frac{V_1^2}{2}}$$

6. (a) $I = \dfrac{V_R}{R} = \dfrac{(2\text{ V})\sin\left((10^3\text{ rads}^{-1})t\right)}{100}$

$$I = (2\times10^{-2}\text{ A})\sin\left((10^3\text{ rads}^{-1})t\right)$$

(b) $X_L = L\omega = (4\text{ H})(10^3\text{ rads}^{-1})$

$$\Rightarrow \quad X_L = 4\times10^3\ \Omega$$

(c) The amplitude of voltage across inductor is

$$V_0 = I_0X_L = (2\times10^{-2}\text{ A})(4\times10^3\ \Omega)$$

$$\Rightarrow \quad V_0 = 80\text{ V}$$

For an ac input, voltage across the inductor leads the current by $90°$ or $\frac{\pi}{2}$ rad. Hence

$$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_L = 80\sin\left\{(10^3 \text{ rads}^{-1})t + \frac{\pi}{2}\right\} \text{V}$$

7. Please remember that the designated or assigned values are the rms values. So, $V_{\text{rms}} = 200$ V.

Since, in case of an ac, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

Here, $X_L = \omega L = (2\pi fL)$

$$X_L = (2\pi)(50)(0.01) = \pi\,\Omega$$

and $R = 1\,\Omega$

$$\Rightarrow \quad \phi = \tan^{-1}(\pi) = \tan^{-1}(3.14) \approx 72.3°$$

Further, $I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|}$

$$\Rightarrow \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_L^2}}$$

Substituting the values, we get

$$I_{\text{rms}} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}}$$

$$\Rightarrow \quad I_{\text{rms}} = 60.67 \text{ A}$$

8. (a) $T = \frac{1}{f} = \frac{1}{60}$ s

$$\omega = 2\pi f = 377 \text{ rads}^{-1}$$

$$X_L = L\omega = (377)(0.04) = 15.08\,\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = 25.05\,\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(0.754)$$

$$\Rightarrow \quad \phi = 37°$$

(b) Amplitudes (maximum value) are,

$$I_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6 \text{ A}$$

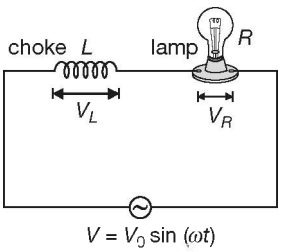
$$(V_0)_R = I_0 R = 120 \text{ V}$$

$$(V_0)_L = I_0 X_L = 90.5 \text{ V}$$

Also, we observe that $V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$

9. For lamp, $V_R = IR = 10 \times 5 = 50$ V

In series, $V^2 = V_R^2 + V_L^2$



$$\Rightarrow \quad V_L = \sqrt{V^2 - V_R^2}$$

$$\Rightarrow \quad V_L = \sqrt{(160)^2 - (50)^2}$$

$$\Rightarrow \quad V_L = 152 \text{ V}$$

Since $V_L = IX_L = 2\pi IfL$

$$\Rightarrow \quad L = \frac{V_L}{2\pi fI}$$

Substituting the values, we get

$$L = \frac{152}{(2\pi)(50)(10)} = 4.84 \times 10^{-2} \text{ H} = 48.4 \text{ mH}$$

Now, when the lamp is operated at 160 V dc and instead of choke, an additional resistance R' is put in series with it then, we have

$$V = I(R + R')$$

$$\Rightarrow \quad 160 = 10(5 + R')$$

$$\Rightarrow \quad R' = 11\,\Omega$$

In case of ac, as the choke has no resistance, power loss in choke is zero.

In case of dc, the loss in additional resistance R' is,

$$P = I^2R' = (10)^2(11) = 1100 \text{ W}$$

10. $I_0 = \frac{V_0}{R} = \frac{2.5}{300} = 8.33 \times 10^{-3}$ A

(a) $I = 8.33 \times 10^{-3} \cos\left[(950 \text{ rads}^{-1})t\right]$

(b) $X_L = L\omega = 0.8 \times 950 = 760\,\Omega$

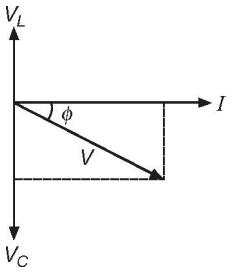
(c) $V_L = (V_L)_0 \cos\left((950 \text{ rads}^{-1})t + \frac{\pi}{2}\right)$

$$\Rightarrow \quad V_L = (8.33 \times 10^{-3} \times 760)\cos\left(950t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad V_L = (6.33 \text{ V})\cos\left[(950 \text{ rads}^{-1})t + \frac{\pi}{2}\right]$$

$$\Rightarrow \quad V_L = -(6.33 \text{ V})\sin\left((950 \text{ rads}^{-1})t\right)$$

11. $X_L = L\omega = 100\ \Omega$, $X_C = \dfrac{1}{C\omega} = 312.5\ \Omega$ and $R = 300\ \Omega$



(a) $I = \dfrac{V}{Z} = \dfrac{120}{\sqrt{(312.5-100)^2 + (300)^2}}$

$\Rightarrow \quad I = \dfrac{120}{368} = 326\ \text{mA}$

(b) Since $X_C > X_L$, so

$V_C > V_L$

$\Rightarrow \quad \tan\phi = \dfrac{X_C - X_L}{R}$

$\Rightarrow$ V lags behind I by ϕ, where

$\phi = \tan^{-1}\left(\dfrac{312.5-100}{300}\right) \cong 35.3°$

So, we have voltage lagging behind current by 35.3°

(c) $V_R = I_R = (0.326)(300) = 97.8\ \text{V}$

$V_L = IX_L = (0.326)(100) = 32.6\ \text{V}$

$V_C = IX_C = (0.326)(312.5) = 102\ \text{V}$

12. (a) $X_L = L\omega = (2)(2\pi(50)) = 200\pi = 628\ \Omega$

(b) $X_L = L\omega$

$\Rightarrow \quad 2 = L(2\pi(50))$

$\Rightarrow \quad L = 6.37\ \text{mH}$

(c) $X_C = \dfrac{1}{C\omega} = \dfrac{1}{2\pi fC} = \dfrac{1}{(314)(2\times10^{-6})} = 1.6\ \text{k}\Omega$

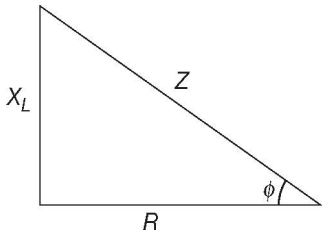
(d) $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{(314)(2)} = 1.6\ \text{mF}$

13. (a) In series, the impedance of the circuit is,

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi fL)^2}$

$\Rightarrow \quad Z = \sqrt{(220)^2 + (2\times3.14\times50\times0.7)^2}$

$\Rightarrow \quad Z = 311\ \Omega$



Since $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{220}{311} = 0.707\ \text{A}$

and $\cos\phi = \dfrac{R}{Z} = \dfrac{220}{311} = 0.707$

So, the power absorbed in the circuit is

$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$

$\Rightarrow \quad P = (220)(0.707)(0.707)\ \text{W}$

$\Rightarrow \quad P = 110.08\ \text{W}$

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, because the choke (having zero resistance) absorbs no power.

$\Rightarrow \quad P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{(220)^2}{220}$

$\Rightarrow \quad P = 220\ \text{W}$

14. Inductive reactance

$X_L = \omega L = (50)(2\pi)\left(35\times10^{-3}\right) \approx 11\ \Omega$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given $V_{rms} = 220\ \text{V}$

Hence, amplitude of voltage

$V_0 = \sqrt{2}\ V_{\text{rms}} = 220\sqrt{2}\ \text{V}$

The amplitude of current is

$I_0 = \dfrac{V_0}{Z} = \dfrac{220\sqrt{2}}{11\sqrt{2}} = 20\ \text{A}$

Phase difference $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right) = \dfrac{\pi}{4}$

In LR circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$I = (20A)\sin\left(\omega t - \dfrac{\pi}{4}\right)$

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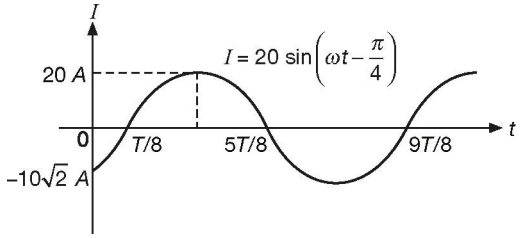
Corresponding I-t graph is shown in figure.



15. Given $R = X_L$

When the plate separation of the capacitor is halved, then C is also halved. So, $X_C = \frac{1}{C\omega}$ is also halved.

Now, the current doubles only when Z becomes half of its initial value.

$$\Rightarrow \quad Z_f = \frac{1}{2} Z_i$$

$$\Rightarrow \quad 2Z_f = Z_i$$

$$\Rightarrow \quad 4\left[R^2 + \left(R - \frac{X_C}{2}\right)^2\right] = R^2 + (R - X_C)^2$$

$$\Rightarrow \quad 4R^2 + 4\left(R - \frac{X_C}{2}\right)^2 = R^2 + (R - X_C)^2$$

$$\Rightarrow \quad 8R^2 - 4RX_C + X_C^2 = 2R^2 - 2RX_C + X_C^2$$

$$\Rightarrow \quad X_C = 3R$$

16. $L = 2\,\text{H}$, $C = 10 \times 10^{-6}\,\text{F}$, $R = 10\,\Omega$ and $V = 100\sin(\omega t)$ volt

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2)(10 \times 10^{-6})}} = 224\ \text{rads}^{-1}$$

(b) $$P = \frac{V_0^2}{2R} = \frac{(100)^2}{2(10)} = 500\ \text{W}$$

(c) Since $P = E_v I_v \cos\phi = E_{\text{rms}} I_{\text{rms}} \cos\phi$

$$\Rightarrow \quad P = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

Since, $\cos\phi = \frac{R}{Z}$ and $I_0 = \frac{E_0}{Z}$

$$\Rightarrow \quad P = \frac{E_0^2 R}{2Z^2} \qquad \ldots(1)$$

Further P is maximum, when Z is minimum i.e., at resonance. So, maximum value of P is

$$P_0 = \frac{E_0^2}{2R} \qquad \ldots(2)$$

As per the question we must have

$$P = \frac{P_0}{2}$$

$$\Rightarrow \quad \frac{E_0^2 R}{2Z^2} = \frac{E_0^2}{4R}$$

$$\Rightarrow \quad Z^2 = 2R^2$$

$$\Rightarrow \quad \left(L\omega - \frac{1}{C\omega}\right)^2 + R^2 = 2R^2$$

$$\Rightarrow \quad \left(L\omega - \frac{1}{C\omega}\right)^2 = R^2$$

$$\Rightarrow \quad (L^2C^2)\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0$$

$$\omega^2 = \frac{(2LC + R^2C^2) \pm \sqrt{(2LC + R^2C^2)^2 - 4L^2C^2}}{2L^2C^2}$$

Substituting values, we get

$$\omega^2 = 51130 \ \text{OR} \ \omega^2 = 48894$$

$$\Rightarrow \quad \omega_1 = 226\ \text{rads}^{-1} \ \text{OR} \ \omega_2 = 221\ \text{rads}^{-1}$$

17. (a) $\omega = 6280$

$$\Rightarrow \quad 2\pi f = 6280$$

$$\Rightarrow \quad 6.28 f = 6280$$

$$\Rightarrow \quad f = 1\ \text{kHz}$$

(b) $$\cos\phi = \cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

(c) $$\cos\phi = \frac{R}{Z}$$

where $Z = \frac{170}{8.5} = 20\ \Omega$

$$\Rightarrow \quad R = 20\frac{\sqrt{3}}{2} = 10\sqrt{3} = 17.32\ \Omega$$

Since this circuit has voltage lagging behind the current, so this must contain a resistance and a capacitance. Hence

$$Z^2 = \frac{1}{C^2\omega^2} + R^2$$

$$\Rightarrow \quad 400 = \frac{1}{4\pi^2 f^2 C^2} + 300$$

$$\Rightarrow \quad 100 = \frac{1}{(2\pi f C)^2}$$

$$\Rightarrow \quad 2\pi f C = 0.1$$

$$\Rightarrow \quad C = \frac{0.1}{(2)(3.14)(1000)} \cong 16\ \mu\text{F}$$

18. (a) The resonance frequency of a rejector LCR circuit or the parallel circuit is given by,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{(1.6\times10^{-2})(250\times10^{-12})}-\frac{(20)^2}{(1.6\times10^{-2})^2}}$$

$$\Rightarrow \quad f=7.96\times10^{4}\ \text{Hz}$$

(b) The circuit impedance at resonance is given by,

$$Z=\frac{L}{CR}=\frac{1.6\times10^{-2}}{(250\times10^{-12})(20)}$$

$$\Rightarrow \quad Z=3.2\times10^{6}\ \Omega$$

19. (a) $f=\dfrac{1}{2\pi\sqrt{LC}}=1000\times10^{6}\ \text{Hz}$

$$\Rightarrow \quad C=\frac{1}{4\pi^2f^2L}=\frac{1}{(4)(10)(10^{18})(400\times10^{-12})}$$

$$\Rightarrow \quad C=62.5\times10^{-12}\ \text{F}=62.5\ \text{pF}$$

(b) $C=\dfrac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad 62.5\times10^{-12}=\frac{8.85\times10^{-12}\,\ell^2}{10^{-3}}$$

$$\Rightarrow \quad \ell^2=\frac{62.5\times10^{-3}}{8.85}$$

$$\Rightarrow \quad \ell=84\ \text{mm}$$

(c) $X_L=L\omega=2\pi fL=(2)(3.14)(10^{9})(400\times10^{-12})$

$$\Rightarrow \quad X_L=2.51\ \Omega$$

20. $f_0=\dfrac{1}{2\pi\sqrt{L_1C_1}}=\dfrac{1}{2\pi\sqrt{L_2C_2}}$

$$\Rightarrow \quad L_1C_1=L_2C_2 \qquad \ldots(1)$$

When connected in series, the new resonant frequency be say f, then

$$f=\frac{1}{2\pi\sqrt{L_SC_S}}$$

where $L_S=L_1+L_2$ and $C_S=\dfrac{C_1C_2}{C_1+C_2}$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{(L_1+L_2)\left(\dfrac{C_1C_2}{C_1+C_2}\right)}}$$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{\dfrac{L_1C_1C_2+L_2C_1C_2}{C_1+C_2}}}=\frac{1}{2\pi\sqrt{L_1C_1\left(\dfrac{C_1+C_2}{C_1+C_2}\right)}}$$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{L_1C_1}}=\frac{1}{2\pi\sqrt{L_2C_2}}=f_0$$

Test Your Concepts-II (Based on Transformer)

1. By applying a dc voltage, no flux variation takes place in the core of transformer. Hence no voltage is induced in the secondary coil.

2. (a) $\dfrac{N_2}{N_1}=\dfrac{V_2}{V_1}=\dfrac{10\times10^{3}}{120}=83.3$

(b) $\eta=\dfrac{V_2I_2}{V_1I_1}$

$$\Rightarrow \quad I_2=\frac{\eta(V_1I_1)}{V_2}$$

But $I_1=\dfrac{V_1}{R_1}=\dfrac{120}{24}=5\ \text{A}$

$$\Rightarrow \quad I_2=\frac{(0.9)(120)(5)}{10\times10^{3}}$$

$$\Rightarrow \quad I_2=54\times10^{-3}\ \text{A}=54\ \text{mA}$$

(c) $Z_2=\dfrac{V_2}{I_2}=\dfrac{10\times10^{3}}{54\times10^{-3}}\cong185\ \text{k}\Omega$

3. Given that, $N_S=25000$ and $N_P=10000$

$$\Rightarrow \quad \frac{N_S}{N_P}=\frac{25}{10}=2.5$$

Hence it is a step-up transformer. Since, we have

$$\frac{\xi_1}{\xi_2}=\frac{N_1}{N_2}$$

$$\Rightarrow \quad \xi_2=\left(\frac{N_2}{N_1}\right)\xi_1=\left(\frac{25000}{10000}\right)(50)\ \text{V}$$

$$\Rightarrow \quad \xi_2=125\ \text{V}$$

4. (a) $R=(5\times10^{-4})(600\times10^{3}\ \text{m})=300\ \Omega$

Since $I_{\text{rms}}=\dfrac{P}{V_{rms}}=\dfrac{5\times10^{6}}{500\times10^{3}}=10\ \text{A}$

So, power loss is

$$P_{\text{loss}}=I_{rms}^2R=(100)(300)=30\ \text{kW}$$

(b) Fraction $f=\dfrac{P_{\text{loss}}}{P_{\text{input}}}=\dfrac{30\times10^{3}\ \text{W}}{5\times10^{6}\ \text{W}}=6\times10^{-3}$

(c) It is impossible to transmit so much power at such a low voltage. Since we know that from Maximum Power Transfer Theorem, the power transferred is maximum for load resistance to be equal to the line resistance of 300 Ω and

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$$P_{max} = \frac{V^2}{4R} = \frac{(4.5\times10^3)^2}{4(300)} = 16.9 \text{ kW}$$

This value happens to be far below 5 MW.

Single Correct Choice Type Questions

1. For series RC circuit impedance,

$$Z = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

$$\Rightarrow \quad Z = \sqrt{10^8 + \frac{1}{(10^{-12})(10^4)}}$$

$$\Rightarrow \quad Z = 10^4\sqrt{2}\ \Omega$$

Ammeter will give a reading which is the virtual value or the value that is the labelled value. So

$$I = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}Z}$$

$$\Rightarrow \quad I = \frac{200\sqrt{2}}{\sqrt{2}(10^4\sqrt{2})}$$

$$\Rightarrow \quad I = 10\sqrt{2} \text{ mA}$$

Hence, the correct answer is (B).

2. At resonance, voltage across resistance is 60 V

$$\Rightarrow \quad 60 = I_0R$$

$$\Rightarrow \quad I_0 = \frac{60}{120} = 0.5 \text{ A}$$

Also, voltage across inductance is 40 V

$$\Rightarrow \quad 40 = I_0X_L$$

$$\Rightarrow \quad 40 = 0.5(L\omega)$$

$$\Rightarrow \quad 80 = L(4\times10^5)$$

$$\Rightarrow \quad L = 0.2 \text{ mH}$$

Since $\omega_0 = \frac{1}{\sqrt{LC}}$

$$4\times10^5 = \frac{1}{\sqrt{0.2\times10^{-3}\ C}}$$

$$\Rightarrow \quad 16\times10^{10} = \frac{1}{C(0.2\times10^{-3})}$$

$$\Rightarrow \quad C = \frac{1}{3.2\times10^7}$$

$$\Rightarrow \quad C = \frac{1}{32}\times10^{-6} \text{ F}$$

$$\Rightarrow \quad C = \frac{1}{32}\ \mu\text{F}$$

Hence, the correct answer is (A).

3. Output current $= \frac{140}{24}$ A

$$\Rightarrow \quad I_S = \frac{140}{24} \text{ A}$$

$$\eta = \frac{\xi_S I_S}{\xi_P I_P} = \frac{24}{240}\frac{\left(\frac{140}{24}\right)}{0.7}$$

$$\Rightarrow \quad \eta = 0.833$$

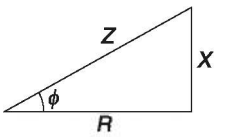
$$\Rightarrow \quad \eta = 83.3\%$$

Hence, the correct answer is (B).

4. According to the problem, we have

$$2Z = \sqrt{3}X$$

From the figure, we get



$$\sin\phi = \frac{X}{Z} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \phi = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

Hence, the correct answer is (A).

5. $I_{rms} = \frac{V_{rms}}{X_C}$

$$\Rightarrow \quad I_{rms} = \left(\frac{300\sqrt{2}}{\sqrt{2}}\right)(C\omega)$$

$$\Rightarrow \quad I_{rms} = (300)(10^{-6})(50)$$

$$\Rightarrow \quad I_{rms} = 15 \text{ mA}$$

Hence, the correct answer is (B).

6. Power factor is $\cos\phi = \frac{R}{Z}$, where

$$Z^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow \quad Z = (100-20)^2 + (40+20)^2$$

$$\Rightarrow \quad Z^2 = (80)^2 + (60)^2$$

$$\Rightarrow \quad Z = 100\ \Omega$$

So, $\cos\phi = \frac{(40+20)}{100} = 0.6$

Hence, the correct answer is (C).

7. An ideal choke coil should have almost zero internal resistance. Otherwise, it will consume some power.
Hence, the correct answer is (A).

8. $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad 10^6 = \frac{1}{2\pi\sqrt{10^{-2}C}}$$

$$\Rightarrow \quad 10^{12} = \frac{1}{4(10)(10^{-2}C)}$$

$$\Rightarrow \quad C = 2.5 \times 10^{-12}\ F$$

$$\Rightarrow \quad C = 2.5\ \text{pF}$$

Hence, the correct answer is (A).

9. At resonance

$$I = \frac{E_0}{R} = \frac{0.1 \times 10^{-3}}{50}$$

$$\Rightarrow \quad I = 2 \times 10^{-6}\ \text{A}$$

$$\Rightarrow \quad I = 2\ \mu\text{A}$$

Hence, the correct answer is (C).

10. The inductive reactance is

$$X_L = \omega L = (5 \times 10^{-3})(2000) = 10\ \Omega$$

The inductive capacitance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)(50 \times 10^{-6})} = 10\ \Omega$$

Since, $X_L = X_C$, so the circuit is in resonance and hence we have

$$Z = Z_{\min} = R = 6 + 4 = 10\ \Omega$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{\left(\frac{20}{\sqrt{2}}\right)}{10} = 1.414\ \text{A}$$

This is also the reading of ammeter.
The reading of the voltmeter is

$$V = I_{\text{rms}} Z_{AB}, \text{ where } Z_{AB} = 4\ \Omega$$

$$\Rightarrow \quad V = I_{\text{rms}} Z_{AB} \approx 5.6 \text{ volt}$$

Hence, the correct answer is (C).

11. When a dc is applied across the coil (an inductor) only resistance is offered to flow of dc
Hence

$$I = \frac{E_0}{R}$$

$$\Rightarrow \quad I = \frac{100}{R}$$

$$\Rightarrow \quad R = 100\ \Omega$$

On application of 100 V ac of frequency 50 Hz we have $I' = 0.5$ A

$$\Rightarrow \quad I' = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + L^2\omega^2}}$$

$$\Rightarrow \quad I' = \frac{E}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

$$\Rightarrow \quad 0.5 = \frac{100}{\sqrt{10^4 + 4(10)(2500)L^2}}$$

$$\Rightarrow \quad 10^4 + 10^5 L^2 = 4 \times 10^4$$

$$\Rightarrow \quad L^2 = \frac{3 \times 10^4}{10^5} = 0.3$$

$$\Rightarrow \quad L = \sqrt{0.3}\ \text{H}$$

So, $R = 100\ \Omega$, $L = \sqrt{0.3}$ H

Hence, the correct answer is (D).

12. Given that $V_0 = 240$ V

$$\Rightarrow \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{240}{\sqrt{2}} = 170\ \text{V}$$

$$\omega = 120\ \text{rads}^{-1}$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{120}{2 \times 3.14} = 19\ \text{Hz}$$

Hence, the correct answer is (A).

13. $P = \frac{E_v^2}{Z}\cos\phi$

$$\Rightarrow \quad 3000 = \frac{(240)^2}{Z}(0.75)$$

$$\Rightarrow \quad Z = \frac{(240)^2}{3000} \times \frac{3}{4}$$

$$\Rightarrow \quad Z = 14.4\ \Omega$$

Hence, the correct answer is (C).

14. When coils are connected in series, we have $R_{eq} = R_1 + R_2$ and $L_{eq} = L_1 + L_2$. Further, we have

$$\text{Power Factor} = \cos\phi = \frac{R}{Z} = \frac{R_1 + R_2}{Z}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{5 + R_2}{14.4}$$

$$\Rightarrow \quad R_2 = 5.8\ \Omega$$

Hence, the correct answer is (B).

15. Since $Z = \sqrt{R_{eq}^2 + L_{eq}^2\omega^2} = \sqrt{R_{eq}^2 + 4\pi^2 f^2 L_{eq}^2}$

$$\Rightarrow \quad (14.4)^2 = (5.8)^2 + L_{eq}^2(4)(10)(2500)$$

$$\Rightarrow \quad 207.36 = 33.64 + 100000\ L_{eq}^2$$

$$\Rightarrow \quad L_{eq} = 0.04\ H$$

$$\Rightarrow \quad L_1 + L_2 = 0.04$$

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$\Rightarrow \quad L_2 = 0.04 - 0.02$

$\Rightarrow \quad L_2 = 0.02\ H$

Hence, the correct answer is (B).

16. $\xi = M\dfrac{dI}{dt}$ (in magnitude)

$$\xi = MI_0\frac{d}{dt}[\sin(\omega t)]$$

$\Rightarrow \quad \xi = MI_0\omega\cos(\omega t)$

$\Rightarrow \quad \xi_{max} = MI_0\omega$

$\Rightarrow \quad \xi_{max} = (0.005)(10)(100\pi)$

$\Rightarrow \quad \xi_{max} = 5\pi$

Hence, the correct answer is (B).

18. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \quad f_0 = \dfrac{1}{2\pi\sqrt{8(0.5\times10^{-6})}}$

$\Rightarrow \quad f_0 = \dfrac{1000}{4\pi} = \dfrac{250}{\pi}$ Hz

Hence, the correct answer is (C).

19. The current in the circuit is

$$I = \frac{V}{Z}$$

$\Rightarrow \quad I = \dfrac{V}{\sqrt{R^2 + \left(\omega L \sim \dfrac{1}{\omega C}\right)^2}}$

If R is increased, then the current will definitely decrease. However, by changing L or C, the current may increase or decrease.

Hence, the correct answer is (A).

20. $\tan\phi = \dfrac{X_L}{R}$

$\Rightarrow \quad \tan 45 = \dfrac{L\omega}{R}$

$\Rightarrow \quad L = \dfrac{R}{\omega} = \dfrac{100}{2\pi(1000)}$

$\Rightarrow \quad L = \dfrac{1}{20\pi}$

Hence, the correct answer is (B).

21. For the given circuit, the current flowing through the capacitor is

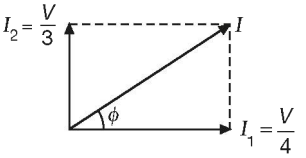
$$I_2 = \frac{V}{X_C} = \frac{V}{3}$$

where, V is the rms value of the ac input.

Similarly, the current I_1 flowing through the resistor is

$$I_1 = \frac{V}{R} = \frac{V}{4}$$

The current I_2 leads the applied voltage by a phase angle of $90°$ whereas the current I_1 is in phase with the applied voltage.



$$\tan\phi = \frac{V/3}{V/4} = \frac{4}{3}$$

$\Rightarrow \quad \phi = 53°$

Hence, the correct answer is (C).

23. Since, we have

$$i = -\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

$\Rightarrow \quad i = \sqrt{2}\sin\left(\omega t + \dfrac{\pi}{4} + \dfrac{\pi}{2}\right)$

Hence, phase difference between V and i is $\dfrac{\pi}{2}$. So, power consumed is zero.

Hence, the correct answer is (D).

24. RMS value $= \sqrt{\langle E^2\rangle}$

$$\langle E^2\rangle = \frac{\int_0^T E^2 dt}{\int_0^T dt}$$

$\Rightarrow \quad \langle E^2\rangle = \dfrac{1}{T}\displaystyle\int_0^T [64\sin^2(\omega t) + 36\sin^2(2\omega t) + 96\sin(\omega t)\sin(2\omega t)]dt$

$\Rightarrow \quad \langle E^2\rangle = \dfrac{1}{T}\left[64\left(\dfrac{T}{2}\right) + 36\left(\dfrac{T}{2}\right) + 0\right]$

$\Rightarrow \quad \langle E^2\rangle = \dfrac{64+36}{2} = 50$

$\Rightarrow \quad$ RMS value $= \sqrt{\langle E^2\rangle} = \sqrt{50} = 5\sqrt{2}$ V

Hence, the correct answer is (A).

Problem Solving Technique(s)

The RMS value can also be calculated using the formula

$$E_{rms} = \sqrt{\frac{8^2 + 6^2}{2}} = 5\sqrt{2} \text{ V}$$

25. Since we know that, for an ac passing through the series combination of inductor and resistor, we have

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow \quad V = \sqrt{(20)^2 + (15)^2} = 25 \text{ V}$$

Since this is the rms value, so the peak value is given by

$$V_0 = \sqrt{2} V_{rms} = 25\sqrt{2} \text{ V}$$

Hence, the correct answer is (C).

26. $M = \sqrt{L_1 L_2}$

When number of turns in the coil are interchanged then M remains same.

Hence, the correct answer is (C).

27. $I_P = \frac{P}{\xi_P} = \frac{6.6 \times 10^3}{220} = 30 \text{ A}$

Further

$$N_S = \frac{\xi_S}{\xi_P} N_P = \frac{4.4 \times 1000 \times 1000}{220} = 20{,}000$$

Also $I_S = \frac{N_P}{N_S} I_P = 1.5 \text{ A}$

Hence, the correct answer is (C).

28. $X_L = L\omega = 2\pi\nu L$

$X_L \propto \nu$ which indicates a graph for a straight line.

Hence, the correct answer is (A).

29. Since the resistance R equals the capacitive reactance, so we have

$$R = X_C = \frac{1}{\omega C}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{2}R \qquad \{\because X_C = R\}$$

$$\Rightarrow \quad I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{2}R} \qquad \ldots(1)$$

When ω becomes $\frac{1}{\sqrt{3}}$ times, X_C will become $\sqrt{3}$ times or $\sqrt{3}R$, so the new impedance is

$$Z' = \sqrt{(R^2) + (\sqrt{3}R)^2} = 2R$$

$$\Rightarrow \quad I_0' = \frac{V_0}{Z'} = \frac{V_0}{2R} = \frac{I_0}{\sqrt{2}}$$

Hence, the correct answer is (B).

30. The power factor is given by

$$\cos\phi = \frac{R}{Z}$$

Where the impedance can be

$$Z = \sqrt{R^2 + L^2\omega^2}$$

OR $\quad Z = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$

OR $\quad Z = \sqrt{(X_L - X_C)^2 + R^2}$

Hence, the correct answer is (B).

31. For an ideal capacitor or an ideal inductor, the phase difference between current and voltage is $90°$.

Hence, the correct answer is (D).

32. These ammeters use the heating property of current to calculate its value and hence can be used to measure both ac and dc.

Hence, the correct answer is (C).

33. Since we know that

$$V = \sqrt{V_C^2 + V_R^2}$$

$$\Rightarrow \quad V_R = \sqrt{V^2 - V_C^2} = \sqrt{(10)^2 - (8)^2} = 6 \text{ V}$$

$$\Rightarrow \quad \tan\phi = \frac{X_C}{X_R} = \frac{V_C}{V_R} = \frac{8}{6} = \frac{4}{3}$$

Hence, the correct answer is (A).

34. At resonance, no phase difference exists between applied voltage and current.

Hence, the correct answer is (B).

35. Applied voltage V is a sine function and we see that the current is leading the voltage by $45°$. So, the circuit should be capacitive in nature.

Since $\phi = 45°$

$$\Rightarrow \quad \tan 45° = \frac{X_C}{R} = \frac{1}{RC\omega}$$

$$\Rightarrow \quad X_C = R$$

$$\Rightarrow \quad \omega C = R$$

$$\Rightarrow \quad C = \frac{R}{\omega} = \frac{R}{100} = 0.01 \text{ R}$$

The best suitable combination for satisfying the given condition is in OPTION (B).

Hence, the correct answer is (B).

36. The average emf during the positive half cycle of an ac is

$$E_{av} = \frac{2E_0}{\pi}$$

Hence, the correct answer is (D).

37. In first case, $X_C = \frac{V}{I} = \frac{220}{0.25} = 880\ \Omega$

In the second case, $R = \frac{V}{I} = \frac{220}{0.25} = 880\ \Omega$

In the combination of P and Q,

$$\tan\phi = \frac{X_C}{R} = 1$$

$\Rightarrow \quad \phi = 45°$

Since the circuit is capacitive in nature, so current leads the voltage by a phase angle of $45°$.

The impedance of the circuit is

$$Z = \sqrt{R^2 + X_C^2} = 880\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{Z} = \frac{220}{880\sqrt{2}} = \frac{1}{4\sqrt{2}}\ \text{A}$$

Hence, the correct answer is (B).

38. $I = \sqrt{(I_C - I_L)^2 + I_R^2}$

$$\Rightarrow \quad I = V\sqrt{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 + \frac{1}{R^2}}$$

$$\Rightarrow \quad I = 100\sqrt{\left(\frac{1}{20} - \frac{1}{10}\right)^2 + \frac{1}{400}}$$

$$\Rightarrow \quad I = 100\sqrt{\frac{1}{400} + \frac{1}{400}}$$

$$\Rightarrow \quad I = \frac{100}{20}\sqrt{2}$$

$$\Rightarrow \quad I = 5\sqrt{2}\ \text{A}$$

Hence, the correct answer is (D).

39. $\frac{N_S}{N_P} = \frac{550}{22000} = \frac{1}{40}$

Hence, the correct answer is (C).

40. $\omega_0 = \frac{1}{\sqrt{LC}}$

C goes to $4C$, so to keep ω_0 the same L must go to $\frac{L}{4}$

Hence, the correct answer is (C).

41. The capacitive reactance is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\ \Omega$$

The ac ammeter measures the rms current, so we have

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{\frac{(200\sqrt{2})}{\sqrt{2}}}{10^4} = 0.02\ \text{A}$$

$\Rightarrow \quad I_{\text{rms}} = 20\ \text{mA}$

Hence, the correct answer is (C).

42. $I_S = \frac{E_S}{Z} = \frac{22}{220} = 0.1\ \text{A}$

Further

$$I_P = \frac{E_S}{E_P} I_S = \frac{22}{220}(0.1) = 0.01\ \text{A}$$

Hence, the correct answer is (A).

43. According to the problem, we have

$$T = \frac{1}{f} = \frac{1}{50}\ \text{s}$$

$$\Rightarrow \quad t = \frac{T}{4} = \frac{1}{200}\ \text{s}$$

$$\Rightarrow \quad t = 5 \times 10^{-3}\ \text{s}$$

$\Rightarrow \quad t = 5\ \text{ms}$

Hence, the correct answer is (A).

44. $I_S = \frac{N_P}{N_S} I_P$

$$I_S = \left(\frac{100}{20}\right)(2)$$

$I_S = 10\ A$

Hence, the correct answer is (D).

45. For the series combination of resistor and a capacitor, we have

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(20)^2 - (12)^2}$$

$\Rightarrow \quad V_C = 16\ \text{V}$

Hence, the correct answer is (B).

46. $I_v \sin\phi$ component of current gives no power consumption and hence is called **wattless current**.

Hence, the correct answer is (B).

47. Since we observe that

$$i = \frac{V}{R}$$

Hence the given circuit is in resonance.

$\Rightarrow \quad V_C = V_L = 200 \text{ V}$

Hence, the correct answer is (B).

48. For steady voltage, we have

$$x = \frac{(10)^2}{R}$$

For AC voltage, we have

$$\frac{x}{2} = \frac{E^2}{2R}$$

$\Rightarrow \quad E = 10V$

Hence, the correct answer is (C).

49. Current in the circuit is maximum at resonance and

$$I_{\max} = \frac{V}{R}$$

$\Rightarrow \quad 6 = \frac{24}{R}$

$\Rightarrow \quad R = 4\ \Omega$

$\Rightarrow \quad I_{DC} = \frac{V}{R+r} = \frac{12}{4+4} = 1.5 \text{ A}$

Hence, the correct answer is (C).

50. $V = V_0 \cos(\omega t)$

$$-L\frac{dI}{dt} = V_0 \cos(\omega t)$$

$\Rightarrow \quad dI = -\frac{V_0}{L}\cos(\omega t)\,dt$

$\Rightarrow \quad I = -\left(\frac{V_0}{L\omega}\right)\sin(\omega t)$

$\Rightarrow \quad I = -I_0 \sin(\omega t)$

Hence, the correct answer is (B).

51. The power consumed in the circuit is

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$\Rightarrow \quad P = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos 60°$

$\Rightarrow \quad P = \left(\frac{220}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$\Rightarrow \quad P = 220 \text{ W}$

Hence, the correct answer is (C).

52. For current to lag behind the voltage, circuit should be inductive in nature i.e. $X_L > X_C$

$\Rightarrow \quad L\omega > \frac{1}{C\omega}$

$\Rightarrow \quad \omega^2 > \frac{1}{LC}$

$\Rightarrow \quad \omega^2 > \omega_0^2$

$\Rightarrow \quad \omega > \omega_0$

Hence, the correct answer is (B).

53. $\tan\phi = \frac{X}{R} = \frac{4}{3}$

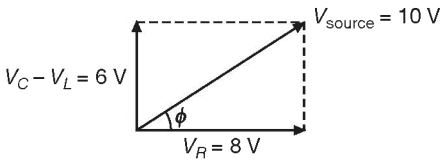
$$\text{Power Factor} = \cos\phi = \frac{3}{5} = 0.6$$

Hence, the correct answer is (B).

54. The voltage of the source is given by

$$V_{\text{source}} = \sqrt{V_R^2 + (V_C - V_L)^2} = 10 \text{ V}$$

Since $V_C > V_L$, hence current leads the voltage.



Power factor of the circuit is

$$\cos\phi = \frac{8}{10} = 0.8$$

Hence, the correct answer is (D).

55. $\tan\phi = \frac{X_L}{R} = \frac{L\omega}{R}$

$\Rightarrow \quad \tan\phi = \frac{\left(\frac{1}{\pi}\right)(2\pi)(200)}{300}$

$\Rightarrow \quad \tan\phi = \frac{4}{3}$

$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$

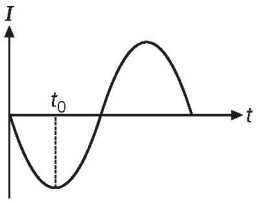
Hence, the correct answer is (D).

56. Current will lead the voltage by $90°$, so we have

$$i = i_0 \cos\left(\frac{\pi t}{2} + 90°\right) = -i_0 \sin\left(\frac{\pi t}{2}\right)$$

So, the current function plot versus time is shown in Figure.

CHAPTER 4



$$t_0 = \frac{T}{4} = \frac{\left(\frac{2\pi}{\omega}\right)}{4} = \frac{\pi}{2\omega}$$

$$\Rightarrow \quad t_0 = \frac{\pi}{2\left(\frac{\pi}{2}\right)} = 1\ \text{s}$$

Hence, the correct answer is (A).

57. $\frac{\xi_S}{\xi_P} = \frac{I_P}{I_S}$

$$\Rightarrow \quad \frac{4.6}{230} = \frac{I_P}{5}$$

$$\Rightarrow \quad I_p = 0.1\ \text{A}$$

Hence, the correct answer is (A).

58. For a resistor, the voltage and current are in the same phase, whereas for an inductor the voltage leads the current by $90°$ or current lags behind the voltage by $90°$. Hence, we have

$$I_R = I_0 \sin(\omega t) \text{ and}$$

$$I_L = I_0 \sin(\omega t - 90°) = -I_0 \cos(\omega t)$$

Hence, the correct answer is (C).

59. For ac of rms value 10 A

$$P_{ac} = I_v^2 R \qquad \ldots(1)$$

For dc of value I

$$P_{dc} = I^2 R \qquad \ldots(2)$$

Equating (1) and (2), we get

$$I = I_v = 10\ A$$

Hence, the correct answer is (A).

60. Resistance does not depend on the frequency of AC.

Hence, the correct answer is (A).

61. Effective current is the rms value. Here 220 V is the labelled value of ac which is also the rms value. Hence

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{220}{100 \times 10^3}$$

$$\Rightarrow \quad I_{\text{rms}} = 2.2\ \text{mA}$$

Hence, the correct answer is (A).

62. The power consumed in the circuit is

$$P = I_{\text{rms}}^2 R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R$$

$$\Rightarrow \quad P = \left[\frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R^2 + \omega^2 L^2}\right] R$$

$$\Rightarrow \quad P = \frac{V_0^2 R}{2\left(R^2 + \omega^2 L^2\right)}$$

Hence, the correct answer is (C).

63. $\xi_S = \left(\frac{N_S}{N_P}\right)\xi_P$

$$\Rightarrow \quad \xi_S = \left(\frac{3000}{1000}\right)(80)$$

$$\Rightarrow \quad \xi_s = 240\ V$$

So, potential difference across each turn of the secondary is

$$\frac{\xi_S}{N_S} = \frac{240}{3000} = 0.08\ V/\text{turn}$$

Hence, the correct answer is (D).

65. $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}}$

$$\Rightarrow \quad \omega_0 = 500\ \text{rads}^{-1}$$

Hence, the correct answer is (A).

66. $I^2 = 16t^2$

$$\langle I^2 \rangle = \frac{\int_0^2 16t^2 dt}{\int_0^2 dt} = \frac{16\frac{t^3}{3}\Big|_0^2}{2} = 8\frac{t^3}{3}\Big|_0^2 = \frac{8(8)}{3}$$

$$\Rightarrow \quad \langle I^2 \rangle = \frac{64}{3}$$

$$\Rightarrow \quad I_{RMS} = \sqrt{\langle I^2 \rangle} = \frac{8}{\sqrt{3}}\ \text{A}$$

Hence, the correct answer is (C).

67. Since current in the capacitor I_C is $90°$ ahead of the applied voltage and current in the inductor I_L lags behind the applied voltage by $90°$. So, there is a phase difference of $180°$ between I_L and I_C. Hence

$$I = I_C - I_L = 0.2\ \text{A}$$

Hence, the correct answer is (C).

68. $V_{rms} = \sqrt{\frac{0^2 + V_0^2}{2}} = \frac{V_0}{\sqrt{2}}$

Hence, the correct answer is (B).

69. The inductive reactance is

$X_L = \omega L$

Since ω is very low, so $X_L \approx 0$

$\Rightarrow \quad V_L \approx 0$

$\Rightarrow \quad V = V_C = V_0$

Hence, the correct answer is (B).

70. A Leclanche cell is a dc voltage source and transformer does not work for dc sources.

Hence, the correct answer is (A).

71. The impedance of this circuit is

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

and $I_0 = \frac{V_0}{Z} = V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

Hence, the correct answer is (B).

72. $I_0 = \frac{E_0}{X_L}$

$\Rightarrow \quad I_0 = \frac{E_0}{L\omega}$

$\Rightarrow \quad I_0 = \frac{220\sqrt{2}}{1(2\pi(50))}$

$\Rightarrow \quad I_0 = \frac{220\sqrt{2}}{100\pi}$

$\Rightarrow \quad I_0 \simeq 1 \text{ A}$

Hence, the correct answer is (C).

73. Power consumed is

$P = E_V I_V \cos\phi$

Also $\phi = \frac{\pi}{2}$

$\Rightarrow \quad P = 0$

Hence, the correct answer is (D).

74. For DC current passing through the resistor, we have

$H_{DC} = I^2 Rt$

For AC current passing through the resistor, we have

$\Rightarrow \quad H_{AC} = I_{rms}^2 Rt = \left(\frac{I}{\sqrt{2}}\right)^2 Rt = \frac{I^2 Rt}{2}$

$\Rightarrow \quad \frac{H_{DC}}{H_{AC}} = \frac{2}{1}$

Hence, the correct answer is (A).

75. $V = V_0 \sin(\omega t) + V_0 \sin(2\omega t)$

$\Rightarrow \quad V = V_1 + V_2$

where $V_1 = V_0 \sin(\omega t)$ and $V_2 = V_0 \sin(2\omega t)$

Now, $I_1 = \frac{V_0}{Z_1}$ and $I_2 = \frac{V_0}{Z_2}$ where

$$Z_1 = \sqrt{R^2 + \frac{1}{C^2\omega^2}} \text{ and } Z_2 = \sqrt{R^2 + \frac{1}{4C^2\omega^2}}$$

Since $Z_2 < Z_1$

$\Rightarrow \quad I_1 < I_2$

Hence, the correct answer is (C).

76. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$\Rightarrow \quad X_C \propto \frac{1}{f}$

So, X_C versus f graph is a rectangular hyperbola

Hence, the correct answer is (B).

77. $\omega_0 = 2\pi(100) = 600 \text{ rad s}^{-1}$ $\{\text{Given } \pi = 3\}$

Further

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \ldots(1)$$

Also $X_C = \frac{1}{C\omega_0} = 60$

$\Rightarrow \quad C = \frac{1}{\omega_0(60)}$

$\Rightarrow \quad C = \frac{1}{600 \times 60}$

$\Rightarrow \quad C = \frac{1}{36 \times 10^{-3}} \text{ F}$

So, put values in (1), we get

$$600 = \frac{1}{\sqrt{L\left(\frac{1}{36 \times 10^{-3}}\right)}}$$

$\Rightarrow \quad 36 \times 10^4 = \frac{36 \times 10^{-3}}{L}$

$\Rightarrow \quad L = 0.1 \text{ H}$

Hence, the correct answer is (A).

CHAPTER 4

78. $I_0 = \frac{E_0}{R}$ (at resonance)

$\Rightarrow \quad I_0 = \frac{300}{10} = 30 \text{ A}$

Hence, the correct answer is (C).

79. The net source voltage is

$V_S = \sqrt{V_R^2 + V_L^2}$

$\Rightarrow \quad V_S = \sqrt{(70)^2 + (20)^2} = 72.8 \text{ V}$

$\Rightarrow \quad \tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{20}{70} = \frac{2}{7}$

Hence, the correct answer is (A).

80. $I = \frac{V}{Z}$ where

$Z^2 = R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2$

$L\omega = 2\pi fL = 120 \times 0.3\pi$

$\Rightarrow \quad L\omega = 36\pi = 113.04$

$\frac{1}{C\omega} = \frac{1}{2\pi fC} = \frac{10^6}{7200\pi}$

$\Rightarrow \quad \frac{1}{C\omega} = 44.23$

$\Rightarrow \quad Z = 85.05\ \Omega$

$\Rightarrow \quad I = \frac{120}{85.05} \cong 1.5 \text{ A}$

Hence, the correct answer is (A).

81. $P = E_V I_V \cos\phi$

$\Rightarrow \quad P = (I_V Z)(I_V)\left(\frac{R}{Z}\right)$

$\Rightarrow \quad P = I_V^2 R$

$\Rightarrow \quad P = (1.5)^2 (50)$

$\Rightarrow \quad P = 112.5 \text{ W}$

Hence, the correct answer is (D).

83. $V = V_0 \sin(2\pi ft)$

$\Rightarrow \quad V = 10 \sin\left(\frac{100\pi}{600}\right)$

$\Rightarrow \quad V = \frac{10}{2} = 5 \text{ V}$

Hence, the correct answer is (C).

84. $\frac{N_S}{N_P} = \frac{22000}{220} = 100$

Hence, the correct answer is (B).

85. Average value of $5\sin 100\omega t$ is zero. But average value of 6 A (which is a constant current) is 6 A. Hence, average value of current is 6 A.

Hence, the correct answer is (B).

86. $P_{av} = E_v I_v \cos\phi = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$

$\Rightarrow \quad P_{av} = \frac{E_0 I_0}{2} \cos\phi$

$\Rightarrow \quad P_{av} = \frac{100 \times 100 \times 10^{-3}}{2} \times \frac{1}{2}$

$\Rightarrow \quad P_{av} = 2.5 \text{ W}$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. The average power consumed in the circuit is

$$\langle P \rangle = \frac{V_0^2 R}{2\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]}$$

$\langle P \rangle = 0$, when $\omega \to 0$ or when $\omega \to \infty$ i.e. $\langle P \rangle$ is zero for extremely small and extremely large frequencies. So, (A) is correct.

The average power consumed in the circuit varies with the frequency, because the impedance Z depends upon the frequency. So, (B) is incorrect.

Also, $\langle P \rangle = \frac{V_0^2}{2R} = P_{\max}$ at resonance i.e. when

$\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow \quad \omega^2 LC = 1$

So, (C) is correct.

Also, $\langle P \rangle = \frac{V_0^2}{4R}$ i.e. $\langle P \rangle = \frac{V_0^2}{4R} = \frac{P_{\max}}{2}$ at two frequencies that are also called half power frequencies, such that $\Delta\omega = \frac{R}{L}$. So, (D) is also correct.

Hence, (A), (C) and (D) are correct.

2. Power factor, $\cos\phi = \frac{R}{Z}$

When circuit is purely resistive, then

$Z = R$

$\Rightarrow \quad \cos\phi = 1$

When circuit is purely capacitive or inductive, then

$R = 0$

$\Rightarrow \quad \cos\phi = 0$

When the difference of inductive reactance and capacitive reactance i.e. $X_L - X_C$ is 1.732 i.e. $\sqrt{3}$ times the resistance R in the circuit, then, we have

$$X_L - X_C = 1.732R = \sqrt{3}R$$

Since, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow \quad Z = \sqrt{R^2 + (\sqrt{3}R)^2} = 2R$$

$$\Rightarrow \quad \cos\phi = \frac{R}{Z} = \frac{R}{2R} = 0.5$$

Hence, (A), (C) and (D) are correct.

3. Mean current $\langle i \rangle = \dfrac{\text{Area}}{\text{Time}}$

$$\langle i \rangle = i_m = \frac{\frac{1}{2} \times i_0 \times \frac{T}{2}}{\frac{T}{2}}$$

$$\Rightarrow \quad i_m = \frac{i_0}{2}$$

As $i = \dfrac{2i_0}{T}t$

$$\Rightarrow \quad i_2 = \frac{\frac{4i_0^2}{T^2}\int_0^{T/2} t^2 dt}{\frac{T}{2}} = \frac{2 \times 4i_0^2}{T^3}\frac{T^3}{3 \times 8} = \frac{i_0^2}{3}$$

$$\Rightarrow \quad i_{\text{rms}} = \sqrt{i^2} = \frac{i_0}{\sqrt{3}}$$

Hence, (B) and (C) are correct.

4. Since we know that

$$P_R = V_R i$$

$$\Rightarrow \quad i = \frac{P_R}{V_R} = \frac{60}{60} = 1 \text{ A}$$

Also, for the given circuit

$$V_L = \sqrt{V^2 - V_R^2}$$

$$\Rightarrow \quad V_L = \sqrt{(100)^2 - (60)}^2$$

$$\Rightarrow \quad V_L = 80 \text{ V} = iX_L = i(2\pi fL)$$

$$\Rightarrow \quad L = \frac{80}{2\pi fi}$$

$$\Rightarrow \quad L = \frac{80}{(2\pi)(50)(1)} = \frac{4}{5\pi} \text{ H}$$

When we connect another resistance R in series, then it should consume 40 V, so that remaining 60 V is used by the tube light.

$$R = \frac{V}{i} = \frac{40}{1} = 40\ \Omega$$

Hence, (A) and (D) are correct.

5. Since $V_L = V_C$

So, $V = \sqrt{(V_L - V_C)^2 + V_R^2} = V_R$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{V_R}{R} = \frac{200}{100} = 2 \text{ A}$$

Hence, (C) and (D) are correct.

6. The voltage across the resistor is

$$V_R = IR = 80 \text{ V}$$

The capacitive reactance is

$$X_C = \frac{V_C}{I} = \frac{100}{2} = 50\ \Omega$$

Voltage across the inductor is

$$V_L = IX_L = 40 \text{ V}$$

The net voltage across the combination is

$$V = V_{\text{rms}} = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\Rightarrow \quad V = \sqrt{(80)^2 + (100 - 40)^2} = 60 \text{ V}$$

So, the peak value of voltage is

$$V_0 = \sqrt{2}V_{\text{rms}} = 60\sqrt{2} \text{ V}$$

Hence, (A), (B) and (C) are correct.

7. $P = \dfrac{E_0 I_0}{2}\cos\phi$

At resonance, $\phi = 0°$

$\Rightarrow \quad \cos\phi = \text{Power Factor} = 1$

Also, $Z = Z_{\text{MIN}} = R$ and

$$P = \frac{I_0^2 R}{2}$$

No power is consumed across the capacitor and the inductor at resonance.

Hence, (A), (C) and (D) are correct.

8. Current through the resistor is

$$I_R = \frac{V_{\text{rms}}}{R} = \frac{200}{100} = 2 \text{ A}$$

CHAPTER 4

Capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(5\times10^3)\left(\frac{1}{\pi}\times10^{-6}\right)}$$

$$\Rightarrow \quad X_C = 100\ \Omega$$

$$\Rightarrow \quad I_C = \frac{V_{rms}}{X_C} = \frac{200}{100} = 2\ \text{A}$$

Since I_C is 90° ahead of the applied voltage and I_R is in phase with the applied voltage, so there is a phase difference of 90° between I_R and I_C too. Hence

$$I = \sqrt{I_R^2 + I_C^2}$$

$$\Rightarrow \quad I = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} \approx 2.83\ \text{A}$$

Hence, (A), (C) and (D) are correct.

9. For voltmeter V_1, we have

$$\sqrt{V_R^2 + V_L^2} = 100\ \text{V} \qquad \ldots(1)$$

For voltmeter V_2, we have

$$|V_L - V_C| = 120\ \text{V} \qquad \ldots(2)$$

Since the applied voltage is 130 V, so we have

$$\sqrt{V_R^2 + (V_L - V_C)^2} = 130\ \text{V} \qquad \ldots(3)$$

Solving these three equations, we get

$$V_R = 50\ \text{V},\ V_L = 86.6\ \text{V} \text{ and } V_C = 206.6\ \text{V}$$

Power factor $\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$

Since $V_C > V_L$, circuit is capacitive in nature.

Hence, (A), (C) and (D) are correct.

10. In series LCR circuit

$$V_{rms}^2 = (V_R)^2 + (V_L - V_C)^2$$

$$(170)^2 = V_R^2 + (V_L - V_C)^2$$

Obviously $V_R \le (170)^2$

Here V_L and V_C may be greater than 170 volt or less than 170 volt

But $|V_L - V_C| \le (170)$ volt

Hence, (A), (C) and (D) are correct.

11. The resonance frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Since $X_L < X_C$

$$\Rightarrow \quad L\omega < \frac{1}{C\omega}$$

$$\Rightarrow \quad \omega^2 < \frac{1}{LC}$$

$$\Rightarrow \quad \omega^2 < \omega_0^2$$

$$\Rightarrow \quad \omega < \omega_0$$

When the frequency is increased from the given value, X_C will further increase, whereas X_L will increase, so $X_C - X_L$ will decrease and hence the net impedance will decrease.

When frequency is increased from the given value, then till the resonant frequency ω_0, the current increase, becomes maximum at resonant frequency and after that it again starts decreasing.

Hence, (A), (C) and (D) are correct.

12. $f = \frac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow \quad f_1 = f_2 = f_3 = \frac{1}{2\pi\sqrt{LC}}$$

Also, we have, from Law of Conservation of Energy,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV^2$$

$$\Rightarrow \quad I_{max} = \sqrt{\frac{C}{L}}V$$

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C}{L}}$$

$$\sqrt{\frac{C_2}{L_2}} = \frac{1}{2}\sqrt{\frac{C}{L}} \text{ and}$$

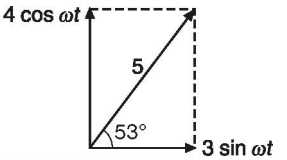
$$\sqrt{\frac{C_3}{L_3}} = 2\sqrt{\frac{C}{L}}$$

$$\Rightarrow \quad I_2 < I_1 < I_3$$

Hence, (A) and (D) are correct.

13. The given function $i = 3\sin\omega t + 4\cos\omega t$ can be re-written as

$$i = 5\sin(\omega t + 53°)$$



So, we have $i_0 = 5$ A

$$\Rightarrow \quad i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 2.5\sqrt{2}\ \text{A}$$

Mean value of current in positive half cycle is

$$\langle i\rangle = \frac{2i_0}{\pi} = \left(\frac{2}{\pi}\right)(5) = \left(\frac{10}{\pi}\right) = 3.2 \text{ A}$$

For $V = V_0 \sin \omega t$, the current $i = 5\sin(\omega t + 53°)$ leads the voltage. Hence, circuit is capacitive in nature.

However, for $V = V_0 \cos \omega t = V_0 \sin(\omega t + 90°)$, the current $i = 5\sin(\omega t + 53°)$ lags behind the voltage and hence the circuit is inductive in nature.

Hence, (A), (B), (C) and (D) are correct.

14. Since, $X_L > X_C$, hence voltage function will lead the current function.

Also, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \quad Z = \sqrt{(10)^2 + (20-10)^2}$

$\Rightarrow \quad Z = 10\sqrt{2}\ \Omega$

$\Rightarrow \quad \cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \phi = 45°$

So, power factor is given by

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

Hence, (A), (B), (C) and (D) are correct.

Reasoning Based Questions

1. At resonant frequency

$$X_L = X_C$$

$\therefore \quad Z = R$ (minimum)

Therefore, current in the circuit is maximum.

Hence, the correct answer is (A).

4. For half cycle $I_{mean} = 0.636 I_0$

or $E_{mean} = 0.636 E_0$

Average value is always defined over a half cycle cause in next half cycle it will be opposite in direction. Hence for one complete cycle, average value will be zero.

Hence, the correct answer is (B).

5. The capacitive reactance of capacitor is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

So, this is infinite for d.c. $(f = 0)$ and has a very small value for a.c. therefore a capacitor block d.c.

Hence, the correct answer is (A).

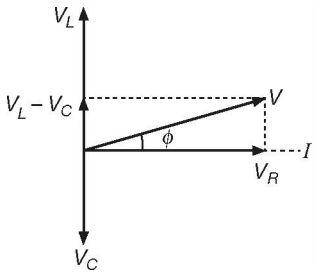
6. $\omega > \omega_0$

Since, $X_L - X_C = L\omega - \frac{1}{C\omega} = \frac{CL\omega^2 - 1}{C\omega} = X_C\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]$

$\Rightarrow \quad X_L - X_C > 0 \qquad \{\because\ \omega > \omega_0\}$

$\Rightarrow \quad \tan\phi = \frac{X_L - X_C}{R}$

$\Rightarrow$ V leads I by ϕ and the circuit will be inductive in nature.



Hence, the correct answer is (A).

8. The maximum value of r.m.s. current is $I_{rms} = \frac{E_{r.m.s.}}{Z} = \frac{E_{r.m.s.}}{R}$. It does not depend upon ω.

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. The given equation $E = 310\sin(314t)$ is compared with the equation $E = E_0 \sin 2\pi ft$, we get

$$2\pi f = 314$$

$\Rightarrow \quad 2 \times \frac{22}{7} \times f = 314$

$\Rightarrow \quad f = \frac{7 \times 314}{2 \times 22}$ cps ≈ 50 cps

Hence, the correct answer is (B).

2. Inductive reactance is

$$X_L = L\omega = 2\pi fL = 2 \times \frac{22}{7} \times 50 \times 0.1 \text{ ohm} = 31.4\ \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi fC} = \frac{1 \times 7}{2 \times 22 \times 50 \times 25 \times 10^{-6}}\ \Omega$$

$\Rightarrow \quad X_C = 127.3\ \Omega$

Net reactance

$$X = X_C - X_L = (127.3 - 31.4)\ \Omega = 95.9\ \Omega$$

$\{\because\ X_C$ and X_L differ in phase by $180°\}$

Hence, the correct answer is (A).

CHAPTER 4

3. Impedance, $Z=\sqrt{R^2+(X_L-X_C)^2}$

$\Rightarrow \quad Z=\sqrt{25^2+95.9^2}\ \Omega=99.1\ \Omega$

Hence, the correct answer is (C).

4. Current, $I_v=\dfrac{E_v}{Z}=\dfrac{E_0}{\sqrt{2}Z}=\dfrac{310}{\sqrt{2}\times 99.1}A=2.21\ A$

Hence, the correct answer is (B).

5. If ϕ be the phase angle, then

$$\tan\phi=\frac{X_C-X_L}{R}=\frac{95.9}{25}=3.84$$

$\Rightarrow \quad \phi=75.4°$

Thus, the current leads the voltage by $75.4°$. This is because the capacitive reactance is greater than inductive reactance and, therefore, the circuit is capacitive in nature.

Hence, the correct answer is (A).

6. The current at any time t is given by

$$I=I_0\sin(\omega t+\phi)$$

$\Rightarrow \quad I=I_0\sin(2\pi ft+\phi)$

where $I_0=\sqrt{2}\times I_v=\sqrt{2}\times 2.21\ A=3.13\ A$

$\Rightarrow \quad I=3.13\sin(314t+1.32) \quad \{\because \phi=75.4°=1.32\ \text{rad}\}$

Hence, the correct answer is (A).

7. Effective voltage across inductor is

$$V_L=I_v\times X_L=2.21\ A\times 31.4\ \Omega=69.4\ \text{V}$$

Effective voltage across capacitor is

$$V_C=I_v\times X_C=2.21\ \text{A}\times 127.3\ \Omega=281.3\ \text{V}$$

Effective voltage across resistor is

$$V_R=I_vR=2.21\ \text{A}\times 25\ \Omega=55.3\ \text{V}$$

Hence, the correct answer is (A).

8. Impedance will be minimum, if $X_L=X_C$

$\Rightarrow \quad 2\pi fL=\dfrac{1}{2\pi fC}$

$\Rightarrow \quad L=\dfrac{1}{4\pi^2f^2C}$

$\Rightarrow \quad L=\dfrac{1\times 7\times 7}{4\times 22\times 22\times 50\times 50\times 25\times 10^{-6}}\ \text{H}$

$\Rightarrow \quad L=0.41\ \text{H}$

Hence, the correct answer is (D).

9. $I_0=\dfrac{E_0}{\sqrt{R^2\left(\dfrac{1}{\omega C}-\omega L\right)^2}}$

For maximum current

$$I_0=\frac{E_0}{R} \qquad \left\{\because \frac{1}{C\omega}-L\omega=0\right\}$$

Hence, the correct answer is (B).

10. $f_0=\dfrac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \quad f_0=\dfrac{250}{3}\ \text{Hz}$

Hence, the correct answer is (D).

12. $\dfrac{1}{2}CV^2=\dfrac{1}{2}LI^2$

$\Rightarrow \quad I=V\sqrt{\dfrac{C}{L}}$

Hence, the correct answer is (A).

13. $f=\dfrac{1}{2\pi\sqrt{LC}}$

Hence, the correct answer is (C).

14. $V_{\text{rms}}=\dfrac{V_m}{\sqrt{2}}=\dfrac{283}{\sqrt{2}}\ \text{V}=\dfrac{283}{1.414}\ \text{V}\approx 200\ \text{V}$

Hence, the correct answer is (C).

15. $X_L=\omega L=2\pi vL=2\times 3.14\times 50\times 25.48\times 10^{-3}\ \Omega$

$\Rightarrow \quad X_L=8\ \Omega$

Hence, the correct answer is (D).

16. $X_C=\dfrac{1}{\omega C}=\dfrac{1}{2\pi vC}$

$\Rightarrow \quad X_C=\dfrac{1}{2\times 3.14\times 50\times 7.96\times 10^{-4}}\approx\dfrac{1}{0.25}=4\ \Omega$

Hence, the correct answer is (B).

17. $Z=\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}=\sqrt{3^2+(8-4)^2}$

$\Rightarrow \quad Z=\sqrt{9+16}\ \Omega=\sqrt{25}\ \Omega=5\ \Omega$

Hence, the correct answer is (D).

18. $I_m=\dfrac{V_m}{Z}=\dfrac{283}{5}\text{A}=56.6\ \text{A}$

Hence, the correct answer is (A).

19. $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\frac{4}{3} = 53.13°$

Hence, the correct answer is (B).

20. $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{56.6}{1.414}\text{A} = 40\text{ A}$

$X_L > X_C$

So, the current lags the voltage

Hence, the correct answer is (D).

21. $(V_C)_{rms} = 4 \times 40\text{ V} = 160\text{ V}$

Hence, the correct answer is (B).

22. $(V_L)_{rms} = 8 \times 40\text{ V} = 320\text{ V}$

Hence, the correct answer is (C).

23. $(V_R)_{rms} = 3 \times 40\text{ V} = 120\text{ V}$

Hence, the correct answer is (A).

24. $P = I_{rms}^2 R = 40^2 \times 3\text{ W} = 4800\text{ W}$

Hence, the correct answer is (A).

25. Power factor $= \cos\phi = \cos 53.13° = 0.6$

Hence, the correct answer is (D).

26. Power input $= V_{rms} I_{rms} \cos\theta$

$\Rightarrow \quad P_{in} = 200 \times 40 \times 0.6\text{ W} = 4800\text{ W}$

It may be noted that the power input is the same as the power dissipated in the resistor. This is because the capacitor and the inductor do not absorb or produce any net power.

Hence, the correct answer is (A).

27. For resonance $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 7.96 \times 10^{-4}}}$$

$$\omega_0 = \frac{10^4}{\sqrt{2028}} = 222.1\text{ rads}^{-1}$$

$$\omega_0 = 2\pi v_0,\ v_0 = \frac{\omega_0}{2\pi} = \frac{222.1}{2 \times 3.14}\text{ Hz} = 35.4\text{ Hz}$$

Hence, the correct answer is (A).

28. At resonance condition

$Z = R = 3\ \Omega$

Hence, the correct answer is (C).

29. At resonance, $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{3} A$

$\Rightarrow \quad I_{rms} = 66.67\ A$

Hence, the correct answer is (A).

30. Power consumed at resonance is

$P = I_{rms}^2 R = 66.67 \times 66.67 \times 3\text{ W} = 13.33\text{ kW}$

Hence, the correct answer is (A).

31. $z = \frac{V_0}{I_0} = \frac{170}{8.5} = 20$, $\phi = \frac{\pi}{6}$ and

$\omega = 6280$

$\tan\phi = \frac{X_C}{R} = \frac{1}{R(C\omega)}$

$\Rightarrow \quad R = \sqrt{3}\frac{1}{C\omega}$

$\Rightarrow \quad C\omega = \frac{\sqrt{3}}{R}$

Since, $Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$

$\Rightarrow \quad Z = \sqrt{R^2 + \frac{R^2}{3}}$

$\Rightarrow \quad Z = \frac{2R}{\sqrt{3}}$

$\Rightarrow \quad R = 17.32\ \Omega$

Hence, the correct answer is (B).

32. $\frac{1}{C\omega} = 10$

$\Rightarrow \quad C = 15.92\ \mu\Omega$

Hence, the correct answer is (A).

33. $\cos\phi = \cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Hence, the correct answer is (C).

34. I_0 is maximum at $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}\text{ rad s}^{-1}$

$\Rightarrow \quad \omega_0 = 4167\text{ rad s}^{-1}$

CHAPTER 4

If f_0 is the frequency corresponding to resonant angular frequency ω_0, then $2\pi f_0 = \omega_0$

$$\Rightarrow \quad f_0 = \frac{\omega_0}{2\pi} = \frac{4167}{2\pi} \text{ Hz} = 663 \text{ Hz}$$

Hence, the correct answer is (D).

35. Maximum current is

$$I = \frac{E_0}{R} = \frac{\sqrt{2} \times 230}{23} \text{ ampere}$$

Hence, the correct answer is (D).

36. Average power $= \frac{1}{2} I_0^2 R$

It is maximum at the same frequency i.e., resonant frequency of 663 Hz for which I_0 is maximum.

Hence, the correct answer is (D).

37. Maximum average power

$$P = \frac{1}{2}(I_0)_{\max}^2 R = \frac{1}{2}(14.1)^2 \times 23 \text{ watt}$$

$$P = 2286 \text{ watt}$$

Hence, the correct answer is (C).

38. At $\omega \simeq \omega_0 \pm \frac{R}{2L}$ $\left\{\text{iff } \frac{R}{2L} \ll \omega_0\right\}$

$$\Rightarrow \quad \Delta\omega = \frac{R}{2L} = \frac{23}{0.24} \text{ rad s}^{-1} = 95.8 \text{ rad s}^{-1}$$

$$\Rightarrow \quad \Delta f = \frac{\Delta\omega}{2\pi} = \frac{95.8}{2 \times 3.14} \text{ Hz} = 15.3 \text{ Hz}$$

Power absorbed is half the peak power at

$$f = 648 \text{ Hz} \text{ and } f_2 = 678 \text{ Hz}$$

Hence, the correct answer is (A).

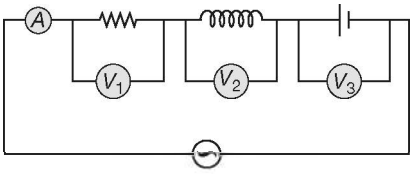
39. At these frequencies, current amplitude is $\frac{1}{\sqrt{2}}$ times $(I_0)_{\max}$ i.e., current amplitude (at half the peak power points) is 10 A.

Hence, the correct answer is (D).

40. $Q = \frac{\omega_0 L}{R} = \frac{4167 \times 0.12}{23} = 21.7$

Hence, the correct answer is (A).

41.



rms values of voltages are

$$V_R = 100 \text{ V}, \; V_L = 125 \text{ V} \text{ and } V_C = 150 \text{ V}$$

Since, $Z = \frac{V_{rms}}{I_{rms}}$

$$\Rightarrow \quad Z = \frac{\sqrt{(V_L - V_C)^2 + V_R^2}}{I_{rms}} = 5\sqrt{17} \; \Omega$$

Hence, the correct answer is (C).

42. Power factor $\cos\phi = \frac{R}{Z} = \frac{100}{26\sqrt{17}} = \frac{4}{\sqrt{17}}$

Hence, the correct answer is (A).

43. $V_{\text{rms}} = 25\sqrt{17} = \frac{E_0}{\sqrt{2}}$

$$\Rightarrow \quad E_0 = 25\sqrt{34} \text{ V}$$

Hence, the correct answer is (D).

44. If $E = E_0 \cos\omega t$, then

$$I = \frac{E_0}{\left|\omega L - \frac{1}{\omega C}\right|} \cos\left(\omega t \pm \frac{\pi}{2}\right), \text{ if } R \text{ is zero.}$$

If $\omega L > \frac{1}{\omega C}$, then –ve sign appears. However, if $\omega L < \frac{1}{\omega C}$, then +ve sign appears.

$$L\omega - \frac{1}{C\omega} = 2 \times \pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}$$

$$\Rightarrow \quad X_L - X_C = 25.12 - 53.08 = -27.96 \; \Omega \simeq -28 \; \Omega$$

The negative sign indicates that the reactance is capacitive in nature.

Current amplitude,

$$I_0 = \frac{E_0}{28} = \frac{1.414 \times 230}{28} \text{ A} = 11.6 \text{ A}$$

Hence, the correct answer is (D).

45. $I_v = \frac{230}{28}$ ampere $= 8.2$ A

Hence, the correct answer is (D).

46. $(E_L)_v = I_v(L\omega) = 25.12 \times 8.2 \text{ volt} \simeq 206 \text{ V}$

Hence, the correct answer is (B).

47. $(E_C)_v = I_v\left(\frac{1}{C\omega}\right) = 53.08 \times 8.2 \text{ volt} = 435.3 \text{ V}$

Hence, the correct answer is (D).

48. Applied rms voltage is

$$E_V = (435.3 - 206) \text{ volt} = 229.3 \text{ V}$$

(The voltages across L and C get subtracted because they are $180°$ out of phase).

Hence, the correct answer is (A).

49. Whatever the value of current in the inductor, the actual voltage leads the current by $\frac{\pi}{2}$.

So, the average power consumed by L is zero.

Hence, the correct answer is (A).

50. For C, voltage lags by $\frac{\pi}{2}$. So, average power consumed by C is zero.

Hence, the correct answer is (A).

51. Total average power absorbed by the circuit is zero.

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (s)
C → (t)
D → (q)

For a series LCR circuit, current lags behind the voltage when $X_L > X_C$, so we have

$$\tan\phi = \frac{X_L - X_C}{R}$$

Also, $V_L = 2\ V_C$

$\Rightarrow \quad IX_L = 2IX_C$

$\Rightarrow \quad X_L = 2X_C$

So, for $\phi = 45°$, we get

$$\tan(45°) = 1 = \frac{2X_C - X_C}{R}$$

$\Rightarrow \quad X_C = R = 20\ \Omega$ and $X_L = 40\ \Omega$

So, (A) → (r) and (B) → (s)
Further

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(20)^2 + (20)^2}$$

$\Rightarrow \quad Z = 20\sqrt{2}\ \Omega$

So, (C) → (t)

Further, when $\phi = 60°$, then

$$\tan(60°) = \frac{X_L - X_C}{R}$$

$\Rightarrow \quad X_L - X_C = \sqrt{3}R$

$\Rightarrow \quad X_L - X_C = 20\sqrt{3}\ \Omega$

$\Rightarrow$ (D) → (q)

2. A → (q)
B → (p)
C → (r)
D → (s)
The resistance is given by

$$R = \frac{V_R}{I} = \frac{40}{2} = 20\ \Omega$$

Since, we know that

$$V_C = IX_C = 2 \times 30 = 60 \text{ V}$$

$$V_L = IX_L = 2 \times 15 = 30 \text{ V}$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2} = 50 \text{ V}$$

3. A → (r)
B → (q)
C → (s)
D → (p)
For a non-ideal inductor, the voltage leads the current. For both ideal inductor and capacitor, phase angle between voltage and current is $90°$.
For series LCR circuit, impedance is minimum at resonance, so there is no phase lag between the voltage and current.
For a non-ideal capacitor, the current leads the voltage.

4. A → (q, s)
B → (r, s)
C → (r, s)
D → (r, s)

Since, $P = I_{\text{rms}}^2 R$, where $I_{\text{rms}} = \frac{E_{\text{rms}}}{Z}$ and the impedance Z is given by $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

By increasing R, current I_{rms} will decrease but the power, $P = I_{\text{rms}}^2 R$ may increase or decrease. Similarly when C (or L) is increased, then Z may increase or decrease and hence P may increase or decrease.

5. A → (p, t)
B → (q)
C → (r, t)
D → (s, t)
For (A), the phase difference between voltage and current is $\phi = 0°$. This is possible when the circuit is only resistive in nature.

CHAPTER 4

For (B), $I = I_0 \sin(\omega t - 90°)$ i.e. voltage leads the current by $\phi = 90°$, which is possible only for an ideal inductor.

For (C), $I = I_0 \sin\left(\omega t + \frac{\pi}{6}\right)$ i.e. current leads the voltage by $\phi = 30°$. This is possible only when circuit is capacitive in nature

For (D), $I = I_0 \sin\left(\omega t - \frac{\pi}{3}\right)$ i.e. voltage leads the current by $\phi = 60°$. This is possible only when the circuit is inductive in nature.

Integer/Numerical Answer Type Questions

1. (a) At resonance frequency $X_L = X_C$, so we have

$$Z = R$$

Since $\cos\phi = \frac{R}{Z}$

$$\Rightarrow \quad \cos\phi = 1$$

(b) $\langle P \rangle = V_{rms} I_{rms} \cos\phi = \left(\frac{300}{\sqrt{2}}\right) \times \frac{\left(\frac{300}{\sqrt{2}}\right)}{300} \times 1$

$$\Rightarrow \quad \langle P \rangle = 150 \text{ W}$$

(c) Average power delivered is still 150 W, because still we have $Z = R$ and $\cos\phi = 1$.

2. Inductive reactance of the inductor is

$$X_L = 2\pi f L = 100\ \Omega$$

Impedance of the circuit is

$$Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow \quad Z = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I_{rms} = \frac{E_{rms}}{Z} = \frac{220}{100\sqrt{2}} = 1.1\sqrt{2} = 1.56 \text{ A}$$

3. The coil consists of an inductance (L) and a resistance (R) and for the dc input only resistance is effective. So,

$$R = \frac{V}{I} = \frac{12}{4} = 3\ \Omega$$

For AC, $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$

$$\Rightarrow \quad L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2\right]$$

$$\Rightarrow \quad L = \frac{1}{\omega}\sqrt{\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2}$$

Substituting the values, we get

$$L = \frac{1}{50}\sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2} = 0.08 \text{ H}$$

$$\Rightarrow \quad L = 80 \text{ mH}$$

When capacitor is connected to the circuit, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here $R = 3\ \Omega$, $X_L = \omega L = (50)(0.08) = 4\ \Omega$ and

$$X_C = \frac{1}{\omega C} = \frac{1}{(50)(2500 \times 10^{-6})} = 8\ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(3)^2 + (4-8)^2} = 5\ \Omega$$

Now, $\langle P \rangle = V_{rms} I_{rms} \cos\phi = (V_{rms})\left(\frac{V_{rms}}{Z}\right)\left(\frac{R}{Z}\right)$

$$\langle P \rangle = \left(\frac{V_{rms}}{Z}\right)^2 R$$

Substituting the values, we get

$$\langle P \rangle = \left(\frac{12}{5}\right)^2 \times 3$$

$$\Rightarrow \quad \langle P \rangle = 17.28 \text{ W}$$

$$\Rightarrow \quad \langle P \rangle = 17 \text{ W}$$

4. Since an AC voltmeter reads the rms value of voltage. The applied rms source voltage is

$$V = V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{50\sqrt{2}}{\sqrt{2}} = 50 \text{ V}$$

Let the reading of voltmeter V_2 be x, then the combined reading of voltmeters V_1 and V_2 equals the applied rms source voltage, so we have

$$\sqrt{x^2 + 40^2} = 50 \text{ V}$$

$$\Rightarrow \quad x = 30 \text{ V}$$

5. $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L}$$

$$\Rightarrow \quad C = \frac{1}{(40)(25 \times 10^{12})(2 \times 10^{-6})}$$

$$\Rightarrow \quad C = 500 \times 10^{-12} \text{ F} = 500 \text{ pF}$$

6. When capacitor is removed, then the circuit is simply an LR circuit, so we have

$$\tan\phi = \frac{X_L}{R}$$

$$\Rightarrow \quad \tan\phi = \tan 45° = \frac{X_L}{R}$$

$$\Rightarrow \quad X_L = R$$

When the inductor is removed, then the circuit simply becomes a CR circuit and then

$$\tan\phi' = \frac{X_C}{R} = \frac{1}{2}$$

$$\Rightarrow \quad X_C = \frac{R}{2}$$

So, the impedance of the complete LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + \left(R - \frac{R}{2}\right)^2} = \frac{\sqrt{5}}{2}R$$

Since, $I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{200}{250} = 0.8 \text{ A}$ and $\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$

$$\Rightarrow \quad P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = 64\sqrt{5} \text{ W} = 143 \text{ W}$$

7. Since $V_0 = 100\sqrt{2}$ volt, so the rms value of voltage across the source is

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

Since $\omega = 1000 \text{ rads}^{-1}$ and the impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where, $X_L = L\omega = (2)(1000) = 2000\ \Omega$ and

$$X_C = \frac{1}{C\omega} = \frac{1}{(10^{-6})(1000)} = 1000\ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(1000)^2 + (2000 - 1000)^2} = 1000\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|} = \frac{100}{1000\sqrt{2}} = \frac{1}{10\sqrt{2}} \text{ A}$$

$$\Rightarrow \quad I_{\text{rms}} = 0.0707 \text{ A}$$

The current will be same everywhere in the circuit, therefore, if potential difference across the inductor, capacitor and resistor is denoted by V_L, V_C and V_R respectively, then

$$V_L = (I_{\text{rms}})X_L = 141 \text{ V}$$

$$V_C = (I_{\text{rms}})X_C = 71 \text{ V}$$

$$V_R = (I_{\text{rms}})R = 71 \text{ V}$$

8. The resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}, \text{ where}$$

$$L = 0.4 \text{ mH} = 0.4 \times 10^{-3} \text{ H and}$$

$$C = 400 \text{ pF} = 4 \times 10^{-10} \text{ F}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{0.4 \times 10^{-3} \times 4 \times 10^{-10}}}$$

$$\Rightarrow \quad f = \frac{10^7}{8\pi} \text{ Hz}$$

The speed of electromagnetic wave produced is

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\Rightarrow \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{\left(\frac{10^7}{8\pi}\right)} = 240\pi$$

$$\Rightarrow \quad * = 240$$

9. Since, at resonance, the angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Now, when $\omega = 2\omega_0 = \frac{2}{\sqrt{LC}}$, then

$$X_L = L\omega = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} \text{ and}$$

$$X_C = \frac{1}{C\omega} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Since by definition, we have

$$Z^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow \quad Z^2 = R^2 + 2.25\left(\frac{L}{C}\right)$$

Further, we know that

$$P = \left(\frac{V^2}{Z}\right)\cos\phi = \left(\frac{V^2}{Z}\right)\left(\frac{R}{Z}\right) = \frac{V^2 R}{Z^2}$$

$$\Rightarrow \quad P = \frac{V^2 R}{R^2 + 2.25\left(\frac{L}{C}\right)}$$

So, energy delivered to the circuit in one cycle is

$$E = PT = P\left(\frac{2\pi}{\omega}\right) = \left(\frac{2\pi RCV^2}{R^2C + 2.25L}\right)\frac{\sqrt{LC}}{2}$$

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$$\Rightarrow \quad E = \frac{4\pi RCV^2\sqrt{LC}}{4R^2C + 9L}$$

Substituting values, we get

$$E = 242 \text{ mJ}$$

10. For DC supply, we have

$$I = \frac{V}{R}$$

$$\Rightarrow \quad R = \frac{V}{I} = 50\ \Omega$$

For AC supply, we have

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$\Rightarrow \quad 1 = \frac{100}{Z}$$

$$\Rightarrow \quad Z = 100\ \Omega$$

Since, we know that for a series LR circuit, the impedance Z is

$$Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow \quad X_L = 50\sqrt{3}\ \Omega \approx 87\ \Omega$$

11. The power factor $\cos\phi$ is given by

$$\cos\phi = \frac{R}{Z}$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow \quad \cos\phi = \frac{60}{\sqrt{60^2 + (100 - 20)^2}}$$

$$\Rightarrow \quad \cos\phi = \frac{60}{100} = 0.6$$

12. At resonance, the current in the circuit is

$$I = \frac{E}{Z_{\min}} = \frac{E_R}{R} = \frac{60}{120} = 0.5 \text{ A}$$

Voltage across inductor is

$$V_L = IX_L = (0.5)(L\omega) = 40 \text{ V}$$

$$\Rightarrow \quad L = \frac{40}{(0.5)(4000)} = 0.02 \text{ H}$$

$$\Rightarrow \quad L = 20 \text{ mH}$$

At resonance, we have

$$X_L = X_C$$

$$\Rightarrow \quad L\omega_0 = \frac{1}{C\omega_0}, \text{ where } \omega_0 = 4000 \text{ rads}^{-1}$$

$$\Rightarrow \quad C = \frac{1}{L\omega_0^2} = \frac{1}{(20 \times 10^{-3})(4000)^2}$$

$$\Rightarrow \quad C = \frac{1}{32} \times 10^{-4} = \frac{25}{8}\ \mu\text{F} = 3.125\ \mu\text{F}$$

13. The impedance of the circuit is given by

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$$

$$\Rightarrow \quad Z = \sqrt{(12)^2 + 4\pi^2 (50)^2 \left(\frac{0.05}{\pi}\right)^2}$$

$$\Rightarrow \quad Z = \sqrt{144 + 25} = 13\ \Omega$$

Current in the circuit is given by

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{130}{13} = 10 \text{ A}$$

Potential difference across resistance is

$$V_R = I_{\text{rms}} R = 10 \times 12 = 120 \text{ V}$$

Inductive reactance of coil is

$$X_L = L\omega = 2\pi fL$$

$$\Rightarrow \quad X_L = 2\pi(50)\left(\frac{0.05}{\pi}\right) = 5\ \Omega$$

Potential difference across inductance is

$$V_L = I_{\text{rms}} X_L = 10 \times 5 = 50 \text{ V}$$

14. From the given voltage and current equations, we conclude that current leads the voltage by a phase angle ϕ given by

$$\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4} = 45°$$

So, the power factor is given by

$$\cos\phi = \cos 45° = \frac{1}{\sqrt{2}}$$

Average power $\langle P \rangle$ dissipated in the circuit is

$$\langle P \rangle = \frac{E_0 I_0}{2}\cos\phi = \frac{(10)(2)}{2}\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \langle P \rangle = 5\sqrt{2} \text{ W} \approx 7 \text{ W}$$

15. Required current for bulb is

$$I_{\text{rms}} = \frac{P}{E_{\text{rms}}} = \frac{10 \text{ W}}{30 \text{ V}} = \frac{1}{3} \text{ A}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{E_{\text{rms}}}{X_C} = \frac{1}{3} \text{ A}$$

$$\Rightarrow \quad I_{rms} = \frac{E_{rms}}{\left(\frac{1}{2\pi fC}\right)} = E_{rms}(2\pi fC)$$

$$\Rightarrow \quad C = \frac{I_{rms}}{2\pi f E_{rms}} = \frac{1}{3(220)2\pi(50)}$$

$$\Rightarrow \quad C = \frac{1}{660 \times \pi \times 100}$$

$$\Rightarrow \quad C = \frac{1}{66 \times 10^3 \times \pi}\ \text{F} = 4.82\ \mu\text{F}$$

$$\Rightarrow \quad C \approx 4.8\ \mu\text{F}$$

16. Since the power consumed across a resistor is

$$P = E_{rms} I_{rms} = \frac{E_{rms}^2}{R}$$

$$\Rightarrow \quad R = \frac{E_{rms}^2}{P} = \frac{220 \times 220}{200} = \frac{22 \times 22}{2} = \frac{484}{2}$$

$$\Rightarrow \quad R = 242\ \Omega$$

The rms value of current is

$$I_{rms} = \frac{P}{E_{rms}} = \frac{200}{220} = \frac{10}{11}\ \text{A} = 0.9\ \text{A}$$

17. The circuit can be thought of as a parallel combination of $3L$, C and R.

Since we have been given that each capacitor has a reactance of $4R$, so we have

$$4R = \frac{1}{(C/4)\omega}$$

$$\Rightarrow \quad R = \frac{1}{C\omega}$$

So, for four capacitors in parallel the equivalent capacitive reactance is

$$X_C = \frac{1}{C_{eq}\omega} = \frac{1}{C\omega} = R$$

Since we have been given that each inductor has a reactance of $\frac{R}{3}$, so we have

$$\frac{R}{3} = L\omega$$

$$\Rightarrow \quad R = (3L)\omega$$

So, for three inductors in series the equivalent inductive reactance is

$$\Rightarrow \quad X_L = L_{eq}\omega = (3L)\omega = R$$

Reading of the ammeter is

$$I_{rms} = E_{rms}\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Since here we see that $X_L = X_C = R$, so we have

$$I_{rms} = \frac{E_{rms}}{R} = \frac{10}{5} = 2\ \text{A}$$

So, the reading of the ammeter is 2 A

18. When current is in phase with the voltage, then we get the condition of resonance. So, we have

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$4\pi^2 f_0^2 LC = 1$$

$$\Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L}$$

$$\Rightarrow \quad C = \frac{1}{4(3.14)^2(50)^2(40 \times 10^{-3})} = 250\ \mu\text{F}$$

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1. Since $I = I_m \sin(100\pi t)$

$$\Rightarrow \quad \frac{I_m}{2} = I_m \sin(100\pi t_1)$$

$$\Rightarrow \quad \frac{\pi}{6} = 100\pi t_1$$

$$\Rightarrow \quad t_1 = \frac{1}{600}\ \text{s}$$

Since $w = 100\pi$

$$\Rightarrow \quad T = \frac{2\pi}{100\pi} = \frac{1}{50}\ \text{s}$$

So, $t_{req} = \frac{T}{4} - t_1$

$$\Rightarrow \quad t_{req} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{\text{i}}{300}\ \text{s} = 3.3\ \text{ms}$$

Hence, the correct answer is (D).

2. As $\phi = \frac{\pi}{4}$, $x_c = R$

Hence, the correct answer is (C).

3. $Q = 1 \times 4200 \times 80 + 2260 \times 10^3\ \text{J}$

$$\Rightarrow \quad Q = (336 + 2260) \times 10^3\ \text{J} = 2596 \times 10^3\ \text{J}$$

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Since $Q = I_{rms} V_{rms} t = 200 \times \frac{200}{20} t = 2000t$

$\Rightarrow \quad t = \frac{2596}{2}$ s $\approx$ 21.6 minute $\approx$ 22 minute

Hence, the correct answer is (C).

4. Inductive Reactance

$$X_L = L\omega = (20 \times 10^{-3})(2\pi \times 50) = 2\pi\ \Omega = 6.28\ \Omega$$

Capacitive Reactance

$$X_C = \frac{1}{C\omega} = \frac{1 \times 10^6}{(120)(2\pi \times 50)} = \frac{1000}{2\pi \times 6}\ \Omega = 26.53\ \Omega$$

$\Rightarrow \quad Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\Rightarrow \quad |Z| = \sqrt{(60)^2 + (20.25)^2} = 63.32\ \Omega$

$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{24}{63.32}$

Since, $\cos\phi = \frac{R}{Z} = \frac{60}{63.32}$

$\Rightarrow \quad \Delta E = \frac{24}{63.32} \times 24 \times \frac{60}{63.32} \times 60 = 5.17 \times 10^2$ J

Hence, the correct answer is (B).

5. Power input is $P_{input} = (V_p)(I_p) = 2300 \times 5$ W

Since $\eta = \frac{P_{out}}{P_{in}} = 0.9$

$\Rightarrow \quad P_{output} = 0.9 \quad P_{input} = V_S I_S$

$\Rightarrow \quad I_S = \frac{0.9 \times 2300 \times 5}{230} = 45$ A

Hence, the correct answer is (C).

6. Capacitive reactance is

$$X_C = \frac{1}{C\omega} = \frac{1 \times 2}{100 \times \sqrt{3}} \times 10^6 = \frac{20}{\sqrt{3}}\ \text{k}\Omega$$

Inductive reactance is

$$X_L = L\omega = 10\sqrt{3}\ \Omega$$

For the C-R_2 path, since, $X_C \gg R_2$, so we have $\phi_2 \to 90°$ and hence I_2 leads V by $90°$

For the L-R_1 path, we have $\tan\phi_1 = \frac{L\omega}{R_1} = \sqrt{3}$ i.e. $\phi_1 = 60°$, hence I_1 lags V by $60°$

So, phase difference between I_1 and I_2 is $150°$.

*No given option is correct.

Please note that, if we had been given $R_2 = 20\ \text{k}\Omega$ instead of $R_2 = 20\ \Omega$, then we would have got $\phi_2 = \tan^{-1}\left(\frac{X_C}{R_2}\right) = 30°$ and then our result would have been $30° + 60° = 90°$

7. Average power, $P_{av} = e_{rms} i_{rms} \cos\phi = \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos\phi$

where, $e_0 = 100$, $i_0 = 20$, $\phi = \frac{\pi}{4}$

$\Rightarrow \quad P_{av} = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \cos 45° = \frac{1000}{\sqrt{2}}$ units

Wattless current is given by

$$i_w = i_{rms} \sin\phi = \frac{i_0}{\sqrt{2}} \sin 45° = 10 \text{ units}$$

Hence, the correct answer is (B).

8. Quality factor is given by

$$Q = \frac{L\omega_0}{R}$$

Hence, the correct answer is (A).

9. Using Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Electrostatic Energy}\\ \text{of capacitor}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{Magnetic Energy}\\ \text{of Inductor}\end{pmatrix}$$

$\Rightarrow \quad \frac{1}{2}(0.2 \times 10^{-6})[(10)^2 - (5)^2] = \frac{1}{2}(0.5 \times 10^{-3}) I^2$

$\Rightarrow \quad I = \sqrt{3} \times 10^{-1}$ A $= 0.17$ A

Hence, the correct answer is (B).

10. Efficiency of the transformer is

$$\eta = 0.9 = \frac{P_s}{P_p}$$

$\Rightarrow \quad V_s I_s = 0.9 \times V_p I_p$

$\Rightarrow \quad I_s = \frac{0.9 \times 2300 \times 5}{230} = 45$ A

Hence, the correct answer is (C).

11. According to the problem, we have

$E_0 = 283$ V, $\omega = 320\ \text{s}^{-1}$, $R = 5\ \Omega$,

$L = 25$ mH $= 25 \times 10^{-3}$ H, $C = 1000\ \mu\text{F} = 10^{-3}$ F

$$\Rightarrow \quad X_L = \omega L = 320 \times 25 \times 10^{-3} = 8\ \Omega$$

$$\Rightarrow \quad X_C = \frac{1}{\omega C} = \frac{1}{320 \times 10^{-3}} = \frac{1000}{320} = 3.125\ \Omega$$

Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

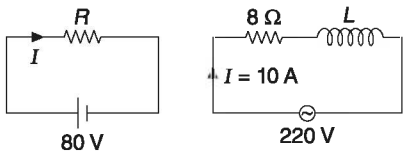
$$\Rightarrow \quad Z = \sqrt{5^2 + (8 - 3.125)^2} \approx \sqrt{49} = 7\ \Omega$$

Since $X_L > X_C$, so the voltage leads the current by a phase angle

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{4.875}{5}\right) \approx 45°$$

Hence, the correct answer is (D).

12. For a dc source $I = 10$ A, $V = 80$ V



Resistance of the arc lamp is

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$$

For an ac source,

$$e_{rms} = 220\ \text{V}$$

$$\nu = 50\ \text{Hz}$$

$$\omega = 2\pi \times 50 = 100\pi\ \text{rads}^{-1}$$

Arc lamp will glow if $I_{rms} = 10$ A,

$$\Rightarrow \quad I_{rms} = \frac{E_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow \quad R^2 + \omega^2 L^2 = \left(\frac{E_{rms}}{I_{rms}}\right)^2$$

$$\Rightarrow \quad 8^2 + (100\pi)^2 L^2 = \left(\frac{220}{10}\right)^2$$

$$\Rightarrow \quad L^2 = \frac{22^2 - 8^2}{(100\pi)^2}$$

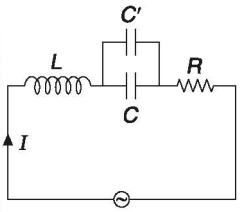
$$\Rightarrow \quad L = \frac{\sqrt{30 \times 14}}{100\pi} = 0.065\ \text{H}$$

Hence, the correct answer is (D).

13. Current in LR circuit is $I = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \frac{\pi}{2}\right)$, i.e., it is sinusoidal in nature.

Hence, the correct answer is (D).

14. Since the current is observed to lead the applied voltage, so the circuit is capacitive in nature. Now to make the power factor 1, effective capacitance should be increased thus the capacitor should be connected in parallel.



$$\cos\phi = 1$$

$$\Rightarrow \quad \phi = 0$$

$$\Rightarrow \quad L\omega = \frac{1}{(C + C')\omega}$$

$$\Rightarrow \quad C + C' = \frac{1}{\omega^2 L}$$

$$\Rightarrow \quad C' = \frac{1}{\omega^2 L} - C$$

$$\Rightarrow \quad C' = \frac{1 - \omega^2 LC}{\omega^2 L} \text{ in parallel}$$

Hence, the correct answer is (D).

15. Given that $R = 200\ \Omega$, $V_{rms} = 220$ V, $\nu = 50$ Hz

When only the capacitor is removed, the phase difference between the current and voltage is $\tan\phi = \frac{X_L}{R}$

$$\Rightarrow \quad \tan 30° = \frac{X_L}{R}$$

$$\Rightarrow \quad X_L = \frac{1}{\sqrt{3}} R$$

When only the inductor is removed, the phase differ

$$\tan 30° = \frac{X_C}{R}$$

$$\Rightarrow \quad X_C = \frac{1}{\sqrt{3}} R$$

Since we see that $X_L = X_C$, therefore the given series LCR circuit is in resonance. Hence the impedance of the circuit is minimum and is given by

$$Z = R = 200\ \Omega$$

The power dissipated in the circuit is

$$P = V_{rms} I_{rms} \cos\phi, \text{ where } I_{rms} = \frac{V_{rms}}{Z}$$

$$\Rightarrow \quad P = \frac{V_{rms}^2}{Z} \cos\phi$$

At resonance, power factor $\cos\phi = 1$

$$\Rightarrow \quad P = \frac{V_{rms}^2}{Z} = \frac{(220\ \text{V})^2}{(200\ \Omega)} = 242\ \text{W}$$

Hence, the correct answer is (A).

CHAPTER 4

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Single Correct Choice Type Problems

1. $I_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}}$

when ω increases, I_{rms} increases so the bulb glows brighter.

Hence, the correct answer is (B).

2. As the current i leads the emf e by $\dfrac{\pi}{4}$, it is an R-C circuit.

$$\tan\phi = \frac{X_C}{R}$$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \frac{\dfrac{1}{\omega C}}{R}$$

$$\Rightarrow \quad \omega CR = 1$$

As $\omega = 100$ rads^{-1}

The product of RC should be $\dfrac{1}{100}$ s^{-1}

Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. At $\omega \approx 0$, $X_C = \dfrac{1}{\omega C} \to \infty$. So, current is nearly zero.

Further at resonance, current and voltage are in phase. This resonance frequency is given by,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}\times 10^{-6}}} = 10^6 \text{ rads}^{-1}$$

So, we observe that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$

At, $\omega = \omega_0 = 10^6$ rads^{-1}, $X_L = X_C$

For $\omega > \omega_0$, $X_L > X_C$ and hence circuit is inductive in nature.

Hence, (A) and (B) are correct.

2. The potential difference between the points X and Y is

$$V_{XY} = V_X - V_Y$$

If ϕ be the phase difference between the two voltages, then the peak value of this potential difference is

$$(V_{XY})_0 = \sqrt{(V_X)_0^2 + (V_Y)_0^2 - 2(V_X)_0(V_Y)_0\cos\phi}$$

From the problem, we see that the phase difference between V_X and V_Y is $\dfrac{2\pi}{3}$, so we have

$$(V_{XY})_0 = \sqrt{V_0^2 + V_0^2 - 2V_0^2\cos\left(\frac{2\pi}{3}\right)} = \sqrt{3}V_0$$

Similarly, we have phase difference between V_Y and V_Z to be $\dfrac{2\pi}{3}$ and hence

$$(V_{XY})_0 = (V_{YZ})_0 = (V_{ZX})_0 = \sqrt{3}V_0$$

$$\Rightarrow \quad (V_{XY})_{rms} = (V_{YZ})_{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}} = \sqrt{\frac{3}{2}}V_0$$

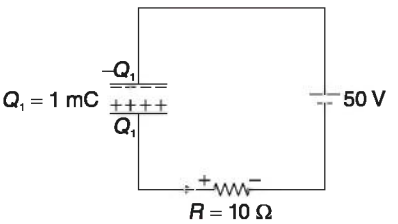
Hence, (B) and (C) are correct.

3. Since $I = \dfrac{dQ}{dt}$

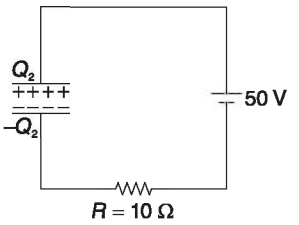
$$\Rightarrow \quad Q = \int Idt = \int (I_0\cos\omega t)dt$$

$$\Rightarrow \quad Q_{max} = \frac{I_0}{\omega} = \frac{1}{500} = 2\times 10^{-3} \text{ C} = 2 \text{ mC}$$

JUST AFTER SWITCHING



IN STEADY STATE



At $t = \dfrac{7\pi}{6\omega}$, $I(t)$ = NEGATIVE

So, current is anti-clockwise sense. Charge supplied by source from $t = 0$ to $t = \dfrac{7\pi}{6\omega}$ is

$$Q=\int_0^{\frac{7\pi}{6\omega}}\cos(500t)\,dt=\left(\frac{\sin 500t}{500}\right)\Bigg|_0^{\frac{7\pi}{6\omega}}$$

$$\Rightarrow\quad Q=\frac{\sin\left(\frac{7\pi}{6}\right)}{500}=-1\text{ mC}$$

Apply Kirchhoff's loop law just after changing the switch to position D, we get

$$50+\frac{Q_1}{C}-IR=0$$

Substituting the values of Q_1, C and R, we get

$$I=10\text{ A}$$

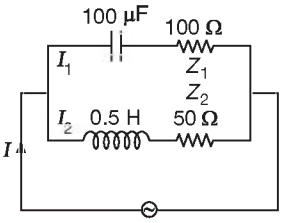
In steady state, we have

$$Q_2=CV=1\text{ mC}$$

So, net charge flown from battery is $\Delta q=2$ mC

Hence, (C) and (D) are correct.

4.



CIRCUIT 1:

The capacitive reactance is

$$X_C=\frac{1}{\omega C}=100\ \Omega$$

$$\Rightarrow\quad Z_1=\sqrt{(100)^2+(100)^2}$$

$$\Rightarrow\quad Z_1=100\sqrt{2}\ \Omega$$

$$\Rightarrow\quad \phi_1=\cos^{-1}\left(\frac{R_1}{Z_1}\right)=45^\circ$$

In this circuit, current leads the voltage by 45°.
So, we have

$$I_1=\frac{V}{Z_1}=\frac{20}{100\sqrt{2}}=\frac{1}{5\sqrt{2}}\text{ A}$$

$$\Rightarrow\quad V_{100\,\Omega}=(100)I_1=(100)\frac{1}{5\sqrt{2}}\text{ V}=10\sqrt{2}\text{ V}$$

CIRCUIT 2:

The inductive reactance is

$$X_L=\omega L=(100)(0.5)=50\ \Omega$$

$$\Rightarrow\quad Z_2=\sqrt{(50)^2+(50)^2}=50\sqrt{2}\ \Omega$$

$$\Rightarrow\quad \phi_2=\cos^{-1}\left(\frac{R_2}{Z_2}\right)=45^\circ$$

In this circuit, voltage leads the current by 45°.
So, we have

$$I_2=\frac{V}{Z_2}=\frac{20}{50\sqrt{2}}=\frac{\sqrt{2}}{5}\text{ A}$$

$$\Rightarrow\quad V_{50\,\Omega}=(50)I_2=50\left(\frac{\sqrt{2}}{5}\right)=10\sqrt{2}\text{ V}$$

Since phase difference between the currents I_1 and I_2 is 90°, so we have

$$I=\sqrt{I_1^2+I_2^2}=0.34$$

Hence, (A) and (C) are correct.

5.

$$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$$

In case (B), capacitance C will be more. Therefore, impedance Z will be less. Hence, current will be more.

Further, $V_C=\sqrt{V^2-V_R^2}$

$$\Rightarrow\quad V_C=\sqrt{V^2-(IR)^2}$$

In case (B), since current I is more. Therefore, V_C will be less.

Hence, (B) and (C) are correct.

Linked Comprehension Type Questions

1. The power consumed is

$$P=Vi$$

$$\Rightarrow\quad i=\frac{P}{V}=\frac{600\times10^3}{4000}=150\text{ A}$$

Total resistance of cables is

$$R=0.4\times20=8\ \Omega$$

SO, power loss in cables is

$$P=i^2R=(150)^2(8)=180000\text{ W}=180\text{ kW}$$

This loss is 30% of 600 kW

Hence, the correct answer is (B).

2. For a step-up transformer, we have

$$\frac{N_p}{N_s}=\frac{V_p}{V_s}$$

$$\Rightarrow\quad \frac{1}{10}=\frac{4000}{V_s}$$

CHAPTER 4

$\Rightarrow \quad V_s = 40000 \text{ V}$

For a step-down transformer, we have

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = \frac{200}{1}$$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r, s, t)
B → (q, r, s, t)
C → (q, p)
D → (q, r, s, t)

For Circuit (p):

I cannot be non-zero in steady state

For Circuit (q):

$V_1 = 0$ and $V_2 = 2I = V$ (also)

For Circuit (r):

$$V_1 = IX_L = I(2\pi fL)$$

$$\Rightarrow \quad V_1 = (2\pi \times 50 \times 6 \times 10^{-3})I = 1.88I$$

$$\Rightarrow \quad V_2 = 2I$$

For Circuit (s):

$V_1 = IX_L = 1.88I$ and

$$V_2 = IX_C = I\left(\frac{1}{2\pi fC}\right)$$

$$\Rightarrow \quad V_2 = \left(\frac{1}{2\pi \times 50 \times 3 \times 10^{-6}}\right)I = (1061)I$$

For Circuit (t):

$V_1 = IR = (1000)I$ and $V_2 = X_C I = (1061)I$

Integer/Numerical Answer Type Questions

1. For the circuit, impedance is

$$Z = \sqrt{R^2 + X_C^2} = R\sqrt{1.25}$$

$$\Rightarrow \quad R^2 + X_C^2 = 1.25R^2$$

$$\Rightarrow \quad X_C = \frac{R}{2}$$

$$\Rightarrow \quad \frac{1}{\omega C} = \frac{R}{2}$$

$$\Rightarrow \quad \text{Time constant} = CR = \frac{2}{\omega} = \frac{2}{500}\text{s} = 4 \text{ ms}$$

CHAPTER 5: ELECTROMAGNETIC WAVES

Single Correct Choice Type Questions

1. $Z = \sqrt{\frac{\mu_r}{\varepsilon_r}} \times \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\Rightarrow \quad Z = \sqrt{\frac{50}{2}} \times 376.6\ \Omega = 1883\ \Omega$

Hence, the correct answer is (C).

2. Since, $\frac{dE}{dz} = -\frac{dB}{dt}$

$\Rightarrow \quad \frac{dE}{dz} = -2E_0 k \sin kz \cos \omega t = -\frac{dB}{dt}$

$\Rightarrow \quad dB = +2E_0 k \sin kz \cos \omega t dt$

$\Rightarrow \quad B = +2E_0 k \sin kz \int \cos \omega t dt$

$\Rightarrow \quad B = +2E_0 \frac{k}{\omega} \sin kz \sin \omega t$

Also, $\frac{E_0}{B_0} = \frac{\omega}{k} = c$

$\Rightarrow \quad B = \frac{2E_0}{c} \sin kz \sin \omega t$

$\Rightarrow \quad \vec{B} = \frac{2E_0}{c} \sin kz \sin \omega t \hat{j}$

Hence, the correct answer is (C).

3. $v = \frac{c}{n} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$\Rightarrow \quad v = \frac{3 \times 10^8}{\sqrt{1.3 \times 2.14}} = 1.8 \times 10^8\ \text{ms}^{-1}$

Hence, the correct answer is (D).

4. $\nu = 1\ \text{BHz} = 10^9\ \text{Hz}$

$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = 0.3\ \text{m} = 30\ \text{cm}$ (radio waves)

Hence, the correct answer is (B).

5. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ are mutually perpendicular to each other and are in phase i.e. they become zero and minimum at the same place and at the same time.
Hence, the correct answer is (C).

6. Frequency $\nu = \frac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \quad \nu = \frac{1}{2 \times 3.14 \times \sqrt{100 \times 10^{-6} \times 400 \times 10^{-12}}}$

$\Rightarrow \quad \nu = \frac{1}{2 \times 3.14 \times 2 \times 10^{-7}}$

$\Rightarrow \quad \lambda = \frac{c}{\nu} = 3 \times 10^8 \times 2 \times 3.14 \times 2 \times 10^{-7} = 377\ \text{m}$

Hence, the correct answer is (C).

7. $u = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} B_{\text{rms}}^2$

$\Rightarrow \quad u = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}\left(\frac{E_{\text{rms}}^2}{c^2}\right) = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} E_{\text{rms}}^2 \varepsilon_0 \mu_0$

$\Rightarrow \quad u = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 = \varepsilon_0 E_{\text{rms}}^2$

$\Rightarrow \quad u = (8.85 \times 10^{-12}) \times (720)^2 = 4.58 \times 10^{-6}\ \text{Jm}^{-3}$

Hence, the correct answer is (B).

9. $I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{EA}{t} = \varepsilon_0 \left(\frac{V}{d}\right)\left(\frac{A}{t}\right)$

$I_D = \frac{(8.85 \times 10^{-12})(400)(60 \times 10^{-4})}{(10^{-3})(10^{-6})}$

$I_D = 1.602 \times 10^{-2}\ \text{A}$

Hence, the correct answer is (B).

11. $\vec{E} \times \vec{B}$ gives direction of wave propagation,

$\Rightarrow \quad (\hat{k} \times \vec{B}) \| \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

Since, $\hat{k} \times \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right) = \frac{\hat{j} - (-\hat{i})}{\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Wave propagation vector should be along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ and direction of magnetic field is along $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$.

Hence, the correct answer is (B).

13. Polarization proves the transverse nature of electromagnetic waves.
Hence, the correct answer is (A).

14. $\lambda_m T = \text{constant}$

$\Rightarrow \quad \lambda_m \times (2.7\ \text{K}) = 2.888 \times 10^{-3}\ \text{Km}$

$\Rightarrow \quad \lambda_m = \dfrac{0.2888}{2.7}\ \text{cm}$

$\Rightarrow \quad \lambda_m = 0.10\ \text{cm} = 1\ \text{mm}$ (For microwave)

Hence, the correct answer is (B).

15. $I_d = \varepsilon_0 A \dfrac{dE}{dt} = \varepsilon_0 \dfrac{A}{d}\dfrac{dV}{dt} \qquad \left\{\because E = \dfrac{V}{d}\right\}$

$\Rightarrow \quad I_d = \dfrac{8.86 \times 10^{-12} \times 3.14 \times (2 \times 10^{-2})^2}{0.1 \times 10^{-3}} \times 5 \times 10^{13}$

$\Rightarrow \quad I_d = 5.56 \times 10^3\ \text{A}$

Hence, the correct answer is (C).

16. $I = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)c$

$\Rightarrow \quad E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2 \times 5 \times 10^{-16}}{8.85}} = 0.61 \times 10^{-6}\ \text{Vm}^{-1}$

Since, $E_0 = \dfrac{V_0}{d}$

$\Rightarrow \quad V_0 = E_0 d = (0.61 \times 10^{-6})(2) = 1.23\ \mu\text{V}$

Hence, the correct answer is (A).

17. $d = \sqrt{2hR} \Rightarrow d \propto \sqrt{h}$

Hence, the correct answer is (B).

18. Given that $E_0 = 100\ \text{Vm}^{-1}$ and $B_0 = 0.265\ \text{Am}^{-1}$

Maximum rate of energy flow is

$S = E_0 \times B_0$

$S = 100 \times 0.265 = 26.5\ \text{Wm}^{-2}$

Hence, the correct answer is (A).

20. $\dfrac{E_0}{B_0} = c = \dfrac{\omega}{k}$

$\Rightarrow \quad E_0 K = B_0 \omega$

Hence, the correct answer is (A).

22. $E = E_0 \sin(\omega t + 6y - 8z)$

$\Rightarrow \quad \hat{s} = \dfrac{8\hat{k} - 6\hat{j}}{10} = \left(\dfrac{4\hat{k} - 3\hat{j}}{5}\right)$

Hence, the correct answer is (A).

24. The intensity of the wave is

$I_{av} = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)c = \dfrac{P}{4\pi R^2}$

$\Rightarrow \quad E_0 = \sqrt{\dfrac{P}{2\pi R^2 \varepsilon_0 c}}$

$\Rightarrow \quad E_0 = \sqrt{\dfrac{3}{2 \times 3.14 \times 100 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$

$\Rightarrow \quad E_0 = 1.34\ \text{Vm}^{-1}$

Hence, the correct answer is (A).

25. $B_0 = \dfrac{E_0}{c} = \dfrac{10^{-3}}{3 \times 10^8} = 3.33 \times 10^{-12}\ \text{T}$

Hence, the correct answer is (B).

26. $E = \dfrac{hc}{\lambda} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{21 \times 10^{-2}}$

$\Rightarrow \quad E = 0.94 \times 10^{-24} \approx 10^{-24}\ \text{J}$

Hence, the correct answer is (D).

28. $|\vec{E}_1| = cB_1$ and $|\vec{E}_2| = cB_2$

Since $\vec{E}_1 \perp E_2$

$\Rightarrow \quad F_{\text{net}} = \dfrac{Q}{\sqrt{2}}\sqrt{E_1^2 + E_2^2}$

$\Rightarrow \quad F_{\text{net}} = \dfrac{10^{-4}}{\sqrt{2}} \times 3 \times 10^8 \times 30 \times 10^{-6}$

$\Rightarrow \quad F_{\text{net}} = \dfrac{90 \times 10^8 \times 10^{-10}}{\sqrt{2}}$

$\Rightarrow \quad F_{\text{net}} \approx 0.6\ \text{N}$

Hence, the correct answer is (A).

30. Infrared radiation produces thermal effect and is detected by pyrometer.

Hence, the correct answer is (B).

31. Electric energy density

$u_e = \dfrac{1}{2}\varepsilon_0 E_{rms}^2$

Since, $E_{rms} = \dfrac{E_0}{\sqrt{2}}$

$\Rightarrow \quad u_e = \dfrac{1}{4}\varepsilon_0 E_0^2$

Hence, the correct answer is (D).

32. Momentum transferred to the mirror in one second is

$F = \dfrac{\Delta p}{\Delta t} = \left(\dfrac{2I}{c}\right)A$

$\Rightarrow \quad F = \dfrac{2S_{av}A}{c} = \dfrac{2 \times 6 \times 40 \times 10^{-4}}{3 \times 10^8}$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = 1.6 \times 10^{-10} \text{ kgms}^{-2}$$

Hence, the correct answer is (D).

34. Amplitude of electric field, $E_0 = B_0 c$

$$\Rightarrow \quad E_0 = 1.6 \times 10^{-6} \times \sqrt{5} \times 3 \times 10^8$$

$$\Rightarrow \quad E_0 = 4.8 \times 10^2 \sqrt{5} \text{ Vm}^{-1}$$

Also $\vec{E} \times \vec{B}$ is along $-\hat{k}$ which is the direction of propagation.

$$\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(-\hat{i} + 2j\right) \text{ Vm}^{-1}$$

Hence, the correct answer is (A).

37. β-rays are not electromagnetic waves.
Hence, the correct answer is (C).

39. Speed of light of vacuum is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Speed of light in any medium is

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\Rightarrow \quad \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}} = \sqrt{\mu_r K}$$

$$\Rightarrow \quad v = \frac{c}{\sqrt{\mu_r K}}$$

Hence, the correct answer is (C).

41. During propagation of a wave from one medium to another, frequency remains constant and wavelength changes

$$\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{4} = 2$$

Since $\mu \propto \frac{1}{\lambda}$

$\Rightarrow$ Wavelength is halved.
Hence, the correct answer is (C).

42. $\vec{E}$, $\vec{B}$ and $\vec{k}$ form a right handed system $\vec{k}$ is along z-axis

As $\hat{i} \times \hat{j} = \hat{k}$

$$\Rightarrow \quad E_x \hat{i} \times B_y \hat{j} = c\hat{k}$$

i.e. E is along x-axis and B is along y-axis.
Hence, the correct answer is (A).

43. EM waves carry momentum and hence can exert pressure on surfaces. They also carry energy, so we have

$$p \neq 0 \text{ and } E \neq 0$$

Hence, the correct answer is (B).

45. Electric field is given by

$$E = \frac{V}{l} = \frac{iR}{l}$$

where, R is the resistance of wire
Magnetic field at the surface of wire or radius a is

$$B = \frac{\mu_0 i}{2\pi a}$$

Hence Poynting vector, directed radially inward is given by

$$S = \frac{EB}{\mu_0} = \left(\frac{iR}{\mu_0 l}\right)\left(\frac{\mu_0 i}{2\pi a}\right) = \frac{i^2 R}{2\pi a l}$$

Hence, the correct answer is (D).

47. The angular wave number is given by

$$k = \frac{2\pi}{\lambda}$$

where λ is the wave length.
The angular frequency is $\omega = 2\pi f$

The ratio $\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c$

$$\Rightarrow \quad \frac{k}{\omega} = \text{constant}$$

Hence, the correct answer is (C).

49. Average energy density of electric field is given by

$$u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

$$\Rightarrow \quad u_e = \frac{1}{4}\left(8.85 \times 10^{-12}\right)(1)^2 = 2.2 \times 10^{-12} \text{ Jm}^{-3}$$

Hence, the correct answer is (B).

51. The pressure due to radiation is

$$P = \frac{I}{c} = \frac{0.5}{3 \times 10^8} = 0.166 \times 10^{-8} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

53. $c = \frac{E}{B}$

$$\Rightarrow \quad B = \frac{E}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8} \text{ T}$$

Hence, the correct answer is (B).

54. $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$\lambda = \frac{c}{\nu} = 2\pi c\sqrt{LC}$$

CHAPTER 5

$\Rightarrow \quad \lambda = 3\times10^{8}\times2\pi\sqrt{1\times10^{-6}\times0.01\times10^{-6}}$

$\Rightarrow \quad \lambda = 3\times10^{8}\times2\times3.14\times10^{-7} = 188.4 \text{ m}$

Hence, the correct answer is (C).

55. Area through which the energy of beam passes is

$$A = (6.328\times10^{-7})^{2} = 4\times10^{-13} \text{ m}^2$$

$$\Rightarrow \quad I = \frac{P}{A} = \frac{10^{-3}}{4\times10^{-13}} = 2.5\times10^{9} \text{ Wm}^{-2}$$

Hence, the correct answer is (B).

57. EM waves transport energy, momentum and information but not charge. EM waves do not possess any charge.

Hence, the correct answer is (B).

58. $B = \frac{E}{c} = \frac{10^{-4}}{3\times10^{8}} = 3.3\times10^{-13} \text{ T}$

Hence, the correct answer is (D).

59. $f = \frac{c}{\lambda}$

$f_1 = \frac{3\times10^{8}}{1} = 300 \text{ MHz}$ and

$f_2 = \frac{3\times10^{8}}{10} \text{ Hz} = 30 \text{ MHz}$

Hence, the correct answer is (A).

61. Intensity of EM wave is given by

$$I = \frac{P}{4\pi R^2} = \langle u\rangle c = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c$$

$\Rightarrow \quad E_0 = \sqrt{\frac{P}{2\pi R^2 \varepsilon_0 c}}$

$\Rightarrow \quad E_0 = \sqrt{\frac{800}{2\times3.14\times(4)^2\times8.85\times10^{-12}\times3\times10^{8}}}$

$\Rightarrow \quad E_0 = 54.77 \text{ Vm}^{-1}$

Hence, the correct answer is (D).

63. $\lambda = \frac{c}{f} = \frac{3\times10^{8}}{8.2\times10^{6}} = 36.5 \text{ m}$

Hence, the correct answer is (A).

65. EM waves travels in the direction perpendicular to $\vec{E}$ as well as $\vec{B}$ and the direction of propagation of the EM wave is along the vector $\vec{E}\times\vec{B}$

Hence, the correct answer is (C).

67. $\nu = 1057 \text{ MHz}$

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{1057\times10^{6}} \text{ m}$$

$\Rightarrow \quad \lambda = 0.28 \text{ m} = 28 \text{ cm (radiowaves)}$

Hence, the correct answer is (A).

68. Population covered is given by

$$\begin{pmatrix}\text{Population}\\ \text{Covered}\end{pmatrix} = (2\pi Rh)\begin{pmatrix}\text{Population}\\ \text{Density}\end{pmatrix}$$

Hence, the correct answer is (C).

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1. Amplitude of electric field, $E_0 = B_0 c$

$\Rightarrow \quad E_0 = 1.6\times10^{-6}\times\sqrt{5}\times3\times10^{8}$

$\Rightarrow \quad E_0 = 4.8\times10^{2}\sqrt{5} \text{ Vm}^{-1}$

Also $\vec{E}\times\vec{B}$ is along $-\hat{k}$ which is the direction of propagation.

$\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z + 6\times10^{15}t)(-\hat{i}+2j) \text{ Vm}^{-1}$

Hence, the correct answer is (A).

2. $B_0 = \frac{E_0}{c} = \frac{6}{3\times10^{8}} = 2\times10^{-8} \text{ T}$

Propagation direction is along $\hat{E}\times\hat{B}$

$\Rightarrow \quad \hat{i} = \hat{j}\times\hat{B}$

$\Rightarrow \quad \hat{B} = \hat{k}$

Hence, the correct answer is (C).

3. $|\vec{E}_1| = cB_1$

$|\vec{E}_2| = cB_2$

Since, $\vec{E}_1 \perp E_2$

$\Rightarrow \quad F_{\text{net}} = \frac{Q}{\sqrt{2}}\sqrt{E_1^2 + E_2^2}$

$\Rightarrow \quad F_{\text{net}} = \frac{10^{-4}}{\sqrt{2}}\times3\times10^{8}\times30\times10^{-6}$

$$\Rightarrow \quad F_{net} = \frac{90 \times 10^8 \times 10^{-10}}{\sqrt{2}}$$

$$\Rightarrow \quad F_{net} \approx 0.6 \text{ N}$$

Hence, the correct answer is (A).

4. $\frac{E_0}{B_0} = c$

$$\Rightarrow \quad B_0 = \frac{E_0}{c}$$

Given that $\vec{E} = E_0 \cos(kz)\cos(\omega t)\hat{i}$

$$\Rightarrow \quad \vec{E} = \frac{E_0}{2}[\cos(kz - \omega t)\hat{i} - \cos(kz + \omega t)\hat{i}]$$

The corresponding magnetic field is

$$\vec{B} = \frac{B_0}{2}[\cos(kz - \omega t)\hat{j} - \cos(kz + \omega t)\hat{j}]$$

$$\Rightarrow \quad \vec{B} = \frac{B_0}{2}[2\sin(kz)\sin(\omega t)]$$

$$\Rightarrow \quad \vec{B} = \frac{E_0}{c}[\sin(kz)\sin(\omega t)]\hat{j}$$

Hence, the correct answer is (C).

5. $E = E_0 \sin(\omega t + 6y - 8z)$

$$\Rightarrow \quad \hat{s} = \frac{8\hat{k} - 6\hat{j}}{10} = \left(\frac{4\hat{k} - 3\hat{j}}{5}\right)$$

Hence, the correct answer is (A).

6. $B_0 = \frac{E_0}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$ T

$\nu = 23.9 \times 10^9$ Hz

$$\Rightarrow \quad \omega = 2\pi\nu = 2 \times 3.142 \times 23.9 \times 10^9$$

$$\Rightarrow \quad \omega = 1.5 \times 10^{11} \text{ s}^{-1}$$

Since, $c = \frac{\omega}{k}$

$$\Rightarrow \quad k = \frac{\omega}{c} = \frac{1.5 \times 10^{11}}{3 \times 10^8}$$

$$\Rightarrow \quad k = 0.5 \times 10^3$$

$$\Rightarrow \quad \vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\hat{i}$$

Hence, the correct answer is (C).

7. In air, EM wave is

$$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$$

$$\Rightarrow \quad \vec{E}_1 = E_{01}\hat{x}\cos[k(z - ct)] \qquad \left\{\because k = \frac{2\pi}{\lambda_0} = \frac{2\pi\nu}{c}\right\}$$

In medium, EM wave is

$$\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$$

$$\Rightarrow \quad \vec{E}_2 = E_{02}\hat{x}\cos\left[2k\left(z - \frac{c}{2}t\right)\right]$$

During refraction, frequency remains unchanged, whereas wavelength changes

$$\Rightarrow \quad k' = 2k \qquad \{\text{from equations}\}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda_0}\right)$$

$$\Rightarrow \quad \lambda' = \frac{\lambda_0}{2}$$

Since, $v = \frac{c}{2}$

$$\Rightarrow \quad \frac{1}{\sqrt{\mu_0 \varepsilon_{r_2}}} = \frac{1}{2} \times \frac{1}{\sqrt{\mu_0 \varepsilon_{r_1}}}$$

$$\Rightarrow \quad \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

Hence, the correct answer is (C).

8. $\vec{E} \times \vec{B}$ gives direction of wave propagation,

$$\Rightarrow \quad (\hat{k} \times \vec{B}) \| \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$$

Since, $\hat{k} \times \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right) = \frac{\hat{j} - (-\hat{i})}{\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Wave propagation vector should be along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ and direction of magnetic field is along $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$.

Hence, the correct answer is (B).

9. Electric field in electromagnetic wave is given by

$$E = E_0 \sin(\omega t_1 - kz_1)$$

Also, $E' = E_0 \sin(\pi + \omega t_1 - kz_2)$

As per question, $E = E' = 0$

$$\Rightarrow \quad \omega t_1 - kz_1 = (\pi + \omega t_1 - kz_2)$$

$$\Rightarrow \quad \pi = k(z_2 - z_1) = \frac{2\pi}{\lambda}|z_2 - z_1|$$

$$\Rightarrow \quad \lambda = 2|z_2 - z_1|$$

$$\Rightarrow \quad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{2|z_2 - z_1|} = \frac{1.5 \times 10^8}{|z_2 - z_1|}$$

Hence, the correct answer is (D).

CHAPTER 5

10. If E is magnitude of electric field, then

$$I=\left(\frac{1}{2}\varepsilon_0 E^2\right)c$$

$$\Rightarrow \quad E=\sqrt{\frac{2I}{c\varepsilon_0}}$$

Also, $B=\dfrac{E}{c}$

So, direction of $\vec{E}\times\vec{B}$ will be along $+\hat{j}$.

Hence, the correct answer is (C).

11. Given that $\vec{B}=B_0\sin(kx+\omega t)\hat{j}$ T

The relation between electric and magnetic field is,

$$c=\frac{E}{B}$$

$$\Rightarrow \quad E=cB$$

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along z-axis is obtained as

$$E=cB_0\sin(kx+\omega t)\hat{k}$$

Hence, the correct answer is (D).

12. Since, $\dfrac{dE}{dz}=-\dfrac{dB}{dt}$

$$\Rightarrow \quad \frac{dE}{dz}=-2E_0k\sin kz\cos\omega t=-\frac{dB}{dt}$$

$$\Rightarrow \quad dB=+2E_0k\sin kz\cos\omega t dt$$

$$\Rightarrow \quad B=+2E_0k\sin kz\int\cos\omega t dt$$

$$\Rightarrow \quad B=+2E_0\frac{k}{\omega}\sin kz\sin\omega t$$

Also, $\dfrac{E_0}{B_0}=\dfrac{\omega}{k}=c$

$$\Rightarrow \quad B=\frac{2E_0}{c}\sin kz\sin\omega t$$

$$\Rightarrow \quad \vec{B}=\frac{2E_0}{c}\sin kz\sin\omega t\hat{j}$$

Hence, the correct answer is (C).

13. $E_{\underset{A}{\text{radiowave}}}<E_{\underset{B}{\text{yellow}}}<E_{\underset{C}{\text{blue}}}<E_{\underset{D}{x\text{-ray}}}$

So, D < B < A < C

Hence, the correct answer is (A).

15. An electromagnetic wave propagating in $+x$ direction means electric field and magnetic field should be function of x and t.

Also, $\vec{E}\perp\vec{B}$ or $\hat{E}\perp\hat{B}$

$$\Rightarrow \quad (\hat{y}-\hat{z})\cdot(\hat{y}+\hat{z})=\hat{y}\cdot\hat{y}-\hat{z}\cdot\hat{z}=0$$

Hence, the correct answer is (D).

16. Intensity of light, $I=u_{av}c$

Also, $I=\dfrac{P}{4\pi r^2}$ and $u_{av}=\dfrac{1}{2}\varepsilon_0E_0^2$

$$\Rightarrow \quad \frac{P}{4\pi r^2}=\frac{1}{2}\varepsilon_0E_0^2c \text{ or } E_0=\sqrt{\frac{2P}{4\pi\varepsilon_0r^2c}}$$

Here, $P=0.1$ W, $r=1$ m, $c=3\times10^8$ ms^{-1}

$$\frac{1}{4\pi\varepsilon_0}=9\times10^9 \text{ NC}^{-2}\text{m}^2$$

$$\Rightarrow \quad E_0=\sqrt{\frac{2\times0.1\times9\times10^9}{1^2\times3\times10^8}}=\sqrt{6}=2.45 \text{ Vm}^{-1}$$

Hence, the correct answer is (D).

17. $f=2\times10^{14}$ Hz

$E_0=27$ Vm^{-1}

Since, $\dfrac{E_0}{B_0}=c$

$$\Rightarrow \quad B_0=\frac{27}{3\times10^8}=9\times10^{-8} \text{ T}$$

Also, $\lambda=\dfrac{c}{f}=\dfrac{3\times10^8}{2\times10^{14}}=1.5\times10^{-6}$ m

$$\Rightarrow \quad B=B_0\sin2\pi\left(\frac{x}{\lambda}-ft\right)$$

$$\Rightarrow \quad B=(9\times10^{-8} \text{ T})\sin2\pi\left(\frac{x}{1.5\times10^{-6}}-2\times10^{14}t\right)$$

Oscillation of B can be along either $\hat{j}$ or $\hat{k}$ direction.

*No given option is correct

18. For electromagnetic wave, direction of propagation, $\vec{E}$ and $\vec{B}$ are transverse in nature.

According to question, $\vec{E}\times\vec{B}$ gives the direction of propagation which is along $+z$ direction. Only OPTION (B) satisfies both conditions

(i) $\vec{E}\cdot\vec{B}=0$ (ii) $(\vec{E}\times\vec{B})$ directed along the z-axis.

Hence, the correct answer is (B).

19. Infrared waves are used to treat muscular strain. Radio waves are used for broadcasting.
X-rays are used to detect fracture of bones. Ultraviolet rays are absorbed by the ozone layer of the atmosphere.
Hence, the correct answer is (A).

20. In an EM wave, energy is equally divided between the electric and the magnetic fields.
Hence, the correct answer is (D).

21. In EM wave, the peak value of electric field (E_0) and peak value of magnetic field (B_0) are related by

$$E_0 = B_0 c$$

$$\Rightarrow \quad E_0 = (20 \times 10^{-9}\ \text{T})(3 \times 10^8\ \text{ms}^{-1}) = 6\ \text{Vm}^{-1}$$

Hence, the correct answer is (C).

22. The direction of polarization is parallel to electric field

$$\Rightarrow \quad \vec{X} \parallel \vec{E}$$

The direction of wave propagation is parallel to $\vec{E} \times \vec{B}$.

$$\Rightarrow \quad \vec{k} \parallel \vec{E} \times \vec{B}$$

Hence, the correct answer is (A).